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BRITISH MUSEUM 7 SE 1900

TRANSACTIONS.

1.—*On some Type Specimens of Lepidoptera and Coleoptera in the Edinburgh Museum of Science and Art.* By PERCY HALL GRIMSHAW, F.E.S., Natural History Department, Edinburgh Museum of Science and Art. Communicated by Dr R. H. TRAQUAIR, F.R.S. (With Plate.)

(Read 17th May, 1897.)

In the year 1819 the University of Edinburgh acquired by purchase a large zoological collection from M. DUFRESNE, of Paris, including a cabinet containing upwards of 12,000 specimens of insects. Some years later (in 1855), when the whole of the University collections were formally transferred to the Science and Art Department, the Dufresne cabinet became public property.

While consulting lately the volume by GODART devoted to the article "Papilio" in the famous *Encyclopédie Méthodique*, I was surprised to see a reference to the "Dufresne cabinet" in one of the descriptions of new species, and it immediately occurred to me that the specimen in our possession must be the actual "type" of the species, which discovery led me to search carefully through the whole of the volumes devoted to insects in this well-known work, with a view to finding as many types as possible. At the same time, I thought it possible that OLIVIER, who wrote his great *Histoire Naturelle des Insectes—Coléoptères* about the same time, might also mention some of DUFRESNE's specimens, and this I found to be as I had anticipated. Altogether, in the two works mentioned, I find no less than twelve references to butterflies and forty to beetles in the Dufresne cabinet. I have been successful in finding *all* the types of butterflies, but in the case of the beetles about half of them are missing. These are not mentioned in the MS. catalogue which accompanied the collection, and hence must either have been removed from the cabinet before it came to Edinburgh, or, *as is much more probable*, must be among the *unnamed* specimens, in which case the types must remain unidentified until such time as all the specimens have been carefully examined and referred to their proper species.

In addition to the twelve species of butterflies already referred to, there are a number of others in the Dufresne collection belonging to species described by GODART as new, and labelled *in the same handwriting*. From LATREILLE's "Avertissement" to the ninth volume of the *Encyclopédie*, we learn that GODART worked through the whole of Dufresne's cabinet, and therefore, although there is no special allusion to these specimens, it seems to me quite fair to regard them as true types.

By the comparison of these original and interesting specimens with others in the Natural History Collections at the British Museum, I have been enabled to clear up many doubtful points in synonymy, etc., and the following paper is the result of my investigations. In submitting it to the Royal Society of Edinburgh, I must express my best thanks to Dr TRAQUAIR, Keeper of the Natural History Department of the Museum, for his kindly encouragement and advice, and also my great indebtedness to Dr A. G. BUTLER, Mr F. A. HERON, Mr C. O. WATERHOUSE, and Mr C. J. GAHAN, who, by their generous assistance, materially contributed to the success of my work in London.

LEPIDOPTERA.

NECTARIA IDEA.

Papilio idea, Clerck, *Icones*, t. 38, f. 1 (1764).
Idea agelia, Godart, *Enc. Méth.*, ix. p. 195, n. 1 (1819).

Godart's types (2 specimens) in Dufresne collection.

RADENA MEGANIRA.

Danais meganira, Godart, *Enc. Méth.*, ix. p. 192, n. 51 (1819).

Types (2 specimens) in Dufresne collection.

TASITIA CLEOTHERA.

Danais cleothera, Godart, *Enc. Méth.*, ix. p. 185, n. 31 (1819).

Type agrees with British Museum specimens from St Domingo. Locality given by Godart (Timor) probably an error.

EUPLŒA PHÆNARETA.

Papilio phænareta, Schaller, *Naturforscher*, xxi. p. 177, pl. 5, f. 1, 2 (1785), ♂.
Danais prothoe, Godart, *Enc. Méth.*, ix. p. 177, n. 1 (1819).

Godart's type (a ♂, labelled "Amboine") in Dufresne collection.

ISAMIA ALOPIA.

Danais alopia, Godart, *Enc. Méth.*, ix. p. 177, n. 4 (1819).

Type (?) in Dufresne collection, a ♂, distinct from *midamus*, Linn.

PENOA ALCATHOE.

Danais alcathoe, Godart, *Enc. Méth.*, ix. p. 178, n. 5 (1819).

A ♀ in the Dufresne collection, labelled "Amboine," is probably the variety mentioned by Godart; it comes very near to Standinger's *butra*.

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PENOA DOUBLEDAYI.

Euplœa doubledayi, Felder, *Reise Novara*, Lep. ii. p. 337 (1867).
,, *alcathoe*, auct.

The specimens in the British Museum collection referred to *alcathoe* do not correspond at all to Godart's description, nor does the figure given by Marshall and de Nicéville (*Butt. of India*, vol. i. pl. ix. fig. 17). Moreover, *alcathoe*, as generally understood, comes from Sikkim, Sylhet, and Further India, while Godart's species was stated to come from Amboyna. Under these circumstances, it seems advisable to revive Felder's name for this species, and to regard Godart's species as distinct and not yet identified.

VONONA EUPHON.

Papilio euphon, Fab., *Ent. Syst. Suppl.*, p. 423 (1798).
Danais baudiniana, Godart, *Enc. Méth.*, ix. p. 181, n. 17 (1819).
,, *euphone*, Godart, t.c., p. 181, n. 18 (1819).

Godart's *baudiniana* is nothing but the well-known Fabrician species, and the locality "Timor" is an error. Godart was evidently unacquainted with *euphon*, as he merely gives a translated description. His types of *baudiniana* (2 specimens) are in the Dufresne collection.

LYCOREA CLEOBÆA.

Heliconia cleobæa, Godart, *Enc. Méth.*, ix. p. 222, n. 58 (1819).

One male and one female (types?) in the Dufresne collection.

CERATINIA EUCLEA.

Heliconia euclea, Godart, *Enc. Méth.*, ix. p. 220, n. 53 (1819).
Ithomia fenestella, Hew., *Ex. Butt.* I., *Ith.*, t. 5, f. 25 (1854).

By an examination of the type in the Dufresne collection the identity of this species, which has long been obscure, with that of Hewitson's *fenestella* is established.

CALISTO HYSIUS.

Satyrus hysius, Godart, *Enc. Méth.*, ix. p. 525, n. 131 (1823?).

It is somewhat curious that, although the second part of the ninth volume of the *Encyclopédie Méthodique* is generally supposed to have been issued in 1823, the MS. Catalogue of the Dufresne collection, *which bears the date* 1818, contains the name of this species, *in the original handwriting*. There are two specimens in the collection, one of each sex, but only the female bears the original label. This label *must* have been written prior to 1819, as in that year the collection came to Edinburgh; and as Godart only describes the female, this specimen is evidently the *type*. The question whether Godart wrote his description some years previous to publication, or whether the generally accepted date of publication is erroneous, is a point difficult to settle. The specimens at the British Museum referred to *hysius* differ slightly from the type. They have the ring at the anal angle of the hind wings much smaller, and that at the apex of the fore wings less sharply defined.

ACRÆA ZETES.

Papilio zetes, Linn., *Syst. Nat.*, i. 2, p. 766, n. 110 (1767).
Acræa zethea, Godart, *Enc. Méth.*, ix. p. 236, n. 21 (1819).

Godart's types (2 ♂ and 1 ♀) in Dufresne collection.

ACRÆA SERENA.

Papilio serena, Fab., *Syst. Ent.*, p. 461, n. 76 (1775).
♀ *Acræa janisca*, Godart, *Enc. Méth.*, ix. p. 233, n. 10 (1819).

As pointed out by Kirby (*Syn. Cat.*, p. 718), Godart's species is the ♀ of *serena*. His type is in the Dufresne collection.

ACRÆA SERVONA.

Acræa servona, Godart, *Enc. Méth.*, ix. p. 239, n. 28 (1819).
♂ *Acræa dejana*, Godm. & Salv., Jameson's *Story of the Rear Column*, p. 431 (1890).

Of this species Godart describes only the female, which is in the Dufresne collection. *Dejana* is merely the male of the same species.

ACRÆA CEPHEUS.

Papilio cepheus, Linn., *Mus. Ulr.*, p. 252 (1764).
Acræa zosteria, Godart, *Enc. Méth.*, ix. p. 232, n. 6 (1819).

Godart's type in Dufresne collection.

ACTINOTE OZOMENE.

Acræa ozomene, Godart, *Enc. Méth.*, ix. p. 241, n. 36 (1819).

Two specimens in the Dufresne collection, of which one bears the original label, and is therefore the type. The specimens in the British Museum are from Quito.

HELICONIUS CYRBIA.

Heliconia cyrbia, Godart, *Enc. Méth.*, ix. p. 203, n. 3 (1819).

Type in Dufresne collection.

HELICONIUS ETHILLA.

Heliconia ethilla, Godart, *Enc. Méth.*, ix. p. 219, n. 49 (1819).

Of this distinct and handsome species, which has long remained in obscurity, the type is represented in the Plate, fig. 2. Two unnamed specimens in the British Museum were found to agree exactly with the type in the Dufresne collection.

EUEIDES ALIPHERA.

Cethosia aliphera, Godart, *Enc. Méth.*, ix. p. 246, n. 7 (1819).

Type in Dufresne collection.

CLOTHILDA BRIAREA.

Argynnis briarea, Godart, *Enc. Méth.*, ix. p. 261, n. 16 (1819).
Anelia numida, Hüb., *Samml. Ex. Schmett.* (1816–1824).

By comparison of the two Dufresne specimens (the types), Godart's species has been proved to be identical with *numida*, Hüb., and not with *pantherata*, Mart., as generally supposed, in spite of the different locality.

PSEUDARGYNNIS HEGEMONE.

Argynnis hegemone, Godart, *Enc. Méth.*, ix. p. 258, n. 7 (1819).
Messaras ? hegemone, Kirby, *Syn. Cat. D.L.*, p. 153 (1871).
Iæra duodecimpunctata, Snell, *Tijdschr. Ent.* (2), vii. p. 15, t. 1, f. 1–3 (1872).
Aterica clorana, Druce, *Trans. Ent. Soc.*, 1874, p. 157.

The type is in the Dufresne collection, and is represented in the Plate, fig. 5. I have compared it with specimens in the British Museum collection from Angola, Zomba, and Nyasa-land.

PHYCIODES THAROS.

Papilio tharos, Cram., *Pap. Ex.*, ii. t. 169, E.F. (1779).
Argynnis tharossa, Godart, *Enc. Méth.*, ix. p. 289, n. 61 (1819).

Godart's type in Dufresne collection.

PHYCIODES ÆGON.

Papilio ægon, Fab., *Spec. Ins.*, ii. p. 130, n. 594 (1781).
Argynnis pygmæa (part), Godart, *Enc. Méth.*, ix. p. 290, n. 63 (1819).

There are two specimens (Godart's types?) in the Dufresne collection with the original label attached, one of which is the Fabrician species, the other *pelops*, Drury.

PHYCIODES PELOPS.

Papilio pelops, Drury, *Ill. Ex. Ent.*, i. t. 19, f. 3, 4 (1773).
Argynnis pygmæa (part), Godart, *Enc. Méth.*, ix. p. 290, n. 63 (1819).

See remarks under previous species.

HYPANARTIA BELLA.

Papilio bella, Fab., *Ent. Syst.*, iii. 1, p. 328, n. 713 (1793).
Vanessa zabulina, Godart, *Enc. Méth.*, ix. p. 301, n. 13 (1819).

One specimen (Godart's type?) in Dufresne collection.

PYRAMEIS CALLIROË.

Hamadryas decora calliroë, Hüb., *Samml. Ex. Schmett.* (1806–1816).
Vanessa vulcania, Godart, *Enc. Méth.*, ix. p. 320, n. 55 (1819).

Specimen (one of Godart's types?) in Dufresne collection labelled "Ténériffe."

PRECIS PELARGA.

Papilio pelarga, Fab., *Syst. Ent.*, p. 513, n. 296 (1775).
Vanessa laodora, Godart, *Enc. Méth.*, ix. p. 314, n. 38 (1819).

There are two specimens in the Dufresne collection labelled *laodora*, one of which is the present species, the other being *galami*, Boisd. The latter bears the original label, but as Godart's description agrees better with true *pelarga*, and as he makes no special mention of Dufresne's specimens, it will perhaps be safer to let Boisduval's name stand, and treat Godart's name as a synonym of the Fabrician species. After all, *pelarga* and *galami* may be only local forms of the same species.

ANARTIA LYTREA.

Vanessa lytrea, Godart, *Enc. Méth.*, ix. p. 299, n. 7 (1819).
Anartia dominica, Skinner, *Tr. Am. Ent. Soc.*, xvi. p. 86 (1889).

By comparison of Dufresne's type specimen with those in the British Museum, it is proved that Skinner's species is identical with *lytrea*, while *chrysopelea*, Hüb., is distinct; the latter name must therefore be restored.

DIDONIS BIBLIS.

Papilio biblis, Fab., *Syst. Ent.*, p. 505, n. 261 (1775).
Biblis thadana, Godart, *Enc. Méth.*, ix. p. 326, n. 1 (1819).

The Dufresne specimen (one of Godart's types?) is a female.

LIBYTHEA MYRRHA.

Libythea myrrha, Godart, *Enc. Méth.*, ix. p. 171, n. 4, (1819).

Types (2 specimens) in the Dufresne collection.

LIBYTHEA TERENA.

Libythea terena, Godart, *Enc. Méth.*, ix. p. 170, n. 2 (1819).

Type in Dufresne collection, from the Antilles.

EUTERPE TEREAS.

Papilio tereas, Godart, *Enc. Méth.*, ix. p. 38, n. 39 (1819).

Type in Dufresne collection.

DISMORPHIA SPIO.

Pieris spio, Godart, *Enc. Méth.*, ix. p. 167, n. 163 (1819).

Types (2 specimens) in Dufresne collection.

XANTHIDIA PYRO.

Pieris pyro, Godart, *Enc. Méth.*, ix. p. 137, n. 60 (1819).

The type of this rare and remarkable species is in the Dufresne collection, and is represented in the Plate, fig. 7. It is not represented in the collections at the British Museum, and unfortunately we have no indication of its *habitat*.

PIERIS JOSEPHINA.

Pieris josephina, Godart, *Enc. Méth.*, ix. p. 158, n. 136 (1819).

Types (1 ♂ and 1 ♀) in Dufresne collection.

This species, which comes from St Domingo and Mexico, is quite distinct from *P. amaryllis*, Fab., which is a native of Jamaica.

HERPÆNIA ERIPHIA.

Pieris eriphia, Godart, *Enc. Méth.*, ix. p. 157, n. 134 (1819).

Type in Dufresne collection. *H. tritogenia*, Klug, is possibly the dry-season form of this species.

BELENOIS GIDICA.

Pieris gidica, Godart, *Enc. Méth.*, ix. p. 131, n. 37 (1819).

The type of this rare and little-known species is figured in the Plate (fig. 4). It is a dry-season form, of which the wet-season form is as yet unknown.

PINACOPTERYX DOXO.

Pieris doxo, Godart, *Enc. Méth.*, ix. p. 123, n. 15 (1819).
„ *charina*, Boisd., *Sp. Gén.*, i. p. 525, n. 128 (1836).
„ *simana*, Hopff., *Ber. Verh. Ak. Berl.*, 1855, p. 640, n. 13.
Pinacopteryx alba, Wallengr., *Lep. Rhop. Caffr.*, p. 10 (1857).
Callosune doxo, Kirby, *Syn. Cat. D.L.*, p. 500 (1871).

The type is in the Dufresne collection, and is figured in the Plate (fig. 6). *Pieris charina*, Boisd., is the dry-season form of this species.

DAPTONURA LIMNORIA.

Pieris limnoria, Godart, *Enc. Méth.*, ix. p. 144, n. 93 (1819).

This is quite distinct from *flippantha*, Fab., which has no orange hind margin in the posterior wings of the male. Godart describes both sexes, but in the Dufresne collection there is only a single (male) specimen labelled *limnoria*, which is figured in the Plate (fig. 3).

DAPTONURA SALACIA.

Pieris salacia, Godart, *Enc. Méth.*, ix. p. 144, n. 91 (1819).

Godart describes both sexes. The single specimen (a type?) in the Dufresne collection is a female.

GLUTOPHRISSA ILAIRE.

Pieris ilaire, Godart, *Enc. Méth.*, ix. p. 142, n. 83 (1819).
Mylothris margarita, Hüb., *Samml. Ex. Schmett* (1816–1841).

Godart's name probably is the earlier one. There is only one specimen in the Dufresne collection, but it is no doubt one of the types.

IXIAS VENILIA.

Pieris venilia, Godart, *Enc. Méth.*, ix. p. 121, n. 7 (1819).

One specimen, a male, in the Dufresne collection, evidently one of the types. *I. venatrix*, Wallace, is *venilia*, Godart, in part, as the latter author apparently described both species as one.

TERACOLUS PHISADIA.

Pieris phisadia, Godart, *Enc. Méth.*, ix. p. 132, n. 40 (1819).
Pontia arne, Klug, *Symb. Phys.*, t. 7, f. 1–4 (1829).
Idmais phisadia et *arne*, Boisd., *Sp. Gén.*, i. p. 587, n. 3, t. 19, f. 2 (1836).
„ *philamene*, Mabille, *Compt. Rend. Soc. Ent. Belg.*, xxxiii. p. cvi. (1880).

Type in Dufresne collection.

PAPILIO ZIDORA.

Acræa zidora, ♀ Godart, *Enc. Méth.*, ix. p. 237, n. 22 (1819).
Papilio ridleyanus, White, *Ann. Nat. Hist.*, xii. p. 262 (1843).

It is somewhat startling to find that the single specimen in the Dufresne collection labelled "zidora, fem.," and described by Godart as the female of his *Acræa zidora*, is nothing else but *Papilio ridleyanus*, White! That he certainly confused the two species is further shown on examining his description of the female. He says:—"Cette bande" [*i.e.*, the "bande fauve ou d'un rouge-cerise"] "s'étend un peu sur les premières ailes du mâle; dans la femelle, au contraire, elle monte beaucoup plus haut, et elle est divisée en cinq taches ovales, dont la supérieure plus petite." Now this description exactly fits *Papilio ridleyanus*, which name must therefore stand as a synonym, and the name *zidora* be restored under the present genus.

Type in Dufresne collection.

PAPILIO TRIOPAS.

Papilio triopas, Godart, *Enc. Méth.*, ix. p. 33, n. 23 (1819).

Type in Dufresne collection.

PAPILIO ZACYNTHUS.

Papilio zacynthus, Fab., *Ent. Syst.*, iii. 1, p. 15, n. 46 (1793).
♂ „ *polymetus*, Godart, *Enc. Méth.*, ix. p. 35, n. 28 (1819).

Godart's species is represented in the Dufresne collection by a single specimen, which is the male of the Fabrician species. It bears the original label, and is probably one of Godart's types.

PAPILIO CRESPHONTINUS.

Papilio cresphontinus, Mart., *Psyche*, t. 3, f. 8; t. 4, f. 10 (1797).
„ *temenes*, Godart, *Enc. Méth.*, ix. p. 63, n. 104 (1819).

One specimen in the Dufresne collection, presumably one of Godart's types.

PAPILIO MACHAONIDES.

Papilio machaonides, Esp., *Ausl. Schmett.*, t. 45, f. 2 (1785–1798).
„ *lycorœus*, Godart, *Enc. Méth.*, ix. p. 63, n. 105 (1819).

Type of Godart's species in the Dufresne collection.

PAPILIO IMERIUS.

Papilio imerius, Godart, *Enc. Méth.*, ix. p. 69, n. 121 (1819).
„ *zetes*, Westw., *Trans. Ent. Soc.*, v. p. 36, t. 3, f. 1, 1* (1847).

Godart's species is not identical, as Kirby supposed (*Syn. Cat. D.L.*, p. 542), with *pelaus*, Fab., but corresponds exactly with *zetes*, Westw. The type in the Dufresne collection has been compared with British Museum specimens of *zetes* from St Domingo.

PAPILIO LEUCASPIS.

Papilio leucaspis, Godart, *Enc. Méth.*, ix. p. 55, n. 85 (1819).

The type of this fine species is in the Dufresne collection.

COLEOPTERA.

CLADOGNATHA SUTURALIS.

Lucanus suturalis, Olivier, *Ent.*, i. 1, p. 16, n. 9, t. 4, f. 12 (1789).

Type in Dufresne collection.

ODONTOLABIS CAMELUS.

Lucanus camelus, Olivier, *Ent.*, i. 1, p. 22, n. 18, t. 5, f. 19 (1789).
Odontolabis camelus, Leuthner, *Trans. Zool. Soc.*, xi. p. 446, pl. xcvi. f. 7, 8 ♂, 9 ♀ (1885).
„ *gouberti*, C. O. Waterhouse, *Ent. Mo. Mag.*, xii. p. 172 (1876).

The type (a male) is in the Dufresne collection. The species is well described and figured by Leuthner (*l.c.*); it comes from the Philippines.

ORYCTES AUGIAS.

Scarabæus augias, Olivier, *Ent.*, i. 3, p. 36, n. 39, t. 24, f. 212 (1789).

Type in Dufresne collection.

HEXODON RETICULATUM.

Hexodon reticulatum, Olivier, *Ent.*, i. 7, p. 4, n. 1, f. 1, a–e (1789).

One of the types of this rare species is in the Dufresne collection.

PHOTINUS VITTATUS.

Lampyris vittata, Olivier, *Ent.*, ii. 28, p. 23, n. 21, t. 3, f. 20 (1790).

Type in Dufresne collection.

PHOTINUS FULGIDUS.

Lampyris fulgida, Olivier, *Ent.*, ii. 28, p. 16, n. 9, t. 2, f. 9, a, b (1790).
„ *rufa*, Olivier, *Ent.*, ii. 28, p. 28, n. 30, t. 3, f. 30 (1790).

From an examination of the type of *rufus*, which is in the Dufresne collection, it appears certainly identical with *fulgidus*, described by Olivier a few pages earlier on. *Fulgidus* is probably an immature form of *rufus*, the original description of which is incorrect. Olivier says, "Corpus subtus pedesque rufa immaculata"; but the type has the *two last* segments of the abdomen blackish.

STENOCARA ARANIPES.

♂ *Pimelia aranipes*, Olivier, *Ent.*, iii. 59, p. 17, n. 22, t. 4, f. 6 (1795).
♀ „ *longipes*, Olivier, *Ent.*, iii. 59, p. 16, n. 20, t. 1, f. 3 (1795), *nec. Fab.*
Stenocara herbsti, Gemminger, *Col. Heft.* vi., 1870.

Type of *aranipes* in Dufresne collection. The ♀ of this species is the same as Olivier's *longipes*, described on the previous page of the *Entomologie*. The latter name, however, was preoccupied by Fabricius, whose species of that name is an *Adesmia*. On this account the species was re-named by Gemminger, but *aranipes* is now the correct name.

EUSCELUS CRIBRARIUS.

Attelabus cribrarius, Olivier, *Ent.*, v. 81, p. 8, n. 5, t. 1, f. 5 (1807).

Types (2 specimens) in the Dufresne collection. This species appears to be rare, and is unrepresented in the British Museum collections. It is represented in the Plate at fig. 9.

SPHENOPHORUS SERICEUS.

Calandra sericea, Olivier, *Ent.*, v. 83, p. 84, n. 14, t. 28, f. 409 (1807).
„ *sericea*, Latr., *Humb. et Bonpl. Voy.*, i. p. 206 (1811).

The types (2 specimens) are in the Dufresne collection. Latreille is usually quoted as the authority for this species, but Olivier's description is earlier by four years.

HOMALONOTUS VALIDUS.

Curculio validus, Olivier, *Enc. Méth.*, v. p. 499, n. 131 (1790).
Rhynchænus validus, Olivier, *Ent.*, v. 83, p. 157, n. 124, t. 15, f. 186 (1807).

Types (2 specimens) in Dufresne collection.

HILIPUS APIATUS.

Rhynchænus apiatus, Olivier, *Ent.*, v. 83, p. 171, n. 144, t. 28, f. 424 (1807).

Type in Dufresne collection.

PHÆDROPUS CANDIDUS.

Curculio candidus, Fab., *Syst. Ent.*, p. 146 (1775).
„ *tomentosus*, Olivier, *Enc. Méth.*, v. p. 536, n. 288 (1790).
„ „ Olivier, *Ent.*, v. 83, p. 343, n. 394, t. 13, f. 155, a, b (1807).

Type of Olivier's species in the Dufresne collection.

CYPHICERUS NOVEMLINEATUS.

Curculio 9-lineatus, Olivier, *Ent.*, v. 83, p. 417, n. 513, t. 26, f. 377 (1807).

Type in Dufresne collection.

CERAMBYX INTERRUPTUS, *Olivier*.

The type of this interesting species, which is represented in the Plate at fig. 8, is in the Dufresne collection, and is probably unique. It has been carefully examined by Mr C. J. Gahan, who considers it the type of a new genus, which he characterises as follows:—

"ZONOTYLUS, *gen. nov.* (Fam. *Cerambycidae*).

"Head rather narrow and slightly concave between the antennary condyles, the latter having each a small angular process at the upper margin just in front of the upper lobe of the eye. Eyes finely facetted, deeply emarginate. Last joint of labial palpi with a distinct pit on the outer surface (maxillary palpi wanting). Antennæ of the female scarcely longer than half the body; third joint a

little longer than first, thickened at the apex; fourth to tenth gradually diminishing in length, fourth towards the apex, and the remaining joints in their whole length dilated and compressed towards the anterior edge, each having a sharp angle at the distal extremity; eleventh joint scarcely longer than tenth, oblique and slightly emarginate at the apex. Prothorax with a stout conical tubercle at the middle of each side; disk raised and somewhat irregularly convex, deeply and very closely punctured with a smooth callosity in the middle; basal margin slightly rounded in the middle, sinuate on each side. Scutellum rather long, and acutely triangular; surface of mesonotum between the scutellum and the stridulating area marked with a transversely elliptical pit. Elytra with sides sub-parallel, apex rounded. Hind legs much longer than either of the two anterior pairs, and with their tibiæ flattened and somewhat dilated; femora of all the legs strongly clavate below the middle, and narrowed again towards the apex. Prosternum furnished with a small narrow tubercle near the posterior end, which is almost vertical; anterior coxal cavities rounded, closed in externally, open behind. Mesosternum emarginate and somewhat bilobed behind, furnished, just in front of the emargination, with a narrow and distinct, but not very strongly raised tubercle; coxal cavities of the middle pair opening outwards to the epimera.

"This genus offers the characters of the group *Stenaspides* of Lacordaire, and in this group seems to come nearest in general structure to *Euryclea*, Thoms., though owing to its narrower form, and the peculiar ivory-like markings of the type species, this relationship would at first sight scarcely be suspected.

"TYPE: *Zonotylus interruptus*, Oliv.

" *Cerambyx interruptus*, Oliv., *Enc. Méth.*, v. p. 307 (1790); *Ent.*, iv., n. 67, p. 35, pl. 17, f. 133.

"The *habitat* of this interesting species is unfortunately still unknown. Both Pascoe and Lacordaire, to whom the species was known only from Olivier's description and figure, surmised that it came from Australia, and was referable to the genus *Bixoresthes*, Pasc., which was founded upon what was supposed to be an Australian, but is now known to be a South African species. Olivier's figure gives a fairly good idea of the general appearance of his species; but neither from his figure nor his description could it be inferred that the markings on the elytra—consisting of a basal spot and two transverse and slightly sinuate bands on each elytron—are raised and present an ivory-like appearance. The rest of the surface of the elytra, except at the shoulders, which are nearly impunctate and somewhat glossy, is finely and very closely punctured and of a dull black colour. The legs are glossy and black with a faint bluish tint." [C. J. GAHAN.]

SAGRA SPLENDIDA.

Sagra splendida, Olivier, *Ent.*, v. 90, p. 497, n. 2, t. 1, f. 2, a, b (1807).

Type in Dufresne collection.

ŒDIONYCHIS FASCIATA.

Galeruca fasciata, Fab., *Suppl. Ent. Syst.*, p. 96 (1798).
Altica fasciata, Olivier, *Ent.*, vi. 93 bis. p. 675, n. 9, t. 1, f. 9 (1808).

The specimen described and figured by Olivier is in the Dufresne collection.

CEPHALODONTA MACULATA.

Hispa maculata, Olivier, *Enc. Méth.*, vii. p. 96, n. 3 (1792).
„ *spinipes*, Fab., *Ent. Syst.*, iv. App., p. 448 (1794); Olivier, *Ent.*, vi. 95, p. 761, n. 4, t. 1, f. 4 (1808).

Olivier's type in the Dufresne collection.

In 1885 Baly, in working through the *Hispidae* for the *Biologia Centrali-Americana*, described a new species as *Cephalodonta maculata*. As this name is pre-occupied by the species just referred to, I propose to re-name that described by Baly as follows:—

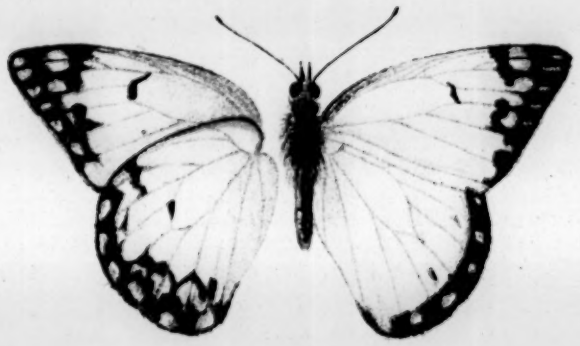
CEPHALODONTA BALYI, n. n.

Cephalodonta maculata, Baly, *Biol. Centr. Am. Col.*, vi. (2), p. 35, t. ii. f. 18 (1885).

Type Specimens of Lepidoptera & Coleoptera in the Edinburgh Museum of Science & Art.



1.



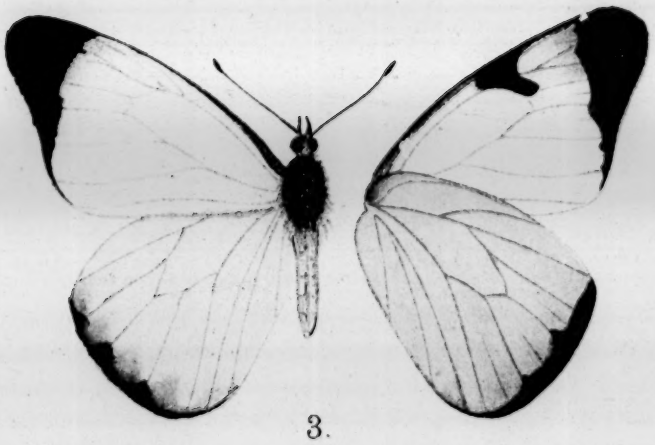
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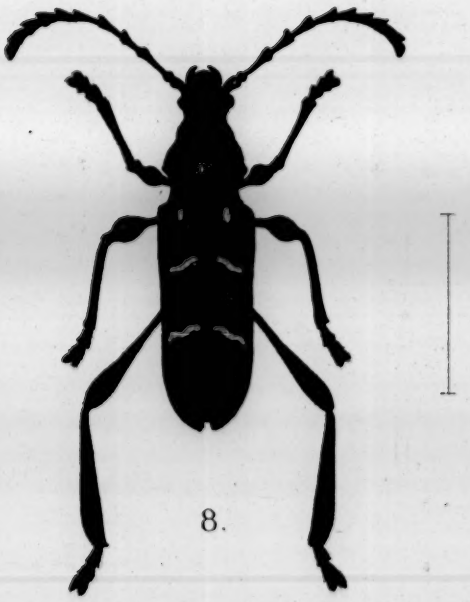
2.



5.



3.



8.



9.



6.



7.

P. H. Grimshaw. del.

McFarlane & Erskine. Lith^rs Edin^r

EUMOLPUS IGNITUS.

Chrysomela ignita, Fab., *Mant.*, i. p. 68 (1787).
Eumolpus ignitus, Olivier, *Ent.*, vi. p. 897, n. 1, t. 1, f. 1 (1808).

The specimen described and figured by Olivier is in the Dufresne collection.

DIABROTICA QUADRIGUTTATA.

Chrysomela quadriguttata, Olivier, *Enc. Méth.*, v. p. 703, n. 62 (1790).
Galeruca albicornis, Fab., *Suppl. Ent. Syst.*, p. 96 (1798)
Crioceris thoracica, Fab., *Syst. El.*, i. p. 457 (1801).
Galeruca thoracica, Olivier, *Ent.*, vi. 93, p. 650, n. 61, t. 4, f. 62, and *Chrys.*, t. 4, f. 53 (1808).

Type of *quadriguttata* in Dufresne collection.

EXPLANATION OF PLATE.

Fig. 1. *Hestina nama*, Dbl. ab.
„ 2. *Heliconius ethilla*, Godart.
„ 3. *Daptonura limnoria*, Godart.
„ 4. *Belenois gidica*, Godart.
„ 5. *Pseudargynnis hegemone*, Godart.
„ 6. *Pinacopteryx doxo*, Godart.
„ 7. *Xanthidia pyro*, Godart.
„ 8. *Zonotylus interruptus*, Olivier.
„ 9. *Euscelus cribrarius*, Olivier.

II.—*On a Melanic Specimen of* Hestina nama, *Doubleday.* By Percy Hall Grimshaw, F.E.S. Communicated by Dr R. H. Traquair, F.R.S. (With figure on Plate of previous Paper.)

(Read 17th May 1897.)

In a collection of Indian butterflies purchased by the Edinburgh Museum in the year 1890, I found a specimen of *Hestina* which is of some interest. After carefully examining it, and comparing it with a figure by Oberthür of a form which he regards as a melanic aberration of the well-known *Hestina nama*, Doubleday, I have come to the conclusion that our example is another and more melanic form of the same species. From Oberthür's figure, which he calls ab. *melanina* (*Études d'Entomologie*, xx., 1896, p. 30, pl. 10, no. 177), the present specimen differs in the fore wing, in the absence of the inner row of whitish spots, in the size and position of the upper spots in the outer series, and in the spot at the anal angle being nearer to the base of the wing. At the base of the cell there is a rich cream-coloured sub-triangular spot, instead of the slender greenish streak. There is no ferruginous border to the hind wing, and the whitish streaks are more elongated, and in the anal half of the wing only very faintly indicated.

For the reference to the figure of *melanina*, I am indebted to the kindness of Mr F. A. Heron, of the British Museum. As the under side is neither described by Oberthür nor represented in his plate, I have thought it advisable to figure the Edinburgh specimen, and to write the following more detailed description :—

Upper side: Fore wing entirely purplish-black, with the exception of a marginal series of whitish spots, the uppermost of which is elongated into the form of a streak, the four next lower becoming shorter and more crescentic towards the anal angle, the lowermost (between the sub-median and the first median nervures) more quadrate, and followed by a short streak at the anal angle, which is, however, further removed from the margin. At the apex of the wing are two much smaller and less distinct whitish streaks. Inner margin dusted for about two-thirds of its length with bluish-grey. A pale streak at base of costal area and a cream-coloured spot at base of cell. Hind wing more reddish in colour, but much darker in the discal area. Light markings confined to a gradually disappearing marginal series of streaks beginning at the costa and extending to the anal angle, only the first three, however, being distinct. Inner margin broadly whitish. *Under side :* Fore wing lighter than upper side, with a greater extension of the whitish markings, those near the apex being especially more distinct. Inner margin paler than on upper side, with an additional bluish-white streak at the anal

angle. Nervures in apical portion of wing strongly margined with chestnut-red; spots at base of wing more distinct than on upper side. Hind wing lighter than on upper side, with all the nervures strongly margined with bright chestnut-red. Marginal series of light markings much larger and more distinct, of a more yellowish colour than those on the fore wing, and practically forming a broad band, interrupted by the red-margined nervures, and dentated externally by the extension inwards of a triangle of chestnut-red between each nervure. Precostal area with four white spots; inner margin creamy-white, dusted with chestnut-red. Cilia of both wings alternately edged with white on both upper and under surfaces. Body, head, and legs marked as in typical *H. nama*.

III.—*On some Nuclei of Cloudy Condensation.* By JOHN AITKEN, F.R.S.
(With a Plate.)

(Read 3rd May, 1897.)

I have to apologise for placing this communication before you in a somewhat incomplete form. My work has been much interrupted in the past, and it will again shortly be stopped for a considerable time. I have therefore decided to place before you the results so far as they are worked out.

PART I.

IONS AND CLOUDY CONDENSATION.

ROBERT VON HELMHOLTZ published in 1887 the results of "Experiments with a Steam Jet."* This investigation was continued, and he and Professor RICHARZ joined forces, and the result of their investigation will be found in the same journal for 1890, under the title—"On the Influence of Chemical and Electrical Action on the Steam Jet, and on the Dissociation of Gases, particularly of Oxygen."†

HELMHOLTZ seems to have begun his work under a wrong impression as to the conclusions of other investigators. In the first of these papers, after describing some results, he says they *cannot* be explained on the dust theory; and after stating that some observers had rejected the dust theory, he adds that "AITKEN, on the other hand, has attempted to explain everything by it." In the second of the papers referred to, the authors say: "We hold that the earlier assertions that cloud is never formed without dust is incorrect." I have great difficulty in understanding how these authors could have made this mistake, because in my first paper it is distinctly stated that cloudy condensation can take place without dust; and a number of experiments are described in which condensation takes place in the presence of the vapours of different substances, such as sulphuric acid, hydrochloric acid, and a number of others,—the very substances one of the authors uses to prove that condensation can take place without dust. I had also shown, that if the supersaturation be great enough, that condensation can take place without nuclei of any kind.

In my first communication on the subject in 1881 I even went further, and said that it was probable that sunshine might cause the formation of nuclei, and allow cloudy condensation to take place where there was no dust. On this point I shall have something further to say in this paper.

* *Weid. Ann.*, xxxii. pp. 1–19, 1887.

† *Weid. Ann.*, xl. pp. 161–202, 1890.

The principal object of the two papers by HELMHOLTZ and RICHARZ is to prove that cloudy condensation may take place without dust, and the authors describe a great number of very beautiful experiments made with the steam jet, showing the change which takes place in its appearance when mixed with air charged with the products of combustion either from flames or from slow combustion, such as damp phosphorus slowly oxidising in the air, or when the jet is mixed with air which had been exposed to the electric discharge. In all these cases the authors show that something is produced which, when it meets the steam jet, causes an increase in the density of the condensation, evidently due to an increase in the number of small water drops. That there is something produced which causes an increase in the density of the steam is evident; but whilst one may go thus far with these authors, yet it is difficult to accept their explanation of the phenomenon. They attribute the condensation not to nuclei, but to the molecular shock produced by the chemical processes going on in the neighbourhood of the jet, or, to use a translation of the authors' words, "The collapse of the unstable condition of the supersaturated water vapour from the jet is determined less by the particular nature of the chemical action than to a far greater extent by the molecular shocks set up in the jet, molecular dissociations and associations, and on the presence of unsaturated compounds, molecular groups with free valencies or 'ions.'"

Whilst accepting the facts of the experiments, namely, that something is produced in these chemical processes which determines dense condensation in the steam jet, yet one has great difficulty in accepting the above explanation. Supposing we even admit that a molecular shock of the kind described could determine condensation in a supersaturated vapour, it must be remembered that the degree of supersaturation in a steam jet in the open air where there is dust is extremely slight,—the particles of water are so close that any strain easily relieves itself. Further, it must be remembered that the vapour in the jet is nearly in equilibrium with the drops of the size present in the jet, and it is very doubtful if it will be at all supersaturated to extremely minute particles of water, as the vapour tension at the surface of extremely small particles is higher than at the surface of larger particles. For these reasons it seems more probable that whatever it is that is produced in these chemical processes, it acts somehow in forming nuclei, and is itself engaged directly in the formation of the centres of condensation.*

Near the end of the second paper already referred to, the authors put in a claim for the importance of these "ions" in producing ordinary cloudy condensation. They

* There is another way of considering the whole question, from which it would appear that there is no such thing as supersaturation in a vapour; that is, no strain in the vapour which either dust or "molecular shocks" can relieve. What is generally called a saturated vapour is one whose tension is equal to the tension of the vapour at a flat surface. Now this tension is not so high as the tension at a surface of extremely small convex curvature; and vapour that is in equilibrium with the vapour at a convex surface is supersaturated to the flat surface; so that saturation is a relative and not an absolute quantity, relative to the curvature of the condensing surface, and a vapour that is supersaturated to a flat surface is not necessarily saturated to a surface of very small curvature. It would thus appear that there is no strain in a vapour till a surface makes its appearance; but after it is formed the lower tension at its surface determines a movement of the vapour molecules towards it.

say: "Apart from the explanation, the fact remains that by rapid and slow combustion an agent is called forth which sets up condensation, and which does not disappear immediately on the cessation of the originating process. Now so much oxidation takes place in so many different ways, that the idea is at once suggested that this agent which is formed by oxidation will be always present in small quantities in the air. We consider it quite possible that the faint condensation which the steam jet suffers in the open air may quite as well be due to the presence of this agent as to that of dust."

This conclusion seemed to be of sufficient importance to demand further investigation. Admitting the existence of these "ions" in the products of combustion, the question resolves itself into this: Do these "ions" persist for any length of time, and so come to play some part in the condensation of a steam jet or of a cloud, or is their action very short-lived? The difficulty of arranging an experiment for testing this point is, that during combustion so many different products are thrown into the air as well as an enormous number of fine dust particles. It is, therefore, difficult to find, by testing this complex mixture, many of the constituents which cause cloudy condensation, whether any particular one is more affected by time than the others. Because, if we keep the gases, new chemical combinations are formed, and the number of dust particles also decreases, and it would be difficult to determine how much of the decrease was due to loss of activity of the "ions." If, therefore, we are going to test the length of time the "ions" of combustion retain their activity, we must use some form of combustion in which no dust is formed, and the products are free from any substance that can form on cooling solid or liquid nuclei. Hydrogen burned in oxygen seemed particularly suitable for the purpose, but as this would involve considerable difficulties, it was determined to try the effect of burning it in filtered air. It was, however, kept in view that the presence of the nitrogen might give rise to complications, but fortunately there was no trouble due to its presence; nor did hydrogen peroxide, ozone, or ammonium nitrate, which are said to be produced when hydrogen is burned, produce any effect. Further, the products from hydrogen burning in air are stated by HELMHOLTZ to be very powerful in increasing the density of the steam jet, and they, therefore, seemed the most suitable substance for an experiment of this kind.

In carrying out this experiment the following arrangement was adopted (see Plate). The hydrogen was prepared from zinc and sulphuric acid placed in the flask, *A*. After being purified, it was burned at the platinum jet, *B*, having a diameter of 1 mm. The air for combustion was drawn through the large cotton-wool filter, *C*, and brought in at the lower end of the tall cylindrical glass combustion chamber, *D*, and taken off at the top. The combustion chamber used for these experiments was a Fresenius Chloride of Calcium Cylinder placed in an inverted position. For the present the flask, *U*, is supposed to be removed, and the combustion chamber, *D*, connected directly by its branch pipe, *V*, with the long glass tube, *E*, through which the products of combustion are carried to the gasometer, *F*, by means of which the air and products of combustion

are drawn through the apparatus. On the pipe, *E*, at a point about 1 m. from the combustion chamber, there is a branch pipe, *G*, which leads through the stop-cock, *H*, into the test-flask, *I*. This flask is provided with an air-pump, *K*, and a filter, *L*. By means of this part of the apparatus a sample of the air passing from the combustion chamber can be tested when desired. To make this test, the flask, *I*, is first cleared of all dust particles by means of the pump, *K*, and filter, *L*. After the air is free from nuclei, the stop-cock, *H*, leading to the pipe from the combustion chamber is opened, and a sample of the products passing in the pipe are drawn into the flask, I. The stop-cock, *H*, is then closed, and the air in the flask expanded by means of the pump, when the density of the condensation observed indicates the number of nuclei in the air from the burning hydrogen. For observing the amount of condensation in the flask, *I*, the dark lantern, *R*, was used, the light being focussed on the contents of the flask by means of the condensing lens, *S*, whilst the cloudy particles were observed through the lens, *T*. When making these experiments the room should be darkened, as the phenomena are then most easily observed.

At first the results were unsatisfactory: there was always considerable clouding, but the density was far from being constant. Efforts were, therefore, made to get everything as perfect as possible. Pure redistilled zinc and the purest sulphuric acid were used for generating the hydrogen, and, to prevent sulphur compounds from coming over with the gas, cupric sulphate* was added to the sulphuric acid, which was used very weak, and mixed with water and cooled before being put in the flask, *A*. The flask, *A*, in which the hydrogen was generated was kept cool by a bath of water, *M*. The hydrogen after leaving the generating flask was passed through the wash-bottle, *N*, filled with solution of lead nitrate. The gas then passed through a plug of cotton-wool, *O*, saturated with a strong solution of lead nitrate, then through a dry cotton-wool filter, *P*. It then passed through a small but very tight cotton-wool filter, *Q*, to regulate the pressure of the gas, as without this obstruction the light was apt to go out, owing to irregular pressure due to the bubbling of the gas. The gas was burned at the platinum jet, *B*, which was made red-hot with a blow-pipe before each experiment to thoroughly cleanse it, and prevent nuclei being driven off by the heat. The combustion chamber was kept cool by surrounding it with a wet cloth.

We shall now turn to the question of filtering the air. This looks a simple matter, but it was found to be one of the most troublesome parts of the experiment. In the apparatus a large cylindrical metal box, *C*, filled with cotton-wool, was used, with a pipe leading off it at the end. This pipe was at first connected with the combustion chamber by means of a short length of india-rubber tube. When the pure hydrogen was burned in this apparatus the flame was scarcely visible, and quite invisible if there was any daylight in the room, even if the combustion cylinder was surrounded with a black surface. Watching the burning hydrogen on one occasion, suddenly a little bright spark

* It was necessary to be constantly adding cupric sulphate to the acid in *A*. If this was not done, considerable condensation took place in the products.

made its appearance in the flame; on now testing the products they were found to have greatly increased in impurity. This at once suggested that some piece of solid matter had got into the flame. I now gently tapped the india-rubber tube by which the filtered air was brought to the combustion chamber, when the flame at once responded, showing a few bright sparks in succession, and the test-flask showed the air to be very full of nuclei. The india-rubber tube was therefore removed, and the filter connected direct to the combustion chamber. On now testing the apparatus, I gently tapped the filter itself, with the result that matters were as bad as before—dense clouding made its appearance in the test-flask. The arrangement of the apparatus was now slightly altered. The combustion chamber, filter, and gas-generating apparatus were put on one table, while the test-flask, with its pump, were put on another, to prevent the vibrations produced by working the pump from shaking the other parts of the apparatus. Further, the cotton in the filter was damped by passing damp air through it the reverse way, and when in use a wet cloth was hung over the entrance end of the filter. This was done to prevent dust rising from the dry wool or metal case.

It may be asked why all these precautions are necessary, when a cotton-wool filter is found to act quite satisfactorily without them when used in the ordinary dust experiments. The reason is very simple. If a little bit of any kind of matter happens to be carried with the air out of the filter in making the ordinary experiments, this means only one centre of condensation, and that drop may not meet the eye of the observer; but when the same bit of matter, particularly if it be organic, passes through the hydrogen flame, at once there are created thousands of centres of condensation, and what in the one case may escape detection gives rise in the other to a very manifest effect.

After many failures, and many alterations and improvements in the apparatus, the result is that, when everything is working right, the test-flask shows the products of combustion of pure hydrogen in dust-free air to be free from nuclei of all kinds. The air remains clear on expansion, and only a few drops are seen falling rapidly; the air is in fact far freer from condensation than the purest air met with in the country. The air was never absolutely free from a single nucleus on expansion, but frequently only a very few drops appeared, and all in view could be easily counted.

From this experiment it may be concluded, 1st, that pure hydrogen burning in dustless air gives rise to no solid or liquid form of dust; and 2nd, that any activity the "ions" may have is extremely short-lived, as they are inactive by the time they are cooled.

Whilst the apparatus, as described, shows that pure hydrogen when burning in dustless air gives rise to no permanent nuclei of condensation, we can, by making a slight alteration in the conditions, show the great importance of dust in the cloudy condensation produced by the products of combustion. If we remove the filter, *C*, and allow the air of the room to go unfiltered to the hydrogen flame, we get very different results, according to the condition of the air. If the day be calm, and the air of the room be but little disturbed, we get a great increase in the density of the condensation

when tested in the flask, *I*, but the density is not very much greater than what the air of the room would give if it had not passed through the hydrogen flame. The concentrated light from the lantern produces a luminous cone shining in the flask. If now, however, we move about things in the room, so as to stir up some dust, the result is remarkable. On now testing the products from the hydrogen flame in the flask the cone of light is no longer visible. So dense is the condensation that the whole flask is filled with a dense white fog, the flask looking like a white ball, the intense heat of the hydrogen flame having produced an enormous number of nuclei out of some of the dust floating in the air.

The conditions of the experiment, in which it is shown that hydrogen burning in filtered air gives rise to no nuclei of condensation, is not, however, quite the same as that in which HELMHOLTZ found these products to be very active, as he mixed them much more quickly with the re-acting agent. The products in his experiment were carried direct to the steam jet. To test the condensing power of the products immediately after combustion, the following alterations were made in the arrangement already described in the apparatus shown in the Plate :—The test flask, *U*, was introduced into the circulating system at a point close to the combustion chamber, so that the products might be tested immediately after combustion. For testing them in this case the steam jet method was employed as in HELMHOLTZ' experiments, only no unfiltered air was allowed to mix with the products. Steam was generated in the flask, *W*, and conveyed by means of a glass tube to the interior of the test-flask, *U*. This tube terminated in a fine jet. With the hydrogen burning and unfiltered air, there was a dense condensation, but when filtered air was used the condensation was extremely slight, and if the flame was low on some trials, it was difficult to say whether there was any condensation or not. In each test the steam was allowed to enter only for a very short time, as the test-flask got heated, and had, therefore, to be cooled before each test. It is evident from these experiments that the products of combustion from pure hydrogen and dustless air are far from being very active in the condensation of the steam jet, even when they are newly formed.

There are other reasons for leading us to conclude that the action of these "ions" is extremely short-lived. It is generally admitted that these "ions" are the cause of electrical conduction in gases, that it is to their presence in the products of combustion that these gases owe their power of discharging electrified bodies. Now we know that the products of combustion soon lose this power; if we bring the products from the flame to the electrified body by means of a short tube, the discharge is rapid, but as we lengthen the tube, the power very rapidly decreases. Again, the air of a room where gas is burning has very little, if any, power of discharging electrified bodies, yet it has enormous numbers of nuclei of condensation, due to the products of combustion. Lord KELVIN and Dr MAGNUS MACLEAN, in a recent communication to the Royal Society of Edinburgh, show that the products of combustion lose their power of discharging bodies in less than fifteen seconds. Further, we know that air

is nearly freed of all nuclei of condensation by the electric discharge, which, while it deposits the dust, rather increases the "ions." It may also be remembered that air passed through the Thermic filter is freed of all nuclei.

But whilst all experimental tests indicate that these "ions," or whatever it is that is produced during combustion, and which gives rise to condensation, are extremely short-lived, and do not play any important part in cloudy condensation in the atmosphere, it would be rash to say that they never do play any part. The conditions of combustion of a laboratory experiment are different from those under which combustion usually takes place; and further, whilst hydrogen may not give any long-lived "ions," some other forms of combustion may, and though they lose their power of discharging electrified bodies, they may yet be capable of causing cloudy condensation.

From many hundreds of observations on the dust and the transparency of the atmosphere, it has been shown that, for the same *relative humidity*, the transparency is proportional to the number of dust particles in the air, except at one place of observation, and under exceptional conditions.* Now this is not likely to be the case if "ions" played any important part in the condensation, as we can hardly expect their action to bear a constant proportion to that of the dust. For the reasons above given, it therefore does not appear at all likely that "ions" play any important part in the ordinary cloudy condensation in the atmosphere.

PART II.

Sunshine and Cloudy Condensation.

In 1894 there was communicated to this Society Part III. of a paper "On the Number of Dust Particles in the Atmosphere." It is there shown that under certain conditions the sun gives rise to a great increase in the number of nuclei. The observations made at Kingairloch show, that when the wind is from the N.W. the number of particles is always very low, being generally under 500 per c.c., and often very much lower when the sky is clouded, but that when the sun came out the numbers rapidly increased as the day advanced, and if the sun kept shining all day, the numbers rose to many thousands in the afternoon, after which the numbers decreased, and again were very low next morning if the wind continued to blow from the N.W. These observations entirely justify the prediction which was made in 1881 on the probable action of sunshine in producing nuclei of condensation.

This action of sunshine on our atmosphere evidently deserves further investigation. A few of the ordinary constituents and impurities of our atmosphere have therefore

* *Proc. Roy. Soc.*, Edin., vol. xx. pp. 66, 93; *Trans. Roy. Soc.*, Edin., vol. xxxvi., Part III., pp. 621, 693.

been tested to see if sunshine acted on them in such a way as to make them active as producers of nuclei of cloudy condensation.

The gases tested were ammonia, nitric acid, nitrous acid, peroxide of hydrogen, sulphurous acid, sulphuretted hydrogen, hydrochloric acid, and chlorine. The result of these tests is that the gases or vapours of all these substances give rise to nuclei of condensation after being acted on by sunshine.

The apparatus used for these experiments consisted of a glass flask, in which the gases were sunned and tested. This flask was provided with an air-pump and a cotton-wool filter, the arrangement being the same as that shown on the right-hand side of the Plate, with the centre tube removed. Different methods were used for getting the gases into the flask; in some cases a little of their solutions in water was introduced into the flask, in others the gases were drawn into the flask through the cotton-wool filter.

In making a test, the first thing to be done was to wash the flask and connecting tubes, and put new cotton-wool into the filter. Filtered air was then pumped in till no drops appeared on expanding the air. If a solution of any of the gases was in the flask, the apparatus was then removed to an open window and exposed to sunshine, generally for one minute, after which it was brought back, the air expanded by means of the pump, and the density of the condensation noted. If the gas was introduced into the flask through the filter, it was sometimes necessary to pass a considerable quantity of vapour when making the first test, as it was found that the vapour that first entered the filter did not pass through, but seemed to combine with the cotton-wool, or perhaps only condensed on it, as after a filter was saturated it acted for some time, supplying nuclei without a fresh quantity of vapour being passed into it.

I regret that the experiments on the action of sunlight on the vapours mentioned are far from being as complete as I could wish; but for the reasons already given, and on account of the difficulty of getting plenty of sunlight, it has not been found possible to get the work done. The following notes are, therefore, very incomplete.

It should be noted at the outset that ordinary air, after being filtered and exposed to sunshine, does not show any cloudy condensation on expansion, only occasionally a drop or two may appear; but when any of the above-named gases are in the air a very different result is obtained. Ammonia has been experimented with both by placing a weak solution in the test-flask, and by drawing the vapour into the flask through the filter. If one drop of ammonia be added to 100 c.c. of water, and put in the flask and sunned for one minute, a great deal of condensation takes place on expansion with even this very weak solution, and if we expose it for five minutes the condensation is much more dense. If we put a weak solution of nitric acid in the flask, say two drops to 100 c.c. of water, we get cloudy condensation on expansion after being sunned; but the action of nitric acid is not so powerful as ammonia; that is to say, that though the solution of nitric acid was double the strength, it did not give so dense a condensation as the vapour from solution of ammonia.

The experiments made with nitrous acid must be looked on with considerable doubt, owing to the uncertainty which hangs round the existence of this gas when produced by the ordinary methods. I shall, therefore, only describe the experiments made, and leave the reader to draw his own conclusions. In experimenting with this gas it was always made outside the test-flask, and drawn in through the cotton-wool filter. The gas was prepared by means of nitric acid and arsenious anhydride. With gas prepared in this way, it is probable some of the effect might be due to nitric acid passing over with the nitrous acid. Another sample of nitrous acid was prepared from weak sulphuric acid and sodium nitrate. Both these samples of nitrous acid gave dense condensation after being exposed to sunshine.

The hydrogen peroxide was tested by placing a little of the ordinary 10 per cent. solution in the test-flask and sunning the contents. It proved to be a powerful generator of nuclei.

Sulphurous acid is one of the most puzzling of all the gases tested, its action is so uncertain. Unlike the others, it almost always gives rise to condensation in the dark. On some days, with even a weak solution in the test-flask, it was impossible to get the condensation to cease entirely. When attempting to clear the air in the flask, even in the dark, a few drops fell at each expansion, and on most days, though the drops nearly ceased to appear, yet on standing a few minutes a considerable number made their appearance when the air was expanded; but in all cases the cloudy condensation was very greatly increased after sunning. The products of combustion from household gas in some cases gave rise to slight condensation after sunning, whilst the products of combustion from anthracite coal, which probably contained a good deal of sulphur, gave dense condensation after being exposed to sunshine. This test might possibly be used for the detection of sulphur compounds in household gas.

The sulphuretted hydrogen tested was prepared from sulphide of iron and weak sulphuric acid. The gas was generated in a glass bottle, and drawn into the test-flask through the cotton-wool filter. This gas gave dense condensation after being exposed to sunshine. In order to see that the effect was not due to vapour of sulphuric acid, the acid used was tested alone before adding the sulphide of iron, and was found to be inactive.

Vapour from hot hydrochloric acid was drawn into the test-flask through the filter and sunned, after which it gave dense condensation on expansion, whilst the same vapour was inactive whilst kept in the dark.

Chlorine is one of the most interesting of the gases tested, as it caused condensation to take place without supersaturation. The gas was prepared from potassium bichromate and hydrochloric acid; it was drawn into the test-flask through the filter. Whilst this gas gave no condensation on being expanded, if kept in the dark, it became fogged on exposure to sunlight without being expanded, and the density of the condensation was but little increased by expanding it.

A point of some importance connected with these nuclei due to the action of light

on gases in the atmosphere is: after they are formed, do they remain active for any length of time, or are they short-lived? Unfortunately, for reasons already given, this point has not been thoroughly worked out; but this much has been ascertained: some of them, such as the hydrogen peroxide nuclei, are very short-lived—fifteen minutes to half-an-hour being sufficient for the air in the flask to lose its power of cloudy condensation; whilst the nuclei from sulphurous acid remain active for a long time, half an hour having no appreciable effect on the density of the condensation when that substance forms the nuclei. In this case the nuclei are probably particles of fine sulphur dust, and their action seems to be as permanent as that of ordinary atmospheric dust.

As might be expected, all the gases do not respond to light rays of the same rate of vibration; but no extensive experiments have been made in this direction. It may, however, be mentioned that whilst sulphurous acid is easily made active by means of the light of burning magnesium, on many of the other gases that light has little if any action. The action of burning magnesium on sulphurous acid makes a convenient lecture experiment, the flask being shown free from cloudy condensation when the moist air and sulphurous acid vapour are expanded, if the room be lit with gas. On now burning two or three inches of magnesium ribbon, and expanding, the air in the flask becomes densely white with cloudy condensation.

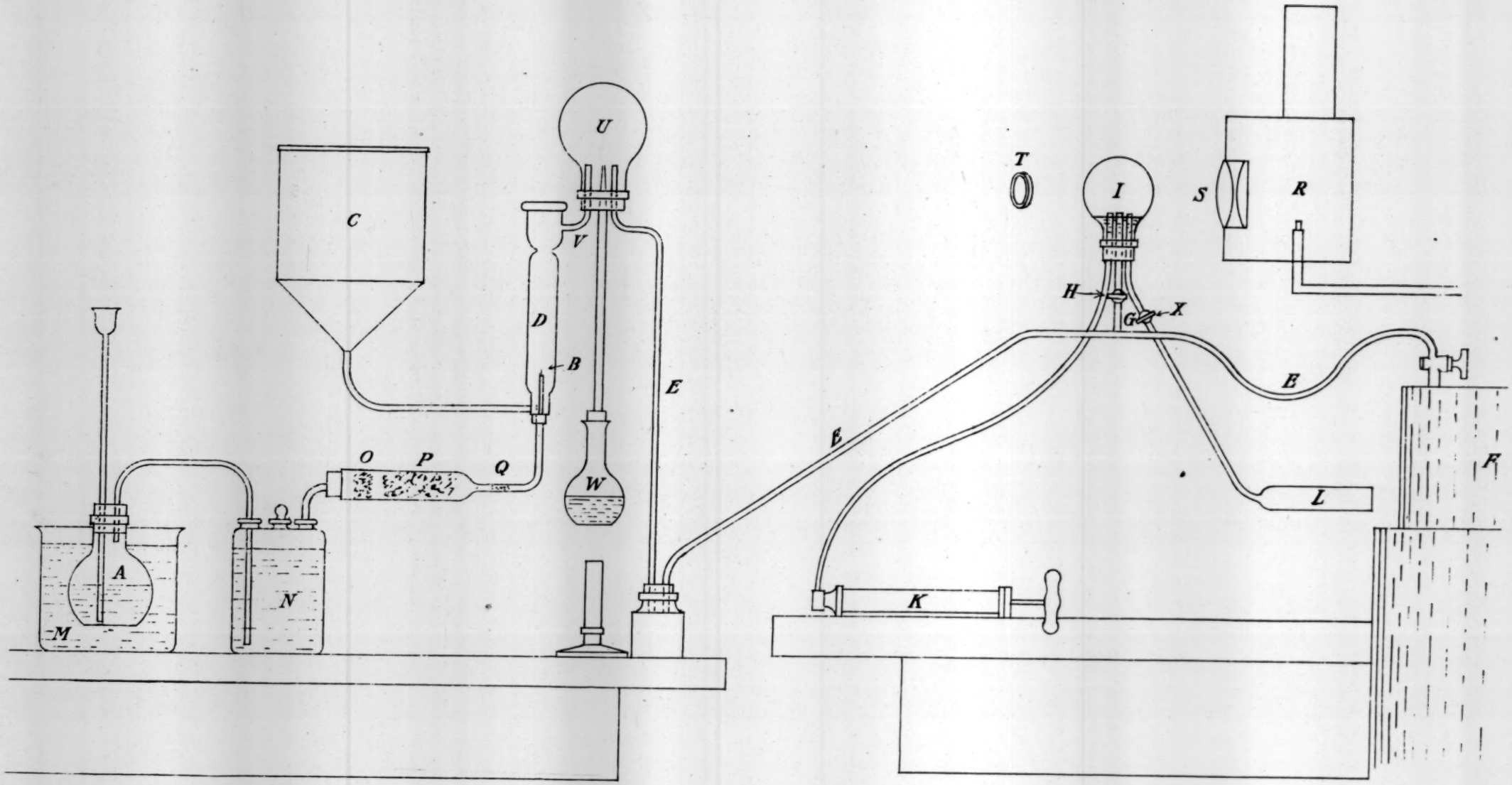
Whilst these experiments show that many gases which are frequently found in the atmosphere form nuclei of condensation after being acted on by sunlight, yet it must be admitted that they throw but little light on the abnormally high dust readings got at Kingairloch, when the wind blew from a pure direction and the sun shone. On looking for a source of impurities in the atmosphere at Kingairloch, I found there was a "mineral well" to the north-west of the place of observation, at a distance of about three miles; and on inquiry I found that there are a number of these wells in the district. The water in these wells contains sulphuretted hydrogen, and it seemed possible that this gas under the action of sunlight might be the cause of the abnormal number of nuclei; but as yet I have not been able to get evidence sufficient to bring in a verdict against it. Since I discovered that the sunlight was in some way connected with the production of these nuclei of condensation in N.W. winds, I have only been once at Kingairloch to test if the mineral springs supplied the gas out of which the nuclei were produced, but unfortunately the weather was never favourable for a test. If the sun shone, the wind was in the wrong direction, and when the wind was N.W., there was no sunshine. Only on one day did I get a minute or two of weak sunshine while the wind was N.W., but on that occasion I got no evidence against the well. The numbers were the same in the air coming to the well as in the air leaving it. The test, however, was too imperfect to put any reliance on, as by the time I got to a small distance to the lee side of the well, to get the air after the sun had time to act on it, the sun was under a cloud. I have also had some of the water bottled and brought here, where it was tested by placing some of it in the test-flask, and sunning

the air and gases over it, but obtained little indication of nuclei being formed. It should, however, be kept in view that it is quite possible the sun may produce the nuclei found at Kingairloch in some other way than from gaseous impurities in the atmosphere, and the phenomenon will require to be further investigated at that place.

These experiments on the effect of sunshine on the gases in the atmosphere show that it is possible for cloudy condensation to take place in the absence of dust. That is, supposing there is any part of the upper air free from dust, it is possible, if any of the gases referred to be present, for the sun to convert them into nuclei of condensation, and permit of clouds forming in dustless air. In the lower atmosphere there always seems to be plenty of nuclei to form cloudy condensation whether the sun shines or not. The only case where almost no nuclei were detected in the air was at the Ben Nevis Observatory, where on one occasion, at least, Mr Rankin found the air nearly free from dust, whilst the top of the mountain was in cloud. All the nuclei there at the time seemed to have been used up to form fog particles, and the air was then in a peculiarly supersaturated condition.

Though it is here shown that the impurities in the atmosphere when acted on by sunshine may permit of cloudy condensation taking place in the absence of what is commonly known as dust, we are not yet in a position to say that they do act in this way, because the impurities may not be in sufficient quantity to give a large enough number of nuclei to produce the cloudy form of condensation; there may be only nuclei sufficient to produce the rainy form of condensation.

Mr. John Aitken on some Nuclei of Cloudy Condensation.



IV.—*The C Discriminant as an Envelope.* By James A. Macdonald, M.A., B.Sc.

(Read 5th July 1897.)

The purpose of this paper is to discuss the conditions under which the C discriminant of a system of curves furnishes a curve which at every point of its length is touched by a curve of the system.

Subsidiary Proposition.

The following proposition will be used :—

If Δ be the discriminant of

$$U \equiv Ac^n + Bc^{n-1} + Dc^{n-2} + \ldots + Nc^2 + Pc + Q = 0 \qquad (1)$$

where A, B, etc., are finite, continuous, single valued, differentiable functions of x and y, the doubled root of $U = 0$ is

$$c = \Delta_A/\Delta_B = \Delta_B/\Delta_D \qquad = \Delta_P/\Delta_Q . \qquad (2)$$

Now, if we give such values to A, B, etc., that one of the functions $\Delta_A \ldots \Delta_P$ vanishes in virtue of $\Delta = 0$, then it follows from (2) that *all* the functions except Δ_Q must vanish. If Δ_Q does not vanish, the doubled root is $c = 0$. Hence P and Q must contain a factor in common with Δ, and this factor is the particular case of the system (1) obtained by putting $c = 0$.

If *all* the functions $\Delta_A \ldots \Delta_Q$ vanish, then since $\Delta_x = \Sigma \Delta_A A_x$ and $\Delta_y = \Sigma \Delta_A A_y$, $\Delta_x = 0$ and $\Delta_y = 0$ all along this part of $\Delta = 0$, that is $\Delta = 0$ has a repeated factor.

Similarly, if $\Delta_A \neq 0$, $\Delta_B = 0$, the rest of the functions must vanish ; the doubled root is $c = \infty$, and A and B have a factor in common with Δ, this factor being a particular case of system (1).*

* The discriminant may always be written in either of the forms :—

$$\Delta \equiv A\chi + B\psi \qquad (a)$$
$$\Delta \equiv P\psi' + Q\chi' \qquad (b)$$

where χ, χ' are determinants whose first columns contain respectively only the coefficients A, B, D, and N, P, Q ; and ψ, ψ' are determinants whose first columns contain respectively only A, B and P, Q.

Differentiating (a) we obtain :—

$$\begin{aligned} \Delta_A &= \chi + A\chi_A + B\psi_A \\ \Delta_B &= \psi + A\chi_B + B\psi_B \\ \Delta_D &= A\chi_D + B\psi_D \end{aligned} \qquad (c)$$
$$\text{etc.}$$

It is evident from (a) that $\Delta = 0$ always passes through the points common to $A = 0$, $B = 0$, and that if these coefficients have a common factor Δ contains this factor. In this case (c) shows that Δ_B, Δ_D, etc., vanish.

If A, B, and D have a common factor, χ also contains this factor, and the same factor is by (a) repeated in Δ ; (c) shows that in this case all the derived functions vanish. If one of the functions B, D vanish identically, it may be considered as divisible by the factors contained in A.

Similar propositions hold with respect to N, P, Q, and in fact all the results obtained above may also be obtained in this way.

The system of curves

$$c^2x^2-2c(y^3-x^3)+(y-x)^2(3y^2+4xy+2x^2)=0 \qquad (3)$$

has for discriminant $\Delta \equiv (y^2-x^2)^3$

$$\Delta_A = -4D = -4(y-x)^2(3y^2+4xy+2x^2)$$
$$\Delta_B = 2B \quad = -4(y^3-x^3)$$
$$\Delta_D = -4A = -4x^2.$$

Along $y=x$ $\Delta_A=0$ $\Delta_B=0$ $\Delta_D \neq 0$, the corresponding value of c is zero, and the part $y-x=0$ of the discriminant is a part of $D=0$ and of $B=0$.

Again, the system $c^3y+c^2(2y+x)+3cx+2x=o$ (4) has for its discriminant $\Delta \equiv 3x(x-4y)^3$,

$$\Delta_A \equiv 0$$
$$\Delta_B = -18x(x-4y)^2$$
$$\Delta_D = +18x(x-4y)^2$$
$$\Delta_E = -6(2x+y)(x-4y)^2.$$

Along the branch $(x-4y)^3=0$ of the discriminant all the functions vanish, and as already indicated $(x-4y)^3$ is a repeated factor.

Since $\Delta_A=0$ for every point of the plane of x y, it is suggested that, regarded as a function of x and y, the expression on the left-hand side of (4) contains a factor independent of x and y. In fact, the expression is identically equal to $(c+2)(c^2y+cx+x)$.

Along the branch $x=0$ of the discriminant all the functions except Δ_E vanish; the corresponding value of c is zero, and this part of the discriminant is contained in D and in E.

It should be noted that the ordinary rule for calculating the value of c does not hold in the last mentioned case; it holds, however, when the new and correct discriminant $x(x-4y)$ is calculated as well as the new values of Δ_A, Δ_B, and Δ_D.

Nature of the Value of C for an Envelope.

With regard to the value of c obtained, we have to remark that if it be independent of x and y in virtue of the relation between the variables along any branch of the discriminant, then that branch must be merely a particular case of the curves given by the c equation, and not an envelope. An envelope may be a particular case of the C equation; but the *corresponding* value of c in this case must be a function of the variables, so that it changes its numerical value as we proceed from point to point of the discriminant.

For example, the curves

$$c^2y+cx+x=0 \qquad (5)$$

and

$$c^2\sin x-2cy+\sin x=0 \qquad (6)$$

have their discriminants respectively $x(x-4y)$ and $y^2-\sin^2 x$.

Both these when equated to zero give constant values for c, and therefore do not supply envelopes.

Envelope and Particular Case of the 'C' Curves simultaneously.

The system (5) consists of straight lines passing through the origin, and two constant values of c *correspond to* any one of these straight lines; but along $x-4y=0$ these two values become equal, hence this locus is included in the discriminant. The curves given by the C equation might all touch each other at a fixed point, and in this case the discriminant if it corresponded to a constant value of c would also touch each curve of the group; but it is plain that it is no more an envelope than the line $x-4y=0$ is an envelope of the group (5).

On the other hand, the system $y=c(x-c)^2$ (7) whose discriminant is $3y(4x^3-27y)$ gives for $y=0$, $c=x$. Now, $y=0$ is a particular case of the curves indicated by $y=c(x-c)^2$, viz., that given by $c=0$; but $c=0$ is not the *corresponding* value of c, and the curve $y=0$ is a true envelope. In fact we may regard the curve $y=0$ as *made up of infinitesimally small pieces of each of the curves* $y=c(x-c)^2$ at their maximum or minimum points.

Main Proposition.

Let us now take the irreducible equation

$$U\equiv Ac^n+Bc^{n-1}+Dc^{n-2}+\ \ldots\ .\ +Pc+Q=0 \qquad (8)$$

A, B, D, etc., being subject to the restrictions already stated

$$U_x=c^nA_x+c^{n-1}B_x+\ \ldots\ .\ +cP_x+Q_x.$$

But

$$c=\Delta_A/\Delta_B=\Delta_B/\Delta_D=\text{ etc.},$$
$$\therefore (\Delta_B)^2=\Delta_A\Delta_D\,,(\Delta_B)^3=(\Delta_A)^2\Delta_E \text{ etc.},$$

all the functions being assumed finite both ways.

Hence

$$U_x=A_x\left(\frac{\Delta_A}{\Delta_B}\right)^n+B_x\left(\frac{\Delta_A}{\Delta_B}\right)^{n-1}+\ \ldots\ .$$
$$=\frac{(\Delta_A)^{n-1}}{(\Delta_B)^n}\left\{\Delta_AA_x+\Delta_BB_x+\ \ldots\ .\ +\Delta_QQ_x\right\}$$

Similarly

$$U_y=\frac{(\Delta_A)^{n-1}}{(\Delta_B)^n}\left\{\Delta_AA_y+\Delta_BB_y\quad \ldots\ .\ +\Delta_QQ_y\right\}$$

i.e. along $\Delta=0$

$$\left.\begin{aligned}U_x&=\frac{(\Delta_A)^{n-1}}{(\Delta_B)^n}\Delta_x\\ U_y&=\frac{(\Delta_A)^{n-1}}{(\Delta_B)^n}\Delta_y\end{aligned}\right\} \qquad (9)$$

Therefore, assuming for the present that Δ_A and Δ_B are both finite both ways, and that the corresponding value of c is variable, we have the following conclusions:— U_x and U_y vanish or do not vanish according as Δ_x and Δ_y respectively vanish or do not vanish; and when Δ_x and Δ_y are not both zero, the curve $\Delta = 0$ touches at every point of its length one of the curves $U = 0$. If, however, Δ_x and Δ_y are zero in consequence of the relation $\Delta = 0$, both U_x and U_y vanish all along $\Delta = 0$ (or part of it if it be a degenerate curve), that is $\Delta = 0$ is (provided c be variable) a locus of multiple points on the system $U = 0$. Now if $\Delta_x = 0$ $\Delta_y = 0$ along a finite part of $\Delta = 0$, Δ must have a repeated factor. Hence, *the occurrence of a repeated factor in the discriminant indicates, under the conditions already stated with respect to* Δ_A, Δ_B *and* c, *a locus of multiple points.*

Locus of Multiple Points not in general an Envelope.

It is easy to show that in general the discriminant curve does not touch the U curves at their multiple points; for if (x, y) be a double point on $U = 0$, we have (m_1 and m_2 being the values of the tangents of the angles which the tangent at this point makes with the x axis)

$$m_1 + m_2 = -2U_{xy}/U_{yy}$$

$$m_1 m_2 = U_{xx}/U_{yy}$$

$$\therefore m_1 + m_2 = -2\frac{\{c^n A_{xy} + c^{n-1} B_{xy} + \dots\}}{\{c^n A_{yy} + c^{n-1} B_{yy} + \dots\}}$$

$$= -2\frac{\Sigma \Delta_A A_{xy}}{\Sigma \Delta_A A_{yy}} \tag{10}$$

$$m_1 m_2 = \frac{\Sigma \Delta_A A_{xx}}{\Sigma \Delta_A A_{yy}} \tag{11}$$

The values for the discriminant are:—

$$(2m =) m_1' + m_2' = -2\frac{\Sigma \Delta_A A_{xy} + \Sigma (\Delta_A)_y A_x}{\Sigma \Delta_A A_{yy} + \Sigma (\Delta_A)_y A_y} \tag{12}$$

and

$$(m^2 =) m_1' m_2' = \frac{\Sigma \Delta_A A_{xx} + \Sigma (\Delta_A)_x A_x}{\Sigma \Delta_A A_{yy} + \Sigma (\Delta_A)_y A_y} \tag{13}$$

The condition that the discriminant may touch one or both branches of the C curve at the double point is easily found. In fact, if $\delta^m = 0$ be a repeated factor of the discriminant, the condition that $\delta = 0$ touch one branch of $U = 0$ at the double point is

$$\frac{2U_{xy}}{U_{yy}} \equiv \frac{U_{xx}}{U_{yy}} \cdot \frac{\delta_y}{\delta_x} + \frac{\delta_x}{\delta_y} \tag{14}$$

and the condition that it touch both branches is

$$\left(\frac{U_{xy}}{U_{yy}}\right)^2 \equiv \left(\frac{\delta_x}{\delta_y}\right)^2 \equiv \frac{U_{xx}}{U_{yy}} \tag{15}$$

the value of c in terms of x and y having been substituted in U_{xx}, U_{xy} and U_{yy} after differentiation. The conditions (14) and (15) may of course be expressed in terms of the discriminant and its derived functions only.

Practical Examination of the Curves $U=0$ *and* $\Delta=0$.

Given therefore a system of curves $U=0$ and the discriminant curve $\Delta=0$, we may draw the following conclusions :—

(1) If the terms P and Q contain a common factor, (or if $P\equiv 0$) this factor is contained in the discriminant and is not an envelope but merely the particular case of the C system obtained by putting $c=0$. Similarly, if A and B contain a common factor, (or if $B\equiv 0$), the corresponding discriminant factor is not an envelope. This factor may be either single or repeated.

(2) The remaining factors of the discriminant are either (a) single or (b) repeated.

(a) Taking the single factors, we must test whether the corresponding values of c are constant or functions of x and y. Whether this is more easily done by finding the value of Δ_A/Δ_B (as in the case where the c equation is of the second degree in c) or by direct substitution, depends on special circumstances. If c be found to be constant the corresponding factor is not envelope; but if c be variable, the curve is a true envelope and is touched at every point of its length by the C curves.

(b) Taking the repeated factors we have to test whether they represent doubled (or in general n-pled) curves, at every point of which $U_x=0$ $U_y=0$ in virtue of $U=0$ or loci of multiple points.*

The value of c discriminates between these two cases. If it be constant, the locus is a particular case of the C curves. An example of this is supplied by the system

$$c^2(x^2+y^2-1)^2+6cy(x^2+y^2-1)-3(x^2-2y^2-1)=0$$

which is discussed below.

If c be variable, the locus is a locus of multiple points, and in general is nothing more.

The following systems of curves illustrate some interesting points.

(1)
$$c^2(x^2+y^2+2x)+2cy+2x=0 \qquad (a)$$
$$\Delta\equiv y^2-2x(x^2+y^2+2x) \qquad (b)$$

$\Delta=0$ has a node at the origin and an asymptote $x=\frac{1}{2}$.

Every circle of the system (a) touches $\Delta=0$ and passes through the origin, and two circles of the system (given by $c=\pm 1$) touch $\Delta=0$ at (0, 0)

(2)
$$c^2x^2+c(x+y)+y^2=0 \qquad (a)$$
$$\Delta\equiv 4x^2y^2-(x+y)^2=0 \qquad (b)$$

* Note also here that the vanishing of the first three or the last three coefficients, in virtue of their containing a common factor, leads to a repeated factor in the discriminant. This case has already been disposed of under (1).

(b) represents two equal hyperbolæ passing through the origin, with asymptotes $x = \pm\frac{1}{2} = y$. The curves (a) are a series of ellipses passing through the origin and touching both parts of $\Delta = 0$ there, as well as at one other point on each.

(3)
$$c^2x^2 + 2c(y^3 - x^3) + (y-x)^2(3y^2 + 4xy + 2x^2) = 0 \qquad (a)$$
$$\Delta \equiv (y^2 - x^2)^3 \qquad (b)$$

$y + x = 0$ is a locus of multiple points ($c = 2x$), $y - x = 0$ is a particular case of the primitive given by $c = 0$.

(4)
$$c^2(x^2 + y^2 - 1)^2 + 6cy(x^2 + y^2 - 1) - 3(x^2 - 2y^2 - 1) = 0 \qquad (a)$$
$$\Delta \equiv 3(x^2 + y^2 - 1)^3 = 0 \qquad (b)$$

The points ($y = 0$, $x = \pm 1$) are on all the curves (a) as well as on (b).

All the curves touch at these points the discriminant which is a particular case of $U = 0$ ($c = +\infty$); $y = \dfrac{1}{4c}$ is a tangent to all the curves at two points. The curve given by $c = -a$ is the image in the x-axis of the curve given by $c = +a$, and as c increases to $+\infty$ and decreases to $-\infty$ the curves and their images gradually close together, and finally give the discriminant.

V.—*On the Fossil Flora of the Yorkshire Coal Field.* (Second Paper.*)

By Robert Kidston, F.R.S.E., F.G.S. (With Three Plates.)

(Read 20th July 1896.)

Among the specimens from the Yorkshire Coal Field which have been collected by Mr W. Hemingway, Barnsley, and submitted to me for examination at various times, are the remains of several cones which are referable to the genus *Sigillariostrobus*, Schimper.

Notwithstanding the great frequency of the genus *Sigillaria* in the Coal Measures, and especially in the Middle Coal Measures, examples of their fructification are very rare. This is the more remarkable, as specimens of *Sigillaria*, showing cone scars, though not common, are occasionally met with. Possibly, however, the apparent rarity of Sigillarian cones is due, in part, to our inability at present to distinguish them in all cases from cones generally placed under the name of *Lepidostrobus*, which latter genus there is every reason to believe comprises cones that belong to several genera of Lycopods.

A *Sigillaria* occurs in the Upper Coal Measures of Somersetshire, which I have recorded under the name of *Sig. tessellata* (but which differs slightly from the typical form), that frequently shows verticils of cone-scars; but I could never refer any of the many forms of *Lepidostrobi*, occurring in the same beds, to that or any other *Sigillaria*.

It is quite possible that the cones I described as *Lepidostrobus* (?) *spinosus*† and *Lepidostrobus squarrosus*‡ may belong to *Sigillaria*, but on this point we cannot speak with any certainty. In the case of the two cones to be described in this paper, no doubt can remain as to their Sigillarian nature.

Although the genus *Sigillaria* was founded by Brongniart in 1822,§ nothing was known of the internal organisation of *Sigillaria* till 1839, when the same distinguished author published his "*Observations sur la structure intérieure du Sigillaria elegans comparée a celle de Lepidodendron et des Stigmaria et a celle des végétaux vivants.*"‖

Previous to the publication of Brongniart's paper no data existed from which the affinities of the genus could be suggested, and owing to imperfect knowledge of the internal structure of fossil and at present living allied plants, some of the structure displayed by the now classic specimen was misinterpreted, and from then till the present time, notwithstanding the description of undoubted Lycopodiaceous Sigillarian cones by Zeiller,¶ some botanists are still very reluctant to give up their old ideas of the Cycadaceous affinities of *Sigillaria*, as originally *suggested* by Brongniart, and still seem to hesitate to place *Sigillaria* amongst the Lycopodiaceæ.

* "On the Fossil Flora of the Yorkshire Coal Field" (first paper), *Trans. Roy. Soc. Edin.*, vol. xxxviii. part ii. No. 5, pp. 203-223, with three plates, 1896.

† *Trans. Roy. Soc. Edin.*, vol. xxxvii. p. 342, pl. iv. figs. 13–14, 1893.

‡ *Trans. Roy. Soc. Edin.*, vol. xxxvii. p. 341, pl. ii. fig. 7; pl. iii. figs. 11–12, 1893.

§ "Sur la classification des végétaux foss.," p. 9, *Mem. Muséum d'hist. nat.*, vol. viii. Paris.

‖ *Archives du Museum*, vol i. pp. 405–461, pls. xxv.–xxxv. (i.–xi.).

¶ Zeiller, *Ann. d. Sc. Nat.* 6e. sér., "Bot.," vol. xix. p. 256, plates xi., xii., 1884; also in *Flore foss. du Bassin houil. d. Valenciennes*, p. 591, 1888.

It may not be without service to review as shortly as possible the evidence on which these two opinions as to the affinities of *Sigillaria* have been founded. Other views which referred *Sigillaria* to the *Cactaceæ* and *Euphorbaceæ* need not be discussed, as they are utterly untenable.

The small specimen of *Sigillaria*, whose internal organisation was described by BRONGNIART in 1839, under the name of *Sigillaria elegans*,* consisted of a small part of a young stem about 2 cm. long and 4 cm. in diameter.

The most of the delicate tissue lying between the central vascular bundle and the bark had disappeared, but the denser cortical portion and the vascular system were excellently preserved. The specimen therefore presented an outer cylinder of bark and an inner cylinder of vascular tissue, between which originally lay a delicate cellular tissue, of which only a few fragments were preserved.

The outer cortical cylinder, constituting the bark, and on whose outer surface were the rhomboidal leaf-cushions, was composed of very fine and very dense fibro-cellular tissue. The more important portion of the stem—the vascular system—formed a perfectly regular hollow cylinder, 13 to 14 mm. in diameter, but the cylinder itself is only about 1 mm. in thickness, and composed of a definite number of bundles, always perfectly equal and similar, and placed beside each other without almost any appreciable interval, but distinctly individualised by their round interior margins, which gave a "festoon" appearance to the inner boundary limit of the vascular system. Each of these bundles is formed of two distinct zones—the inner and primary bundles forming the "festoon" structure, the outer and much larger portion the exogenously developed zone.

In transverse section the primary bundles have the form of a segment of a circle whose convexity points towards the centre of the stem, and are composed of vessels irregularly placed without any order, whose walls are transversely or obliquely barred or even reticulated. The larger vessels occupy the inner portion of the bundle, while the smaller elements are placed externally.

The exogenous zone is composed of vessels disposed in radiating series, sometimes separated by a narrow interval, which was occupied by the medullary rays, though these were generally destroyed in the specimen. The vessels on the inner portion of the exogenous zone are smaller than those on its outer surface; the smallest vessels of the secondary zone being in contact with the smaller vessels of the primary bundles, and these vessels of smaller size form by their contact, but without any confusion, a line of demarcation between the primary and the exogenously developed bundles.

Outside of the exogenous zone, and situated close to it, are small isolated lenticular or circular bundles composed of uniform tissue, but smaller and disposed without any order. These are the foliar bundles.

* This specimen was named *S. elegans* by BRONGNIART in error. The fossil is his *Sig. Menardi*, *Hist. d. végét. foss.*, pl. clviii. fig. 6 (? not fig. 5), which again is only a young condition of *Sigillaria Brardii*, Bgt., the type of the *Clathraria* section of *Sigillaria*. See ZEILLER, *Ann. d. Scienc. Nat.*, 6e. sér., "Bot.," vol. xix. p. 259, 1884; WEISS, *Sitz-Bericht d. Gesell. natur. Freunde.* Berlin, No. 5, 1886, p. 70.

Examining the structure of these various parts more in detail, the *primary bundle* is composed of very long tubular utricules, irregular in size, of which the smaller are external. These utricules or vessels,* disposed without order, differ among themselves, not only by their size but by their length. Their walls are marked by transverse bars or spiral striæ, numerous and fine, but very variable, some vessels having one kind of marking, some the other, and occasionally the two forms of marking occur on the same vessel. In the vessels of smaller size the bars are generally more oblique, and form spirals. Rarely the thickenings form hexagonal areolations, but all these various forms of cell-thickening appear to occur on the same vessel. The smallest vessels are situated on the exterior surface, and contain spiral fibres.

In the *exogenous zone* the bundles are composed of a uniform tissue, consisting of tubular radiating vessels disposed in regular parallel radiating series, which are sometimes contiguous or separated by narrow medullary rays. The vessels next the primary bundles are smallest, and increase in size as one proceeds outwards, and become almost as large as the large vessels of the primary bundles. They are hexagonal in form, but irregular in section. Their walls are marked by transverse bars (*scalariform*), perfectly regular, and form a series on each face of the vessel on all sides. In tangential section the medullary rays are seen to be composed of plates of cellular tissue of little vertical height, generally of one series of cells, but occasionally the medullary rays are of more than one layer in thickness.

The foliar bundles which spring from the outer surface of the primary zone are entirely composed of scalariform vessels, smaller than the external vessels of the secondary bundles, and do not show any regular arrangement.

The *cortical envelope* is composed of two different layers, which are intimately connected, and pass almost insensibly into each other. The inner layer is formed of elongated prosenchymatous cells, very dense, terminated by oblique extremities, and of which many contiguous cells have the same height, so that their terminations form zigzag lines. They are placed in regular uniform radiating series, the walls being without punctations. The outer layer is formed of parenchymatous tissue, more or less regular, the smooth cells being closely placed without lacunæ, and are not arranged in radiating series, nor are they parallel to the surface. In the most external zone the walls appear to be thickened, and form the surface of the leaf-cushion.

After describing *Sigillaria elegans*, *Anabathera pulcherrima*, and *Stigmaria ficoides*, BRONGNIART says :—"These fossil stems ally themselves then, on the one part to the *Coniferæ* and *Cycadaceæ*, by the disposition and uniformity of their ligneous or vascular tissue, and on the other part to the vascular cryptogams by the constancy of the structure of the walls of the vessels." †

* These, like those in ferns and lycopods, are not true vessels. They communicate with each other by lateral openings, and are not continuous as in true vessels, but the septæ forming the individual cells or utricules remain intact.

† BRONGNIART, *Observations sur la Structure inter du* S. elegans, p. 426.

Brongniart then proceeds to consider the structure of *Lepidodendron Harcourtii*,* the only *Lepidodendron* whose internal organisation was then known.

In *Lepidodendron Harcourtii* the primary vascular bundle consists of a *closed* ring of scalariform tissue, sharply defined on its inner side, where the larger-sized vessels are placed. On its outer surface the vessels become much smaller, and form a curious crenulate line caused by bay-like hollows with dividing projections. The elements composing the bundle are disposed without any definite order, and there are no medullary rays. The foliar bundles spring from the outer smaller vessels.

From the structure exhibited by this species, Brongniart came to the conclusion that *Lepidodendron* belonged to the vascular cryptogams, as the arrangement of the vessels was such as one constantly observes in Lycopods. In contra-distinction to this, he believed that the disposition of the ligneous tissue (secondary xylem) of *Sigillaria elegans*, composed of radiating series of vessels, was a character foreign to all cryptogams and characteristic of Dicotyledons, and for these and other reasons, which Brongniart fully gives, he believed the affinities of *Sigillaria elegans* were with the gymnosperms. To show, however, that his opinions were not free from doubt, he says :—"But in the meantime it may be difficult to establish this in a positive manner, because there are numerous differences between this plant and the gymnosperms which we know," and of these differences he gives a summary which need not be repeated here, but concludes with the following suggestive sentence :—"All these circumstances lead to the conclusion that the *Sigillaria* and *Stigmaria* constitute a special and extinct family probably belonging to the great division of the Dicotyledonous gymnosperms, but of whose fruit and leaves we are still ignorant," and he further asks whether *Stigmaria* may not be the root of *Sigillaria*.

The great point on which Brongniart founded his conclusions that *Lepidodendron* was *Lycopodiaceous*, and *Sigillaria gymnospermous*, was the absence of a secondary xylem in the former and its presence in the latter, accompanied with medullary rays and a radial arrangement of the vascular elements.

It is a curious circumstance that *Lepidodendron Harcourtii*, the species examined by Brongniart while he was instituting his comparison between the internal structure of *Sigillaria* and *Lepidodendron*, is one of the very few *Lepidodendra* which has not yet yielded any specimens showing the development of the secondary xylem. In regard to the absence of secondary xylem in *Lepidodendron Harcourtii*,† Professor Williamson remarks :—"No specimen of it has yet been found showing a trace of secondary xylem.

* Witham, "On the *Lepidodendron Harcourtii*," *Trans. Nat. Hist. Soc. Northumb., Durham, and Newcastle*, vol. ii., read March 1832. *Ibid.*, "Internal Structure of Fossil Vegetables found in the Carboniferous and Oolitic Deposits of Great Britain," pp. 51 and 75, pl. xii. figs. 1–7, and pl. xiii., Edin., 1833. Lindley and Hutton, *Lepidodendron Harcourtii*, *Fossil Flora*, vol. ii. p. 45, pls. xcviii. and xcix., 1833.

† Williamson, "On the Light Thrown upon the Question of the Growth and Development of the Carboniferous Arborescent Lycopods by a Study of the Details of their Organisation," *Mem. and Proc. Manchester Lit. and Phil. Soc.*, 4th ser. vol. ix., session 1894–5, p. 47, 1895. See also Brongt., *Hist. d. végét. foss.*, vol. ii. p. 37, pls. xx. and xxi., 1837 ; Bertrand, "Remarques sur le *Lepidodendron Harcourtii* de Witham," *Travaux et Mémoires des Facultés de Lille*, vol. ii. *Mémoir* No. 6, pls. i.–x. Lille, 1891.

It appears probable that in this respect the type resembles *L. Wunschianum*, viz., that the exogenous zone, or secondary xylem, only made its appearance at an advanced age of growth, and that we have as yet obtained no specimen sufficiently advanced to have entered upon that stage."

Since BRONGNIART wrote his memoir, little has been added to our knowledge of the internal organisation of *Sigillaria*, the only contribution being by MM. RENAULT and GRAND 'EURY, "Recherches sur les végétaux Silicifies d'Autun: Étude du *Sigillaria spinulosa*."* This memoir contains some important anatomical discoveries. The *primary vascular bundles* form an inner circle, but do not touch each other laterally; they are, however, in contact with the secondary xylem or "ligneous cylinder," which is slightly swollen at the point where they come in contact with it. The form of the primary bundles is exactly similar to the corresponding portion in *Sigillaria elegans*. In transverse section the bundle is in the form of a crescent, of which the convexity formed by the larger vessels faces inwards, while the slightly concave part composed of smaller vessels faces the slight prominences of the secondary xylem. The large vessels are scalariform or reticulated, but as they approach the secondary xylem they become smaller, and assume the form of spiral vessels or false tracheids. These smaller vessels give rise to the foliar bundles.

The *secondary xylem* consists of a cylinder of many regular radiating series of medium-sized vessels separated by medullary rays, but without any division into component bundles as in *Sigillaria elegans*. The vessels which compose this secondary xylem are elongate and barred on all surfaces (scalariform), as in *Sigillaria elegans*. These vessels are disposed in long radiating lines and separated by the slender medullary rays or by the foliar bundles.

The *Medullary rays*, which are composed of one, or at most of two, rows of laterally placed smooth-walled muriform cells, have a considerable vertical height.

The *Bark*, according to RENAULT, is composed of three layers: an *inner* and delicate cellular tissue which is usually destroyed; a *middle*, of coarse Dictyoxyloid structure, which he names the *suberous layer*. This suberous layer is formed of bands of elongated cells with smooth resistant walls, which anastomise longitudinally and transversely, and thus form a strong mesh-like structure, whose meshes are filled with more delicate prismatic cellular tissue. The suberous layer appears to be formed of several zones of this Dictyoxyloid tissue, which, under certain conditions of preservation, separate from each other. In comparison to the size of the vascular system, this middle zone of the bark bulks largely in the size of the stem.

The third or *outermost layer* of the bark—the *Epiderm*—is formed of polyhedric cells, which become elongate as they approach the cellular rays which fill the meshes of the suberous layer, and especially so where they join the mesh-bands. The epiderm forms a continuous layer extending over all the outer surface of the plant, and obscures

* *Mém. Presentes par divers savants a l'acad. d. Sciences*, vol. xxii. Paris, 1875. Plates. See also note on p. 62 of this communication.

more or less the prominent projections of the suberous meshes. To these projections, not wholly obscured by the epiderm, are due the *striæ* on the cortical surface of the stem. It also extends over the leaf-scars, and blends with the cells which form the cushions.

The *foliar bundles*, which spring from the outer surface of the primary bundles, at first rise up vertically in the parenchymatous inner layer of the bark, but at a certain height they bend outwards and penetrate the suberous layer, following the course of the cellular tissue which fills the meshes of the Dictyoxyloid tissue.

The foliar bundles, at first circular, become enlarged and lunate in form as they approach the surface of the stem. The foliar bundle is always accompanied by two lunate lacunæ parallel to its direction, of which their vertical section is an ellipse, limited by a cellular envelope. These two organs, of which it is difficult to ascertain their function, were each penetrated longitudinally by a tolerably large canal, of which it has been impossible to study the structure. Perhaps they originally contained some gummy substance. It is these lacunæ which give rise to the lateral arcs that are seen, one on each side of the foliar bundles of the leaf-scar.*

The authors of this memoir arrive at a conclusion very similar to that propounded by BRONGNIART, that "by the most essential characters the *Sigillaria* have the organisation of Dicotyledonous stems, and particularly of gymnosperms, and, above all, to the *Cycadaceæ*," an opinion they have since seen grounds to modify, though apparently not entirely to resign; for in a communication, "Sur les fructification des sigillaires," † after giving the description of a cone which RENAULT refers to the *Clathrarian* section of *Sigillaria*, and in which he describes the occurrence of pollen sacs on the under surface of the basal portion of the bracts on each side of the medial nerve, he arrives at the conclusion that the *Clathrarian* and *Leiodermarian Sigillaria* are Phanerogamous gymnosperms allied to the Cycads, and that the *Sigillaria* with *ribbed stems* are cryptogams. I am not aware that this interesting specimen has yet been figured, nor

* *Note.*—In the earlier part of this paper (p. 34) I have remarked that the *Sigillaria elegans*, whose internal structure was described by BRONGNIART in 1839, was now recognised as his *Sig. Menardi* (*in part*), *Hist. d. végét. foss.*, pl. clviii. fig. 6. This again is considered to be only a young condition of *Sig. Brardii*, Brongt., and to this last mentioned species the *Sigillaria spinulosa*, Germar, must be united as only representing a different state or condition of growth. See WEISS and STERZEL, *Die Sigillarien der preussischen Steinkohlen-und Rothliegenden-Gebiete*, II. "Die Gruppe der Sub-sigillarien," *Abhand der König Preuss. geol. Landesanstalt*, Neue Folge, Heft. 2, Berlin, 1893, p. 84 *et seq.* KIDSTON "On *Sigillaria Brardii*, Brongt., and its Variations," *Proc. Roy. Phys. Soc. Edin.*, vol. xiii. p. 233, pl. vii., 1896. But there are great structural differences in the bark of the specimen described by BRONGNIART and that described by RENAULT and GRAND 'EURY, differences which preclude the possibility of these two forms of *cortex* belonging to the same species.

Two questions therefore arise:—(*First*) Are the specimens figured by RENAULT and GRAND 'EURY in their memoir on the internal structure of *Sigillaria spinulosa*, pl. i. figs. 2–3, really referable to that plant or one of the forms of *Sig. Brardii*? and (*secondly*) granted they do belong to this species, is it proved beyond doubt that the vascular axis described under the name of *Sigillaria spinulosa* really belongs to the bark found associated with it? From the crushed and broken condition of the axis and bark shown in their fig. 1, pl. i., the absolute proof that all these parts belong to the same stem appears to be wanting, and their relative position might be only accidental. These remarks are made from an examination of the figures and descriptions which accompany them only, as I have not had an opportunity of studying the original specimens.

† *Comptes rendus*, 7th Dec. 1885.

have I had the pleasure of examining it, but as there is absolutely no evidence adduced to show that the fructification under discussion belongs to *Sigillaria*, I cannot accept the deductions drawn from it as throwing any light upon the affinities of that genus,* especially when cones undoubtedly belonging to *Sigillaria*, which have been described by ZEILLER, and other specimens possessed by myself presently to be described, have an entirely different structure.

Mons. GRAND 'EURY, in the introduction to his *Géologie et Paléontologie du Bassin Houiller du Gard*,† after reviewing the evidence for and against the cryptogamic position of *Sigillaria*, places them among the vascular cryptogams; but this opinion is scarcely upheld in the body of the work, for we find under *Sigillariostrobus fastigiatus* ‡ the following remark:—"For me, in any case, there is not a doubt that the celebrated silicified *Sigillaria elegans* from AUTUN, which is the *Sigillaria Brardii* with the structure of a Dicotyledonous gymnosperm, has not been reproduced by spores."

To sum up, it is seen then that those authors who regard *Sigillaria*, in whole or in part, as Cycadaceous, and *Lepidodendron* as Lycopodiaceous, base their opinions largely, if not entirely, on the presence of an exogenous zone in *Sigillaria*, and the absence of such a zone in *Lepidodendron*.

Within the last few years much has been done to elucidate the structure of *Lepidodendron*, especially by the late Professor WILLIAMSON, who, from the rich material from the beds containing plant structure in Yorkshire and Lancashire, in England, from Pettycur and the Island of Arran, in Scotland, has described in detail the internal structure of many species of *Lepidodendron*, in which he has proved beyond all doubt the presence in this genus also of a secondary exogenously developed zone of vascular tissue. This exogenous zone does not appear in all species at the same relative age; in some it occurs in comparatively small branches, as in *Lepidodendron selaginoides*, Carr. (? Sternb.), while in others, like *Lepidodendron Wunschianum*, from Laggan Bay, Arran, it only appears in advanced age. *Lepidodendron Wunschianum* may therefore be taken as serving to point out the general structure of *Lepidodendron* when compared with that of *Sigillaria*, especially as its anatomy, from very young twigs to old stems, is known.

In the earliest condition the *vascular cylinder* of *Lepidodendron Wunschianum* consists of a solid circular bundle devoid of any medulla. As growth proceeds a small medulla appears in the centre of the vascular bundle, and as the medulla increases in size, the vascular bundle is carried outwards, and eventually, when all increase of the medulla has ceased, the primary vascular bundle appears as a zone of considerable size, composed of scalariform tissue of tolerably equal diameter, the transverse bars of which are again connected with fine transverse lines.

* See also in regard to the affinities of *Sigillaria*:—RENAULT, *Cours d. botan. foss. Première Année*, 1881, pp. 125, 151. RENAULT, "Structure comparée de quelques tiges de la flore carbonifère," *Nouvelles Archives du Muséum de Paris*, 2e. sér., vol. ii. p. 213, 1879. DAWSON, "On New Plants from the Erian and Carboniferous, and on the Characters and Affinities of Palæozoic Gymnosperms," Peter Redpath Museum, M'Gill University, Montreal—Notes on Specimens, 1890, *Canadian Record of Science*, January 1890, p. 19 *et seq.*

† St Etienne, 1890, pp. 196–197.

‡ GRAND 'EURY, *loc. cit.*, p. 258.

Professor WILLIAMSON concludes that the addition to the primary vascular bundle (which is not separated into distinct lateral bundles) must have arisen through a conversion into vessels of the cells of the central or medullary parenchyma by a centripetal process of development.*

Outside of this primary bundle we have the *exogenously* developed centrifugal xylem, composed of small scalariform tissue, radially disposed, between which run the medullary rays. In the specimen figured by WILLIAMSON,† of which the above is a short description, the primary xylem forms a circular band, 4·5 mm. thick, while the secondary xylem is 20 mm. thick.

The *cortex* consists of several layers; the innermost is composed of small-celled parenchyma. Outside of this is a broad zone of very uniform parenchyma, composed of small cells passing externally into another zone of uniform tissue, but composed of cells with thicker walls and of larger size. This is enclosed in the usual zone of prosenchyma, which in the arborescent stems attained to a considerable thickness from increase by age.‡

The *leaf bundles* in *Lepidodendron Wunschianum* offer no peculiarity. They spring from the outer surface of the primary bundle, and pass out in an upward course to the leaves.

It is therefore clearly proved that *Lepidodendron* possessed an exogenous growth quite similar to *Sigillaria*, and especially like that described in *Sigillaria spinulosa*, where the secondary xylem is not separable into component bundles, as in the *Sigillaria elegans* described by BRONGNIART.

A further difference has been presumed to exist between the primary bundles of *Lepidodendron* and *Sigillaria*, as illustrated by BRONGNIART, RENAULT, and GRAND 'EURY, in that the primary bundle in *Lepidodendron* forms a continuous closed ring, while in *Sigillaria* it is formed of a number of separate bundles arranged in a circle; but in the figure of *Sigillaria spinulosa*, given by SOHNS-LAUBACH,§ taken from a specimen presented to the late Professor WILLIAMSON by M. RENAULT, at one part of the primary bundle circle, these bundles are shown to be united, and form part of a ring quite similar to that seen in *Lepidodendron Harcourtii* or *Lepidodendron Wunschianum*. As illustrating this point more fully, I figure here the central portion of a transverse section of another specimen of *Sigillaria spinulosa*,—for which I am also much indebted to Mons. RENAULT,—that shows still more clearly that the isolated primary bundles have evidently been separated from what appears to have been originally a closed vascular ring.

* "On the Organisation of the Fossil Plants of the Coal Measures," part x. p. 497, *Phil. Trans.*, part ii., 1880.

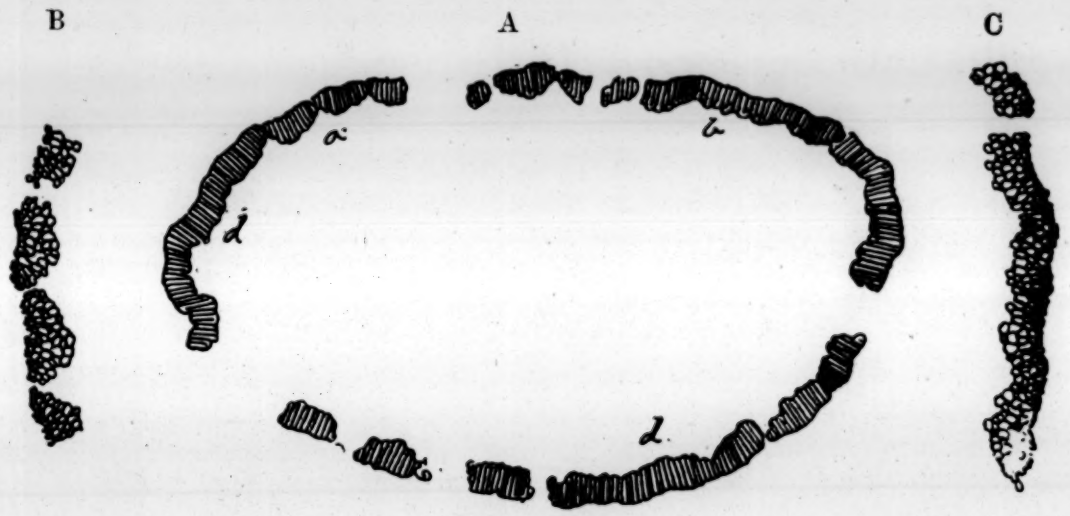
† *Loc. cit.*, *Mem.* x., pl. xiv. fig. 6.

‡ WILLIAMSON, "General Morphological and Histological Index to the Author's Collective Memoirs on the Fossil Plants of the Coal Measures," part ii. p. 16, *Mem. and Proc. M'ter. Liter. and Phil. Soc.*, session 1892–93, 1893. *Note.*—*If* the Halonial branch referred here to *Lepidodendron Wunschianum* belongs to this species, then the plant is a *Lepidophloios*. This does not, however, in the slightest alter the position of the question under discussion as to the occurrence of an exogenous zone in *Lepidodendron*, as it is present in *Lepidodendron selaginoides* and other species.

§ *Fossil Botany*, English edit., p. 253, fig. 29, 1891.

One is led, therefore, to believe that the primary vascular bundle in *Sigillaria* was a closed vascular ring quite similar to that found in *Lepidodendron*; but that in *Sigillaria*, by the increase in the size of the pith core (which we know in *Lepidodendron* increased in size up to a certain period), the original closed bundle became broken up and carried outwards by the increasing volume of the pith, instead of persisting as a closed ring, as in *Lepidodendron*.

That the presence or absence of a secondary xylem is no proof of a Cycadaceous affinity is abundantly shown in the case of *Lepidodendron*, which possesses a secondary



Sigillaria spinulosa.

A, Primary vascular zone × 8; B, part marked *a* in fig. A, showing isolated bundles × 17; C, part marked *b* in fig. A, showing union of primary bundles × 17 and formation of the *Lepidodendron* type of primary xylem. This is further seen at *d* in fig. A.

xylem, and whose fruit is in the form of cones, bearing macro- and microspores. But this phenomenon of exogenously developed xylem is not entirely unknown amongst recent Cryptogams, for "a secondary thickening, starting from a cambial layer, which produces secondary wood and secondary cortex, is found (only) in the *Isoetes*.*

* A. DE BARY, *Comparative Anatomy of the Vegetative Organs of the Phanerogams and Ferns*, English edit., p. 623, 1884.

Note.—On the structure of *Sigillaria* and *Lepidodendron* the following additional works may be consulted:—H. GRAF ZU SOLMS-LAUBACH, *Fossil Botany, being an Introduction to Palæophylology from the Standpoint of the Botanist*, English translation, Oxford, 1891. Sir WM. DAWSON, "On the Conditions of the Deposition of Coal, more especially Illustrated by the Coal Formation of Nova Scotia and New Brunswick," *Quart. Journ. Geol. Soc.*, May 1866, vol. xxii. p. 95. *Ibid.*, *Geological History of Plants*, New York, 1888. *Ibid.*, *Acadian Geology*, 2nd edit., London, 1868. Sir W. HOOKER, "On the Vegetation of the Carboniferous Period, as Compared with that of the Present Day," *Mem. Geol. Survey of Great Britain*, vol. ii. part ii. p. 387, 1848. BINNEY, *Quart. Journ. Geol. Soc.*, 1862, vol. xviii. p. 106. *Ibid.*, *Phil. Trans.*, vol. clv. pp. 579 and 591, 1865. *Ibid.*, *Palæont. Soc.*, vol. xxix. pp. 97 and 147, 1875. P. VAN TIEGHEM, "Sur quelques points de l'anatomie des cryptogames vasculaires," *Bull. Soc. Bot. de France*, vol. xxx. p. 169, 1883. *Ibid.*, *Traité de Botanique*, 1884. WILLIAMSON, *Memoirs*, "On the Organisation of the Fossil Plants of the Coal Measures," in *Phil. Transactions*, No. ii. 1872; No. iii., 1872; No. ix., 1878; No. x., 1880; No. xi., 1881; No. xii., 1881; No. xvi., 1889. *Ibid.*, "General, Morphological, and Histological Index to the Author's Collective Memoirs on the Fossil Plants of the Coal Measures," part ii., *Mem. and Proc. M'ter. Liter. and Phil. Soc.*, vol. vii., series 4, session 1892–93, 1893. *Ibid.*, "On the Light thrown upon the Question of Growth and Development of the Carboniferous Arborescent Lepidodendra by a Study of the Details of their Organisation," *Mem. and Proc. M'ter. Liter. and Phil. Soc.*, session 1894–95, series 4, vol. ix., 1895.

Fructification.

Goldenberg was the first to refer to *Sigillaria*, as its fructifications contain cones which he found associated with their stems. These he described in his "Flora Saræpontana Fossilis" in Heft. i. (1855) and Heft. ii. (1857). The cone he figures on pl. x. fig. 1 he thought might belong to *Sigillaria tessellata*, and that on pl. x. fig. 2 to *Sigillaria intermedia*; and for these fructifications Schimper founded the genus *Sigillariostrobus*.* Some other portions of Sigillarian cones are shown by Goldenberg on his pl. b figs. 18–25, and pl. iv. fig. 3 (p. 1, Heft. ii.).

The specimens figured by Goldenberg consisted of fragments: that given on pl. b fig. 18 exhibits a portion of a cone, some of the bracts of which have been removed, causing the thick axis to be seen. Figs. 19–25 show the more minute details of the structure of the bracts, but one cannot learn from them any accurate knowledge as to the structure or position of the sporangia.

The figure on pl. iv. fig. 3 shows the basal part of a cone with a portion of its supporting pedicel. Remains of two grass-like leaves lie on each side of the specimen, and are probably the remains of the foliage of the parent plant.

The specimens given on pl. x. figs. 1–2 show the upper portions of the cones. Enlarged drawings of the lanceolate bracts, with their rhomboidal expanded base, on which the spores are seen, are added. When these cones were referred to the *Sigillariæ* the only ground for the assumption was their association with *Sigillarian* stems—quite insufficient data for such important conclusions; and for thirty years their systematic position remained uncertain, till, in 1884, Mons. Zeiller announced the discovery of specimens agreeing with the cones referred to *Sigillaria* by Goldenberg, on the pedicel of one of which the Sigillarian leaf-scars were seen.

This discovery confirmed the conclusions arrived at by Goldenberg as to his cones being the fructification of *Sigillaria*.†

In 1884 Zeiller gave a preliminary description of these interesting specimens, in which he says:—"It is impossible to discover any trace of a sporangium in which the macrospores were contained; the position which they occupy, grouped most frequently at the base of each bract, only permit it to be supposed, with considerable probability, that they had been inclosed in the fold which is present in the wedge-shaped basal portion of the bract, and covered by a tissue, from whose destruction they had been set free, similar to what occurs to-day in *Isoetes*. The affinity indicated by Goldenberg (that the *Sigillaria* were an arborescent form of *Isoetes*‡) appears to me, then, well founded."§

* *Traité d. paléont. végét.*, vol. ii. p. 105, pl. lxvii. figs. 12–24, 1870.

† Feistmantel, in 1876, under the name of *Sigillariostrobus Goldenbergi*, appears to include all the Sigillarian cones described by Goldenberg, but the accuracy of this course is open to serious doubt. *Vers. d. böhmischen Kohlen-Ablager*, iii. Abth. p. 31, Cassel, 1876.

‡ Goldenberg, *Flora Saræpont. foss.*, Heft. i. p. 25.

§ *Comptes rendus*, 30th June 1884.

All the spores which M. ZEILLER observed were *macrospores*; but, as he points out, even had *microspores* been present in the specimens he examined, their mode of preservation was such that, from the minute size of *microspores*, it would have been impossible to observe them.

In a subsequent communication ZEILLER describes and figures in detail his Sigillarian cones from the coal field of Nord and Pas-de-Calais.*

Of the various cones he describes and figures, the most interesting is *Sigillariostrobus Tieghemi*, Zeiller,† for the pedicel of this specimen shows the true *Sigillaria* leaf-scar. The determination of the Sigillarian affinity of this specimen carries with it all the other examples he describes, as well as those formerly described by GOLDENBERG.

Two figures of *Sigillariostrobus Tieghemi* are given, both of which show the basal portion of the cone borne on a stout leafy peduncle, which increases in width as it approaches the base of the cone. On the lower part of the peduncle of fig. 1 pl. xi. the acicular leaves have been removed, showing the stem to be flexuously ribbed and transversely barred beneath the characteristic Sigillarian leaf-scar. Each leaf is provided with a medial nerve, contained between two very close and parallel longitudinal folds, whose origin is marked by a prominent point, corresponding to the two lateral cicatricules which lie one on each side of the vascular cicatricule. This cone may possibly be referable to *Sigillaria polyploca*, Boulay, but certainly in this identification is wanting.

At the summit of the peduncle the leaves assume the form of bracts. They are inserted obliquely on the axis, single nerved, oval lanceolate, acute at the point, and suddenly contracted at the base. Between the bracts, and still in position, one can see the macrospores sometimes in great numbers, about 2 mm. in diameter, perfectly smooth, but marked by a triradiate ridge, the arms of which are connected by a semicircular line. The macrospores are disseminated without order. "One cannot see any trace of an envelope on the bracts, but on some of them can be discovered above the line which separates the limb from the claw a slight arched line, which might well correspond to the attachment of a membrane which originally covered the bodies in question."‡ M. ZEILLER also points out that these "bodies" much resemble the macrospores of *Isoetes*. That these "bodies" are macrospores seems to be clear beyond all doubt. Structurally they are identical with the macrospores of the cones of *Lepidodendron*.

In all the cones ZEILLER examined, he only found these large spores. SCHIMPER thought he saw sufficient difference in the size of the spores figured by GOLDENBERG to treat them as macrospores and microspores,§ though he was not certain of this interpretation. He gives as the size of his macrospores 1·5 to 2 mm., and the microspores as 1 mm., in

* "Cones de Fructification de Sigillaires," *Ann. d. Scienc. Nat.*, 6e. sér., "Bot.," vol. xix. pp. 256–280, pls. xi.–xii., 1884. See also ZEILLER, *Flore foss. Bassin houil. d. Valenciennes*, pp. 591–608, pls. lxxxix.–xc., 1886 and 1888.

† *Ann. d. Sc., loc. cit.*, pl. xi. figs. 1, 1*a*, 4, 4*a*, 4*b*.

‡ *Loc. cit.*, pp. 264–265.

§ SCHIMPER, *Traité d. paléont. végét.*, p. 105.

diameter; but, as far as I am aware, these measurements are not derived from GOLDENBERG's original specimens, but from his figures, which one can scarcely regard as a reliable source for such minute measurements.

Although microspores have not yet been observed in the cones of *Sigillaria*, it does not follow that they are *isosporous*; for the minute size of microspores, either in recent or fossil Lycopods, would make their detection with certainty very difficult in the only state of preservation in which the Sigillarian cones are known to us.

ZEILLER further suggests that perhaps certain cones bore macrospores, others microspores, which may account for the absence of spores between the bracts of his *Sigillaria nobilis*.* This condition in the case of *Sigillariostrobus nobilis* may be equally explained, and I think more probably, by the state of maturity at which the cone had arrived before mineralisation took place: the spores might have been shed at maturity or been imperfectly developed. Several of my specimens show no indications of macrospores. This, of course, is only a suggestion, but it is equally valid to supposing the cone contained only microspores. From certain appearances presented by a small fragment of a Sigillarian cone from the Kilmarnock Coal Field, I am inclined to think that the cones of *Sigillaria* were *heterosporous*, though I cannot speak definitely on this point, but on a subsequent page I give the evidence on which I have formed this opinion.

Mons. ZEILLER refigures and describes the specimens to which reference has already been made in his complete work on the Carboniferous Flora of the Valenciennes Coal Field,† and adds there the figure and description of another species, the *Sigillariostrobus Crépini*.‡

In this the bracts are arranged on the axis in a gentle spiral, or are verticillate. This character does not yet appear to be clearly determined. In form the bracts are elongate rhomboidal, and divided into a limb and claw. The claw portion of the bract bends downwards, but the limb is directed upwards; thus a knee-like angle is formed where the limb and the claw of the bract merge together. The margins of the bract are bent up on each side of the medial nerve, and form a sort of sack or spoon-like structure; this is sometimes empty, but is occasionally occupied by an ovoid body. Mons. ZEILLER suggests that these ovoid bodies may be microspore sporangia. They are too large to be macrospores, and had they been sporangia containing such, the probability is that some of the remains of the macrospores would have been preserved. These "ovoid bodies" are further said to be more or less enveloped by the bract which bears them; frequently only their edge can be seen, and their margins are often imperfect on account of portions of the structure, which appears to have been very delicate, having adhered to or been imbedded in the matrix.

* *Loc. cit.*, p. 267, pl. xii. figs. 1, 2, 2a.

† *Études des Gites Mineraux de la France, Bassin houiller de Valenciennes. Flore fossile du Bassin houiller de Valenciennes*, Paris, plates i.–xciv., 1886, text, 1888.

‡ *Ibid.*, p. 605, pl. lxxvii. figs. 2–3.

From the structures here described, the spoon-like form of the bract, and the mode of attachment of the "ovoid body," which appears from the enlarged figures to be attached to the "knee" of the bract, I am afraid this cone cannot be placed in the genus *Sigillariostrobus*; and especially, in the light of the specimen given on my pl. ii. fig. 3, ZEILLER's cone, if my interpretation of it is correct, would require a new genus to be created for its reception.

One frequently finds on stems of certain *Sigillariæ* curious scars, occasionally placed in the hollow between the ribs, sometimes on the ribs and in the clathrate forms, inserted between the leaf-cushions. These scars form a girdle round the stem, often of little width, but occasionally of greater vertical extent. These are the scars left by the fallen cones. In the majority of cases the cones appear to have been pedicellate;* but in some members of the Clathrarian section, as on *Sigillaria* (*Ulodendron*) *discophora*, and *Sigillaria* (*Ulodendron*) *Taylori*, the cones were sessile, and from the pressure of the base of the cone on the bark, a cup-like depression was formed, which, after the cone had fallen, increased in size with the increase of girth of the stem, in the same way that we see our initials cut in the bark of a tree enlarge with the growth of the trunk.

The mode of arrangement and disposition of these cone-scars on the stems of *Sigillariæ* afford important specific distinctions. Some have supposed these scars were due to adventitious roots, but RENAULT has shown that small branchlets, bearing leaf cicatrices, were attached to such scars on a specimen he describes from the Comentry Coal Field,† so therefore the aerial-root hypothesis must be entirely abandoned. When we add to this the discovery of pedicellate Sigillarian cones, the dimensions of whose stems agree with that of the scars in question, the chain of evidence in favour of these curious scars frequently found on the stems of *Sigillaria*, being the scars left by the fall of deciduous pedicellate cones, seems complete.

Some other writers than those already referred to have figured cones which they believed to be those of *Sigillaria*. Of some of these, with only the figures and descriptions to guide one in forming an opinion of their systematic position, there must rest considerable doubt as to the accuracy of placing them in *Sigillariostrobus*.

Mr BINNEY gives a woodcut of a fossil which he regards as the fructification of *Sigillaria*,‡ but neither his description nor figure show any evidence that his fossil had any connection with *Sigillaria*, much less that it was a cone.

In the *Versteinerungen der böhmischen Kohlen-Ablagerungen*, part iii. pp. 32–33, 1876, FEISTMANTEL describes two species of *Sigillariostrobus*—*S. Cordai*, pl. xi. fig. 4, and *S. Feistmanteli*, pl. xi. figs. 1–3. These cones, from the fragments figured, must have been of very large size, much larger than any we can definitely refer to this genus. In no case is the complete bract shown, only the basal expanded portion. One has considerable difficulty in accepting, without great reservation, the systematic position

* See also KIDSTON, "On *Sigillaria Brardii*, Brongt., and its Variations," *Proc. Roy. Phys. Soc.*, vol. xiii. p. 233.

† *Flore foss. Bassin houil. d. Comentry*, pp. 540–541.

‡ *Phil. Trans.*, 1865, p. 595, fig. 6.

assigned by FEISTMANTEL to these fossils. I cannot think that the specimens figured as *Sigillariostrobus Feistmanteli* (*loc. cit.*, pl. xi. figs. 1–2) are specifically identical with that given by GOLDENBERG (*Flora Saræp. foss.*) on his pl. B figs. 18–25.

GRAND 'EURY gives the restoration of a cone that he names *Sigillariostrobus rugosus*,* and which he believes is referable to *Sigillaria lepidodendrifolia*, Brongt. One would like, however, to be in possession of the original data from which the figure is produced. According to Mons. ZEILLER, the specimen from which the restoration was made is very incomplete, and badly preserved in the upper portion.† The *Sigillariostrobus mirandus*, Grand 'Eury,‡ of which the figure also appears to be a restoration, seems to be of even more doubtful Sigillarian affinity.

The stem of Sigillaria seems to have very rarely branched, and never to have produced the much dichotomised ramification so characteristic of the *Lepidodendra*. In some cases the *Sigillarian* stem appears to have been a narrow conical trunk, as shown in GOLDENBERG'S § and other figures.‖ GOLDENBERG'S stem was over 18 feet in height, and must, therefore, have attained considerable age. We can easily imagine on such a specimen the foliage would be confined to the upper portion, it having fallen from the lower and older part of the tree. That the leaves were articulated, and shed after a shorter or longer interval, is, I think, indicated by the clearly defined leaf-scar they leave behind them. If the leaves were articulated, as I believe they were, this may account for the rarity of specimens being found with their foliage attached.

All *Sigillariæ* had not, however, these thick, short, cactus-like stems, for GOLDENBERG figures portions of two other stems which show little or no diminution in their girth in the parts which have been preserved.¶ In the Museum, Newcastle-on-Tyne, there are two very good decorticated stems of this type, and another in a similar condition in the Sunderland Museum.

The Sunderland Museum specimen is specially interesting, as it shows most beautifully how the ribs increase in number as the stem advances in age. The portion of the trunk which is preserved is about 6 feet 6 inches high. It is slightly "bottle-shaped" at the base, where it measures about 5 feet in circumference. In this part of the stem there are 29 broad ribs. About one-third up the stem many of these wide ribs bifurcate, and when about two-thirds from the top, there are 40 ribs, with a circumference of stem of about 3 feet. All these additional ribs have not, however, arisen by a division of the primary basal ribs, for about two-thirds from the base new ribs with narrow, pointed extremities** are inserted between the older ones; thus about 6 inches below the broken-over extremity there are 45 ribs, though the stem is considerably smaller in

* *Flore Carb. du Depart. de la Loire et du centre de la France*, p. 160, pl. xiv. fig. 4, 1877.

† *Ann. d. Sc. Nat.*, 6e. sér., "Bot.," vol. xix. p. 257.

‡ *Loc. cit.*, p. 160, pl. xiv. fig. 5.

§ *Loc. cit.*, pl. B fig. 13.

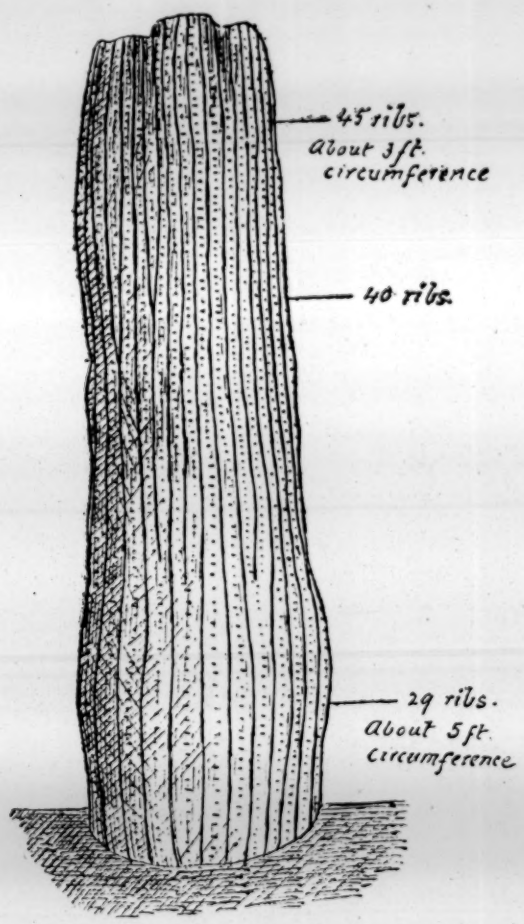
‖ GRAND 'EURY, *Geol. and Paléont. du Bassin houil. du Gard*, 1890, pl. xiii. figs. 7, 8, 9, 10.

¶ *Loc. cit.*, pl. x. fig. 6.

** See ZEILLER, *Flore foss. Bassin houil. d. Valenciennes*, pl. lxxviii. fig. 3 (*Sig. lævigata*). *Ibid.*, pl. lxxxv. fig. 1 (*S. tessellata*).

circumference than at the base. It is seen, then, that the number of leaves borne is not proportional to the size of the stem in girth. A transverse section of such a specimen (presuming the internal organisation was preserved) would show at the base a much less number of leaf traces than a section made immediately below the broken-over summit, and a few feet above its base.

In the upper portion of the trunk, five curious furrow-like grooves appear, and, deepening, give to the broken-over extremity of the tree the appearance as if it were about to divide into five branches. These grooves may be due to shrinkage, but they seem to be



Ribbed Stem of Sigillaria Height about 6ft 6 in
Sunderland Museum

too regular for that. I am not aware, however, that a dichotomising specimen of the ribbed *Sigillariæ* (Rhytidolepis section) has ever been discovered, though we know that the Favularia section occasionally, though apparently very rarely, produced dichotomised branches. Stur figures such an example.* Mr George Wild, Bardsley, has also shown me a very fine example of a dichotomously branched stem of *Sigillaria tessellata* from the Diamond Shaft, Bardsley Colliery, Lancashire. It was found in a bed of shale, about 50 feet below the New Mine (Middle Coal Measures).†

* *Die Culm Flora*, Heft. ii. pl. xxv. figs. 2–3 (*Sigillaria Eugenii*, Stur).

† Wild, "On Section of Shaft sunk through the Middle Coal Measures at Bardsley Colliery, and an interesting Discovery of Calamites," *Manchester Geol. Soc.*, Feb. 2, 1886.

As already mentioned, it is very rarely that one finds specimens of *Sigillariæ* with the leaves still attached. I only possess two with the foliage connecting with the stem. One, the *Sigillaria* (*Ulodendron*) *discophora*, König, sp., where the leaves are lanceolate, single nerved, about 1½ inch long and ⅛ inch wide; the other, *Sigillaria camptotænia*, Wood, sp., where the foliage is of the long grass-like type, about ⅛ inch wide; but they are so broken on my example that their full length cannot be determined, but they must have been of considerable length. From the descriptions and figures of several writers, this appears to have been the prevailing type of leaf among the *Sigillariæ*, and is, in fact, the *Cyperites bicarinata*, L. and H.,* though the description by these authors seems to be erroneous, in so far as they ascribe to the leaf two sub-lateral veins.

Though Sigillaria occurs in the Permian Formation, it is essentially a Carboniferous genus, and even here it is rare, except in the Upper Carboniferous. From the Lower Carboniferous of Britain I only know of two species,—the *Sigillaria* (*Ulodendron*) *Taylori*, Carr, and the *Sigillaria Youngiana*, Kidston.† The former belongs to the Clathrarian section, and occurs in both the divisions of the Lower Carboniferous; and the latter, belonging to the Rhytidolepis section, is only known by a single specimen, which was discovered in the Carboniferous Limestone Series of Scotland by Dr JOHN YOUNG of the Hunterian Museum, Glasgow, and is the only example of a ribbed Sigillaria I have seen from Lower Carboniferous rocks. In the Middle Coal Measures the *Sigillariæ*, especially the ribbed forms, appear to attain their maximum period of development in Britain.

A prevalent idea seems to have taken possession of many geologists that every trunk found in Carboniferous rocks belongs to *Sigillaria*, and several trunks occurring in the Lower Carboniferous have been publicly announced as such; but in no case have any of these stems, so far as I have been able to trace, ever shown any characters, either on their outer surface or impressed upon the surrounding matrix, in support of this popular but erroneous belief. On the other hand I have in two cases been able to prove conclusively that certain large stems which occurred in the Calciferous Sandstone Series belonged to *Lepidodendron Veltheimianum*, Sternb. I possess a specimen which was sent to me as one of these so-called *Sigillaria*, but in receiving portion of the enveloping rock the leaf-scars of *Lepidodendron Veltheimianum* were clearly impressed upon it.‡ Between this impression and the ribbed core a layer of coaly matter was inserted. In certain *Lepidodendra* the bark of old stems splits into longitudinal clefts by the increase of the stem in girth, and it is the casts of such fossils which give rise to these so-called *Sigillaria*. Sir WILLIAM DAWSON called attention to these old irregularly ribbed stems of *Lepidodendron* in his "Report on the Fossil Land Plants of the Lower Carboniferous and Millstone Grit Formations of Canada," in 1873, when he pointed out

* *Fossil Flora*, vol. i. pl. xliii. figs. 1–2.

† *Proc. Roy. Phys. Soc.*, vol. xii. p. 261, pl. vi. figs. 2, 2*a*, 1894.

‡ From the left bank of the Water of Leith, a little above Spylaw House, Colinton, Midlothian, Reg. Nos. 67 and 68.

how these longitudinally furrowed stems simulate the ribbed character of *Sigillaria.** A similar warning has been given by the late Professor WILLIAMSON† in regard to the Arran stems, of which some published accounts refer them to the *Sigillariæ*, although all the evidence in the case is diametrically opposed to such a supposition.

DESCRIPTION OF SPECIMENS.

Sigillariostrobus, Schimper.

1870. *Sigillariostrobus*, Schimper. *Traité d. paléont végét.*, vol. ii. p. 105.
1888. *Sigillariostrobus*, Zeiller. *Flore foss. Bassin houil. d. Valenciennes*, p. 391.

Description.—Cones borne on deciduous peduncles, with few leaves or the leaves reduced to bract-like structures, cylindrical, composed of a ligneous axis bearing spirally arranged sporiferous bracts. Bracts rhomboidal, acute, or long-lanceolate, acute with an expanded rhomboidal base, caducous. Macrospores contained in the hollow inflated base of the bract, 0·75 mm. to 2·00 mm. in diameter, smooth, or the surface apiculate; under surface marked by three ridges radiating from a central point (microspores probably present, but their existence not yet definitely proved).

Whether the cones of *Sigillaria* bore microspores as well as macrospores is at present not conclusively settled, and in an earlier part of this paper reference has been made to the opinions expressed on this point by Mons. ZEILLER.‡ I am, however, strongly of opinion that the *Sigillariæ* were heterosporous, and this view is very much founded on the specimen figured on pl. ii. fig. 1.

This fragment shows a small portion of a Sigillarian cone which has apparently been split longitudinally; the upper part of the bracts being embedded in the matrix, they only show their basal extremities exposed on the surface of the rock. The part which came off this fossil must have contained the axis and other half of the cone, but this unfortunately has not been found.

In examining this specimen the first point that arrests one's attention is the great difference in the size of the bases of the lower and upper bracts. The bases of the lower bracts marked *a*, pl. ii. fig. 1, are about 3 mm. wide, while the upper ones marked *b*, *c*, *e*, on the figure are about 5 mm. wide. The bases of the smaller and lower bracts are covered with small, smooth macrospores (fig. 1*a* × 4) about 0·75 mm. in diameter (fig. 1*aa* × 8). These lower bracts bear the macrosporangia.

The most interesting part of this fossil is, however, the upper bracts. These, as already stated, have larger bases, and are in an equally fine state of preservation. When their upper surface is examined under the microscope it is found to be distinctly

* *Geological Survey of Canada.* Montreal, 1873, p. 41.

† WILLIAMSON, "On the Organisation of the Fossil Plants of the Coal Measures," part x., *Phil. Trans.*, 1880, part ii. p. 494.

‡ *Ante*, p. 43.

granulated (fig. 1*c*). These granular roughnesses measure about 0·20 mm.,* and are covered by a thin cellular envelope, the size of the cells forming this layer being such that from three to four cells equal the size of the individual granular roughnesses. The cells are therefore about 0·05 mm. in their longest diameter (pl. ii. fig. 1*d*).

Some of these larger sporangia are seen in side view on pl. i. fig. 1*d d'*.

On the exposed surface of some of the bases of these larger upper bracts is a subrhomboidal mark with a central point, and below it the indication of a semicircular area (fig. 1*b*). This may represent the point of attachment of the bract to the axis of the cone, but of this I am not certain, as it might perhaps represent the part where dehiscence took place. This structure is well seen on fig. 1 at *e* and *b*.

It is from the examination of this specimen that I have come to the conclusion that most probably the cones of *Sigillaria* were heterosporous, for I do not see what other explanation can be given than that the granular structure is caused by contained microspores covered by a delicate cellular envelope. I do not claim for this interpretation of the fossil an absolute certainty, but dealing with bodies so minute as microspores† preserved in the manner of the specimen under discussion, I scarcely see how we can expect more conclusive evidence. This interesting specimen, which occurs on a small slab with a piece of the bark of *Sigillaria camptotænia*, Wood, sp. (but whose association may be merely accidental), was collected by Mr JOHN RORRISON from the "Major Coal," at No. 3 Pit, Springhill, Crosshouse, Ayrshire (Lower Coal Measures), and communicated to me by the Rev. D. LANDSBOROUGH, Kilmarnock, to both of whom I am much indebted.

Sigillariostrobus rhombibractiatus, Kidston, n. sp.

Pl. I. figs. 1–8 ; Pl. II. figs. 10–11.

Cone cylindrical, elongated, stalked, caducous, apex blunt ; bracts caducous, rhomboidal, single-nerved, with an acuminate point, finely ciliate on margin, placed spirally on axis. Pedicel without ribs, bearing distant lanceolate bract-like leaves, which become more numerous at the base of the cone, round which they are closely placed ; pedicel thickly covered with small acicular points. Macrospores large, rough, with small apiculi, and having a triradiate ridge on lower surface, 1·5 mm. to 2 mm. in diameter.

Remarks.—This species is represented by over a dozen specimens from Monckton Main Colliery, all of which have been collected by Mr HEMINGWAY. Of these the more interesting examples are here figured and described.

* If I am correct in regarding the cause of these "roughnesses" to be contained microspores—presumably they were still united in groups of four, as is frequently seen in the cones of *Lepidostrobus*—then the size of the individual microspores would be less than 0·10, for the thickness of the containing envelope adds to their apparent size.

† The size of the *microspores* in a cone of *Lepidostrobus* showing structure is 0·02 mm.

Description of Specimens.—The largest specimen I have is not so well preserved as those figured, but measures in complete length rather over 9 inches. This example, except in giving the complete length of the cone, is not of any special interest, the structural details being much better shown on other specimens. (Reg. No. 2242.)

Pl. I. fig. 1.

This fossil shows the lower portion of the cone, from the upper remaining part of which the bracts have been shed. The shedding of the bracts at maturity seems to be a characteristic of Sigillarian cones, and one of the distinguishing points between them and *Lepidostrobus.*

In some cases the slabs are thickly covered with bracts and portions of denuded axes. On fig. 1 pl. i. the bracts are spread out, and evidently the cone has attained to maturity, as macrospores occur on the surface of, and others are shadowed through, the substance of the bracts. At the base of the cone are the spirally arranged narrow setaceous bracts, quite dissimilar to the fruiting bracts. As far as can be observed, there is little transition between the narrow linear leaf-like bracts and the wide rhomboidal sporangiferous bracts; but the former seem to cease suddenly when the sporangiferous bracts take their place. The pedicel is without ribs, but is covered with closely placed small thorn-like points.

Pl. I. fig. 2.

This example shows more distinctly the rhomboidal acuminate form of the bracts still in position. The stem is faintly striated, but cannot be said to be ribbed, and like that in all the other specimens shows also the small apiculi. The bracts show well their ciliate margin, but the cilia are too small to show clearly on figures given in natural size —in which the matrix is represented—but they are very distinctly observable on the originals when examined with a hand lens. A few fragments of the grass-like leaves, which are probably the foliage of the plant which bore these cones, are seen on the slab. These generally accompany the cones.

On the pedicel, and especially at the base of the cone, are seen the lanceolate, leaf-like bracts, which do not seem to leave, as far as I have been yet able to observe, any distinct scar, but their base appears to be carried down the stem in a decurrent manner.

Pl. I. fig. 3.

This specimen does not show the individual bracts so well as the other figured examples, but I believe it is referable to the same species. The pedicel, which is long, is not so thick as on the other specimens. One cannot help thinking that such pedicellate cones must have been pendant—such slender pedicels could scarcely have supported the cone in an upright position.

Pl. I. fig. 4.

The bracts have almost entirely been shed from this example, only a few at the base of the cone remaining attached to the axis, which in this naked condition shows well by the scars of the fallen bracts their spiral arrangement. At *a* on fig. 4 is one of the lanceolate bract-like structures (shown enlarged at fig. 4*a*), which congregate round the base of the cone. The pedicel is covered with the small apiculi, which are also indicated on fig. 4*a*.

The slab on which this cone occurs has many isolated bracts scattered over its surface, some of which, as well as a fragment of another cone and pedicel, are shown in the figure.

Pl. I. fig. 5.

Here two cones are seen to spring from the top of the same pedicel. This is probably an abnormal condition. The cone to the right is complete, and shows well its narrow cylindrical form. Its size, however, is probably less than it would have been under more normal circumstances. The cone to the left seems to have its upper part broken over.

Pl. I. figs. 6, 6*a*, and 7, 7*a*.

These figures show two isolated bracts, with portions enlarged to illustrate the delicate cilia which occur on the margins. These cilia are generally simple and more numerous and longer on the expanded portion of the bract than on its upper region. The bracts occasionally exhibit a slight keel, and are single nerved.

Pl. I. fig. 8.

This exhibits very beautifully the small thorn-like apiculi on the pedicel, not only when seen from above, but also in profile at the margin of the pedicel. An enlarged drawing is given at fig. 8*a*. These minute thorn-like structures are extremely numerous, and occur on all the specimens which show the pedicel. In form and structure they have the appearance of very minute thorns. On fig. 8 at *b* a fragment of one of the bract-like leaves occurs.

Pl. II. fig. 10.

Here there is a tolerably complete cone, with portion of its pedicel. This has a faint indication of ribs, but this may be due to a slight collapsing of the pedicel during mineralisation, as it is absent from the other specimens. One of the bract-like leaves is seen on the pedicel a short distance below the base of the cone.

Pl. II. fig. 11.

Two of the macrospores magnified 10 times.* These vary in size from 1·5 mm. to 2 mm. Their upper surface is covered with short, blunt apiculi, and their lower surface bears a triradiate ridge, around which the spore is generally smooth. This triradiate ridge is formed by the mutual pressure of the other three spores, which were developed within the same mother cell, and the smooth parts between the arms of the triradiate ridges represent the points where the macrospores were in contact with each other. I have specimens of somewhat similar macrospores from another source, still united in groups of four.†

Affinities of Species.

Sigillariostrobus rhombibractiatus, in the form of bracts, shows a slight approach to *Sigillariostrobus Tieghemi*, Zeiller,‡ but is easily separated from the latter species by its much more truly rhomboidal bracts, with ciliate margin and the apiculate macrospores.

The pedicel of *Sigillariostrobus rhombibractiatus* is also destitute of ribs, but is finely apiculate, with minute thorn-like points. In addition to these differences, Zeiller's cone is larger and more robust than our species.

Middle Coal Measures.

Locality.—Monckton Main Colliery, near Barnsley, Yorkshire.
Horizon.—Barnsley Thick Coal. (W. Hemingway.)

Sigillariostrobus ciliatus, Kidston, n. sp.

Pl. II. figs. 2–9.

Description.—Cone cylindrical, pedicellate. Bracts spirally placed on the ligneous axis, linear-lanceolate acute with expanded rhomboidal base, single nerved, margins thickly beset with short, stout cilia. Sporangia walls formed by the hollow basal portion of the bract. Macrospores large, about 1·5 mm. in diameter, apiculate, with triradiate ridge on under surface. Peduncle indistinctly ribbed, and roughened with very small thorn-like apiculi.

Remarks.—Of this cone I have seen a less number of specimens from Yorkshire than of *Sigillariostrobus rhombibractiatus*, but a single bract from the Forest of Wyre has

* These macrospores have a great similarity to those figured in the *Proc. Roy. Phys. Soc.*, vol. ix. p. 109, pl. iii. fig. 7 (Triletes vii.), but it is improbable that they belong to the same species, as macrospores of different species have evidently a great similarity in form and structure, as seen in the case of those belonging to the two cones described in this paper.

† Binney and Kidston, *Proc. Roy. Phys. Soc.*, vol. ix. p. 113, pl. v. figs. 16*d* and 16*e*, 1887.

‡ Zeiller, *Ann. d. Scienc. Nat.*, 6e. sér., "Bot.," vol. xix. pp. 263 and 266, pl. xi. figs. 1 and 4, 1884; also *Flore foss. Bassin houil. d. Valenciennes*, p. 593, pl. lxxxix. figs. 2–3, 1886 and 1888.

been sent me by Mr T. Crosbee Cantrill (pl. ii. fig. 7). All the Yorkshire examples come from Wooley Colliery, Darton, and for which I am entirely indebted to Mr W. Hemingway. The form of the bracts and the ciliate margins easily distinguish this from all other described species.

Description of Specimens.

Pl. II. fig. 2.

This example shows the base and the apex of the cone. Unfortunately the central part connecting these two portions was lost. It shows the peduncle with distinct ribbing on its upper region, but this is not so clearly seen as the peduncle is followed downwards.

Pl. II. fig. 3.

This small fragment is the most interesting of all the specimens, as it shows the form and structure of the macrosporangia. There is also another example which shows the sporangia (No. 2268), but it does not exhibit them so distinctly.

Two of the sporangia, marked *a* on fig. 3, are enlarged four times at fig. 3*a*. The sporangia are evidently formed by a hollowing of the basal portion of the bract, and its walls seem to be only the modified upper and lower surface of the bract, which again unite at the point indicated by the letter *a* (fig. 3*a*) to form the upper lanceolate extension of the limb. This structure is quite analogous to that found in *Isoetes*.

Pl. II. fig. 4.

This shows the basal portion of a cone. The bracts are removed from the upper part of the specimen which exhibits the axis. One of the bracts, by displacement, is seen to extend past the others at *b*, and thus shows distinctly its form. Numerous macrospores occur scattered among the bracts, two of which, magnified ten times, are drawn at fig. 9 pl. ii.

Pl. II. figs. 5, 6, 7, and 8.

These give figures of isolated bracts and portions of the margins enlarged to show the cilia, which vary a little in their size and regularity.

Middle Coal Measures.

Locality.—Wooley Colliery, Darton, near Barnsley, Yorkshire.
Horizon.—Barnsley Thick Coal. (W. Hemingway.)
Locality.—"Road Section," Cooper's Mill, Dowles Valley, Forest of Wyre.
Horizon.—(?). (T. Crosbee Cantrill.)

Sporangium.

Pl. II. fig. 12.

I give here, natural size, a sporangium containing numerous small smooth macrospores. This sporangium is larger than those of *Sigillariostrobus ciliatus*, while the macrospores are smaller and smooth, their size being about 0·75 mm. On another small piece of shale from the same locality (No. 1273) there are more or less perfect remains of about a dozen and a half of similar sporangia.

They may possibly be *Sigillarian*, but no remains occur along with them to throw any light on their affinities. Two of the macrospores are enlarged ten times at fig. 12*a* and *b*.

These sporangia are evidently similar in nature to those described by Lesquereux under the name of *Sporocystis*.*

Affinities of Sigillaria.

Goldenberg believed that *Sigillaria* represented an arborescent form of Isoetes,† and though at the time he expressed that opinion, satisfactory evidence in its support was wanting. Subsequent discoveries have, however, proved the accuracy of his suggestion. Two years after he stated these views on the affinities of *Sigillaria*, he said:—"The great agreement which the *Sigillaria* bespeak with our quillworts, as well in their internal structure as in their fructification, points to the decision that the *Sigillariæ* were nearly related to the *Isoetes*, and in the highest probability represented an arborescent form of these plants in bygone times."‡

The *Sigillariæ*, though they differ from *Isoetes* in their arborescent dimensions and in their fruit being in the form of pedicellate cones, show so great an agreement with them in the structure of their sporangia that their affinities with *Isoetes* is very close. Of course the presence of microspores in the cones of *Sigillaria* is not absolutely certain, but the evidence already accumulated points to their cones having been heterosporous.

Zeiller, in his interesting communication to which reference has been already made,§ says, when discussing the affinities of *Sigillaria*:—"In conclusion, the *Sigillariæ* appear to me to deserve to be considered as constituting in the *Lycopodiaceæ*, in some respects, an intermediate group between the *Lepidodendron* proper and the *Isoetes*, on account of the affinities which they present towards the *Isoetes* in the arrangement of their sporangia and probable mode of dissemination of the spores, and,

* *Coal Flora*, pl. lxix. figs. 11–14.
† *Flora Saræp. foss.*, Heft. i. p. 25, 1855.
‡ *Ibid.*, Heft ii. p. 1, 1857.
§ *Ann. d. Scienc. Nat.*, 6e. sér., "Bot.," vol. xix. p. 278, 1884.

on the other hand, with the *Lepidodendron* in the structure of the leaf-scars and the anatomical details of the stem.

M. ZEILLER further points out what he believes to be another point of difference between *Sigillaria* and *Lepidodendron*, viz., the stalked deciduous cones of *Sigillaria*, and it must be admitted that, as far as we know, no *Lepidodendron* had stalked deciduous cones; but neither were all Sigillarian cones stalked, for in *Sigillaria* (*Ulodendron*) *discophora*, König, sp., and *Sigillaria* (*Ulodendron*) *Taylori*, Carr, sp., the cones have evidently been sessile, and it was from the pressure of the bases of these deciduous sessile cones on the bark that the cup-like depressions were made. But further, in these the cones were borne in two vertical opposite rows, not in verticils, as in the majority of *Sigillariæ*.

Also in *Lepidodendron Veltheimianum*, Sternb., and *Lepidodendron Landsburgii*, Kidston,* the fructification consisted of sessile deciduous cones, disposed likewise in two opposite vertical rows, which produced the same cup-like depressions on the stem. The case of deciduous cones is, however, very exceptional in *Lepidodendron*, whereas in *Sigillaria* it seems to be the normal condition. In *Lepidophloios*, and probably in all the species, we find the fructification in the form of deciduous stalked cones. The most important distinction, however, between all these genera is the structure of the *sporangia*, and in this *Sigillaria* and *Lepidodendron* are essentially distinct; but the other structural points in which they differ cannot, of course, be ignored in any system of classification.

On the other hand, it must be pointed out that there do not appear to be any differences in regard to the internal organisation of the stems of *Lepidodendron* and *Sigillaria* which would enable us to separate the stems of these two genera from each other without the aid of additional characters. We may also place *Lepidophloios* in the same category, for we now know as a fact that the stem the late Professor WILLIAMSON described under the name of *Lepidodendron fuliginosum* is the stem of *Lepidophloios acerosus*, L. and H., sp.†

All these fossil Lycopods form a closely-connected group of genera,—*Lepidodendron* and *Lepidophloios* pointing their affinities in the direction of *Lycopodium* and *Selaginella*, and *Sigillaria* in the direction of *Isoetes*, but each one differing in some important respect from any recent genus.

Sigillaria Sol., Kidston, n. sp.

Pl. III. fig. 6.

Description.—Stem ribbed, furrows straight. Leaf-scars occupying about half the width of the ribs, sub-orbicular, lateral angles not prominent, upper margin slightly notched, lower margin rounded. Vascular cicatrice punctiform, slightly above centre of

* *Trans. Roy. Soc. Edin.*, vol. xxxvii. p. 338, pl. iii. figs. 9–10, 1893.

† LINDLEY and HUTTON, *Fossil Flora*, vol. i., pl. vii. fig. 1, pl. viii. (= *Lepidodendron acerosum*).

scar, and flanked by two very slightly outward-directed oval cicatricules. Immediately above the leaf-scar is a small transversely elongated little cicatricule, above which is a short transverse lunate depression, which is again surmounted by a larger straight or slightly lunate hollow, from each extremity of which arises a bent band of slightly curved oblong depressions, decreasing in size as they reach the centre of the rib, where the two halves of the semicircle meet. Outer surface of ribs otherwise smooth.

Remarks.—Pl. iii. fig. 6 shows the fossil natural size; and at fig. 6*a* one of the leaf-scars and the surmounting ornamentation is shown in outline, also natural size.

This specimen I received in 1888 from Mr C. Bradshaw, Sheffield, to whom my thanks are due. The species is evidently very rare, as I have not again met with the plant.

Middle Coal Measures.

Locality.—Kilnhurst Pit, Rotherham, Yorkshire.
Horizon.—Barnsley Thick Coal.

Sigillaria semipulvinata, Kidston, n. sp.

Pl. III. figs. 1–5.

Description.—Leaf-cushions almost contiguous, transversely rhomboidal, hexagonal or longer than broad; in transversely rhomboidal forms, lateral angles prominent, acute; in others, generally rounded. Lower margin rounded or truncate; extreme upper portion occupied by the leaf-scar. *Leaf-scar* occupying the entire apex of the cushion, rhomboidal, or transversely rhomboidal, lateral angles prominent; upper margin flatly arched or slightly notched; lower margin rounded, or rounded with slightly concave sides; cicatricules placed slightly above the centre, central punctiform, the two lateral oblong and slightly directed outwards. On the leaf-cushion are occasionally seen two faint rows of transverse lines, which descend obliquely from the base of the leaf-scar. Subcortical layer striated longitudinally.

Remarks.—Though this species varies in the proportional size of the leaf-cushion to the size of the leaf-scar, and in the cushion and leaf-scar being sometimes transversely expanded, or in the cushion being longer than broad, a definite character runs through all these variations which at once connect them as different conditions of the same species,—forms whose existence depends entirely on circumstances of growth, as shown by these variations sometimes occurring on the same individual.

Description of Specimens.

Pl. III. fig. 1.

This illustrates what I regard as the typical form of the species. A few of the leaf-scars and cushions are given in outline, natural size, at fig. 1*a*. This example shows a

younger state of the plant than any of the other specimens. The leaf-cushions are transversely rhomboidal, and the leaf-scar of a similar form occupies its upper portion. There is no part of the cushion above the leaf-scar, and in this *Sigillaria semipulvinata* differs from almost all the other species of the *Sigillaria Brardii* type. The lateral angle of the leaf-scar meets the margin of the leaf-cushion about half-way between its lateral angle and the apex of the cushion. The leaf-cushions are not strictly contiguous, but are separated by a slight interval; therefore, though the arrangement of the leaf-cushions is clathrate, still, as they do not rest on each other as in the typical *Clathraria*,* the species shows some relationship to the *Leiodermaria* section, where the leaf-scars are more or less distant.

Pl. III. fig. 2.

This shows an older form of the plant. The leaf-cushions are more distantly placed here than in fig. 1. There is also seen in fig. 2 a somewhat irregular longitudinal splitting of the bark in lines following the direction of the upward rows of leaf-cushions. This is evidently brought about by a fissuring of the bark caused by increase of the stem in girth, and it gives a *Favularia*-like appearance to the specimen which is directly derived from what was originally a *Clathrate* or *Leiodermarian* form.

Pl. III. fig. 3.

This shows still more clearly the *Favularia* arrangement of the leaf-cushions, and also illustrates the variations in their size. Two of the upper leaf-cushions and accompanying leaf-scars are given at fig. 3*a* × 2, and at 3*b* and 3*c*, natural size. Here the leaf-cushion is not much larger than seen on figs. 1 and 2, but at the lower part of this fossil (fig. 3) a considerable increase in the size of the leaf-cushion is observable (fig. 3*c*), in which increase the leaf-scar also participates. Two rows of faint transverse lines are present on the larger cushions (fig. 3*a*).

Pl. III. fig. 4.

This is apparently from a somewhat aged specimen. The leaf-cushions are practically contiguous, and show below the scar two rows of faint transverse lines (fig. 4*a*). The leaf-cushion here is fully longer than broad.

Pl. III. fig. 5.

This represents a similar condition to that shown at fig. 4. On the lower portion of the fossil the leaf-cushions are contiguous; on the upper part, more or less separated.

The study of this species shows the caution that is necessary in bestowing names on fragments of *Sigillaria*. If figs. 1 and 4 are compared, they look at first sight very

* As the leaf-cushions appear to become more distant with age, probably the younger conditions would represent true *Clathraria*.

distinct, but as seen from the specimens described above, beyond all doubt they only represent different ages of the same plant, and these differences are quite unworthy of any name to distinguish them as *varieties or forms.*

Sigillaria semipulvinata shows some similarity to *Sigillaria decorata*, Weiss,* and *Sigillaria subornata*, Weiss,† in both of which the leaf-scar is situated towards the top of the cushion.

In both of these species the leaf-cushion is much smaller and the leaf-scar of a different form, but the character which at once distinguishes *Sigillaria semipulvinata* from these two species is the position of the leaf-scar in regard to the cushion.

In *Sigillaria decorata*, Weiss, and *Sigillaria subornata*, Weiss, the lateral angle of the leaf-scar is on the same line with the lateral angle of the leaf-cushion which it joins, whereas in *Sigillaria semipulvinata* the lateral angle of the leaf-scar meets the upper receding boundary line of the cushion some distance above its lateral angle; or to put it otherwise, the lateral angles of the leaf-scar of *Sigillaria decorata* and *Sigillaria subornata* meet in the lateral angles of the leaf-cushion, whereas in *Sigillaria semipulvinata* they do not.

Distribution.—Though *Sigillaria semipulvinata* is rare, I have seen it from several localities, and my thanks are due to those whose names are mentioned in the appended list of distribution for specimens of this interesting *Sigillaria.*

Middle Coal Measures.

Locality.—Cinderford, Yorkshire.
Horizon.—(?). (Late Mr E. Tindal, Leeds.)
Locality.—Low Moor, Yorkshire.
Horizon.—Black Bed Coal. (Late Mr J. W. Davis, Halifax.)
Locality.—Wharncliffe, Woodmoor Colliery, Carlton, near Barnsley, Yorkshire.
Horizon.—Kent's Thick Coal. (Mr W. Hemingway.)
Locality.—Venture Pit, Burmantofts, Leeds.
Horizon.—Better Bed Coal. (Mr S. W. Bond.)
Locality.—Great Bridge, Worcestershire.
Horizon.— (?). (Mr C. Beale.)

* Weiss and Sterzel, *Die Sigillarien d. preuss. Steink. u. Rothl., Gebiete*, ii. "Die Gruppe der Sub-sigillarien," p. 207, pl. xxvii. fig. 105, 1893, *Abhandl. d. König. Preuss. geol. Landesanstalt*, neue folge, Heft. ii. Berlin.

† *Ibid.*, p. 209, pl. xxvii. fig. 106, 1893.

EXPLANATION OF PLATES.

Plate I.

Fig. 1. *Sigillariostrobus rhombibractiatus*, n. sp. Lower portion of cone and pedicel, showing axis denuded of bracts on upper part. Natural size. *Loc.* Monckton Main Colliery, near Barnsley, Yorkshire. *Hor.* Barnsley Thick Coal. Middle Coal Measures. Mr W. Hemingway, Collector. Reg. No. 2263. P. 51.

Fig. 2. *Sigillariostrobus rhombibractiatus*, n. sp. Lower portion of cone, showing the setaceous-like bracts at its base and on the upper part of the pedicel. Natural size. *Loc.* Monckton Main Colliery, near Barnsley, Yorkshire. *Hor.* Barnsley Thick Coal. Middle Coal Measures. Mr W. Hemingway, Collector. Reg. No. 2260. P. 51.

Fig. 3. *Sigillariostrobus rhombibractiatus*, n. sp. Portion of cone attached to its pedicel. Natural size. *Loc.* Monckton Main Colliery, near Barnsley, Yorkshire. *Hor.* Barnsley Thick Coal. Middle Coal Measures. Mr W. Hemingway, Collector. Reg. No. 2272. P. 51.

Fig. 4. *Sigillariostrobus rhombibractiatus*, n. sp. Cone mostly denuded of bracts, and exhibiting their spiral arrangement on the naked axis. Natural size. *Loc.* Monckton Main Colliery, near Barnsley, Yorkshire. *Hor.* Barnsley Thick Coal. Middle Coal Measures. Mr W. Hemingway, Collector. Reg. No. 2259. P. 52.

4*a*. Portion of pedicel of fig. 4, immediately below cone, showing leaf-like bract and small tubercles on pedicel; slightly enlarged.

Fig. 5. *Sigillariostrobus rhombibractiatus*, n. sp. Two cones attached to the same pedicel. Natural size. *Loc.* Monckton Main Colliery, near Barnsley, Yorkshire. *Hor.* Barnsley Thick Coal. Middle Coal Measures. Mr W. Hemingway, Collector. Reg. No. 2271. P. 52.

Fig. 6. *Sigillariostrobus rhombibractiatus*, n. sp. Isolated bract. Natural size. *Loc.* Monckton Main Colliery, near Barnsley, Yorkshire. *Hor.* Barnsley Thick Coal. Middle Coal Measures. Mr W. Hemingway, Collector. Reg. No. 2262. P. 52.

6*a*. Portion enlarged to show ciliated margin of bract.

Fig. 7. *Sigillariostrobus rhombibractiatus*, n. sp. A bract. Natural size. *Loc.* Monckton Main Colliery, near Barnsley, Yorkshire. *Hor.* Barnsley Thick Coal. Middle Coal Measures. Mr W. Hemingway, Collector. Reg. No. 2260. P. 52.

7*a*. Portion enlarged to show ciliated margin of bract.

Fig. 8. *Sigillariostrobus rhombibractiatus*, n. sp. Portions of two cones attached to their pedicels. Natural size. *Loc.* Monckton Main Colliery, near Barnsley, Yorkshire. *Hor.* Barnsley Thick Coal. Middle Coal Measures. Mr W. Hemingway, Collector. Reg. No. 1272. P. 52.

8*a*. Portion of pedicel enlarged to show small thorn-like hairs with which it is beset.

Plate II.

Fig. 1. *Sigillariostrobus*, sp. Portion of cone—*a*, *a'*, *a''*, *a'''*, sporangia, showing macrospores; *b*, *c*, sporangia; *d*, sporangia seen laterally. Natural size. *Loc.* No. 3 Pit, Springhill, Crosshouse, Ayrshire. *Hor.* Major Coal. Lower Coal Measures. Mr John Rorrison, Collector. Reg. No. 1573. P. 49.

1*a*. Macrosporangia, *a* on fig. 1, showing smooth macrospores, × 4.

1*aa*. Macrospore, × 8.

1*b*. Microsporangia (?), *b* in fig. 1, × 4.

1*c*. Surface of 1*b* more highly magnified to show granular appearance of surface.

1*d*. Surface of 1*b* still more highly magnified to show the cellular structure of the sporangium wall, smaller than, and extending over, the granular surface of the sporangium.

Fig. 2. *Sigillariostrobus ciliatus*, n. sp. Cone attached to its ribbed axis. Natural size. *Loc.* Woolley Colliery, Darton, near Barnsley, Yorkshire. *Hor.* Barnsley Thick Coal. Middle Coal Measures. Mr W. Hemingway, Collector. Reg. No. 1192. P. 54.

Fig. 3. *Sigillariostrobus ciliatus*, n. sp. Small fragment of cone split longitudinally, showing the sporangia containing macrospores. Natural size. *Loc.* Woolley Colliery, Darton, near Barnsley, Yorkshire. *Hor.* Barnsley Thick Coal. Middle Coal Measures. Mr W. Hemingway, Collector. Reg. No. 2144. P. 54.

3*a*. Two of the sporangia of figs. 3, × 4, showing the sporangia and contained macrospores.

Fig. 4. *Sigillariostrobus ciliatus*, n. sp. Portion of cone with numerous macrospores scattered amongst the bracts. Natural size. *Loc.* Woolley Colliery, Darton, near Barnsley, Yorkshire. *Hor.* Barnsley Thick Coal. Middle Coal Measures. Mr W. Hemingway, Collector. Reg. No. 2143. P. 54.

Fig. 5. *Sigillariostrobus ciliatus*, n. sp. Bract from cone. Natural size. *Loc.* Woolley Colliery, Darton, near Barnsley, Yorkshire. *Hor.* Barnsley Thick Coal. Middle Coal Measures. Mr W. Hemingway, Collector. Reg. No. 2267. P. 54.

Fig. 6. *Sigillariostrobus ciliatus*, n. sp. Bract. Natural size. *Loc.* Woolley Colliery, Darton, near Barnsley, Yorkshire. *Hor.* Barnsley Thick Coal. Middle Coal Measures. Mr W. Hemingway, Collector. Reg. No. 1177. P. 54.

Fig. 7. *Sigillariostrobus ciliatus*, n. sp. Bract. Natural size. *Loc.* "Road Section," Cooper's Mill, Dowles Valley, Forest of Wyre, Worcestershire. *Hor.* Middle Coal Measures. Mr T. Crosbee Cantrill, Collector. Reg. No. 2117. P. 54.

7*a*. Portion of base enlarged to show the cilia.

Fig. 8. *Sigillariostrobus ciliatus*, n. sp. Bract. Natural size. *Loc.* Woolley Colliery, Darton, near Barnsley, Yorkshire. *Hor.* Barnsley Thick Coal. Middle Coal Measures. Mr W. Hemingway, Collector. Reg. No. 1178. P. 54.

8*a*. Portion of margin enlarged to show cilia.

Fig. 9. *Sigillariostrobus ciliatus*, n. sp. Macrospores from cone, fig. 4 × 10. *Loc.* Woolley Colliery, Darton, near Barnsley, Yorkshire. *Hor.* Barnsley Thick Coal. Middle Coal Measures. Mr W. Hemingway, Collector. Reg. No. 2143. P. 53.

Fig. 10. *Sigillariostrobus rhombibractiatus*, n. sp. Cone showing setaceous bract on axis below cone. Natural size. *Loc.* Monckton Main Colliery, near Barnsley, Yorkshire. *Hor.* Barnsley Thick Coal. Middle Coal Measures. Mr W. Hemingway, Collector. Reg. No. 2273. P. 52.

Fig. 11. *Sigillariostrobus rhombibractiatus*, n. sp. Macrospores, × 10. *Loc.* Monckton Main Colliery, near Barnsley, Yorkshire. *Hor.* Barnsley Thick Coal. Middle Coal Measures. Mr W. Hemingway, Collector. P. 53.

Fig. 12. *Sporangium.* Natural size. *Loc.* Woolley Colliery, Darton, near Barnsley, Yorkshire. *Hor.* Barnsley Thick Coal. Middle Coal Measures. Mr W. Hemingway, Collector. Reg. No. 1669. P. 55.

12*ab*. Two of the smooth macrospores, × 10.

Plate III.

Fig. 1. *Sigillaria semipulvinata*, n. sp. Showing the leaf-cushions separated by only a very slight interval, and assuming a "Clathrate" disposition. Natural size. *Loc.* Cinderfield, Yorkshire. *Hor.* Coal Measures. Mr E. Tindal, Collector. Reg. No. 1408. P. 57.

1*a*. Outline figure of leaf-scars and cushions. Natural size.

Fig. 2. *Sigillaria semipulvinata*, n. sp. Showing the leaf-cushions separated by a very slight space, and assuming an arrangement characteristic of the *Favularia*. Natural size. *Loc.* Low Moor, Yorkshire. *Hor.* Black Bed Coal. Middle Coal Measures. The late Mr J. W. Davis, F.G.S., Collector. Reg. No. 1410. P. 58.

2*a*. Leaf-scar and cushion, × $1\frac{1}{2}$.

Fig. 3. *Sigillaria semipulvinata*, n. sp. Showing the same characters as the last, but the cushions on the lower part of the specimen are much enlarged. Natural size. *Loc.* Low Moor, Yorkshire. *Hor.* Black Bed Coal. Middle Coal Measures. The late Mr J. W. Davis, F.G.S., Collector. Reg. No. 1409. P. 58.

3*a* and 3*b*. Leaf-scars and cushions. Natural size.

3*c*. Leaf-scar and cushion, × 2.

Fig. 4. *Sigillaria semipulvinata*, n. sp. With large cushions arranged as in "Clathraria." Natural size. *Loc.* Great Bridge, near Dudley. *Hor.* Middle Coal Measures. Mr C. Beale, Collector. Reg. No. 832. P. 58.

4*a*. Leaf-cushion and scar, × 1½.

Fig. 5. *Sigillaria semipulvinata*, n. sp. Showing large cushions separated by a slight interval on the upper part of specimen, but touching each other on lower portion of the fossil. Natural size. *Loc.* Wharncliffe, Woodmoor Colliery, Carlton, near Barnsley, Yorkshire. *Hor.* Kent's Thick Coal. Middle Coal Measures. Mr W. Hemingway, Collector. Reg. No. 2258. P. 58.

5*a*. Leaf-scar and cushion. Natural size.

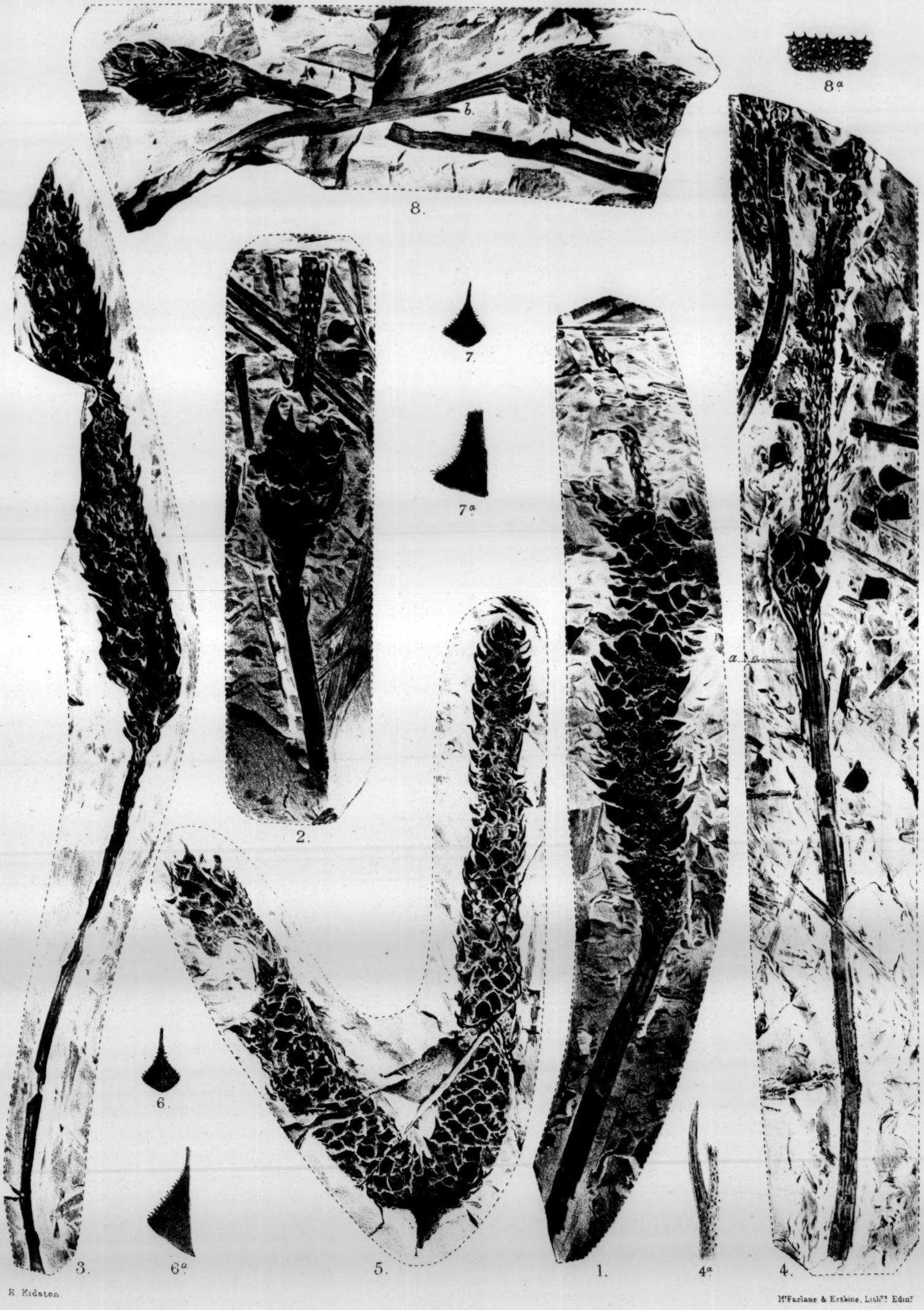
Fig 6. *Sigillaria sol.*, n. sp. Natural size. *Loc.* Kilnhurst, near Rotherham, Yorkshire. *Hor.* Barnsley Thick Coal. Middle Coal Measures. Mr C. Bradshaw, Collector. Reg. No. 1407. P. 56.

6*a*. Leaf-scar and ornamentation on rib above it. Natural size.

Note,—Pl. i. figs. 1, 2, 3, 4, 5, 8; pl. ii, figs. 1, 2, 4, 10; pl. iii. figs. 1, 2, 3, 4, 5, 6, are from photographs; all the other figures and enlargements are from drawings. All the original specimens are in the author's collection.

Note.—Since this paper was written, a further contribution on the affinities of *Sigillaria* has appeared from the pen of Mons. R. Renault in "Bassin houil. et perm. d'Autun et d'Epinac." Fasc iv. *Flore foss.*, deux partie, 1896, pp. 239–245 (Aug. 1897).

KIDSTON ON THE FOSSIL FLORA OF THE YORKSHIRE COAL FIELD. (SECOND PAPER). PLATE I.

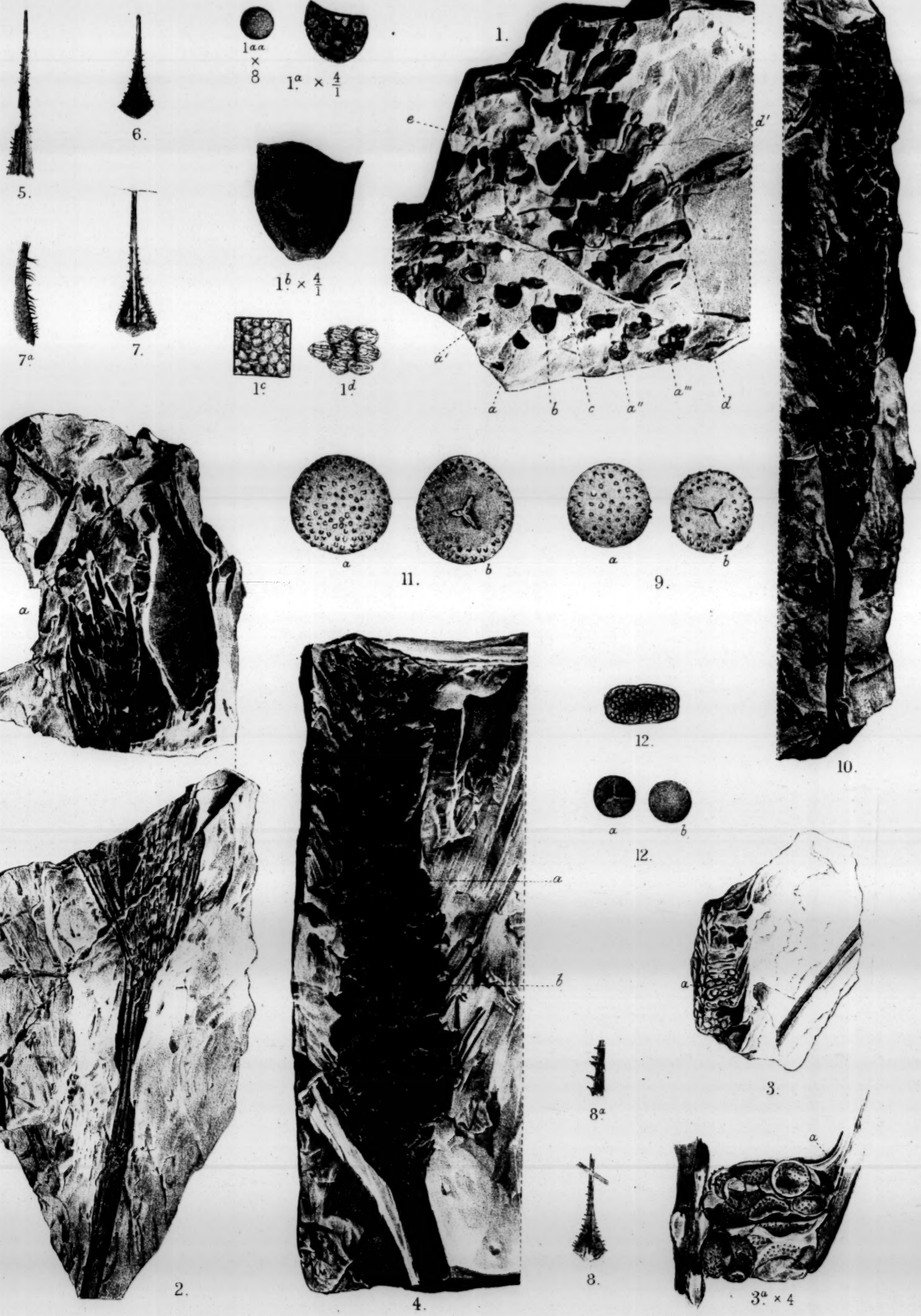


R. Kidston.

M'Farlane & Erskine, Lith.rs Edinr.

Figs. 1–8. SIGILLARIOSTROBUS RHOMBIBRACTIATUS. Kidston, n. sp.

KIDSTON ON THE FOSSIL FLORA OF THE YORKSHIRE COAL FIELD. (SECOND PAPER). PLATE II.

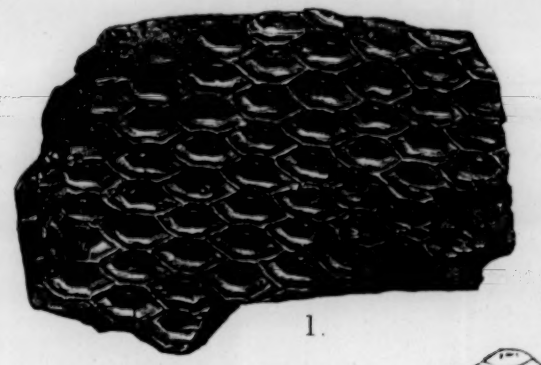


R. Kidston.

McFarlane & Erskine, Lith^rs Edin^r

Fig. 1. SIGILLARIOSTROBUS, sp. Figs. 2–9. SIGILLARIOSTROBUS CILIATUS. Kidston, n. sp.

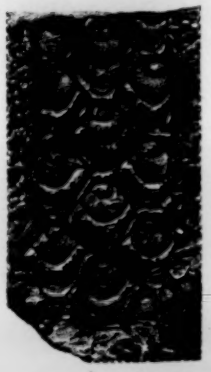
KIDSTON ON THE FOSSIL FLORA OF THE YORKSHIRE COAL FIELD. (SECOND PAPER). PLATE III.



1.



1a



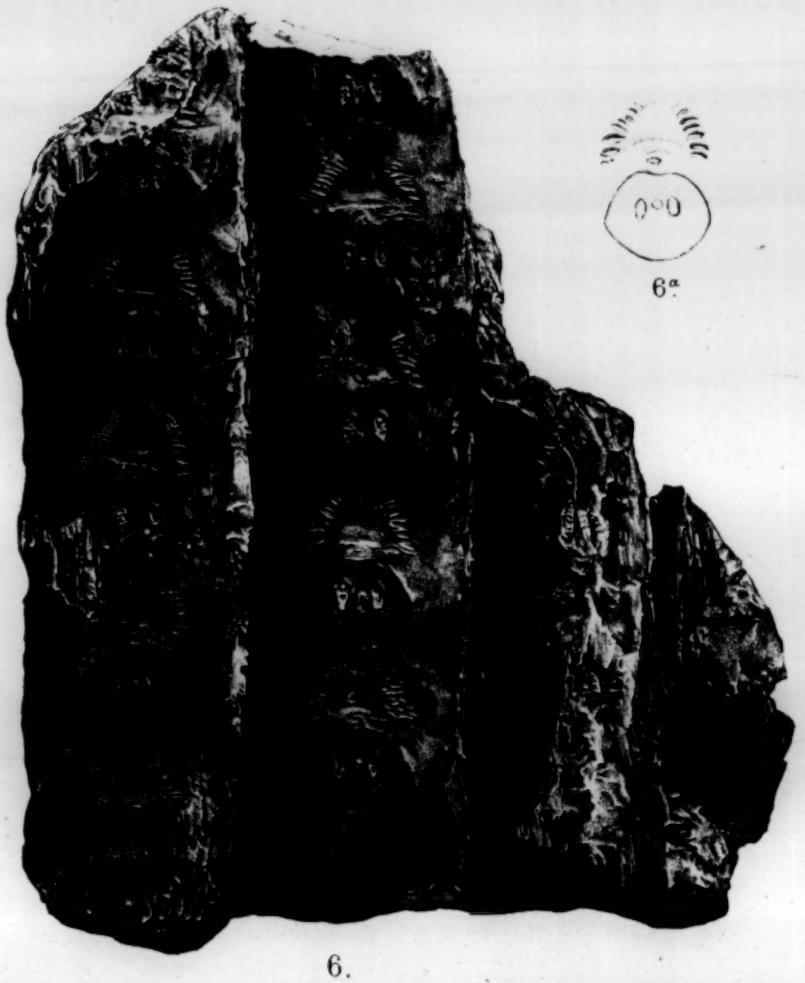
4.



4a × 1½



2a × 1½



6.

6a



2.



5.



3b



3c



3a × 2



5a



3.

R. Kidston.

M.cFarlane & Erskine, Lith.rs Edin.r

Fig. 1–5. SIGILLARIA SEMIDIVINATA, Kidston. Fig. 6. SIGILLARIA SOL, Kidston, n. sp.

VI.—*The Meteorology of Edinburgh.* By Robert C. Mossman, F.R.S.E., F.R. Met. Soc. (With Four Plates.)

(Read 1st March 1897.)

PART II.

Preliminary.

The first part of this paper was communicated to the Society on June 1, 1896, and published in the *Transactions* (vol. xxxviii. part iii., No. 20, pp. 681–755), the data there discussed being mean values of the climatic elements for each day in the year.

In the present paper an attempt will be made to focus the results deduced from an examination and reduction of the various meteorological registers kept in Edinburgh from 1731 to 1736 and from 1764 to the present time, with special reference to secular and other weather changes.

The condensed results of a number of minor papers dealing with subjects which have, in many cases, formed part of the daily routine of observation during the last ten years have also been included. Attention may also be called to the list of remarkable atmospheric occurrences, such as phenomenal gales, snow-storms, auroras, etc., which is contained in the appendix. In presenting this paper my warm thanks must be expressed to Dr Buchan, from whom I received invaluable advice when points of difficulty arose in the reduction of the observations.

Barometric Pressure.

The preparation of Table I., showing the mean monthly and annual air pressure since 1769, has been a work of considerable labour. This was more especially the case with the observations taken prior to the establishment of the Scottish Meteorological Society in 1856. During the last forty years these observations have been examined and checked by the Secretary, who further tested the instruments at the Society's stations. The errors of the barometers were thus known and allowed for in the calculation of the monthly means, while any accidental displacement of the mercury or other injury was at once apparent on comparing the returns with those made at stations in the vicinity of Edinburgh. The values for the period 1856 to 1896 were accordingly extracted from the *Journals* of the Scottish Meteorological Society, and entered in the table, any blanks in the observations being made good from the records of contiguous stations by interpolation and differentiation. No such easy method of dealing with the older observations presented itself,

as the values from 1769 to 1853 were, with the exception of those taken by PLAYFAIR* from 1794 to 1799, entirely unreduced and uncorrected. There were thus the accumulated data of eighty years awaiting discussion. As the work of reduction proceeded, it became evident from the numerous anomalies and discrepancies disclosed by an inspection of the monthly means, that the preparation of monthly isobaric charts for Scotland must be attempted for the greater part of the first fifty years covered by the investigation, with a view to the elimination of discordances. In this connection the numerous manuscript observations kept at various places in Scotland, and kindly lent by the Royal Society of Edinburgh and the Scottish Meteorological Society, proved of the highest value. I have specially to thank Dr BUCHAN for placing a large mass of material at my disposal.

The following are the additional stations whose data were utilised in the preparation of the monthly isobaric charts, the values for Edinburgh being calculated from Registers III., IV., VI., VII., X., XI., XIV., XVI., XVII., XVIII.†

Kept at.	Years.	Hours of Observation.	Remarks.
Selkirk, ...	1769–1780	?	The means were collected by HOY and are contained in his MS. registers.
Kirkcaldy, . . .	1775–1778	8 A.M. and noon.	Means calculated from MS.
Branxholm, . . .	1774–1783	...	*Trans. Roy. Soc. Edin.*, vol. i. p. 204.
Glendoich, . . .	1783–1801 1810–1816	9 A.M.	Means calculated from MS.
Gordon Castle, . .	1781–1827	8 A.M.	*Jour. Scot. Met. Soc.*, vol. v. p. 73.
Dunfermline, . . .	1799–1826	9 A.M.	Means calculated from MS.
Carlisle, ...	1801–1824	8 A.M., 1 P.M. and 9 P.M.	*Trans. Roy. Soc. Edin.*, vol. xi. p. 429.
Kinfauns Castle, . .	1811–1834	8 A.M. and 10 P.M.	Means calculated from MS.
Lasswade, . . .	1828–1843	8 A.M. and 10 P.M.	Means calculated from MS.
Dollar, ...	1836–1842	9.15 A.M. and 8.30 P.M.	Means calculated from MS.
Aberdeen, . . .	1829–1841	8 A.M. and 9 P.M.	From Abstracts given in *Aberdeen Journal*.

Much labour was expended in ascertaining approximately the instrumental error of the above instruments, and in their reduction to 32° and sea-level, the height above the sea being known in each case. The values from the above stations were then entered month by month, on small maps of Scotland. The entries include the following:—

1. The mean barometric pressure corrected and reduced to sea-level, and corrected for instrumental errors.
2. The rise or fall of pressure from the previous month.
3. The rise or fall of pressure from the corresponding month of the previous year.
4. The prevailing wind at Edinburgh and such places as observed the wind direction.

The monthly means had also corrections applied to them so as to bring them to the

* *Trans. Roy. Soc. Edin.*, vols. iv. p. 213, and v. p. 193. † *Trans. Roy. Soc. Edin.*, vol. xxxviii. pp. 682–683.

mean of Edinburgh, on the assumption that the distribution of pressure over the country was normal. These corrections were obtained from Dr BUCHAN'S paper on "The Mean Atmospheric Pressure of the British Isles."* Although but little weight was attached to the values thus corrected, they were of much interest when viewed in connection with anomalies in the barometric gradients over the country. Maps were prepared for a period of thirty-seven years, viz., from 1781 to 1817. It was not necessary to adopt this tedious process after 1817, as from that date the instruments were on the FORTIN principle, and carefully observed. From an examination of the results thus graphically shown by the data delineated on the maps, the elimination of errors was rendered comparatively easy. I believe that the means thus obtained give a close approximation to the average pressure for the period under discussion. The observations utilised from 1817 to 1856 were the following:—From 1817 to 1826 the means were computed from the Calton Hill Observatory, where daily readings were taken at 8 A.M. and 10 P.M. These were printed monthly *in extenso* in the *Scots Magazine* for the years to which they refer. ADIE'S observations given in the *Edinburgh Journal of Science* were adopted for the period 1827 to 1832, while the Royal Society's observations were employed from January 1833 to October 1834, and again from 1839 to 1852, the hiatus being filled in from a register kept at Lasswade, six miles S.E. of Edinburgh. The Lasswade means were calculated from 1828 to 1843, so as to allow of the determination of the instrumental correction by comparison with Edinburgh. Means were also computed for part of this period from the Dollar register, which furnished an additional check. The hours of observation were, at Lasswade, 8 A.M. and 10 P.M., and at Dollar, 9.15 A.M. and 8.30 P.M. The observations at the rooms of the Royal Society from 1839 to 1852 were taken at 10 A.M., and were deficient on Sundays and holidays. It was, therefore, necessary to interpolate values for the missing days. The height of the barometer for these days was found from the contemporaneous registers kept by ALEX. ADIE till 1850 and continued for some years thereafter at his place of business. As the Royal Society observations were made only once a day, it was necessary to reduce ADIE'S 10 A.M. and 10 P.M. readings, in order to obtain corrections to be applied so as to bring the former series to the mean of 10 A.M. and 10 P.M. This was accordingly done. The reason ADIE'S observations were not utilised for the actual means is that there was no attached thermometer. The readings could not, therefore, be reduced to 32°. The means for 1853 to 1856 were obtained from Sir Henry James'† abstracts taken in Edinburgh by the Royal Engineers. From 1856 down to the present time the 9 A.M. and 9 P.M. observations made at the Edinburgh stations of the Scottish Meteorological Society have, as already stated, been employed. Every effort has been made to make the results comparable by reducing or otherwise correcting the means to those of 9 A.M. and 9 P.M. For many years the hours were 8 A.M. and 10 P.M., or 10 A.M. and 10 P.M.; observations taken at these hours differ

* *Jour. Met. Soc.*, vol. vi. pp. 14–18.

† Abstracts from Meteorological Observations taken at the stations of the Royal Engineers.

but little from readings taken at 9 A.M. and 9 P.M. so that no corrections were made. With reference to the monthly means from 1769 to 1816, it was not considered desirable to attempt any reduction to 9 A.M. and 9 P.M., as the hours of observation could not be ascertained for some periods. The limit of error arising from this disturbing factor must be small, as the Edinburgh observations were checked against the isobars drawn month by month for the E. of Scotland. In any case, the departure from the true mean due to this deficiency would not exceed 0·012 inch.

Table I. shows the means of each month and year reduced to 32° and mean sea-level, as well as decadal and monthly means for the whole period, viz., 1770 to 1896. The annual mean was 29·858 inches, being highest (29·962 inches) in 1864 and lowest (29·706 inches) in 1789, showing a difference of 0·256 inch in the annual means. The highest monthly mean was that of May, which is 29·940 inches, and the lowest that of December, which is 29·800 inches, there being thus a difference of 0·140 inch between the highest and lowest monthly means. It is to be observed that the average pressure of November is practically the same as that of December, the difference being only 0·001 inch.

The highest mean pressure of any month was 30·361 inches in March 1840, and the lowest was 29·186 inches in January 1791, the difference being 1·175 inch. The month showing the greatest range among the means is February, the highest mean being 30·337 inches, in 1891, and the lowest 29·202 inches, in 1776, a difference of 1·135 inches. The least variation is in July, the highest mean being 30·153, in 1825, and the lowest, 29·633 inches, in 1798, a difference of 0·520 inch.

The absolutely highest barometric pressure during the 127 years under review was 31·071 inches, at 9 A.M. on January 9, 1896, and the lowest 27·451 inches, at 10 P.M. on January 26, 1884, giving a difference of 3·620 inches. The highest and lowest pressures are given for each month since 1840 in Tables II. and III. Table IV. gives the extreme range of pressure during the last fifty-seven years, for each month. The greatest monthly range was 3·035 inches in January 1884, and the lowest 0·515 in July 1852. The mean monthly range is greatest (1·611 inch) in January and least (0·935 inch) in June and July. It is to be observed that the differences between the values given in Tables II. and III. do not always agree with the values in Table IV. This is due to the entry in the former tables of extra readings taken during periods of high and low pressure, whereas the table of monthly range has been compiled from the bi-diurnal observations taken at 9 A.M. and 9 P.M. The results given in Tables I. to IV. are further summarised in Table V., while Table VI. shows all the sea-level pressures above 30·90 inches or below 28·20 inches experienced in Edinburgh from 1770 to 1896.

Mean Temperature of the Air.

Table VII. shows the mean temperature of the air in shade, 4 feet above grass, and at a height of 250 feet above mean sea-level, from 1764 to 1896. From 1764 to June 1781 the values given are those taken by HOY at Hawkhill House, St Andrew Square,

the Pleasance, and, for a short time, at Mertown. They have been reduced and otherwise corrected to the mean of the maximum and minimum by Dr BUCHAN, so that it was only necessary to correct them to a height of 250 feet by applying a reduction equal to 1° for each 270 feet. After having the small correction of 0·6° applied they were entered in the table.

Considerable labour was involved in the reduction of the observations taken from June 1781 to December 1821. It will, therefore, be necessary to go into the processes involved in the reduction of the earlier registers with some degree of elaboration. The best observations throughout this period are undoubtedly those made by ADIE in Merchant Court from 1795 to June 1805, the hours being 8 A.M. and 8 P.M.

The uncorrected values for the months and the years are given by FORBES. They have been brought to the mean of the maximum and minimum by applying the corrections given in the first part of this inquiry.* The corrections there given were tested by a number of methods, but the values were so accordant that it was decided not to make any alteration. A comparison of the *Edinburgh Advertiser* 8 A.M. and 8 P.M. readings from 1795 to 1804 with ADIE's corrected mean values gave the following plus corrections, which were applied to the *Edinburgh Advertiser* record from 1787 to 1806.

Jan.	Feb.	Mar.	Apr.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.
0°·9	1°·4	1°·8	2°·1	2°·2	2°·2	2°·0	1°·5	1°·0	0°·5	0°·4	0°·6

Some change was made in the exposure of the instruments in 1806, the corrections applied from that time till 1821 being those already given.† The means for the period 1787 to 1831 have been computed, and are given in Table VIII. Another change took place in the instruments or their exposure in 1824, but a fresh table of corrections was not made, as the observations were not utilised after 1821.

Another register is available for the period 1785 to 1816, the temperatures taken "before sunrise" and "at noon" being given *in extenso*, in the *Edinburgh Magazine* and afterwards in the *Scots Magazine*. The station was at Duddingston, near the foot of Arthur's Seat, from 1785 to January 1793, "within one mile of the Castle of Edinburgh" from 1793 to May 1798, and then at Barnton, three and a half miles west of Edinburgh, till 1816. The means have been computed and are given in Tables IX. and X. The averages utilised for the calculation of mean temperatures are those taken before sunrise, some little doubt attaching to the noon observations, especially in hot, sunny weather. The corrections were obtained by a comparison with ADIE'S and the *Edinburgh Advertiser* records, the latter being the values for the five years 1788 to 1792. The corrections thus obtained were applied to the observations at Duddingston from 1785 to January 1793. The observations taken within one mile of the Castle from February 1793 to May 1798 were corrected by means of a comparison with ADIE for the three years 1795 to 1797, and those taken at Barnton till 1816 from a comparison with ADIE for the five years 1800 to 1804.

* *Trans. Roy. Soc. Edin.*, vol. xxxviii. p. 686. † *Trans.*, vol. xxxviii. p. 687.

The following are the plus corrections for each series :—

Years.	Jan.	Feb.	Mar.	Apr.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.
	°	°	°	°	°	°	°	°	°	°	°	°
1785–1793 February	1·6	3·3	4·6	6·6	6·6	6·8	6·3	6·1	5·1	3·9	2·0	1·6
1793–1798 May	2·0	3·3	4·6	3·9	3·8	4·1	4·9	5·1	4·3	2·9	1·6	1·8
1798–1816	1·2	2·0	3·2	4·5	5·5	6·0	6·8	6·8	5·7	3·7	1·9	1·3

A register was kept in Edinburgh by Mr GEORGE WATERSTON from 1799 to 1850 (see Table XI.). The hours of observation were 8 A.M., 2 P.M., and 10 P.M. They have been utilised from 1806 to 1820, and were corrected by comparing them with ADIE'S mean temperatures for the ten years 1821–30. The following are the monthly corrections obtained after smoothing the curve :—

Jan.	Feb.	Mar.	Apr.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.
−2°·2	−1°·8	−1°·5	−1°·6	−2°·0	−2°·2	−2°·5	−2°·3	−2°·2	−2°·0	−2°·2	−2°·4

From June 1781 to December 1784 no observations are known to have been made in Edinburgh. It was therefore necessary to interpolate from the records of contiguous stations. A register was kept at Branxholm from 1775 to 1783, the results being given in vol. I. of the *Trans. Roy. Soc. Edin.* A comparison of the means there given with HOY'S corrected values from 1775 to June 1781 gave the following smoothed corrections which were applied to the observations made from June 1781 to December 1783, as follows :—

Jan.	Feb.	Mar.	Apr.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.
+3°·9	+3°·0	+2°·7	+2°·1	+1°·8	+1°·5	+2°·2	+2°·4	+1°·6	+2°·2	+3°·4	+3°·6

A register was kept at Glendoich from May 1783 to 1817, from which means (see Table XII.) have been calculated from May 1783 to 1794, the corrections to Edinburgh mean temperature being obtained by a comparison with the *Edinburgh Advertiser* means from 1788 to 1793, thus :—

Jan.	Feb.	Mar.	Apr.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.
−0°·7	−0°·6	−0°·6	−0°·6	−0°·6	−0°·9	−1°·0	−1°·5	−1°·4	−1°·3	−0°·9	−0°·6

The corrected means from all these sources having been obtained, final means were calculated from them. For example, from 1799 to 1804 the means given in Table VII. are the average of the corrected means deduced from ADIE'S and the *Edinburgh Advertiser* 8 A.M. and 8 P.M. observations, along with the Barnton observations, all being brought to the mean of the maximum and minimum by the corrections already given. The results are remarkably accordant in the majority of cases. FORBES' adopted temperatures were utilised as a check from 1805 to 1820. The Kinfauns Castle record was further brought to the mean of the Edinburgh record for the period 1813 to 1821 by a comparison

with ADIE's for the five years 1822 to 1826. These two last-mentioned registers not being deduced from observations taken in the city, were only employed as a check on the other registers.

The means from 1822 to 1896 given in Table VII. were derived from the following sources. From 1822 to 1850 ADIE'S mean temperature values, as reduced by FORBES, were employed, but the means were recomputed from 1824 to 1831 and from 1840 to 1850. During the latter period some blanks were made good by interpolating from WATERSTON'S register. The means for these years will accordingly be found to differ in some months from those given in FORBES' paper. From October 1849 to January 1853 the means were obtained from a record kept by ALEX. ADIE & SONS. From February 1853 to 1855 the observations taken by the Royal Engineers were utilised, while from 1856 the returns from the Edinburgh stations of the Scottish Meteorological Society were employed.

The station was in Melbourne Place from May 1858 to December 1861.* The returns from this station are too high, owing to radiation from the surrounding buildings. They have accordingly been corrected by the smoothed values calculated from the data given in the under-mentioned report.† The corrections which are all minus, have been severally ascertained for the maximum, minimum, and mean temperatures, as follows :—

	Jan.	Feb.	Mar.	Apr.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.
	°	°	°	°	°	°	°	°	°	°	°	°
Maximum, .	2·9	3·0	2·3	2·5	2·5	3·2	3·3	3·5	3·1	2·7	2·5	3·0
Minimum, .	2·5	2·4	2·3	2·5	2·5	2·5	2·5	2·5	2·7	2·6	2·6	2·5
Mean, . .	2·7	2·7	2·3	2·5	2·5	2·8	2·9	3·0	2·9	2·6	2·5	2·8

With regard to the observations of the past thirty-five years, for a few months interpolations had to be made from Leith or Inveresk. When this was found necessary the values were corrected for height.

The mean temperature for the period is 46°·8, or reduced to sea-level, 47°·7, the correction being 1° for every 276 feet. The highest mean annual temperature was 49°·6 in the years 1779 and 1846, and the lowest 43·8 in 1879, giving a range in the annual means of 5°·8. The warmest month was July 1779, with a mean temperature of 65°·2 or 6°·6 above the average, and the coldest month January 1814, the mean being 26°·5 or 10°·3 below the average, the extreme range in the monthly means being 38°·7.

The following table shows the highest and lowest mean monthly temperatures during the last 133 years :—

* In part 1 of the paper it was erroneously stated that the observations were made at this station from 1853 to 1856.

† (See *Quarterly Report of the Meteorological Society of Scotland*, for the quarter ending 31st March 1862, p. 7.)

Month.	Highest.	Year.	Lowest.	Year.	Range.
	°		°		°
January,	43·8	1796	26·5	1814	17·3
February,	47·2	1779	29·8	1838	17·4
March,	46·5	1779, 1841	34·2	1785	12·3
April,	49·8	1792, 1798	38·9	1837	10·9
May,	55·8	1833	45·1	1810	10·7
June,	61·9	1846	51·5	1860	10·4
July,	65·2	1779	54·4	1879	10·8
August,	63·7	1779	52·6	1830	11·1
September,	59·5	1846	48·2	1807	11·3
October,	52·7	1831	42·0	1817	10·7
November,	46·7	1818	34·0	1807	12·7
December,	47·8	1843	31·0	1878	16·8
Year,	49·6	1779, 1846	43·8	1879	5·8

The mean warmest month is July, 58°·6, and the coldest January, 36°·8, the range being 21°·8.

In the years 1854 and 1857 the mean temperature was above the average in each month, while it was below the average in each month in the years 1816 and 1879, both of these years being most disastrous from an agricultural point of view. The longest spell of cold was from April 1859 to January 1861, only one month in this period, viz., May 1860 having a mean temperature in excess of the average. The coldest five year period was from 1812 to 1816, and the warmest from 1777 to 1781, the excess or defect of temperature being the same in each case, viz., 1°·2.

Table XIII. shows the extremes in the mean and absolute daily temperature. The table is incomplete from 1770 to 1821. For this period the values given are (1) the extreme maximum and minimum temperatures observed by Hoy at Hawkhill from 1770 to 1776, the observations being made several times a day from 8 A.M. to midnight; (2) the observations taken from 1785 to 1798 were "near the foot of Arthur's Seat" or "near the Castle," the hours of observation being "before sunrise" and "at noon"; (3) the lowest and highest mean daily temperature from 1795 to 1804 taken by Adie at 8 A.M. and 8 P.M., and corrected to mean temperatures; (4) the absolute minimum temperatures from 1803 to 1821 given in the *Edinburgh Advertiser* register. From 1822 to 1896 the observations were taken first by Adie till 1850, and under the auspices of the Scottish Meteorological Society from 1856 to 1896. The hiatus from 1851 to 1855 was made good from records kept by Adie & Son and the Royal Engineers. During the last seventy-five years the highest mean temperature of any day was 75°·5 on August 5, 1868, and the lowest 12°·4 on December 24, 1860, showing an extreme range of 63°·1 between the mean temperatures deduced from the average of the daily maximum and minimum readings. The earliest date of highest mean temperature was May 19 in the year 1888, and the latest date September 2 in the year 1824.

The corresponding dates for the lowest mean daily temperature were November 22, 1880, and March 26, 1872. The range between the extreme daily temperatures was greatest 56°·0 in 1826 and least 34°·5 in 1883. The absolute maximum temperature in the seventy-five years under review was 87°·7 on August 5, 1868, and the lowest 5°·0, this value being recorded on January 31, 1845, January 29, 1848, and December 24, 1860, the latter observation being taken at Marchhall, which is within 200 yards of the place where observations are now made. The extreme range in the shade temperature was 82°·7. The earliest date at which the maximum temperature occurred was April 30 in the year 1862, and the latest, September 25 in 1895, the next latest being on September 8, 1890. The extreme dates on which the absolute minimum took place were November 7, 1868, and March 24, 1834. The annual range was greatest, 77°, in 1826 and 1848, and least, 48°, in 1862. The lowest absolute maximum was 70°·0 on April 30, 1862, and the highest minimum, 24°·5, on December 28, 1863.

Tables XIV. to XXIV. give the reduction of nearly all the temperature observations taken in Edinburgh.

Table XIV. shows the highest mean daily temperature in each month from 1857 to 1896, Table XV. gives the lowest mean temperature, and Table XVI. the range. Table XVII. shows the greatest daily range of temperature during this period.

Table XVIII. gives a general synopsis of the thermometric observations from 1840 to 1896. Table XIX. summarises some of the data contained in the above tables.

Table XX. gives all the instances of a maximum temperature below 25°·1 and of a minimum temperature above 60°·9.

Tables XXI. to XXIV. give the results of the reduction of ADIE'S observations taken at Canaan Cottage. The original observations are given *in extenso* in the *Edinburgh Journal of Science*. Table XXI. shows the average maximum, minimum, and mean, temperatures, and the mean daily range of temperature.

Table XXII. gives the extreme shade temperatures and the extreme range of temperature.

Table XXIII. gives the highest night minimum and lowest day maximum, and Table XXIV. the extremes in the mean daily temperatures. The date of the occurrence is given in each instance.

Temperature Variability 1840 *to* 1896.

The mean daily variability of temperature is given in Table XXV. In the calculation of the values, the mean temperature was assumed to be the arithmetical mean of the daily maxima and minima. The calculation of the variability of temperature consists in extracting the difference between the day to day values. Thus, if the mean temperatures of two successive days were respectively 60° and 55°, the difference, viz., 5°, would represent the variability. Table XXVI. summarises the data given in Table XXV., along with some additional particulars.

The mean annual variability of temperature is 2°·85, being highest, 3°·24, in January and lowest, 2°·52, in July, thus showing a difference of 0°·72. The greatest variability was 3°·38 in 1843, and the least 2°·50 in 1860, the range in the annual means being less than 1 degree. The greatest variability of any month was 4°·9 for November 1847, while the low value of 1°·6 was recorded in the Julys of 1853 and 1854, the Augusts of 1858 and 1860, and in September 1861. The greatest daily rise of temperature occurred on March 17, 1892, whose mean temperature was 15°·1 higher than that of the 16. August 29, 1869, on the other hand, was 15°·5 colder than the previous day. The daily observations for fifty-seven years were gone over, each rise or fall of 10° or more in the mean temperatures being extracted. The number of such cases was 230, viz., 129 rises and 101 falls (see Table XXVI.). The greatest number was 14 in 1843, and the least 1 in 1857, 1859, 1861, 1862, 1883, and 1891. In six of the years there was no fall of 10°, and in four of the years no rise of 10°. The greatest number of 10° rises was in 1843 and 1845, when nine cases were recorded, while the maximum number of 10° falls, viz., six, occurred in 1880. As the variability of temperature at stations on the Continent is as a rule calculated from observations taken at stated hours, and not from the mean of the maximum and minimum, Table XXVII. has been prepared. This Table gives the mean daily temperature variability for the hours of 9 A.M. and 9 P.M. which are then compared with the values deduced by taking the daily means of the maximum and minimum. Table XXVIII. shows the means deduced from the 8 A.M. observations taken by Hoy at Hawkhill House, and Kirkcaldy, while corresponding values for the period 1731 to 1736 are discussed in another section. It has been shown that the variability of temperature is subject to a diurnal range,* but unfortunately the Edinburgh records are sadly defective in data from which hourly values could be calculated for this or any other climatic element, with the single exception of sunshine.

Rainfall.

Table XXIX. shows the monthly and annual rainfall in Edinburgh for 120 years and six months. The values from 1770–76 were taken by Hoy at Hawkhill. Mr Hoy was also the observer during 1780 and the first half of 1781 when he removed to Gordon Castle. From 1785 to 1794 the observations were deduced from the *Edinburgh Magazine* record, the gauge being at Duddingston till January 1793, and thereafter "within one mile of the Castle." The values from 1795 to 1805 and from 1822 to 1850 are those taken by Mr Adie, and given by Forbes in his *Climate of Edinburgh*. The late Mr Leslie commenced his long series of rainfall observations in 1850, the station being Charlotte Square, where the record is still continued. The returns from this station have been utilised for the period 1851–96.

From 1805 to 1821 rainfall was not systematically observed at any one station during the whole period; but values have been obtained from measurements made at the Royal Observatory, and at other places in Edinburgh. I am indebted

* *Jour. Scot. Met. Soc.*, vol. x. p. 150.

to Mr G. J. Symons, F.R.S., for copies of some of the earlier rainfall observations. When no observations were available for the City, the Barnton register was utilised. It is to be particularly observed that the process adopted of dovetailing one rainfall record into the other introduces a slight element of error, the precipitation, as a whole, increasing the nearer the station is to the high grounds surrounding Arthur's Seat, the Blackford Hill, and the Pentlands (see *Jour. Scot. Met. Soc.*, vol. x. p. 16).* The records, however, approximate closely to the mean rainfall of Charlotte Square, as shown by the observations taken there during the last forty-five years.

The mean annual rainfall is 25·86 inches, the wettest year being 1872, with a rainfall of 38·96 inches, and the driest, 1826 (the year of the short crop), with a downfall of only 15·27 inches. These amounts are respectively 51 per cent. above, and 41 per cent. below the mean. The wettest month is July; the mean daily fall being ·091 inch, and the driest month March, the average being ·049 inch.

The wettest month was September 1785 with a rainfall of 10·69 inches, and the driest March 1781 with a rainfall of 0·03 inch. The mean annual number of days with 0·01 inch or more of rain, taking the observations of the last twenty years (1877-96), is 190, distributed throughout the year as follows :—

Jan.	Feb.	Mar.	Apr.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.
16	14	15	14	14	14	18	19	16	17	17	16

The greatest number of days with rain in the period 1856–96 was twenty-nine in July 1882, and the least, two for March 1856.

Droughts and Heavy Rains.

Since the year 1770, as already stated in last section, rainfall observations have been taken in Edinburgh or its immediate vicinity without a break, there being always one or more rain-gauges at work in different parts of the city. During thirty-four years, however, viz., from 1777–79, 1781–83, 1817–23, and from 1833–55, the rainfall measurements were only made weekly or monthly. Waterston for a year or two gave the amounts recorded during great falls, but they have not been utilised. The material available for examination in connection with this inquiry was thus restricted to the ninety-two years during each of which the gauge was examined daily, and the amount, if any, measured. The period under discussion ends with 1895.

Before stating the more prominent results of an investigation into droughts it seems desirable to give an answer to the question, "What is a drought?" Mr Symons, our greatest authority on rainfall matters, has solved the problem by dividing droughts into two classes, viz., absolute and partial. He defines the former as periods of more than fourteen consecutive days absolutely without rain, and the latter as periods of more than twenty-eight consecutive days, the aggregate rainfall of which does not exceed one-hundredth of an inch per day. The examination has been confined in the present

* The mean annual rainfall for the twenty-five years, 1866–90, at various places in Edinburgh was as follows :— Charlotte Square, 26·71 inches; Cumin Place, 30·13 inches; Blacket Place, 29·86 inches; and Napier Road, 28·97 inches.

instance to the former class, viz., absolute droughts. The total number of these during the ninety-two years under review was 65. Their distribution throughout the year (as will be seen on looking at Table) is somewhat irregular, June having the greatest number with 10, closely followed by February and March with 9 each. The minimum is reached in Autumn, November having only 2, and October 3. The secondary minimum in April and May is of interest as is the sharp drop after July. We may state that the droughts have been entered to those months in which they commenced.

	Jan.	Feb.	Mar.	Apr.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.	Year.
Number of Droughts,	3	9	9	5	4	10	7	3	5	3	2	5	65
Mean duration, days,	20	18	20	19	21	18	19	18	19	17	16	18	18·6

As regards individual years, the greatest number of droughts observed was three in 1786, 1825, 1829 and 1867, while none occurred from February 1787 to July 1795, a period of eight years and three months. A recent instance of a long spell without one was from August 1876 to May 1884, or seven years and nine months. The longest period without rain occurred in 1786, when none fell for thirty-three days, viz., between May 24 and June 25. The water supply in Edinburgh fell short during this year, the community being put to much inconvenience thereby. Other long spells without rain were from March 13 to April 11, 1825, and from June 24 to July 22, 1869, periods of thirty and twenty-nine days respectively. Of the sixty-five droughts recorded, sixteen exceeded twenty days while four lasted a month. Nearly all the dry periods occurred in early spring. Only on one occasion during the three months October to December did a drought last for a longer time than seventeen days. As to the atmospheric causes concurring in such long dry periods, little can be said. We know that droughts are due to the unwonted prevalence and persistence of anti-cyclonic systems over Western Europe, but to say more than this would be to enter on the ground of pure speculation.

With regard to heavy rains, all falls of an inch or more in the twenty-four hours were extracted for the ninety-two years under consideration (see Table XXXI.). An inch a day in this part of the country is looked on as a heavy rainfall, being equivalent to 101 tons or 22,623 gallons of water per acre. The total number of cases as will be seen from the following Table was 165, giving an average of very nearly two per

Month.	Jan.	Feb.	Mar.	Apr.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.	Year.
Falls of 1·00 inch or more,	7	6	5	6	15	12	21	33	20	18	15	7	165

annum. The heavy falls were distributed among the years in a most capricious manner. For instance, there were eight such rains in the year 1808, while 1809 and 1877 had seven each. On the other hand not a single case was observed from September 1884 to August 1889. The number of heavy falls during thunderstorms was one in May, two in June, ten in July, and four in August. It would thus appear that in Edinburgh, at any

rate, thunderstorm rains usually fall short of an inch. August stands out prominently for its rainstorms with thirty-three falls exceeding an inch. The period known as the Lammas Floods shows to what an extent these heavy downpours have obtruded themselves upon public notice, and that long before the days of rain-gauges. July comes second to August with twenty-one cases, while February and April have only six each and March but five. It is of interest to note that two of the six heavy rains in February occurred within a week. The seasonal distribution was spring, twenty-seven cases; summer, sixty-six; autumn, fifty-three; and winter, twenty. From an examination of the daily weather reports it was seen that the majority of notable downpours took place during the passage of small shallow depressions moving slowly eastwards. Sometimes the depression remained almost stationary for days. Enormous quantities of rain were then precipitated, 7 inches, for example, falling in five days during August 1877. In a few cases, principally in winter, the rain was general over the country, but as a rule the western parts of the country were not affected by the cyclonic storms which gave the heavy rains on the east coast. The general direction of the wind during the rainstorm was noted, the percentage frequency being as follows, viz. :—

N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.
5	15	32	9	3	9	21	6

The maximum number of cases took place with winds from the east, a well marked secondary maximum being observed with winds from the west. If we weigh the observations so as to allow for the relative frequency of the winds during the ninety-two years, we obtain quite a different windrose, as it is called. The overwhelming preponderance of sea-winds during the occurrence of heavy rains now becomes apparent, while the secondary maximum with west winds—a maximum due to the frequency with which these winds blow—vanishes. Thus, approximately, in 1,000 days of wind there will be ten rains exceeding an inch with a north-east wind, and nine with an east wind, while only two cases may be expected with a south wind. The values for the other winds are N. 6, S.E. 6, S.W. 3, W. 3, N.W. 4. The comparative infrequency of heavy rains with south-east winds is doubtless due to the fact that they have been deprived of much of their moisture by the Lammermoors over which they had previously passed. In Aberdeenshire, as shown by Dr Buchan, the south winds blow against the cold slopes of the Grampians with the result that there they are by far the wettest. Mr Symons has stated that there is no part of the British Isles, however dry, where 4 inches of rain may not fall in twenty-four hours. The Edinburgh record bears this statement out, for although there are only three rains exceeding 3 inches in the ninety-two years, yet one was above 4 inches, no less a quantity than 4·20 inches having fallen on December 9th, 1787. On that occasion there was a great flood in Leith Harbour, greater than ever remembered. The flood was as high at low water as at ordinary full tide. Much damage was done to the shipping, while several casualties involving loss of life were reported from Leith and other parts of the country. It cannot be too strongly urged

on observers to have rain-gauges capable of holding at least 4 inches of rain, otherwise important facts of interest to meteorologists and engineers alike will be irretrievably lost. The following are the maximum daily rainfalls noted in each month during the period under review.

Month.	Jan.	Feb.	Mar.	Apr.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.
Rainfall, .	1·59	1·80	1·55	1·71	1·50	1·54	2·95	2·56	3·80	2·50	2·89	4·20
Date, . .	10/1809	3/1809	16/1891	5/1808	14/1795	26/1874	13/1879	18/1797	24/1785	5/1775	18/1795	9/1787

Direction of the Wind.

Table XXXII. shows the number of days on which each wind prevailed, from June 1731 to May 1736, and from 1764 to 1896; for the months and the year. From 1764 to 1769 the only values available are the summaries of east and west wind prepared by Hoy. The former includes observations from N., N.E., E., and S.E., the latter those from S., S.W., W., and N.W. As 50 per cent. of the winds in Edinburgh are from the S.W. and W., and 25 per cent. from N.E. and E., it follows that the above method of reducing the wind observations to the two principal directions gives a close approximation to the truth. The registers employed are those utilised in the preparation of daily values.* From 1781 till the commencement of Waterston's observations in their complete form in 1805, the direction of the wind was not systematically observed. It was therefore necessary to interpolate from the Glendoich and Dunfermline registers, which in some measure help to supply the deficiency. Table XXXIII. shows the mean percentage frequency of the winds for the months and the year for the 133 years 1764–1896. The mean values for 100 years are given in Part I., and are herewith compared with the longer record.

Percentage Frequency.

	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm or W.
133 Years, .	4	7	18	5	5	15	35	7	4
100 „ .	4	7	16	7	6	17	32	7	4

The means for the two periods are thus essentially the same. The observations were made twice a day during nearly the whole of the 133 years.

In their reduction the values were resolved to eight points by counting N.N.E., for example, along with N.; S.S.E. along with S.; E.N.E. along with E., and so on. This was done in order to make the observations taken prior to 1856 comparable with those given in the *Scottish Meteorological Society's Journal* during the last forty years.

* *Trans. Roy. Soc. Edin.*, vol. xxxviii. p. 691.

A disturbing element is introduced owing to some observers entering calms and variable winds, whilst others always give a direction, which in still weather was probably the point from which the wind last blew. Tables XXXIV. and XXXV. were accordingly prepared so as to make the results as uniform as possible. In these tables the percentage frequency was resolved to two directions as described above, calms being eliminated.

Looking at Table XXXV., it will be seen that the years with the greatest percentage of west wind were 1854, 1887 and 1798, with values of 79·1 per cent., 77·8 per cent., and 76·2 per cent. respectively.

The effect of wind upon the temperature of the air is very apparent. Thus, in 1854 the mean temperature was above the average in every month, and in 1798 in ten months. In 1887 the excess was not so noticeable. The years with the greatest percentage of east wind were 1768, 47·5 per cent.; 1829, 47·3 per cent.; and 1816, 46·0 per cent.; the prevalence of polar winds, as was to be expected, resulting in a marked fall of temperature during these years.

Mean Relative Humidity.

The mean relative humidity has been determined from the bi-daily observations made with the dry and wet bulb thermometer, the hours of observation being 9 A.M. and 9 P.M. The period under discussion is the thirty-five years 1862–1896. The mean

Mean Relative Humidity, 1862–1896.

Month.	Mean.	Highest.	Year.	Lowest.	Year.	Range.
	%	%		%		%
January, . . .	86·8	96	1879	81	1895 1896	15
February, . . .	86·4	97	1879	78	1895	19
March, . . .	84·1	96	1871 1879	77	1868	19
April, . . .	80·5	93	1872	73	1868 1896	20
May, ...	78·1	92	1872	72	1881	20
June, ...	77·4	94	1875	67	1884	27
July, ...	79·1	85	1870	74	1869 1878	11
August, . . .	81·5	90	1877	74	1869	16
September, . . .	82·6	92	1875	74	1869	18
October, . . .	85·8	92	1882	80	1867 1892	12
November, . . .	86·8	94	1876 1881	78	1869	16
December, . . .	86·4	92	1876	83	1867	9
Year, ...	83·0	97	February 1879	67	June 1884	30

annual humidity is 83 (Saturation = 100). The air is driest in June, which has a mean humidity of 77·4 per cent., and dampest in January and November with 86·8 per cent., the range being thus 9·4 per cent. As regards individual months, the dampest was

February 1879 with a mean humidity of 97, the driest being June 1884 with a humidity of 67. In June 1878 the mean humidity was 69, but in no other month did the mean fall below 70. The month showing the greatest difference between the means is June, the highest being 94 per cent. in 1875, and the lowest 67 per cent. in 1884, the difference being 27, and the month of least difference, December, the highest being 92 per cent. in 1876 and the lowest 83 per cent. in 1867, the difference being only 9 per cent. A completely saturated atmosphere is of rare occurrence, not more than two or three cases occurring on an average in the year, while in some years no such high value was attained. During the past six years a RICHARD hair hygrometer has furnished a continuous record, the lowest value thus registered being 18 per cent. on February 8th, 1895. An examination of the hygrograms shows that a humidity below 35 per cent. is of very rare occurrence, even with the shade temperature over 80°.

Thunderstorms. (*See Table XXXVI.*)

During the period 1770–1896, 811 thunderstorms were observed in Edinburgh, or at the rate of six per annum. Of these, 34 took place in winter, 145 in spring, 549 in summer, and 83 in autumn. The months of greatest frequency were June with 169, July with 229, and August with 151; on the other hand, November and December had only 7 each, while February had 10, and March 11. During the six months, April to September, 741 thunderstorms were observed, being 91 per cent. of the whole. Thunderstorms begin to diminish after the Lammas floods, few being observed after the 13th of August. The absolute minimum covered the nineteen days ending with December 5th without a single thunderstorm during the 127 years. Lightning without thunder is comparatively rare; the average annual number of days being only one. Sheet lightning rises to a maximum in September, there being 18 cases in that month during the period under review. A secondary maximum occurs in December. The winter thunderstorms and other electrical phenomena are no doubt associated with deep cyclonic systems;—the explanation being that in the winter months, 'warm, moist, ascending, and cold, dry descending currents are most frequently brought into close proximity during the great Atlantic storms of the season.'* The diurnal distribution of thunderstorms is well marked (see Table XXXVII.), 64 per cent. being observed during the six hours ending with 5 P.M.; the maximum taking place in the two hours ending 3 P.M.; and the minimum in the early morning hours. Lightning without thunder, on the other hand, is essentially a nocturnal phenomenon, nearly all the cases taking place in the five hours ending with 11 P.M. Thunderstorms appear to diminish at 1 P.M.; this being doubtless due to the loose way in which certain observers use the word noon. Entries of thunderstorms at noon have all been put down as having occurred in the hour ending noon, whereas half of such entries should have been entered to one o'clock. It was not until the investigation was completed that this anomalous result presented itself.

The mean annual number of thunderstorms, as already remarked, is six, the year with

* *Ency. Brit.*, Art. 'Meteorology,' Buchan.

the greatest number being 1872, when twenty were experienced. During that year pressure was lower and the rainfall greater than in any other year, with perhaps the exception of 1789. Only one thunderstorm was recorded in the years 1773, 1780, 1784, 1796, and 1801. During comparatively recent years, 1844, 1851, 1859, and 1865 had two, but there is no record of a year without any. The months with the greatest number of thunderstorms were August 1831, and July 1893, which had eight each.

Thunderstorms appear to be on the increase, the mean number from 1770 to 1809 being 4·5 per annum. In the forty years ending with 1849, the number rose to 6·3 per annum, while during the period 1850 to 1889 a further increase to 9 per annum was recorded. During the six years ending with 1895, the mean annual number was ten. The increase can hardly be accounted for by the assumption that the early observers systematically neglected to record this meteor. Only for about twenty years are we dependent on one weather register for our information.

The annual totals have been smoothed by BLOXAM's method, taking continuous sets of five. The results were projected on a chart which was originally prepared in connection with a paper on "Sunspots and Auroras." On comparing the two curves, little of a definite nature can be made out, it being very doubtful whether thunderstorms are phenomena of a fortuitous nature or are in some way connected with sunspots. There is some reason to think thunderstorms are subject to a long cycle, a wave crest of which we have lately passed. The wave shows distinct minima in 1802 and 1864, and maxima in 1829 and 1882.

With the view of ascertaining the damage done by thunderstorms to life and property, every instance of a severe storm was examined, the newspaper reports for the days characterised by disturbances of an exceptional nature being extracted. The result of the inquiry is, that damage to property took place in thirteen thunderstorms, twenty-six people in all being injured, and only two killed. Of the very severe thunderstorms, seven occurred in June, three in July, two in August, and one in January, the latter occurring on January 26, 1792, when George Watson's Hospital was struck.

The worst storm on record appears to have been that of July 22, 1873, when an observer of the Scottish Meteorological Society counted in one hour 680 flashes of lightning with their accompanying thunder-claps. This gives a rate of fully eleven per minute.

During recent years, the severest storm experienced was that of August 12, 1884, when the Earl of Lauderdale was killed. For notices of these storms see Appendix.

Snow.

Table XXXVIII. gives the number of days on which snow fell for each month, and the year from 1770 to 1896. Values are also given showing the results grouped by winters, with date of first and last snowfall. The total number of days on which snow fell was 2664, giving an average of 21 per annum. The snowiest year was 1782

(the black auchty-twa), with forty-seven entries, closely followed by 1838 with forty-six days, and 1814 with forty-five days. On the other hand, snow fell on only three days in 1856, the number being below ten in eleven years. Grouping the results by winters, a slightly different arrangement obtains, the snowiest being the winter of 1836–37 with forty-nine days, while the winter of 1850–51 had but two snowfalls. With the exception of a little sleet on September 23, 1893, no snow fell in the months of June, July, August and September.

The greatest number of cases in each month is as follows :—

	Jan.	Feb.	Mar.	Apr.	May.	Oct.	Nov.	Dec.
Number, . . .	15	14	16	10	5	4	7	13
	...	1772	...	...	...	...	...	...
	...	1795	...	...	...	...	...	...
Year, ...	1823	1855	1812	1837	1802	1895	1807	1874

The snowiest month was thus March, 1812, with sixteen days on which snow fell.

The earliest date of first snow was October 1 (see Table XXXIX.) in the year 1817, and the latest January 31, in the winters of 1850–51 and 1857–58. The latest date of last snowfall was May 30, 1808, and the earliest January 17, 1853. The mean date of first snowfall is November 22, and the mean date of latest fall, April 10.

Hail.

Table XL. shows the number of times hail fell during the 127 years 1770–1896, for each month and the year. The mean annual number of days with hail is ten, the maximum being thirty-two days in 1824, and the minimum one day in 1848. The greatest number of days in each month is shown in the following Table :—

	Jan.	Feb.	Mar.	Apr.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.
Year, . .	...	...	...	...	...	...	...	1809	...	...	1795	...
	...	...	...	...	...	...	...	1820	...	...	1801	...
	...	...	...	...	1805	...	...	1889	...	...	1821	...
	1808	1843	1824	1803	1894	1895	1891	1891	1889	1819	1824	1784
Number, .	7	5	9	9	7	4	3	2	3	4	4	7

Hail does not seem to be associated with thunderstorms, few cases being observed in summer.

Gales.

Table XLI., showing the number of gales, must be looked on as a tolerable approximation to the truth. As the entries depend on personal and not instrumental observation, the results are not strictly comparable. The greatest number of gales was seventy-two in 1818, and the least number, five in 1856. The mean annual number is twenty-nine.

Fog or Mist.

This is also an unsatisfactory Table (No. XLII.), although every effort has been made to eliminate entries of "haze" by comparing the Edinburgh records with those from contiguous stations. The foggiest year was 1808, with thirty-eight entries, while in 1784 no fog was reported.

Auroras.

Table XLIII. shows the number of auroras observed in Edinburgh from 1773 to 1781, and from 1800 to 1896.

I have to thank PROFESSOR COPELAND for permission to examine the records of the Edinburgh Royal Observatory from 1862 to 1894. Many notices have also been obtained from the published records of that institution.

The year of maximum auroral frequency was 1871, with twenty-one auroras, closely followed by 1870 with nineteen notices. The maximum observed in one month was six in March 1871.

Lightning.

Table XLIV. shows the number of cases of lightning without thunder recorded from 1807 to 1835, and from 1868 to 1896. During the other years this phenomenon was not systematically recorded, as there are only about a dozen entries. The greatest number of cases was six in 1818 and 1884. The maximum in any month was three in February 1818, and again in September 1884. Sheet lightning is a comparatively common occurrence in winter, being frequently seen during severe gales, especially when accompanied by a low barometer.

Hourly Sunshine Values.

Table XLV. shows the distribution of bright sunshine throughout the day for the months, seasons, and the year. The results are derived from the records of a Campbell-

Stokes sunshine recorder, which occupies a good exposure at my meteorological station in the south side of Edinburgh. The hourly values have been tabulated for the six years ending with July 1896, the means given in the Table being for this period. Looking at the seasonal values, it will be seen that about four per cent. more sunshine is recorded after noon than before it, except in winter, when the afternoon hours are sunnier than the forenoon by nearly ten per cent. There is little doubt that the relatively greater clearness of the afternoons in winter is due to the prevalence of fog and haze during the morning hours. It will be observed that there is a well-marked seasonal swing in the hour characterised by the greatest amount of sunshine, which approximates closely to the time of highest mean temperature. Attention may also be drawn to the slow rate at which the sky clears in summer, compared with other seasons of the year. Thus in April, the mean amount of sunshine for the hours ending 7 A.M. and 11 A.M. is 3·7 hours and 13·1 hours, respectively, while in June the corresponding values are 9·2 hours and 10·4 hours. This is probably due to the condensation accompanying the strong ascending currents so prevalent during summer.

In Table XLVI. the number of days with different percentages of sunshine is shown for the six years ending with July 1896. It will be seen from the maximum values that on practically cloudless days in summer at least ten per cent. of the possible sunshine is lost, owing to haze at the horizon; while in winter the amount so lost is about 25 per cent. Days with from 1 to 10 per cent. of the possible sunshine are the most frequent at all seasons of the year, sunless days excepted. The latter are at a maximum in winter when no sunshine is recorded in 42 per cent. of the cases.

Rainband Observations.

Observations of the thickness of the rainband in the spectrum of sunlight have been made three or four times a day since August 1887. The hours of observation were 9 A.M., noon, 3 P.M., and 6 P.M., the latter observation being dispensed with in the winter owing to lack of sunlight. The instrument employed was a direct vision spectroscope, which was pointed to the N.W. at an angle of from 40° to 50°. The scale was an arbitrary one, ranging from 0 to 6. The rainband was compared with the lines B, b, and F, to which values corresponding to 1, 2, and 3 were given. The following are the means for the ten years ending July 1896 :—

Jan.	Feb.	Mar.	Apr.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.	Year.
1·14	0·92	1·14	0·98	1·15	1·18	1·28	1·27	1·10	1·14	1·12	1·12	1·13

There is little doubt that the rainband spectroscope is a valuable auxiliary to the ordinary instruments for forecasting weather. The following Table gives certain particulars for the days on which rainband observations were made during the three years 1888–90 :—

Rainband.	Days.	Rain fell Days.	Per cent. of Cases followed by Rain within 24 Hours.
0·0	44	9	20
0·5	146	44	27
1·0	347	146	42
1·5	256	143	56
2·0	123	93	76
2·5 and upwards	66	60	91

It will be seen that there is a regular rise in the frequency of rainfall with an increasing rainband. The principal drawback to the forecasting value of these spectroscopic indications lies in the fact that nearly two-thirds of the readings are normal. It will be seen on reference to the Table that the chances of rain or no rain, with values corresponding to 1·0 and 1·5 on our mental scale, are pretty evenly balanced. Under such circumstances the observer must turn to his other instruments for guidance in framing his prognostications. Many cases occurred during the ten years under review when a thick rainband was observed with a clear sky, and a thin one with a cloudy sky, the accompanying weather being wet in the one case and dry in the other. One point specially noticed is that days on which hail fell are characterised by low rainband values, while the same may be said regarding days with snow. An elaborate investigation into the whole subject was commenced some time ago, but it has not been found possible to include the results in this paper.

Solar and Terrestrial Radiation.

The following tabular statement shows the more prominent results deduced from the reduction of the daily observations taken in the south side of Edinburgh during the nine years 1888–96. The solar radiation thermometer is at a height of four feet above the ground, and the terrestrial radiation at a height of a quarter of an inch over short grass.

It will be seen that solar radiation is at a maximum in May, and at a minimum in December; while terrestrial radiation is at a maximum in November, and at a minimum in June. The greatest excess of sun over shade temperature occurred on March 27, 1892, viz., 76°·8; while on May 22, 1890, the grass minimum fell 12°·6 below the minimum in shade. A few cases have been observed when slight inversions of the normal condition of affairs took place, the air at the time being nearly saturated and the sky densely overcast. The maximum excesses of sun over shade were observed in spring or early summer on days when showers and bright sunshine alternated.

	Black Bulb in Vacuo.				Bright Bulb on Grass.			
			Excess over Shade Max.				Minus difference from Shade.	
	Maximum.	Mean.	Mean.	Greatest.	Minimum.	Mean.	Mean.	Greatest.
	°	°	°	°	°	°	°	°
January, . . .	84·5	56·1	13·8	45·2	7·5	29·4	4·3	10·7
February, . . .	103·3	70·3	26·7	61·4	7·0	29·3	3·8	10·6
March, ...	115·0	84·6	38·5	76·8	17·3	30·6	4·1	12·2
April, ...	129·0	96·6	44·3	65·7	20·0	33·9	4·2	12·0
May, ...	133·0	107·5	48·5	67·7	27·2	39·5	3·8	12·6
June, ...	139·3	111·1	47·7	71·0	31·3	45·2	2·8	11·1
July, ...	137·9	111·2	46·6	69·5	34·3	47·8	2·6	8·5
August, . . .	134·0	111·0	46·6	67·2	33·3	47·7	3·3	8·9
September, . . .	126·5	99·5	39·1	60·4	28·1	43·4	4·2	8·8
October, . . .	112·5	82·3	30·1	52·9	21·2	36·0	4·6	10·2
November, . . .	103·5	64·9	17·4	48·9	19·3	33·4	4·8	9·7
December, . . .	82·0	51·8	8·6	32·0	8·7	30·2	4·5	12·4
Year, ...	139·3	87·2	34·2	76·8	7·0	37·2	3·9	12·6

Reduction of the Observations taken in Edinburgh, from June 1731 *to May* 1736.—(The observations are given *in extenso* in *Medical Essays and Observations*, vol. i. to v. Edin., 1748, 3rd ed.)

This register seems to have been kept with much care and regularity. The observations were made twice a day, the first nearly always at 9 A.M., the second between 2 and 7 P.M., but as a rule either at 4 or 5 P.M. The observations made include readings of pressure, humidity, temperature, wind direction and force, and a condensed state of the weather at the time. The daily rainfall was also measured from June 1731 to May 1735. The observations, it may be remarked, are adapted to the Julian or old style.

Pressure.

The barometer is described as a simple portable one, with a tube about a fourth of an inch in diameter. The scale was probably of wood. The instrument was kept in a chamber at a height of 270 feet above the level of the sea, the height being determined experimentally by carrying the instrument to the sea-shore during an anti-cyclonic period. The values given in the Table below have been corrected and reduced to 32° and sea-level. There was no attached thermometer, but a mean value of 60° was assumed, and the corrections for reducing observations made with instruments having wooden scales applied.* The values may be looked upon as tolerable approximations. The mean annual pressure was 29·877 inches. The highest mean pressure was 30·204 inches in May 1733, and the lowest 29·530 inches in January 1736, showing a range of 0·674 inches between the mean monthly pressures.

* Simmond's Meteorological Tables, p. 23.

Mean Pressure, 5 Years.

Jan.	Feb.	Mar.	Apr.	May	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.	Year.
Ins.	Ins.	Ins.	Ins.	Ins.	Ins.	Ins.	Ins.	Ins.	Ins.	Ins.	Ins.	Ins.
29·813	29·743	29·778	29·944	29·967	30·021	29·923	29·913	29·840	29·876	29·961	29·749	29·877

Temperature.

The thermometer was placed along with the hygrometer in a perforated case freely exposed to the air on the outside of a window facing north. As the observer says, "neither the sun, or rain, nor the fire and company in the chamber can have any bad effect on the instruments within it, and the air has open free access to them."* The instrument was filled with alcohol, and graduated into inches and tenths. "The freezing point is at 8 inches and 2 tenths, and the heat of a man in health raises the spirit to 22 inches 2 tenths." The conversion of the values to Fahrenheit's scale is thus rendered an easy matter, as a change of 14·0 inches in the reading of the thermometer is equivalent to an alteration in temperature of 66°·6, the normal blood heat being 98°·6. The highest temperature recorded during the five years under consideration was 78° at 6 P.M. on June 30, 1734 (New Style), and the lowest, 19°·5 at 9 A.M. on January 8, 1732 (New Style), thus giving an extreme range of 58°·5 at the hours of observation. The following are the highest and lowest temperatures recorded during the five years.

	Jan.	Feb.	Mar.	Apr.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.
	°	°	°	°	°	°	°	°	°	°	°	°
Highest, .	51·5	50·5	64 5	64·5	70·5	78·0	73·5	76·0	64·5	62·5	49·5	54·0
Lowest, .	22·0	22·5	28·0	33·5	36·5	41·5	49·5	48·5	40·0	31·5	27·0	19·5
Range, .	29·5	28·0	36·5	31·0	34·0	36·5	24·0	27·5	24·5	31·0	22·5	34·5

In the reduction of the observations, for the purpose of obtaining mean monthly values, the morning reading was alone employed. The 9 A.M. values were accordingly extracted and averaged, Table XLVII. containing the corrected means for the five years.

Table XLVIII. contains the observations brought to the mean of the maximum and minimum, the corrections being found from a comparison of the 9 A.M. readings with the mean temperature deduced from the average of the maxima and minima, for the years 1888–1896. The following are the monthly corrections thus obtained after smoothing the curve :—

Jan.	Feb.	Mar.	Apr.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.	Year.
+0°·6	+0°·8	+0°·7	−0°·0	−0°·4	−0°·2	−0°·0	+0°·2	+0°·3	+0°·3	+0°·4	+0°·3	+0°·2

* *Medical Essays*, vol. i. p. 8.

The mean annual temperature for the period was 47°·0, being highest, 59°·9, in July and lowest, 36°·8, in January, a difference of 23°·1 between the mean monthly averages. The warmest month during the five years was July 1734, 61°·8, and the coldest, February 1736, 33°·6, showing a range in the mean monthly temperatures of 28°·2.

Rainfall. (*Table XLIX.*)

The rainfall was measured from June 1731 to May 1735. The gauge was 28 inches in diameter, and was placed on the top of a garden wall. Precautions were taken to prevent loss through evaporation, and the measurements were made, as a rule, every day.

The wettest month was March 1735, with 5·38 inches, and the driest, May 1733, with only 0·08 inch of rain.

Variability of Temperature. (*Table L.*)

The mean daily temperature variability has been determined from the observations made at 9 A.M. The average for the period was 3°·4, being greatest, 5°·3, in October 1731, and least, 2°·3, in September 1733. The mean varied from 4°·0 in December to 3°·1 in May.

Humidity.

The hygrometer, or rather hygroscope, consisted of a piece of whip-cord with a plummet appended. The cord was alternately baked in an oven and saturated with moisture, before the scale was graduated. The operation was repeated four times until the difference in the length of the cord when fully dried to its length when saturated with moisture was constant at 4·5 inches. The point of greatest dryness on the scale was fixed at five-tenths of an inch, the scale extending to five inches, which was the point indicated in a completely saturated atmosphere. The instrument was inclosed in the perforated case containing the thermometer. Although this method of observation is crude, it may be of interest to give the results, as affording a tolerable approximation to the seasonal distribution of this element of climate. The mean annual humidity on this scale was 2·11, being at a maximum in December, viz., 2·47 inches, and at a minimum in May, viz., 1·70 inches. The seasonal variation in humidity was, therefore, virtually the same as during the last thirty-five years.

Wind Direction.

The number of days the wind blew from the eight principal points of the compass is shown in Table XXXII. which summarise the results of all the wind observations taken in Edinburgh. During the five years under review, the mean percentage frequency was

N. 5, N.E. 8, E. 12, S.E. 9, S. 9, S.W. 20, W. 28, N.W. 9. The wind vane on the steeple of St Giles' Cathedral was the instrument employed in the determination of the direction.

Gales.

In addition to the direction the force of the wind is also given. The scale ranged from 0 to 4. The days on which the force was entered as 3 or above were picked out for the five years. The total was 154, equal to an annual average of 31. Their distribution throughout the year is shown in the following table :—

Jan.	Feb.	Mar.	Apr.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.
13	33	19	9	12	1	6	4	14	12	15	16

Fog or Mist.

The total number of fogs recorded was 57, an average of 11 per annum. They were distributed throughout the year as follows :—

Jan.	Feb.	Mar.	Apr.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.
6	0	7	11	0	3	1	2	5	3	11	8

The cold weather fogs of winter and those associated with the easterly winds of spring manifest themselves very clearly during the period under review.

Thermal Windrose.

The mean temperature of the winds is given in Table LII., the observations utilised being those taken at 9 A.M. The 8 A.M. observations taken by Hoy at Hawkhill for seven years, 1770–1776, have also been analysed with reference to the temperature of the various winds (see Table LIII.). The values given in the Tables refer to the months and the seasons, and it may be pointed out that the latter means are not the averages of the months comprised in the season but have been derived by taking the gross totals and dividing by the number of days, which gives the true average temperature of the wind. A comparison of the seasonal values derived from these old registers with similar means calculated from the 9 A.M. and 9 P.M. observations from July 1887 to June 1894 * gives the following results :—

	Spring.		Summer.		Autumn.		Winter.	
	Coldest.	Warmest.	Coldest.	Warmest.	Coldest.	Warmest.	Coldest.	Warmest.
	°	°	°	°	°	°	°	°
1731–36	N.W. 43·0	S.W. 48·8	N.E. 55·0	S. 61·0	N.W. 40·3	E. 46·8	N. 32·0	S.W. 40·8
1770–76	N.W. 38·9	S.W. 46·4	N. 55·7	S. 59·4	N.W. 42·6	N.E. 49·8	N. 33·4	S.W. 39·2
1887–94	N. 40·9	S.W. 47·2	E. 52·7	S.W. 58·6	N. 42·3	S.W. 50·5	N. 33·7	S.W. 43·3

* *Trans.*, vol. xxxviii. p. 750.

Hence the relative temperature of the winds has not appreciably changed during the last 160 years. The results are very accordant except the direction of the warmest point in Autumn, which was respectively E. from 1731–36, and N.E. from 1770–1776, while it was S.W. from 1887 to 1894. I incline to the belief that the unusual warmth of the sea winds during the earlier years is to be accounted for by the undue prevalence of anti-cyclonic weather in these months. It is evident that when we are calculating the mean temperature of a wind from a few values that the result will largely depend on the type of weather which predominated during the time the wind in question prevailed.

The number of observations tabulated in the calculation of the windrose from 1731 to 1736 was 1,826, from 1770 to 1776, 2,557 were employed, while during the seven years ending June 1894, 5,114 were utilised, so that it is evident that the latter average gives the closest approximation.

An inspection of the thermal windroses for the three periods will reveal many points of similarity (see Plate IV.).

Hygrometric Windrose (*Table LIV.*).

The mean relative humidity of the winds has been already determined from the 9 A.M. and 9 P.M. readings of the dry and wet bulb thermometer for the seven years ending June 1894. Values have been calculated for the five years 1731–36 with a view of ascertaining whether any change has taken place in the humidity recorded with the various winds. We cannot compare the actual means, but the following Table showing the dampest and driest directions for the four seasons may be of interest :—

	Spring.		Summer.		Autumn.		Winter.	
	Dampest.	Driest.	Dampest.	Driest.	Dampest.	Driest.	Dampest.	Driest.
1731–36	E.	N.W.	N.E.	S.	S.E.	N.W.	N.E.	N.W.
1887–94	E.	N.W. N.	N.E. E.	N.W.	N.E. E. S.E.	S.W. N.W.	E. S.E.	S.W.

There has, therefore, been no change of any importance in the wind with which our greatest and least humidities are experienced. Taking the mean annual values, the dampest wind from 1731–36 was N.E., and the driest N.W., while during recent years the points were E. and N.W. respectively. Sea winds were thus damp, and land winds dry, a result entirely in accordance with recent observations (see Plate IV.).

General Results.

An examination of the facts apparent from a comparison of the reduced values for the five years 1731–1736 with observations taken during recent years, shows conclusively that no appreciable alteration has taken place in the climate of the east of Scotland

* *Trans. Roy. Soc. Edin.*, vol. xxxviii. p. 751.

during at least the last 165 years. The seasonal distribution of pressure, temperature, wind and rain is the same now as at the beginning of the eighteenth century, and so far as we can ascertain there has been no change in the annual means of the more prominent elements of climate, while the prevalent weather of special months does not appear to have altered in the slightest. These conclusions are entirely in accordance with what we should expect. As is well known, the climate of a place is largely determined by the prevailing winds; these in turn are simply the result of the distribution of the weight of the earth's atmosphere over the globe. The latter is determined by the position and extent of the land and water surfaces, and as these have not materially altered within the last 200 years it may fairly be assumed that the circulation of the air and the climatic results springing therefrom are practically unchanged. Local influences, more especially drainage and deforesting, produce slight changes in climate; but so far as Edinburgh is concerned no alteration appears to have taken place during the last century and a half.

Does the Weather Move in Cycles?

Tables of continuous five year averages of the more important climatic elements have been calculated with the primary object of giving an answer to this question. The method adopted was as follows:—The mean temperature of the five Januarys 1764–68 was calculated and found to be 1°·7 below the normal temperature of the month; the difference, 1°·7 was accordingly entered in Table LV. opposite the year 1766 which is the middle year of the series. Then the mean of the five Januarys 1765–69 was similarly ascertained, and entered in the Table opposite 1767, and so on for each of the 129 groups of five year periods embraced in the 133 Januarys, means above the normal being entered in heavy and those below it in italic type. The eleven months and the year were similarly dealt with. The data discussed comprise temperature, pressure, wind direction, and rain (see Tables LV. to LVIII.). With regard to rainfall, the inquiry has been extended back to the year 1766 by differentiating during the missing years from registers kept at Peebles, Dumfries, and Branxholm. For copies of these registers I am indebted to Mr G. J. Symons, F.R.S. The hiatus thus completed comprises the years 1766–1769, 1777–79, July 1781 to Dec. 1784. The Peebles register was employed during most of this period, the rainfall of that spot approximating closely to the mean rainfall of Edinburgh during years which are common to both series. The mean monthly rainfalls for the period 1766–1896 were then ascertained, and the percentage of excess or defect calculated for each of the five year groups. Similar values were computed showing the percentage excess or defect of east and west winds. The winds were resolved to these two points by including S., S.W., W., and N.W. winds under west, and N., N.E., E., and S.E. winds under east. During some years calms were entered; these were, however, eliminated from the discussion. It is not necessary to give the table showing the percentage excess or defect of east winds as they are simply the converse of west winds.

The annual values of the non-instrumental phenomena during the last 127 years, 1770–1896, have also been discussed (Table LIX.). The results are graphically shown along with other data on Plate III., the monthly departures of pressure, temperature, wind, and rain being shown on Plates I. to III.

It may be here mentioned that we do not at present intend to discuss at any length or with any degree of elaboration the peculiarities, resemblances, and contrasts shown by an inspection of the diagrams, but merely to point out some of the more prominent features.

Dr Buchan, in the results of an investigation into the mean temperature of the N.E. of Scotland,* says :—"The tendency of types of high and low temperature to be prolonged through terms of years, very unequal as regards duration, is shown, both as regards the months and the year, in a manner so decidedly as to suggest no appearance of a temperature cycle." I have only to add in this connection that the above remark is equally applicable to pressure, wind, and rain, as well as to the non-instrumental phenomena. The most casual glance at the diagrams will establish the truth of this assertion. There is, apparently, no periodicity in the recurrence of weather. If such a period could be found weather-forecasting would be a very simple matter, as it would only be necessary to have observations over one of the periods. Our weather, as is well known, is the result of the distribution of cyclonic and anti-cyclonic areas over western Europe and the adjacent parts of the Atlantic. The average path pursued by, and rate of motion of these areas, are known, but they are subject to many irregularities. In winter, for example, the normal condition of pressure in our immediate vicinity is low to the N. and W., and high to the S. and E. The result of this pressure distribution is a predominance of warm equatorial winds, the atmospheric flow being from the Atlantic Ocean towards the interior of the Eurasian continent. In some winters, however, as in that of 1895, the normal distribution of pressure is reversed, with the result that the whole wind system of Europe passes from N.E. to S.W., the prevailing winds being therefore from the N. and E. Little or nothing is at present known regarding the causes concurring in the production of these weather anomalies. All that can be done in the meantime is to steadily accumulate and reduce observations.

The following is a condensed abstract of the prominent weather conditions prevailing during the past 133 years, the time under discussion being divided when possible into periods of twenty years.

1766 to 1780.—This period was characterised by low pressure, there being also a great depression of temperature till 1776. The cold was pretty evenly partitioned throughout the months and the seasons, a noticeable feature, however, being the mildness of the Decembers. From 1777 to the end of this period very warm weather prevailed which culminated in 1779. Rainfall was above the average during the time, the excess being largely brought about by the wetness of the autumns and winters. After 1770, equatorial winds predominated. Snowfall was about normal, but hail,

* *Jour. Scot. Met. Soc.*, vol. ix. p. 227.

thunderstorms, and fog, were distinctly below the average. Gales were slightly in excess during the years of low pressure.

1781 to 1800.—Pressure was above the mean till 1787, and then below the normal as a whole. Temperature was low during the period of high barometer, but thereafter much above the average. Speaking broadly, the temperature was below the normal from October to March, and above it during the other half of the year, the exceptional warmth of the summers being a striking feature. The unusual depression of temperature during some of the Marches and the Decembers is also of interest, the cold being brought about by the unusual excess of polar winds during these months. During most of the period rainfall was in excess. There was a marked deficit of westerly winds till about 1794. Snowstorms were frequent. Hail was just the average, and thunderstorms, gales, and fog, very much below their normal frequency.

1801 to 1820.—The weather of this period was characterised by a low barometer, a low mean temperature, a deficiency of rainfall, and a marked deficiency of westerly winds. Warm summers continued to prevail till about 1808, but thereafter the depression of temperature manifested itself in a prominent degree throughout the months and the seasons. In the heart of this great cold occurred some of the worst harvests of the century. The outstanding feature of the meteorology of the period under review was, however, the frequency of hyperborean storms of the first order, these snowstorms being of a severity, extent, and duration which have not been equalled since. Hail was above, but thunderstorms on the whole below, the average. Gales were greatly above the normal, while fogs were rare, except from 1805 to 1811.

1821 to 1840.—The characteristic features of this period were a rather high pressure, normal rainfall, and excess of temperature. West winds were above the normal from 1820 to 1826 and from 1831 to 1836. Snowstorms show a decided excess from 1836 to 1840, during which time polar winds prevailed with a low temperature. Pressure was also low, and rainfall above the average during the time. With regard to the non-instrumental phenomena, hail, thunderstorms, gales, and fog, were all above their average frequency.

1840 to 1860.—Low pressure prevailed with a very high temperature and small rainfall. The wet Junes and dry Aprils, Septembers, and Decembers, are striking features of the meteorology of this period. West winds show a marked excess after 1848. Snow and hail storms were infrequent; thunderstorms about the average; and fog much in excess of the normal. Gales were of common occurrence till 1850.

1861 to 1880.—Pressure was above the average with but few and unimportant interruptions. Temperature was below the average from 1861 to 1866, and after 1876. During most of the time cold summers prevailed, the winters on the whole being mild. Rainfall was much above the average. West winds were greatly in excess during the first cold period, but in defect during the second spell of low temperature. Snowstorms were on the whole infrequent. Thunderstorms show an enormous excess after 1868

with a slight dip during the time of maximum cold. Gales were below, but fog much above the normal.

1881 to 1894.—Pressure was much above the normal; the mean temperature, however, being just about the average during the ten years ending 1890, when the warm winters were balanced by the cold summers. A drought prevailed during most of the time. West winds were in excess, thunderstorms much above average, and gales above the normal till 1888. During most of the time fog was uncommon.

Frost Days.

Table LX. shows the number of times the minimum temperature in shade fell to or below 32° in each month during eighty-one years; the data from 1802 to 1823 were obtained from the *Edinburgh Advertiser* record, while ADIE'S observations were utilised from 1824 to 1831 and from 1840 to 1851. The values from 1857 to 1896 are from the observations taken by the Edinburgh observers of the Scottish Meteorological Society. The total number of frosts recorded was 5294, equal to an average of 65 per annum. The annual number varied from 108 in 1879 to 33 in 1822. The maximum in one month was 29 in January 1814.

Table LXI. shows the values grouped according to winters, with date of first and last frost. The maximum number of frosts was in the winter of 1878-79 with 116 cases, and the minimum in the winter of 1821–22 with 28 cases. The mean date of first frost is October 23, and the mean date of last frost, April 26. The earliest date of first frost was September 22 in 1844, and the latest December 4 in 1811. The latest date of last frost was June 8, 1814, and the earliest March 12, 1874.

Table LXII. shows the values for each day in the year.

Table LXIII. shows the number of times the minimum temperature fell to 20° or below. The total number of cases was 239, equal to an average of 3 per annum. The greatest number in any year was 19 in 1881, while there were twenty-two years without any. The maximum in one month was 14 in January 1814, closely followed by January 1881 with 13 instances. The earliest date was October 15, 1824 with a minimum of 20°·0, and the latest April 2, 1831, when the temperature fell to 17°·0.

APPENDIX OF REMARKABLE ATMOSPHERIC PHENOMENA.

The appended catalogue of phenomenal atmospheric occurrences in Edinburgh has been compiled from a variety of sources. Some of the notices prior to 1740 have been obtained from such works as CHAMBERS' *Domestic Annals*, LOW's *Natural Phenomena and Chronology of the Seasons*, and SHORT's *General Chronological History of the Air*. The method generally adopted, subsequent to 1764, was to examine the manuscript notes of the various Edinburgh observers, extract anything of interest, and go to the newspapers for further particulars. Copious extracts were made from such papers as the *Caledonian Mercury*, *Scots Magazine*, *Edinburgh Magazine*, *Edinburgh Advertiser*, and *Scotsman*. Condensed abstracts were then prepared and entered in the catalogue. The primary object in compiling this list is to place on record the more noteworthy and remarkable atmospheric occurrences. In this way, should any apparently unwonted phenomenon occur, we shall at once be able to form an opinion as to whether it is unprecedented or otherwise. In dealing with such a long period it would be sanguine to imagine that every occurrence of a phenomenal nature has been brought to light. If any omissions come to the reader's notice, I should be much obliged for a reference. Care has been taken to avoid giving facts that are readily apparent from an inspection of the various tables scattered throughout the paper.

Year.	Phenomenon.	REMARKS.
1575 and 1582	Drought	MAITLAND informs us that in these years there was such scarcity of water that the Magistrates strictly prohibited all the brewers from drawing any out of the town wells, "but to fetch what they had occasion for from the South Loch or Meadows."
1595	Snow	March 10. Commenced "ane horrible tempest of snaw, whilk lay upon the ground till the 14[th] of April thereafter."
1595	Dearth	Dearth owing to failure of the harvest.
1596	Storms	July 1 to August 6. Severe gales; no less than sixty-six ships lost at Leith.
1598	Eclipse	February 17. Total eclipse of sun between 9 and 10 A.M.
1609	Storm	January 5. Severe storm; people lifted off the ground by the violence of the wind.
1614–15	Frost	Very intense frost. "In February the Tay was frozen over so strongly as to admit of passage for both man and horse."
1615	Snow	March 2. A great snowstorm; all communication stopped throughout the country.
1624–25	Frost	Hard frost from Martinmas 1624, which lasted till February 23, 1625.
1625	Storm	March 28–30. Severe storm; many vessels lost at Leith.
1625	Rains	Heavy rains prevailed from the middle of May till the end of June, doing serious injury to the crops.
1627	Rains	July. Great falls of rain.
1633	Snow	February 7. "There began a great storm of snow, with horrible high winds." The ordinary ebb and flow of the tide interrupted for twenty-four hours at Leith and other places on the East Coast.
1634–35	Frost and Storms	The winter is described as "the most tempestuous and stormy that has been seen in Scotland these sixty years past." Snow lay from the 9th of December to the 9th of March, the fall being particularly heavy from January 26 to February 16.

Year.	Phenomenon.	REMARKS.
1652	Total Solar Eclipse	March 29. Eclipse observed between 8 and 11 A.M., the sky being perfectly clear. This day was long known as "Mirk Monday."
1652	Hot summer	September. Very hot summer and plentiful harvest.
1655	Storms	February. Severe and protracted storms, followed by a frost which continued till April.
1655	Rains	August very wet, threatening the crops with destruction.
1655	Storm	December 10. Great gale from N.E.; many ships lost and much damage on land.
1659	Thunderstorm	September 1. Great thunderstorm with very heavy rain. Sixteen mills on the Water of Leith were destroyed.
1664	Comet	December. Remarkable comet, "in the head the breadth of ane reasonable man's hand, and sprang out in the tail the length of five or six ells."
1667	Drought	Severe in summer; grass burned up.
1668	Storm	October. Violent storm; many ships lost.
1673	Rains	Very wet summer.
1674	Snowstorm	February 20 to March 4. Great fall of snow, long remembered as the "Thirteen Drifty Days."
1675	Frost	December 18. Great cold; "the most aged never remembered the like." Ale froze.
1681	Drought	From March to June 24. Severe drought, with continuance of searching easterly winds.
1683–84	Frost	Severe frost from November to March.
1684	Snow	Gale at end of October with snow and thunder.
1698	Cold Spring	An "unkindly cold and winter-like spring;" great want of food and seed; sheep and cattle died in great numbers.
1709	Dearth	May. There was at this time a dearth of victual in Scotland.
1715	Eclipse	April 22. Total eclipse of the sun at 9 A.M. The darkness lasted over three minutes.
1717	Thunderstorm	June 10. Severe thunderstorm. A man and woman were killed instantaneously, and a gentleman so severely scorched that he died in a few hours.
1722	Gale	September 1. A high wind shook the crops in the Lothians, doing particular damage to the pease.
1723	Drought	Summer remarkably dry and sultry, with little wind.
1732	Snow	May 1. A great fall of snow.
1732	Frost	May 2. Ice so strong as to bear man and horse. Lambs succumbed to the excessive cold.
1732	Gale and Lightning.	September 10. Violent hurricane of wind and rain between 5 and 6 P.M. Very vivid lightning, "so that it appeared as if the whole horizon had been in a flame (which continued for about four minutes); the like has not been seen here in the memory of the oldest man living."—*Caledonian Mercury*.
1736	Gale	November 12. Great gale from N.W.
1736	Frost	November 12–18. "Frost so severe that in 24 hours after it began persons were walking on the lake."
1736	Aurora	November 13. Brilliant aurora.
1738	Frost	In December 1738 and January and February 1739, very severe frost. Snow lay deep on the ground for six weeks.
1740	Hurricane	January 14. Hurricane from W.S.W. commenced at 1 A.M. accompanied with lightning. Sheet lead torn from roof of St Giles' Cathedral, and blown like paper through the air. Great damage to property; many chimneys blown down, and streets strewn with tiles and slates. Trees which had stood at Penicuik for 200 years blown to the ground.
1740	Snowstorm	May 4. Great quantity of snow.
1744	Thunderstorm	August 13. Severe thunderstorm; several people and cattle stunned; very heavy rain and hail fell, flooding streets and cellars. The steeple of Liberton church was struck by lightning, and in the east end of the church a smooth round hole was made in one of the windows by a hailstone, some of which were nearly 2 inches in diameter.

Year.	Phenomenon.	REMARKS.
1765	Snowstorm	January 24. Severe snowstorm; many lives lost in the Border counties.
1767	Thunderstorm	January 3. Thunderstorm from 9 P.M. continued till early morning of 4th.
1768	Snowstorm	January 2. Great fall of snow, with thunder and lightning late on evening of 2nd and morning of 3rd.
1768	Sudden Thaw	January 14. "Owing to a sudden thawing of the ice on the Water of Leith, it came down in great quantities into the harbour and did much damage to vessels there."
1768	Thunderstorm	July 30. Severe thunderstorm at 11 A.M. lasting one and a half hours. Royal Infirmary struck, the glass in four windows being broken. Three men injured.
1769	Aurora	October 24. Red aurora seen in south at 8 P.M.
1769	Comet	Well observed in August.
1773	Hurricane	January 20. Severe W.S.W. gale, blowing a perfect hurricane between 3 and 5 A.M. A stack of chimneys on the west gable of a house situated in Gosford's Close, Lawnmarket, fell and killed three persons. A whole range of the heavy stone balustrade of the (then) New Bridge was fairly shifted from its position, carried down to the foot pavement in a regular order, and most of the stones broken.
1773	Aurora	July 26.
1773	Aurora	September 11.
1774	Aurora	March 3, 14.
1774	Aurora	August 10.
1774	Aurora	November 13 and 14.
1774	Aurora	December 25.
1775	Aurora	February 21.
1776	Frost	Very severe frost all January. Ice on lochs 12 to 16 inches thick at beginning of February. On January 31 it is recorded that on this morning, at Hawkhill House, the milk froze in milking the cow.
1776	Aurora	March 28.
1776	Aurora	April 16, 20.
1778	Aurora	February 25.
1778	Aurora	March 31.
1778	Aurora	April 15.
1778	Aurora	October 13, 14, 19.
1779	Aurora	February 10, 13, 14, 15.
1779	Aurora	March 24, 25.
1779	Aurora	April 8, 9, 22.
1779	Aurora	July 15.
1780	Aurora	February 29.
1780	Aurora	November 19, 22, 23.
1780	Aurora	December 19.
1781	Aurora	April 25.
1783	Thunderstorms	July 2 and 10. Very severe.
1784–85	Protracted cold	"A meteorological correspondent assures us from observation, that from the 18th of October 1784 till the present time, which is a period of 143 days, there have been only 26 in which the thermometer has not been from 1 to 18 degrees and a half below the freezing point, which is a more constant succession of cold weather than has been known in this climate. Last year there were 89 days of frost, and in the year 1779 there were 84; in 1763 there were 94 days of frost, and in the celebrated winter of 1739 there were only 103, which are 12 fewer than in the present winter."—*Edinburgh Magazine.*
1786	Thunderstorm	July 26. Great storm from 3 to 7 P.M., with heavy rain and hail.
1786	Earthquake	August 11. Slight shock in Edinburgh and Leith; severe in other places.
1787	Rainstorm	December 9. Great rainstorm, 4·20 inches falling within 24 hours. Much damage done in Leith harbour.
1789	Earthquake	September 30. Slight shock.

Year.	Phenomenon.	REMARKS.
1791	Gale	January 13. Heavy gale of wind from 4 to 5.30 A.M., attended with rain and flashes of very vivid lightning from the S.W.
1792	Lightning	January 20. A flash of lightning came down the chimney into the porter's room in Watson's Hospital, doing slight damage.
1795	Great frost	January 20–27. Continued snowstorm. Mail coaches delayed. February 9. In the High Street a woman was dangerously wounded on the head owing to a huge mass of snow falling off the roof of one of the houses. February 11. Very heavy snowfall; so deep was the snow that the hackney coaches were frequently obliged to draw with four horses. Mail coaches snowed up. February 12. The snow lies excessively deep in the streets of Edinburgh and in the neighbourhood. Three hundred soldiers and labourers employed by the Magistrates to clear the roads to the coal-hills. February 14. A gentle thaw commences, with the thermometer from 34 to 40 degrees; this, however, is soon again succeeded by frost. Frost broke up on 3rd of March, having lasted 53 days.
1795	Gale and rainstorm	November 18. Severe N.E. gale with great rainstorm, supposed to have been the worst for 30 years. About 10 A.M. the Water of Leith rose to such a height that the low grounds adjacent to it were submerged; bridge at Bonnington Mills swept away; ground floors of houses in back of Canongate, Cowgate, etc., submerged; roads impassable. Meadow near Hope Place like large lake. P.M., snow.
1796	Storm	January 23. Severe storm from S.S.W. that blew down trees and unroofed houses.
1796	Lunar rainbow	December 27. This evening, about five minutes before ten o'clock, there was observed in the neighbourhood of Edinburgh a most beautiful prismatic rainbow of considerable extent, in the north-west quarter of the horizon, directly opposite to the moon, then two days past full, and shining very dazzlingly from the south-east through cold, stormy, flying clouds or showers. This phenomenon, which is believed to be a very unusual one, continued with little alteration for more than five minutes, differing nothing in appearance from a faint *solar* rainbow, the red, yellow, and green colours, and even a shade of the blue or purple being distinctly marked, without any resemblance whatever to an Aurora Borealis.
1797	Thunderstorm	July 14. Sharp thunderstorm; "a flash of lightning darted down the chimney and entered a room on the ground floor of a house in the Water of Leith village near this city." A girl eleven years of age was burnt in a severe manner. A number of copper and iron articles which were near the chimney changed colour.
1799	Snowstorm	February 9. This day "was remarkable for the most violent storm of wind and snow that is remembered in this country."—PLAYFAIR.
1799	Cold summer	The period from the 20th of March to the 20th October was characterised by a great depression of temperature, so much so that the harvest was not generally got in till the end of November, and in high grounds till nearly the end of December.
1800	Snowstorm	January 2. Heavy fall of snow accompanied by a strong gale from the S.E. Snow lying from 2 to 3 feet in depth. Great damage on east coast; many vessels lost. It was computed that 80 seamen belonging to the port of Aberdeen alone perished on this occasion.
1800	Frost	February 7 to 14. Severe frost; the new basin at Leith was nearly covered with ice. Severe snowstorms in England, the London mail due on the 14th not arriving till Wednesday the 17th.
1801	Rainstorm	September 4. Exceedingly heavy shower of rain at 7 P.M. "The heaviest shower in my remembrance."—WATERSTON.
1801	Earthquake	September 7. Slight shock of earthquake felt in Edinburgh at 6 A.M. Beds, tables, chairs, etc., shook violently in some houses. The motion was from N. to S.

Year.	Phenomenon.	REMARKS.
1801	Meteor	December 5. A little before midnight, a large meteor, with a globular head and a long tail, was seen, the whole atmosphere being surrounded with a blaze of light, so that the smallest object could have been picked up on the streets. It was seen for about two seconds.
1801	Aurora	December 5. Very fine red and violet aurora. "During the evening a whizzing kind of noise was heard in the air, exactly similar to the sound which always accompanies the electric spark from the glass cylinder to the conductor. During the time when the coruscations were most vivid, the top of St Giles' steeple seemed to emit rays of light in all directions (St Elmo's Fire?), in every respect similar to a glass jar when surcharged with the electric fluid."
1803	Gales	From January 8 to 10 a severe gale blew, doing much damage to the shipping along the east coast.
1806	Thunderstorm	August 9. A storm, exceeding in violence perhaps anything in remembrance, was experienced at Edinburgh and the neighbourhood. The thunder and lightning continued, without intermission, from 2 o'clock in the afternoon till past 8 o'clock in the evening. The lightning was forked and extremely vivid, and the peals of thunder tremendously loud. The rain fell in torrents, and continued to fall till 5 o'clock on Sunday morning. The storm was preceded by a heavy gust of wind, which seemed to darken the atmosphere by the quantity of dust it hurled into the air. The morning was very sultry, and the thermometer stood at 73° in the shade. During the storm a most violent squall of wind arose from the south-west, which overset and sunk a pleasure boat, belonging to a gentleman in South Queensferry, then near the island of Inchcolm. The owner of the boat, his servant, a skipper, and two tradesmen, all residing in Queensferry, were on board, and all unfortunately perished. On Sunday different boats and expresses were despatched from Queensferry in quest of them. The Ferry Custom-house boat found one of the oars, the water ballast-box, and two deals, used as tables. A vessel off St Abb's Head had her mast shivered. WATERSTON describes this as "one of the worst storms in my remembrance."
1807	Gale	September 6. Strong northerly gale with very heavy rain. Much corn swept away in vicinity of Edinburgh.
1807	Comet	October 4. Comet observed. It continued visible till the beginning of November.
1807	Frost	November. An exceptionally cold month, mean temperature 34°·0. "The quantity of snow fallen and the number of frosty days this month, as also the circumstance of the Clyde being frozen at Glasgow, and the Tweed at Kelso, are said to be unprecedented in the memory of the oldest inhabitant so early as November."—WATERSTON.
1808	Snow	April 8 and 22. Heavy snowfall, the depth in Edinburgh being over half-a-foot.
1808	Thunderstorm and hail	May 7. The hailstones to-day were of uncommon size, some being half-an-inch in circumference.
1808	Great heat	July 13–15. Very hot, the thermometer varying from 76° to 86° in the shade. In London the temperature rose to 100°.
1808	Snow and meteor	October 14. Heavy snow fell in morning to the depth of 6 inches. At 7.30 P.M. a meteor passed over the city.
1808	Gale	October 21. Heavy S.W. gale. Building at foot of the Mound containing model panorama of the Battle of Trafalgar blown down.
1808	High tides	November 17–20. The tides at Leith were of uncommon height. Tides equally great are on record, but *four successive tides* of such height and impetus no one recollects to have observed.
1808	Snow	Dec. 23, 24. Heavy snowfall; depth on the average being 9 inches.
1809	Great cold	January . By the end of December, the large quantity of snow which had fallen in that month had disappeared from off the ground. The wind, however, remained chiefly at E. and N.E. On 2nd January, the cold

Year.	Phenomenon.	REMARKS.
1809	Great cold—*contd.*	became pretty severe, and it continued so for several succeeding days, accompanied with much drifting snow, and some hail. On the 7th, the wind veering for some time towards the south, a gentle thaw commenced. This continued till the 12th, when frost again set in. The quantity of snow near Edinburgh, was, at this time, nothing to what occurred to the north of the Forth. Between Queensferry and Kinross, it lay from 6 to 10 feet deep for many days. On Wednesday the 18th, in the evening, the frost became exceedingly intense, the mercury in Fahrenheit's thermometer falling as low as 11° or 21° below the freezing point in the neighbourhood of this city. At Foxhall, about eight miles west from Edinburgh, in a window exposed to the current of air from the N.E., it was observed as low as 6°, or 26° below the freezing point. During the three following days, the thermometer indicated from 22° to 28°. Sunday the 22nd was one of the coldest days in the remembrance of the present generation. At 8 o'clock in the morning, in this city, the mercury stood at 11°. A little way from town it was observed at 8°. In Queen Street, exposed to the north, it was as low as 6°. In the evening it was perhaps still colder; for, at Foxhall, it was noticed as low as 5°, or 27° below the freezing point. The large basin of the new harbour at Leith, though filled with *salt* water, was so completely frozen over, that the sailors could pass from ship to ship upon the ice. From the 22nd to the 25th, the thermometer varied from 15° to 25°. January 26. The intensity of the cold began this day to abate. Snow fell copiously, drifting in some places to the depth of many feet. The ice on the lakes in this neighbourhood has been observed to be from 18 to 22 inches thick. January 27. In the morning the mercury rose 15° above the freezing point; and a breeze springing up from the S.W., the snow began to disappear rapidly. January 29. "Squalls from S.W., accompanied with heavy showers of rain, have produced so rapid a solution of the immense quantity of snow which covered the high grounds, that all the meadows are flooded, and the level parts of the country around Edinburgh appear as if spotted with small lakes."—NEILL.
1809	Snow	May 29. A heavy fall of snow and hail has rendered the whole country around Edinburgh quite white. The snow and hail continuing at intervals on the 30th and 31st, in some places, to the south of this, lay on the ground a foot and a half deep.
1809	Thunderstorm	August 3. At half past 7 P.M. a thunderstorm passed over Leith and Edinburgh. The lightning killed a boy at the former place.
1809	Meteor	August 11. At half past 9 P.M. a meteor was seen in the north-west. It appeared about the same time at Glasgow.
1809	Thunderstorm	August 13. Violent thunderstorm with torrents of rain at 1 P.M. Streets inundated, and part of a garden wall at the west end of Queen Street washed away.
1810	Snowstorm	January 15. On this day a heavy fall of snow came on. On the following morning the depth around Edinburgh was 18 inches. So much has not fallen in such a short space of time for fourteen years past, since the remarkable winter of 1795. The fall was but local, extending little beyond Dunbar to the S. and Glasgow to the W. Thick fog on the 20th, which covered trees and shrubs with beautiful frost crystals.
1810	Snow	May 4–7. A good deal of snow and hail has fallen, with the wind from E. and N.E.
1810	Meteor	August 28. A few minutes past 12 P.M. a very brilliant meteor appeared here, in the S.W., and rapidly proceeded in a north-easterly direction. Its apparent size and shape were those of a large tun or hogshead. It had stripes or bands of bright light along its sides, which continued a short way beyond the ball, and formed a sort of fringed tail. No noise was heard.

Year.	Phenomenon.	REMARKS.
1810	Gale and lightning	December 20. After a rapid thaw and change of wind, at half past 10 P.M., with a strong south-westerly gale, and heavy showers of rain, there occurred a great deal of lightning. It continued with little intermission for several hours. No thunder was heard.
1811	Snow and frost	April 7 and 8. Intense frost, with showers of hail and snow, and a strong gale from N.W., continued for these two days. The mercury in the thermometer was several times at 24°, and once as low as 20°, or 12° below the freezing point. Vegetation had been proceeding rapidly, and has thus met with a very severe check.
1811	Thunderstorm	On the afternoon of the 8th June, a thunder-cloud passed over the south side of the city of Edinburgh, making frequent discharges of electric matter into the earth. A house situated in Fountainbridge Street, at the head of the Lothian Road, unfortunately became the conductor of one of these discharges. The fluid penetrated one of the chimneys shattering many large stones, and projecting some of them violently to a distance. The fluid passed through several apartments in the house, conducted chiefly by the bell wires, which it twisted, melted, and oxidated in its progress. The gilding on a picture-frame was partly converted into a purple oxide; and a large oil-painting was completely disfigured, the oil and colours having undergone chemical changes. A maid-servant was slightly struck by the fluid, as appeared from arborescent marks on one of her shoulders, and at the same time she was severely scorched by her clothes catching fire.
1811	Thunderstorm	June 25. This morning, another thunder cloud passed in the vicinity of this city, making very frequent discharges. A labouring man, going to his work at Craigleith quarry, a little before 6 o'clock, was suddenly and forcibly thrown to the ground, where he lay stunned for some time.
1811	Comet	August 30. A faint, nebulous comet was seen this evening, which continued to grow in brightness till about November 8, when it was of great magnitude and brilliancy.
1811	Tides	October 31. Exceptionally high tide at Leith; much damage done.
1811	High tides	On the night between Friday the 1st and Saturday the 2nd of November the waters of the North Sea rose to a very unusual height. The rise exceeded 20 feet in the Firth of Forth.
1812	Protracted Snow-storm	March 19. Early on the morning of the 19th a sudden and heavy fall of snow took place. In about three hours it lay near a foot thick all around Edinburgh. All kinds of country labour were therefore completely stopped. March 21. A strong gale from N.E., with continued snow, has rendered most of the roads in this neighbourhood impassable. In many places the snow, where drifted, lies 8 feet deep on the roads, hiding hedges and walls from the view. The mail-coaches could no longer make their way, even with six horses. In the valleys about Arthur's Seat hills, the snow, in some hollows, is from 15 to 20 feet deep. March 22. The London mail came into and left town this day on horseback, the roads being so blocked up by snow as to be totally impassable to coaches. March 23. The snow ceased; but this evening an intense frost set in, the mercury in Fahrenheit's thermometer falling to 23°, or 9° below freezing point. March 26. The severe frost still continues, the mercury at 8 this morning standing at 24°, and having been observed, more early, as low as 21°, or 11° below freezing. "Even now (27th March) all the lakes and pools are thickly frozen over, and to see boys skating on the North Loch ditches, on Good Friday, is perhaps rather a novelty."—NEILL.
1812	High Tide	October 21. Very high tide at Leith. Streets inundated.
1812	Snow and Frost	December 10. Heavy fall of snow, followed by intense cold. On the 12th, in the evening, the mercury in Fahrenheit's thermometer stood at $13\frac{1}{2}°$, or $18\frac{1}{2}°$ below the freezing point.

Year.	Phenomenon.	REMARKS.
1813	Drought	September. This month and the last have seldom been surpassed for dryness, sunshine, and warmth. During a period of ten weeks only two rainy days occurred.
1814	Great Frost	On Sunday, 2d January, there was a good deal of rain, but towards evening the atmosphere became clear, accompanied by such intense cold during the night that next morning boys were venturing on the ice which covered the mill-pond at Canonmills. The snow soon after began to fall, and it lay on the open fields about Edinburgh nearly 16 inches deep on an average; where drifted, it was from 3 to 6 feet in depth. The cold was very great, the temperature ranging from 17° to 22°, but at 8 A.M. of the 15th January it was, at Canonmills, as low as 10°·5, while during the early morning it had, in the New Town, fallen as low as 9°. On the 18th, at 8 A.M., a reading of 11° was recorded, and it had at the same time been observed 3° lower in the neighbourhood. At Glasgow it is stated to have fallen to – 5°. The Firth of Forth was nearly covered with floating ice from Queensferry upwards. From the interstices which remained free the vapour ascending from the water became suddenly condensed, producing the appearance of smoke rising from the surface, exactly as is described to happen in Hudson's Bay and West Greenland, at the edges of the ice. Many birds were benumbed with the cold, and easily surprised and caught. The frost continued unabated till the 24th of January, when a gentle thaw commenced. The ice in the vicinity of Edinburgh varied from 7 inches in the large lochs to over a foot on the rivers. During February, alternate frosts and thaws succeeded each other, but the ice which covered the lakes on the 3rd of January did not wholly disappear till the 24th of February, having thus lasted 52 days. The effect of the severe weather was to delay the London mails considerably,—the one on the 13th, due at 7 A.M., not arriving till past 5 P.M. On the 14th and 15th it was also much behind time. From the 17th to the 21st it was one day later, and all the other mails, except from Glasgow, were also one or two days behind. The London mails due on the 22nd, 23rd, 24th, and 25th did not reach Edinburgh till the evening of the 26th, a circumstance unprecedented, there never before having been more than three London mails due at one time. Two still remained due on the 27th, and there was always one behind till after the 31st. Much damage was done to bridges, etc., on the breaking-up of the ice on the Esk and other places.
1814	Heat	August 25. The shade temperature at the Calton Hill observatory rose to 84° at 2 P.M.
1814	Aurora	September 11. Very bright aurora. So bright was the light that it was possible to read a book of a large type.
1814	Lunar Rainbow	September 15. Fine lunar rainbow.
1814	Aurora	September 11. At 7.30 P.M. a column of light was observed stretching along the northern hemisphere from S.W. to N.E., in appearance not unlike the Milky Way.
1814	Frost	November 20. Sharp frost. During the following night the temperature fell to 21°. Next morning the margins of the lochs around Edinburgh were covered with skaters.
1815	Snow and Frost	A severe storm of snow and frost continued from the 20th of January to the end of month.
1815	Gale	January 27 and 28. Severe E.N.E. gale with very high tide at Leith. Solid masonry on the pier broken down by the huge waves.
1815	Aurora	September 26. Aurora.
1815	Thunderstorm	November 24. Thunderstorm with hail.
1816	Snow	April 18. Heavy snowstorm.
1816	Earthquake	August 13. Slight shock at 11 P.M.
1816	Aurora	September 24. Brilliant aurora, assuming the form of a vast luminous arch of purple or red colours. Slight tremulous motions of the light were seen at intervals. The arch extended from N.E. to S.W.

Year.	Phenomenon.	REMARKS.
1817	Aurora	February 8. Brilliant aurora.
1817	Thunderstorm	February 16.
1817	Thunderstorm	June 10. Severe thunderstorm, with torrents of rain and hail from 11 to 12 o'clock. The lightning struck several buildings, including Messrs Ballantine & Co.'s printing office, and a hat factory adjoining. Several people injured.
1817	Thunderstorm	August 26. Severe thunderstorm with heavy rain. The water flowing down the Cowgate for some time presented the appearance of a rapid river. The parapet of the Earl of Moray's pleasure ground was undermined, nearly 30 yards of it giving way.
1817	Lunar rainbow	August 31.
1818	Gale	January 12. Severe gale. Lead stripped off the dome of St George's church, and several other buildings injured.
1818	Gale	January 14 and 15. Another heavy gale from S.W. to N.W. Turret and other ornaments upon the tower of Bishop Sandford's chapel at West end of Princes Street blown down.
1819	Comet	July 1. A very large comet seen, described as "not much inferior in magnitude and brilliancy to the celebrated comet of 1811."
1819	Snow	December 9 and 10. Snow 6 inches deep.
1819	Snow	December 28 and 29. Heavy snowstorm; a foot in depth where not drifted.
1820	Solar eclipse	January 7. "Great eclipse of the sun; weather so thick, hardly visible here. I had some distinct glimpses of it, however, after two o'clock, half-an-hour past the middle of the eclipse."—WATERSTON.
1820	Snow	January 16. Snow 9 inches deep.
1820	Snowstorm	January 19. Snow 18 inches deep, but 2 to 3 feet in drifts.
1820	Gale	January 22. Heavy gale. A caravan weighing about six tons and containing several wild animals blown over at Wombell's menagerie on the Mound.
1821	Gale	November 4. "On the morning of the 4th we had a most severe gale from the N.E., which did great damage to the shipping on the east coast of Scotland."—WATERSTON.
1822	Thunderstorm	July 21. Severe thunderstorm, accompanied by heavy showers of pieces of clear ice. House in Gibb's Entry, Nicolson Street, struck. Lower parts of town flooded.
1822	Gale	September 11. Severe S.W. gale; much damage at sea.
1822	Aurora	November 7. Beautiful auroral arch to north.
1822	Aurora	November 8. Brilliant aurora.
1823	Snow	January 12–27. Heavy snow showers almost every day. Depth on 23rd nearly a foot. "We have had a longer continuance of snow on the ground this month than at any time since January 1814."—WATERSTON.
1823	Snowstorm	February 1–4. Great snowstorm from E.N.E., the heaviest since 1795. During the first week hardly any mails arrived in Edinburgh. On the 9th no less than twenty-two mails were due at the Post Office, six of these London. Hundreds of men had to be employed clearing the roads, the snow where not drifted averaging two feet in depth.
1823	Lunar eclipse	July 23. Total eclipse of the moon.
1824	Gale	October 9–13. Severe gale from N.E. About 150 vessels stranded or lost on the eastern coasts of Scotland and England.
1824	Gale	December 29. Heavy S.W. gale in Edinburgh; blew down house walls, trees, etc.
1825	Thunder	January 2. Some thunder in the morning.
1825	Rapid barometric fall	January 18. Barometer fell an inch in about 10 hours.
1825	Rapid barometric rise	February 6. In twelve hours the barometer rose something more than an inch.
1825	Aurora	March 19. Very fine aurora. (See *Edin. Phil. Jour.*, vol. xiii. pp. 178–179.)
1825	Snow	May 27. Snow and sleet. Pentland hills white on the 28th.

Year.	Phenomenon.	REMARKS.
1825	Drought	July. Very warm and dry; slight showers on the 1st and 15th. "The river Tay hardly ever remembered lower than it has been this month."—WATERSTON.
1825	Aurora	August 17. Aurora seen at 10 P.M.
1825	Aurora	September 11. Aurora at 10 P.M., just after a thunderstorm.
1825	Aurora	October 7. Aurora in the evening, observed synchronously in the north of Scotland.
1825	Aurora	November 3 and 4, that of the 4th being of great beauty.
1825	Meteor	November 14. At 8 P.M. a large meteor was seen to pass from E. to W. through a space equal to 25°. It left a luminous trail behind.
1826	Gale	February 13. Severe S.S.W. gale.
1826	Aurora	March 29. Brilliant aurora.
1826	Heat	June 24–30. "Hotter than any seven successive days in my remembrance."—WATERSTON.
1826	Early harvest	July. Cutting began in the Edinburgh district about the 10th, and by the 29th many fields were cleared. "Harvest mostly finished by middle of August, about the time it usually commences in a tolerably early season."—WATERSTON.
1826	Thunderstorm	August 27. Thunderstorm with very heavy rain.
1826	Dearth	September. "Owing to the sudden rise and acknowledged deficiency of the oats, barley, and peas crops, Government, by an extraordinary Order of Council, have allowed the importation of these articles, the quarterly average struck immediately before not allowing the ports to open. Even hay from Holland has been imported into different parts."—WATERSTON.
1826	Double crop	October. "It is rather remarkable this season that in more than one place two distinct crops of barley have come to maturity over the same ground in succession, one after the other."—WATERSTON.
1826	Snowstorm	November 25 and 26. Dreadful storm of wind and snow from the N.N.W. Several vessels lost. From thirty to forty people perished in different parts of the country, and many thousand sheep.
1827	Aurora	January 9. Aurora.
1827	Snowstorm	March 3–4. Great snowstorm. Snow fell to the depth of nearly 4 feet in 24 hours, with strong east wind; many lives lost both on land and sea.
1827	Snowstorm	April 23 and 24. Snow fell to the depth of nearly 2 feet, with stormy east wind.
1827	Aurora	September 25. Remarkable aurora.
1827	Gale	October 22–23. Severe gale with very high sea at Leith, doing damage to the harbour.
1827	Eclipse	November 3. Lunar eclipse.
1827	Thunderstorm	December 15. Thunderstorm.
1828	Aurora	September 15. Aurora.
1828	Aurora	September 29. Bright aurora.
1828	Lightning	December 31. A great deal of lightning.
1829	Snow	January 24. Snow fell to the depth of a foot.
1829	Aurora	February 18. Brilliant aurora.
1829	Aurora	April 19. Bright aurora.
1829	Rainfall	July. A very wet month, rainfall $6\frac{1}{2}$ inches, half of which fell on the 4th and 5th, when the precipitation amounted to 3·80 inches.
1829	Thunderstorm	July 30. Severe thunderstorm with very heavy rain.
1829	Rainfall	August. Wettest month for many years. WATERSTON gives the rainfall as 8·75 inches. The rain exceeded an inch on the following days:—3rd, 1·90 inch; 4th, 1·40 inch; 19th, 1·11 inch; 22nd, 1·11 inch; and 27th, 1·12 inch.
1829	Aurora	August 19, 20 and 26. Bright aurora.
1829	Lunar eclipse	September 2. Eclipse of the moon, nearly total.
1829	Aurora	September 25. Bright aurora.
1829	Gale	October 13 and 14. Severe gale from the N.E. Several vessels stranded or lost on the east coast, and a very high tide at Leith. 2·50 inches of rain fell.
1829	Gale	November 25. E.N.E. gale, many ships lost.

Year.	Phenomenon.	REMARKS.
1829	Aurora	October 6. Luminous auroral arch.
1829	Aurora	October 22. Bright aurora.
1829	Thunder	November 4. Thunder.
1829	Aurora	November 11. Bright aurora.
1829	Aurora	December 12. Bright aurora.
1831	Aurora	January 7. Bright aurora at 7 P.M.
1831	Aurora	January 11. "Bright aurora at 11 P.M. when I observed it northward and to enlighten St Giles' spire most beautifully. This aurora was seen in Paris."—GAIRDNER.
1831	Aurora	March 4. Aurora to northward.
1831	Aurora	March 7. Serpentine Aurora.
1831	Aurora	March 11-12. At 10 P.M. a tint of light as if the rising moon waning N.W. Zenith in one mass from the horizon, and at 4 A.M. on the 12th streamers were seen to dart up beyond the height of St Giles' in same direction.
1831	Rainstorm	July 15. "Between 3 and 5 o'clock one of the heaviest rains ever experienced in this country."—GAIRDNER.
1832	Thunderstorm	June 13. Alarming thunderstorm, several houses struck and people injured.
1832	Rain	October 8. Heavy rain, 2·2 inches, with strong N.E. wind.
1832	Rain	October 12. Two inches of rain, wind N.E.
1833	Gale	February 20. Severe N.E. gale, many vessels lost. Four or five fishing boats with several men lost in the Firth of Forth.
1833	Aurora	March 17. "At 8 P.M. it was said there was a glowing aurora in the zenith which lasted only 10 or 15 minutes."—GAIRDNER.
1833	Rain	June 11. Two inches of rain.
1833	Aurora	August 18. Bright aurora.
1833	Gale	August 30. Violent N.E. gale; much damage done to the ripe grain by shaking.
1833	Aurora	September 18. Bright aurora.
1833	Aurora	October 12. Splendid aurora.
1833	Lightning	December 2. Lightning to N.W. at midnight.
1833	Thunderstorm	December 6. "At 4 A.M. I was awoke with thunder and lightning with wind."—GAIRDNER.
1833	Lunar Eclipse	December 26. Total eclipse of the moon.
1834	Thunderstorm	January 18. Thunderstorm.
1834	Lightning	January 23. Very squally, with lightning.
1834	High Tide	October 4. The water in Leith dock rose to within a foot of the edge of the quay, and presented the novel spectacle of vessels lying to appearance almost out of the water. Some houses in Baltic Street flooded.
1834	Aurora	October 6. Bright aurora.
1834	Aurora	December 22. Bright aurora.
1835	Storm	January 19. Storm from N.E. did much damage along coast.
1835	Aurora	February 7. At 7 P.M. splendid aurora to N. and W. consisting of two oval amphitheatres. At 8 P.M. it began to blow and rain.
1835	Comet	October 10. HALLEY'S comet distinctly visible in the Great Bear.
1835	Aurora	November 18. Bright aurora N.W. to S.E. at 9.30 P.M.
1836	Gale	January 21 to 23. Severe gale; several persons injured through falling masonry.
1836	Gale and Lightning	January 27 to 29. Severe gale with lightning. Some damage done to the Presbytery Hall, North St David Street, and other buildings.
1836	Thunderstorm	January 30. Thunderstorm.
1836	Storm	March 17. Violent storm, with hail and sleet.
1836	Thunderstorm	July 5. Violent thunderstorm, attended with damage to crops and loss of life in many places. Continued from 10 A.M. to 8 P.M.
1836	Snowstorm	October 28. Snow fell this evening to the depth of 4 or 5 inches. Hard frost continued till the 31st. "Such an early appearance of winter has not been observed for many years, probably since 1782. Harvest has been much protracted, and in some places oats are hardly ripe yet."—WATERSTON.

Year.	Phenomenon.	REMARKS.
1836	Aurora	October 18. Remarkable aurora; bright red and white.
1837	Gale	February 19. Violent S.W. gale. Thunder in the evening.
1837	Snowstorm	Mar. 11. Six inches of snow fell.
1837	Rainstorm	August 2 to 4. "More than 3 inches of rain fell."—WATERSTON.
1837	Aurora	October 6. Bright aurora.
1837	Aurora	October 18. Reddish aurora.
1837	Lunar Eclipse	October 13. Total eclipse of moon.
1838	Frost	In January and February the frost was very hard. WATERSTON says with reference to January, "We have had a longer continuance of frost and snow than any month since February 1823."
1838	Rainstorm	September 6. Two inches of rain.
1839	Ice Accidents	January 18. Skating at Duddingston; three people drowned by the ice giving way.
1839	Aurora	January 14, 16, 19. Bright aurora.
1839	Aurora	September 3 and 4. Bright aurora.
1839	Aurora	October 13. Bright aurora.
1839	Earthquake	October 23. At 10.15 P.M. a sharp shock of earthquake was felt. It was accompanied with no noise and lasted about four seconds.
1841	Aurora	December 14. Bright aurora.
1842	Aurora	February 11. Bright aurora.
1843	Gale	July 3. Severe gale from W.; fruit trees and bushes stripped, and the wall fruit which was fast ripening destroyed. "So wild a tempest has not been experienced at this season for at least twenty years."
1843	Thunderstorm	July 5. Violent thunderstorm; house struck in Lothian Road.
1843	Storm	October 11. Violent storm from the E. did much damage.
1843	Storm	October 28. Severe storm from E.; very injurious to shipping.
1845	Gale	January 25. Severe S.W. gale.
1845	Rainstorm	October 3. Severe rainstorm.
1846	Thunderstorm	June 22. Severe thunderstorm.
1846	Thunderstorm	July 5. Severe thunderstorm.
1846	Gale	November 20. Stormy S.E. gale.
1847	Aurora	September 27. Brilliant aurora.
1849	Meteor	December 18. Bright meteor.
1851	Snow	June 1. Pentland Hills covered with fresh snow.
1852	Aurora	February 18. Fine aurora at 10 P.M.
1852	Aurora	February 19. At 10.30 P.M. aurora over the whole sky, all shooting up to a point very nearly to the true pole, and of red and green colours.
1852	Thunderstorm	December 24. Gale of wind with lightning and thunder, from 4 to 5 A.M. (25th).
1854	Comet	March 29. Brilliant comet in the west this evening.
1854	Thunderstorm	May 9. Thunderstorm with very large hail.
1858	Thunderstorm	June 17. Severe thunderstorm with torrents of rain. A portion of the new road from St Leonards to Duddingston gave way at a point about 200 yards to the east of Samson's Ribs. The lightning struck a house in the Pleasance.
1859	Aurora	August 29. Most brilliant purple aurora australis at 1 A.M.
1860	Storm	October 3. Severe storm; windows of the Church of Scotland Normal School blown in. Scaffolding at a church under repair blown away. Gable end of an old house blown in. Three large trees were blown down in the Meadows and had their trunks broken across, twisted and shattered remarkably. Wind veered from W.S.W. to W.N.W.
1860	Great Frost	December 22 to 28. Great cold prevailed during this period, mean temperature being only 20°·3. The following are the maximum, minimum and mean temperatures for the week :—

Year.	Phenomenon.	REMARKS.
1860	Great Frost—*continued*	Dec. / Max. / Min. / Mean. 22 / 30·1 / 21·5 / 25·8 23 / 30·0 / 21·6 / 25·8 24 / 19·8 / 5·0 / 12·4 25 / 19·0 / 13·0 / 16·0 26 / 23·0 / 8·8 / 15·9 27 / 26·7 / 19·8 / 23·4 28 / 29·2 / 17·0 / 23·1 25·4 / 15·2 / 20·3 Dense fog prevailed with few interruptions. See *Jour. Scot. Met. Soc.* for quarter ending 30th December 1860, pp. 6–14.
1861	Rainstorm	September 23. Great rainstorm; 2·40 inches of rain fell in two hours.
1862	Gale	October 14. Very severe gale in morning.
1862	Aurora	November 17. Aurora in N.W.
1862	Aurora	December 24. Aurora in W.
1863	Storm	February 4. Great storm all day with thunder, lightning, and hail.
1863	Aurora	April 15. Aurora in N.
1863	Aurora	May 8. Aurora in N.W.
1863	Aurora	November 5. Aurora near horizon.
1863	Aurora	December 9. Aurora in S. extending from horizon to zenith.
1864	Gale	October 22. Strong gale with heavy rain. Several buildings damaged, including St Mary's Roman Catholic Chapel, Broughton Street. The Water of Leith higher than it had been for seven or eight years.
1864	Gale	February 13. Severe storm of wind and rain. Much damage to property.
1866	Meteor	November 13. At 2 h. 53 m. (Sidereal time) a very bright meteor descended from near the zenith in a N.W. direction, at an angle of 75° to the horizon.
1867	Aurora	January 11. Aurora in N.W.
1867	Aurora	February 8. Aurora in N.W.
1867	Darkness	September 10. A very dark cloud passed between 11 and 12 noon, so dark that gas had to be lighted.
1867	Aurora	October 2. Aurora in N.W.
1867	Aurora	October 29. Aurora in N.W. near horizon.
1868	Lightning	January 15. Lightning from 7 to 8 P.M.
1868	Hurricane	January 24 (Windy Friday). Great storm of wind reaching the force of a tropical hurricane from 1 to 4 P.M. Gable in Duke Street blown down, cabs overturned, etc. "Many buildings much damaged, 21 instances of injured masonry being reported, the first at 12.15 and the last at 4.30 P.M. In the hour ending 2.20 P.M. nine of the 21 buildings were damaged."—(See *Scot. Met. Soc. Jour.*, vol. ii. pp. 169–180.)
1868	Thunderstorm	February 1. Thunderstorm at 8 A.M.
1868	Aurora	April 27. Aurora between 10 and 11 P.M.
1868	Aurora	October 19. Bright aurora in N.W.
1868	Thunderstorm	November 4. Thunderstorm with hail.
1869	Lightning	March 1. Lightning from 7 to 10 P.M. with small snow.
1869	Aurora	May 13. Very beautiful aurora; radiating from the zenith towards the horizon in all directions.
1869	Storm	June 15. Severe gale from N.E. At 11 A.M. the wind, which was S.S.E., of a sudden chopped round to the N.E., and without warning blew a violent gale. A tremendous sea was raised on the east coast, many shipwrecks occurring with serious loss of life.
1870	Snow	February 25. Very heavy snowfall; depth 13½ inches where not drifted.
1870	Snowstorm	February 27. Severe snowstorm, drifts forming in the streets to the depth of 3 to 4 feet, while the average depth where not drifted was 20 inches.

Year.	Phenomenon.	REMARKS.
1870	Aurora	August 28. Low circular arc to N.N.W
1870	Aurora	August 29. Low circular arc to N.W.
1870	Aurora	September 3. Low arc to N.N.W.
1870	Aurora	September 24. Auroral light, sky clouded
1870	Aurora	September 25. Auroral light, sky clouded.
1870	Aurora	October 14. Low arc to N.N.W. with transverse streamers shooting from N. to S. over all the sky.
1870	Aurora	October 20. Red and green aurora chiefly to N.E.
1870	Aurora	October 25. Grand aurora, red, green and blue over all the sky.
1871	Aurora	February 12 Transverse band, S. and S.E. of zenith, occasionally red and radiating.
1871	Aurora	March 9. Low auroral arch N., 10° W.
1871	Aurora	March 14. Auroral gleam to N.W.
1871	Aurora	March 16. Auroral arch to N.W.
1871	Aurora	March 17. Auroral arch to N.N.W.
1871	Aurora	March 27. Auroral arch, low to N.W.
1871	Aurora	March 28. Auroral arch, low to N.W.
1871	Aurora	April 9. General auroral light through sky heavily clouded; said to have been a red aurora elsewhere.
1871	Aurora	April 12. Upward shooting beams of aurora N. to N.W.
1871	Aurora	April 20. Long low arc of auroral glow to N.W.
1871	Aurora	April 28. Elliptical arches, low to N.W. and N.
1871	Aurora	May 8. Long low auroral arc N. and N.W.
1871	Aurora	August 6. Midnight auroral arc to the N.W.; mean rising to N.E.
1871	Aurora	August 21. Auroral arc, bright and large from W., round by N.W. to N. and N.N.E.; a few dark clouds in front.
1871	Aurora	September 7. Auroral lights N. and N.W., in upward shooting beams.
1871	Aurora	October 14. Auroral light to the N.W. amongst clouds.
1872	Gale	January 1. Severe gale. Property damaged and many people injured.
1872	Thunderstorm	October 30. A thunderstorm of unusual violence from 5 to 6 P.M., with heavy rain and hail causing much flooding.
1873	Thunderstorm	March 14. Thunder and lightning at 8 P.M.
1873	Thunderstorm	July 22. Very severe thunderstorm. An observer of the Scottish Meteorologicial Society counted in one hour 680 flashes of lightning with their accompanying thunderclaps. This gives a rate of fully 11 per minute.
1874	Thunderstorm	June 25. Thunderstorm from 11.35 A.M. to 12.35 P.M. No less than 81 flashes of lightning were observed, of which 43 were seen in the first 26 minutes.
1874	Gale	October 21. Worst gale since the hurricane of January 1868. Much damage done to property all over the city. Several people injured.
1874	Thunderstorm	December 30. Snow showers, with thunder and lightning at 10.45 P.M.
1876	Mock Suns	May 10. Solar halo and mock suns seen.
1877	Great Rainfall	August 18 to 22. Very heavy and continuous rain with east (N.E. to S.E.) winds, 7·07 inches fell in the five days, the amounts for each of the days being as follows, 18th, 1·54 inch; 19th, 0·89 inch; 20th, 1·88 inch; 21st, 1·96 inch; 22nd, 0·80 inch. The Water of Leith overflowed its banks, houses in Warriston Crescent being flooded to the depth of 7 feet. All over the town extensive inundations took place in low-lying situations.
1878	Thunderstorm	June 27. Severe thunderstorm; the lightning struck several buildings.
1878–79	Frost	Very cold winter; skating and curling carried on at Duddingston uninterruptedly for 12 weeks.
1879	Gale	March 4. Severe gale; several buildings damaged, and people injured.
1879	Gale and Rain	July 13. N.E. gale and heavy rain, no less a quantity than 2·95 inches falling within the 24 hours. Flooding throughout the town.
1879	Gale	December 28. Heavy gale; Tay Bridge blown down.
1879–80	Frost	Skating for eight weeks on Duddingston Loch.
1880	Lunar Rainbow	January 1. Fine lunar rainbow between 11 o'clock and midnight, the colours being well defined.

Year.	Phenomenon.	REMARKS.
1880–81	Frost	During this winter there was skating and curling at Duddingston for thirteen weeks.
1881	Fog	January 4. Very dense fog. At night objects could hardly be distinguished at a few yards distance. Vehicular traffic much impeded.
1881	Snow	January 17 and 21. Heavy snowstorms. Traffic seriously deranged.
1881	Rain	August 10. A quarter of an inch of rain fell in 15 minutes.
1881	Storm	October 14. Severe N.E. gale; occasioned much damage of property as well as lamentable loss of life.
1881	Gale	November 15 to 22. Continued gale; numerous casualties.
1881	Meteor	November 15. Brilliant meteor at 5.52 P.M.
1882	Gale	January 5. Severe gale. An unusually large amount of damage done to property throughout the city. Several people injured.
1882	Thunderstorm	March 9. Thunderstorm at 4 P.M.
1882	Gale and Snowstorm	December 5. Heavy east gale with snow. Tramway service suspended.
1883	Gale	January 24. Strong east gale; average velocity of wind 56·6 miles per hour, rising to 70 miles an hour in gusts. Several buildings damaged.
1884	Low Pressure	January 26. Barometer fell to 27·451 inches at 10 P.M., being the lowest recorded since the commencement of pressure observations in 1769. The fluctuations previous to this great depression were remarkable, the following being the corrected readings, 9 A.M. of the 21st, 30·243; 5 P.M. of the 23rd, 28·466; 4 P.M. of the 24th, 29·583; 10 P.M. of the 26th, 27·451; and 11 P.M. of the 28th, 29·598 inches.
1884	Thunderstorm	August 12. Thunderstorm of tropical severity for several hours. The storm was general over the country. Very heavy rain fell.
1884	Gale	February 10. Severe gale "from 5 to 5.20 P.M. the gusts exceeded in force any previous storm here."—BLACKWOOD.
1885	Meteor	April 15. Intensely blue meteor in east at 10.20 P.M.
1885	Meteors	November 27. Grand meteoric shower. A modest computation would be 50 per minute from 5 to 8 P.M. when it reached its maximum.
1886	Thunderstorm	January 16 and 17. Thunderstorm both days, a very unusual circumstance at this season.
1886	Snowstorm	March 2. Severe snowstorm from E. Trains blocked on east coast. On March 4th there were taken out of Edinburgh 1434 cart-loads of snow.
1886	Low Pressure	December 8. Barometer at 32° and sea-level fell to 27·651 inches at 7.30 P.M., having fallen 1·52 inch since 10 P.M. of the previous day.
1887	Heat and Drought	June. A very dry and hot month. No rain after 14th.
1887	Thunderstorm	September 2. At 10 A.M. during a thunderstorm 0·29 inch of rain fell in fourteen minutes, while on the 4th 0·33 inch of rain fell in ten minutes.
1888	Snowstorm	February 19. Fierce N.E. gale with snow and hail squalls.
1888	Snowstorm	March 15. Snowstorm from E.
1888	Fall of Temperature	March 22. Temperature fell 7° in fifteen minutes at 9 A.M.
1888	Gale	November 16. Heaviest gale experienced for many years. Average velocity of wind from 9 A.M. to noon over 50 miles per hour. Very stormy till the 25th.
1889	Earthquake	January 18. Slight shock of earthquake at 6.50 A.M.
1889	Rise in Pressure	February 4. Barometer rose 1·26 inch in 24 hours.
1890	Lightning	January 5. Lightning at 9.10 P.M.
1890	Mock Sun	September 4. Mock sun at 6.30 P.M.
1890	Sunless Weather	December. The sun only shone for seven hours; four and a half hours being recorded on 1st.
1891	Aurora	March 31. Aurora, single arch, no streamers.
1891	Aurora	November 6. Faint aurora.
1892	Squall	February 1. Severe squall at 7 A.M. Wind veered from S.W. to W. Temperature fell 9° and barometer rose 0·06 inch in three minutes (see *Journ. Scot. Met. Soc.*, vol. ix. p. 237).
1892	Thunderstorm	February 16. Thunderstorm at 8.33 A.M. with hail.
1892	Aurora	March 1, 2, 3. Aurora.

Year.	Phenomenon.	REMARKS.
1892	Aurora	May 1. Aurora.
1892	Lightning	December 6. Lightning at 10.4 P.M.
1893	Meteor	September 5. Very fine meteor at 11.38 P.M., travelled from S.E. to E.N.E.
1893	Snow and Sleet	September 23. Observer at Blacket Place reports "slight showers of rain and sleet." The report from Leith is "slight snow and hail at 11.5 A.M."
1893	Gale	December 8. Heavy gale from S.W. Average wind velocity from 10 A.M. to 6 P.M. was 44 miles per hour, and that in a somewhat sheltered situation.
1894	Low Temperature	January 6. In the twelve hours ending 9 P.M. the highest temperature recorded was 17°·8.
1894	Increase in Pressure	February 12. The barometer rose 0·31 inch between 4 and 5 A.M.
1894	Rainfall	February. An exceptionally wet, stormy month; rainfall, 6·81 inches. Thunderstorms occurred on the 24th at 8 A.M., and on the 25th at 3.30 P.M. Lightning was seen on the latter date at 8.5 P.M.
1894	Thunderstorm	March 11. Thunder at times, 2.25 to 3 P.M.
1894	Rise in Temperature	March 29. Temperature rose 30° in six hours ending with noon.
1894	Aurora	May 7. Aurora after 7 P.M. Pale white streamers.
1894	Snow	May 20. Snow and hail. Pentland Hills white to base at 8 P.M.
1894	Rain from a Cloudless Sky	November 16. A shower of rain fell from a cloudless sky at 10.45 P.M.
1895	Frost	January and February. Very severe weather prevailed throughout, with hard frost, but little snow, in the immediate vicinity of Edinburgh (see *Jour. Scot. Met. Soc.*, vol. x. pp. 163–173).
1895	Lightning	January 24. Lightning at 8.40 P.M.
1895	Cold	February 8. Minimum in shade 11°·9, lowest in February since February 5, 1823.
1895	Aurora	March 13. Aurora in south at 9 P.M. stretching from W. to E.
1895	Rain	June 19. Thunderstorm with heavy rain; 0·53 inch of rain fell in 39 minutes, of which 0·40 inch fell in 21 minutes.
1895	Rain	August 22. Between 4.15 and 5.7 A.M. 0·86 inch of rain fell.
1895	Snow	October. Snow fell on the 24th, 25th, 26th, and 28th, covering the ground to a depth of 2 inches.

TABLE I.

Showing the Mean Barometric Pressure of the Air in Edinburgh from 1769 *to* 1896. *Corrected to* 32° *and Reduced to Mean Sea-Level.*

NOTE.—During six months the Observations were incomplete. These months are marked with an asterisk.

Year.	Jan.	Feb.	Mar.	April.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.	Year.
	ins.	ins.	ins.	ins.	ins.	ins.	ins.	ins.	ins.	ins.	ins.	ins.	ins.
1769	?	29·602*	29·928	29·806	29·959	29·837	29·976*	29·598*	?	?	29·838*	29·625	29·797*
1770	29·977	29·851	29·778	29·734	29·900	29·697	29·943	29·994	29·703	29·616	29·577	29·602	29·781
1771	29·691	29·938	29·945	30·021	29·842	30·055	29·881	29·758	29·952	29·632	30·003	29·513	29·853
1772	29·771	29·551	29·637	29·900	30·129	30·006	29·894	29·795	29·784*	29·364*	29·492	29·880	29·767
1773	29·679	29·842	30·144	29·792	29·911	29·905	30·013	29·944	29·565	29 644	29·706	29·734	29·823
1774	29·642	29·640	29·925	29·773	30·022	29·817	29·842	29·845	29·766	30·060	29·944	30·141	29·868
1775	29·758	29·552	29·716	30·040	30·109	29·974	29·794	29·737	29·733	29·757	29·915	29·952	29·836
1776	29·909	29·202	29·854	30·060	30·070	29·798	29·815	29·796	29·855	29·988	29·807	29·865	29·835
1777	29·859	29·719	29·766	30·065	29·784	29·824	29·868	29·886	30·009	29·754	29·836	29·869	29·853
1778	29·710	29·720	29·797	29·779	29·799	29·925	29·837	30·026	30·014	29·704	29·616	29·759	29·807
1779	30·111	29·889	30·021	29·716	29·698	30·076	29·959	30·124	29·859	29·980	29·726	29·746	29·909
1780	29·999	29·702	29·714	29·660	29·827	29·907	29·791	30·107	29·704	29·990	29·936	30·266	29·883
1781	29·977	29·622	30·155	29·861	30·086	29·893	29·950	29·777	29·864	30·121	29·758	29·736	29·900
1782	29·628	29·973	29·627	29·858	29·702	29·942	29·923	29·988	29·878	30·059	29·950	29·985	29·876
1783	29·410	29·659	29·768	30·201	30·009	29·844	29·949	29·888	29·696	29·827	29·932	29·999	29·848
1784	29·941	29·835	29·874	29·695	29·991	29·796	29·841	29·894	29·899	30·202	29·768	29·963	29·892
1785	29·812	30·014	30·252	30·167	29·971	30·087	29·780	29·784	29·747	29·877	29·662	29·977	29·928
1786	29·589	29·878	30·023	30·050	29·862	30·053	30·023	29·826	29·641	30·065	30·039	29·521	29·889
1787	30·123	29·697	29·676	30·007	29·946	29·828	29·771	29·900	29·863	29·696	29·770	29·794	29·839
1788	29·977	29·714	29·734	30·055	30·094	30·058	29·926	29·901	29·830	30·128	29·938	29·994	29·946
1789	29·636	29·420	29·788	29·644	29·839	29·770	29·794	29·918	29·764	29·644	29·673	29·583	29·706
1790	29·949	29·959	30·217	29·934	29·941	29·980	29·660	29·717	29·849	29·816	29·774	29·667	29·872
1791	29·186	29·789	29·990	29·810	29·903	29·877	29·716	29·978	30·063	29·692	29·626	29·542	29·764
1792	29·742	29·920	29·582	29·849	29·893	29·911	29·766	29·898	29·652	29·794	29·865	29·642	29·793
1793	29·916	29·666	29·869	29·967	30·150	29·884	29·970	29·878	29·960	29·811	29·923	29·710	29·892
1794	29·909	29·614	29·871	29·818	29·973	30·075	29·950	29·909	29·874	29·740	29·662	29·937	29·861
1795	30·163	29·756	29·827	29·739	30·135	29·953	30·003	29·866	30·054	29·500	29·818	29·803	29·885
1796	29·434	29·802	30·133	30·092	29·803	29·871	29·645	30·015	29·942	29·769	29·867	29·937	29·859
1797	30·020	30·189	29·881	29·836	29·795	29·909	29·839	29·716	29·702	29·809	29·920	29·603	29·852
1798	29·798	29·938	29·995	29·895	30·074	30·046	29·633	29·985	29·666	29·761	29·550	29·968	29·859
1799	29·904	29·657	29·901	29·716	29·851	30·027	29·745	29·625	29·741	29·695	29·693	30·094	29·804
1800	29·505	29·970	29·926	29·578	29·834	29·974	30·047	30·011	29·720	29·700	29·551	29·586	29·775
1801	29·713	29·718	29·726	29·986	29·891	30·047	29·840	30·115	29·930	29·741	29·673	29·490	29·822
1802	29·378	29·665	29·957	29·865	30·077	29·762	29·754	29·923	29·991	29·684	29·821	29·701	29·840
1803	29·889	29·709	30·042	29·829	29·814	29·970	30·075	29·962	30·082	30·048	29·528	29·601	29·879
1804	29·568	30·088	29·697	29·775	29·808	30·005	29·832	29·854	30·053	29·643	29·961	29·958	29·854
1805	29·654	29·721	29·869	29·910	29·949	29·978	29·901	29·883	29·934	29·994	30·234	29·633	29·888
1806	29·434	29·751	29·845	30·149	30·018	30·081	29·806	29·772	29·972	29·884	29·601	29·364	29·806
1807	29·972	29·621	30·093	29·912	29·888	30·001	29·883	29·863	29·764	29·809	29·533	29·819	29·846
1808	29·684	30·067	30·209	29·847	29·886	29·993	29·990	29·874	29·859	29·654	29·839	29·817	29·893
1809	29·612	29·624	30·098	29·896	29·920	29·915	29·942	29·708	29·729	30·064	29·949	29·399	29·821
1810	30·102	29·796	29·703	29·878	30·017	30·065	29·767	29·830	30·027	29·929	29·541	29·664	29·860
1811	29·901	29·461	30·125	29·764	29·815	29·916	30·056	29·873	30·098	29·613	29·904	29·662	29·849
1812	29·835	29·514	29·817	29·972	29·912	29·951	29·986	30·042	29·960	29·436	29·873	30·071	29·864
1813	30·031	29·556	30·017	29·970	29·789	30·046	29·832	30·036	29·996	29·766	29·668	29·861	29·881
1814	29·736	30·026	29·850	29·840	30·086	30·059	29·941	29·862	30·054	29·760	29·688	29.641	29·879
1815	29·920	29·664	29·576	29·971	29·879	29·889	30·037	29·816	29·886	29·802	29·977	29·747	29·847
1816	29·613	29·840	29·798	29·814	29·870	29·889	29·692	29·912	29·841	29·850	29·754	29·634	29·767
1817	29·629	29·708	29·670	30·298	29·774	29·806	29·737	29·676	29·962	30·058	29·837	29·582	29·814
1818	29·605	29·615	29·471	29·910	30·002	29·967	30·030	30·083	29·757	29·844	29·829	30·070	29·849
1819	29·679	29·627	29·900	29·830	29·978	29·833	29·967	30·025	29·941	29·883	29·808	29·772	29·854
1820	29·955	30·071	29·920	29·947	29·779	29·966	29·972	29·776	29·947	29·648	29·921	30·018	29·910

TABLE I.—*continued.*

Year.	Jan.	Feb.	Mar.	April.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.	Year.
	ins.	ins.	ins.	ins.	ins.	ins.	ins.	ins.	ins.	ins.	ins.	ins.	
1821	29·966	30·278	29·487	29·635	29·926	30·219	29·918	29·931	29·796	29·817	29·631	29·318	29·827
1822	30·102	29·867	29·788	29·984	30·092	30·103	29·790	29·825	29·980	29·627	29·518	30·078	29·896
1823	29·874	29·386	29·768	29·903	20·853	29·888	29·762	29·795	29·881	29·699	30·058	29·567	29·786
1824	29·962	29·854	29·861	29·922	30·052	29·994	29·951	29·889	29·880	29·654	29·498	29·600	29·843
1825	30·124	30·055	30·134	29·997	30·014	29·921	30·153	29·857	29·836	29·853	29·637	29·556	29·920
1826	30·081	29·688	30·052	29·899	30·166	30·257	29·903	29·874	29·920	29·782	29·898	29·846	29·947
1827	29·795	30·078	29·543	29·967	29·733	29·831	29·974	30·054	29 938	29·775	29·971	29·628	29·857
1828	29·906	29·703	29·853	29·755	29·902	29·921	29·667	29·784	29·904	29·982	29·800	29·788	29·830
1829	29·926	29·980	30·112	29·529	30·057	30·003	29·701	29·813	29·664	29·928	30·056	30·190	29·913
1830	30·076	29·689	29·992	29·650	29·865	29·772	29·811	29·774	29·630	30·135	29·741	29·706	29·820
1831	29·838	29·652	29·854	29·787	30·019	29·885	29·951	29·909	29·841	29·616	29·735	29·628	29·810
1832	29·929	29·984	29·795	30·048	29·952	29·849	30·068	29·812	30·010	29·891	29·782	29·777	29·908
1833	30·270	29·382	29·978	29·719	30·047	29·725	29·987	29·911	29·876	29·730	29·736	29·437	29·816
1834	29·514	29·950	30·051	30·219	29·991	29·873	29·989	29·824	30·012	29·880	29·911	30·185	29·950
1835	30·028	29·616	29·896	30·127	29·879	30·044	29·942	29·948	29·585	29·736	29·903	30·164	29·906
1836	29·795	29·842	29·401	29·863	30·312	29·764	29·831	29·976	29·839	29·730	29·534	29·762	29·804
1837	30·004	29·816	30·046	29·876	29·948	29·916	29·889	29·932	29·856	29·889	29·644	29·834	29·888
1838	30·104	29·789	29·692	29·698	29·968	29·822	29·910	29·768	29·977	29·911	29·612	29·950	29·850
1839	29·759	29·746	29·762	30·034	30·016	29·923	29·830	29·904	29·525	30·070	29·694	29·637	29·825
1840	29·529	29·914	30·361	30·043	29·896	29·835	29·752	29·843	29·664	29·985	29·543	30·192	29·880
1841	29·785	29·866	29·741	29·817	29·804	29·916	29·773	29·778	29·722	29·570	29·701	29·570	29·754
1842	29·987	29·835	29·706	30·187	29·854	29·989	29·896	29·968	29·888	29·943	29·709	29·871	29·903
1843	29·564	29·765	29·922	29·734	29·843	29·881	29·879	29·885	30·173	29·627	29·721	30·172	29·847
1844	29·927	29·573	29·754	30·030	30·237	29·869	29·859	29·723	30·042	29·620	29·795	30·113	29·878
1845	29·728	29·927	29·996	29·867	29·925	29·820	29·859	29·795	29·868	29·840	29·589	29·626	29·820
1846	29·625	29·833	29·650	29·752	29·873	29·952	29·778	29 912	29·953	29·541	29·905	29·847	29·802
1847	29·828	29·882	29·999	29·683	29·819	29·902	30·032	29·935	29·794	29·875	29·843	29·707	29·858
1848	29·939	29·412	29·579	29·766	29·989	29·708	29·837	29·740	29·930	29·763	29·829	29·771	29·772
1849	29·718	30·042	30·038	29·693	29·932	29·963	29·833	29·873	30·004	29·827	29·741	29·993	29·888
1850	29·966	29·675	30·171	29·646	29·837	29·912	29·900	29·810	30·053	29·763	29·689	29·866	29·857
1851	29·561	29·931	29·701	29·877	30·018	29·894	29·787	29·969	30·179	29·832	29·940	30·169	29·905
1852	29·427	29·899	30·206	30·140	29·875	29·628	29·969	29·725	29·930	29·813	29·505	29·434	29·796
1853	29·527	29·758	29·919	29·751	30·008	29·825	29·707	29·877	29·884	29·587	29·956	30·012	29·818
1854	29·592	30·018	30·068	30·035	29·702	29·822	29·887	29·917	30·022	29·736	29·782	29·624	29·850
1855	30·150	29·840	29·615	29·951	29·873	29·935	29·852	29·850	30·018	29·538	30·058	29·790	29·872
1856	29·437	39·803	30·094	29·764	29·857	29·913	29·866	29·869	29·770	30·111	30·037	29·629	29·846
1857	29·734	29·910	29·806	29·938	29·962	30·016	29·862	30·041	29·871	29·813	30·134	30·028	29·926
1858	30·149	30·022	29·865	29·800	29·810	30·032	29·891	29·946	29·908	29·901	29·954	29·733	29·918
1859	29·930	29·750	29·734	29·742	30·018	29·950	30·064	29·850	29·725	29·662	29·854	29·642	29·827
1860	29·513	29·924	29·640	29·960	29·837	29·643	29·986	29·571	29·884	29·801	29·935	29·700	29·783
1861	30·058	29·702	29·551	30·191	30·076	29·960	29·611	29·785	29·720	29·953	29·531	30·034	29·848
1862	29·691	30·058	29·738	29·893	29·805	29·765	29·709	29·862	30·020	29·689	29·919	29·816	29·830
1863	29·568	30·078	29·786	29·874	29·990	29·810	30·123	29·792	29·639	29·688	29·884	29·898	29·844
1864	30·080	29·930	29·669	30·066	30·032	29·910	30·012	30·138	29·868	30·022	29·816	29·996	29·962
1865	29·566	29·894	29·942	30·168	29·918	30·213	29·896	29·844	30·122	29·644	29·826	30·042	29·923
1866	29·64)	29·596	29·758	29·976	30·032	29·915	29·944	29·752	29 619	30·122	29·851	29·753	29·830
1867	29·692	29·906	29·932	29·658	29·989	30·027	29·815	29·823	29·902	29·746	30·196	29·908	29·883
1868	29·736	29·816	29·756	29·834	29·868	30·004	30·024	29·782	29·868	29·772	29·949	29·318	29·811
1869	29·785	29·697	29·885	29·916	29·881	30·020	29·950	30·058	29·578	29·989	29·818	29·772	29·862
1870	29·892	29 886	30·085	30·046	29·952	30·072	29·982	30·044	29·979	29·662	29·679	29·980	29·938
1871	29·764	29·868	29·944	29·818	30·105	30·000	29·738	29·945	29·955	29·912	30·022	29·958	29·919
1872	29·432	29·727	29·768	29·920	29·856	29·804	29·962	29·986	29·722	29·628	29·554	29·482	29·737
1873	29·500	30·118	29·831	30·078	29·934	29·913	29·817	29·825	29·868	29·780	29·876	30·038	29·882
1874	29·808	29·903	30·036	29·766	30·003	30·100	29·908	29·687	29·756	29·678	27·876	29·817	29·861
1875	29·718	30·112	30·177	30·049	29·916	29·831	30·002	29·993	29·987	29·787	29·846	29·978	29·950
1876	30·168	29·685	29·420	29·841	30·184	29·949	29·994	29·884	29·749	29·887	29·874	29·420	29·838
1877	29·669	29·737	29·696	29·846	29·884	29 934	29·792	29·795	30·066	29·817	29·422	29·864	29·794
1878	30·032	30·142	30·006	29·876	29·734	29·929	30·027	29·780	29·870	29·602	29·823	29·692	29·876
1879	30·079	29·530	29·934	29·766	30·018	29·727	29·716	29·727	29·851	30·094	30·244	30·170	29·905
1880	30·252	29·590	30·063	29·834	30·110	29·929	29·838	30·058	29·916	29·978	29·804	29·767	29·928

TABLE I.—*continued.*

Year.	Jan.	Feb.	Mar.	April.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.	Year.
	ins.	ins.	ins.	ins.	ins.	ins.	ins.	ins.	ins.	ins.	ins.	ins.	
1881	29·920	29·823	29·792	30·028	30·052	29·899	29·863	29·750	30·008	30·046	29·666	29·802	29·887
1882	30·145	30·029	29·794	29·785	30·030	29·820	29·714	29·819	29·857	29·834	29·558	29·628	29·835
1883	29·758	29·866	29·974	30·014	29·927	29·964	29·785	29·857	29·764	29·852	29·632	30·028	29·868
1884	29·794	29·768	29·810	29·872	29·878	30·017	29·943	29·943	29·928	29·950	30·077	29·676	29·888
1885	29·796	29·468	30·010	29·783	29·712	30·000	29·982	29·982	29·723	29·724	29·876	30·037	29·842
1886	29·554	30·090	29·895	29·877	29·893	29·934	29·887	29·887	29·949	29·796	29·792	29·492	29·829
1887	29·809	30·177	30·032	29·984	30·046	30·182	29·921	29·921	29·870	30·039	29·665	29·702	29·946
1888	30·115	30·043	29·631	29·875	29·940	29 981	29·902	29·902	30·141	29·936	29·671	29·807	29·912
1889	30·049	29·830	29·910	29·738	29·823	30·067	29·743	29·743	30·000	29·668	30·082	30·013	29·889
1890	29·602	30·242	29·684	29·816	29·847	29·874	29·800	29·800	30·007	29·950	29·736	30·104	29·872
1891	29·991	30·337	29·758	30·040	29·790	30·084	29·672	29·672	29·822	29·635	29·785	29·738	29·860
1892	29·744	29·776	30·047	30·012	29·926	29·940	29·808	29·808	29·800	29·700	29·921	29·891	29·864
1893	30·007	29·540	30·008	30·192	30·058	29·988	29·928	29·928	29·739	29·730	29·966	29·739	29·902
1894	29·650	29·755	29·813	29·877	29·940	29·971	29·832	29·826	30·229	29·926	29·786	29·854	29·872
1895	29·766	30·162	29·631	29·850	30·116	30·045	29·784	29·765	30·053	29·800	29·784	29·691	29·870
1896	30·222	30·202	29·633	30·054	30·266	29·908	29·868	29·985	29·660	29·702	30·121	29·682	29·942
						Decennial Means.							
1771–80	29·813	*29·675*	29·852	29·881	29·919	**29·929**	29·870	29·901	29·824	29·787	29·798	29·872	29·843
1781–90	29·804	*29·777*	29·910	**29·947**	29·945	29·925	29·862	29·859	29·803	29·944	29·827	29·822	29·869
1791–1800	29·758	29·820	29·898	29·830	29·941	**29·953**	29·831	29·888	29·837	*29·727*	29·748	29·782	29·834
1801–10	29·751	29·776	29·924	29·905	29·927	**29·982**	29·879	29·878	29·934	29·845	29·767	*29·645*	29·851
1811–20	29·790	*29·708*	29·814	29·932	29·888	29·922	29·925	29·910	**29·944**	29·766	29·826	29·806	29·851
1821–30	29·981	29·838	29·859	29·824	29·966	**29·991**	29·863	29·878	29·843	29·825	29·781	*29·728*	29·864
1831–40	29·877	29·769	29·884	29·941	**30·003**	29·864	29·915	29·883	29·818	29·843	*29·709*	29·857	29·864
1841–50	29·807	29·781	29·856	29·817	29·911	29·891	29·865	29·842	**29·943**	*29·737*	29·752	29·853	29·838
1851–60	*29·702*	29·886	29·865	29·895	29·896	29·866	29·887	29·861	**29·919**	29·780	29·916	29·776	29·854
1861–70	*29·771*	29·856	29·810	29·963	29·953	**29·970**	29·907	29·888	29·831	29·829	29·847	29·852	29·873
1871–80	29·842	29·838	29·888	29·880	**29·974**	29·912	29·880	29·868	29·874	29·919	29·833	*29·819*	29·869
1881–90	29·854	29·933	29·853	29·877	29·915	**29·974**	29·854	29·860	29·925	29·879	*29·775*	29·829	29·877
1891–96	29·897	29·962	29·815	30·004	**30·016**	29·989	29·815	29·831	29·884	29·749	29·894	*29·766*	29·885
Means 1770-1896 127 Yrs.	29·818	29·813	29·864	29·895	**29·940**	29·932	29·876	29·875	29·874	29·810	29·801	*29·800*	29·858

Table II.

Showing the Highest Barometric Pressure in each Month from 1840 *to* 1896.

At 32° and Mean Sea-Level.

Year.	Jan.	Feb.	Mar.	April.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.
	ins.	ins.	ins.	ins.	ins.	ins.	ins.	ins.	ins.	ins.	ins.	ins.
1840	30·415	30·717	30·725	30·407	30·390	30·126	30·315	30·305	30·161	30·618	30·454	30·803
1841	30·318	30·662	30·479	30·297	30·524	30·236	30·206	30·250	30·315	30·262	30·437	30·121
1842	30·567	30·419	30·375	30·427	30·516	30·434	30·357	30·336	30·524	30·551	30·527	30·501
1843	30·408	30·353	30·406	30·397	30·453	30·255	30·274	30·280	30·602	30·383	30·356	30·429
1844	30·515	30·135	30·482	30·376	30·601	30·176	30·190	30·318	30·337	30·332	30·340	30·551
1845	30·284	30·252	30·441	30·582	30·326	30·230	30·364	30·403	30·352	30·417	30·357	30·498
1846	30·504	30·381	30·521	30·277	30·271	30·356	30·187	30·350	30·546	30·300	30·596	30·540
1847	30·560	30·456	30·712	30·188	30·582	30·515	30·426	30·451	30·448	30·349	30·428	30·428
1848	30·599	30·218	30·208	30·128	30·363	30·198	30·471	30·041	30·366	30·372	30·632	30·575
1849	30·166	30·649	30·361	30·377	30·326	30·184	30·426	30·198	30·595	30·440	30·227	30·867
1850	30·499	30·265	30·621	30·490	30·271	30·381	30·222	30·277	30·557	30·416	30·322	30·410
1851	30·190	30·520	30·469	30·237	30·479	30·331	30·267	30·380	30·637	30·404	30·437	30·629
1852	30·190	30·732	30·797	30·377	30·329	29·939	30·232	30·372	30·324	30·417	30·134	30·185
1853	30·301	30·195	30·256	30·287	30·362	30·282	30·116	30·394	30·500	30·972	30·398	30·517
1854	30·179	30·530	30·902	30·514	30·236	30·248	30.271	30·389	30·510	30·511	30·377	30·343
1855	30·653	30·291	30·524	30·513	30·355	30·420	30·188	30·261	30·624	30·131	30·485	30·297
1856	30·595	30.581	30·750	30·453	30·402	30·280	30·250	30·319	30·305	30·586	30·671	30·380
1857	30·430	30·470	30.660	30·262	30·352	30·414	30·264	30·393	30·495	30·406	30·810	30·622
1858	30·693	30·538	30·545	30·395	30·434	30·386	30·262	30·385	30·393	30·586	30·621	30·257
1859	30·740	30·521	30·305	30·303	30·347	30·252	30·286	30·277	30·218	30·106	30·717	30·684
1860	30·255	30·782	30·564	30·524	30.505	30·186	30·319	29·947	30·411	30·295	30·649	30·290
1861	30·374	30·508	30·212	30·536	30·349	30·242	30·005	30·184	30·208	30·373	30·294	30·583
1862	30·448	30·693	30·377	30·467	30·304	30·259	30·203	30·341	30·363	30·521	30·555	30·225
1863	30·546	30·756	30·486	30·287	30·496	30·316	30·616	30·246	30·236	30·356	30·566	30·456
1864	30·726	30·616	30·186	30·536	30·416	30·286	30·416	30·686	30·416	30·556	30·776	30·676
1865	30·176	30·616	30·276	30·536	30·456	30·636	30·436	30·316	30·556	30·386	30·536	30·856
1866	30·486	30·356	30·486	30·616	30·586	30·326	30·376	30·186	30·256	30·616	30·276	30·456
1867	30·216	30·556	30·846	30·216	30·326	30·636	30·356	30·096	30·516	30·316	30·616	30·406
1868	30·356	30·396	30·526	30·386	30·326	30·436	30·486	30·216	30·486	30·166	30·606	30·136
1869	30·326	30·131	30·370	30·540	30·320	30·304	30·216	30·336	30·433	30·334	30·313	30·670
1870	30·656	30·608	30·560	30·440	30·412	30·570	30·305	30·408	30·515	30·620	30·561	30·673
1871	30·394	30·406	30·616	30·221	30·416	30·351	30·021	30·368	30·458	30·448	30·305	30·431
1872	29·926	30·213	30·260	30·432	30·340	30·268	30·188	30·328	30·184	30·354	30·408	29·978
1873	30·282	30·623	30·276	30·508	30·426	30·345	30·122	30·092	30·480	30·434	30·602	30·574
1874	30·507	30·607	30·879	30·427	30·440	30·691	30·256	30·564	30·297	30·513	30·348	30·401
1875	30·570	30·506	30·732	30·576	30·480	30·406	30·406	30·244	30·432	30·208	30·580	30·574
1876	30·677	30·240	30·224	30·426	30·584	30·263	30·456	30·290	30·375	30·390	30·387	30·197
1877	30·496	30·348	30·254	30·410	30·450	30·293	30·150	30·240	30·550	30·627	30·337	30·620
1878	30·624	30·545	30·685	30·280	30·136	30·180	30·436	30·382	30·190	30·269	30·396	30·382
1879	30·504	30·295	30·543	30·339	30·520	30·155	30·044	30·162	30·297	30·570	30·526	30·685
1880	30·650	30·365	30·516	30·546	30·402	30·480	30·246	30·376	30·392	30·454	30·433	30·452
1881	30·778	30·498	30·346	30·305	30·769	30·268	30·138	30·170	30·461	30·496	30·188	30·400
1882	30·866	30·664	30·486	30·596	30·467	30·383	30·375	30·363	30·390	30·670	30·341	30·140
1883	30·538	30·632	30·734	30·690	30·446	30·362	30·304	30·270	30·570	30·468	30·300	30·620
1884	30·547	30·290	30·321	30·272	30·404	30·318	30·189	30·216	30·520	30·736	30·661	30·419
1885	30·433	30·189	30·638	30·242	30·183	30·387	30·441	30·285	30·160	30·455	30·443	30·611
1886	30·146	30·499	30·589	30·402	30·360	30·312	30·282	30·254	30·634	30·549	30·661	30·476
1887	30·451	30·700	30·506	30·598	30·459	30·484	30·356	30·337	30·558	30·595	30·286	30·280
1888	30·691	30·574	30·509	30·332	30·566	30·377	30·055	30·286	30·519	30·511	30·134	30·390
1889	30·580	30·456	30·507	30·038	30·074	30·488	30·447	30·223	30·499	30·388	30·613	30·733
1890	30·250	30·735	30·543	30·278	30·304	30·366	30·220	30·218	30·454	30·450	30·323	30·593
1891	30·795	30·662	30·146	30·429	30·327	30·392	30·407	30·016	30·164	30·751	30·731	30·614
1892	30·411	30·619	30·621	30·457	30·499	30·383	30·384	30·255	30·377	30·379	30·437	30·272
1893	30·485	30·295	30·447	30·653	30·582	30·434	30·221	30·331	30·330	30·376	30·553	30·643
1894	30·759	30·466	30·509	30·424	30·519	30·477	30·342	30·323	30·568	30·497	30·528	30·642
1895	30·675	30·711	30·242	30·408	30·616	30·486	30·230	30·185	30·389	30·558	30·590	30·447
1896	31·071	30·689	30·190	30·450	30·537	30·170	30·325	30·327	30·362	30·543	30·644	30·392

Table III.

Showing the Lowest Barometric Pressure in each Month from 1840 *to* 1896.

At 32 and Mean Sea-Level.

Year.	Jan.	Feb.	Mar.	April.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.
	ins.	ins.	ins.	ins.	ins.	ins.	ins.	ins.	ins.	ins.	ins.	ins.
1840	28·241	28·615	29·589	29·392	29·254	29·300	29·163	28·846	28·928	29·065	28·401	29·105
1841	28·653	29·042	29·020	29·264	28·901	29·462	29·436	29·336	28·835	28·820	28·620	28·825
1842	28·786	28·885	29·037	29·277	28·787	29·154	29·204	29·593	29·295	28·703	28·901	29·029
1843	28·082	28·997	29·349	29·276	29·354	28·913	29·384	29·271	29·698	28·770	28·995	29·513
1844	29·044	28·952	28·798	29·603	29·823	29·480	29·156	29·001	29·615	28·850	28·885	29·490
1845	28·624	29·452	29·048	29·124	29·404	29·109	29·246	29·283	29·026	29·142	28·413	28·500
1846	28·790	29·036	28·870	29·087	29·017	29·216	29·178	29·323	29·303	28·771	28·920	28·963
1847	28·850	29·225	29·239	28·984	29·278	29·329	29·726	29·248	28·635	29·091	28·984	28·439
1848	29·297	28·580	28·535	29·342	29·203	29·269	28·996	29·316	29·485	29·182	28·949	28·810
1849	29·287	28·920	29·349	29·232	29·115	29·487	29·266	29·153	29·217	29.364	29·142	29·407
1850	29·217	28·695	29·539	28·775	29·241	29·251	29·168	29·293	29·323	28·893	28·340	28·785
1851	29·019	29·134	28·999	29·250	29·565	29·465	28·855	29·576	29·131	28·851	29·194	29·449
1852	28·710	28·962	29·389	29·627	29·391	29·231	29·714	29·038	29·226	29·027	28·685	28·051
1853	28·720	29·222	29·504	28·707	29·566	29·200	29·098	28·987	28·832	29·095	29·584	29·584
1854	28·960	29·069	29·526	29·115	28·854	29·306	29·303	29·460	29·516	29·068	28·676	28·682
1855	29·646	29·407	28·751	28·859	29·423	29·145	29·485	29·420	29·332	28·740	29·168	28·884
1856	28·606	29·243	29·916	29·214	29·226	29·486	20·274	29·414	28 963	29·427	29·587	28 854
1857	28·829	29·196	28·474	29·300	29·499	29·324	29·402	29 674	29·899	29·095	28·977	29·246
1858	29·555	29·031	28·985	28·869	29·287	29·666	29·318	29·368	29·449	28·798	28·860	29·020
1859	28·939	29·078	28·920	29·000	29·766	29·582	29·247	29·113	29·227	28·998	28·455	28·846
1860	28·464	28·673	28·602	29·676	29·335	29·102	29·606	28·909	29·236	29·070	29·072	28·866
1861	29·421	28·908	28·852	29·813	29·467	29·625	29·212	29·371	28·903	29 217	29·092	29·091
1862	29·097	29·252	29·003	29·279	29·477	29·169	29·391	29·341	29·519	28·621	29·029	29·339
1863	28·646	29·396	29·216	29·251	29·396	29·466	29·716	29·456	28·736	29·756	29·136	28·926
1864	29·416	29·106	29·086	29·416	29·766	29·716	29·616	29·586	29·536	28·786	28·636	29·546
1865	28·466	29·216	29·356	29·926	29·678	29·836	29·626	29·616	29·756	28·876	28·916	29·046
1866	29·766	29·166	28·866	29·446	29·376	29·436	29·206	29·306	29·216	29·536	29·366	28·846
1867	28·816	28·556	28·916	28·886	29·526	29·486	29·236	29·496	29·486	29·036	29·256	29·096
1868	29·016	29·616	28·796	28·856	29·366	29·656	29·526	29·276	29·066	29·246	28·906	28·446
1869	28·708	28·678	28·910	29·143	29·188	29·450	29·387	29·493	28·940	29·127	29·144	28·331
1870	28·547	29·080	29·186	29·397	29·092	29·517	29·514	29·537	28·854	28·482	28·900	28·843
1871	28·493	29·208	29·192	29·138	29·462	29·474	29·187	29·463	29·345	29·120	29·393	29·180
1872	28·215	29·186	29·125	29·360	28·900	29·288	29·648	29·464	29·086	29 004	28·578	28·716
1873	28·232	28·872	29·080	29·632	29·320	29·424	29·488	29·410	29·306	28·795	28·564	29·146
1874	28·898	28·898	29·316	28·647	29·554	29·632	29·522	29·300	29·207	28·660	28·707	28·720
1875	28·630	29·670	29·447	28·997	29·276	29·074	29·399	29·528	29·180	29·174	29·044	29·069
1876	29·476	28·881	28·260	28·995	29·560	29·371	29·392	29·026	29·229	28·725	29·357	28·497
1877	28·744	29·102	29·178	28·908	28·803	29·220	29·157	29·322	28·930	28·804	28·255	29·053
1878	29·133	29·530	28·892	28·831	29·254	29·286	29·641	29·318	28·983	28·862	29·064	28·823
1879	29·462	28·871	29·320	29·053	29·531	29·410	29·016	28.893	29·126	29·058	29·415	29·075
1880	29·406	28·642	28·761	29·100	29·410	29·480	29·380	29·412	29·501	29·212	28·766	28·802
1881	28·891	28·722	29·003	29·509	29·163	29·282	29·420	28·881	29·456	28·418	28·171	28·604
1882	28·853	28·872	28·990	29·131	29·102	29·300	29·090	29·169	29·004	29·111	28·817	28·957
1883	28·732	28·946	28·875	29·276	29·478	29·450	29·227	29·086	29·070	28·922	28·594	28·757
1884	27·451	28·807	29·050	29·177	28·914	29·530	29·372	29·464	29·287	28·855	29·237	28·874
1885	28·349	28·548	29·223	29·009	29·156	29·208	29·610	29·099	29·033	28·805	28·802	28·891
1886	28·865	28·916	29·054	28·839	29·189	29·474	29·103	29·483	29·361	28·778	29·125	27·651
1887	28·779	29·271	29·075	29·105	29·212	29·680	29·253	29·205	29·025	29·133	28·835	29·069
1888	29·069	29·342	28·859	29·372	29·106	29·502	29·277	29·424	29·681	29·189	28·960	28·975
1889	29·215	28·999	29·031	29·228	29·473	29·705	29·365	29·127	29·453	28·660	28·827	29·078
1890	28·726	29·514	29·049	29·213	29·381	29·307	29·375	29·011	29·340	29·231	28·660	29·323
1891	29·138	29·640	29·038	29·431	29·187	29·534	29·426	28·698	28·801	28·370	28·363	28·558
1892	29·150	28·708	29·188	29·454	29·297	29·258	29.168	29·209	29·221	28·820	29·249	29·145
1893	29·244	28·668	29·272	29·691	29·470	29·244	29·254	29·147	28·796	28·992	28·510	28·569
1894	28·939	28·319	28·789	29·288	29·466	29·561	29·188	29·074	29·885	28·646	28·802	28·121
1895	29·000	29·508	28·643	28·996	29·660	29·517	29·399	29·245	29·249	28·823	28·357	28·789
1896	28·857	29·525	28·299	29·524	29·755	29·399	29·510	29·551	28·821	28·907	29·081	28·769

Table IV.

Showing the Monthly Range of Pressure. From Observations made daily at 9 a.m. and 9 p.m.

Year.	Jan.	Feb.	Mar.	April.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.
	ins.	ins.	ins.	ins.	ins.	ins.	ins.	ins.	ins.	ins.	ins.	ins.
1840	2·174	2·102	1·146	1·015	1·136	0·826	1·152	1·459	1·232	1·553	2·053	1·698
1841	1·665	1·620	1·459	1·033	1·623	0·774	0·770	0·914	1·480	1·442	1·817	1·296
1842	1·781	1·534	1·338	1·150	1·729	1·280	1·153	0·743	1·229	1·848	1·626	1·472
1843	2·326	1·356	1·057	1·121	1·099	1·342	0·890	1·109	0·904	1·613	1·361	0·916
1844	1·471	1·183	1·684	0·773	0·788	0·696	1·034	1·317	0·722	1·482	1·455	1·061
1845	1·660	0·801	1·393	1·458	0·922	1·121	1·118	1·120	1·326	1·275	1·944	1·998
1846	1·714	1·345	1·651	1·190	1·257	1·140	1·109	1·027	1·243	1·509	1·776	1·577
1847	1·710	1·231	1·473	1·204	1·304	1·186	0·700	1·203	1·813	1·258	1·444	1·949
1848	1·302	1·638	1·673	0·786	1·160	0·929	1·475	0·725	0·881	1·190	1·683	1·727
1849	0·879	1·729	1·012	1·145	1·211	0·697	1·160	1·045	1·578	1·076	1·085	1·460
1850	1·282	1·570	1·082	1·715	1·080	1·130	1·054	0·974	1·234	1·523	1·982	1·625
1851	1·171	1·393	1·470	0·997	0·914	0·866	1·512	0·804	1·506	1·553	1·243	1·180
1852	1·420	1·770	1·408	0·750	0·938	0·758	0·515	1·334	1·098	1·390	1·449	2·134
1853	1·581	0·873	0·742	1·580	0·796	1·082	1·018	1·407	0·648	0·877	0·819	0·933
1854	1·219	1·461	1·376	1·399	1·382	0·942	0·968	0·929	0·994	1·443	1·703	1·662
1855	1·007	0·884	1·773	1·654	0·932	1·275	0·705	0·841	1·286	1·391	1·317	1·413
1856	1·989	1·338	0·834	1·234	1·176	0·794	0·802	0·905	1·342	1·159	1·084	1·568
1857	1·601	1·174	2·021	1·850	0·853	1·090	0·862	0·719	1·096	1·311	1·833	1·376
1858	1·138	1·507	1·560	1·526	1·147	0·720	0·889	1·017	0·944	1·788	2·211	1·237
1859	1·801	1·443	1·285	1·303	0·531	0·670	1·039	1·164	0·991	1·108	2·262	1·838
1860	1·791	2·109	1·962	1·848	1·170	1·084	0·713	1·038	1·175	1·225	1·577	1·424
1861	0·953	1·600	1·360	0·723	0·884	0·617	0·793	0·813	1·255	1·156	1·202	1·492
1862	1·351	1·441	1·361	1·188	0·827	1·090	0·812	1·000	1·044	1·900	1·472	1·150
1863	1·900	1·360	1·270	1·030	1·100	0·850	0·900	0·790	1·500	1·600	1·430	1·530
1864	1·310	1·510	1·100	1·120	0·650	0·570	0·800	1·100	0·880	1·770	2·140	1·130
1865	1·710	1·400	0·920	0·610	0·780	0·800	0·810	0·700	0·800	1·510	1·620	1·810
1866	1·720	1·190	1·620	1·170	1·210	0·890	1·170	0·880	1·040	1·080	0·910	1·610
1867	1·400	2·000	1·930	1·330	0·800	1·150	1·120	0·600	1·030	1·280	1·360	1·310
1868	1·920	1·780	1·730	1·530	0·960	0·780	0·960	0·940	1·420	0·920	1·700	1·690
1869	1·630	1·460	1·470	1·400	1·150	0·930	0·750	0·860	1·480	1·230	1·110	2·180
1870	2·030	1·530	1·330	1·200	1·430	1·050	0·770	0·870	1·580	2·130	1·590	1·760
1871	1·870	1·220	1·410	1·060	0·860	0·830	0·860	1·180	1·110	1·260	0·940	1·190
1872	1·700	0·990	1·040	1·000	1·270	0·980	0·540	0·864	1·098	1·268	1·798	1·274
1873	2·023	1·842	1·201	0·878	1·095	0·874	0·608	0·758	1·174	1·674	1·976	1·452
1874	1·400	1·707	1·512	1·661	0·842	1·050	0·724	1·250	1·015	1·758	1·637	1·658
1875	1·857	0·847	1·286	1·530	1·024	1·295	1·014	0·664	1·185	1·081	1·392	1·404
1876	1·170	1·294	1·930	1·416	1·000	0·793	1·031	1·240	1·133	1·591	1·012	1·638
1877	1·692	1·185	1·019	1·453	1·655	1·031	0·980	0·901	1·142	1·823	1·982	1·586
1878	1·452	0·962	1·703	1·461	0·900	0·899	0·721	1·058	1·176	1·388	1·261	1·462
1879	1·036	1·406	1·162	1·264	0·983	0·737	1·019	1·264	1·166	1·693	1·098	1·444
1880	1·249	1·724	1·715	1·445	0·954	0·963	0·866	0·964	0·986	1·242	1·667	1·610
1881	1·910	1·772	1·356	0 776	1·605	1·000	0·693	1·444	1·005	2·070	2·017	1·796
1882	2·022	1·789	1·510	1·473	1·364	1·102	1·254	1·120	1·280	1·579	1·518	1·177
1883	1·809	1·615	1·768	1·350	0·963	0·850	1·048	1·110	1·360	1·542	1·690	1·736
1884	3·035	1·445	1·243	1·069	1·469	0·816	0·826	0·682	1·189	1·513	1·377	1·498
1885	1·916	1·352	1·333	1·131	0·974	1·149	0·818	1·158	1·061	1·583	1·578	1·609
1886	1·230	1·482	1·453	1·505	1·118	0·829	1·134	0·751	1·227	1·737	1·526	2·565
1887	1·594	1·414	1·446	1·467	1·213	0·804	1·103	1·132	1·533	1·462	1·743	1·445
1888	1·622	1·232	1·650	0·960	1·460	0·853	0·778	0·862	0·838	1·322	1·119	1·414
1889	1·365	1·457	1·476	0·810	0·629	0·783	1·082	1·098	1·046	1·728	1·550	1·655
1890	1·524	1·198	1·494	1·065	0·923	1·041	0·845	1·140	1·083	1·219	1·502	1·270
1891	1·614	0.871	1·081	0·998	1·140	0·858	0·980	1·030	1·301	2·327	2·226	1·992
1892	1·251	1·745	1·347	0·959	1·132	0·921	1·166	0·986	1·158	1·558	1·115	1·114
1893	1·205	1·607	1·159	0·902	1·107	1·160	0·917	1·126	1·447	1·384	2·048	2·074
1894	1·754	2·147	1·720	1·038	1·023	0·855	1·114	1·129	0·707	1·835	1·706	2·521
1895	1·778	1·177	1·579	1·359	0·950	0·959	0·831	0·937	1·140	1·543	1·963	1·616
1896	2·164	1·164	1·730	0·922	0·768	0·718	0·815	0·776	1·528	1·636	1·563	1·633
Means 1840–96	1·611	1·437	1·418	1·210	1·077	0·935	0·935	0·999	1·173	1·481	1·566	1·558

TABLE V.

Pressure at 32° and Mean Sea-Level.

Years 1770–1896.	Highest Mean.		Lowest Mean.		Difference.	Range of Pressure. 1840–1896.				Difference.
	Inches.	Year.	Inches.	Year.	Inches.	Greatest.	Year.	Least.	Year.	
January, . . .	30·270	1833	29·196	1791	1·074	3·035	1884	0·879	1849	2·156
February, . . .	30·337	1891	29·202	1776	1·135	2·147	1894	0·801	1845	1·346
March, . . .	30·361	1840	29·401	1836	0·960	2·021	1857	0·742	1853	1·279
April, ...	30·298	1817	29·529	1829	0·769	1·848	1860	0·750	1852	1·098
May, ...	30·312	1836	29·698	1779	0·614	1·729	1842	0·581	1859	1·148
June, ...	30·257	1826	29·628	1852	0·629	1·342	1843	0·570	1864	0·772
July, ...	30·153	1825	29·633	1798	0·520	1·512	1851	0·515	1852	0.997
August, . . .	30·138	1864	29·571	1860	0·567	1·459	1840	0·600	1867	0·859
September, . . .	30·229	1894	29·525	1839	0·704	1·813	1847	0·648	1852	1·165
October, . . .	30·202	1784	29·436	1812	0·766	2·327	1891	0·877	1853	1·450
November, . . .	30·244	1879	29·422	1877	0·822	2·262	1859	0·814	1853	1·448
December, . . .	30·266	1780	29·318	{1821 1868}	0·948	2·565	1886	0·916	1843	1·649
Year, ...	29·962	1864	29·706	1789	0·256	3·035	Jan. 1884	0·515	July 1852	2·520

Years 1770–1896.	Highest Pressure. 1840–1896.				Range.	Lowest Pressure. 1840–1896.				Range.
	Maximum.	Year.	Minimum.	Year.		Maximum.	Year.	Minimum.	Year.	
January, . . .	31·071	1896	29·926	1872	1·145	29·646	1855	27·451	1884	2·195
February, . . .	30·782	1860	30·131	1869	0·652	29·670	1875	28·319	1894	1·351
March, . . .	30·902	1854	30·146	1891	0·756	29·916	1856	28·260	1876	1·656
April, ...	30·690	1883	30·038	1889	0·652	29·926	1865	28·647	1872	1·279
May, ...	30·769	1881	30·074	1889	0·695	29·823	1844	28·787	1842	1·036
June, ...	30·691	1874	29·939	1852	0·752	29·836	1865	28·913	1843	0·923
July, ...	30·616	1863	30·005	1861	0·611	29·726	1847	28·855	1851	0·871
August, . . .	30·686	1864	29·947	1860	0·739	29·674	1857	28·698	1891	0·976
September, . . .	30·637	1851	30·160	1885	0·477	29·899	1857	28·635	1847	1·264
October, . . .	30·751	1891	29·972	1853	0·779	29·536	1866	28·370	1891	1·166
November, . . .	30·810	1857	30·134	1852	2·676	29·587	1856	28·171	1881	1·476
December, . . .	30·867	1849	29·978	1872	0·889	29·584	1853	27·651	1886	0·933
	31·071	Jan. 9 1896	29·926	Jan. 9 1872	1·145	29·926	April 3 1865	27·451	Jan. 26 1884	2·475

TABLE VI.

High and Low Pressures.

The following Table shows all the sea-level pressures above 30·90 inches or below 28·20 inches during the period 1770–1896.

High Pressure.		Low Pressure.	
Date.	Reading.	Date.	Reading.
	Inches.		Inches.
1774 November 23, . . .	30·902	1773 January 18, . . .	28·089
1789 January 6, ...	30·937	1789 January 18, . . .	28·070
1808 February 25, . . .	31·004	1818 March 5, ...	28·198
1820 January 9, ...	31·058	1839 January 7, ...	28·112
1825 January 9, ...	30·961	1843 January 13, . . .	28·082
1834 December 15, . . .	30·950	1852 December 27, . . .	28·051
1835 January 2, ...	30·941	1881 November 27, . . .	28·171
1854 March 4, ...	30·902	1884 January 26, . . .	27·451
1896 January 9, ...	31·071	1886 December 8, . . .	27·651
		1894 December 22, . . .	28·121

Table VII.

Showing the Mean Temperature of the Air in Edinburgh from 1764–1896.

Note.—The means where not the average of the maximum and minimum values have been corrected. The height above mean sea-level is 250 feet.

	Jan.	Feb.	March.	April.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.	Year.
	°	°	°	°	°	°	°	°	°	°	°	°	°
1764, . .	36·3	38·0	38·7	44·0	52·2	55·6	59·9	57·6	51·0	46·5	38·4	36·1	46·2
1765, . .	39·8	32·9	40·0	44·5	51·9	53·7	58·5	56·8	51·7	47·2	37·1	35·6	45·8
1766, . .	34·7	34·5	38·1	45·8	45·8	54·0	58·9	59·5	51·6	46·6	43·0	37·6	45·8
1767, . .	31·7	41·1	38·9	44·8	48·7	53·1	56·4	59·8	54·6	45·7	43·0	39·3	46·4
1768, . .	33·2	38·2	40·2	46·5	52·5	54·5	58·3	58·7	51·0	47·0	40·1	39·1	46·5
1769, . .	35·3	36·6	40·6	45·5	50·4	54·4	60·1	56·3	53·9	45·7	40·1	40·4	46·7
1770, . .	39·9	41·1	35·8	41·5	47·9	53·3	57·1	58·2	55·1	44·4	38·3	37·6	45·8
1771, . .	33·8	38·2	36·5	41·7	49·5	54·3	57·4	56·3	51·0	47·2	42·1	41·7	45·8
1772, . .	32·6	32·6	37·8	42·7	48·6	56·1	58·0	57·4	51·0	49·0	42·4	39·6	45·6
1773, . .	38·5	36·2	43·0	45·4	47·9	54·0	56·2	58·3	51·3	46·1	39·2	36·5	46·0
1774, . .	30·1	36·7	38·2	43·6	45·5	54·0	56·8	56·7	52·1	48·7	39·0	37·7	44·9
1775, . .	38·3	39·9	40·2	47·2	53·0	55·1	59·7	57·5	53·4	45·9	38·5	39·1	47·3
1776, . .	29·2	36·7	42·1	46·5	49·4	54·3	59·6	56·7	51·5	47·8	41·0	38·1	46·1
1777, . .	35·4	35·2	40·1	42·6	51·2	53·7	57·5	59·2	55·8	48·8	42·9	38·8	46·9
1778, . .	37·8	39·5	40·1	44·0	53·1	59·1	61·2	58·7	51·3	42·6	40·8	43·4	47·6
1779, . .	37·6	47·2	46·5	47·1	51·0	58·1	65·2	63·7	56·0	48·8	40·9	33·1	49·6
1780, . .	28·4	35·1	44·7	42·0	53·2	57·0	60·7	63·2	57·4	45·9	38·8	39·5	47·2
1781, . .	36·3	40·3	44·5	47·5	51·9	59·8	60·4	58·6	52·7	48·5	43·4	41·1	48·8
1782, . .	39·4	34·7	37·8	40·7	47·2	57·2	60·1	56·1	51·4	44·0	35·6	35·9	45·0
1783, . .	37·1	38·9	37·5	48·5	49·9	54·2	63·2	58·4	53·6	47·2	41·2	37·1	47·2
1784, . .	32·2	34·8	35·0	41·1	55·4	53·5	58·5	56·4	54·7	46·4	39·7	34·0	45·2
1785, . .	38·2	32·8	34·2	49·2	50·6	60·7	58·3	54·1	54·3	45·7	43·2	36·1	46·4
1786, . .	36·0	37·0	35·4	45·1	49·7	57·6	56·4	58·7	51·1	44·0	39·0	36·3	45·5
1787, . .	40·0	43·8	44·4	43·9	49·7	53·8	60·0	60·0	53·6	48·0	38·0	36·8	47·7
1788, . .	38·8	37·2	37·4	49·2	50·2	57·2	60·3	58·8	54·4	47·8	42·4	32·2	47·2
1789, . .	34·6	40·2	34·6	43·8	53·2	56·9	60·9	61·6	55·0	47·6	41·0	43·9	47·8
1790, . .	39·4	44·7	43·2	42·6	52·8	58·8	59·0	57·8	52·8	48·8	39·9	37·9	48·1
1791, . .	38·8	39·3	43·9	47·5	52·0	56·9	58·6	58·4	54·7	46·7	41·2	32·7	47·6
1792, . .	34·8	39·8	40·9	49·8	48·6	53·7	58·4	60·3	51·0	46·2	44·5	37·6	47·1
1793, . .	37·4	40·1	37·6	40·4	49·5	53·9	60·0	57·8	52·9	51·7	41·0	40·6	46·9
1794, . .	38·2	43·0	43·2	46·8	50·6	58·4	60·7	56·4	52·2	47·0	40·8	40·2	48·1
1795, . .	29·9	31·6	37·5	45·2	49·4	52·9	57·6	59·3	57·0	50·8	37·9	42·9	46·0
1796, . .	43·8	40·5	39·0	48·9	49·0	55·8	57·6	59·5	55·1	44·8	39·2	31·8	47·1
1797, . .	40·7	43·8	39·5	44·6	52·0	54·4	60·9	58·0	53·8	44·7	38·5	40·0	47·6
1798, . .	38·4	38·9	40·6	49·8	53·3	60·8	60·6	59·4	54·9	48·2	39·0	35·8	48·3
1799, . .	37·2	36·2	38·0	41·1	48·5	55·4	58·0	55·9	54·6	45·1	40·3	35·1	45·4
1800, . .	35·3	36·3	38·5	46·9	51·1	55·5	61·6	59·7	55·1	47·7	40·1	36·2	47·0
1801, . .	39·1	39·9	42·5	46·3	52·0	57·4	58·9	60·4	56·2	49·4	40·0	34·0	48·0
1802, . .	36·9	37·8	42·0	46·6	49·0	55·5	56·3	60·1	55·1	49·6	41·5	37·9	47·4
1803, . .	35·6	37·7	41·9	46·5	50·1	55·6	62·8	59·1	52·5	47·2	39·0	38·5	47·2
1804, . .	40·2	36·4	38·7	43·0	54·1	59·6	59·1	58·6	51·1	49·4	41·6	36·2	47·3
1805, . .	37·2	38·9	42·5	46·3	47·9	53·8	59·3	59·4	56·4	46·3	42·1	36·7	47·2
1806, . .	35·1	36·7	39·3	41·5	50·0	56·6	57·5	58·8	55·5	49·5	42·9	39·9	46·9
1807, . .	36·3	35·7	39·3	44·7	47·8	55·1	61·0	59·8	48·2	50·6	34·0	35·1	45·6
1808, . .	35·2	35·5	37·3	41·6	54·3	56·3	62·5	60·4	54·1	43·6	40·1	35·4	46·1
1809, . .	36·3	38·7	42·5	40·4	52·0	55·1	57·3	57·4	52·8	51·0	39·8	36·6	46·7
1810, . .	36·8	36·2	36·6	44·5	45·1	55·8	57·2	58·0	55·8	48·4	39·1	35·7	45·8

TABLE VII.—*continued.*

	Jan.	Feb.	March.	April.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.	Year.
	°	°	°	°	°	°	°	°	°	°	°	°	°
1811, . .	33·6	37·7	43·5	43·6	52·2	54·8	59·3	56·7	54·8	51·5	43·8	35·9	47·3
1812, . .	36·3	39·4	36·8	40·7	49·0	55·3	57·0	57·2	53·3	47·3	39·8	34·9	45·6
1813, . .	35·6	39·7	43·3	44·8	49·3	55·9	59·3	57·5	53·4	44·3	37·7	37·3	46·5
1814, . .	26·5	35·1	37·7	48·2	47·7	52·9	59·4	57·2	54·4	45·4	38·9	36·6	45·0
1815, . .	33·5	41·6	41·5	45·3	52·0	55·9	58·2	57·8	53·6	47·7	37·6	33·8	46·5
1816, . .	35·4	35·8	36·4	40·9	48·2	53·2	55·7	55·6	50·9	46·3	38·5	35·2	44·3
1817, . .	38·7	40·3	39·6	44·5	45·4	54·7	57·2	54·1	53·8	42·0	44·7	35·6	45·9
1818, . .	37·2	35·5	37·1	40·8	50·3	58·8	60·0	56·5	52·9	52·4	46·7	38·9	47 3
1819, . .	37·7	36·4	42·2	45·0	50·6	54·8	59·5	62·7	53·7	46·4	37·5	33·3	46·7
1820, . .	30·4	40·0	40·9	46·9	50·4	55·0	59·0	56·3	52·2	43·9	41·6	39·1	46·3
1821, . .	39·1	40·2	42·8	48·9	47·6	52·9	57·5	58·7	57·4	50·6	43·0	41·1	48·3
1822, . .	39·0	40·6	43·7	45·5	52·4	59·2	58·0	57·0	50·3	47·8	44·0	36·1	47·8
1823, . .	31·1	34·4	40·5	42·4	51·3	53·3	56·4	55·6	51·9	44·8	44·6	37·3	45·3
1824, . .	39·8	39·0	39·6	45·2	50·1	56·6	59·9	57·2	54·6	45·8	40·8	38·4	47·2
1825, . .	39·1	39·0	41·2	46·6	50·7	56·7	61·4	60·0	56·9	50·1	38·5	39·0	48·3
1826, . .	31·6	41·8	41·8	46·8	51·8	61·4	62·0	61·3	54·6	50·0	38·8	41·0	48·6
1827, . .	35·4	34·0	40·1	45·0	50·8	56·1	58·4	55·2	55·0	50·1	42·8	42·2	47·1
1828, . .	39·4	40·1	42·8	45·2	51·2	56·9	57·6	57·0	54·6	48·4	44·8	43·4	48·4
1829, . .	32·1	38·8	39·6	41·9	51·6	56·3	56·5	54·0	50·3	46·0	39·6	36·0	45·2
1830, . .	34·3	36·0	44·2	46·6	49·8	52·0	57·7	52·6	52·2	48·5	42·6	35·4	46·0
1831, . .	34·7	38·6	42·3	45·0	48·8	58·0	59·4	60·1	55·3	52·7	40·2	41·8	48·1
1832, . .	39·1	40·6	41·8	45·8	48·7	55·9	57·7	57·4	54·0	49·7	41·4	40·6	47·7
1833, . .	34·7	39·5	38·9	44·4	55·8	55·6	58·9	54·6	53·0	48·9	41·8	40·3	47·2
1834, . .	41·4	40·5	43·0	45·0	52·3	56·9	59·3	58·4	54·0	48·8	43·2	42·2	48·7
1835, .	37·9	39·6	40·6	44·6	49·0	54·3	57·6	59·1	52·2	45·6	42·4	38·8	46·8
1836, . .	38·1	37·2	39·6	42·8	50·9	55·9	56·0	54·9	50·0	45·1	40·0	38·9	45·7
1837, . .	35·0	38·9	34·8	38·9	48·0	56·0	59·6	55·6	51·7	49·2	40·1	41·0	45·7
1838, . .	30·6	29·8	39·1	41·1	46·0	54·5	59·2	56·9	53·5	47·4	37·9	40·5	44·7
1839, . .	35·5	37·8	38·2	43·5	49·1	55·8	58·6	56·6	53·4	47·4	42·3	38·1	46·4
1840, . .	39·2	37·5	40·8	48·4	48·2	55·2	59·9	59·1	51·4	46·3	41·6	36·6	46·7
1841, . .	33·4	37·9	46·5	45·0	52·0	53·5	56·2	57·3	54·6	44·6	39·0	38·9	46·6
1842, . .	35·0	40·0	42·4	46·0	51·7	57·1	56·5	60·7	55·1	45·6	40·9	45·6	48·0
1843, . .	39·4	34·3	42·3	45·6	47·1	52·2	58·9	57·9	57·1	44·9	44·1	47·8	47·6
1844, . .	41·2	36·2	41·4	49·5	48·8	55·0	56·9	55·7	53·1	47·1	43·1	33·0	46·8
1845, . .	36·6	35·4	36·6	45·2	48·1	56·8	54·8	55·7	54·1	49·4	43·6	38·7	46·2
1846, . .	42·1	44·9	43·0	44·7	53·2	61·9	59·3	59·9	59·5	48·1	44·7	34·4	49·6
1847, . .	36·2	35·7	42·0	43·2	50·6	56·0	61·6	57·6	51·5	49·1	45·6	39·6	47·4
1848, . .	33·6	40·4	41·6	43·8	55·5	55·0	59·2	54·1	53·8	46·7	40·2	40·5	47·0
1849, . .	36·8	42·1	42·6	42·9	51·0	52·8	56·8	57·1	52·6	45·0	41·6	36·8	46·5
1850, . .	31·5	41·8	42·4	46·4	48·3	58·4	59·1	56·8	52·4	45·3	42·9	39·7	47·1
1851, . .	40·6	40·8	41·0	43·7	50·4	55·4	56·1	56·6	52·5	50·2	37·3	40·7	47·1
1852, . .	39·0	40·2	40·1	48·6	51·0	54·8	64·0	60·0	54·0	46·3	42·8	42·2	48·6
1853, . .	38·8	33·8	37·8	45·6	49·3	58·2	59·2	57·8	53·7	48·4	43·2	37·0	46·9
1854, . .	36·9	39·9	45·3	46·4	50·9	56·5	59·6	60·5	57·0	47·9	42·0	39·9	48·6
1855, . .	37·5	30·6	37·5	45·2	46·3	57·2	61·6	59·7	54·1	47·5	41·2	37·3	46·3
1856, . .	36·8	41·4	41·1	45·8	48·1	58·8	59·4	58·6	52·7	51·4	42·8	40·4	48·1
1857, . .	37·9	41·1	40·9	44·9	51·2	55·8	60·9	60·7	56·9	51·3	45·6	46·5	49·5
1858, . .	40·6	36·2	40·8	42·9	50·9	58·5	55·5	56·9	54·8	44·7	39·5	39·6	46·8
1859, . .	39·9	39·8	43·3	40·6	49·9	53·6	58·6	57·0	52·8	45·6	38·8	33·3	46·1
1860, . .	34·5	33·6	38·4	40·7	50·6	51·5	57·5	55·0	51·2	46·4	38·5	33·4	44·3
1861, . .	36·3	39·0	41·6	43·8	49·8	54·9	56·6	58·4	53·5	49·5	38·0	37·0	46·5
1862, . .	38·1	40·3	37·4	45·0	50·6	53·4	54·5	56·4	52·6	47·2	36·8	42·4	46·2
1863, . .	38·7	41·2	43·0	44·6	49·4	55·6	58·0	56·0	50·0	46·8	44·1	41·4	47·4
1864, . .	36·0	33·3	37·2	46·3	51·0	54·1	57·0	54·8	53·0	45·4	41·3	39·4	45·7
1865, . .	35·0	33·4	36·6	45·7	50·6	57·0	58·8	56·5	58·1	46·0	41·2	43·2	46·8
1866, . .	40·0	37·6	37·2	42·7	48·0	57·0	58·0	56·2	53·1	49·4	42·1	41·8	46·9
1867, . .	32·8	42·8	37·0	46·4	47·8	56·6	56·2	59·0	54·4	47·1	41·8	40·4	46·9
1868, . .	38·0	43·0	44·0	46·6	53·2	57·4	62·2	59·8	54·7	45·5	39·4	41·4	48·8
1869, . .	40·2	42·4	38·0	47·6	45·5	54·4	60·0	57·1	54·6	47·8	42·0	36·9	47·2
1870, . .	36·5	34·8	39·2	48·8	52·7	57·6	60·8	58·8	55·2	47·1	40·0	35·2	47·2

TABLE VII.—*continued.*

	Jan.	Feb.	March.	April.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.	Year.
	°	°	°	°	°	°	°	°	°	°	°	°	°
1871, . .	35·4	41·8	43·3	42·8	50·4	52·7	58·0	59·8	52·6	47·2	38·2	37·7	46·7
1872, . .	38·5	40·6	41·4	44·6	46·4	57·4	59·1	56·9	52·1	45·6	41·4	39·5	47·0
1873, . .	40·6	36·0	39·0	44·7	48·3	54·9	60·0	57·9	52·4	45·3	42·5	42·0	47·0
1874, . .	40·7	39·0	44·6	47·2	46·6	55·6	60·1	56·8	53·9	47·8	41·6	31·8	47·1
1875, . .	40·9	37·4	40·6	46·6	52·4	55·4	57·0	58·6	54·4	47·4	39·9	40·0	47·6
1876, . .	40·4	37·1	38·9	44·4	48·8	55·4	59·2	57·5	52·6	50·4	40·8	40·4	47·2
1877, . .	39·2	41·2	38·4	41·6	46·4	56·6	57·8	55·4	51·1	47·4	43·3	40·0	46·5
1878, . .	38·5	42·4	41·0	46·0	51·4	56·8	61·0	58·8	54·8	48·9	38·0	31·0	47·4
1879, . .	31·2	34·8	37·4	40·6	46·2	52·4	54·4	56·4	52·0	45·6	39·8	34·8	43·8
1880, . .	37·0	43·2	41·6	45·8	49·6	55·5	57·6	60·8	55·4	44·0	39·4	37·8	47·3
1881, . .	29·1	35·7	38·5	42·8	51·3	54·8	58·4	55·2	53·8	44·4	46·3	38·7	45·8
1882, . .	42·0	43·3	44·4	44·1	50·0	54·3	57·8	57·1	57·3	48·4	40·0	33·6	47·2
1883, . .	39·1	41·2	36·8	45·8	48·8	53·8	56·2	57·5	54·0	47·6	41·6	41·1	47·0
1884, . .	41·3	40·7	42·3	44·5	49·6	55·0	57·0	58·5	54·8	48·6	41·8	38·0	47·7
1885, . .	37·0	40·6	40·4	43·7	46·6	54·7	60·0	54·8	52·5	43·8	41·0	39·0	46·2
1886, . .	34·8	35·2	38·0	42·6	48·1	54·3	58·4	58·1	53·8	50·5	44·8	35·2	46·2
1887, . .	39·2	39·7	39·2	42·8	49·1	58·5	61·3	58·1	52·6	45·1	39·8	36·8	46·8
1888, . .	39·3	35·9	36·5	42·8	50·2	51·7	55·2	55·7	52·5	47·9	43·2	41·4	46·0
1889, . .	39·7	37·4	40·4	43·1	52·0	57·8	58·4	57·2	53·0	46·2	44·0	39·6	47·4
1890, . .	41·6	37·8	42·8	44·6	51·0	55·1	56·6	57·1	58·3	49·4	41·6	35·3	47·6
1891, . .	36·8	43·4	38·2	42·8	48·0	55·0	58·9	56·8	56·0	47·8	41·4	39·7	47·1
1892, . .	36·6	37·4	37·4	43·8	50·8	54·4	55·9	57·2	51·9	43·8	43·1	34·8	45·6
1893, . .	37·6	39·8	43·9	47·8	54·0	58·6	57·8	61·6	53·6	49·0	40·6	42·0	48·9
1894, . .	38·0	40·4	44·1	47·6	47·1	54·7	59·0	56·9	52·2	46·4	46·0	40·9	47·8
1895, . .	31·8	31·2	41·0	46·0	53·0	56·6	57·2	59·4	58·6	44·2	43·4	37·7	46·7
1896, . .	41·2	42·0	42·4	48·8	54·8	56·7	58·4	56·8	53·3	43·2	42·0	39·0	48·2
Decennial Means.													
1764–70, .	*35·8*	37·5	38·9	44·7	49·9	54·1	**58·5**	58·1	52·7	46·2	40·0	38·0	46·2
1771–80, .	*35·2*	37·7	40·9	44·3	50·2	55·6	**59·2**	58·8	53·1	47·1	39·0	39·2	46·7
1781–90, .	37·2	38·4	38·4	45·2	51·1	57·0	**59·7**	58·0	53·4	46·8	40·3	*37·1*	46·9
1791–1800, .	37·4	38·9	40·1	46·1	50·4	55·8	**59·4**	58·5	54·1	47·3	40·2	*37·3*	47·1
1801–10, .	36·9	37·2	40·3	44·1	48·2	56·1	**59·2**	**59·2**	53·8	48·5	40·0	*36·6*	46·8
1811–20, .	*34·5*	38·1	39·9	44·1	49·5	55·1	**58·5**	57·2	53·3	46·7	40·7	36·1	46·1
1821–30, .	*36·1*	38·4	41·6	45·4	50·7	56·1	**58·5**	56·9	53·8	48·2	41·9	39·0	47·2
1831–40, .	*36·6*	38·0	39·9	43·9	48·7	55·8	**58·2**	57·3	52·8	48·1	41·1	39·9	46·7
1841–50, .	*36·4*	38·8	42·1	45·2	50·6	55·9	**57·9**	57·3	54·4	46·6	42·5	39·5	47·3
1851–60, .	38·2	*37·7*	40·6	44·4	49·9	56·0	**59·2**	58·3	54·0	48·0	41·2	39·0	47·2
1861–70, .	*37·2*	38·8	39·1	45·8	49·9	55·8	**58·2**	57·3	53·9	47·2	40·7	39·9	47·0
1871–80, .	38·2	39·3	40·6	44·4	48·6	55·2	**58·4**	57·9	53·1	47·0	40·5	*37·5*	46·7
1881–90, .	38·3	38·7	39·9	43·7	49·7	55·0	**57·9**	56·9	54·3	47·2	42·4	*37·9*	46·8
1891–96, .	*37·0*	39·0	41·2	46·1	51·3	56·0	57·8	**58·1**	54·3	45·7	42·8	39·0	47·4
Means.													
1764–1896, .	*36·8*	38·3	40·3	44·8	49·9	55·7	**58·6**	57·8	53·6	47·2	40·9	38·3	46·8

Table VIII.

Reduction of the "Edinburgh Advertiser" Observations, showing the Mean Temperature deduced from Observations made at 8 a.m. and 8 p.m.

Year.	Jan.	Feb.	Mar.	April.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.	Year.
	°	°	°	°	°	°	°	°	°	°	°	°	°
1787	?	?	?	?	?	54·4	58·0	58·5	52·6	47·9	38·5	37·8	?
1788	38·8	37·2	36·4	47·0	56·8	55·6	59·3	58·4	54·4	46·5	43·0	31·8	47·1
1789	34·6	38·6	33·8	41·5	51·5	54·2	58·6	60·2	53·5	46·3	39·8	42·0	46·2
1790	38·8	42·7	42·2	39·8	50·4	55·9	58·3	56·8	52·2	49·0	39·6	36·5	46·8
1791	37·7	36·5	41·9	44·2	49·4	54·6	56·6	56·1	54·0	46·5	39·9	31·4	45·7
1792	33·9	36·9	38·2	48·4	45·5	50·8	55·8	57·3	50·2	46·1	44·6	37·0	45·4
1793	36·2	39·5	36·0	40·5	48·8	52·3	57·6	55·0	51·2	50·3	40·5	39·7	45·6
1794	37·0	41·1	41·7	46·0	47·6	56·8	58·2	55·4	51·2	45·7	40·5	39·2	46·7
1795	30·2	30·2	36·1	43·0	47·1	51·2	56·1	57·6	54·9	49·8	37·2	40·4	44·5
1796	41·7	37·6	36·7	45·8	47·9	54·3	56·2	59·1	54·6	44·9	38·5	30·5	45·6
1797	38·7	40·9	36·0	41·2	49·6	52·3	58·8	56·8	55·5	44·0	38·4	39·2	46·0
1798	37·6	37·3	38·9	48·2	52·4	57·5	58·5	57·6	53·2	47·6	38·6	35·9	46·9
1799	36·6	34·8	35·8	39·7	46·8	54·3	56·6	55·3	55·6	44·5	39·9	34·8	44·6
1800	34·5	35·4	37·4	45·0	49·8	53·3	59·3	57·6	53·8	47·0	40·5	36·2	45·8
1801	38·5	39·0	41·8	46·1	48·5	55·2	56·8	59·4	55·8	49·6	39·8	33·5	47·0
1802	35·6	36·5	40·3	44·9	47·0	53·8	54·3	58·6	54·0	48·8	41·1	37·8	46·1
1803	34·6	36·2	40·0	45·3	47·3	52·5	60·3	57·2	50·8	46·2	39·0	38·0	45·6
1804	40·1	35·1	37·2	40·8	50·8	57·0	57·1	56·5	55·6	48·5	41·3	35·8	46·3
1805	36·7	36·3	40·8	44·5	45·5	50·8	57·2	58·1	55·1	46·1	42·6	37·8	46·0
1806	35·9	37·0	37·3	41·8	45·7	52·1	51·8	55·2	50·8	46·8	42·0	39·0	44·6
1807	35·2	33·5	33·0	40·3	46·0	50·6	56·6	56·0	54·2	46·5	31·9	34·6	43·2
1808	34·6	33·5	34·7	38·5	52·1	52·7	58·4	57·5	51·5	40·9	38·5	35·2	44·0
1809	29·5	36·2	39·4	37·6	48·6	53·6	55·1	54·8	50·3	49·0	39·3	36·1	44·1
1810	35·4	34·0	33·5	40·8	41·8	51·5	52·9	52·7	49·4	43·8	37·5	34·6	42·3
1811	34·2	35·8	37·5	41·0	49·2	52·8	56·0	53·5	51·3	50·2	42·6	35·5	45·0
1812	34·8	38·5	34·5	38·3	46·8	53·3	55·3	55·6	52·8	46·8	38·7	34·4	44·2
1813	34·8	38·1	41·2	42·5	48·2	54·0	58·2	55·8	53·1	43·1	35·6	35·6	45·0
1814	24·4	33·1	35·3	46·2	45·0	50·3	57·1	54·5	53·8	43·4	37·8	36·1	43·1
1815	32·0	39·2	39·0	42·8	46·3	52·2	52·3	53·8	50·3	45·6	35 0	32·4	43·4
1816	32·9	32·3	33·6	36·2	44·5	49·6	52·0	51·1	47·8	43·0	36·3	33·0	41·0
1817	36·3	37·2	35·8	42·1	41·8	49·8	52·4	51·1	50·8	39·2	43·8	33·1	42·8
1818	34·6	32·9	34·3	38·2	48·3	57·0	58·3	55·0	51·8	51·2	46·1	39·0	45·6
1819	37·4	36·4	40·0	42·9	47·6	52·5	56·4	60·0	52·5	45·8	37·0	32·5	45·1
1820	31·3	37·8	37·7	44·5	48·8	53·0	55·5	55·6	50·8	43·2	41·6	38·2	44·8
1821	36·7	37·0	38·8	42·8	45·0	50·0	55·6	56·0	54·2	47·9	41·8	39·5	45·4
1822	37·2	38·8	39·6	42·8	48·8	55·7	55·5	54·2	49·8	45·9	42·6	35·6	45·5
1823	32·0	32·7	37·2	40·0	47·5	48·5	51·2	51·8	49·0	44·8	43·8	36·9	43·0
1824*	40·5	42·0	40·6	47·6	52·4	58·4	61·2	58·3	56·0	48·2	42·8	40·8	49·1
1825	41·7	42·0	43·8	48·7	53·2	59·8	64·7	62·0	59·1	51·6	41·2	41·7	50·8
1826	38·2	44·7	44·4	48·6	55·0	64·4	65·4	62·8	56·5	51·0	41·8	43·2	51·3
1827	37·6	36·0	39·9	46·8	52·6	57·2	60·0	57·2	55·8	51·2	43·8	43·8	48·5
1828	41·2	40·6	43·4	46·1	52·2	58·8	61·0	59·0	56·4	48·8	46·2	45·4	49·9
1829	35·2	39·4	40·8	42·7	52·6	57·4	58·6	55·6	51·8	47·8	41·6	37·8	46·8
1830	36·3	36·5	45·0	47·0	51·4	53·2	58·8	54·3	52·7	49·3	43·9	37·0	47·1
1831	37·3	39·9	42·6	47·1	51·6	60·8	61·2	61·0	55·6	53·8	41·4	43·6	49·7

* A change was made in the position of the thermometers during this year.

Table IX.

Reduction of the "Edinburgh Magazine" and "Scots Magazine" Registers.
Hour of Observation.—"Before Sunrise."

Year.	Jan.	Feb.	Mar.	April.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.	Year.
	°	°	°	°	°	°	°	°	°	°	°	°	°
1785	37·0	30·1	30·5	?	?	?	?	?	?	?	40·7	33·7	?
1786	33·6	33·4	30·1	?	?	?	?	?	?	39·9	36·4	33·7	?
1787	36·6	40·5	39·4	?	?	?	?	?	?	43·7	35·1	33·5	?
1788	36·4	32·4	32·0	42·6	45·9	49·9	53·0	51·6	48·4	44·8	39·5	30·3	42·2
1789	32·2	37·1	29·0	37·4	46·1	50·5	54·9	55·6	50·3	44·5	39·8	43·6	43·4
1790	37·5	42·1	37·9	36·6	46·5	52·6	51·5	51·1	47·2	44·2	37·8	37·2	43·5
1791	37·3	37·4	39·5	42·1	45·7	50·2	52·4	53·2	49·5	42·5	40·0	31·6	43·4
1792	33·3	38·0	37·2	42·4	42·8	47·6	52·6	55·8	45·7	42·0	42·1	36·1	43·0
1793	(35·6)	36·0	33·8	36·2	44·2	49·2	55·4	54·1	49·3	49·7	39·4	39·0	43·5
1794	36·6	40·4	39·4	41·7	47·7	53·8	56·3	50·9	47·9	44·9	39·0	38·8	44·8
1795	26·0	28·5	32·3	41·1	45·4	48·1	51·7	53·4	52·8	48·3	36·2	42·0	42·2
1796	41·9	38·2	35·2	45·2	46·1	50·8	52·2	52·7	50·0	41·7	38·5	31·0	43·6
1797	39·9	42·5	36·8	41·5	48·8	50·7	57·4	54·5	48·5	42·6	36·3	38·2	44·8
1798	37·0	36·1	36·7	45·7	49·2	54·6	53·8	52·5	50·4	44·7	36·5	33·4	44·2
1799	36·0	34·4	35·0	36·2	42·0	48·7	50·7	47·8	47·9	41·4	38·1	34·0	41·0
1800	34·0	34·5	35·0	42·8	45·8	49·7	55·2	53·0	50·2	45·0	38·4	35·0	43·2
1801	38·2	38·1	39·4	40·9	48·2	51·6	53·3	52·2	49·3	45·3	38·4	33·8	44·1
1802	35·3	36·0	38·6	41·1	43·1	48·9	49·0	52·7	48·7	46·1	39·6	36·4	43·0
1803	34·3	35·6	38·5	40·2	44·6	50·0	55·9	52·6	48·0	43·8	36·9	37·3	43·1
1804	38·9	34·6	35·4	38·5	49·0	53·7	51·7	52·2	51·0	45·7	39·1	34·7	43·7
1805	34·9	36·0	37·9	40·4	41·6	47·1	53·0	52·1	49·6	41·6	39·6	36·6	42·5
1806	35·0	35·0	36·5	38·9	45·0	51·1	53·4	52·8	48·7	45·4	41·9	40·7	43·7
1807	35·7	34·8	32·6	35·9	44·0	48·4	52·8	53·0	41·4	46·7	32·7	35·5	41·1
1808	35·1	34·4	35·1	36·7	48·7	53·3	57·2	55·1	46·6	38·5	36·5	34·7	42·7
1809	28·8	36·8	37·3	35·5	45·0	50·4	53·3	52·6	47·3	47·2	37·9	35·7	42·3
1810	35·3	33·3	32·8	39·0	40·0	50·6	51·6	51·7	46·9	42·7	37·6	33·7	41·3
1811	32·9	36·1	36·8	40·7	47·4	(51·9)	54·7	51·6	47·4	47·3	40·3	35 4	43·5
1812	34·7	37·5	34·6	37·2	44·8	53·0	51·0	48·7	47·8	44·8	37·8	34·5	42·2
1813	34·6	39·3	40·4	41·0	46·3	55·1	56·3	50·8	48·8	41 5	37·0	37·5	44·0
1814	26·9	34·8	35·5	44·5	45·4	47·7	56·7	53·0	48·9	41·0	37·9	35·7	42·3
1815	34·4	40·8	39·4	42·0	49·1	51·6	55·3	55·0	48·5	45·6	35·8	34·2	44·3
1816	34·5	34·4	35·5	37·9	44·4	49·3	49·9	49·8	45·4	43·9	37·8	33·2	41·3

TABLE X.

Reduction of the "Edinburgh Magazine" and "Scots Magazine" Registers.
Hour of Observation—"Noon."

Year.	Jan.	Feb.	Mar.	April.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.	Year.
	°	°	°	°	°	°	°	°	°	°	°	°	°
1785	?	?	?	57·1	60·0	71·8	69·5	62·2	61·3	51·2	?	?	?
1786	?	?	?	52·0	55·9	63·5	65·3	62·2	55·6	?	?	?	?
1787	?	?	?	47·9	55·8	58·2	62·0	66·1	60·6	55·7	42·1	37·3	?
1788	40·3	38·4	41·0	52·3	60·1	60·8	63·5	66·1	59·1	51·3	45·6	33·6	51·0
1789	35·5	41·4	39·2	49·4	57·0	60·4	65·5	68·7	60·3	51·6	43·9	45·2	51·5
1790	41·4	47·6	49·0	46·1	53·6	59·5	61·1	60·9	55·4	53·1	43·1	38·9	50·8
1791	40·1	41·9	47·9	48·1	56·3	61·0	64·1	63·4	61·0	52·5	44·5	36·3	51·4
1792	37·2	45·2	45·5	52·2	54·0	58·8	61·4	64·8	55·4	49·2	47·9	39·7	50·9
1793	(39·5)	42·3	41·1	46·2	55·5	58·2	64·6	63·1	58·0	53·4	44·1	41·8	50·6
1794	40·7	45·4	49·4	53·6	54·1	63·7	65·0	60·5	55·7	51·6	45·2	42·4	52·3
1795	32·0	34·4	41·6	48·2	53·6	56·6	60·8	62·9	63·3	55·9	43·2	45·6	49·8
1796	46·4	43·0	43·6	56·8	53·7	60·2	60·0	66·8	60·1	51·7	43·3	36·3	51·8
1797	42·9	50·4	47·5	50·8	57·6	58·1	63·7	62·6	59·0	51·6	44·5	42·3	52·6
1798	42·0	44·0	47·5	58·1	62·2	66·1	67·3	64·2	61·3	52·3	42·3	37·4	53·7
1799	40·3	39·2	41·6	44·1	50·2	62·5	61·5	58·9	57·4	50·6	43·6	35·9	48·8
1800	35·9	39·2	43·1	54·0	54·9	61·6	69·5	67·0	60·1	52·6	43·6	38·8	51·7
1801	41·1	43·5	46·9	55·5	58·6	65·9	63·0	69·2	62·8	55·1	44·2	36·9	53·6
1802	38·4	41·8	47·4	53·2	59·7	62·1	60·7	65·2	61·8	53·2	44·0	40·0	52·3
1803	37·3	41·3	46·7	55·8	58·5	62·6	71·6	65·0	62·2	52·8	42·1	39·8	53·0
1804	41·9	41·3	42·5	47·3	58·4	65·3	68·9	64·5	63·5	54·0	43·7	38·1	52·4
1805	38·8	41·6	49·9	53·8	53·4	63·0	67·9	67·6	64·9	54·2	49·1	41·4	53·8
1806	39·3	42·6	46·2	52·2	58·9	65·8	64·0	64·7	61·7	54·5	47·6	44·0	53·5
1807	40·6	42·4	44·9	47·0	54·5	62·3	67·4	65·0	55·6	54·3	41·6	38·7	51·2
1808	38·7	40·9	43·0	47·1	62·5	64·8	70·9	67·5	60·9	49·6	42·7	37·7	52·2
1809	34·7	42·8	47 5	48·5	62·8	64·2	66 5	64·7	60·0	55·7	44·1	39·0	52·5
1810	40·3	41·4	42·9	51·1	55·5	67·5	65·8	64·6	64·2	55·0	43·5	38·5	52·5
1811	37·6	41·8	50·4	51·4	59·9	(68·8)	70·3	65·8	64·8	57·0	50·0	39·5	54·8
1812	39·8	44·4	43·2	48·1	56·5	67·7	66·7	66·1	62·8	54·1	44·2	38·2	52·6
1813	40·8	45·5	50·7	52·9	57·8	71·2	71·6	69·9	63·7	51·8	44·7	41·7	55·2
1814	34·9	42·1	44·6	56·3	58·4	65·9	72·3	67·1	67·3	53·8	44·6	41·0	54·0
1815	39·5	47·0	49·5	56·3	59·9	66·0	69·6	68·7	64·8	55·3	45·1	41·4	55·3
1816	40·7	41·4	43·8	48·3	57·5	65·6	65·1	64·1	58·0	53·7	44·7	39·3	51·8

Table XI.

Reduction of Waterston's Register 1800–1850.

Year.	Jan.	Feb.	Mar.	April.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.
	°	°	°	°	°	°	°	°	°	°	°	°
1800, . . .	33·0	34·3	37·3	47·2	50·5	55·0	61·3	60·6	54·0	47·5	40·3	36·7
1801, . . .	39·1	39·7	42·5	47·2	51·0	57·2	57·0	61·5	55·4	49·6	39·0	34·0
1802, . . .	38·6	38·0	42·0	46·6	48·6	56·3	55·8	60·9	53·8	50·0	40·5	38·6
1803, . . .	36·6	37·8	41·7	47·0	50·0	55·6	62·8	59·6	52·0	47·6	38·3	38·4
1804, . . .	39·3	36·2	38·0	42·3	54·1	59·5	59·5	58·3	57·8	50·2	41·0	35·5
1805, . . .	36·8	37·3	43·2	46·4	48·5	54·8	60·8	60·7	57·6	47·3	43·3	38·1
1806, . . .	36·3	37·5	39·8	42·1	51·0	57·8	58·8	60·1	56·7	50·5	44·1	41·3
1807, . . .	37·5	36·5	39·8	45·3	48·8	56·3	62·5	61·1	49·4	51·6	35·2	36·5
1808, . . .	36·4	36·3	37·8	42·1	55·3	57·5	64·0	61·7	55·3	44·6	41·3	36·8
1809, . . .	31·5	39·5	43·0	41·0	53·0	56·3	58·8	58·7	54·0	52·0	41·0	38·0
1810, . . .	38·0	37·0	37·1	45·1	46·1	57·0	58·7	59·3	57·0	49·4	40·3	37·1
1811, . . .	34·8	38·5	44·0	44·3	53·2	57·0	61·8	59·0	57·0	53·5	46·0	38·3
1812, . . .	38·5	41·2	38·3	42·3	51·0	57·5	59·5	58·5	55·5	49·3	42·0	37·3
1813, . . .	38·2	42·5	45·2	46·3	51·0	57·8	61·2	59·5	54·5	46·5	40·5	40·0
1814, . . .	31·5	38·2	39·8	47·0	49·0	55·8	62·8	58·5	56·5	48·5	41·5	39·0
1815, . . .	36·0	42·8	43·8	46·5	53·0	58·0	60·0	60·0	57·0	49·7	40·5	36·5
1816, . . .	37·2	37·2	39·0	42·0	50·0	55·0	57·5	58·0	53·9	50·0	40·8	36·8
1817, . . .	41·0	42·0	41·0	46·3	48·0	57·0	59·0	57·5	55·5	44·0	46·0	37·0
1818, . . .	39·0	38·3	40·0	43·0	52·7	61·0	61·7	59·2	54·6	54·0	48·8	40·0
1819, . . .	39·0	37·0	42·0	46·4	52·8	57·7	61·5	55·0	55·2	48·5	39·5	34·0
1820, . . .	32·5	40·5	43·2	48·0	53·2	57·5	61·0	59·0	53·5	46·7	44·3	41·2
1821, . . .	40·5	40·7	42·8	48·8	49·5	55·6	60·8	59·5	55·0	51·2	44·8	42·5
1822, . . .	41·7	40·8	45·3	48·0	54·3	61·0	60·0	59·5	53·0	49·5	46·3	38·7
1823, . . .	35·6	36·6	42·5	45·3	53·0	55·3	58·5	58·0	54·5	47·3	47·2	40·2
1824, . . .	42·2	41·2	41·0	48·0	51·3	56·7	61·5	58·3	56·2	47·9	42·9	40·2
1825, . . .	41·0	40·8	41·8	48·2	50·7	58·3	61·7	61·2	58·2	50·8	40·3	40·0
1826, . . .	36·0	42·3	40·2	47·5	53·0	63·5	64·9	63·7	56·2	51·1	41·5	43·3
1827, . . .	38·3	36·5	41·5	47·0	53·2	59·0	62·2	60·0	57·0	52·8	44·6	44·1
1828, . . .	41·5	41·5	44·0	46·5	52·6	59·5	60·0	59·0	56·9	53·3	46·3	45·1
1829, . . .	35·6	40·6	41·2	44·5	54·3	59·2	60·0	57·5	52·6	48·5	42·2	37·0
1830, . . .	37·0	38·5	45·8	48·6	52·6	55·3	61·2	56·4	54·0	50·8	44·4	37·3
1831, . . .	37·3	40·5	44·0	46·7	51·5	60·8	62·4	61·2	56·0	54·7	42·0	43·6
1832, . . .	40·2	41·8	43·5	47·6	51·6	58·7	59·2	59·2	56·3	51·2	42·8	41·8
1833, . . .	36·5	41·0	41·2	47·5	57·8	58·5	58·0	56·5	55·3	50·4	43·1	41·0
1834, . . .	42·5	41·0	45·0	46·0	55·0	58·5	60·0	60·0	54·0	50·2	44·5	43·0
1835, . . .	38·0	41·5	41·5	47·0	52·0	57·3	61·0	61·0	53·0	47·0	44·5	40·5
1836, . . .	40·0	37·6	41·0	46·0	53·3	59·0	59·5	58·0	52·7	47·5	41·2	40·7
1837, . . .	37·5	40·0	36·3	42·5	51·5	59·5	62·5	58·6	55·3	52·0	42·7	43·7
1838, . . .	34·0	32·0	40·7	44·6	50·3	56·5	61·5	60·0	55·0	49·0	41·0	42·0
1839, . . .	37·0	38·8	39·5	45·8	51·0	57·5	61·5	57·0	55·5	49·8	45·0	39·5
1840, . . .	40·0	39·0	41·5	50·0	50·5	58·5	60·0	61·0	53·8	48·0	43·8	39·5
1841, . . .	35·5	39·3	46·8	46·5	54·5	56·8	59·0	59·0	56·8	47·0	41·3	40·8
1842, . . .	37·0	41·0	44·0	47·0	54·8	60·0	60·5	63·0	57·0	47·0	42·3	47·3
1843, . . .	40·5	36·0	42·0	48·0	49·0	55·0	62·0	61·5	59·2	46·3	42·0	48·0
1844, . . .	39·8	35·3	40·8	50·8	51·0	58·8	60·3	58·5	55·8	49·3	44·8	35·3
1845, . . .	37·5	35·5	38·0	47·2	52·0	60·0	59·0	59·0	55·0	49·8	44·0	38·8
1846, . . .	43·3	43·5	42·8	46·3	54·0	65·8	63·3	63·0	60·3	50·5	45·2	35·8
1847, . . .	37·5	36·0	43·5	46·0	53·8	60·0	64·0	60·0	52·0	50·3	46·5	40·0
1848, . . .	34·5	40·0	42·0	45·5	56·7	56·0	61·0	58·0	56·0	48·5	41·0	40·8
1849, . . .	38·5	42·0	43·3	44·5	53·0	58·0	61·5	60·0	55·5	47·0	43·8	38·5
1850, . . .	34·0	43·0	42·5	48·8	51·0	59·0	60·5	...	...	....	...	...
1800–09, . . .	36·6	37·3	40·5	44·7	51·1	56·6	60·1	60·2	54·6	49·1	40·6	37·4
1810–19, . . .	36·5	38·8	41·0	45·3	50·7	57·4	60·4	59·5	55·7	49·3	42·6	37·6
1820–29, . . .	38·2	40·1	42·0	47·2	52·3	58·6	61·1	59·6	55·3	49·9	44·1	41·2
1830–39, . . .	37·3	38·4	41·4	46·2	52·7	57·8	60·8	58·7	54·7	50·2	43·0	41·3
1840–49, . . .	38·4	37·9	42·5	47·2	52·9	58·9	61·1	60·3	56·1	48·4	43·5	40·5
Means, . . .	37·4	38·5	41·5	46·1	51·8	57·9	60·7	59·7	55·3	49·4	42·8	39·6

Table XII.

Mean Temperature at Glendoich.

Year.	Jan.	Feb.	Mar.	April.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.
	°	°	°	°	°	°	°	°	°	°	°	°
1783 . .	?	?	?	?	51·5	53·5	63·0	59·6	53·6	48·3	42·8	37·5
1784 . .	32·0	35·1	35·6	41·9	?	?	?	57·6	55·6	47·2	40·5	33·6
1785 . .	38·5	33·0	35·5	49·6	52·6	61·3	62·5	58·9	53·3	47·3	43·5	36·5
1786 . .	36·0	37·5	36·3	46·5	50·0	60·0	60·6	59·6	53·0	45·6	40·3	38·2
1787 . .	40·9	42·5	44·5	46·2	52·5	55·5	59·5	60·0	54·6	48·3	42·2	32·0
1788 . .	39·3	37·0	37·3	49·0	55·2	55·2	60·2	60·7	55·6	46·6	44·0	33·2
1789 . .	31·7	41·0	36·0	44·9	52·5	57·3	60·2	61·7	59·7	48·8	41·5	43·9
1790 . .	39·5	45·3	43·6	43·6	53·0	53·2	58·9	58·5	53·6	51·0	38·3	35·6
1791 . .	40·2	40·9	45·0	46·3	52·5	57·3	62·5	59·0	56·0	49·0	40·6	33·3
1792 . .	39·0	41·3	42·2	49·0	51·3	57·6	60·5	60·3	52·7	47·5	40·5	39·7
1793 . .	38·2	39·2	39·0	43·9	49·5	59·6	64·5	58·8	55·8	52·5	48·5	41·9
1794 . .	38·5	42·6	44·3	44·5	55·2	60·2	65·6	59·0	57·6	53·7	42·3	39·7

Table XIII.

Showing the Extremes in the Mean and Absolute Daily Temperatures in Edinburgh from 1822 *to* 1896, *by means of the Self-Registering Maximum and Minimum Thermometers in Shade* 4 *feet above Grass and* 250 *feet above Mean Sea-Level. For Description of Position of Thermometers from* 1770 *to* 1821 *see Page* 67.

Year.	Highest Mean Daily Temperature.	Date.	Lowest Mean Daily Temperature.	Date.	Difference or Range.	Absolute Maximum Temperature.	Date.	Absolute Minimum Temperature.	Date.	Difference or Range.
	°		°		°	°		°		°
1770	...	...	...	...	...	81	Aug. 5	...	...	...
1771	...	...	...	...	...	71	July 24	22	April 15	49
1772	...	...	...	...	...	75	June 11	12	Feb. 6	63
1773	...	...	...	...	...	77	Aug. 5	23	Feb. 12	44
1774	...	...	...	...	...	72	July 23	18	Jan. 12, Dec. 10	54
1775	...	...	...	...	...	76	June 16	22·5	Jan. 26	53·5
1776	...	...	...	...	...	76	Aug. 3	11	Jan. 31	65
1777	...	...	...	...	...	...	...	...	...	...
1778	...	...	...	...	...	...	...	...	...	...
1779	...	...	...	...	...	...	...	...	...	...
1780	...	...	...	...	...	...	...	...	...	...
1781	...	...	...	...	...	...	...	...	...	...
1782	...	...	...	...	...	...	...	...	...	...
1783	...	...	...	...	...	...	...	...	...	...
1784	...	...	...	...	...	...	...	...	...	...
1785	...	...	...	...	...	89	June 27	18	Mar. 1	71
1786	...	...	...	...	...	81	June 5	11	Jan. 2	70
1787	...	...	...	...	...	77	July 4	17	Dec. 27	60
1788	...	...	...	...	...	78	June 17	14	Dec. 15	64
1789	...	...	...	...	...	81	Aug. 19	17	Jan. 11	64
1790	...	...	...	...	...	73	June 22	22	Dec. 1	51
1791	...	...	...	...	...	77	July 15	18	Dec. 11	59
1792	...	...	...	...	...	77	Aug. 12	22	Jan. 12	55
1793	...	...	...	...	...	82	July 14	23	Jan. 16	59
1794	...	...	...	...	...	76	July 6	13	Jan. 27	63
1795	69·5	Aug. 12	17·3	Jan. 29	52·2	72	July 6	9	Jan. 29, 31	63
1796	68·7	June 20	18·9	Dec. 24	49·8	79	Aug. 18	16	Dec. 24	63
1797	68·3	July 14	24·4	Nov. 29	43·9	74	May 25, June 14	20	Nov. 24	54
1798	69·7	June 28	21·9	Dec. 28	47·8	73	July 4	14	Dec. 28	59
1799	67·7	June 21, 22	20·4	Dec. 31	47·3	...	...	19	Dec. 31	...
1800	70·8	July 24	23·4	Dec. 30	47·4	...	...	15	Jan. 1	...
1801	70·0	Aug. 19	25·4	Dec. 19	44·6	...	...	21	Jan. 25	...
1802	68·0	Aug. 17	25·3	Jan. 1, 6, 7	42·7	...	...	21	Jan. 15	...
1803	77·8	July 18	20·3	Jan. 13	57·5	...	...	19	Jan. 13	...
1804	67·0	Sept. 14	21·4	Dec. 31	45·6	...	...	17	Jan. 7	...
1805	...	...	...	...	...	...	...	26	Dec. 10, 12	...
1806	...	...	...	...	...	...	...	20	Feb. 1	...
1807	...	...	...	...	...	...	...	14	Dec. 7	...
1808	...	...	...	...	...	...	...	18	Nov. 27	...
1809	...	...	...	...	...	...	...	9	Jan. 22	...
1810	...	...	...	...	...	...	...	16	Feb. 14, 16	...
1811	...	...	...	...	...	...	...	16	Jan. 29	...
1812	...	...	...	...	...	...	...	16	Dec. 12	...
1813	...	...	...	...	...	...	...	19	Nov. 26	...
1814	...	...	...	...	...	...	...	10	Jan. 16	...
1815	...	...	...	...	...	...	...	17	Jan. 24, Dec 16	...
1816	...	...	...	...	...	...	...	18	Feb. 7	...
1817	...	...	...	...	...	...	...	21	Dec. 22	...
1818	...	...	...	...	...	...	...	22	Feb. 2	...
1819	...	...	...	...	...	...	...	16	Dec. 10	...
1820	...	...	...	...	...	...	...	10	Jan. 17	...
1821	...	...	...	...	...	...	...	18	Jan. 2	...
1822	65·5	June 5	25·0	Dec. 28, 29	40·5	80	June 13	18	Dec. 28	62
1823	66·0	Aug. 11	19·0	Feb. 5	47·0	75	Aug. 11	11	Feb. 5	64
1824	72·0	Sept. 2	22·5	Dec. 5	49·5	85	Sept. 2	16	Dec. 5	69
1825	70·5	July 14	27·0	Jan. 5 Feb. 4	43·5	83	July 30, 31	19	Nov. 10	64
1826	74·0	June 28	18·0	Jan. 16	56·0	87	June 24, 26	10	Jan. 16	77
1827	63·5	July 16, Sept. 16	19·0	Jan. 3	44·5	77	July 16	14	Jan. 3	63
1828	66·0	June 27	23·0	Jan. 11	43·0	76	Aug. 27	15	Jan. 11	61

TABLE XIII.—*continued.*

Year.	Highest Mean Daily Temperature.	Date.	Lowest Mean Daily Temperature.	Date.	Difference or Range.	Absolute Maximum Temperature.	Date.	Absolute Minimum Temperature.	Date.	Difference or Range.
	°		°		°	°		°		°
1829	65·0	Aug. 8	22·5	Jan. 22	42·5	75	July 13	15	Jan. 22, 25	60
1830	68·0	July 26	22·0	Dec. 24	46·0	81	July 28	15	Dec. 25, 26	66
1831	67·0	July 29, 31	26·0	Feb. 4	41·0	76	July 31	19	Feb. 4	57
1832	65·0	Aug. 10	29·5	Jan. 3, 27	35·5	75	Aug. 10	24	Jan. 27	51
1833	67·5	July 28	27·0	Jan. 15	40·5	75	July, 17, 29	23	Jan. 16	52
1834	68·5	Aug. 12	31·0	Feb. 21, Nov. 24	37·5	77	Aug. 12	20	Mar. 24	57
1835	66·5	Aug. 11	28·0	Jan. 20	38·5	77	Aug. 4, 10	22	Jan. 21	55
1836	65·0	June 15	28·0	Dec. 26	37·0	76	June 15	24	Jan. 19	52
1837	64·0	July 6	23·0	Jan. 11	41·0	73	July 10	16	Jan. 12	57
1838	68·5	July 12	18·0	Jan. 21	50·5	84	Sept. 9	13	Feb. 13	71
1839	66·5	June 17	24·0	Feb. 21	42·5	87	June 17	13	Jan. 30	74
1840	67·5	Aug. 21	27·5	Dec. 24	40·0	78	Aug. 9	21	Jan. 30, Feb. 27	57
1841	67·0	Aug. 20	21·5	Jan. 9	45·5	79	June 10	8	Jan. 9	71
1842	68·0	Aug. 13	26·5	Jan. 16	41·5	79	July 23	18	Jan. 16, 17	61
1843	67·0	July 14	24·0	Feb. 15	43·0	77	July 14	16	Feb. 15, 17	61
1844	66·0	July 22, 25	24·5	Feb. 21	41·5	77	Sept. 1	13	Feb. 27	64
1845	69·0	June 12	18·0	Jan. 31	51·0	79	June 12, 13	5	Jan. 31	74
1846	71·5	June 5	25·5	Dec. 25	46·0	84	June 5	16	Dec. 18	68
1847	75·5	July 12	25·5	Feb. 8, 9, Dec. 31	50·0	83	July 14	17	Feb. 8, 9	66
1848	68·5	July 13	17·5	Jan. 29	51·0	82	July 13	5	Jan. 29	77
1849	65·5	July 10	20·0	Jan. 2	45·5	78	June 5	19	Jan, 4, 6	59
1850	68·5	June 24	19·5	Jan. 17	49·0	78	July 23	12	Jan. 18	66
1851	64·5	June 29	29·5	Dec. 3	35·0	75	June 29	23	Dec. 3	52
1852	74·5	July 5, 6	30·5	Feb. 20	44·5	85	July 6	24	Nov. 30	60
1853	63·5	June 23	24·0	Feb. 13	39·5	75·5	June 23	19·0	Feb. 11	56·5
1854	62·5	July 12, 31	21·5	Jan. 2	41·0	75·0	July 12	18·5	Jan. 2	56·5
1855	66·5	July 22	20·5	Feb. 16	46·0	79·5	June 12, Sept. 9	14·5	Feb. 16	65·0
1856	67·5	Aug. 2	23·5	Dec. 3	44·0	81·9	Aug. 2	18·6	Dec. 3	63·3
1857	70·6	June 27	27·2	Jan. 29	43·4	81·6	June 28	20·0	Jan. 29	61 6
1858	68·4	June 16	26·0	Feb. 2	42·4	78·2	June 16	20·5	Mar. 8	57·7
1859	69·1	Aug. 18	23·5	Dec. 19	45·6	78·7	July 12	18·7	Dec. 19	60·0
1860	65·0	July 15	12·4	Dec. 24	53·6	71·7	July 16	8·8*	Dec. 24	62·9
1861	65·5	Aug. 28	25·2	Jan. 8	40·3	71·1	June 14	16·3	Jan. 8	54·8
1862	61·0	July 16, 31	25·5	Mar. 3	36·5	70·0	April 30	22·0	Mar. 3	48·0
1863	66·5	July 10	29·8	Dec. 28	36·7	75·2	July 11, 12	24·5	Dec. 28	50·7
1864	68·0	July 18	22·0	Feb. 24	46·0	79·0	May 18, July 18	18·0	Mar. 10	61.0
1865	65·5	June 8	25·5	Feb. 15, 17	40·0	78·0	June 22	18·0	Feb. 15, 17	60·0
1866	70·0	June 7	26·0	Jan. 11	44·0	82·7	July 12	20·0	Mar. 5	62·7
1867	66·5	July 10	22·0	Jan. 1	44·5	76·7	July 10, Aug. 14	16·0	Jan. 1	60·7
1868	75·5	Aug. 5	28·5	Jan. 10	47·0	87·7	Aug. 5	23·0	Nov. 7	64·7
1869	67·5	July 11	26·5	Dec. 27	41·0	80·7	Aug. 28	20·0	Dec. 2	60·7
1870	72·5	July 24	21·5	Dec. 23	51·0	84·7	July 23	16·0	Dec. 23	58·7
1871	70·0	Aug. 11	28·0	Jan. 28	42·0	79·7	Aug. 11	23·0	Nov. 13	56·7
1872	68·6	July 5	30·5	Mar. 26	38·1	79·7	July 5	23·0	Mar. 26	56·7
1873	71·8	July 21	28·9	Jan. 29	42·9	82·9	July 21	22·2	Feb. 24	60·7
1874	69·2	Aug. 18	23·6	Dec. 29	45·6	81·3	July 18	13·6	Dec. 29	67·7
1875	67·3	Aug. 17	23·2	Jan. 1	44·1	76·8	July 7	14·0	Jan. 1	62·8
1876	70·0	July 16	26·4	Jan. 9	43·4	86·7	July 16	21·3	Feb. 14	65·4
1877	63·8	July 30	26·8	Dec. 25	37·0	72·0	June 14	22·4	Feb. 28	49·6
1878	71·8	June 28	19·2	Dec. 14	52·6	83·7	July 20	9·0	Dec. 14	74·7
1879	67·5	Aug. 12	18·4	Dec. 3	49·1	78·0	Aug. 12	7·5	Dec. 4	70·5
1880	69·8	Aug. 11	26·8	Nov. 22	43·1	77·6	Aug. 11	20·6	Nov. 21	57·0
1881	68·4	July 14	17·1	Jan. 17	51·3	79·2	May 30, 31	9·4	Jan. 17	69·8
1882	67·8	Aug. 12	13·2	Dec. 15	54·6	81·0	Aug. 12	6·4	Dec. 15	74·6
1883	65·1	July 3	30·6	Mar. 23	34·5	75·0	July 3	24·5	Mar. 15	50·5
1884	70·3	Aug. 24	29·0	Nov. 30	41·3	79·9	June 27	23·0	Nov. 30	56·9
1885	71·0	July 26	26·0	Nov. 18 Dec. 7	35·0	82·2	July 24	19·8	Nov. 18	62·4
1886	68·0	July 2	21·4	Jan. 19	46·6	80·7	July 2	12·2	Jan. 19	68·5
1887	69·9	July 8	28·6	Mar. 13	41·3	83·2	June 18	21·0	Feb. 9	72·2
1888	64·5	May 19	28·5	Jan. 19	36·0	76·8	May 19	18·3	Feb. 16	58·5
1889	67·0	Aug. 1	25·8	Feb. 10	41·2	78·4	June 26	20·8	Mar. 4	57·6
1890	67·6	Aug. 5	27·9	Dec. 19	39·7	76·0	Sept. 8	23·2	Dec. 14	52·8
1891	64·6	July 17	28·0	Jan. 18	36·6	79·8	Sept. 12	20·3	Mar. 9	59·5
1892	67·4	June 9	22·6	Dec. 25	44·8	80·1	June 9	14·0	Feb. 19	66·1
1893	72·0	Aug. 15	23·0	Jan. 6	49·0	85·9	June 18	15·0	Jan. 6	70·9
1894	67·2	July 6	21·5	Jan. 6	45·7	77·5	July 6	13·9	Jan. 6	63·6
1895	68·6	Aug. 17	20·3	Feb. 7	48·3	78·3	June 26, Sept. 25	11·9	Feb. 8	66·4
1896	68·2	July 20	30·8	Dec. 1	37·4	78·1	May 11	23·8	Dec. 1	54·3

* At Marchhall, Newington, the temperature fell to 5°·0.

Table XIV.

Showing the Highest Mean Daily Temperature in Edinburgh from 1857 *to* 1896.
Height above Sea 250 *feet.*

Note.—The Mean Temperature was assumed to be the mean of the daily maximum and minimum values.

Year.	Jan.	Feb.	Mar.	April.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.
	°	°	°	°	°	°	°	°	°	°	°	°
1857, . .	49·4	49·4	47·9	55·0	60·7	70·6	67·6	65·6	65·0	57·9	52·2	52·7
1858, . .	48·3	47·2	52·2	55·6	61·5	68·4	63·1	62·7	62·9	54·1	48·7	47·7
1859, . .	45·8	48·9	52·6	57·9	59·8	65·3	68·5	69·1	59·4	60·3	45·5	45·4
1860, . .	45·6	42·2	47·0	50·0	58·0	60·5	65·0	61·5	64·4	53·4	44·6	44·8
1861, . .	50·8	51·0	48·9	50·2	59·0	65·0	60·7	65·5	60·0	59·0	49·0	50·2
1862, . .	45·5	50·4	47·5	57·5	56·5	58·0	61·0	60·5	59·0	59·0	48·5	48·5
1863, . .	46·8	47·5	53·4	50·6	57·5	61·5	66·5	63·0	56·3	52·5	51·8	48·2
1864, . .	46·4	45·0	44·0	58·3	65·7	61·5	68·0	64·0	57·5	51·0	48·5	52·0
1865, . .	45·0	45·5	48·5	54·0	60·0	65·5	64·5	63·0	65·0	56·0	48·5	51·0
1866, . .	49·5	45·5	50·0	51·0	59·0	70·0	70·0	65·0	60·0	57·0	52·0	53·0
1867, . .	48·0	49·0	46·5	52·0	57·0	63·0	66·5	66·5	61·5	57·0	50·0	51·0
1868, . .	49·0	51·5	50·5	55·0	60·5	65·0	69·0	75·5	70·0	53·0	52·0	50·0
1869, . .	48·0	51·5	44·0	57·5	51·0	62·5	67·5	66·0	60·0	59·5	54·0	46·0
1870, . .	42·0	43·0	48·0	57·0	59·0	65·0	72·5	66·0	60·0	56·5	50·0	47·0
1871, . .	42·5	51·2	52·8	47·5	60·5	58·0	64·0	70·0	63·0	58·5	45·5	46·0
1872, . .	48·0	47·5	51·5	54·5	57·6	67·4	68·6	64·2	62·0	55·8	52·0	51·4
1873, . .	50·6	45·6	48·7	52·4	55·0	64·5	71·8	63·0	64·0	53·6	49·5	49·5
1874, . .	45·5	45·5	52·9	60·8	60·4	62·8	69·2	65·6	60·0	56·8	56·5	43·2
1875, . .	49·1	46·8	50·1	56·0	61·0	62·8	65·4	67·3	64·4	57·0	52·8	49·2
1876, . .	49·0	46·7	48·4	57·5	59·0	66·2	70·0	64·6	58·0	61·4	52·2	49·0
1877, . .	45·5	46·8	46·3	50·9	54·2	61·8	63·8	62·5	59·0	58·2	51·2	48·0
1878, . .	46·7	50·9	49·6	55·6	58·2	71·8	68·8	65·4	63·6	59·8	43·6	45·4
1879, . .	41·8	49·8	46·3	47·8	53·2	58·9	64·4	67·5	61·0	53·2	49·2	46·4
1880, . .	51·2	48·4	48·0	53·4	61·1	64·6	62·5	69·8	63·8	55·9	50·6	51·2
1881, . .	45·2	44·3	51·4	53·6	65·4	64·6	68·4	64·3	58·6	57·2	55·4	50·0
1882, . .	49·1	51·8	50·9	52·9	56·8	59·2	64·8	67·8	59·5	54·4	47·9	48·2
1883, . .	46·4	47·6	47·9	53·6	56·6	60·4	65·1	64·4	58·0	56·3	53·2	49·8
1884, . .	50·7	49·2	55·3	51·7	58·9	67·9	66·0	70·3	62·4	53·8	55·6	47·6
1885, . .	48·0	49·4	47·4	56·2	55·2	61·8	71·0	62·0	60·0	49·9	54·1	51·0
1886, . .	49·0	45·0	53·8	53·7	57·8	61·6	68·0	65·4	63·4	61·2	54·6	44·7
1887, . .	50·8	49·0	48·5	50·6	59·2	66·7	69·9	64·2	59·6	53·9	45·9	49·1
1888, . .	51·6	48·0	50·2	50·3	64·5	58·7	62·6	61·4	58·2	60·3	50·8	51·3
1889, . .	47·8	49·9	52·3	52·5	61·5	64·2	65·8	67·0	62·6	51·3	53·2	50·1
1890, . .	49·3	47·5	51·8	51·6	60·0	60·9	62·0	67·6	63·0	59·7	50·2	51·9
1891, . .	46·2	50·8	51·6	50·9	56·9	64·4	64·6	62·8	67·1	58·0	47·8	47·7
1892, . .	50·4	47·2	55·3	53·8	60·6	67·4	64·0	65·5	57·6	53·8	51·0	48·6
1893, . .	47·8	49·2	53·4	58·4	60·6	71·6	63·0	72·0	63·2	60·4	52·6	51·6
1894, . .	48·4	49·3	52·9	55·0	54·6	63·8	67·2	61·6	57·8	56·6	56·7	53·8
1895, . .	37·0	41·4	50·0	55·4	62·8	65·2	63·6	68·6	66·7	57·2	49·2	45·4
1896, . .	48·2	51·0	50·0	55·8	61·5	67·1	68·2	64·1	60·8	58·3	51·0	50·6
Highest, .	51·6	51·8	55·3	60·8	65·7	71·8	72·5	75·5	70·0	61·4	56·7	53·8
Lowest, .	37·0	41·4	44·0	47·5	51·0	58·0	60·7	60·5	56·3	49·9	43·6	43·2
Range, .	14·6	10·4	11·3	13·3	14·7	13·8	11·8	15·0	13·7	11·5	13·1	10·6

TABLE XV.

Showing the Lowest Mean Daily Temperature in Edinburgh from 1857 *to* 1896.
Height above Sea 250 *feet.*

NOTE.—The mean Temperature was assumed to be the mean of the daily maximum and minimum values.

Year.	Jan.	Feb.	Mar.	April.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.
	°	°	°	°	°	°	°	°	°	°	°	°
1857, . .	27·2	32·7	34·7	37·5	43·2	47·0	54·5	54·4	50·6	44·0	34·2	37·4
1858, . .	32·2	26·0	27·8	35·6	42·4	51·8	50·4	51·3	50·2	39·3	32·1	35·9
1859, . .	31·7	33·5	36·2	34·9	40·5	47·2	47·0	53·1	49·0	31·4	33·2	23·5
1860, . .	27·6	25·1	33·5	36·2	39·5	45·0	52·5	53·1	48·0	38·0	31·8	12·4
1861, . .	25·2	33·3	37·8	39·6	37·0	50·1	51·6	53·5	47·4	41·0	30·8	27·0
1862, . .	30·4	29·5	25·5	32·5	42·7	49·0	48·1	52·5	45·6	38·0	28·8	36·0
1863, . .	31·8	33·8	31·5	38·5	42·2	47·4	50·2	47·0	45·0	38·8	32·8	29·8
1864, . .	24·0	22·0	28·4	38·0	41·0	47·2	51·0	48·5	49·0	36·5	35·5	30·5
1865, . .	27·0	25·5	32·0	39·0	41·0	47·0	50·5	50·5	51·0	37·0	34·5	37·0
1866, . .	26·0	28·5	27·5	36·0	38·0	46·5	50·0	51·0	48·0	41·5	32·0	32·5
1867, . .	22·0	23·5	30·5	40·5	38·5	48·5	48·5	53·0	49·0	36·5	35·0	30·5
1868, . .	28·5	34·5	34·5	38·0	40·0	52·0	55·0	53·0	46·5	37·0	30·0	30·0
1869, . .	28·0	30·5	32·5	35·0	41·0	44·0	53·5	47·5	49·0	33·5	30·0	22·5
1870, . .	28·5	25·5	30·0	43·1	42·5	51·0	54·0	50·5	46·5	42·0	34·0	21·5
1871, . .	28·0	31·4	32·1	36·5	38·5	47·5	52·5	52·0	41·5	38·5	31·5	29·5
1872, . .	31·0	35·0	30·5	36·5	41·5	50·2	54·0	52·9	43·4	39·0	35·6	31·5
1873, . .	28·9	29·2	31·6	39·4	38·2	48·8	54·3	51·0	46·2	39·5	34·5	30·5
1874, . .	31·5	28·0	31·2	39·1	41·8	51·2	55·0	51·0	47·6	38·5	32·4	23·6
1875, . .	23·2	29·6	34·2	41·0	46·6	46·2	52·0	54·4	49·1	40·2	33·2	31·4
1876, . .	26·4	29·0	31·3	33·8	41·6	48·2	53·4	52·1	44·8	42·0	31·3	30·9
1877, . .	28·8	26·9	29·5	34·8	39·2	52·0	52·3	50·2	46·2	32·5	34·4	26·8
1878, . .	27·9	34·6	31·8	36·7	42·9	45·0	54·6	51·8	48·0	36·2	31·8	19·2
1879, . .	24·6	27·6	26·6	34·0	38·6	45·8	48·1	51·5	46·4	37·8	29·6	18·4
1880, . .	28·7	39·4	37·1	40·8	43·7	48·8	54·2	55·2	48·7	31·6	26·8	27·0
1881, . .	17·1	30·0	27·6	34·5	41·2	45·4	53·4	47·6	50·1	34·8	35·3	29·8
1882, . .	35·0	34·4	36·2	36·0	42·2	45·0	54·3	51·2	46·5	39·7	34·9	13·2
1883, . .	32·8	32·8	30·6	40·2	38·1	48·8	51·1	53·4	45·0	40·6	36·3	33·8
1884, . .	31·1	31·0	34·6	39·1	42·0	46·0	51·0	49·6	50·4	37·6	29·0	30·0
1885, . .	28·4	31·6	33·8	38·9	38·9	48·6	53·6	45·0	41·8	37·0	26·0	26·0
1886, . .	21·4	30·0	27·6	36·8	37·3	44·9	50·2	51·4	45·6	44·5	38·2	27·4
1887, . .	29·4	28·8	28·6	37·6	41·7	46·3	52·3	50·2	44·2	35·8	32·4	30·5
1888, . .	28·5	28·7	29·5	36·5	42·0	41·2	49·5	51·6	43·2	40·1	34·9	28·6
1889, . .	32·3	25·8	29·2	36·9	46·0	52·0	49·5	52·1	44·0	40·0	32·0	31·4
1890, . .	30·5	32·5	33·1	38·2	45·0	49·3	51·2	50·7	50·1	36·4	29·2	27·9
1891, . .	28·0	37·2	29·0	35·9	41·5	45·6	52·8	51·2	47·2	37·4	32·9	33·4
1892, . .	28·7	23·1	29·1	34·7	42·2	44·8	49·6	48·0	46·0	34·3	35·0	22·6
1893, . .	23·0	29·7	33·7	38·5	44·7	48·4	52·8	55·0	41·4	36·4	32·8	29·2
1894, . .	21·5	33·2	39·0	41·6	41·8	45·1	54·8	49·0	46·1	35·6	36·4	30·8
1895, . .	24·6	20·3	32·2	38·8	43·0	48·8	51·2	52·4	51·0	34·2	38·6	28·9
1896, . .	32·4	31·0	35·8	41·3	44·9	48·6	53·5	51·8	45·6	34·4	30·4	30·8
Highest, .	35·0	39·4	39·0	43·1	46·6	52·0	55·0	55·2	51·0	44·5	38·6	37·0
Lowest, .	17·1	20·3	25·5	32·5	37·0	41·2	47·0	45·0	41·5	31·4	26·0	12·4
Range, .	17·9	19·1	13·5	10·6	9·6	10·8	8·0	10·2	9·5	13·1	12·6	24·6

Table XVI.

Showing the Extreme Range in the Mean Daily Temperatures in Edinburgh from 1857 *to* 1896.

Year.	Jan.	Feb.	Mar.	April.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.
	°	°	°	°	°	°	°	°	°	°	°	°
1857, . .	22·2	16·7	13·2	17·5	17·5	23·6	13·1	11·2	14·4	13·9	18·0	15·3
1858, . .	16·1	21·2	24·4	20·0	19·1	16·6	11·7	11·4	12·8	14·8	16·2	11·8
1859, . .	14·1	15·4	16·4	23·0	19·3	18·1	21·5	16·0	10·4	28·9	12·3	21·9
1860, . .	18·0	17·1	13·5	13·8	18·5	15·5	12·5	8·4	16·4	15·4	12·8	32·4
1861, . .	25·6	17·7	11·1	10·6	22·0	14·9	9·1	12·0	12·6	18·0	18·2	23·2
1862, . .	15·1	20·9	22·0	25·0	13·8	9·0	12·9	8·0	13·4	21·0	19·7	12·5
1863, . .	15·0	13·7	21·9	12·1	15·3	14·1	16·3	16·0	11·8	13·7	19·0	18·4
1864, . .	22·4	23·0	15·6	20·8	24·7	14·3	17·0	15·5	8·5	14·5	13·0	21·5
1865, . .	18·0	20·0	16·5	15·0	19·0	18·5	14·0	12·5	14·0	19·0	14·0	14·0
1866, . .	23·5	17·0	22·5	15·0	21·0	23·5	20·0	14·0	12·0	15·5	20·0	20·5
1867, . .	26·0	15·5	16·0	11·5	18·5	14·5	18·0	13·5	12·5	20·5	15·0	20·5
1868, . .	20·5	17·0	16·0	17·0	20·5	13·0	14·0	22·5	23·5	16·0	22·0	20·0
1869, . .	20·0	21·0	11·5	22·5	10·0	18·5	14·0	18·5	11·5	26·0	24·0	19·5
1870, . .	13·5	17·5	18·0	14·0	16·5	14·0	18·5	15·5	13·5	14·5	16·0	25·5
1871, . .	14·5	19·8	20·7	11·0	22·0	10·5	11·5	18·0	21·5	20·0	14·0	16·5
1872, . .	17·0	12·5	21·0	18·0	16·1	17·2	14·6	11·3	18·6	16·8	16·4	19·9
1873, . .	21·7	16·4	17·1	13·0	16·8	15·7	17·5	12·0	17·8	14·1	15·0	19·0
1874, . .	14·0	17·5	21·7	21·7	18·6	11·6	14·2	14·6	12·4	18·3	24·1	19·6
1875, . .	25·9	17·2	15·9	15·0	14·4	16·6	13·4	12·9	15·3	16·8	19·6	17·8
1876, . .	22·6	17·7	17·1	23·7	17·4	18·0	16·6	12·5	13·2	19·4	20·9	18·1
1877, . .	16·7	19·9	16·8	16·1	15·0	9·8	11·5	12·3	12·8	25·7	16·8	21·2
1878, . .	18·8	16·3	17·8	18·9	15·3	26·8	14·2	13·6	15·6	23·6	11·8	26·2
1879, . .	17·2	22·2	19·7	13·8	14·6	13·1	16·3	16·0	14·6	15·4	19·6	28·0
1880, . .	22·5	9·0	10·9	12·6	17·4	15·8	8·3	14·6	15·1	24·3	23·8	24·2
1881, . .	28·1	14·3	23·8	19·1	24·2	19·2	15·0	16·7	8·5	22·4	20·1	20·2
1882, . .	14·1	17·4	14·7	16·9	14·6	14·2	10·8	16·6	13·0	14·7	13·0	35·0
1883, . .	13·6	14·8	17·3	13·4	18·5	11·6	14·0	11·0	13·0	15·7	16·9	16·0
1884, . .	19·6	18·2	20·7	12·6	16·8	21·9	15·0	12·7	12·0	16·2	26·6	17·6
1885, . .	19·6	17·8	13·6	17·3	16·3	13·2	17·4	17·0	18·2	12·9	28·1	25·0
1886, . .	27·6	15·0	26·2	16·9	20·5	16·7	17·8	14·0	17·8	16·7	16·4	17·3
1887, . .	21·4	20·2	19·9	13·0	17·5	20·5	17·6	14·0	15·4	18·1	13·5	18·6
1888, . .	23·1	19·3	20·7	13·8	22·5	17·5	13·1	9·8	15·0	20·2	10·7	22·5
1889, . .	15·5	24·1	23·1	15·6	15·5	12·2	16·3	14·9	18·6	11·3	21·2	18·7
1890, . .	18·8	15·0	18·7	13·4	15·0	11·6	10·8	16·9	12·9	23·3	21·0	24·0
1891, . .	18·2	13·6	22·6	15·0	15·4	18·8	11·8	11·6	19·9	20·6	14·9	14·3
1892, . .	21·7	24·1	26·2	19·1	18·4	22·6	14·4	17·5	11·6	19·0	16·0	16·0
1893, . .	24·8	19·5	19·7	19·9	15·9	23·2	10·2	17·0	21·8	24·0	19·8	22·4
1894, . .	26·9	16·1	13·9	13·4	12·8	18·7	12·4	12·6	11·7	21·0	20·3	23·0
1895, . .	12·4	21·1	17·8	16·6	19·8	16·4	12·4	16·2	15·7	23·0	10·6	16·5
1896, . .	15·8	20·0	14·2	14·5	16·6	18·5	14·7	12·3	15·2	24·9	20·0	19·8
Highest, .	26·9	24·1	26·2	23·7	24·7	26·8	21·5	22·5	23·5	28·9	28·1	35·0
Lowest, .	12·4	9·0	10·9	10·6	10·0	9·0	8·3	8·0	8·5	11·3	10·6	11·8
Range, . .	14·5	15·1	15·3	13·1	14·7	17·8	13·2	14·5	15·0	17·6	17·5	23·2

Table XVII,

Showing the Greatest Daily Range of Temperature in each Month from 1857 *to* 1896.

Year.	Jan.	Feb.	Mar.	April.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.
	°	°	°	°	°	°	°	°	°	°	°	°
1857, . .	15·6	16·8	16·9	37·1	24·7	26·6	25·6	21·8	28·3	18·7	16·3	16·3
1858, . .	17·9	17·0	18·3	21·7	21·7	28·1	19·6	19·8	22·7	21·0	15·0	17·1
1859, . .	18·3	20·6	21·2	23·8	25·2	26·0	23·3	25·2	21·2	18·6	16·2	13·9
1860, . .	18·1	15·9	18·0	29·0	28·2	22·0	22·7	26·5	23·3	15·8	19·5	14·8
1861, . .	21·6	17·8	20·7	21·6	27·0	21·0	17·6	15·2	18·0	17·0	18·0	17·5
1862, . .	15·0	12·0	18·0	25·0	22·6	22·4	22·4	20·2	20·0	21·0	16·0	16·5
1863, . .	20·4	23·0	19·8	21·0	23·0	19·0	30·3	21·0	14·7	16·0	18·2	20·5
1864, . .	17·0	15·5	21·0	22·5	31·0	20·0	24·0	22·0	18·0	20·0	18·0	13·0
1865, . .	16·9	15·0	19·0	39·0	19·0	30·0	24·0	20·0	22·0	19·0	18·0	17·0
1866, . .	23·0	16·0	22·0	23·0	36·0	28·0	30·0	22·0	23·0	17·0	17·0	22·0
1867, . .	16·0	15·0	18·0	22·0	26·0	26·0	26·4	21·0	20·0	17·0	17·0	18·0
1868, . .	15·7	17·7	20·7	21·7	26·0	32·0	32·7	26·7	24·7	16·7	16·0	18·7
1869, . .	15·7	16·7	20·7	32·3	34·7	28·3	31·0	28·7	22·7	25·0	20·7	18·7
1870, . .	20·0	15·7	24·7	33·3	22·7	21·7	27·7	31·0	26·7	22·0	18·0	16·0
1871, . .	13·0	14·6	24·0	21·0	30·0	25·3	23·0	26·0	24·0	23·0	21·0	23·0
1872, . .	23·0	19·0	26·0	20·7	20·9	26·8	25·7	18·5	16·0	23·7	26·3	17·7
1873, . .	15·4	16·3	18·9	24·4	21·4	24·5	22·9	20·3	24·3	25·6	25·0	27·0
1874, . .	18·0	20·0	25·9	25·2	23·9	22·5	24·3	27·2	20·6	17·1	17·2	19·9
1875, . .	18·5	14·0	17·1	31·7	22·4	19·9	27·9	18·8	24·5	16·9	18·2	14·9
1876, . .	18·5	12·9	16·8	18·3	28·9	31·6	33·3	27·2	18·4	16·9	12·4	15·7
1877, . .	17·5	16·5	23·6	27·8	24·5	25·2	22·9	25·3	22·1	21·7	23·0	17·0
1878, . .	19·8	15·3	18·6	24·8	26·5	26·6	29·7	25·4	19·2	23·5	16·5	20·5
1879, . .	16·3	15·9	20·8	20·3	24·7	19·0	20·9	21·0	20·3	22·7	19·6	19·8
1880, . .	15·6	16·3	25·7	27·4	23·7	27·4	24·6	23·6	22·6	18·8	22·4	16·9
1881, . .	19·8	17·6	27·6	23·8	30·2	29·4	21·7	27·4	22·2	20·5	21·1	21·1
1882, . .	19·4	18·6	20·5	20·4	22·2	24·0	19·1	20·4	26·0	23·0	16·8	29·0
1883, . .	16·5	17·6	18·9	20·4	23·2	22·8	22·4	23·4	30·0	21·0	16·9	18·6
1884, . .	16·3	16·8	24·3	24·2	30·3	27·7	21·9	24·3	20·8	16·4	18·3	13·5
1885, . .	14·1	17·2	21·7	22·7	23·3	24·6	26·9	24·2	23·0	20·6	22·5	21·0
1886, . .	18·3	18·0	17·4	33·8	29·5	28·5	25·3	23·0	23·0	19·0	19·3	17·0
1887, . .	17·9	23·0	21·2	23·1	25·4	32·6	30·1	24·2	20·4	21·2	16·4	15·6
1888, . .	13·7	20·8	23·4	24·6	27·6	28·8	23·8	22·9	26·0	19·3	13·3	18·6
1889, . .	16·4	19·4	19·4	19·5	27·0	28·1	26·2	29·0	22·0	20·5	18·8	16·2
1890, . .	15·6	17·8	20·2	26·6	26·2	27·4	22·3	21·9	28·2	21·9	19·8	16·0
1891, . .	20·0	23·0	21·0	22·8	32·4	31·6	22·1	21·4	25·4	17·1	19·7	16·8
1892, . .	13·4	18·2	26·0	33·1	26·0	25·3	25·5	24·7	20·0	18·6	19·1	18·8
1893, . .	16·1	17·5	28·0	24·6	23·4	28·5	25·3	25·6	23·5	18·9	19·8	16·0
1894, . .	18·5	17·7	31·4	23·4	26·2	22·9	21·4	19·8	21·0	21·2	13·8	19·2
1895, . .	14·0	20·5	18·2	22·5	26·0	29·3	22·2	20·3	26·0	21·8	18·1	17·5
1896, . .	16·1	18·9	18·8	27·6	33·2	26·0	23·7	22·9	17·9	17·6	14·9	15·3
Greatest, .	23·0	23·0	31·4	39·0	36·0	32·6	33·3	31·0	30·0	25·6	26·3	29·0

Table XVIII.

Synopsis of Thermometric Observations made in Shade 4 feet above Grass from 1840 *to* 1896. *The Observations are from Registering Thermometers at a Height of* 250 *feet above Mean Sea-Level.*

Year.	JANUARY.						FEBRUARY.						MARCH.					
	Maximum.	Minimum.	Range.	Mean of all the Highest.	Mean of all the Lowest.	Mean Daily Range.	Maximum.	Minimum.	Range.	Mean of all the Highest.	Mean of all the Lowest.	Mean Daily Range.	Maximum.	Minimum.	Range.	Mean of all the Highest.	Mean of all the Lowest.	Mean Daily Range.
	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°
1840, . .	52·0	21·0	31·0	45·1	33·4	11·7	50·0	21·0	29·0	43·3	31·7	11·6	60·0	23·0	37·0	49·1	32·5	16·6
1841, . .	51·0	8·0	43·0	39·1	27·7	11·4	53·0	26·0	27·0	42·7	33·1	9·6	66·0	28·0	38·0	54·1	38·9	15·2
1842, . .	47·0	18·0	29·0	40·1	29·8	10·3	51·0	22·0	29·0	46·0	34·0	12·0	60·0	29·0	31·0	49·5	35·2	14·3
1843, . .	55·0	20·0	35·0	45·7	33·1	12·6	56·0	16·0	40·0	38·8	29·8	9·0	61·0	21·0	40·0	50·1	34·5	15·6
1844, . .	57·0	23·0	34·0	50·4	31·9	18·5	55·0	13·0	42·0	45·1	27·1	18·0	68·0	23·0	45·0	51·9	30·9	21·0
1845, . .	53·0	5·0	48·0	42·8	30·4	12·4	49·0	18·0	31·0	41·9	29·0	12·9	57·0	16·0	41·0	44·6	28·6	16·0
1846, . .	59·0	25·0	34·0	47·6	36·5	11·1	64·0	28·0	36·0	52·9	36·9	16·0	63·0	17·0	46·0	51·9	34·1	17·8
1847, . .	53·0	21·0	32·0	42·4	30·0	12·4	51·0	17·0	34·0	43·1	28·3	14·8	66·0	21·0	45·0	49·1	35·0	14·1
1848, . .	54·0	5·0	49·0	40·4	26·9	13·5	57·0	15·0	42·0	47·4	33·4	14·0	61·0	26·0	35·0	49·5	33·6	15·9
1849, . .	55·0	19·0	36·0	42·5	31·2	11·3	54·0	24·0	30·0	47·8	36·3	11·5	58·0	24·0	34·0	48·7	36·4	12·3
1850, . .	45·0	12·0	33·0	35·9	27·2	8·7	55·0	25·0	30·0	46·0	37·5	8·5	56·0	22·0	34·0	48·8	35·9	12·9
1851, . .	55·0	26·0	29·0	44·7	36·5	8·2	54·0	27·0	27·0	45·8	35·7	10·1	52·0	25·0	27·0	46·8	35·2	11·6
1852, . .	51·0	25·0	26·0	42·4	35·5	6·9	53·0	26·0	27·0	46·1	34·4	11·7	62·0	27·0	35·0	46·7	36·1	10·6
1853, . .	54·0	25·0	29·0	43·5	34·2	9·3	45·0	19·0	26·0	37·8	29·9	7·9	52·0	25·5	26·5	42·6	32·9	9·7
1854, . .	53·0	18·5	34·5	41·2	32·6	8·6	53·0	27·0	26·0	45·6	34·3	11·3	59·0	32·5	26·5	51·1	39·4	11·7
1855, . .	52·0	22·5	29·5	41·7	33·4	8·3	41·0	14·5	26·5	35·1	26·0	9·1	47·5	29·0	18·5	42·8	32·3	10·5
1856, . .	49·8	24·7	25·1	40·1	34·3	5·8	56·4	28·1	28·3	45·8	37·8	7·9	51·1	29·4	21·7	46·0	37·1	8·9
1857, . .	53·0	20·0	33·0	42·9	33·6	9·3	53·8	28·8	25·0	46·3	36·7	9·2	54·4	27·8	26·6	46·0	36·6	9·4
1858, . .	53·3	28·2	25·1	46·0	36·0	10·0	51·3	22·5	28·8	41·6	31·6	10·0	61·3	20·5	40·8	46·8	35·6	11·2
1859, . .	50·9	27·5	23·4	45·2	34·6	10·6	52·8	26·6	26·2	45·3	34·3	11·0	56·1	28·8	27·3	49·0	37·4	11·6
1860, . .	50·6	22·8	27·8	38·5	30·5	8·0	46·7	17·8	28·9	38·3	28·9	9·4	51·0	26·7	24·3	44·1	32·8	11·3
1861. . .	51·8	16·3	35·5	40·2	32·4	7·8	52·3	26·0	26·3	43·4	34·7	8·7	54·4	29·1	25·3	48·3	34·9	13·4
1862, . .	52·2	28·6	23·6	43·6	36·0	7·6	57·0	26·2	30·8	46·1	37·9	8·2	54·6	22·0	32·6	44·5	35·1	9·4
1863, . .	53·0	25·5	27·5	43·5	33·9	9·6	59·0	26·5	32·5	47·0	35·3	11·7	58·5	25·0	33·5	48·5	37·4	11·1
1864, . .	50·0	19·5	30·5	41·1	30·9	10·2	49·0	20·0	29·0	37·6	29·0	8·6	51·0	18·0	33·0	42·5	31·9	10·6
1865, . .	51·0	20·0	31·0	39·6	30·4	9·2	49·0	18·0	31·0	38·0	28·7	9·3	54·0	27·0	27·0	41·9	31·2	10·7
1866, . .	56·0	21·0	35·0	45·2	34·9	10·3	51·7	25·0	26·7	41·9	33·2	8·7	58·7	20·0	38·7	43·0	31·5	11·5
1867, . .	52·7	16·0	36·7	37·0	28·6	8·4	53·7	29·0	24·7	47·2	38·4	8·8	54·7	24·0	30·7	41·4	32·5	8·9
1868, . .	53·7	25·0	28·7	42·2	33·9	8·3	54·7	31·0	23·7	47·2	38·8	8·4	56·7	28·0	28·7	49·7	38·4	11·3
1869, . .	51·7	25·0	26·7	45·0	35·5	9·5	56·7	28·0	28·7	47·5	37·2	10·3	50·0	26·0	24·0	44·0	31·9	12·1
1870, . .	46·7	25·0	21·7	40·6	32·4	8·2	47·7	19·0	28·7	39·6	30·1	9·5	53·7	22·0	31·7	46·1	32·3	13·8
1871, . .	49·0	24·0	25·0	38·4	32·3	6·1	54·5	27·8	26·7	46·1	38·7	7·4	61·7	25·2	36·5	50·0	37·8	12·2
1872, . .	52·7	24·0	28·7	44·0	33·0	11·0	53·7	28·0	25·7	46·1	35·2	10·9	61·7	23·0	38·7	47·5	35·4	12·1
1873, . .	54·8	24·0	30·8	44·0	36·1	7·9	49·2	22·2	27·0	40·4	31·6	8·8	54·9	26·0	28·9	43·2	34·7	8·5
1874, . .	53·0	28·6	24·4	45·4	36·0	9·4	58·0	19·6	38·4	45·5	32·6	12·9	61·0	27·4	33·6	50·7	38·5	12·2
1875, . .	53·9	14·0	39·9	45·8	36·0	9·8	51·0	24·5	26·5	40·9	33·8	7·1	56·0	28·0	28·0	45·5	35·7	9·8
1876, . .	53·0	22·3	30·7	44·7	35·0	9·7	53·9	21·3	32·6	42·5	32·5	10·0	54·0	24·0	30·0	44·7	33·0	11·7
1877, . .	52·0	25·0	27·0	43·7	34·8	8·9	54·5	22·4	32·1	47·2	35·2	12·0	53·1	23·0	30·1	44·5	32·4	12·1
1878, . .	52·4	17·5	34·9	43·2	33·8	9·4	55·8	29·5	26·3	47·1	37·8	9·3	57·8	24·5	33·3	47·3	34·7	12·6
1879, . .	46·2	16·5	29·7	35·8	26·7	9·1	49·4	21·4	28·0	38·9	30·7	8·2	54·2	17·0	37·2	43·0	31·9	11·1
1880, . .	54·7	23·0	31·7	40·6	33·5	7·1	55·1	32·0	23·1	48·2	38·2	10·0	57·0	28·2	28·8	48·3	34·8	13·5
1881, . .	47·0	9·4	37·6	34·1	24·1	10·0	48·8	21·7	27·1	39·9	31·6	8·3	57·8	15·0	42·8	44·6	32·4	12·2
1882, . .	53·0	29·1	23·9	46·2	37·7	8·5	57·1	31·2	25·9	48·7	38·0	10·7	59·1	30·2	28·9	50·2	38·5	11·7
1883, . .	52·7	25·9	26·8	43·9	34·3	9·6	54·1	28·0	26·1	46·1	36·4	9·7	53·0	24·5	28·5	42·6	30·9	11·7
1884, . .	53·0	26·0	27·0	45·8	36·8	9·0	51·6	24·8	26·8	45·7	35·6	10·1	66·6	28·4	38·2	48·5	36·1	12·4
1885, . .	51·7	22·0	29·7	40·6	33·3	7·3	54·1	24·4	29·7	45·9	35·4	10·5	53·3	27·0	26·3	46·7	34·0	12·7
1886, . .	51·4	12·2	39·2	39·2	30·4	8·8	49·0	21·0	28·0	39·5	31·0	8·5	56·5	18·9	37·6	42·7	33·3	9·4
1887, . .	53·8	21·8	32·0	44·0	34·3	9·7	57·5	20·0	37·5	45·6	33·8	11·8	57·0	20·5	36·5	44·8	33·5	11·3
1888, . .	55·1	24·3	30·8	43·2	35·4	7·8	50·5	18·3	32·2	39·9	31·9	8·0	55·2	23·8	31·4	41·4	31·6	9·8
1889, . .	54·9	26·0	28·9	44·0	35·5	8·5	54·2	21·6	32·6	42·3	32·4	9·9	56·0	20·8	35·2	46·0	34·7	11·3
1890, . .	53·8	25·0	28·8	46·7	36·8	9·9	52·9	25·9	27·0	42·9	32·7	10·2	59·5	25·4	34·1	48·6	37·0	11·6
1891, . .	51·0	24·0	27·0	40·8	32·7	8·1	61·1	27·5	33·6	50·3	36·4	13·9	55·2	20·3	34·9	44·4	32·0	12·4
1892, . .	52·6	24·8	27·8	40·8	32·4	8·4	52·3	14·0	38·3	42·3	32·6	9·7	63·2	21·6	41·6	43·7	30·9	12·8
1893, . .	52·4	15·0	37·4	41·5	33·6	7·9	54·9	25·1	29·8	44·8	34·9	9·9	67·0	24·9	42·1	51·3	36·5	14·8
1894, . .	51·7	13·9	37·8	42·8	33·3	9·5	54·5	26·1	28·4	45·7	35·1	10·6	64·3	30·1	34·2	51·3	36·9	14·4
1895, . .	41·0	19·0	22·0	36·2	27·5	8·7	46·3	11·9	34·4	37·1	25·4	11·7	55·3	27·0	28·3	45·9	36·2	9·7
1896. . .	52·9	23·9	29·0	45·5	36·9	8·6	52·9	27·7	25·2	47·0	37·6	9·4	56·1	29·4	26·7	48·8	36·0	12·8

TABLE XVIII.—*continued.*

Year.	APRIL.						MAY.						JUNE.					
	Maximum.	Minimum.	Range.	Mean of all the Highest.	Mean of all the Lowest.	Mean Daily Range.	Maximum.	Minimum.	Range.	Mean of all the Highest.	Mean of all the Lowest.	Mean Daily Range.	Maximum.	Minimum.	Range.	Mean of all the Highest.	Mean of all the Lowest.	Mean Daily Range.
	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°
1840, . .	76·0	31·0	45·0	57·6	39·2	18·4	74·0	29·0	45·0	55·5	40·9	14·6	72·0	37·0	35·0	64·2	46·2	18·0
1841, . .	64·0	28·0	36·0	53·1	37·0	16·1	76·0	28·0	48·0	61·4	42·7	18·7	79·0	35·0	44·0	61·8	45·2	16·6
1842, . .	72·0	29·0	43·0	55·8	36·1	19·7	67·0	36·0	31·0	60·0	43·3	16·7	77·0	39·0	38·0	66·5	47·7	18·8
1843, . .	65·0	26·0	39·0	53·6	37·6	16·0	69·0	35·0	34·0	53·7	40·5	13·2	71·0	39·0	32·0	59·7	44·7	15·0
1844, . .	71·0	31·0	40·0	58·2	40·8	17·4	68·0	30·0	38·0	58·4	39·2	19·2	74·0	40·0	34·0	64·1	46·0	18·1
1845, . .	64·0	28·0	36·0	54·3	36·1	18·2	69·0	33·0	36·0	55·8	40·5	15·3	79·0	38·0	41·0	65·7	48·0	17·7
1846, . .	64·0	25·0	39·0	52·9	36·6	16·3	70·0	31·0	39·0	65·3	41·1	24·2	84·0	43·0	41·0	73·2	50·6	22·6
1847, . .	61·0	29·0	32·0	52·7	33·7	19·0	77·0	32·0	45·0	59·0	42·2	16·8	76·0	38·0	38·0	66·3	45·8	20·5
1848, . .	65·0	26·0	39·0	53·1	34·5	18·6	78·0	35·0	43·0	67·6	43·3	24·3	75·0	37·0	38·0	63·5	46·4	17·1
1849, . .	67·0	23·0	44·0	49·8	36·1	13·7	70·0	37·0	33.0	59·0	43·1	15·8	78·0	34·0	44·0	62·0	43·6	18·4
1850, . .	61·0	31·0	30·0	54·0	39·1	14·9	66·0	26·0	40·0	56·6	39·9	16·7	76·0	38·0	38·0	67·2	49·7	17·5
1851, . .	59·0	30·0	29·0	51·3	36·2	15·1	72·0	32·0	40·0	59·2	41·6	17·6	75·0	32·0	43·0	64·4	46·3	18·1
1852, . .	71·0	33·0	38·0	56·9	40·3	16·6	68·0	37·0	?	?	?	?	72·0	43·0	?	?	?	?
1853, . .	60·5	32·0	28·5	51·1	40·2	10·9	69·5	31·5	38·0	56·3	42·3	14·0	75·5	44·5	31·0	65·2	51·2	14·0
1854, . .	58·0	30·0	28·0	52·5	40·3	12·2	64·0	39·0	25·0	57·8	44·0	13·8	74·0	41·0	33·0	63·4	49·7	13·7
1855, . .	64·0	30·5	33·5	53·0	37·5	15·5	70·0	30·0	40·0	52·5	39·9	12·6	79·5	41·5	38·0	65·2	49·4	15·8
1856, . .	61·1	35·7	25·4	51·5	40·8	10·7	64·6	36·4	28·2	54·0	43·0	11·0	73·6	43·5	30·1	62·6	49·8	12·8
1857, . .	63·6	26·5	37·1	49·4	37·5	11·9	70·9	38·8	32·1	58·7	44·5	14·2	81·6	43·0	38·6	67·8	50·5	17·3
1858, . .	65·6	26·7	38·9	52·6	38·1	14·5	66·4	35·1	31·3	58·7	43·5	15·2	78·2	43·8	34·4	65·5	51·5	14·0
1859, . .	68·9	25·5	43·4	48·8	34·4	14·4	68·2	33·6	34·6	59·2	43·2	16·0	69·8	41·2	28·6	62·4	47·9	14·5
1860, . .	57·7	29·5	28·2	46·8	34·9	11·9	69·7	36·0	33·7	57·5	43·7	13·8	65·8	40·8	25·0	56·7	46·3	10·4
1861, . .	63·1	31·5	31·6	50·0	37·6	12·4	67·1	29·5	37·6	57·5	42·4	15·1	71·1	40·5	30·6	60·5	49·3	11·2
1862, . .	70·0	26·0	44·0	51·6	38·4	13·2	64·0	31·4	32·6	58·0	43·2	14·8	65·2	42·0	23·2	59·4	47·3	12·1
1863, . .	57·0	30·0	27·0	51·4	37·8	13·6	64·5	36·5	28·0	56·0	42·9	13·1	68·0	42·0	26·0	63·8	47·5	16·3
1864, . .	69·0	30·0	39·0	53·1	39·5	13·6	79·0	32·0	47·0	60·0	42·0	18·0	68·0	47·5	30·5	61·2	47·0	14·2
1865, . .	70·0	29·0	41·0	53·4	38·0	15·4	69·0	36·0	33·0	56·9	44·2	12·7	78·0	40·0	38·0	66·1	48·0	18·1
1866, . .	59·7	27·0	32·7	48·9	36·5	12·4	76·7	30·0	46·7	56·6	39·4	17·2	79·7	37·0	42·7	65·0	48·9	16·1
1867, . .	58·7	36·0	22·7	52·0	40·9	11·1	67·7	32·0	35·7	53·4	42·3	11·1	75·7	43·0	32·7	63·9	49·2	14·7
1868, . .	62·7	32·0	30·7	52·8	40·5	12·3	72·7	32·0	40·7	60·4	45·9	14·5	78·7	42·0	36·7	65·1	49·6	15·5
1869, . .	70·7	26·0	44·7	56·2	39·4	16·8	68·7	30·0	38·7	53·4	37·6	15·8	71·7	37·0	34·7	62·6	46·2	16·4
1870, . .	72·7	33·0	39·7	55·8	41·8	14·0	69·7	36·0	33·7	59·9	45·5	14·4	74·7	45·0	29·7	64·5	50·8	13·7
1871, . .	58·7	30·0	28·7	48·3	37·4	10·9	72·7	27·0	45·7	59·3	41·4	17·9	70·7	39·0	31·7	61·1	44·3	16·8
1872, . .	62·7	30·0	32·7	51·1	38·0	13·1	64·0	30·0	34·0	53·5	39·4	14·1	75·7	38·7	37·0	63·8	51·0	12·8
1873, . .	57·3	32·9	24·4	50·3	39·1	11·2	62·4	30·8	31·6	55·1	41·6	13·5	71·4	37·5	33·9	62·5	47·3	15·2
1874, . .	69·0	34·2	34·8	53·6	40·8	12·8	66·3	35·0	31·3	51·5	41·8	9·7	71·7	40·9	30·8	63·4	47·6	15·8
1875, . .	70·3	33·8	36·5	53·2	40·1	13·1	70·2	39·8	30·4	59·2	45·6	13·6	73·4	40·3	33·1	62·5	48·2	14·3
1876, . .	65·2	26·9	38·3	50·3	38·5	11·8	66·5	32·4	34·1	55·8	41·9	13·9	75·8	42·4	33·4	64·1	46·8	17·3
1877, . .	59·2	29·0	30·2	47·5	35·7	11·8	64·0	31·5	32·5	53·3	39·6	13·7	72·0	42·8	29·2	64·6	48·7	15·9
1878, . .	62·5	28·8	33·7	52·7	39·2	13·5	71·5	33·5	38·0	59·0	43·7	15·3	81·3	36·4	44·9	65·2	48·3	16·9
1879, . .	54·2	28·7	25·5	46·5	34·7	11·8	62·3	29·2	33·1	54·2	38·3	15·9	66·0	37·0	29·0	58·4	46·5	11·9
1880, . .	60·1	31·2	28·9	52·6	39·1	13·5	71·6	32·9	38·7	57·1	42·2	14·9	72·6	38·2	34·4	63·2	47·8	15·4
1881, . .	58·5	25·2	33·3	50·2	35·5	14·7	79·2	34·0	45·2	59·3	43·3	16·0	78·4	37·5	40·9	62·5	47·1	15·4
1882, . .	60·4	28·5	31·9	50·3	37·9	12·4	66·8	35·4	31·4	57·9	42·1	15·8	67·6	38·6	29·0	61·1	47·5	13·6
1883, . .	60·0	33·1	26·9	52·8	38·9	13·9	66·8	29·2	37·6	56·1	41·5	14·6	66·6	38·5	28·1	60·3	47·3	13·0
1884, . .	63·8	28·8	35·0	52·1	36·9	15·2	72·9	34·0	38·9	57·8	41·5	16·3	79·9	41·1	38·8	62·2	47·7	14·5
1885, . .	65·7	32·0	33·7	52·6	38·5	14·1	62·0	30·1	31·9	53·3	39·9	13·4	70·6	39·0	31·6	61·0	48·4	12·6
1886, . .	70·6	31·5	39·1	50·3	37·1	13·2	72·6	31·7	40·9	55·2	41·0	14·2	75·6	37·1	38·5	62·7	45·9	16·8
1887, . .	59·0	29·0	30·0	49·8	35·4	14·4	68·0	29·1	38·9	57·1	41·1	16·0	83·2	40·9	42·3	67·1	49·8	17·3
1888, . .	59·6	28·3	31·3	49·6	36·1	13·5	76·8	33·0	43·8	58·6	41·9	16·7	69·3	36·9	32·4	59·6	43·9	15·7
1889, . .	61·0	31·5	29·5	48·1	37·9	10·2	74·0	39·8	34·2	59·6	44·5	15·1	78·4	39·4	39·0	66·6	49·0	17·6
1890, . .	64·0	28·4	35·6	52·6	36·5	16·1	71·0	36·3	34·7	57·9	44·0	13·9	71·4	38·1	33·3	62·5	47·7	14·8
1891, . .	60·8	26·6	34·2	50·2	35·5	14·7	73·1	31·0	42·1	56·1	40·0	16·1	72·9	40·6	32·3	61·6	48·3	13·3
1892, . .	68·0	26·0	42·0	52·0	35·7	16·3	69·9	36·0	33·9	58·6	43·1	15·5	80·1	37·8	42·3	62·0	46·8	15·2
1893, . .	69·5	34·0	35·5	56·1	39·8	16·3	70·5	36·9	33·6	61·5	46·5	15·0	85·9	43·6	42·3	66·8	50·5	16·3
1894, . .	63·8	34·3	29·5	54·6	40·7	13·9	64·1	32·0	32·1	54·2	40·0	14·2	74·1	38·7	35·4	62·0	47·4	14·6
1895, . .	61·9	29·4	32·5	52·6	39·4	13·2	74·8	38·5	36·3	62·0	44·1	17·9	78·3	37·6	40·7	65·3	48·0	17·3
1896, . .	65·9	34·5	31·4	55·7	41·9	13·8	78·1	37·4	40·7	64·0	45·7	18·3	75·5	44·4	31·1	62·8	50·6	12·2

TABLE XVIII.—*continued.*

Year.	JULY.						AUGUST.						SEPTEMBER.					
	Maximum.	Minimum.	Range.	Mean of all the Highest.	Mean of all the Lowest.	Mean Daily Range.	Maximum.	Minimum.	Range.	Mean of all the Highest.	Mean of all the Lowest.	Mean Daily Range.	Maximum.	Minimum.	Range.	Mean of all the Higest.,	Mean of all the Lowest.	Mean Daily Range.
	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°
1840, . .	71·0	40·0	31·0	64·1	47·8	14·3	78·0	40·0	38·0	67·7	50·6	16·1	68·0	39·0	29·0	59·2	43·6	15·6
1841, . .	73·0	41·0	32·0	64·2	48·3	15·7	75·0	40·0	35·0	65·5	49·2	16·3	77·0	37·0	40·0	61·8	47·3	14·5
1842, . .	79·0	40·0	39·0	65·4	47·5	17·9	78·0	40·0	38·0	69·1	52·3	16·8	72·0	39·0	33·0	62·3	47·9	14·4
1843, . .	77·0	43·0	34·0	67·5	50·3	17·2	76·0	43·0	33·0	66·2	49·6	16·6	75·0	32·0	43·0	66·2	48·0	18·2
1844, . .	74·0	39·0	35·0	65·4	48·4	17·0	76·0	37·0	39·0	64·5	46·9	17·6	77·0	31·0	46·0	60·9	45·3	15·6
1845, . .	70·0	38·0	32·0	63·0	46·6	16·4	77·0	40·0	37·0	64·6	46·7	17·9	72·0	31·0	41·0	64·8	43·4	21·4
1846, . .	79·0	46·0	33·0	67·3	51·3	16·0	75·0	42·0	33·0	68·3	51·5	16·8	78·0	39·0	39·0	68·9	50·0	18·9
1847, . .	83·0	42·0	41·0	71·0	52·2	18·8	77·0	37·0	40·0	67·7	47·6	20·1	66·0	31·0	35·0	61·1	41·9	19·2
1848, . .	82·0	38·0	44·0	69·1	49·4	19·7	70·0	36·0	34·0	63·4	44·8	18·6	76·0	35·0	41·0	60·5	47·0	13·5
1849, . .	73·0	44·0	29·0	64·6	49·1	15·5	70·0	40·0	30·0	64·2	50·1	14·1	66·0	37·0	29·0	58·0	47·2	10·8
1850, . .	78·0	40·0	38·0	67·8	50·1	17·7	74·0	36·0	38·0	65·9	47·7	18·2	71·0	36·0	35·0	60·4	44·3	16·1
1851, . .	72·0	38·0	34·0	63·7	48·5	15·2	70·0	43·0	27·0	63·7	49·5	14·1	66·0	34·0	32·0	60·2	44·8	15·4
1852, . .	85·0	50·0	?	?	?	?	75·0	49·0	?	?	?	?	74·0	35·0	39·0	61·8	46·3	15·5
1853, . .	71·5	49·5	22·0	65·3	53·1	12·2	70·0	45·0	25·0	63·8	51·8	12·0	66·0	41·0	25·0	59·4	48·0	11·4
1854, . .	75·0	48·5	26·5	65·9	53·4	12·5	71·5	48·0	23·5	67·9	53·2	14·7	71·5	43·0	28·5	63·0	51·0	12·0
1855, . .	79·5	50·5	29·0	68·3	54·9	13·4	73·0	48·0	25·0	66·6	52·8	13·8	60·5	38·6	30·9	60·5	47·8	12·7
1856, . .	76·7	43·7	33·0	67·1	52·6	14·5	81·9	44·9	37·0	65·2	52·8	12·4	67·0	40·9	26·1	58·1	48·1	10·0
1857, . .	75·6	45·7	29·9	69·8	52·9	16·9	76·0	49·0	27·0	67·7	54·5	13·2	77·9	41·6	36·3	63·9	50·7	13·2
1858, . .	69·4	42·7	26·7	61·5	49·5	12·0	68·5	43·6	24·9	62·7	51·1	11·6	68·6	38·8	29·8	61·4	48·2	13·2
1859, . .	78·7	41·5	37·2	65·9	51·3	14·6	77·3	45·8	31·5	67·1	50·9	16·2	66·3	41·3	25·0	59·7	45·8	13·9
1860, . .	71·7	44·7	27·0	65·1	49·9	15·2	69·2	41·7	27·5	61·9	48·0	13·9	67·6	35·7	31·9	59·0	43·4	15·6
1861, . .	69·9	42·0	27·9	63·8	49·5	14·3	70·2	46·9	23·3	64·7	52·0	12·7	64·0	38·0	26·0	58·7	48·3	10·4
1862, . .	68·0	42·8	25·2	60·7	48·3	12·4	68·0	44·0	24·0	61·8	51·1	10·7	65·0	37·3	27·7	58·8	46·5	12·3
1863, . .	75·2	39·5	35·7	66·6	49·3	17·3	69·0	39·0	30·0	62·5	49·5	13·0	60·0	41·0	19·0	54·7	45·2	9·5
1864, . .	79·0	44·0	35·0	64·0	50·0	14·0	74·0	40·0	34·0	62·6	47·1	15·5	65·0	40·0	25·0	59·6	46·4	13·2
1865, . .	76·0	43·0	33·0	66·4	51·3	15·1	68·0	43·0	25·0	62·8	50·2	12·6	74·0	43·0	31·0	65·1	51·1	14·0
1866, . .	82·7	44·0	38·7	65·5	50·5	15·0	71·7	41·0	30·7	62·2	50·1	12·1	67·7	38·0	29·7	59·9	46·6	13·3
1867, . .	76·7	45·0	31·7	62·3	50·0	12·3	76·7	48·0	28·7	65·0	53·0	12·0	66·7	41·0	25·7	59·7	49·2	10·5
1868, . .	85·8	47·0	38·7	70·6	53·7	16·9	87·7	48·0	39·7	66·8	52·9	13·9	81·7	42·0	39·7	60·5	48·9	11·6
1869, . .	79·7	42·0	37·7	68·9	51·0	17·9	80·7	35·0	45·7	65·6	48·6	17·0	68·7	40·0	28·7	60·7	48·4	12·3
1870, . .	84·7	45·0	39·7	68·0	53·5	14·5	79·0	42·0	37·0	67·1	50·5	16·6	69·7	36·0	33·7	63·1	47·4	15·7
1871, . .	72·7	45·0	27·7	65·9	50·2	15·7	79·7	41·0	38·7	68·6	51·0	17·6	71·7	37·0	34·7	59·5	45·7	13·8
1872, . .	79·7	43·7	36·0	65·6	52·6	13·0	73·5	45·7	27·7	62·7	51·1	11·6	66·2	34·7	31·5	57·6	46·6	11·0
1873, . .	82·9	45·3	37·6	66·9	53·1	13·8	71·4	43·8	27·6	63·8	52·0	11·8	76·2	38·8	37·4	58·8	46·0	12·8
1874, . .	81·3	47·2	34·1	66·5	53·7	12·8	76·0	40·8	35·2	63·0	50·5	12·5	67·8	40·4	27·4	60·2	47·6	12·6
1875, . .	76·8	43·6	33·2	64·3	49·7	14·6	73·0	46·3	26·7	64·5	52·7	11·8	72·5	42·6	29·9	60·4	48·5	11·9
1876, . .	86·7	44·7	42·0	66·5	51·9	14·6	74·8	42·2	32·6	63·8	51·2	12·6	62·7	40·8	21·9	57·7	47·5	10·2
1877, . .	69·5	45·0	24·5	64·5	51·2	13·3	71·0	40·5	30·5	61·6	49·1	12·5	67·5	38·0	29·5	60·3	41·9	18·4
1878, . .	83·7	44·3	39·4	69·3	52·6	16·7	74·2	43·5	30·7	66·4	51·2	15·2	69·6	43·0	26·6	61·3	49·6	11·7
1879, . .	71·0	40·3	30·7	59·7	49·0	10·7	78·0	44·0	34·0	63·4	49·5	13·9	68·2	37·0	31·2	59·1	44·8	14·3
1880, . .	72·5	44·7	27·8	64·4	50·9	13·5	77·6	48·4	29·2	68·2	53·3	14·9	72·5	41·8	30·7	62·4	49·7	12·7
1881, . .	75·8	42·5	33·3	65·3	51·5	13·8	76·0	38·7	37·3	62·1	48·4	13·7	64·8	39·0	25·8	59·4	48·2	11·2
1882, . .	74·4	47·9	26·5	63·9	51·7	12·2	81·0	43·2	37·8	63·4	50·8	12·6	68·5	34·6	33·9	59·7	43·7	16·0
1883, . .	75·0	42·4	32·6	62·9	49·4	13·5	72·8	45·0	27·8	64·2	50·8	13·4	69·1	36·0	33·1	60·7	48·4	12·3
1884, . .	75·6	42·8	32·8	62·9	51·2	11·7	78·5	42·8	35·7	65·3	51·6	13·7	66·8	40·4	26·4	60·5	49·0	11·5
1885, . .	82·2	44·9	37·3	68·5	51·6	16·9	71·0	40·5	30·5	61·7	47·9	13·8	69·0	31·6	37·4	59·8	45·2	14·6
1886, . .	80·7	42·6	38·1	66·1	50·8	15·3	73·8	41·5	32·3	65·5	50·7	14·8	67·7	36·1	31·6	59·6	48·0	11·6
1887, . .	81·4	40·5	40·9	69·5	53·2	16·3	76·1	42·6	33·5	66·0	50·1	15·9	67·2	35·9	31·3	59·1	46·1	13·0
1888, . .	73·0	40·2	32·8	61·8	48·6	13·2	70·9	41·1	29·8	62·9	48·6	14·3	69·0	38·3	30·7	59·7	45·3	14·4
1889, . .	76·4	43·2	33·2	64·5	49·0	15·5	74·1	42·4	31·7	63·2	50·8	12·4	69·6	34·6	35·0	59·6	46·4	13·2
1890, . .	70·1	43·5	26·6	64·3	48·8	15·5	71·8	40·1	31·7	63·5	49·7	13·8	76·0	42·3	33·7	66·2	50·4	15·8
1891, . .	73·3	45·4	27·9	65·7	52·1	13·6	69·5	40·4	29·1	63·1	50·6	12·5	79·8	42·4	37·4	62·2	49·8	12·4
1892, . .	71·8	43·4	28·4	62·4	49·4	13·0	71·3	39·2	32·1	63·8	50·5	13·3	64·5	38·2	26·3	58·2	45·6	12·6
1893, . .	72·5	44·2	28·3	64·4	51·1	13·3	84·0	42·6	41·4	69·6	53·7	15·9	72·7	34·9	37·8	60·6	46·7	13·9
1894, . .	77·5	46·5	31·0	65·7	52·2	13·7	68·1	44·4	23·7	63·7	50·1	13·6	65·9	37·2	28·7	58·3	46·0	12·3
1895, . .	74·7	45·5	29·2	64·0	50·3	13·7	77·0	44·6	32·4	65·8	53·1	12·7	78·3	41·0	37·3	66·4	50·8	15·6
1896, . .	75·3	43·3	32·0	65·1	51·8	13·8	73·1	43·7	29·4	63·8	49·8	14·0	66·5	39·9	26·6	58·8	47·8	11·0

TABLE XVIII.—*continued.*

Year.	OCTOBER.						NOVEMBER.						DECEMBER.					
	Maximum.	Minimum.	Range.	Mean of all the Highest.	Mean of all the Lowest.	Mean Daily Range.	Maximum.	Minimum.	Range.	Mean of all the Highest	Mean of all the Lowest.	Mean Daily Range.	Maximum.	Minimum.	Range.	Mean of all the Highest.	Mean of all the Lowest.	Mean Daily Range.
	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°
1840, . .	61·0	30·0	31·0	53·0	39·5	13·5	55·0	27·0	28·0	47·7	35·6	12·1	54·0	22·0	32·0	41·7	31·5	10·2
1841, . .	60·0	28·0	32·0	49·6	39·0	10·6	55·0	23·0	32·0	44·2	33·8	10·4	51·0	23·0	28·0	43·9	34·0	9·9
1842, . .	65·0	25·0	40·0	55·3	37·9	17·4	54·0	20·0	34·0	46·6	35·2	11·4	60·0	30·0	30·0	51·3	39·9	11·4
1843, . .	65·0	25·0	40·0	51·5	38·3	13·2	60·0	26·0	34·0	53·2	35·1	18·1	62·0	32·0	30·0	56·5	39·0	17·5
1844, . .	62·0	28·0	34·0	53·8	40·4	13·4	62·0	28·0	34·0	48·1	38·1	10·0	44·0	20·0	24·0	37·1	28·9	8·2
1845, . .	71·0	30·0	41·0	56·7	42·0	14·7	61·0	24·0	37·0	50·2	36·9	13·3	55·0	23·0	32·0	46·1	31·2	14·9
1846, . .	64·0	30·0	34·0	54·8	41·4	13·4	60·0	22·0	38·0	50·5	39·0	11·5	55·0	16·0	39·0	41·2	27·6	13·6
1847, . .	68·0	34·0	34·0	55·6	42·5	13·1	60·0	25·0	35·0	53·0	38·2	14·8	57·0	21·0	36·0	45·9	33·3	12·6
1848, . .	64·0	28·0	36·0	52·2	41·1	11·1	53·0	30·0	23·0	45·8	34·7	11·1	59·0	21·0	38·0	45·5	35·5	10·0
1849, . .	66·0	28·0	38·0	51·8	38·2	13·6	58·0	21·0	37·0	47·0	36·3	10·7	49·0	21·0	28·0	40·5	33·2	7·3
1850, . .	?	?	?	?	?	?	?	?	?	?	?	?	?	?	?	?	?	?
1851, . .	63·0	33·0	30·0	56·0	44·4	11·6	51·0	25·0	26·0	42·5	32·1	10·4	57·0	23·0	34·0	44·5	36·9	7·6
1852, . .	62·0	32·0	30·0	52·2	40·5	11·7	59·0	24·0	35·0	47·7	37·8	9·9	56·0	26·0	30·0	47·5	36·9	10·6
1853, . .	58·0	34·0	24·0	52·7	44·2	8·5	56·0	30·0	26·0	47·8	38·7	9·1	53·0	32·5	20·5	41·2	32·8	8·4
1854, . .	62·5	32·0	30·5	53·6	42·3	11·3	58·5	31·0	27·5	46·9	37·2	9·7	54·5	27·5	27·0	45·3	34·6	10·7
1855, . .	62·5	29·7	32·8	53·1	41·7	11·4	55·2	29·9	25·3	45·0	37·6	7·4	49·4	22·9	26·6	41·2	33·4	7·8
1856, . .	67·9	40·1	27·8	55·0	48·6	6·4	56·2	25·1	31·1	48·1	38·4	9·7	56·3	18·6	37·7	45·1	36·6	8·3
1857, . .	63·4	36·5	26·9	56·7	46·8	9·8	58·9	28·8	30·1	50·2	41·9	8·3	56·7	32·3	24·4	51·1	42·8	8·3
1858, . .	61·2	30·5	30·7	51·0	38·4	12·6	51·1	24·8	26·3	43·6	35·4	8·2	51·1	27·5	23·6	44·2	35·0	9·2
1859, . .	65·5	24·4	41·1	50·8	40·2	10·6	51·1	26·9	24·2	43·9	33·8	10·1	49·6	18·7	30·9	36·5	30·1	6·4
1860, . .	57·9	27·2	30·7	51·6	41·2	10·4	47·9	26·1	21·8	42·3	34·5	7·8	51·8	8·8	43·0	36·9	30·0	6·9
1861, . .	62·0	33·4	28·6	54·4	44·5	9·9	54·5	22·4	32·1	43·4	32·6	10·8	54·0	22·5	31·5	40·7	33·4	7·3
1862, . .	63·0	32·0	31·0	53·7	40·8	12·9	54·0	24·5	29·5	41·5	32·1	9·4	53·0	32·1	20·9	47·3	37·6	9·8
1863, . .	58·0	31·5	26·5	51·8	41·8	10·0	57·0	26·5	30·5	49·1	39·1	10·0	56·0	24·5	31·5	46·8	36·1	10·7
1864, . .	57·0	31·0	26·0	51·0	39·7	11·3	56·0	29·0	27·0	46·7	35·9	10·8	58·0	25·0	33·0	43·9	34·8	9·1
1865, . .	63·0	29·0	34·0	51·8	40·1	11·7	54·0	28·0	26·0	46·7	35·7	11·0	56·0	32·0	24·0	48·1	38·3	9·8
1866, . .	61·7	33·0	28·7	54·3	44·5	9·8	56·7	29·0	27·7	46·5	37·7	8·8	55·7	28·0	27·7	46·4	37·2	9·2
1867, . .	62·7	32·0	30·7	52·3	41·9	10·4	53·7	31·0	22·7	45·8	37·9	7·9	53·7	27·0	26·7	44·3	36·4	7·9
1868, . .	58·7	30·0	28·7	50·9	40·1	10·8	59·7	23·0	36·7	43·4	35·4	8·0	54·7	26·0	28·7	45·4	37·5	7·9
1869, . .	64·7	31·0	33·7	52·9	42·6	10·3	57·7	25·0	32·7	47·1	36·9	10·2	55·7	20·0	35·7	41·8	32·0	9·8
1870, . .	66·7	33·0	33·7	52·8	41·5	11·3	54·7	29·0	25·7	44·9	35·1	9·8	50·7	16·0	34·7	38·7	31·7	7·0
1871, . .	64·7	28·0	36·7	53·5	40·9	12·6	51·9	23·0	28·9	43·7	32·7	11·0	53·7	25·0	28·7	43·8	31·6	12·2
1872, . .	58·7	28·7	30·0	49·7	41·4	8·3	60·7	30·7	30·0	45·9	36·9	9·0	55·7	24·7	31·0	43·8	35·2	8·6
1873, . .	58·4	28·3	30·1	52·2	38·4	13·8	56·4	26·3	30·1	48·2	36·9	11·3	54·0	27·6	26·4	46·8	37·3	9·5
1874, . .	63·0	32·8	30·2	53·6	42·1	11·5	61·7	27·8	33·9	45·9	37·3	8·6	50·0	13·6	36·4	36·4	27·3	9·1
1875, . .	62·9	31·8	31·1	52·1	42·6	9·5	57·9	25·7	32·2	44·4	35·4	9·0	53·5	27·9	25·6	44·2	35·9	8·3
1876, . .	66·1	38·5	27·6	54·6	46·3	8·3	55·6	26·6	28·9	44·8	36·8	8·0	52·1	28·4	23·7	43·7	37·2	6·5
1877, . .	62·3	29·4	32·9	53·9	40·9	13·0	57·9	28·0	29·9	48·3	38·3	10·0	52·0	23·8	28·2	44·6	35·4	9·2
1878, . .	65·2	30·0	35·2	54·9	42·9	12·0	47·3	26·5	20·8	42·9	33·1	9·8	47·8	9·0	38·8	35·7	26·2	9·5
1879, . .	60·2	29·0	31·2	52·8	38·5	14·3	55·5	24·6	30·9	44·7	35·0	9·7	51·7	7·5	44·2	40·6	28·9	11·7
1880, . .	61·8	24·3	37·5	49·4	38·5	10·9	55·8	20·6	35·2	44·7	34·1	10·6	55·5	21·4	34·1	41·7	33·8	7·9
1881, . .	65·0	28·3	36·7	50·3	38·5	11·8	61·4	29·0	32·4	51·6	41·0	10·6	52·6	25·2	27·4	43·7	33·7	10·0
1882, . .	65·3	29·9	35·4	53·7	43·2	10·5	54·4	29·0	25·4	44·9	35·0	9·9	52·0	6·4	45·6	38·4	28·8	9·6
1883, . .	61·0	32·6	28·4	54·3	41·0	13·3	57·0	28·3	28·7	46·5	36·7	9·8	52·9	28·8	24·1	46·1	36·1	10·0
1884, . .	59·7	31·2	28·5	53·7	43·6	10·1	57·8	23·0	34·8	46·5	37·0	9·5	53·3	25·4	27·9	42·3	33·6	8·7
1885, , .	55·3	27·4	27·9	49·3	38·4	10·9	60·8	19·8	41·0	45·9	36·0	9·9	55·0	20·0	35·0	43·7	34·3	9·4
1886, . .	68·7	35·0	33·7	55·5	45·5	10·0	57·1	30·0	27·1	50·6	39·0	11·6	52·1	19·8	32·3	40·0	30·0	9·7
1887, . .	60·4	28·5	31·9	51·1	39·1	12·0	51·8	28·8	23·0	43·6	36·2	7·4	50·9	23·9	27·0	40·7	32·8	7·9
1888, . .	63·1	32·2	30·9	53·9	41·9	12·0	57·0	30·9	26·1	47·0	39·3	7·7	56·7	22·7	34·0	45·6	37·1	8·5
1889, . .	58·2	36·1	22·1	51·4	40·8	10·6	59·1	29·3	29·8	49·2	38·8	10·4	55·9	26·4	29·5	44·6	34·6	10·0
1890, . .	64·9	29·3	35·6	55·1	43·8	11·3	57·3	25·1	32·2	47·1	36·0	11·1	54·9	23·2	31·7	38·7	31·9	6·8
1891, . .	61·9	32·1	29·8	53·8	41·9	11·9	53·0	27·7	25·3	45·8	37·1	8·7	55·9	28·8	27·1	44·0	35·4	8·6
1892, . .	59·8	25·9	33·9	49·1	38·6	10·5	55·8	30·1	25·7	48·1	38·1	10·0	53·1	16·9	36·1	39·4	30·2	9·2
1893, . .	65·2	29·0	36·2	55·4	42·5	12·9	55·1	28·4	26·7	45·3	36·0	9·3	53·9	25·0	28·9	46·3	37·8	8·5
1894, . .	64·4	28·1	36·3	52·6	40·1	12·5	60·3	30·4	29·9	50·6	41·3	9·3	56·2	27·6	28·6	45·6	36·2	9·4
1895, . .	62·8	27·4	35·4	50·1	38·4	11·7	55·5	33·3	22·2	47·8	39·0	8·8	51·9	25·5	26·4	41·9	33·5	8·4
1896, . .	63·5	28·8	34·7	48·7	37·8	10·9	53·7	26·3	27·7	46·0	38·1	7·9	56·2	23·8	32·4	42·7	35·2	7·5

Table XIX.

Abstract of Temperature Observations.

	Mean Temperature, 1764–1896.					Extremes, 1840–96.					Daily Mean Temp., 1795–1804, 1821–1850, 1857–1896.		
	Highest.	Year.	Lowest.	Year.	Range.	Highest.	Date.	Lowest.	Date.	Range.	Highest.	Lowest.	Range.
	°		°		°	°		°		°	°	°	°
January, .	43·8	1796	26·5	1814	17·3	59·0	30, 31. 1846	5·0	31. 1845 29. 1848	54·0	51·6	16·5	35·1
February, .	47·2	1779	29·8	1838	17·4	64·0	28. 1846	11·9	8. 1895	52·1	55·0	19·0	36·0
March, . .	46·5	1779 1845	34·2	1785	12·3	68·0	31. 1844	15·0	2. 1881	53·0	62·0	25·0	37·0
April, . .	49·8	1792 1798	38·9	1837	10·9	76·0	28. 1840	23·0	17. 1849	53·0	60·8	26·5	34·3
May, . .	55·8	1833	45·1	1810	10·7	79·2	30, 31. 1881	26·0	9. 1850	53·2	67·0	35·0	32·0
June, . .	61·4	1826	51·5	1860	9·9	85·9	18. 1893	32·0	4. 1851	53·9	74·0	41·2	32·8
July, . .	65·2	1779	54·4	1879	10·8	86·7	16. 1876	38·0	16. 1845 2. 1848 4. 1851	48·7	77·8	47·0	30·8
August, .	63·7	1779	52·6	1830	11·1	87·7	5. 1868	35·0	30. 1869	52·7	75·5	45·0	30·5
September, .	59·5	1846	48·2	1807	11·3	81·7	6. 1868	31·0	22. 1844 23. 1845 27. 1847	50·7	72·0	35·5	36·5
October, .	52·7	1831	42·0	1817	10·7	71·0	14. 1845	24·3	20. 1880	46·7	62·0	29·5	32·5
November, .	46·7	1818	34·0	1807	12·7	62·0	17. 1844	19·8	18. 1885	42·2	56·7	24·0	32·7
December, .	47·8	1843	31·0	1878	16·8	62·0	11. 25. 1843	6·4	15. 1882	55·6	55·0	12·4	43·6
Annual Mean and Extreme,	49·6	1779 1846	43·8	1879	5·8	87·7	Aug. 5, 1868	5·0	Jan. 31, 1845 Jan. 29, 1848	82·7	77·8 July 18, 1803	12·4 Dec. 24 1860	65·4

Table XX.

Showing the Low Day Maxima (25° *or below*) *and the High Night Minima* (61°·0 *or above*) *recorded in Edinburgh from* 1840–1896.

Low Day Maxima.			High Night Minima.		
		°			°
1849	December 28, . . .	25·0	1842	August 13, ...	61·0
1850	January 17, . . .	25·0	1850	June 24, ...	61·0
1860	December 24, . . .	19·8	1857	August 20, ...	63·0
1860	December 25, . . .	19·0	1857	July 13, ...	61·2
1860	December 26, . . .	23·0	1868	August 5, ...	63·0
1864	February 24, . . .	24·0	1870	July 24, ...	61·8
1879	December 3, . . .	25·0	1872	July 4, ...	61·3
1881	January 16, . . .	25·0	1872	July 21, ...	62·1
1881	January 17, . . .	24·8	1875	August 17, ...	61·6
1882	December 13, . . .	24·5	1878	June 28, ...	62·2
1882	December 15, . . .	20·0	1881	July 14, ...	61·0
1895	February 7, . . .	24·5	1881	August 11, ...	62·2
			1890	August 5, ...	63·3
			1893	June 17, ...	61·8
			1893	August 16, ...	61·4
			1896	July 20, ...	62·5

Table XXI.

Reduction of Adie's Observations from 1824–1831, *showing the Mean Maximum, Minimum, and Average Temperature, and the Mean Daily Range of Temperature from* 1824–1831.

	1824.				1825.			
	Max.	Min.	Mean.	Daily Range.	Max.	Min.	Mean.	Daily Range.
	°	°	°	°	°	°	°	°
January,	43·6	36·1	39·8	7·5	43·4	34·7	39·1	8·7
February,	43·8	34·3	39·0	10·5	44·5	33·4	39·0	11·1
March,	45·9	33·4	39·6	12·5	48·4	34·0	41·2	14·4
April,	54·9	35·5	45·2	19·4	56·8	36·4	46·6	20·4
May,	60·9	39·3	50·1	21·6	60·4	41·0	50·7	19·4
June,	67·0	46·3	56·6	20·7	67·6	45·8	56·7	21·8
July,	69·8	50·0	59·9	19·8	71·7	51·1	61·4	20·6
August,	66·6	47·8	57·2	18·8	68.6	51·5	60·0	17·1
September,	63·0	46·2	54·6	16·8	65·7	48·1	56·9	17.6
October,	51·9	39·6	45·8	12·3	57·5	42·8	50·1	14·7
November,	46·8	34·7	40·8	13·1	44·6	32·5	38·5	12·1
December,	42·7	34·1	38·4	8·6	42·4	35·6	39·0	6·8
	1826.				1827.			
January,	35·3	27·8	31·6	7·5	39·8	31·0	35·4	8·8
February,	47·5	36·0	41·8	11·5	38·5	29·4	34·0	9·1
March,	49·5	34·2	41·8	15·3	46·0	34·2	40·1	11·8
April,	55·5	38·0	46·8	17·5	52·6	37·5	45·0	15·1
May,	62·3	41·3	51·8	21·0	58·4	43·2	50·8	15·2
June,	72·2	50·6	61·4	21·6	65·4	46·9	56·1	18·5
July,	72·6	51·4	62·0	21·2	67·8	49·1	58·4	18·7
August,	71·0	51·6	61·3	19·4	62·6	47·9	55·2	14·7
September,	63·7	45·6	54·6	18·1	62·0	48 0	55·0	14·0
October,	58·2	41·7	50·0	16·5	55·7	44·6	50·1	11·1
November,	44·4	33·1	38·8	11·3	48·1	37·5	42·8	10·6
December,	44·8	37·3	41·0	7·5	47·0	37·5	42·2	9·5
	1828.				1829.			
January,	43·2	35·7	39·4	7·5	36·3	27·9	32·1	8·4
February,	45·2	35·0	40·1	10·2	43·6	34·0	38·8	9·6
March,	49·3	36·4	42·8	12·9	46·2	33·1	39·6	13·1
April,	52·6	37·8	45·2	14·8	48·9	34·9	41·9	14·0
May,	59·3	43·1	51·2	16·2	61·1	42·2	51·6	18·9
June,	64·9	48·9	56·9	16·0	63·8	48·8	56·3	15·0
July,	64·9	50.3	57 6	14·6	63·3	49·7	56·5	13·6
August,	65·3	48·7	57·0	16·6	61·0	47·1	54·0	13·9
September,	61·9	47·2	54·6	14·7	57·9	42·7	50·3	15·2
October,	55·7	41·2	48·4	14·5	52·8	39·1	46·0	13·7
November,	49·3	40·4	44·8	8·9	44·4	34·7	39·6	9·7
December,	47·3	39·4	43·4	7·9	40·2	31·8	36·0	8·4
	1830.				1831.			
January,	38·3	30·3	34·3	8·0	38·3	31·1	34·7	7·2
February,	41·6	30·5	36·0	11·1	43·7	33·6	38·6	10·1
March,	51·3	37·2	44·2	14·1	48·3	36·6	42·3	11·7
April,	54·7	38·6	46·6	16·1	51·1	38·9	45·0	11·2
May,	55·9	43·6	49·8	12·3	57·5	40·1	48·8	17·4
June,	60·2	43·8	52·0	16·4	65·8	50·3	58·0	15·5
July,	64·9	50·5	57·7	14·4	66·2	52·6	59·4	13·6
August,	59·8	45·5	52·6	14·3	66·2	54·0	60·1	12·2
September,	58·8	45·5	52·2	13·3	61·2	49·4	55·3	11·8
October,	55·4	41·6	48·5	13·8	57·7	47·7	52·7	10·0
November,	47·4	37·7	42·6	9·7	45·2	35·5	40·2	9·7
December,	39·6	31·3	35·4	8·3	46·0	37·7	41·8	8·3

Table XXII.

Showing the Extreme Temperature from 1824 *to* 1831, *with the Monthly Range of Temperature.*

Year.	January.					February.					March.				
	Max.	Date.	Min.	Date.	Range.	Max.	Date.	Min.	Date.	Range.	Max.	Date.	Min.	Date.	Range.
	°		°		°	°		°		°	°		°		°
1824, .	54	9	23	16	31	51	7	26	15	25	58	19	23	4	35
1825, .	53	27	23	5	30	52	12	22	5	30	60	27	25	4	35
1826, .	48	21	10	16	38	54	6	29	18	25	72	10	25	17	47
1827, .	52	6	14	3	38	52	26	20	16, 18	32	58	31	18	5	40
1828, .	53	21	15	11	38	57	26	22	16	35	58	17	26	6	32
1829, .	44	1	15	22, 25	29	52	7	23	23	29	57	19	21	15	36
1830, .	47	4	20	19	27	57	25	17	22	40	64	28	29	5	35
1831, .	47	3	20	25, 26	27	55	10	19	4	36	55	27	28	24	27
	April.					May.					June.				
1824, .	70	21	24	1, 15	46	73	27, 28	29	21	44	82	7	37	19	45
1825, .	67	7	25	19	42	72	18	32	28	40	81	12, 16	37	6	44
1826, .	66	21	27	30	39	72	21	28	1	44	87	24, 26	39	5	48
1827, .	64	29	27	25	37	69	21	31	11, 12	38	74	12	39	7	35
1828, .	65	29	26	7	39	66	15	36	8	30	75	27	42	6	33
1829, .	56	17, 18	27	1	29	70	23, 29	36	2	34	72	20	36	6	36
1830, .	73	30	17	2	56	64	15	33	14	31	70	30	35	23	35
1831, .	64	15	27	4	37	70	31	28	8	42	72	2	45	6, 7	27
	July.					August.					September.				
1824, .	82	14	37	31	45	71	1	37	22	44	85	2	28	29	57
1825, .	83	30, 31	41	8	42	76	20	44	12	32	72	1, 16	34	23	38
1826, .	80	...	39	24	41	81	19	44	28	37	71	26	35	15	36
1827, .	77	16	41	12	36	73	23	38	6	35	73	16	38	20	35
1828, .	72	3, 11	42	29	30	76	27	38	16, 17	38	71	8, 25	33	14	38
1829, .	75	13	42	10	33	71	8	36	16	35	65	1	35	16, 25	30
1830, .	81	28	40	11	41	68	3	36	27	32	64	27	36	30	28
1831, .	76	31	44	22	32	74	22	48	18	26	71	4	42	28	29
	October.					November.					December.				
1824, .	62	1	20	15	42	56	2	26	6	30	52	18	16	5	36
1825, .	70	3	28	22	42	62	21	19	10	33	53	15	23	31	30
1826, .	70	20, 23	29	30, 31	41	59	1	21	27	38	54	7	24	6	30
1827, .	67	24	31	29	36	56	13	23	24	33	54	5	24	29	30
1828, .	66	12	29	18	37	57	21	26	11	31	57	12	32	26, 29	25
1829, .	61	11	29	15	32	55	3	23	18	32	54	6	23	27	31
1830, .	61	20	30	17	31	58	2, 3	28	24	30	48	16	15	25, 26	33
1831, .	67	7	38	28	29	58	1	22	20	36	54	11	29	28	25

TABLE XXIII.

Highest Night Minimum and Lowest Day Maximum.

	1824.				1825.			
	Highest Minimum.	Date.	Lowest Maximum.	Date.	Highest Minimum.	Date.	Lowest Maximum.	Date.
	°		°		°		°	
January,	49	10	36	16	46	30	31	5
February,	44	8	39	15, 27	44	12	31	4
March,	45	20	34	3	47	10	41	16
April,	49	20	39	1	50	15	46	12
May,	49	24	51	20	48	9	47	24
June,	53	28	59	20	58	12	55	19
July,	58	23	57	30	61	14	57	10
August,	54	8	57	21	60	20	63	29
September,	60	3	43	29	56	25	57	14
October,	52	8	43	13, 29	50	2	42	20
November,	48	17	37	30	41	1	35	10, 28
December,	49	13, 19	29	5	43	18	31	31
	1826.				1827.			
January,	40	21	26	12, 16	45	29	24	3
February,	45	3	42	18, 19	37	5, 6	30	18, 19
March,	42	1	42	26	47	23, 24	33	4
April,	47	8	42	27	48	30	36	24
May,	50	15	51	3	51	22, 29	46	10
June,	66	28	61	5	55	17	56	5
July,	60	5, 29	55	21	57	25	62	1
August,	61	18	64	12	54	3, 10	50	16
September,	55	17	54	18	57	17	51	20
October,	52	16	47	28	52	26	46	28, 29
November,	44	11	34	27	50	13, 14	35	22, 24
December,	47	11, 29	32	5	51	26	35	29
	1828.				1829.			
January,	48	21	31	11, 16	38	10	30	22
February,	49	5, 27	32	14	45	15	32	18
March,	48	13	37	6	45	20	36	13
April,	50	29	42	27	42	15	39	10
May,	49	31	50	7, 21	49	29	51	6
June,	57	27	53	15	57	3	55	28
July,	57	18	58	14	59	14	54	5
August,	54	20	58	14	59	8	49	13
September,	58	24	55	13, 14	55	1	51	18
October,	54	13	43	29	48	19	45	31
November,	52	21	38	11	46	12	37	18
December,	47	13	41	28	50	6	30	27

TABLE XXIII.—*continued.*

	1830.				1831.			
	Highest Minimum.	Date.	Lowest Maximum.	Date.	Highest Minimum.	Date.	Lowest Maximum.	Date.
	°		°		°		°	
January,	37	26	32	19	41	3	32	16, 31
February,	48	25	26	6	48	10	30	1
March,	53	29	39	16	44	5, 20	40	24, 25
April,	48	28	36	2, 3	46	27	40	1
May,	52	17	43	9	48	20	43	6
June,	51	29	54	15	55	13	58	6
July,	61	25, 26	54	12	59	29	59	14, 15
August,	54	3, 4	49	28	58	4, 6	60	28
September,	56	2	52	21	59	4	57	8, 9
October,	49	20	47	26	57	7, 19	51	29
November,	51	1	42	30	48	1	33	19, 20
December,	43	16	27	24	43	13, 25	35	28

Table XXIV.

Mean Daily Temperature.

The mean temperature is the average of the Minimum and Maximum.

	1824.					1825.				
	Highest.	Date.	Lowest.	Date.	Range.	Highest.	Date.	Lowest.	Date.	Range.
	°		°		°	°		°		°
January,	49·0	26	30·5	16	18·5	48·5	30	27·0	5	21·5
February,	47·0	8	32·5	15	14·5	48·0	12	27·0	4	21·0
March,	51·0	18	30·5	4	20·5	50·0	9, 27	33·5	4	16·5
April,	58·0	29	31·5	1	26·5	54·0	21	38·0	18	16·0
May,	59·0	30	40·5	20	18·5	58·5	6	43·0	28	15·5
June,	64·0	7	50·0	19	14·0	69·5	12	49·0	19	20·5
July,	67·0	14	50·0	31	17·0	70·5	14, 31	54·0	9, 10	16·5
August,	61·0	26	50.0	22	11·0	68·0	20	54·5	12	13·5
September,	72·0	2	35·5	29	36·5	62·5	18	49·5	28	13·0
October,	56·0	1	33·5	15	22·5	60·0	3	37·5	25	22·5
November,	51·5	17	32·0	30	19·5	49·0	21	27·0	10	22·0
December,	50·5	13	22·5	5	28·0	46·5	18	27·0	31	19·5
	1826.					1827.				
January,	44·0	21	18·0	16	26·0	48·0	29	19·0	3	29·0
February,	47·5	3	35·5	18	12·0	42·5	26	25·0	18	17·5
March,	62·0	10	35·5	17	26·5	51·0	23	27·5	5	23·5
April,	53·5	8	37·0	27	16·5	55·5	30	33·0	24	22·5
May,	58·0	15	42·5	10	15·5	59·0	21	40·5	10, 11	18·5
June,	74·0	28	50·0	5	24·0	63·0	10	50·0	3	13·0
July,	69·5	5	53·0	20	16·5	63·5	16	53·0	12	10·5
August,	70·5	18, 19	56·0	12	14·5	61·0	3	49·0	16	12·0
September,	62·5	17	48·0	15	14·5	63·5	16	44·5	20	19·0
October,	61·0	23	41·0	28	20·0	57·5	16	38·5	29	19·0
November,	48·5	1	27·5	27	21·0	53·0	13	29·0	24	24·0
December,	50·0	11	28·5	5	21·5	52·5	26	29·5	29	23·0
	1828.					1829.				
January,	50·5	21	23·0	11	27·5	40·5	1	22·5	22	18·0
February,	52·5	27	30·0	14	22·5	47·5	12	28·0	18	19·5
March,	53·0	13	31·5	6	21·5	51·0	20	30·0	15	21·0
April,	57·5	29	38·0	8	19·5	47·0	15	33·5	1	13·5
May,	57·0	26, 31	45·5	20	11·5	59·5	29	45·0	2	14·5
June,	66·0	27	48·0	6	18·0	62·5	3	46·0	6	16·5
July,	64·0	3	50·5	29	13·5	63·5	14	50·5	5	13·0
August,	64·5	27	51·5	16	13·0	65·0	8	48·5	13	18·5
September,	64·5	25	44·0	14	20·5	60·0	1	45·0	16	15·0
October,	58·5	12	36·0	29	22·5	54·0	19	38·0	23	16·0
November,	54·5	21	32·0	11	22·5	48·5	3	30·0	18	18·5
December,	48·5	30	36·5	9	12·0	52·0	6	26·5	27	25·5
	1830.					1831.				
January,	41·0	4	26·0	19	15·0	44·0	3	26·5	25	17·5
February,	52·5	25	23·0	6	29·5	51·5	10	26·0	4	25·5
March,	54·0	26	34·5	16	19·5	48·5	20	34·0	24	14·5
April,	58·0	28	26·5	2	31·5	52·0	16	39·5	1	12·5
May,	57·5	17	42·0	9	15·5	58·5	31	36·0	6	22·5
June,	59·0	28	46·0	19	13·0	62·5	13	55·0	3	7·5
July,	68·0	26	50·0	11	18·0	67·0	31	53·5	22	13.5
August,	61·0	3	46·0	28	15·0	64·0	22	55·0	28	9·0
September,	59·0	2	46·0	21	13·0	65·0	4	50·5	28	14·5
October,	55·0	20	39·5	30	15·5	60·0	19	45·0	28	15·0
November,	53·5	1	35·5	24	18·0	53·0	23	27·5	20	25·5
December,	45·5	16	22·0	24	23·5	46·5	25	32·0	28	14·5

Table XXV.

Showing the Mean Daily Variability of Temperature in Edinburgh from 1840–1896.

	Jan.	Feb.	Mar.	Apr.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.	Year.
	°	°	°	°	°	°	°	°	°	°	°	°	°
1840, . .	2·7	2·6	2·5	2·8	2·7	3·5	2·1	3·4	2·4	2·5	2·5	3·1	2·73
1841, . .	4·2	2·1	2·4	2·4	3·7	2·5	2·3	2·7	2·9	3·4	3·0	3·5	2·93
1842, . .	2·5	2·7	3·1	2·3	2·5	2·9	2·6	3·6	3·1	3·8	3·0	3·5	2·97
1843, . .	4·0	3·0	4·5	3·0	2·7	2·3	3·6	3·0	4·2	3·3	4·4	2·5	3·38
1844, . .	3·4	3·1	3·9	3·0	3·3	2·7	2·4	2·6	2·9	3·8	3·0	2·1	3·02
1845, . .	4·1	4·3	4·2	3·1	2·3	2·4	2·7	2·5	3·4	3·1	3·6	3·2	3·24
1846, . .	3·6	2·9	2·9	3·9	3·0	3·1	3·0	2·3	3·1	3·0	3·3	3·7	3·15
1847, . .	3·5	3·4	2·4	2·8	3·2	2·8	2·5	3·6	3·5	3·5	4·9	3·3	3·28
1848, . .	4·4	4·7	2·9	2·7	3·2	2·8	3·0	2·1	3·5	2·6	3·4	3·8	3·26
1849, . .	3·6	3·0	3·8	2·3	2·8	2·4	2·3	3·5	2·9	3·3	3·6	3·0	3·02
1850, . .	4·4	3·3	3·0	2·8	3·1	3·7	2·7	3·2	2·7	3·2	3·5	2·8	3·20
1851, . .	4·4	3·5	2·3	2·4	2·6	2·7	2·2	2·8	2·3	2·9	2·6	3·7	2·87
1852, . .	3·5	3·5	2·7	2·9	2·5	2·6	2·8	2·0	2·3	2·0	2·9	3·5	2·77
1853, . .	2·6	2·4	2·0	2·5	3·1	3·8	1·6	2·1	2·4	2·9	3·9	3·9	2·77
1854, . .	3·1	2·9	2·7	2·4	2·1	2·7	1·6	2·3	2·9	3·5	3·9	3·7	2·82
1855, . .	3·3	1·7	1·7	3·1	3·2	3·4	2·9	2·8	3·7	4·4	2·5	3·7	3·03
1856, . .	2·5	2·7	2·3	2·1	2·1	3·0	2·8	3·1	2·5	2·6	4·0	3·9	2·80
1857, . .	3·2	2·9	2·1	2·5	2·9	3·9	2·5	2·2	2·8	2·0	2·4	2·6	2·67
1858, . .	3·8	2·8	3·3	3·3	2·9	2·6	2·1	1·6	2·7	2·5	2·1	2·2	2·66
1859, . .	2·2	2·6	3·0	2·3	2·5	2·9	3·4	2·2	2·0	2·4	2·5	2·7	2·56
1860, . .	2·9	3·2	2·0	2·0	3·7	2·6	2·6	1·6	1·8	2·5	2·2	2·9	2·50
1861, . .	2·7	2·3	2·3	2·4	2·9	2·6	1·8	2·3	1·6	2·7	3·9	3·1	2·55
1862, . .	2·0	3·0	2·2	4·0	2·4	2·1	3·0	2·3	2·5	3·1	3·1	2·7	2·70
1863, . .	2·8	3·3	3·4	2·8	2·8	1·9	2·7	2·4	2·2	2·1	3·2	3·6	2·77
1864, . .	2·4	2·8	2·2	3·3	2·9	2·2	2·3	2·5	1·9	3·0	2·3	2·8	2·55
1865, . .	2·0	3·1	1·7	3·2	3·3	3·7	2·4	2·3	2·6	2·5	3·0	3·0	2·72
1866, . .	3·9	2·1	3·2	2·5	3·0	3·2	2·6	2·5	2·6	2·1	3·2	4·3	2·93
1867, . .	3·6	2·7	1·9	2·4	2·8	2·4	2·4	2·5	2·0	3·2	3·5	3·8	2·77
1868, . .	3·6	3·3	3·3	2·6	3·3	2·5	3·4	2·8	2·9	2·8	2·9	3·6	3·08
1869, . .	3·5	3·2	2·0	3·6	2·4	2·9	3·5	3·1	2·7	3·4	4·0	3·9	3·18
1870, . .	2·1	2·7	2·6	3·2	2·7	3·1	2·4	2·2	2·6	2·5	2·6	3·5	2·68
1871, . .	2·5	2·8	4·2	3·1	3·3	3·6	2·0	3·0	2·2	3·7	2·6	2·1	2·92
1872, . .	3·4	2·2	3·0	2·4	2·8	2·6	2·6	2·5	2·2	2·6	2·7	3·4	2·70
1873, . .	3·3	2·3	2·7	1·9	2·5	2·6	2·8	2·5	3·5	2·7	2·9	4·0	2·81
1874, . .	3·5	2·6	2·6	3·8	2·3	2·7	2·7	2·4	3·3	3·5	3·9	3·3	3·05
1875, . .	4·3	2·8	2·4	3·0	2·5	2·3	2·4	2·4	2·4	2·2	2·7	2·7	2·68
1876, . .	4·0	3·3	3·1	4·1	2·5	2·8	2·7	2·7	2·4	2·1	3·2	2·8	2·98
1877, . .	3·0	2·9	2·4	3·0	2·2	2·3	2·4	2·0	2·7	4·1	2·9	3·5	2·78
1878, . .	3·2	2·8	2·6	2·3	2·6	2·4	1·9	3·0	2·5	2·5	2·5	3·0	2·61
1879, . .	3·2	2·7	3·4	2·3	3·4	2·5	2·2	2·2	2·3	3·4	3·3	3·8	2·90
1880, . .	2·9	2·2	2·5	2·2	2·5	2·0	2·9	2·6	2·6	3·0	4·3	2·7	2·70
1881, . .	3·3	2·7	3·2	2·3	2·9	2·5	2·7	2·2	1·8	2·3	3·9	3·6	2·79
1882, . .	2·9	3·2	3·6	2·6	2·1	2·0	1·8	2·5	2·2	1·8	2·2	3·9	2·57
1883, . .	3·1	2·4	2·7	2·9	2·5	2·3	2·2	2·6	2·2	2·9	2·3	3·1	2·60
1884, . .	3·2	3·2	3·2	2·2	3·1	3·6	2·3	3·4	2·5	3·0	2·9	3·0	2·97
1885, . .	2·8	2·7	3·5	2·6	2·0	2·5	3·1	3·2	2·7	2·2	3·4	2·7	2·87
1886, . .	3·4	2·4	2·2	2·6	3·8	3·2	2·6	2·5	3·1	2·3	3·7	2·8	2·88
1887, . .	2·7	3·1	2·7	2·7	3·4	3·4	3·1	2·3	2·3	3·6	2·6	2·9	2·90
1888, . .	3·2	2·9	2·6	2·3	3·9	2·7	2·8	2·4	2·3	3·3	2·9	3·6	2·91
1889, . .	3·6	3·3	3·7	2·0	3·4	2·2	2·3	1·9	2·2	2·1	3·6	3·6	2·83
1890, . .	3·3	2·7	3·8	2·4	3·3	2·4	1·9	2·2	2·7	3·9	2·9	2·8	2·86

TABLE XXV.—*continued.*

	Jan.	Feb.	Mar.	Apr.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.	Year.
	°	°	°	°	°	°	°	°	°	°	°	°	°
1891, . .	3·1	3·6	3·0	2·1	2·7	2·6	2·4	2·0	2·5	2·4	2·3	3·2	2·76
1892, . .	2·8	2·9	3·7	2·5	2·7	3·4	2·7	2·8	2·4	2·5	3·7	3·4	2·97
1893, . .	2·9	2·9	2·4	3·1	2·3	3·2	2·2	2·1	3·1	3·3	3·4	3·5	2·87
1894, . .	4·4	3·6	2·4	2·7	2·4	2·2	2·1	2·0	1·9	3·1	3·3	3·2	2·78
1895, . .	2·0	2·9	2·6	2·7	2·7	2·7	2·1	2·4	2·4	3·0	2·7	2·9	2·59
1896, . .	4·0	3·2	3·0	2·2	2·8	2·6	2·3	2·8	2·7	3·0	3·1	2 8	2·88
Max., . .	4·4	4·7	4·5	4·1	3·9	3.9	3·6	3·6	4·2	4·4	4·9	4·3	3·38
Min., . .	2·0	1·7	1·7	1·9	2·0	1·9	1·6	1·6	1·6	1·8	2·1	2·1	2·50
Range, . .	2·4	3·0	2·8	2·2	1·9	2·0	2·0	2·0	2·6	2·6	2·8	2·2	0·88
Decennial Means.													
1841–50, .	**3·77**	3·25	3·31	2·83	2·98	2·76	*2·71*	2·91	3·22	3·30	3·57	3·14	3·15
1851–60, .	3·15	2·82	2·41	2·55	2·76	3·02	2·47	*2·27*	2·54	2·77	2·90	**3·28**	2·75
1861–70, .	2·86	2·85	2·48	3·00	2·85	2·66	2·65	2·49	*2·36*	2·74	3·17	**3·43**	2·79
1871–80, .	**3·33**	2·66	2·89	2·81	2·66	2·58	*2·46*	2·53	2·61	2·98	3·10	3·13	2·81
1881–90, .	3·15	2·86	3·12	2·46	3·04	2·68	2·48	2·52	*2·40*	2·74	3·04	**3·20**	2·81
Means, 1840–1896, .	**3·24**	2·91	2·84	2·71	2·83	2·76	*2·52*	2·54	2·61	2·90	3·14	3·22	2·85

TABLE XXVI.

Variability of Temperature.

	Years 1840–96. Highest.	Year.	Lowest.	Year.	Range.	Greatest Rise.	Date.	Greatest Fall.	Date.	No. of Changes of 10° or more. Rises.	Falls.	Total.
	°		°			°		°				
Jan., .	4·4	{1848, '50 / 1851, '94}	2·0	{1862, '65 / 1895}	2·4	14·0	{26. 1841 / 30. 1848}	12·0	29. 1848	23	8	31
Feb., .	4·7	1848	1·7	1855	3·0	14·0	3. 1845	12·5	2. 1843	9	7	16
Mar., .	4·5	1843	1·7	1855, '65	2·8	15·1	17. 1892	12·8	1. 1879	11	10	21
Apr., .	4·1	1876	1·9	1873	2·2	14·0	26. 1845	13·3	29. 1886	11	11	22
May, .	3·9	1888	2·0	1885	1·9	13·0	5. 1841	15·1	5. 1860	15	5	20
June, .	3·9	1857	1·9	1863	2·0	10·6	26. 1894	14·2	14. 1884	2	11	13
July, .	3·6	1843	1·6	1853, '54	2·0	11·5	21. 1843	13·5	27. 1885	8	5	13
Aug., .	3·6	1842	1·6	1858, '60	2·0	12·5	25. 1847	15·5	29. 1869	4	5	9
Sept., .	4·2	1843	1·6	1861	2·6	13·5	30. 1843	11·2	28. 1873	7	2	9
Oct., .	4·4	1855	1·8	1882	2·6	14·5	18. 1849	12·5	28. 1871	9	10	19
Nov., .	4·9	1847	2·1	1858	2·8	13·0	{21. 1866 / 15. 1871}	13·1	10. 1874	14	17	31
Dec., .	4·3	1866	2·1	1871	2·2	14·2	17. 1882	13·4	24. 1860	16	10	26
	3·38	1843	2·50	1860	0·88	15·1	Mar. 17, 1892	15·5	Aug. 19, 1869	129	101	230

Table XXVII.

Mean Daily Variability of Temperature in Edinburgh from 1865–1869, *from Daily Observations made at* 9 *a.m. and* 9 *p.m. and Compared with Means Deduced from the Average of the Max. and Min.*

	Jan.	Feb.	Mar.	Apr.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.	Year.
	°	°	°	°	°	°	°	°	°	°	°	°	°
Mean 9 a.m., . . .	4·5	4·3	3·4	3·4	3·1	3·7	3·3	3·3	2·8	4·4	4·8	4·8	3·83
,, 9 p.m., . . .	4·2	4·0	3·3	3·4	3·1	3·6	3·3	2·6	3·1	3·9	4·4	4·5	3·62
,, 9 a.m. and 9 p.m., .	4·4	4·2	3·4	3·4	3·1	3·6	3·3	3·0	3·0	4·1	4·6	4·6	3·7
Mean variability from Max. and Min.	3·3	2·9	2·4	2·9	3·0	2·9	2·9	2·6	2·6	2·8	3·3	3·7	2·9
Difference, ...	−1·1	−1·3	−1·0	−0·5	−0·1	−0·7	−0·4	−0·4	−0·4	−1·3	−1·3	−0·8	−0·8
Smoothed Difference, . .	−1·1	−1·1	−0·9	−0·5	−0·4	−0·4	−0·5	−0·4	−0·7	−1·0	−1·2	−1·1	−0·8

Table XXVIII.

Comparison of Mean Variability of Temperature at Hawkhill (Edinburgh) and Kirkcaldy for the Years 1776–1777. *Hour of Observation,* 8 *a.m.*

	Jan.	Feb.	Mar.	Apr.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.	Year.
	°	°	°	°	°	°	°	°	°	°	°	°	°
Hawkhill, ...	3·4	3·4	3·8	3·4	2·2	3·4	2·4	2·4	3·3	2·8	4·6	3·7	3·2
Kirkcaldy, ...	3·5	3·4	3·8	3·7	2·4	2·9	2·4	2·4	2·5	3·4	4·7	3·9	3·2
Difference, ...	+0·1	0·0	0·0	+0·3	+0·2	−0·5	0·0	0·0	−0·8	+0·6	+0·1	+0·2	0·0

TABLE XXIX.

Showing the Monthly and Annual Rainfall in Edinburgh for 120 *Years and* 6 *Months.*

Year.	Jan.	Feb.	Mar.	April.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.	Year.
	ins.	ins.	ins.	ins.	ins.	ins.	ins.	ins.	ins.	ins.	ins.	ins.	ins.
1770	·73	1·19	·84	1·99	2·44	2·68	1·74	1·34	3·36	1·20	6·78	3·59	27·88
1771	1·04	1·17	·54	·44	1·38	·48	1·85	3·23	1·74	5 59	3·76	·97	22·19
1772	2·68	1·39	1·68	1·30	2 02	3·00	3·69	2·71	3·26	3·51	5·66	1·28	32·18
1773	3·53	1 15	1·23	3·63	1·83	·87	1·41	1·28	3·68	2 95	3·37	3·91	28·84
1774	2 78	2·02	·86	1·74	3·49	3·87	1·51	4·82	2·93	1·30	2·18	2·69	30 19
1775	4·59	3·01	1·59	·58	1·42	1·21	5·81	2·36	3·82	5·31	3·62	·76	34·07
1776	3·26	2·36	1·46	1·21	·63	2·37	3·08	2·41	2 75	1·73	2·75	2·08	26·09
1777	...	...	...	...	...	...	...	...	...	...	...	...	...
1778	...	...	...	...	...	...	...	...	...	...	...	...	...
1779	...	...	...	...	...	...	...	...	...	...	...	...	...
1780	·72	·88	1 30	3·38	3·15	1·95	1·29	·62	4·65	2·78	1·29	·67	22·68
1781	1·78	2·72	·03	1·43	2·09	2·17	...	...	...	...	...	...	...
1782	...	...	...	...	...	...	...	...	...	...	...	...	...
1783	...	...	...	...	...	...	...	...	...	...	...	...	...
1784	...	...	...	...	...	...	...	...	...	...	...	...	...
1785	1·50	2·31	·52	·35	·94	1·11	2·72	2·00	10·69	2·82	4·42	1·27	30·65
1786	2·54	1·19	·63	·30	2·40	·22	5·50	1·71	1·24	3·85	2·30	1·53	23·41
1787	·14	1·25	2 12	1·36	4·73	3·22	5·00	1·82	·60	2·46	·83	8·42	31·95
1788	1·03	2·20	1·73	1·78	·86	2·13	2·63	1·17	3·28	·40	1·07	1·15	19·43
1789	4·77	1·02	1·24	1·02	1·15	1·14	2·69	1·53	2·04	3·46	5·21	3·93	29·20
1790	1·96	1·75	·85	2·60	2·42	2·90	2·02	3·13	2·65	2·18	2·49	2·57	27·52
1791	2·36	2·15	·69	2·97	1·82	2·53	1·38	3·40	1·28	3·96	3·49	1·39	27·42
1792	1·40	1·67	2·88	1·37	3·21	5·13	4·09	3·40	3·00	4·30	2·50	4·05	37·00
1793	1·53	2·25	3·14	1·13	1·06	1·48	1·14	2·50	·51	1·52	2·14	2·30	20·70
1794	1·40	2·19	1·00	2·12	1·88	1·07	2·16	1·80	3·14	3·58	4·46	3·92	28·72
1795	2·81	3·87	1·37	3·01	1·20	3·92	2·42	3·62	1·12	4·87	4·58	3·81	36·60
1796	3·28	1·40	·43	1·09	1·43	1·03	2·77	·45	2·21	1·19	1·31	1·06	17·65
1797	1·32	·67	1·20	1·47	1·96	2·18	5·19	4·50	2·99	3·24	1·20	1·26	27·18
1798	1·80	·55	1·52	1·56	1·62	2·53	2·10	2·99	2·28	2·15	2·07	1·41	22·58
1799	·89	1·57	·47	2 15	3·27	·87	2·60	5·66	4·02	1·99	1·79	1·23	26·51
1800	3·26	·49	1·34	2 05	2·50	·53	·40	1·26	2·53	3·33	·98	2·91	21·58
1801	1·75	1·44	·82	·60	1·99	·20	5·25	·88	2·66	1·59	1·06	2·17	20·41
1802	·71	1·87	·69	·73	·86	2·21	4·19	2·13	2·37	2·43	2·09	1·02	21·30
1803	·80	1·56	·74	1·16	1·13	1·35	·86	2·00	1·82	1·00	2·26	1·13	15·81
1804	3·72	·57	2·58	2·04	1·58	1·32	1·86	3·91	·74	2·37	1·92	1·96	24·57
1805	·65	1·58	·67	·64	1·01	1·38	1·48	2·83	2·66	1·33	·38	1·57	16·18
1806	2·66	1·18	·48	·74	2·23	·20	2·74	2·65	·98	1·92	4·47	1·71	21·96
1807	·69	·51	1·26	2·06	1·71	·60	1·29	2·59	4·39	3·68	2·21	1·31	22·30
1808	·72	2·16	·72	2·93	1·92	2·61	5·17	4·83	2·46	2·03	·72	2·80	29·07
1809	2·76	3·16	·21	2·01	2·14	2·98	2·39	5·56	2·94	1·19	1·32	3·24	29·90
1810	1·47	1·34	3·16	1·46	1·84	1·92	3·82	3·14	·22	1·22	4·50	2·82	26 91
1811	1·61	3·30	1·37	1·72	3·35	3·68	2·77	2·12	1·70	3·43	3·90	3·69	32·64
1812	1·47	3·59	3·10	1·10	2·10	2·24	1·34	3·40	1·08	2·82	3·97	·89	27·10
1813	·83	2·26	·25	2·03	3·21	1·44	2·58	·86	1·23	2·94	1·45	1·07	20·15
1814	·86	·63	1·65	2·90	·49	1·41	2·59	2 23	1·30	1·43	3·70	3·10	22·29
1815	1·50	1·46	2·22	·89	3·01	2·29	2·18	1·37	1·90	2·84	·56	1·61	21·83
1816	2·04	1·01	1·07	1·27	2·18	1·91	5·22	2·26	2·96	1·94	·95	2·43	25·24
1817	1·79	1·53	·87	·19	2·44	4·80	3·85	5·25	·85	1·55	2·70	3·66	29·48
1818	2·49	·81	1·76	·60	1·80	2·00	3·40	·70	1·80	1·10	2·60	2·52	21·58
1819	3·50	1·79	·84	3·10	2·32	1·64	1·48	1·93	1·43	3·75	2·35	2·93	27·06
1820	·51	1·22	1·10	·52	4·20	3·40	1·30	2·70	1·21	2·66	1·44	2·41	22·67

TABLE XXIX.—*continued.*

Year.	Jan.	Feb.	Mar.	April.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.	Year.
	ins.	ins.	ins.	ins.	ins.	ins.	ins.	ins.	ins.	ins.	ins.	ins.	ins.
1821	2·50	·54	2·46	2·60	1·85	·61	1·51	1·47	1·58	1·43	4·22	2·94	23·71
1822	1·23	2·50	3·57	1·41	1·80	1·36	4·53	2·36	1·27	2·39	2·12	1·60	26·14
1823	2·23	3·85	·66	1·68	2·35	1·00	4·25	3·87	1·82	3·10	1·07	4·38	30·26
1824	·87	1·70	1·34	·57	·63	2·01	1·58	1·50	1·62	4·73	4·38	3·88	24·81
1825	1·31	·69	·43	1·41	3·25	2·05	·15	1·89	2·85	2·19	3·91	1·99	22·12
1826	·55	1·77	1·33	1·52	1·25	·30	2·31	1·83	1·01	1·38	·76	1·26	15·27
1827	3·33	1·58	4·84	2·74	1·28	1·62	2·27	4·89	1·15	4·97	1·02	2·90	32·59
1828	1·70	·98	1·18	1·42	1·85	·81	4·57	3·43	2·31	·86	3·94	2·18	25·23
1829	2·49	1·61	·32	3·35	·77	2·03	4·48	6·80	1·77	2·53	2·48	1·33	29·96
1830	·95	1·21	1·78	2·28	1·96	2·54	6·57	6·69	3·63	·16	3·13	2·35	33·25
1831	·66	3·88	1·97	1·54	·69	1·41	2·44	4·03	1·55	2·15	2·95	1·26	24·53
1832	·61	1·42	1·29	1·21	1·35	2·89	1·14	3·64	·92	5·53	·95	2·28	23·23
1833	·57	2·53	1·43	1·34	·79	3·48	1·53	1·16	2·37	1·13	·71	3·84	20·88
1834	3·28	·86	1·65	·44	·51	1·45	3·20	1·18	4·50	1·23	1·22	1·52	21·04
1835	1·08	2·48	2·28	·79	2·04	1·02	1·37	1·99	5·43	2·09	2·76	1·89	25·22
1836	4·06	1·62	3·79	1·54	·56	2·50	6·53	2·45	2·81	1·66	3·05	2·46	33·03
1837	1·23	2·14	1·28	1·61	1·53	2·86	4·54	4·13	1·73	2·02	2·03	1·67	26·77
1838	2·47	1·21	2·76	1·78	2·90	5·16	2·45	2·97	4·00	1·55	3·06	·73	31·04
1839	1·76	1·45	1·47	·33	·47	3·91	3·51	1·77	3·09	2·38	1·65	1·66	23·45
1840	3·72	1·58	·43	·19	3·97	2·75	3·46	1·99	2·39	2·01	2·33	·68	25·50
1841	1·23	1·64	·60	1·14	1·14	1·56	3·87	3·64	2·63	4·53	2·28	1·96	26·22
1842	1·01	1·11	3·44	·15	1·45	·97	1·53	1·36	1·45	·98	1·63	1·79	16·87
1843	1·69	1·38	·99	1·87	2·99	2·26	3·59	1·40	·89	4·20	2·20	·34	23·80
1844	1·23	1·72	2·42	·40	·15	2·71	2·39	2·11	2·70	·82	3·92	·37	20·94
1845	1·77	·61	1·67	·40	2·24	3·08	1·72	3·48	1·77	6·14	1·70	2·04	26·62
1846	2·64	1·60	·97	2·88	1·27	3·59	4·17	5·01	3·35	3·60	1·74	·72	31·54
1847	·51	·79	·13	1·25	4·77	1·79	1·37	·91	1·25	3·48	1·64	4·88	22·77
1848	1·26	5·21	2·80	1·06	·60	6·04	1·36	2·00	1·45	4·56	2·42	1·84	30·60
1849	2·84	·97	1·05	1·64	1·66	2·45	2·58	2·31	2·02	1·74	1·50	1·45	22·21
1850	1·62	2·84	·14	·88	3·14	1·18	1·63	2·20	1·83	1·16	2·61	1·21	20·44
1851	2·89	·59	3·30	2·06	·53	2·17	3·00	4·25	1·40	1·02	·91	·66	22·78
1852	3·27	2·01	·63	·43	1·92	2·80	1·90	4·30	2·20	2·18	3·42	6·45	31·51
1853	1·78	1·58	·42	·57	1·10	6·90	2·50	3·32	1·82	3·26	·76	1·62	25·63
1854	3·02	·61	1·01	·34	2·45	3·15	1·85	1·34	·87	1·44	3·04	1·77	20·89
1855	·78	1·24	1·05	·55	1·89	2·48	3·89	2·84	·44	2·60	1·43	1·20	20·34
1856	2·45	2·27	·24	1·93	3·12	2·97	2·00	3·54	5·15	·71	1·42	2·68	28·48
1857	1·53	·45	2·04	1·85	1·69	3·92	1·34	2·26	4·65	1·20	2·35	1·64	24·92
1858	1·47	1·02	1·57	·70	1·63	2·69	3·94	2·20	2·00	4·07	1·60	1·46	24·35
1859	2·34	1·44	2·96	2·77	·21	2·06	3·21	·77	1·72	3·44	2·70	2·35	25·97
1860	3·97	1·60	1·74	·56	1·80	3·58	1·21	2·45	3·16	2·85	2·88	7·65	33·45
1861	·75	1·47	2·31	1·46	·73	2·70	3·47	3·65	4·75	2·31	4·00	1·02	28·62
1862	3·83	·90	4·64	1·32	3·71	2·80	2·70	3·70	2·10	3·42	2·00	2·80	33·92
1863	3·44	1·22	·74	2·03	1·61	3·50	·65	3·47	2·65	2·19	1·91	2·22	25·63
1864	1·25	2·14	3·10	1·16	2·13	1·20	2·15	·80	3·40	6·90	1·79	2·07	28·09
1865	2·29	1·70	·99	·30	3·65	·41	3·20	3·41	·55	3·96	1·60	1·59	23·65
1866	2·49	3·50	1·85	1·37	1·50	1·27	3·34	2·73	2·95	1·23	2·71	2·29	27·23
1867	5·62	1·68	1·17	2·71	3·71	2·80	5·68	2·64	1·53	1·50	·74	1·26	31·04
1868	3·61	2·08	1·95	3·28	1·81	·48	·34	4·30	3·27	2·13	1·45	3·87	28·57
1869	2·84	2·67	·79	1·01	2·64	1·74	·73	·76	4·33	1·48	1·42	1·82	22·23
1870	1·68	5·70	1·11	·43	1·31	2·25	1·65	1·29	1·84	1·76	·69	2·40	22·11
1871	1·25	2·41	1·07	4·55	·83	1·90	2·80	2·56	2·55	2·45	2·87	1·63	26·87
1872	3·63	2·02	3·30	1·70	3·46	3·13	3·58	3·28	5·80	3·38	3·60	2·08	38·96
1873	2·32	1·38	1·60	·21	2·70	1·21	2·80	4·53	4·46	3·07	2·47	1·44	28·19
1874	1·74	·70	1·73	·90	1·50	1·60	3·34	4·87	1·75	2·42	3·11	2·10	25·76
1875	2·74	1·17	·90	·67	·75	2·00	3·26	1·13	2·67	2·34	4·92	1·80	24·35
1876	·80	3·42	3·08	3·41	1·01	2·60	1·22	3·40	4·02	2·32	3·64	6·73	35·65
1877	5·17	1·85	1·67	2·98	2·21	1·89	4·57	8·33	1·25	2·50	2·01	1·37	35·80
1878	2·56	·50	·58	1·43	2·71	2·41	·76	4·02	2·80	1·79	2·92	2·22	24·90
1879	1·29	1·78	2·30	2·22	1·74	5·16	5·78	2·44	1·65	·92	1·85	1·39	28·52
1880	·47	1·50	1·54	3·17	·76	1·55	3·40	·40	2·77	3·20	3·35	2·75	24·86

TABLE XXIX.—*continued.*

Year.	Jan.	Feb.	Mar.	April.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.	Year.
	ins.	ins.	ins.	ins.	ins.	ins.	ins.	ins.	ins.	ins.	ins.	ins.	ins.
1881	1·00	2·80	1·50	1·07	1·65	1·75	3·20	5·65	3·45	1·95	2·40	1·80	28·22
1882	1·20	1·50	2 28	2·50	2·65	2·78	3·73	1·44	1·77	2·65	2·83	4·90	30·23
1883	2·24	1·02	1 07	1·42	·72	1·80	4·25	3·20	2·25	2·03	1·33	1·00	22·33
1884	3·78	1·07	1·60	·80	2·70	·57	4·41	2·30	2·23	1·10	1·30	2·80	24·66
1885	1·13	1·74	1·48	1·87	2·13	·40	·88	2·45	2·40	1·30	1·40	·40	17·58
1886	3·00	1·20	1·70	1·20	4·11	1·48	2·87	·74	2·42	3·58	1·55	2·20	26·05
1887	·60	·93	1·50	1·00	1·70	·29	2·10	1·69	4·21	1·33	3·35	1·10	19·80
1888	1·62	1·35	3·17	1·30	·78	2·53	5·65	1·80	·51	1·10	4·20	·85	24·86
1889	·60	·80	·60	2·50	1·80	1·25	3·66	5·05	·80	3·40	·65	1·20	22·31
1890	3·20	·90	1·28	·70	1·56	2·80	2·27	3·40	2·17	2·49	4·56	1·44	26·77
1891	·49	·13	2·70	·26	1·50	·48	2·67	4·40	4·01	1·73	1·37	4·49	24·23
1892	·90	1·83	1·05	·98	2·69	2·75	1·00	4·52	1·01	3·36	1·34	·98	22·41
1893	·54	2·31	·31	1·52	1·31	1·95	2·50	2·90	1·36	2·80	1·33	2·10	20·93
1894	2·02	6·38	1·23	1·55	2·80	2·65	2·08	3·50	·30	3·00	1·10	1·64	28·25
1895	1·60	·20	2·72	·97	1·00	2·67	4·58	4·37	·82	3·18	2·48	2·13	26·72
1896	·60	·81	1·34	·88	·44	3·24	4·18	1·76	3·75	3·50	·50	2·25	23·35
Max.	5·62	6·38	4·84	4·55	4·77	6·90	6·57	8·33	10·69	6·90	6·78	8·42	38·96
Year.	1867	1894	1827	1871	1847	1853	1830	1877	1785	1864	1770	1787	1872
Min.	·14	·13	·13	·15	·15	·20	·15	·40	·22	·16	·38	·34	15·27
Year.	1787	1891	1781	1842	1844	1801, 1806	1825	1880	1810	1830	1805	1843	1826

Decennial Means.

1770-76 1780 1785-90	2·234	1·636	*1·185*	1·549	2·062	1·939	2·924	2·152	**3·335**	2·824	3·266	2·487	27·593
1791-00	2·005	1·681	*1·404*	1·892	1·995	2·127	2·425	2·958	2·308	**3·013**	2·452	2·334	26·594
1801-10	1·593	1·537	*1·133*	1·437	1·641	1·477	2·905	**3·052**	2·124	1·876	2·093	1·973	22·841
1811-20	1·660	1·760	*1·423*	1·432	2·510	2·481	**2·671**	2·282	1·546	2·446	2·362	2·431	25·004
1821-30	1·716	1·643	1·791	1·898	1·699	*1·433*	3·222	**3·473**	1·901	2·364	2·703	2·481	26·324
1831-40	1·944	1·917	1·835	*1·077*	1·481	2·743	**3·017**	2·531	2·879	2·175	2·070	1·799	25·468
1841-50	1·580	1·787	1·421	*1·167*	1·941	2·563	2·421	2·442	1·934	**3·121**	2·164	1·660	24·201
1851-60	2·345	1·281	1·496	*1·176*	1·634	**3·272**	2·484	2·727	2·341	2·277	2·051	2·748	25·832
1861-70	**2·780**	2·306	1·865	*1·507*	2·280	1·915	2·391	2·675	2·737	2·688	1·831	2·134	27·109
1871-80	2·197	*1·673*	1·777	2·124	1·787	2·345	3·151	**3·496**	2·972	2·439	3·074	2·351	29·386
1881-90	1·837	*1·331*	1·618	1·436	1·980	1·565	**3·302**	2·772	2·221	2·093	2·357	1·769	24·281
1891-96	1·025	1·943	1·558	*1·027*	1·623	2·290	2·835	**3·575**	1·875	2·928	1·353	2·286	24·313
Means.	1·950	1·698	1·530	*1·494*	1·901	2·168	**2 815**	2·797	2·396	2·517	2·378	2·211	25·855
Mean Daily Fall.	·063	·061	·049	·050	·061	·072	·091	·090	·080	·081	·079	·071	·071

Table XXX.

Absolute Droughts of more than 14 *Days.*

Year.	Commenced.	Terminated.	Days.	Year.	Commenced.	Terminated.	Days.
1772, . .	June 20	July 5	15	1829, . .	February 27	March 17	19
1780, . .	July 28	August 20	24	,,	May 15	June 3	20
1786, . .	April 14	April 28	15	,,	December 30	January 13	15
,,	May 24	June 25	33	1830, . .	September 26	October 20	26
,,	October 13	October 29	17	1857, . .	June 13	June 28	16
1787, . .	January 5	January 28	24	,,	February 6	February 25	20
1795, . .	July 1	July 20	20	1858, . .	March 6	March 23	18
1796, . .	August 10	August 25	16	1861, . .	April 8	April 26	19
1797, . .	February 14	March 4	19	,,	December 17	January 2, 1362	17
1798, . .	December 28	January 12, 1799	16	1864, . .	July 5	July 21	17
1799, . .	June 6	June 22	17	,,	September 27	October 16	20
,,	December 6	January 1, 1800	27	1865, . .	February 9	February 23	15
1800, . .	March 13	March 30	18	,,	March 14	March 28	15
,,	July 8	August 1	25	,,	June 22	July 6	15
1801, . .	April 17	May 11	25	1867, . .	June 15	July 2	18
,,	June 8	June 25	18	,,	October 29	November 13	16
1803, . .	March 7	March 22	16	1869, . .	June 24	July 22	29
1804, . .	February 12	February 29	18	,,	August 16	September 4	20
1805, . .	May 21	June 4	15	1870, . .	July 21	August 4	15
1806, . .	October 4	October 20	17	1871, . .	March 17	March 31	15
1809, . .	March 2	March 23	22	1872, . .	April 2	April 21	20
1810, . .	September 24	October 9	16	1873, . .	February 7	February 25	19
1811, . .	November 18	December 3	16	1875, . .	April 9	April 26	18
1813, . .	September 17	October 3	17	1876, . .	August 11	August 27	17
1814, . .	January 17	February 1	16	1884, . .	May 21	June 4	15
1817, . .	March 31	April 20	21	,,	December 21	January 4	15
1824, . .	January 2	January 20	19	1837, . .	June 15	July 2	18
1825, . .	February 10	February 25	16	1889, . .	June 15	July 2	18
,,	March 13	April 11	30	,,	November 6	November 21	16
,,	July 16	July 31	16	1894, . .	March 15	March 30	16
1826, . .	June 12	June 26	15	,,	September 6	September 20	15
,,	July 22	August 9	19	1895, . .	February 7	February 22	16
1828, . .	February 28	March 14	16				

TABLE XXXI.

Falls of 1 *Inch or more of rain in* 24 *Hours.*
Years 1770-1776, 1780–*June* 1781, 1785–1817, 1824–1831, 1854–96.

Year.	Date.	Amount.	Year.	Date.	Amount.	Year.	Date.	Amount.	Year.	Date.	Amount.
		ins.			ins.			ins.			ins.
1770	Nov. 7	2·30	1797	Oct. 20	1·30	1813	May 16	1·02	1874	June 26	1·54
,,	,, 9	1·06	,,	Dec. 18	1·03	1814	April 24	1·30	,,	July 24	1·72
,,	,, 14	1·20	1798	Jan. 26	1·03	,,	July 29	1·50	,,	Aug. 14	1·53
1771	Oct. 7	1·42	,,	June 20	1·40	,,	Nov. 18	1·10	,,	Nov. 30	1·32
,,	,, 26	1·13	1799	May 10	1·05	1815	May 11	1·12	1875	Nov. 7	1·28
,,	Nov. 17	1·80	,,	June 4	1·00	1816	July 8	1·30	1876	Aug. 31	1·52
1772	Jan. 12	1·07	,,	Aug. 17	1·75	1827	April 24	1·30	,,	Dec. 31	1·34
,,	May 26	1·00	,,	Sept. 13	1·70	,,	Aug. 16	1·26	1877	Jan. 1	1·00
,,	Nov. 1	1·80	,,	,, 17	2·16	,,	Oct. 11	1·66	,,	,, 30	1·05
1774	May 24	1·40	1800	May 17	1·15	1828	July 12	2·00	,,	April 9	1·29
,,	Aug. 17	1·20	1801	May 23	1·01	,,	Nov. 30	1·60	,,	July 17	1·35
,,	Dec. 4	1·01	1802	Feb. 19	1·07	1829	April 10	1·18	,,	Aug. 18	1·54
1775	July 30	1·13	1803	Aug. 9	1·15	,,	July 5	1·09	,,	,, 20	1·88
,,	Sept. 10	1·01	,,	,, 28	1·12	,,	Aug. 3	1·80	,,	,, 21	1·94
,,	Oct. 5	2·50	1804	Oct. 22	1·62	,,	,, 22	1·29	1878	,, 28	1·13
1785	July 31	1·07	,,	Nov. 13	1·00	,,	,, 26	1·00	1879	Mar. 17	1·28
,,	Sept. 6	1·36	1806	May 2	1·15	,,	Oct. 14	1·05	,,	June 21	1·31
,,	,, 17	3·70	,,	Nov. 9	2·25	1830	July 30	1·80	,,	July 13	2·95
,,	,, 24	3·80	1807	Sept. 6	3·51	,,	Aug. 15	1·51	1880	April 31	1·56
,,	Oct. 13	1·57	1808	April 5	1·71	,,	,, 27	1·02	,,	July 7	1·12
,,	Nov. 5	1·46	,,	May 6	1·05	,,	Sept. 21	1·27	1881	Jan. 29	1·24
1787	May 12	1·44	,,	June 8	1·00	1831	Aug. 30	1·07	,,	Feb. 13	1·44
,,	July 26	1·58	,,	July 24	1·55	1854	June 18	1·25	,,	Aug. 23	1·80
,,	Dec. 9	4·20	,,	,, 28	1·55	1856	Sept. 8	1·06	,,	,, 25	1·21
,,	,, 19	1·05	,,	Aug. 4	1·14	1857	June 7	1·25	1882	May 7	1·20
1788	Sept. 21	1·13	,,	,, 5	1·15	,,	Sept. 13	1·40	1884	Aug. 13	1·70
1789	Aug. 29	1·13	,,	Oct. 14	1·50	1858	June 17	1·43	,,	Sept. 6	1·46
1790	Nov. 24	1·35	1809	Jan. 10	1·59	,,	Aug. 30	1·43	1889	Aug. 19	1·41
1791	June 17	1·13	,,	Feb. 3	1·80	1859	Nov. 5	1·12	1890	,, 12	1·94
,,	Aug. 16	1·17	,,	June 1	1·00	1861	Sept. 23	2·40	,,	Sept. 30	1·19
,,	Oct. 22	1·07	,,	July 5	1·15	1864	Oct. 20	2·43	1891	Mar. 16	1·55
1792	July 13	1·18	,,	Aug. 12	1·14	1864	Oct. 23	1·50	,,	Sept. 20	1·94
1794	Sept. 2	1·00	,,	Sept. 8	1·20	1865	May 30	1·29	1893	June 22	1·00
,,	Oct. 6	1·15	,,	,, 17	1·40	,,	Oct. 18	1·35	1894	Feb. 11	1·03
,,	,, 10	1·07	1810	Jan. 15	1·25	1866	Sept. 29	1·40	,,	,, 16	1·47
1795	May 14	1·50	,,	Feb. 14	1·10	1867	July 22	1·30	,,	Aug. 2	1·24
,,	,, 15	1·40	,,	Mar. 9	1·01	1869	Sept. 12	1·70	1895	July 26	1·63
,,	July 23	1·45	,,	,, 10	1·10	1871	Aug. 21	1·22	1896	July 8	1·23
,,	Nov. 17	2·63	,,	June 19	1·00	1872	May 15	1·01		FROM ADIE.	
1797	July 30	2·00	,,	Aug. 8	1·10	,,	July 27	1·13	1795	Nov. 18	2·89
,,	Aug. 18	2·30	,,	,, 15	1·30	,,	Oct. 22	1·07	1797	July 30	2·63
,,	Oct. 18	1·66	1812	Mar. 21	1·21	1873	Oct. 1	1·15	,,	Aug. 18	2·56

Table XXXII.

Showing the Number of Days the Wind Blew from the Eight Principal Directions in Edinburgh for each Month during 138 *Years.*

JANUARY.																			
Year.	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm or Var.	Year.	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm or Var.
1732	...	3	3	9	5	3	8	...	...	1828	...	...	10	5	...	8	8	...	...
1733	...	...	1	1	5	20	3	1	...	1829	1	1	10	4	...	...	4	4	7
1734	...	...	...	...	6	12	12	1	...	1830	3	2	4	4	1	6	6	2	3
1735	3	...	2	1	1	5	17	2	...	1831	1	2	5	5	...	3	7	6	2
1736	2	1	...	5	12	6	5	...	...	1832	...	1	4	3	1	12	6	...	4
1764	...	...	9	...	...	...	22	...	...	1833	...	1	5	2	2	3	11	2	5
1765	...	...	15	...	...	...	16	...	...	1834	...	1	4	...	3	15	6	1	1
1766	...	...	4	...	...	...	27	...	...	1835	1	1	2	1	...	2	23	1	...
1767	...	...	11	...	...	...	20	...	...	1836	...	1	2	1	...	2	22	3	...
1768	...	...	15	...	...	...	16	...	...	1837	2	6	1	3	5	3	8	3	...
1769	...	...	9	...	...	...	22	...	...	1838	...	...	9	6	5	5	4	2	...
1770	3	...	...	...	1	15	8	4	...	1839	2	1	1	...	1	4	15	6	1
1771	4	...	2	1	1	12	6	5	...	1840	...	2	1	2	1	9	14	...	2
1772	5	2	7	2	...	3	9	2	1	1841	2	2	2	3	...	2	12	4	4
1773	5	...	1	2	2	10	10	...	1	1842	...	1	1	6	2	6	10	2	3
1774	4	1	4	1	...	5	10	3	3	1843	...	...	1	1	1	9	14	5	...
1775	...	...	5	9	3	10	4	...	...	1844	...	...	1	1	...	3	22	3	1
1776	2	2	10	4	3	4	2	...	4	1845	...	1	1	5	3	8	10	3	...
1777	1	7	5	...	2	1	10	5	...	1846	...	...	6	2	...	7	13	...	3
1778	2	3	3	1	5	5	10	1	1	1847	2	...	3	13	2	5	2	...	4
1779	...	3	3	...	2	7	13	2	1	1848	...	...	2	6	4	4	12	1	2
1780	2	6	7	3	...	...	6	1	6	1849	...	3	1	2	2	5	14	1	3
1781	...	2	11	...	...	4	11	2	1	1850	...	1	13	2	5	6	4	...	...
1782	...	...	5	...	...	...	26	...	...	1851	...	...	...	5	13	8	4	1	...
1783	...	...	5	...	...	...	26	...	...	1852	2	...	...	...	4	6	17	2	...
1784	1	2	3	2	...	2	14	7	...	1853	3	1	3	4	3	15	1	1	...
1785	6	3	5	1	2	10	3	1	...	1854	1	1	3	4	7	8	5	2	...
1786	4	1	8	...	...	7	8	3	...	1855	4	4	5	...	...	4	6	4	4
1787	2	3	2	1	...	9	12	2	...	1856	1	2	4	5	4	7	3	3	2
1788	4	2	1	...	...	3	18	3	...	1857	2	5	2	1	1	7	8	3	2
1789	4	4	10	1	...	5	4	3	...	1858	4	2	1	1	1	8	6	5	3
1790	1	...	3	2	...	8	13	4	...	1859	...	...	...	...	2	13	15	1	...
1791	1	1	3	...	4	13	7	2	...	1860	2	3	2	1	8	5	9	...	1
1792	5	3	10	...	...	1	8	4	...	1861	...	1	4	...	3	9	12	1	1
1793	5	3	2	...	...	4	13	4	...	1862	1	...	1	2	10	5	9	1	2
1794	...	...	...	...	...	5	18	8	...	1863	...	3	1	...	5	12	6	4	...
1795	11	10	...	...	...	3	5	2	...	1864	...	...	1	2	10	15	2	1	...
1796	...	...	3	1	1	6	20	...	...	1865	2	2	5	2	3	6	6	5	...
1797	...	...	8	...	...	2	20	1	...	1866	2	...	2	2	2	7	13	3	...
1798	...	...	1	...	...	5	22	3	...	1867	4	2	8	1	3	2	8	3	...
1799	...	1	6	...	...	5	19	...	...	1868	6	3	4	2	5	9	2	...	...
1800	1	1	19	...	1	1	8	...	...	1869	1	1	3	3	10	5	6	2	...
1801	...	1	2	...	...	...	27	1	...	1870	4	2	...	4	...	2	8	10	1
1802	...	...	9	...	...	...	22	...	...	1871	...	...	3	6	3	8	10	1	...
1803	...	...	17	...	...	...	14	...	...	1872	...	6	...	4	2	14	1	4	...
1804	...	...	6	4	...	5	12	...	4	1873	...	...	2	4	2	11	12	...	...
1805	...	1	5	7	3	6	4	...	5	1874	1	...	1	...	2	7	19	...	1
1806	1	4	2	...	...	5	13	6	...	1875	...	2	3	2	4	6	12	...	2
1807	...	...	4	...	...	...	23	4	...	1876	3	...	3	1	3	1	14	4	2
1808	2	1	...	...	1	10	14	2	1	1877	...	4	1	4	5	9	7	1	...
1809	...	2	22	...	...	...	6	1	...	1878	...	2	1	...	2	2	22	2	...
1810	...	1	9	...	...	3	17	1	...	1879	1	1	1	11	4	1	10	1	1
1811	...	...	9	...	...	11	8	3	...	1880	...	...	2	1	1	6	18	3	...
1812	2	...	1	1	2	10	9	6	...	1881	...	4	3	3	3	2	10	6	...
1813	...	...	2	2	3	23	1	...	...	1882	...	1	2	...	1	7	18	2	...
1814	1	8	...	4	2	8	2	6	...	1883	...	...	1	7	3	8	11	1	...
1815	1	4	2	4	...	13	5	2	...	1884	...	...	2	2	1	3	22	1	...
1816	2	1	3	4	3	4	11	3	...	1885	...	3	3	5	2	5	7	3	3
1817	...	...	1	1	3	7	16	3	...	1886	2	5	3	2	...	4	11	4	...
1818	...	1	...	2	3	12	13	...	...	1887	...	2	1	...	3	10	11	1	3
1819	1	1	1	2	5	11	5	5	...	1888	1	...	3	...	2	5	12	4	4
1820	...	9	1	...	...	3	12	5	1	1889	1	...	4	1	2	1	20	1	1
1821	...	1	7	1	...	7	13	1	1	1890	...	...	2	1	1	8	17	1	1
1822	1	1	...	2	...	4	16	6	1	1891	3	...	2	2	1	4	15	2	2
1823	1	2	11	8	1	2	2	1	3	1892	1	...	2	2	2	3	15	4	2
1824	...	...	...	1	1	8	14	5	2	1893	2	1	6	1	1	1	14	2	3
1825	2	1	...	...	...	9	15	2	2	1894	2	1	3	1	3	7	11	1	2
1826	...	...	5	3	1	13	2	4	3	1895	7	3	5	3	1	...	8	3	1
1827	1	2	8	1	...	7	8	3	1	1896	3	...	2	...	...	2	20	3	1

TABLE XXXII.—*continued.*

FEBRUARY.

Year.	N.	NE.	E.	SE.	S.	SW.	W.	NW.	Calm or Var.	Year.	N.	NE.	E.	SE.	S.	SW.	W.	NW.	Calm or Var.
1732	1	1	1	1	3	10	12	...	...	1828	...	...	2	9	2	10	4	...	2
1733	1	1	...	3	1	17	4	1	...	1829	...	1	5	1	1	1	11	2	6
1734	...	...	...	...	4	12	8	4	...	1830	1	2	...	3	2	11	4	3	2
1735	1	1	...	...	1	13	11	1	...	1831	1	2	2	1	1	9	7	5	...
1736	4	4	3	6	1	1	4	6	...	1832	...	...	...	3	2	16	3	1	4
1764	...	...	10	...	...	...	19	...	...	1833	...	1	6	3	2	9	5	1	1
1765	...	...	13	...	...	...	15	...	...	1834	...	...	3	1	1	6	14	1	2
1766	...	...	5	...	...	...	20	...	3	1835	...	...	2	...	...	3	22	1	...
1767	...	...	12	...	...	...	16	...	...	1836	1	6	2	...	1	4	10	5	...
1768	...	...	15	...	...	...	14	...	...	1837	2	2	...	1	1	8	10	4	...
1769	...	...	5	...	...	...	23	...	...	1838	6	1	7	3	2	...	5	3	1
1770	3	...	...	...	...	16	6	3	...	1839	...	2	...	...	2	8	13	2	1
1771	1	3	7	2	4	5	5	1	...	1840	...	...	4	10	1	6	5	1	2
1772	3	4	6	1	2	2	11	...	...	1841	2	1	7	5	1	6	2	2	2
1773	1	...	3	3	8	7	6	...	...	1842	...	...	...	4	1	12	8	2	1
1774	1	...	...	...	4	13	8	2	...	1843	1	9	10	...	...	1	3	4	...
1775	1	...	1	2	4	13	6	...	1	1844	2	5	1	2	...	4	13	2	...
1776	...	2	2	2	4	15	1	1	2	1845	1	2	4	2	1	3	9	2	4
1777	2	3	6	1	1	3	9	3	...	1846	1	...	...	1	1	8	16	1	...
1778	1	...	2	1	1	8	13	1	1	1847	1	3	3	1	...	3	9	7	1
1779	...	...	...	...	3	10	13	...	2	1848	...	1	2	1	...	9	13	3	...
1780	7	...	2	2	...	2	9	3	4	1849	...	...	1	...	...	4	22	1	...
1781	...	...	2	1	1	3	14	5	2	1850	...	1	2	...	2	6	14	3	...
1782	...	...	18	...	...	...	10	...	...	1851	2	1	1	...	5	12	7	...	...
1783	...	...	12	...	...	...	16	...	...	1852	4	1	...	1	1	1	19	2	...
1784	4	5	3	5	1	2	6	3	...	1853	8	3	7	2	1	1	4	2	...
1785	7	6	6	2	...	...	5	2	...	1854	2	...	...	1	1	6	11	7	...
1786	2	6	4	1	...	1	11	3	...	1855	3	3	8	3	...	3	2	2	4
1787	...	...	3	...	...	9	12	4	...	1856	1	2	4	2	4	6	1	6	3
1788	...	1	11	4	2	8	3	...	...	1857	...	1	...	...	3	11	11	1	1
1789	2	...	3	...	2	6	9	6	...	1858	4	4	6	2	1	4	1	3	3
1790	...	1	...	...	...	4	20	3	...	1859	...	...	...	...	6	8	11	2	1
1791	4	...	3	1	...	5	11	4	...	1860	2	2	...	...	1	4	17	3	...
1792	2	2	8	1	...	6	5	5	...	1861	1	4	1	1	7	9	4	...	1
1793	1	...	3	...	...	9	13	2	...	1862	1	2	6	1	7	2	7	2	...
1794	...	...	8	1	...	8	10	1	...	1863	2	1	...	1	6	13	4	1	...
1795	7	1	13	...	...	2	1	4	...	1864	...	2	7	2	2	4	5	5	2
1796	...	...	8	1	...	3	16	1	...	1865	2	1	8	6	...	5	6	...	...
1797	...	...	...	...	...	2	26	...	...	1866	2	3	3	2	...	4	9	6	...
1798	...	1	1	...	...	...	26	...	...	1867	1	...	3	...	4	5	14	1	...
1799	...	...	14	...	...	1	13	...	...	1868	...	...	...	...	2	20	6	1	...
1800	...	...	21	...	1	...	5	1	...	1869	1	...	1	2	4	6	12	2	...
1801	...	1	6	1	...	...	20	...	...	1870	2	1	4	7	1	4	3	6	...
1802	...	...	3	...	...	...	24	...	1	1871	1	1	7	3	...	4	10	2	...
1803	...	...	2	...	...	...	26	...	...	1872	...	1	4	6	1	13	4	...	...
1804	7	2	2	...	...	5	6	4	3	1873	2	2	4	1	...	1	16	2	...
1805	2	3	...	3	1	5	11	1	2	1874	2	2	3	3	3	4	8	2	1
1806	...	...	...	3	2	8	12	1	2	1875	2	2	8	1	1	...	8	2	4
1807	2	2	1	...	...	...	22	1	...	1876	5	2	6	1	1	2	8	3	1
1808	4	2	...	...	...	...	19	2	2	1877	2	...	1	...	1	3	17	4	...
1809	...	...	11	...	...	...	16	1	...	1878	...		1	1	2	4	19	...	1
1810	1	2	1	2	...	2	15	5	...	1879	1	,	4	8	1	1	10	...	...
1811	2	1	1	5	5	7	6	1	...	1880	...	1	2	1	1	8	14	2	...
1812	...	1	2	1	3	10	9	3	...	1881	...	4	4	5	3	2	2	7	1
1813	...	1	...	...	3	21	1	1	1	1882	...	...	1	1	1	5	17	3	...
1814	2	...	3	5	...	13	2	3	...	1883	1	...	...	3	4	6	12	2	...
1815	...	1	1	7	1	12	6	...	...	1884	...	3	...	6	2	9	6	3	...
1816	...	3	...	2	3	5	9	7	...	1885	1	...	1	3	...	12	7	3	1
1817	1	...	...	...	...	13	5	9	...	1886	2	3	4	3	2	5	5	1	3
1818	...	...	...	2	3	14	6	3	...	1887	...	2	2	1	2	12	5	2	2
1819	3	...	2	4	2	6	4	7	...	1888	1	4	7	...	...	...	15	2	...
1820	...	3	4	1	...	9	10	2	...	1889	3	1	3	1	1	...	14	5	...
1821	...	2	1	7	...	3	10	3	2	1890	1	1	10	2	...	...	11	...	3
1822	...	...	...	...	4	13	10	...	1	1891	1	...	2	...	...	...	16	2	7
1823	4	2	5	1	3	4	8	1	...	1892	2	2	5	1	2	...	10	3	4
1824	4	5	5	2	1	6	5	...	1	1893	1	1	3	2	3	1	12	2	3
1825	...	...	...	...	1	10	10	4	3	1894	...	...	1	2	2	6	15	1	1
1826	...	...	...	1	2	20	4	1	...	1895	3	2	7	3	1	...	3	5	4
1827	3	9	1	1	1	2	4	6	1	1896	1	...	3	3	2	3	15	1	1

TABLE XXXII.—*continued.*

MARCH.																			
Year.	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm or Var.	Year.	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm or Var.
1732	2	...	7	...	...	4	12	6	...	1828	2	2	1	...	...	10	9	5	2
1733	3	2	5	8	3	3	3	4	...	1829	2	7	7	2	2	2	2	4	3
1734	...	...	...	4	5	10	10	2	...	1830	...	1	3	4	...	12	10	...	1
1735	2	6	2	6	2	2	6	5	...	1831	...	1	...	4	2	10	6	3	5
1736	...	4	5	8	3	3	5	3	...	1832	1	...	1	1	...	9	9	5	5
1764	...	...	13	...	...	...	18	...	...	1833	4	3	11	2	1	3	6	1	...
1765	...	...	9	...	...	...	22	...	...	1834	...	...	3	...	...	5	19	3	1
1766	...	...	14	...	...	...	17	...	...	1835	...	1	8	...	...	2	19	1	...
1767	...	...	12	...	...	...	19	...	...	1836	...	1	4	2	...	5	17	2	...
1768	...	...	22	...	...	...	9	...	...	1837	5	8	2	1	...	...	9	6	...
1769	...	...	9	...	...	...	22	...	...	1838	2	2	5	1	2	6	5	6	2
1770	6	...	5	...	...	8	5	7	...	1839	1	4	3	8	1	3	8	2	1
1771	5	1	6	5	2	5	5	2	...	1840	2	2	9	...	...	...	4	6	8
1772	1	5	12	4	5	2	2	...	...	1841	...	...	1	4	2	10	11	2	1
1773	3	1	2	1	5	5	13	1	...	1842	1	2	...	1	...	10	11	4	2
1774	...	7	8	2	5	3	4	...	2	1843	2	2	...	6	1	6	6	2	6
1775	1	...	3	...	3	12	8	4	...	1844	1	1	4	1	1	8	8	4	3
1776	...	2	4	2	2	14	7	...	...	1845	1	3	5	3	1	6	8	4	...
1777	5	6	2	...	3	3	7	5	...	1846	2	2	2	3	1	8	9	3	1
1778	1	7	8	...	3	3	8	1	...	1847	4	4	7	2	1	3	4	2	4
1779	3	3	3	...	...	2	16	1	3	1848	2	6	1	2	2	6	7	1	4
1780	...	1	1	...	3	12	12	2	...	1849	1	2	5	2	1	2	16	2	...
1781	2	1	7	1	2	8	6	2	2	1850	3	...	4	1	3	6	9	5	...
1782	...	...	13	...	...	...	18	...	...	1851	1	2	2	4	7	6	5	4	...
1783	...	...	17	...	...	...	14	...	...	1852	4	3	6	2	2	3	7	4	...
1784	6	7	5	2	2	...	5	4	...	1853	6	4	4	3	4	3	4	3	...
1785	15	5	1	...	...	1	6	3	...	1854	...	...	2	1	5	8	12	3	...
1786	3	6	6	1	...	3	8	4	...	1855	2	1	3	2	5	4	5	6	3
1787	...	1	2	2	2	6	16	1	...	1856	3	7	5	1	6	2	4	2	1
1788	8	1	12	6	...	2	2	...	...	1857	2	3	7	1	2	2	9	1	4
1789	13	3	10	...	...	...	1	4	...	1858	9	2	1	1	1	6	5	6	...
1790	...	...	7	...	...	3	15	6	...	1859	...	2	...	1	...	7	19	2	...
1791	2	...	2	...	...	6	18	3	...	1860	3	...	2	...	3	8	14	1	...
1792	8	...	6	...	...	8	9	...	...	1861	1	1	...	1	3	8	17	...	...
1793	...	1	9	4	...	8	5	4	...	1862	1	7	14	1	3	2	3	...	...
1794	2	...	8	...	2	8	11	...	...	1863	...	3	5	3	3	8	4	5	...
1795	4	1	11	...	...	7	4	4	...	1864	2	3	8	2	1	3	9	2	1
1796	1	...	11	3	...	...	13	3	...	1865	4	2	3	2	4	4	6	6	...
1797	...	...	14	3	...	...	14	...	...	1866	2	5	8	3	...	3	5	5	...
1798	1	3	5	...	...	1	20	1	...	1867	4	4	7	6	1	3	4	2	...
1799	...	1	19	...	...	...	9	2	...	1868	...	1	...	...	7	13	8	2	...
1800	...	...	17	...	...	1	8	5	...	1869	4	5	4	2	2	1	8	5	...
1801	...	1	5	...	...	...	21	4	...	1870	3	4	4	5	1	1	1	10	2
1802	...	...	2	...	...	...	29	...	...	1871	1	2	3	1	2	6	15	1	...
1803	...	...	5	...	...	...	26	...	...	1872	...	7	5	3	2	9	4	1	...
1804	1	3	12	2	5	1	6	...	1	1873	1	6	12	2	...	4	4	2	...
1805	...	...	2	5	4	10	4	4	2	1874	4	...	2	1	...	1	17	5	1
1806	3	2	10	1	1	1	6	3	4	1875	2	3	7	4	1	1	9	3	1
1807	2	1	16	...	...	...	11	1	...	1876	4	2	3	2	...	2	12	5	1
1808	...	2	13	9	...	...	4	...	3	1877	...	1	4	4	2	...	15	4	1
1809	2	2	15	1	...	1	10	...	...	1878	...	2	2	...	1	2	18	6	...
1810	1	3	11	2	...	1	7	6	...	1879	...	3	6	7	...	4	11	...	...
1811	...	3	5	2	...	10	10	1	...	1880	...	8	9	2	...	1	10	1	...
1812	4	10	1	1	...	9	3	3	...	1881	...	4	...	5	1	2	15	4	..
1813	...	4	2	1	1	12	8	3	...	1882	1	1	3	...	...	2	22	2	...
1814	...	7	7	5	1	8	3	...	...	1883	8	3	5	1	1	1	6	5	1
1815	1	...	...	1	2	19	7	1	...	1884	...	...	4	3	5	10	4	3	2
1816	1	2	4	9	1	7	4	3	...	1885	1	3	4	1	1	4	5	12	...
1817	1	...	1	2	2	5	14	6	...	1886	4	3	4	3	5	3	5	3	1
1818	1	3	1	...	4	9	9	4	...	1887	1	...	1	1	3	2	10	8	5
1819	...	3	4	1	1	8	12	2	...	1888	3	4	7	...	...	...	9	7	1
1820	6	1	1	2	...	4	15	2	...	1889	2	...	4	4	1	1	16	2	1
1821	2	...	4	5	...	4	12	3	1	1890	2	1	5	...	3	2	17	...	1
1822	2	...	...	1	1	12	14	1	...	1891	6	2	3	...	2	1	9	5	3
1823	3	...	1	3	2	9	11	1	1	1892	...	3	6	4	2	6	4	4	2
1824	3	4	...	...	1	9	5	2	7	1893	1	...	4	...	...	4	16	2	4
1825	...	...	4	8	2	4	6	2	5	1894	1	3	2	1	2	5	13	2	2
1826	2	4	2	6	1	6	3	5	2	1895	6	3	4	1	3	12	2	...	...
1827	3	1	2	...	...	5	15	5	...	1896	...	2	2	...	1	4	20	2	...

TABLE XXXII.—*continued.*

APRIL.

Year.	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm or Var.	Year.	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm or Var.
1732	1	3	11	3	2	3	2	5	...	1828	3	8	2	3	1	5	6	1	1
1733	4	10	4	2	3	4	1	2	...	1829	3	6	6	2	1	3	5	4	...
1734	...	1	7	3	...	7	12	...	...	1830	...	4	...	2	...	10	6	4	4
1735	1	1	8	2	4	6	7	1	...	1831	...	8	10	1	...	4	4	1	2
1736	1	6	...	3	5	2	7	6	...	1832	...	9	8	2	...	6	3	1	1
1764	...	...	14	...	...	...	16	...	...	1833	...	1	7	1	1	5	15	...	...
1765	...	...	13	...	...	...	17	...	...	1834	1	2	13	...	...	1	10	1	2
1766	...	...	15	...	...	...	15	...	...	1835	4	1	7	1	...	1	14	2	...
1767	...	...	20	...	...	...	10	...	...	1836	6	3	3	2	...	2	10	4	...
1768	...	...	13	...	...	...	14	...	...	1837	2	2	10	3	6	3	1	2	1
1769	...	...	15	...	...	...	15	...	...	1838	4	3	5	1	...	4	6	6	1
1770	5	3	2	3	1	4	9	3	...	1839	1	...	4	8	1	2	8	4	2
1771	2	1	5	...	1	6	10	5	...	1840	1	6	4	1	...	6	8	2	2
1772	1	3	8	1	1	5	5	6	...	1841	1	4	4	3	1	6	8	2	1
1773	1	2	1	1	5	10	9	1	...	1842	2	11	9	1	...	...	4	1	2
1774	1	2	4	4	3	5	8	2	1	1843	1	1	4	2	2	7	8	3	2
1775	...	1	1	...	5	6	16	1	...	1844	...	2	2	...	...	8	12	4	2
1776	4	2	1	...	3	5	13	...	2	1845	1	6	10	1	...	6	3	...	3
1777	2	6	8	1	3	1	7	2	...	1846	1	7	10	5	1	1	2	2	1
1778	3	6	4	...	2	5	8	2	...	1847	2	4	4	2	...	2	11	2	3
1779	...	...	3	...	1	6	16	4	...	1848	1	5	7	2	1	2	5	4	3
1780	3	5	4	4	4	2	5	3	...	1849	1	3	9	4	2	3	5	1	2
1781	...	3	15	...	7	4	...	1	...	1850	1	2	11	8	...	1	6	1	...
1782	...	...	27	...	...	...	3	...	...	1851	...	7	6	2	3	2	5	5	...
1783	...	...	12	...	...	...	18	...	...	1852	2	7	8	2	...	1	6	1	3
1784	1	1	6	1	...	3	12	6	...	1853	...	1	4	2	2	4	16	1	...
1785	5	...	...	...	...	...	21	4	...	1854	...	4	10	...	...	1	12	3	...
1786	2	2	16	...	...	5	3	2	...	1855	1	2	2	1	1	1	12	7	3
1787	4	1	9	...	...	1	12	3	...	1856	1	4	2	3	5	3	7	4	1
1788	2	...	1	1	1	6	14	5	...	1857	5	6	4	...	1	5	5	4	...
1789	3	...	5	1	...	6	11	4	...	1858	3	4	6	...	...	...	8	9	...
1790	1	4	10	9	...	3	3	...	...	1859	3	4	7	1	1	3	7	4	...
1791	...	3	19	...	1	3	3	1	...	1860	4	2	12	2	2	1	6	1	...
1792	1	...	10	2	...	8	8	1	...	1861	...	1	13	3	1	...	8	3	1
1793	2	1	17	...	...	4	2	4	...	1862	1	1	3	1	...	3	5	5	11
1794	1	...	7	...	...	9	11	2	...	1863	...	...	2	2	1	9	6	3	7
1795	...	...	11	4	1	6	8	...	...	1864	...	...	10	3	...	2	11	2	2
1796	7	1	1	...	7	7	6	1	...	1865	...	2	8	4	1	4	10	1	...
1797	1	...	16	...	...	...	11	2	...	1866	1	2	13	5	2	2	5	...	...
1798	...	...	14	1	...	...	14	1	...	1867	2	...	4	2	4	5	12	1	...
1799	1	...	13	3	...	...	12	1	...	1868	4	4	2	...	1	12	6	1	...
1800	1	1	5	1	...	6	14	2	...	1869	3	3	4	1	3	8	8	...	...
1801	1	...	9	1	...	...	16	3	...	1870	1	5	...	1	...	4	5	13	1
1802	...	...	3	...	...	...	27	...	...	1871	1	7	11	3	...	1	6	1	...
1803	...	...	7	...	...	...	23	...	...	1872	3	6	3	2	1	4	9	2	..
1804	4	4	8	...	...	6	1	3	4	1873	3	5	10	1	...	1	8	2	...
1805	1	6	3	2	1	7	2	1	7	1874	2	2	3	1	2	3	12	4	1
1806	4	1	10	2	...	1	10	2	...	1875	2	2	8	...	...	2	12	1	3
1807	1	6	6	...	...	...	16	1	...	1876	3	3	7	2	1	...	10	4	...
1808	7	2	2	...	1	2	8	7	1	1877	...	3	14	6	3	...	3	1	...
1809	2	8	10	...	...	...	4	6	...	1878	...	1	12	6	2	3	6	...	...
1810	...	...	8	3	...	8	9	2	...	1879	...	8	12	1	3	...	4	1	1
1811	2	2	10	2	1	2	7	4	...	1880	1	5	7	2	2	1	8	4	...
1812	5	7	9	2	...	1	3	3	...	1881	...	11	7	1	3	...	3	4	1
1813	2	11	...	1	...	9	4	3	...	1882	2	1	12	5	...	3	5	1	1
1814	...	2	2	4	4	12	4	2	...	1883	2	2	7	4	1	2	8	3	1
1815	1	11	3	2	...	3	7	3	...	1884	3	6	9	4	1	4	2	1	...
1816	2	4	8	6	1	2	4	3	...	1885	...	4	7	2	1	9	3	3	1
1817	4	7	2	2	...	2	5	8	...	1886	1	8	8	...	...	4	6	1	2
1818	1	9	10	3	6	...	1	...	...	1887	1	3	1	4	1	4	5	9	2
1819	...	8	5	1	3	6	3	4	...	1888	2	7	3	2	2	2	8	4	...
1820	2	1	4	...	...	6	13	4	...	1889	2	1	12	1	2	2	9	1	...
1821	...	...	5	3	...	5	12	2	3	1890	4	...	10	1	3	1	8	3	...
1822	1	4	5	2	4	6	5	2	1	1891	2	2	11	2	2	...	4	3	4
1823	3	3	10	...	...	2	7	3	2	1892	3	3	9	...	1	2	10	1	1
1824	2	6	5	...	1	2	9	4	1	1893	1	4	9	1	2	...	8	1	4
1825	2	2	5	...	...	7	9	2	3	1894	1	3	8	9	1	3	3	...	2
1826	1	3	1	...	1	6	13	4	1	1895	3	2	9	...	1	3	10	1	1
1827	...	6	4	2	1	9	3	1	4	1896	1	1	2	...	...	1	17	8	...

TABLE XXXII.—*continued.*

MAY.

Year.	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm or Var.	Year.	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm or Var.
1732	2	2	6	3	6	4	5	3	...	1828	...	4	12	1	...	3	9	1	1
1733	1	6	15	2	...	1	5	1	...	1829	2	6	6	1	...	6	4	2	4
1734	1	7	7	...	1	4	7	4	...	1830	...	6	9	2	...	7	5	...	2
1735	2	6	3	1	...	3	8	8	...	1831	...	6	12	2	1	5	...	3	2
1736	2	13	8	1	...	3	2	2	...	1832	3	10	6	1	...	2	6	2	1
1764	...	...	10	...	...	...	21	...	...	1833	...	1	7	4	...	8	11	...	...
1765	...	...	18	...	...	...	13	...	...	1834	...	...	10	1	...	5	13	...	2
1766	...	...	17	...	...	...	14	...	...	1835	...	1	14	2	2	3	9	...	...
1767	...	...	16	...	...	...	15	...	...	1836	...	11	11	1	...	4	...	4	...
1768	...	...	12	...	...	...	19	...	...	1837	1	7	7	2	...	3	4	6	1
1769	...	...	19	...	...	...	12	...	...	1838	1	11	8	2	...	2	2	1	4
1770	2	6	15	...	3	...	3	2	...	1839	2	6	9	1	1	2	5	4	1
1771	4	5	6	1	4	7	4	...	...	1840	3	8	9	1	...	4	4	2	...
1772	4	7	8	1	1	...	8	...	2	1841	1	2	6	1	1	8	8	2	2
1773	1	8	10	...	...	1	10	1	...	1842	...	6	3	5	1	6	5	2	3
1774	7	10	5	...	...	3	3	2	1	1843	...	5	17	...	1	3	3	...	2
1775	2	1	3	...	1	7	15	2	...	1844	2	2	18	1	...	2	2	2	2
1776	6	4	4	...	2	3	10	2	...	1845	2	7	14	...	...	1	4	2	1
1777	1	2	4	3	2	5	12	2	...	1846	...	2	5	3	...	5	11	3	2
1778	...	3	7	1	6	7	6	1	...	1847	...	2	10	3	1	5	7	...	3
1779	...	3	7	...	2	7	6	1	5	1848	...	1	8	...	...	6	9	1	6
1780	...	...	4	2	5	10	9	1	...	1849	1	4	13	2	...	4	4	1	2
1781	...	3	12	4	3	5	1	1	2	1850	1	2	10	2	1	4	11	..	...
1782	...	...	15	...	...	...	16	...	...	1851	5	1	3	...	2	4	16	...	...
1783	...	...	12	...	...	...	19	...	...	1852	5	7	6	...	1	2	8	2	...
1784	4	1	3	2	...	6	13	2	...	1853	...	1	22	3	2	...	1	2	...
1785	...	2	3	...	...	2	19	5	...	1854	1	1	...	3	5	7	14	...	...
1786	...	1	5	2	...	3	18	2	...	1855	3	10	3	3	1	3	3	5	...
1787	2	1	8	4	...	2	12	2	...	1856	...	9	2	1	5	1	11	...	2
1788	...	5	9	1	1	7	7	1	...	1857	1	7	9	2	2	2	6	2	...
1789	...	...	10	7	2	7	4	1	...	1858	5	4	11	...	2	2	2	3	2
1790	1	1	17	2	...	2	8	...	...	1859	...	4	14	4	3	1	4	1	...
1791	3	1	5	3	...	4	13	2	...	1860	1	...	6	4	7	3	10	...	...
1792	3	2	6	...	1	8	7	4	...	1861	4	6	3	2	...	1	15	...	...
1793	...	...	6	2	1	8	4	10	...	1862	...	2	2	4	1	5	7	2	8
1794	1	2	7	...	...	4	11	6	...	1863	...	3	5	3	...	5	9	4	2
1795	3	2	6	...	...	5	14	1	...	1864	...	2	10	3	...	8	7	1	...
1796	2	2	10	4	1	...	12	...	...	1865	1	2	8	5	4	6	4	1	...
1797	1	...	9	...	...	1	17	3	...	1866	...	1	9	6	...	2	11	2	...
1798	...	...	10	...	...	...	20	1	...	1867	5	6	12	1	2	1	3	1	...
1799	...	1	12	2	...	1	11	4	...	1868	2	2	2	2	12	7	4	...	...
1800	...	...	14	...	...	2	15	...	...	1869	4	6	14	...	...	...	6	1	...
1801	...	...	13	1	...	1	16	...	...	1870	2	4	3	2	...	2	7	8	3
1802	...	...	15	...	...	...	16	...	...	1871	3	6	...	7	3	...	2	8	2
1803	...	...	5	...	...	...	26	...	...	1872	...	9	2	1	...	5	12	2	...
1804	...	1	6	...	3	12	3	2	4	1873	...	3	12	1	...	1	13	1	...
1805	2	4	11	1	...	1	2	8	2	1874	2	4	15	1	...	2	4	2	1
1806	1	8	12	1	...	2	4	1	2	1875	1	2	6	...	...	2	15	4	1
1807	...	...	20	...	...	...	11	...	...	1876	2	4	10	1	1	...	5	6	2
1808	1	...	7	2	2	9	5	...	5	1877	...	6	10	2	3	...	10	...	...
1809	...	1	13	...	...	...	15	2	...	1878	...	2	10	4	2	4	8	...	1
1810	...	...	14	2	...	...	13	2	...	1879	3	4	9	1	...	1	10	2	1
1811	...	10	6	6	...	6	3	...	...	1880	...	3	11	1	...	2	13	1	...
1812	...	4	10	4	...	4	7	2	...	1881	1	6	6	...	...	3	9	3	3
1813	...	13	4	2	...	5	5	2	...	1882	...	2	9	3	1	6	5	3	2
1814	...	10	12	1	...	6	2	...	...	1883	1	8	2	...	1	5	11	3	...
1815	2	2	7	3	1	7	7	2	...	1884	...	5	4	1	2	7	7	3	2
1816	...	2	11	4	...	...	9	5	...	1885	...	3	5	2	1	4	8	8	...
1817	...	5	8	2	...	3	12	1	...	1886	1	6	4	5	1	3	6	2	3
1818	...	8	15	...	2	2	3	1	...	1887	2	2	5	2	4	1	4	7	4
1819	1	3	9	4	1	5	7	1	...	1888	...	2	6	4	1	4	6	6	2
1820	...	1	15	1	...	7	5	2	...	1889	1	2	16	3	4	1	2	1	1
1821	4	2	10	3	...	2	7	2	1	1890	...	2	15	2	4	...	6	2	...
1822	...	...	14	2	1	3	5	...	6	1891	6	3	8	1	2	1	4	3	3
1823	1	1	8	3	2	4	12	...	...	1892	...	2	9	...	1	3	11	2	3
1824	1	6	13	1	1	2	2	1	4	1893	2	1	10	1	3	2	10	...	2
1825	...	5	15	2	1	2	1	1	4	1894	5	3	6	1	2	2	8	4	...
1826	...	...	20	2	...	3	6	...	...	1895	3	5	8	2	1	...	9	2	1
1827	...	5	8	4	2	6	5	...	1	1896	6	6	4	...	...	1	9	3	2

TABLE XXXII.—*continued.*

JUNE.

Year.	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm or Var.	Year.	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm or Var.
1731	2	6	5	1	1	2	9	4	...	1828	...	4	5	...	2	8	8	2	1
1732	...	5	7	1	...	2	11	4	...	1829	2	6	2	2	2	3	6	4	3
1733	2	...	11	2	1	7	7	...	...	1830	1	6	4	...	...	4	3	7	5
1734	1	8	7	4	2	1	5	2	...	1831	...	6	5	2	1	9	2	3	2
1735	...	6	4	1	2	1	13	3	...	1832	...	8	6	1	...	...	9	2	4
1764	...	...	16	...	...	...	14	...	...	1833	1	...	7	2	2	5	11	...	2
1765	...	...	13	...	...	...	17	...	...	1834	1	...	3	1	...	6	17	2	...
1766	...	...	16	...	...	...	14	...	...	1835	1	2	13	...	1	2	10	1	...
1767	...	...	14	...	...	...	16	...	...	1836	...	...	4	2	2	10	12	...	...
1768	...	...	8?	...	...	...	12?	...	...	1837	4	2	...	...	5	10	5	1	3
1769	...	...	12	...	...	...	18	...	...	1838	1	3	7	1	...	8	4	1	5
1770	1	6	1	...	4	6	11	1	...	1839	1	9	7	1	...	4	5	1	2
1771	4	8	6	...	...	...	10	2	...	1840	1	2	4	2	...	6	10	4	1
1772	3	4	5	...	3	7	7	1	...	1841	2	4	9	1	...	2	9	1	2
1773	5	8	2	...	3	3	9	...	...	1842	...	7	6	2	...	3	9	3	...
1774	2	3	2	2	6	3	9	3	...	1843	...	5	16	1	...	...	4	3	1
1775	1	10	10	1	...	3	5	...	...	1844	1	3	6	1	...	7	6	2	4
1776	1	7	2	...	3	6	9	2	...	1845	1	...	3	1	1	7	9	6	2
1777	1	3	4	...	3	12	7	...	...	1846	...	1	4	2	2	3	12	...	6
1778	...	1	4	...	3	8	11	...	3	1847	2	3	5	2	...	4	9	3	2
1779	...	3	16	1	...	3	3	1	3	1848	...	1	16	1	...	2	8	...	2
1780	1	1	3	1	2	4	15	1	2	1849	3	3	...	2	...	1	12	4	5
1781	...	...	21	...	...	...	9	...	...	1850	...	1	5	...	...	3	18	3	...
1782	...	...	5	...	...	...	25	...	...	1851	...	...	5	1	3	5	15	1	...
1783	...	...	12	...	...	...	18	...	...	1852	6	3	4	1	3	2	9	1	1
1784	1	2	9	3	...	4	10	1	...	1853	...	...	6	6	1	3	12	2	...
1785	1	1	5	1	...	7	11	4	...	1854	3	1	4	4	4	5	7	1	1
1786	...	...	17	1	...	5	7	...	...	1855	1	2	4	1	4	4	11	2	1
1787	1	2	8	2	3	3	8	3	...	1856	4	4	11	...	2	2	4	1	2
1788	...	1	17	1	...	...	8	3	...	1857	...	3	5	...	1	13	1	6	1
1789	2	2	14	...	...	6	3	3	...	1858	...	2	3	1	2	10	8	3	1
1790	...	...	4	1	...	1	21	3	...	1859	1	4	6	3	1	1	11	2	1
1791	1	7	8	2	...	5	6	1	....	1860	...	...	14	1	4	7	3	1	...
1792	...	2	13	1	1	4	8	1	...	1861	2	3	15	2	1	3	4	...	...
1793	1	...	2	2	2	8	9	6	...	1862	...	1	3	1	1	4	13	4	3
1794	...	1	11	2	...	9	4	3	...	1863	...	2	5	3	2	7	6	1	4
1795	1	1	18	...	1	4	3	2	...	1864	1	1	3	2	3	4	14	2	...
1796	...	1	4	1	...	7	15	2	...	1865	...	...	6	6	1	6	10	1	...
1797	...	...	15	...	...	...	13	2	...	1866	2	4	6	2	2	3	10	1	...
1798	...	...	6	...	...	2	22	...	...	1867	4	4	2	1	1	2	14	2	...
1799	...	2	5	1	...	1	19	2	...	1868	2	2	1	2	2	6	15	...	...
1800	...	2	8	...	...	...	16	4	...	1869	4	4	4	...	1	2	10	5	...
1801	...	1	7	...	...	1	21	...	...	1870	2	3	2	2	...	...	14	6	1
1802	...	...	8	...	...	...	21	...	1	1871	3	7	4	5	...	2	1	2	6
1803	...	...	9	...	...	...	21	...	...	1872	2	...	8	...	6	...	12	1	1
1804	1	1	5	...	...	8	7	4	4	1873	...	2	8	2	...	4	10	3	1
1805	2	4	6	1	...	1	9	3	4	1874	4	3	6	...	1	4	8	4	...
1806	2	1	5	..	1	6	10	2	3	1875	...	3	5	...	1	2	13	2	4
1807	...	...	10	...	...	...	20	...	...	1876	1	6	4	1	2	1	11	4	...
1808	...	3	12	...	...	3	10	2	...	1877	...	1	10	1	1	4	12	1	...
1809	...	2	9	...	...	...	19	...	...	1878	5	7	8	1	...	1	7	1	...
1810	...	3	4	5	...	1	13	4	...	1879	2	4	9	2	1	5	7	...	...
1811	...	4	8	...	1	12	4	1	...	1880	2	4	12	...	...	2	8	...	2
1812	...	1	9	...	1	10	8	1	...	1881	...	4	5	3	...	4	11	2	1
1813	...	14	6	5	...	2	...	3	...	1882	3	5	1	2	4	6	2	7	...
1814	2	2	13	3	...	2	6	2	...	1883	3	2	9	...	2	5	9	...	...
1815	...	9	6	2	1	7	4	1	...	1884	2	5	3	...	...	...	7	7	6
1816	2	3	6	3	3	3	2	8	...	1885	1	7	1	2	...	4	9	4	2
1817	...	4	4	4	2	8	7	...	1	1886	2	4	7	...	...	2	10	4	1
1818	1	1	1	6	4	3	14	...	...	1887	1	2	7	1	3	1	7	6	2
1819	1	...	2	2	3	8	13	1	...	1888	...	5	14	2	2	...	2	2	3
1820	3	2	9	...	...	2	10	4	...	1889	1	11	1	2	3	10	...	...	2
1821	2	3	23	...	...	...	1	...	1	1890	1	...	6	2	2	1	17	...	1
1822	...	...	12	...	...	4	10	...	4	1891	...	3	13	2	1	1	5	3	2
1823	1	3	8	...	...	3	12	2	1	1892	2	2	6	1	2	1	12	1	3
1824	1	2	2	2	...	4	17	1	1	1893	4	1	7	2	2	2	8	2	2
1825	1	1	4	...	1	10	6	2	5	1894	2	3	8	1	2	2	9	3	...
1826	...	4	4	...	1	2	14	1	4	1895	1	2	10	1	1	1	12	2	...
1827	1	1	4	...	...	8	13	2	1	1896	3	3	10	1	1	1	9	2	...

TABLE XXXII.—*continued.*

JULY.																			
Year.	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm or Var.	Year.	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm or Var.
1731	4	5	4	...	4	1	12	1	...	1828	3	8	7	3	2	3	2	1	2
1732	2	3	2	2	1	5	10	6	...	1829	5	6	3	3	1	5	6	...	2
1733	1	4	4	1	1	1	15	4	...	1830	1	5	2	1	1	10	5	3	3
1734	3	1	8	2	...	3	8	6	...	1831	...	6	1	1	...	7	6	4	6
1735	3	5	2	4	3	4	7	3	...	1832	2	2	7	...	...	4	5	4	7
1764	...	...	12	...	...	...	19	...	...	1833	...	...	6	1	1	5	12	...	6
1765	...	...	9	...	...	...	22	...	...	1834	...	3	18	...	2	4	2	1	1
1766	...	...	16	...	...	...	15	...	...	1835	...	...	7	3	...	8	12	1	...
1767	...	...	14	...	...	...	17	...	...	1836	...	2	3	2	1	5	14	4	...
1768	...	...	18	...	...	...	13	...	...	1837	5	3	3	2	2	7	2	5	2
1769	...	..	12	...	...	...	19	...	...	1838	2	1	7	...	1	6	10	4	...
1770	5	4	8	...	4	3	6	1	...	1839	...	1	5	2	2	9	8	1	3
1771	1	3	3	...	...	9	13	2	...	1840	1	2	2	...	...	5	11	4	6
1772	5	4	2	...	2	8	9	1	...	1841	1	3	6	1	...	3	8	4	5
1773	3	3	5	...	2	6	12	...	...	1842	1	3	7	1	2	4	5	4	4
1774	...	2	1	...	7	8	10	3	...	1843	2	2	1	2	...	7	9	6	2
1775	3	1	5	2	7	5	5	2	1	1844	1	...	6	4	2	3	10	2	3
1776	1	2	7	2	4	8	6	1	...	1845	1	2	6	3	1	4	8	5	1
1777	...	1	10	...	1	4	12	...	3	1846	...	1	4	2	2	9	10	3	...
1778	...	1	3	...	3	8	11	1	4	1847	...	...	11	...	...	1	16	...	3
1779	...	1	4	2	2	5	9	1	7	1848	...	...	2	...	2	5	20	1	1
1780	1	...	6	1	...	4	16	1	2	1849	1	6	2	...	...	7	8	4	3
1781	...	...	14	...	...	...	17	...	...	1850	...	1	10	...	4	4	9	3	...
1782	...	...	8	...	...	...	23	...	...	1851	...	2	7	...	2	4	16	...	...
1783	...	...	8	...	...	...	23	...	...	1852	4	9	7	1	2	...	7	1	...
1784	...	5	1	...	3	9	12	1	...	1853	...	...	1	2	5	9	11	3	...
1785	...	1	2	4	3	13	5	3	...	1854	2	4	2	...	3	14	4	2	...
1786	1	...	...	...	...	2	23	5	...	1855	...	6	7	2	1	2	9	1	3
1787	1	...	9	1	3	3	13	1	...	1856	4	...	8	...	...	11	...	7	1
1788	...	...	1	2	2	12	13	1	...	1857	2	1	1	...	...	10	13	3	1
1789	...	1	5	2	1	14	7	1	...	1858	1	4	2	3	2	7	8	4	...
1790	1	1	1	2	...	15	9	2	...	1859	...	...	11	...	1	2	16	1	...
1791	...	...	4	...	3	8	9	7	...	1860	1	6	9	1	1	...	13	...	...
1792	...	...	4	3	1	13	5	5	...	1861	...	1	6	4	2	3	15	...	...
1793	...	...	7	3	...	8	12	1	...	1862	...	1	...	2	1	8	15	3	1
1794	...	...	7	2	...	7	12	3	...	1863	...	3	4	...	...	3	16	4	1
1795	3	3	7	2	2	1	11	2	...	1864	2	...	8	1	1	4	14	1	...
1796	1	2	3	...	...	4	18	3	...	1865	...	...	8	5	2	5	11	...	...
1797	...	...	7	...	1	1	22	...	...	1866	1	6	8	4	...	3	9	...	...
1798	...	...	2	...	...	4	25	...	...	1867	6	4	8	2	1	1	7	2	...
1799	...	...	16	...	...	...	15	...	...	1868	4	6	2	1	2	6	9	1	...
1800	...	2	3	...	...	...	26	...	...	1869	4	2	2	...	8	2	13	...	...
1801	...	...	13	1	...	...	16	1	...	1870	...	2	4	4	1	2	8	7	3
1802	...	...	14	...	...	...	16	...	1	1871	...	1	1	3	3	5	15	3	...
1803	...	...	9	...	...	...	22	...	...	1872	3	...	10	1	4	1	11	1	...
1804	1	4	11	...	...	2	7	2	4	1873	1	1	2	...	2	8	12	3	2
1805	...	3	8	...	2	2	11	3	2	1874	1	1	2	1	3	2	16	3	2
1806	1	4	9	1	...	2	8	2	4	1875	3	5	8	...	...	...	10	3	2
1807	...	...	5	...	...	...	26	...	...	1876	3	1	2	...	...	...	18	6	1
1808	...	4	16	2	...	...	7	...	2	1877	3	...	...	2	5	5	15	1	...
1809	...	...	13	...	...	...	16	2	...	1878	1	5	9	1	...	2	10	3	...
1810	...	4	5	3	...	1	16	2	...	1879	1	6	5	1	...	2	15	1	...
1811	1	2	4	2	2	3	16	1	...	1880	...	5	12	...	...	1	12	1	...
1812	1	4	5	...	3	8	9	1	...	1881	...	1	...	...	...	2	21	6	1
1813	1	4	7	3	2	7	6	1	...	1882	...	2	2	...	1	11	12	3	...
1814	...	2	...	1	1	15	9	3	...	1883	1	4	4	1	4	6	6	4	1
1815	...	9	4	...	...	3	15	...	...	1884	...	1	...	...	...	2	21	6	1
1816	1	9	7	2	4	...	7	1	...	1885	...	2	2	...	1	11	12	3	...
1817	3	3	1	5	2	4	10	...	3	1886	1	4	4	1	4	6	6	4	1
1818	3	5	1	2	1	4	15	...	...	1887	1	1	1	...	1	4	17	5	1
1819	...	...	11	2	...	4	10	4	...	1888	...	4	11	3	2	3	6	1	1
1820	...	7	13	1	...	...	9	1	...	1889	3	1	10	2	1	...	11	3	...
1821	...	4	5	1	1	3	11	1	5	1890	1	...	6	2	2	4	15	1	...
1822	...	3	11	2	...	1	13	...	1	1891	2	2	4	...	3	2	12	3	3
1823	...	1	6	3	...	9	7	2	3	1892	...	1	12	1	...	...	15	...	2
1824	1	...	3	3	...	...	19	4	1	1893	1	5	7	1	1	1	10	2	3
1825	1	1	11	2	4	1	4	1	6	1894	1	3	8	...	1	2	14	1	1
1826	...	1	4	2	1	9	8	2	4	1895	2	1	6	1	2	2	15	2	...
1827	...	...	7	1	1	8	13	1	...	1896	7	1	4	...	1	1	13	2	2

TABLE XXXII.—*continued.*

AUGUST.

Year.	N.	NE.	E.	SE.	S.	SW.	W.	NW.	Calm or Var.	Year.	N.	NE.	E.	SE.	S.	SW.	W.	NW.	Calm or Var.
1731	5	3	15	1	1	1	2	3	...	1828	6	3	7	2	1	4	6	2	...
1732	1	3	4	3	1	5	9	5	...	1829	2	5	3	3	...	3	5	6	4
1733	...	1	4	1	1	7	11	6	...	1830	...	9	7	...	1	2	6	2	4
1734	1	1	4	1	1	6	15	2	...	1831	1	1	4	1	...	4	8	4	8
1735	...	3	4	3	1	8	12	...	...	1832	1	6	1	1	...	5	12	1	4
1764	...	...	12	...	...	...	18	...	1	1833	1	...	4	...	...	1	22	...	3
1765	...	...	18	...	...	...	13	...	...	1834	...	1	6	...	3	6	10	...	5
1766	...	...	7	...	...	...	24	...	...	1835	...	1	10	4	...	2	12	2	...
1767	...	...	5	...	...	...	26	...	...	1836	...	...	5	4	2	3	13	4	...
1768	...	...	14	...	...	...	17	...	...	1837	1	2	9	2	...	6	5	4	2
1769	...	...	11	...	...	...	20	...	...	1838	2	2	2	1	...	4	12	3	5
1770	1	4	2	...	1	11	10	2	...	1839	2	5	1	...	1	13	4	2	3
1771	2	2	1	...	3	9	12	2	...	1840	...	3	3	1	...	5	15	2	2
1772	1	1	2	2	2	7	15	1	...	1841	...	2	2	2	...	8	13	2	2
1773	3	2	5	2	4	3	12	...	...	1842	...	1	5	...	1	12	5	2	5
1774	4	2	1	2	5	7	9	1	...	1843	...	1	3	1	1	9	9	2	5
1775	...	2	3	2	9	9	5	1	...	1844	1	...	4	1	...	2	15	4	4
1776	...	4	3	4	2	6	10	1	1	1845	2	4	4	1	...	3	7	5	5
1777	...	2	2	...	2	10	10	1	4	1846	...	3	10	1	1	3	7	1	5
1778	1	1	3	1	2	5	17	...	1	1847	...	1	4	1	2	5	12	3	3
1779	1	1	8	2	2	9	3	1	4	1848	...	1	5	1	1	3	11	2	7
1780	...	2	16	2	1	3	3	...	4	1849	..	...	4	1	1	4	11	1	9
1781	...	...	15	...	...	...	16	...	...	1850	...	...	2	...	1	6	13	9	...
1782	...	...	5	...	..	...	26	...	...	1851	...	...	9	2	1	9	8	2	...
1783	...	...	13	...	...	...	18	...	...	1852	2	6	2	...	1	5	14	1	...
1784	3	2	3	...	1	4	16	2	...	1853	2	1	1	2	2	5	11	7	...
1785	2	3	7	1	1	4	9	4	...	1854	2	...	...	1	1	8	13	6	...
1786	...	...	8	2	...	5	14	2	...	1855	...	1	1	1	2	6	15	3	2
1787	...	2	5	...	...	1	18	5	...	1856	...	2	5	1	5	6	7	3	2
1788	1	2	2	...	...	8	17	1	...	1857	...	4	12	1	1	2	8	...	3
1789	1	...	6	3	...	16	4	1	...	1858	...	1	5	1	3	10	8	3	...
1790	...	...	6	2	...	5	16	2	...	1859	...	1	4	...	2	7	17	...	...
1791	1	1	5	3	4	2	9	6	...	1860	...	3	2	1	3	6	16	...	...
1792	...	2	20	...	2	4	3	...	...	1861	...	...	...	1	2	7	20	1	...
1793	...	...	5	1	2	7	16	...	...	1862	...	1	1	7	1	1	15	...	5
1794	5	...	4	..	3	4	9	6	...	1863	...	4	1	1	1	8	11	4	1
1795	2	...	4	1	1	6	15	2	...	1864	...	1	8	4	2	5	10	...	1
1796	1	...	2	...	1	7	19	1	...	1865	1	1	8	6	1	6	8	...	...
1797	...	...	2	...	...	6	18	5	...	1866	...	2	6	4	2	5	11	1	...
1798	...	...	3	1	...	1	23	3	...	1867	2	1	1	...	3	10	14	...	...
1799	...	...	8	...	1	4	18	...	...	1868	2	4	5	...	6	6	8	...	...
1800	...	4	9	...	...	...	18	...	...	1869	4	2	1	...	2	2	18	2	...
1801	...	...	12	...	...	...	19	...	...	1870	2	9	3	...	...	2	6	9	...
1802	...	...	8	...	...	...	23	...	...	1871	...	...	2	2	2	7	15	3	...
1803	...	...	5	...	...	...	26	...	...	1872	2	...	9	3	5	...	12	...	...
1804	...	2	...	...	...	5	17	1	6	1873	...	1	8	...	1	6	15	...	...
1805	1	1	2	1	...	7	14	4	1	1874	...	2	6	1	...	4	17	1	...
1806	1	1	2	1	1	5	13	4	3	1875	1	...	8	1	...	1	14	2	4
1807	...	...	8	...	...	...	23	...	...	1876	4	1	6	...	2	2	9	1	6
1808	...	1	8	1	1	4	14	1	1	1877	...	2	11	4	2	1	10	1	...
1809	...	1	15	...	...	...	15	...	...	1878	...	3	13	6	...	...	8	...	1
1810	...	1	8	...	...	2	18	2	...	1879	...	2	8	2	1	4	14	...	...
1811	3	...	3	...	1	8	11	5	...	1880	...	1	17	...	1	3	8	1	...
1812	1	9	10	3	1	3	3	1	...	1881	1	7	4	1	...	3	8	7	...
1813	1	2	3	...	2	8	10	5	...	1882	1	1	...	...	2	4	15	6	2
1814	...	...	2	6	...	7	12	4	...	1883	1	1	1	...	2	3	19	4	...
1815	1	...	2	...	2	12	13	1	...	1884	...	1	3	2	4	11	6	2	2
1816	1	4	7	3	...	2	9	4	1	1885	...	11	7	1	1	2	6	2	1
1817	1	3	3	8	3	7	6	...	...	1886	...	2	2	2	2	2	19	1	1
1818	2	10	2	4	1	4	6	2	...	1887	2	3	6	...	...	4	11	4	1
1819	...	...	14	...	...	11	3	3	...	1888	1	1	5	1	2	7	11	2	1
1820	...	...	4	...	2	9	14	2	...	1889	1	1	3	1	...	3	21	...	1
1821	1	1	10	...	...	4	13	...	2	1890	...	2	6	1	...	4	15	2	1
1822	1	1	...	2	1	6	11	2	7	1891	1	2	7	2	1	3	11	3	1
1823	...	...	1	...	3	9	11	5	2	1892	...	1	5	1	...	3	19	...	2
1824	1	3	9	...	...	1	10	2	5	1893	2	...	4	1	1	...	14	3	6
1825	...	...	6	...	...	4	9	4	8	1894	...	...	5	1	...	3	18	4	...
1826	...	1	3	...	4	11	8	1	3	1895	2	1	1	2	...	4	18	...	3
1827	1	2	7	2	1	3	6	6	3	1896	6	3	3	...	...	...	15	3	1

TABLE XXXII.—*continued.*

SEPTEMBER.																			
Year.	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm or Var.	Year.	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm or Var.
1731	1	...	1	2	3	7	14	2	...	1828	2	2	5	3	2	6	2	...	8
1732	3	2	3	5	...	7	8	2	...	1829	2	2	2	2	1	5	6	5	5
1733	2	...	3	2	4	10	8	1	...	1830	...	4	...	1	2	14	5	2	2
1734	2	1	1	1	...	7	14	4	...	1831	...	6	2	6	1	8	5	...	2
1735	2	1	2	1	...	8	12	4	...	1832	...	...	3	...	...	6	12	6	3
1764	...	...	3	...	...	...	27	...	...	1833	1	...	6	2	2	4	10	1	4
1765	...	...	10	...	...	...	20	...	...	1834	...	1	14	...	1	4	6	...	4
1766	...	...	8	...	...	...	22	...	...	1835	1	...	7	4	...	6	12	...	...
1767	...	...	8	...	...	...	22	...	...	1836	1	7	2	1	2	4	9	4	...
1768	...	...	12	...	...	...	18	...	...	1837	1	4	5	7	...	7	2	3	1
1769	...	...	9	...	...	...	21	...	...	1838	2	3	4	...	1	4	5	1	10
1770	2	4	9	2	4	4	4	1	...	1839	...	...	2	...	5	7	8	1	7
1771	6	2	6	...	4	5	5	2	...	1840	1	5	2	...	...	8	10	2	2
1772	3	2	4	1	3	7	9	1	...	1841	...	1	8	4	2	6	4	1	4
1773	...	...	2	3	4	12	9	...	...	1842	2	8	3	1	...	2	8	2	4
1774	2	2	7	4	7	2	6	...	...	1843	3	2	4	2	1	2	12	2	2
1775	5	4	5	1	5	1	9	...	...	1844	1	2	13	...	...	...	12	1	1
1776	....	6	3	4	1	5	7	4	...	1845	1	1	5	1	2	6	7	2	5
1777	...	...	1	2	2	5	12	1	7	1846	1	...	7	2	2	3	8	...	7
1778	4	1	2	1	3	4	9	2	4	1847	1	...	3	1	1	7	9	6	2
1779	...	1	2	...	4	8	7	2	6	1848	2	2	6	...	1	3	11	2	3
1780	...	...	6	5	2	10	7	...	...	1849	...	2	14	3	1	1	6	...	3
1781	...	...	10	...	...	...	20	...	...	1850	2	3	3	6	11	1	4	...	...
1782	...	...	12	...	...	...	18	...	...	1851	...	3	7	...	2	3	14	1	...
1783	...	...	7	...	...	...	23	...	...	1852	7	2	1	1	3	2	11	3	...
1784	...	...	6	1	...	8	14	1	...	1853	...	3	3	3	4	9	4	4	...
1785	...	1	8	5	...	8	5	3	...	1854	...	1	...	2	...	6	17	3	1
1786	...	...	...	1	...	6	18	5	...	1855	3	1	5	...	...	6	12	...	3
1787	1	...	10	2	...	6	10	1	...	1856	2	6	6	2	1	7	2	3	1
1788	...	...	8	2	...	3	15	2	...	1857	1	5	4	...	2	8	3	...	7
1789	2	2	7	...	...	5	14	...	...	1858	...	1	1	2	4	9	12	1	...
1790	4	1	1	...	...	5	13	6	...	1859	...	2	1	...	6	6	14	1	...
1791	2	...	10	1	...	6	7	4	...	1860	1	4	1	...	3	5	15	1	...
1792	5	...	7	...	...	3	13	2	...	1861	...	2	...	...	1	10	11	6	...
1793	..	...	5	3	...	4	15	3	...	1862	3	...	2	2	5	6	10	2	...
1794	3	...	10	1	1	6	9	...	...	1863	...	...	6	3	1	3	9	3	5
1795	3	1	4	4	3	3	9	3	...	1864	...	...	4	1	1	8	14	...	2
1796	...	...	5	5	...	1	19	...	...	1865	...	...	3	2	2	8	13	2	...
1797	1	...	4	...	...	3	17	5	...	1866	1	2	2	2	10	6	6	1	...
1798	...	1	5	...	...	2	20	2	...	1867	2	4	2	...	8	3	10	1	...
1799	...	...	9	3	...	2	15	1	...	1868	4	8	4	2	3	3	4	2	...
1800	...	1	11	...	1	5	11	1	...	1869	1	2	2	...	10	4	10	1	...
1801	...	...	18	...	...	...	12	...	...	1870	1	...	6	3	1	7	9	3	...
1802	...	...	8	...	...	...	22	...	...	1871	1	7	7	3	...	4	5	3	...
1803	...	...	5	...	...	...	25	...	...	1872	2	1	8	...	1	...	14	4	...
1804	1	1	5	...	...	10	7	2	4	1873	2	2	4	...	...	4	16	2	...
1805	...	1	2	2	...	6	11	3	5	1874	1	...	2	1	1	2	20	1	2
1806	...	2	2	...	4	9	10	1	2	1875	...	4	11	1	...	4	8	2	...
1807	1	3	6	...	...	...	18	2	...	1876	6	3	5	2	2	2	6	3	1
1808	2	3	5	1	1	2	11	2	3	1877	2	1	4	6	...	1	10	5	1
1809	...	...	13	1	...	...	16	...	...	1878	...	2	5	...	1	6	14	2	...
1810	...	...	10	...	...	...	20	...	...	1879	...	1	4	2	...	5	17	1	...
1811	2	8	3	4	2	2	6	3	...	1880	...	2	4	2	1	4	16	1	...
1812	4	...	6	1	...	10	8	1	...	1881	...	3	8	3	2	2	12	...	...
1813	1	5	6	2	1	9	6	...	...	1882	2	1	3	2	4	9	5	4	...
1814	...	6	5	1	1	13	1	3	...	1883	...	2	9	2	2	1	7	5	2
1815	1	1	2	5	2	10	6	3	...	1884	...	...	3	3	...	7	11	...	6
1816	1	9	...	4	...	6	6	3	1	1885	...	4	..	1	...	6	11	8	...
1817	1	2	2	6	5	7	2	5	...	1886	1	3	4	2	3	4	10	1	2
1818	2	1	4	7	5	7	3	1	...	1887	1	2	3	1	5	4	7	6	1
1819	...	2	...	...	...	7	16	5	...	1888	4	...	5	...	...	...	12	1	8
1820	4	...	3	...	3	2	18	...	...	1889	2	2	9	...	...	3	10	2	2
1821	1	...	...	3	1	4	17	2	2	1890	...	...	3	2	2	1	17	1	4
1822	...	2	9	3	...	2	10	1	3	1891	3	...	2	...	2	6	14	...	3
1823	1	...	3	1	2	6	14	1	2	1892	...	...	3	...	2	3	20	1	1
1824	1	2	5	1	...	14	2	5	...	1893	3	1	2	1	...	5	14	2	2
1825	1	1	3	3	2	10	5	3	2	1894	3	2	14	1	...	1	4	3	2
1826	...	1	7	2	2	7	8	2	1	1895	2	1	...	...	1	1	16	3	6
1827	...	1	9	1	4	5	7	2	1	1896	1	3	10	1	1	1	10	1	2

TABLE XXXII.—*continued.*

OCTOBER.

Year.	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm or Var.	Year.	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm or Var.
1731	1	1	1	6	8	7	6	1	...	1828	...	...	...	4	3	9	11	1	3
1732	...	3	2	7	4	9	5	1	...	1829	4	2	1	3	...	8	11	2	...
1733	...	1	1	2	1	12	8	6	...	1830	1	...	...	...	2	4	10	5	9
1734	3	3	1	1	3	6	7	7	...	1831	...	1	1	1	1	22	4	...	1
1735	1	...	8	5	1	5	10	1	...	1832	...	1	...	2	...	10	12	2	4
1764	...	...	4	...	...	...	27	...	...	1833	...	1	8	2	1	6	10	...	3
1765	...	...	3	...	...	...	28	...	...	1834	...	...	...	...	1	6	20	4	...
1766	...	...	9	...	...	...	22	...	...	1835	...	2	1	2	...	8	13	5	...
1767	...	...	...	...	...	...	31	...	...	1836	4	...	3	5	...	6	12	1	...
1768	...	...	17	...	..	...	14	...	...	1837	1	...	2	...	4	5	14	5	...
1769	...	...	14	...	...	...	17	...	...	1838	1	2	2	1	1	6	12	2	4
1770	...	...	5	...	2	11	13	...	...	1839	...	2	6	3	3	9	2	1	5
1771	2	1	...	2	4	15	7	...	...	1840	1	2	3	1	1	1	10	10	2
1772	1	1	6	5	6	8	4	...	...	1841	...	7	6	2	1	1	8	5	1
1773	2	...	...	...	10	5	14	...	...	1842	2	1	1	1	...	1	13	10	2
1774	1	...	3	3	3	7	14	...	...	1843	3	2	2	1	1	4	13	4	1
1775	1	2	...	1	1	7	15	3	1	1844	...	3	1	6	4	4	8	3	2
1776	...	4	3	2	2	8	11	1	...	1845	...	2	1	2	2	3	17	2	2
1777	...	2	8	1	4	6	9	...	1	1846	1	4	2	6	3	5	7	...	3
1778	3	2	6	3	5	2	1	1	8	1847	...	3	9	5	1	5	7	1	...
1779	...	...	4	...	4	10	7	3	3	1848	3	7	2	4	2	5	5	1	2
1780	...	...	3	...	4	11	10	...	3	1849	1	3	4	...	...	9	8	5	1
1781	...	...	1	...	...	...	30	...	...	1850	3	1	1	...	2	8	10	6	...
1782	...	...	14	...	...	...	17	...	...	1851	2	...	...	...	3	14	12	...	...
1783	...	...	...	...	...	...	31	...	...	1852	6	2	3	1	2	2	13	1	1
1784	2	...	5	4	4	2	11	3	...	1853	2	1	2	5	6	6	6	3	...
1785	1	2	1	1	2	4	17	3	...	1854	2	2	...	...	2	10	7	4	4
1786	...	...	16	1	...	4	8	2	...	1855	1	2	2	1	4	5	12	2	2
1787	...	...	10	3	1	6	9	2	...	1856	...	2	2	3	5	7	3	5	4
1788	2	...	4	...	...	5	17	3	...	1857	2	1	6	1	1	5	6	3	6
1789	3	1	8	4	...	6	9	...	...	1858	...	1	5	...	1	7	14	2	1
1790	2	...	10	2	3	6	7	1	...	1859	2	2	4	4	4	5	8	2	...
1791	1	1	11	7	...	2	7	2	...	1860	...	2	2	...	2	6	18	1	...
1792	2	...	15	5	...	4	4	1	...	1861	...	...	...	2	9	8	10	2	...
1793	1	...	4	...	...	14	12	...	...	1862	...	...	4	2	2	2	21	...	...
1794	2	...	...	3	1	6	12	7	...	1863	...	2	4	3	3	6	12	...	1
1795	...	1	1	10	5	8	4	2	...	1864	...	...	15	4	...	2	8	...	2
1796	...	...	...	1	...	1	27	2	...	1865	2	2	12	4	...	1	6	4	...
1797	...	...	6	3	...	3	12	7	...	1866	1	2	1	4	5	5	10	3	...
1798	...	...	6	...	...	3	22	...	...	1867	...	...	3	1	4	8	15	...	...
1799	...	...	7	...	...	2	15	7	...	1868	...	...	2	1	6	6	14	2	...
1800	...	...	5	...	...	2	22	2	...	1869	3	6	1	2	2	2	8	7	...
1801	3	1	4	...	...	...	20	3	...	1870	2	3	8	...	3	2	12	1	...
1802	...	...	5	...	...	...	26	...	...	1871	1	1	7	...	1	6	14	1	...
1803	...	...	13	...	...	...	18	...	..	1872	3	...	4	1	2	1	18	2	...
1804	3	...	3	5	2	8	8	...	2	1873	1	...	2	...	...	10	18	...	...
1805	3	3	5	6	...	2	4	1	7	1874	1	...	3	1	3	3	14	5	1
1806	2	2	6	5	1	5	6	2	2	1875	...	1	14	2	2	1	8	1	2
1807	1	...	6	...	...	...	23	1	...	1876	2	2	10	2	2	4	6	2	1
1808	2	2	...	...	2	9	8	7	1	1877	...	1	1	3	3	4	16	3	...
1809	...	...	13	...	...	...	18	...	...	1878	2	...	1	6	8	4	7	3	...
1810	...	1	8	...	...	1	21	...	...	1879	...	1	5	1	...	1	22	1	...
1811	1	1	2	8	3	10	6	...	...	1880	1	7	4	1	...	...	7	10	1
1812	1	8	1	3	2	8	4	3	1	1881	...	3	7	8	1	3	...	9	...
1813	...	5	10	5	1	5	1	4	...	1882	1	1	6	4	5	4	7	1	2
1814	...	...	...	5	2	10	10	4	...	1883	1	1	1	2	1	7	14	4	...
1815	...	5	3	6	4	11	2	...	...	1884	3	1	2	...	1	4	14	5	1
1816	3	1	7	7	1	3	7	2	...	1885	1	7	5	2	...	2	5	9	...
1817	5	7	2	8	4	3	1	...	1	1886	...	6	7	2	4	4	8	...	...
1818	...	...	4	8	2	13	4	...	...	1887	1	1	...	...	...	4	17	6	2
1819	3	4	4	...	3	5	9	3	...	1888	1	...	1	1	2	9	12	5	...
1820	3	1	5	2	5	2	11	2	..	1889	1	1	9	4	3	2	9	1	1
1821	...	...	...	3	...	7	20	...	1	1890	2	...	1	...	1	2	22	3	...
1822	1	3	6	6	3	8	3	...	1	1891	...	...	1	3	8	4	8	1	6
1823	1	1	4	5	2	10	7	...	1	1892	4	1	3	2	1	3	15	2	...
1824	1	2	6	1	1	5	10	4	1	1893	1	...	2	...	...	2	23	...	3
1825	2	1	2	1	3	7	11	3	1	1894	3	2	6	5	2	...	9	2	2
1826	...	...	1	4	2	10	11	..	3	1895	7	2	3	...	1	...	16	1	1
1827	1	3	10	2	2	3	5	2	3	1896	7	2	2	1	...	2	11	5	1

TABLE XXXII.—*continued.*

NOVEMBER.																			
Year.	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm or Var.	Year.	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm or Var.
1731	...	3	2	1	...	3	16	5	...	1828	2	4	2	5	3	6	5	2	1
1732	3	1	...	6	3	3	10	4	...	1829	1	3	2	2	...	7	8	3	4
1733	1	...	...	1	4	12	11	1	...	1830	...	1	2	5	1	14	6	1	...
1734	1	1	...	3	6	13	6	...	...	1831	...	1	1	1	1	6	12	6	2
1735	1	1	...	6	10	4	5	3	...	1832	1	...	...	3	4	7	12	1	2
1764	...	...	15	...	...	...	15	...	...	1833	...	...	1	...	1	5	18	2	3
1765	...	...	7	...	...	...	23	...	...	1834	...	...	6	3	...	3	17	1	...
1766	...	...	5	...	...	...	25	...	...	1835	1	...	8	4	2	3	12	...	...
1767	...	..	6	...	...	...	24	...	...	1836	1	4	2	2	...	3	15	3	...
1768	...	...	8	...	...	...	22	...	...	1837	1	...	2	2	2	7	12	4	...
1769	...	...	6	...	...	...	24	...	...	1838	...	3	6	6	2	1	8	2	2
1770	5	1	5	3	2	8	4	2	...	1839	1	2	6	6	1	6	5	...	3
1771	2	...	1	...	5	17	3	2	...	1840	2	2	...	4	1	5	8	4	4
1772	3	1	2	4	5	8	6	1	...	1841	2	...	...	2	2	3	11	1	9
1773	1	...	1	2	4	12	9	1	...	1842	1	4	5	5	...	4	6	...	5
1774	3	1	4	4	...	4	10	3	1	1843	...	1	1	2	2	10	11	2	1
1775	4	3	6	5	2	6	2	2	...	1844	...	...	9	2	...	5	7	1	6
1776	...	2	1	4	7	11	3	2	...	1845	...	..	2	6	1	3	14	2	2
1777	...	...	...	...	1	3	26	...	...	1846	2	1	1	5	4	4	8	1	4
1778	1	2	5	2	3	5	7	...	5	1847	2	...	...	...	2	8	13	1	4
1779	4	4	1	...	1	3	8	2	7	1848	2	...	1	...	1	4	16	4	2
1780	1	3	2	...	...	4	15	3	2	1849	1	...	...	2	2	10	8	1	6
1781	...	...	7	...	...	...	23	...	...	1850	3	...	1	1	4	4	17	...	...
1782	...	...	19	...	...	...	11	...	...	1851	3	...	...	.	1	4	15	7	...
1783	...	...	10	...	...	...	20	...	...	1852	2	4	1	2	2	8	9	2	...
1784	1	1	3	1	...	9	12	3	...	1853	2	...	2	2	7	5	5	7	...
1785	1	1	3	...	...	2	19	4	...	1854	3	5	2	...	1	8	6	5	...
1786	8	3	9	4	1	1	3	1	...	1855	3	4	4	2	3	7	...	2	5
1787	14	...	3	1	1	1	6	4	...	1856	5	...	...	...	2	3	8	9	3
1788	1	1	1	5	2	7	11	2	...	1857	1	2	6	3	3	4	3	1	7
1789	3	...	8	1	...	11	7	...	...	1858	...	1	2	3	5	4	14	1	...
1790	3	1	11	3	1	3	7	1	...	1859	...	...	1	1	5	10	12	1	...
1791	3	2	5	1	2	5	11	1	...	1860	...	...	9	10	4	1	4	1	1
1792	1	2	3	...	...	11	8	5	...	1861	4	3	2	1	2	7	10	1	...
1793	2	...	20	1	...	2	3	2	...	1862	1	...	...	3	2	14	5	...	5
1794	1	...	7	6	..	7	7	2	...	1863	...	1	3	1	2	4	16	...	3
1795	7	...	3	1	6	4	4	5	...	1864	3	1	7	2	2	3	8	2	2
1796	3	1	6	...	2	2	16	...	...	1865	2	2	5	3	1	5	8	4	...
1797	...	...	10	...	...	1	17	2	...	1866	1	...	...	1	1	4	18	5	...
1798	...	...	9	..	...	...	19	2	...	1867	4	1	...	...	2	4	14	5	...
1799	...	...	4	1	...	...	24	1	...	1868	4	1	6	4	2	2	7	4	...
1800	...	...	1	...	...	1	28	...	...	1869	1	...	...	...	...	1	12	16	...
1801	...	2	3	...	...	...	20	5	...	1870	...	...	3	...	1	4	18	4	...
1802	...	...	16	...	...	...	14	...	...	1871	5	3	6	2	...	2	7	5	...
1803	...	...	10	...	...	...	19	...	1	1872	...	5	5	1	2	8	7	2	...
1804	2	2	6	8	1	3	4	...	4	1873	2	2	6	1	1	2	13	3	...
1805	2	...	1	...	1	8	13	2	3	1874	2	2	4	1	2	1	12	4	2
1806	...	1	3	1	1	6	16	1	1	1875	4	4	5	3	2	...	7	3	2
1807	2	3	4	...	...	...	15	6	...	1876	2	2	9	2	1	4	8	1	1
1808	...	3	6	2	1	4	9	...	5	1877	...	...	..	2	3	14	10	1	...
1809	3	...	12	...	...	...	15	...	...	1878	4	4	3	...	...	4	8	7	...
1810	1	7	10	...	1	1	9	1	...	1879	1	4	5	...	...	1	15	4	...
1811	...	...	3	...	...	14	10	3	...	1880	2	2	...	...	2	8	12	4	...
1812	1	2	5	7	...	8	3	4	...	1881	...	...	...	3	2	10	12	3	...
1813	...	...	...	4	3	10	8	5	...	1882	2	...	3	1	2	8	9	4	1
1814	...	4	1	1	2	4	15	3	...	1883	2	...	1	...	3	5	16	2	1
1815	2	1	...	2	1	10	11	3	...	1884	2	2	...	3	1	8	6	7	1
1816	3	3	1	2	...	8	6	6	1	1885	...	2	4	6	1	8	5	2	2
1817	...	...	1	4	6	9	7	3	...	1886	3	...	1	...	...	12	10	4	...
1818	1	8	9	2	8	2	...	...	...	1887	1	2	4	1	3	8	4	3	4
1819	3	4	1	...	...	5	13	4	...	1888	...	2	9	3	2	1	12	...	1
1820	1	1	5	1	2	4	10	6	...	1889	1	2	2	1	1	4	18	...	1
1821	1	1	2	3	1	6	16	...	...	1890	...	...	3	2	3	2	17	1	2
1822	...	...	...	3	4	17	5	...	1	1891	1	...	6	1	...	2	13	2	5
1823	...	1	2	1	2	7	13	...	4	1892	...	...	4	5	5	3	10	...	3
1824	1	2	...	1	1	6	16	3	...	1893	5	3	3	...	2	2	9	4	2
1825	3	1	2	...	1	7	10	4	2	1894	...	...	...	1	3	10	12	2	2
1826	2	...	1	1	..	6	9	7	4	1895	2	3	6	2	2	2	12	...	1
1827	1	...	3	...	2	5	12	5	2	1896	2	...	4	2	1	3	15	2	1

TABLE XXXII.—*continued.*

DECEMBER.

Year.	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm or Var.	Year.	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm or Var.
1731	3	...	1	2	2	4	15	4	...	1828	...	...	5	1	...	16	4	...	5
1732	3	...	4	5	6	6	5	2	...	1829	1	2	2	5	4	6	2	1	8
1733	...	...	...	...	4	16	10	1	...	1830	3	2	4	7	...	4	5	3	3
1734	...	1	1	2	10	10	5	2	...	1831	...	...	...	2	3	14	10	...	2
1735	2	3	1	3	10	1	8	3	...	1832	...	...	1	3	...	13	11	2	1
1764	...	...	18	...	...	...	13	...	...	1833	1	...	2	..	...	6	20	1	1
1765	...	...	14	...	...	...	17	...	...	1834	...	...	2	1	...	5	23	...	...
1766	...	...	15	...	...	..	15	...	...	1835	2	1	7	...	...	4	16	1	...
1767	...	...	13	...	...	...	18	...	...	1836	3	4	3	1	...	5	12	3	...
1768	...	...	12	...	...	...	19	...	...	1837	4	3	3	...	...	12	6	3	...
1769	...	...	4	...	...	...	27	...	...	1838	...	1	...	2	1	13	12	1	1
1770	4	...	...	...	3	15	7	2	...	1839	...	...	4	8	3	5	6	...	5
1771	...	2	2	4	9	7	6	1	...	1840	...	3	3	1	5	4	8	3	4
1772	1	...	4	...	5	14	5	1	1	1841	1	...	2	...	...	8	16	1	3
1773	3	4	8	4	...	6	5	1	...	1842	...	...	1	1	...	10	17	...	2
1774	1	1	4	4	5	9	4	2	1	1843	...	...	...	...	...	11	18	1	1
1775	...	1	1	...	8	12	8	1	..	1844	...	...	8	14	1	2	4	...	2
1776	...	3	2	2	5	10	6	1	2	1845	4	...	...	...	...	6	16	4	1
1777	2	2	5	...	1	8	10	3	...	1846	7	2	...	...	...	3	13	5	1
1778	...	...	4	2	1	3	20	1	...	1847	...	...	2	7	4	5	9	2	2
1779	1	...	...	3	1	4	10	5	7	1848	...	...	1	6	4	6	11	...	3
1780	2	3	7	1	3	4	10	1	...	1849	2	2	5	8	...	1	9	3	1
1781	...	...	13	...	...	...	18	...	...	1850	...	...	...	1	8	18	4	...	...
1782	...	...	13	...	...	...	18	...	...	1851	1	...	...	1	3	22	4	...	...
1783	...	...	17	...	...	...	14	...	...	1852	2	2	3	2	2	13	7	...	...
1784	7	11	8	...	...	...	4	1	...	1853	3	3	2	6	5	8	2	2	...
1785	7	5	4	4	1	3	5	2	...	1854	1	...	...	...	...	8	16	6	...
1786	8	2	4	...	...	7	9	1	...	1855	1	2	...	3	2	14	5	3	1
1787	12	1	7	...	1	4	5	1	...	1856	4	1	1	...	4	7	8	4	2
1788	11	4	4	1	...	1	6	4	...	1857	1	...	...	...	4	14	8	2	2
1789	...	...	2	1	1	13	12	2	...	1858	...	...	...	..	6	16	8	1	...
1790	6	1	...	1	...	6	8	9	...	1859	3	3	1	3	3	11	5	1	1
1791	9	...	1	...	...	4	12	5	...	1860	4	1	5	4	6	...	5	2	4
1792	3	1	2	1	2	8	12	2	...	1861	3	1	...	...	2	7	11	1	6
1793	...	...	8	1	...	7	12	3	...	1862	...	...	4	2	2	10	6	5	2
1794	1	...	9	1	2	14	1	3	...	1863	2	...	1	...	2	15	10	1	...
1795	1	1	1	2	5	5	16	...	...	1864	...	1	10	3	2	4	11	...	...
1796	2	1	6	...	...	1	17	4	...	1865	...	1	4	4	1	7	12	2	...
1797	...	1	3	...	...	5	19	3	...	1866	1	1	...	1	4	10	11	3	...
1798	...	1	15	1	...	2	12	...	...	1867	...	...	2	1	2	10	12	4	...
1799	...	2	20	3	...	...	6	...	...	1868	1	2	4	4	6	5	7	2	...
1800	1	...	10	...	...	1	19	...	...	1869	4	1	3	...	2	3	15	3	...
1801	3	2	8	...	...	...	13	5	...	1870	3	...	11	...	...	3	10	4	...
1802	...	...	8	...	...	...	23	...	...	1871	3	...	3	1	1	7	15	1	...
1803	...	...	15	...	...	...	16	...	...	1872	1	...	4	6	12	...	7	1	...
1804	...	2	12	1	2	3	6	...	5	1873	2	...	...	...	2	5	19	3	...
1805	6	1	...	1	...	6	13	4	...	1874	5	3	2	1	...	2	11	3	4
1806	2	2	...	...	...	17	5	2	3	1875	...	...	4	...	1	2	20	3	1
1807	1	2	6	...	...	...	21	1	...	1876	...	2	11	5	2	3	6	1	1
1808	6	...	4	5	...	2	8	4	2	1877	...	1	...	4	2	4	18	2	...
1809	2	1	2	...	...	2	21	3	...	1878	2	2	2	1	...	2	20	2	...
1810	2	3	...	1	...	4	19	2	...	1879	...	1	...	...	2	7	20	1	...
1811	2	...	2	1	1	6	14	5	...	1880	...	3	1	1	1	...	15	9	1
1812	...	6	2	11	8	4	...	...	...	1881	...	1	2	1	2	6	17	2	...
1813	...	7	...	3	2	13	6	...	...	1882	...	5	3	2	1	3	13	4	...
1814	2	2	4	6	...	11	2	4	...	1883	3	4	...	...	...	3	18	3	...
1815	1	...	1	5	2	6	9	7	...	1884	1	2	...	1	4	5	14	3	1
1816	1	1	3	5	2	9	8	2	...	1885	...	3	2	1	...	4	14	6	1
1817	4	...	1	4	3	9	2	8	...	1886	1	4	...	2	2	5	10	7	...
1818	1	2	2	4	4	9	3	6	...	1887	1	...	1	...	2	5	15	7	...
1819	1	4	4	3	...	5	9	5	...	1888	1	...	1	1	3	4	14	...	7
1820	...	5	9	4	...	5	7	...	1	1889	...	...	2	...	3	4	18	1	3
1821	...	1	1	2	4	6	15	1	1	1890	3	...	12	3	2	...	7	...	4
1822	...	...	4	6	3	11	4	...	3	1891	1	...	1	...	2	6	15	1	5
1823	...	...	1	1	1	12	10	3	3	1892	2	2	3	2	1	2	12	2	5
1824	...	1	...	...	...	13	13	4	...	1893	..	1	2	1	2	11	10	2	2
1825	1	1	4	5	1	9	1	4	5	1894	1	...	2	2	1	4	15	4	2
1826	...	...	...	6	2	8	8	2	5	1895	...	...	5	6	1	1	14	2	2
1827	...	2	...	...	2	17	8	1	1	1896	1	3	6	2	1	7	6	4	1

TABLE XXXII.—*continued.*

ANNUAL VALUES.

Year.	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm or Var.
1732	18	27	50	44	31	60	98	38	...
1733	15	25	48	25	28	110	86	28	..
1734	12	24	36	21	38	91	109	34	...
1735	18	33	36	33	35	60	116	34	...
1764	...	...	137	...	...	...	229	...	...
1765	...	...	137	...	...	...	225	...	3
1766	.	...	131	...	...	...	230	...	...
1767	...	...	131	...	...	...	234	...	...
1768	...	...	169	...	...	...	187	...	...
1769	...	...	125	...	...	...	240	...	...
1770	37	28	52	8	25	101	86	28	...
1771	33	28	45	15	37	97	86	24	...
1772	31	34	66	21	35	71	90	14	4
1773	28	28	40	18	47	80	118	5	1
1774	26	31	43	26	45	69	95	21	9
1775	18	25	43	23	48	91	98	16	3
1776	14	40	42	26	38	95	85	15	11
1777	14	34	55	8	25	61	131	22	15
1778	16	27	51	12	37	63	121	11	27
1779	9	19	51	8	22	74	111	23	48
1780	17	21	61	21	24	66	117	16	23
1781	...	...	145	...	...	...	212	...	8
1782	...	...	154	...	...	...	211	...	...
1783	..	...	125	...	..	...	240	...	...
1784	30	37	55	21	11	49	129	34	...
1785	45	30	45	19	9	54	125	38	...
1786	28	21	93	13	1	49	130	30	...
1787	37	11	76	17	11	51	133	29	...
1788	29	17	71	23	8	62	131	25	...
1789	33	13	88	20	6	95	85	25	...
1790	19	10	70	24	4	61	140	37	...
1791	27	16	76	18	14	63	113	38	...
1792	30	14	104	13	7	78	90	30	...
1793	12	5	88	17	5	83	116	39	...
1794	16	3	78	16	9	87	115	41	...
1795	42	21	79	24	24	54	94	27	...
1796	17	8	59	16	12	39	198	17	...
1797	3	1	94	6	1	24	206	30	...
1798	1	6	7.	3	...	20	245	13	...
1799	1	7	133	13	1	16	176	18	...
1800	3	11	123	1	3	19	190	15	...
1801	7	9	100	4	...	2	221	22	...
1802	...	...	96	...	...	...	266	...	3
1803	...	..	102	...	...	...	262	...	1
1804	20	22	76	20	13	68	84	18	45
1805	19	27	45	29	12	61	98	34	40
1806	17	28	61	15	11	67	113	27	26
1807	10	17	92	...	...	...	229	17	...
1808	24	23	73	22	9	45	117	27	26
1809	9	17	148	2	...	3	171	15	...
1810	5	25	88	18	1	24	177	27	...
1811	13	31	56	30	16	91	101	27	...
1812	19	52	61	34	20	85	66	28	1
1813	5	66	40	28	18	124	56	27	1
1814	7	43	49	42	13	109	68	34	...
1815	10	43	31	37	16	113	92	23	...
1816	17	42	57	51	18	49	82	47	3
1817	20	31	26	46	30	77	87	43	5
1818	12	48	49	40	43	79	77	17	...
1819	13	29	57	19	18	81	104	44	...
1820	19	31	73	12	12	53	134	30	2
1821	11	15	68	31	7	51	147	15	20
1822	6	14	61	29	21	87	106	12	29
1823	15	14	60	26	18	79	114	19	22
1824	16	33	48	12	7	70	122	35	23
1825	13	14	56	21	16	80	87	32	46
1826	5	14	48	27	17	101	94	29	30
1827	11	32	63	14	16	78	99	34	18
1828	18	35	58	36	16	88	74	15	26
1829	25	47	49	30	12	49	70	37	46
1830	10	42	35	29	10	98	71	32	38
1831	3	40	43	27	11	101	71	35	34
1832	8	37	37	20	7	90	100	27	40
1833	8	8	70	19	13	60	151	8	28
1834	2	8	82	7	11	66	157	14	18
1835	10	10	86	21	5	44	174	15	...
1836	16	39	44	23	8	53	146	37	...
1837	29	39	44	23	25	71	78	46	10
1838	21	32	62	24	15	59	85	32	35
1839	10	32	48	37	21	72	87	24	35
1840	12	37	44	23	9	59	107	40	35
1841	12	26	53	28	10	63	110	27	36
1842	9	44	41	28	7	70	101	32	33
1843	12	30	59	18	10	69	110	34	23
1844	9	18	73	33	8	48	119	28	30
1845	14	28	55	25	12	56	112	37	26
1846	15	23	51	32	17	59	116	19	33
1847	14	20	61	37	14	53	108	27	31
1848	10	24	53	23	18	55	128	20	35
1849	11	28	58	26	9	51	123	24	35
1850	13	12	62	21	41	67	119	30	...
1851	14	16	40	15	45	93	121	21	...
1852	46	46	41	13	23	45	127	20	5
1853	26	18	57	40	42	68	77	37	...
1854	17	19	23	16	29	89	124	42	6
1855	22	38	44	19	23	59	92	37	31
1856	25	39	50	18	43	62	58	47	24
1857	16	38	56	9	20	82	87	25	32
1858	26	26	43	14	28	83	94	41	10
1859	9	22	49	17	34	74	139	18	3
1860	18	23	64	24	44	46	130	11	6
1861	15	23	44	17	33	72	137	15	9
1862	8	15	40	28	35	62	116	24	37
1863	4	22	37	20	26	93	109	30	24
1864	8	11	91	29	24	62	113	16	12
1865	14	15	78	49	20	63	100	26	...
1866	14	27	58	36	28	54	118	30	...
1867	34	26	52	15	35	54	127	22	...
1868	29	33	32	18	54	95	90	15	...
1869	34	32	39	10	44	36	126	44	...
1870	22	33	48	28	8	33	101	81	11
1871	19	35	54	36	15	52	115	31	8
1872	16	35	62	28	38	55	111	20	1
1873	14	24	70	12	8	57	156	21	3
1874	25	19	49	12	17	35	158	34	16
1875	15	28	87	14	12	21	137	26	26
1876	35	28	76	19	17	19	113	39	20
1877	7	20	56	38	30	45	143	24	2
1878	14	30	67	26	18	34	147	26	3
1879	9	38	68	36	12	32	155	12	3
1880	6	41	81	11	9	36	141	37	4
1881	2	48	46	33	17	39	120	53	7
1882	12	20	45	20	22	68	130	40	8
1883	23	27	40	20	24	52	137	36	6
1884	11	26	30	25	21	70	120	41	22
1885	4	49	41	26	8	71	92	63	11
1886	18	48	48	22	23	54	106	32	14
1887	12	20	32	11	27	59	113	64	27
1888	14	29	72	17	18	35	119	34	28
1889	18	22	75	20	21	31	148	17	13
1890	14	6	79	18	23	25	169	14	17
1891	26	14	60	13	24	30	126	28	44
1892	14	17	67	19	19	29	153	20	28
1893	23	18	59	11	17	31	148	22	36
1894	18	20	63	26	17	45	132	26	18
1895	38	25	64	21	12	17	145	23	20
1896	38	24	52	10	8	26	160	36	12

Table XXXIII.

Direction of the Wind. Mean Monthly Percentages, 1764–1896.

	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm or Var.		N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm or Var.
Jan.	4·1	4·7	13·1	6·2	5·9	18·5	37·1	7·2	3·2	Aug.	2·6	5·5	17·7	3·8	3·8	14·7	40·8	6·5	4·6
Feb.	4·6	4·8	13·2	6·0	5·0	19·4	36·2	7·8	3·0	Sept.	3·8	5·7	17·3	5·2	5·4	15·2	36·7	6·0	4·7
Mar.	6·4	7·2	17·6	6·0	4·5	14·6	32·0	8·8	2·9	Oct.	3·8	4·5	14·0	7·0	6·2	15·8	38·4	7·1	3·2
April	5·3	10·5	24·6	5·6	3·8	11·1	27·8	8·3	3·0	Nov.	5·2	4·3	13·2	6·0	5·3	16·6	37·6	7·8	4·0
May	3·8	11·0	29·5	5·4	3·6	10·4	27·2	5·9	3·2	Dec.	5·0	4·1	12·8	6·2	5·6	20·0	35·8	6·9	3·5
June	3·7	9·1	24·1	3·9	3·7	12·3	33·2	6·4	3·6										
July	3·2	6·8	19·6	3·7	4·4	13·9	38·6	6·2	3·6	Mean.	4·3	6·5	18·1	5·4	4·7	15·1	35·3	7·1	3·5

Table XXXIV.

Percentage Frequency of Wind Direction Decennial Means. East Wind includes N., N.E., E., S.E.; West Wind, S., S.W., W., N.W. Calms and Variables have been excluded.

Year.	January.		February.		March.		April.		May.		June.	
	E.	W.	E.	W.	E.	W.	E.	W.	E.	W.	E.	W.
1764–69, . . .	33·9	66·1	35·9	64·1	42·5	57·5	51·7	48·3	49·5	50·5	46·5	53·5
1770–79, . . .	34·8	65·2	23·2	76·8	40·7	59·3	31·0	69·0	50·0	50·0	43·2	56·8
1780–89, . . .	36·6	63·4	43·0	57·0	51·6	48·4	44·7	55·3	39·6	60·4	45·3	54·7
1790–99, . . .	25·8	74·2	28·7	71·3	40·6	59·4	53·3	46·7	40·7	59·3	38·7	61·3
1800–09, . . .	36·8	63·2	28·7	71·3	44·7	55·3	41·0	59·0	46·8	53·2	34·4	65·6
1810–19, . . .	22·9	77·1	19·9	80·1	33·6	66·4	53·7	46·3	59·4	40·6	45·5	54·5
1820–29, . . .	34·6	65·4	29·6	70·4	32·2	67·8	40·5	59·5	51·9	38·1	40·9	59·1
1830–39, . . .	29·6	70·4	23·6	76·4	32·9	67·1	48·4	51·6	59·6	40·4	40·4	59·6
1840–49, . . .	25·3	74·7	31·9	68·1	34·9	65·1	51·6	48·4	49·2	40·8	44·7	55·3
1850–59, . . .	28·8	71·2	29·3	70·7	34·8	65·2	48·5	51·5	53·9	46·1	36·0	64·0
1860–69, . . .	25·8	74·2	25·0	75·0	39·8	60·2	44·4	55 6	49·3	50·7	39·3	60·7
1870–79, . . .	25·4	74·6	37·8	62·2	40·1	59·9	54·4	45·6	51·5	48·5	46·3	53·7
1880–89, . . .	21·4	78·6	26·8	73·2	35·8	64·2	54·8	45·2	45·7	54·3	48·4	51·6
1890–96, . . .	26·8	73·2	33·7	66·3	30·2	69·8	51·0	49·0	53·9	46·1	48·0	52·0
Means.												
1764–1896, . .	29·1	70·9	29·5	70·5	38·3	61·7	47·5	52·5	51·4	48·6	42·3	57·7

Year.	July.		August.		September.		October.		November.		December.		Year.	
	E.	W.	E.	W.	E.	W.	E.	W.	E.	W.	E.	W.	E.	W.
1764–69, . . .	43·5	56·5	36·2	63·8	27·8	72·2	25·3	74·7	26·1	73·9	41·5	58·5	38·2	61·8
1770–79, . . .	31·9	68·1	26·3	73·7	36·4	63·6	24·9	75·1	30·3	69·7	24·7	75·3	33·2	66·8
1780–89, . . .	24·0	76·0	34·7	65·3	32·0	68·0	28·0	72·0	38·6	61·4	51·3	48·7	39·1	60·9
1790–99, . . .	26·1	73·9	25·5	74·5	32·7	67·3	26·0	74·0	39·0	61·0	33·9	66·1	34·7	65·3
1800–09, . . .	41·8	58·2	28·1	71·9	22·9	67·1	32·9	67·1	32·5	67·5	34·3	65·7	36·2	63·8
1810–19, . . .	38·1	61·9	37·9	62·1	38·1	61·9	44·2	55·8	31·1	68·9	32·6	67·4	38·2	61·8
1820–29, . . .	46·2	53·8	31·5	68·5	31·9	68·1	31·1	68·9	21·6	78·4	25·2	74·8	35·7	64·3
1830–39, . . .	36·9	63·1	35·1	64·9	36·3	63·7	19·4	80·6	28·5	71·5	24·9	75·1	34·7	65·3
1840–49, . . .	30·9	69·1	27·8	72·2	42·7	57·3	35·4	64·6	26·1	73·9	27·6	72·4	36·5	63·5
1850–59, . . .	34·1	65·9	23·8	76·2	30·9	69·1	25·3	74·7	24·9	75·1	16·4	83·6	32·1	67·9
1860–69, . . .	39·6	60·4	28·1	71·9	24·6	75·4	28·8	71·2	29·8	70·2	25·2	74·8	33·3	66·7
1870–79, . . .	31·3	68·7	40·8	59·2	37·2	62·8	30·1	69·9	35·3	64·7	26·3	73·7	38·0	62·0
1880–89, . . .	27·6	72·4	30·6	69·4	33·3	66·7	34·3	65·7	23·2	76·8	16·8	83·2	33·2	66·8
1890–96, . . .	38·4	61·6	29·1	70·9	30·5	69·5	29·4	70·6	28·4	71·6	30·7	69·3	36·0	64·0
Means.														
1764–1896, . .	34·6	65·4	31·0	69·0	33·5	66·5	30·3	69·7	29·9	70·1	28·4	71·6	35·6	64·4

Table XXXV.

Showing the Mean Annual Percentage Frequency of East (N., N.E., E., S.E.) and West Winds (S., S.W., W., N.W.) from 1764 to 1896.

Year.	Direction.		Year.	Direction.		Year.	Direction.	
	E.	W.		E.	W.		E.	W.
1764	37·4	62·6	1811	35·6	64·4	1858	30·7	69·3
1765	37·8	62·2	1812	45·5	54·5	1859	26·8	73·2
1766	36·3	63·7	1813	38·2	61·8	1860	35·8	64·2
1767	35·9	64·1	1814	38·6	61·4			
1768	47·5	52·5	1815	33·2	66·8			
1769	34·2	65·8	1816	46·0	54·0	1861	27·8	72·2
1770	34·2	65·8	1817	34·2	65·8	1862	27·7	72·3
			1818	40·8	59·2	1863	24·3	75·7
1771	33·2	66·8	1819	32·3	67·7	1864	39·3	60·7
1772	42·0	58·0	1820	37·1	62·9	1865	42·7	57·3
1773	31·3	68·7				1866	37·0	63·0
1774	35·4	64·6	1821	36·2	43·8	1867	34·8	65·2
1775	30·1	69·9	1822	32·8	67·2	1868	30·6	69·4
1776	34·4	65·6	1823	33·5	66·5	1869	31·5	68·5
1777	31·7	68·3	1824	31·8	68·2	1870	37·0	63·0
1778	31·4	68·6	1825	32·6	67·4			
1779	27·5	72·5	1826	28·1	71·9			
1780	35·0	65·0	1827	34·6	65·4	1871	40·3	59·7
			1828	43·2	56·8	1872	38·6	61·4
1781	40·6	59·4	1829	47·3	72·7	1873	33·2	66·8
1782	42·5	57·5	1830	35·5	64·5	1874	30·0	70·0
1783	34·3	65·7				1875	42·4	57·6
1784	39·1	60·9	1831	34·1	65·9	1876	45·7	54·3
1785	38·1	61·9	1832	31·3	68·7	1877	33·3	66·7
1786	42·5	57·5	1833	31·2	68·8	1878	37·8	62·2
1787	38·6	61·4	1834	28·5	71·5	1879	41·7	58·3
1788	38·3	61·7	1835	34·8	65·2	1880	38·4	61·6
1789	41·9	58·1	1836	33·3	66·7			
1790	33·7	66·3	1837	38·0	62·0			
			1838	42·1	57·9	1881	36·0	64·0
1791	37·5	62·5	1839	38·4	61·6	1882	27·2	72·8
1792	43·8	56·2	1840	35·1	64·9	1883	30·6	69·4
1793	33·4	66·6				1884	26·7	73·3
1794	31·0	69·0	1841	36·0	64·0	1885	33·9	66·1
1795	45·5	54·5	1842	36·7	63·3	1886	38·7	61·3
1796	27·3	72·7	1843	34·8	65·2	1887	22·2	77·8
1797	28·5	71·5	1844	39·6	60·4	1888	39·1	60·9
1798	23·8	76·2	1845	36·0	64·0	1889	38·4	61·6
1799	42·2	57·8	1846	36·4	63·6	1890	33·6	66·4
1800	37·8	62·2	1847	39·5	60·5			
			1848	33·2	66·8			
1801	32·9	67·1	1849	37·3	62·7	1891	35·2	64·8
1802	26·5	73·5	1850	29·6	70·4	1892	34·6	65·4
1803	28·0	72·0				1893	33·7	66·3
1804	43·0	57·0	1851	23·3	76·7	1894	36·6	63·4
1805	36·9	63·1	1852	40·4	59·6	1895	42·9	57·1
1806	35·7	64·3	1853	38·6	61·4	1896	35·0	65·0
1807	32·6	67·4	1854	20·9	79·1			
1808	41·8	58·2	1855	36·8	63·2			
1809	48·2	51·8	1856	38·6	61·4	Means.		
1810	37·3	62·7	1857	35·7	64·3	1764–1896.	35·6	64·4

TABLE XXXVI.

Days with Thunderstorms.

Year.	Jan.	Feb.	Mar.	April.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.	Year.
1770, . .	...	...	...	...	...	2	...	...	1	...	...	...	3
1771, . .	1	...	...	...	1	1	...	...	...	...	...	...	3
1772, . .	...	...	...	...	...	2	2	2	1	...	...	...	7
1773, . .	...	1	...	...	...	...	...	...	...	...	...	...	1
1774, . .	...	...	...	...	...	1	...	1	1	...	...	...	3
1775, . .	...	...	...	...	...	1	3	1	...	...	...	...	5
1776, . .	...	...	...	...	...	2	3	1	...	...	...	...	6
1777, . .	...	...	...	...	...	1	...	...	2	...	...	...	3
1778, . .	...	...	...	...	1	3	1	...	...	...	...	...	5
1779, . .	...	...	...	...	1	1	2	2	1	1	...	...	8
1780, . .	...	...	...	...	...	...	...	...	...	...	1	...	1
1781, . .	...	...	...	...	1	2	...	3	...	...	...	...	6
1782, . .	...	...	...	...	...	2	1	...	...	...	...	...	3
1783, . .	...	...	...	...	...	...	4	3	1	...	...	...	8
1784, . .	...	...	...	...	...	...	...	...	1	...	...	...	1
1785, . .	...	...	...	...	...	...	3	...	1	...	...	...	4
1786, . .	...	...	...	...	...	...	1	...	1	...	...	...	2
1787, . .	...	...	...	...	1	...	3	1	1	...	...	...	6
1788, . .	...	...	...	2	...	...	...	1	1	...	...	...	4
1789, . .	...	...	...	...	2	1	6	1	...	...	...	...	10
1790, . .	...	...	...	...	1	2	2	...	...	...	...	...	5
1791, . .	...	...	...	1	...	1	...	...	...	...	...	...	2
1792, . .	...	...	...	...	...	2	1	1	...	...	...	...	4
1793, . .	...	...	...	...	...	1	2	...	...	...	...	...	3
1794, . .	...	...	...	1	3	...	...	1	...	...	...	...	5
1795, . .	...	...	...	...	...	...	...	3	...	...	...	...	3
1796, . .	...	...	...	...	...	...	...	1	...	...	...	...	1
1797, . .	...	...	...	...	1	1	2	...	1	...	...	...	5
1798, . .	...	...	...	...	1	4	4	...	1	...	...	...	10
1799, . .	...	...	...	...	...	1	2	...	...	...	1	...	4
1800, . .	...	...	...	1	1	...	...	...	2	...	...	...	4
1801, . .	...	...	...	...	1	...	...	...	...	...	...	...	1
1802, . .	...	...	...	...	...	1	1	1	...	...	...	...	3
1803, . .	...	...	...	1	...	...	2	...	...	...	...	...	3
1804, . .	...	...	...	...	...	1	1	2	...	...	...	...	4
1805, . .	...	...	...	...	...	...	2	2	...	...	...	...	4
1806, . .	...	...	...	...	...	...	3	2	...	...	...	...	5
1807, . .	...	...	...	1	...	...	2	2	...	...	...	...	5
1808, . .	...	...	...	...	4	...	5	1	1	...	...	...	11
1809, . .	...	...	...	...	2	1	1	5	...	...	...	...	9
1810, . .	...	...	...	...	...	...	...	3	1	...	...	...	4
1811, . .	...	...	...	...	2	2	...	1	...	...	...	...	5
1812, . .	...	...	...	...	...	1	...	3	...	...	...	...	4
1813, . .	...	...	...	...	1	1	4	...	...	...	...	...	6
1814, . .	...	...	...	...	...	...	2	...	...	...	...	...	2
1815, . .	...	...	...	...	2	2	...	1	...	...	...	...	5
1816, . .	...	1	...	...	1	1	3	1	...	...	...	...	7
1817, . .	...	1	...	...	2	3	2	2	...	...	...	...	10
1818, . .	1	...	...	...	...	2	2	...	2	...	...	...	7
1819, . .	...	...	...	...	...	...	2	1	...	...	...	...	3
1820, . .	...	...	...	...	5	...	1	2	...	...	...	...	8

TABLE XXXVI.—*continued.*

Year.	Jan.	Feb.	Mar.	April.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.	Year.
1821, . .	...	...	...	...	2	...	2	2	...	...	...	...	6
1822, . .	...	...	1	2	2	3	5	...	...	...	...	...	13
1823, . .	...	...	...	...	1	2	1	...	...	...	...	...	4
1824, . .	...	...	...	...	...	...	1	2	1	...	...	...	4
1825, . .	1	...	...	1	...	3	1	...	3	...	...	1	10
1826, . .	...	...	...	1	1	1	2	1	...	...	...	...	6
1827, . .	2	...	...	...	1	...	4	...	...	...	...	1	8
1828, . .	...	...	...	...	1	...	3	5	2	...	...	...	11
1829, . .	...	...	...	...	...	5	4	...	...	...	...	...	9
1830, . .	...	...	...	1	...	2	1	1	...	...	1	...	6
1831, . .	...	...	...	...	1	1	2	8	...	...	...	...	12
1832, . .	...	...	...	...	...	4	1	...	...	1	...	...	6
1833, . .	...	...	...	2	2	1	2	...	...	...	...	1	8
1834, . .	...	...	...	...	...	1	...	...	...	...	...	...	1
1835, . .	...	...	...	...	...	4	1	3	...	...	...	...	8
1836, . .	...	...	...	2	...	1	2	...	1	...	...	...	6
1837, . .	...	1	...	...	3	...	5	3	...	1	...	...	13
1838, . .	...	...	...	...	...	1	3	...	...	...	...	...	4
1839, . .	...	...	...	...	...	...	2	1	3	...	...	...	6
1840, . .	...	...	...	...	1	1	1	...	...	...	...	...	3
1841, . .	1	...	1	...	...	2	...	...	1	...	...	...	5
1842, . .	...	...	...	...	3	1	...	1	1	...	...	...	6
1843, . .	...	...	1	...	...	...	3	1	...	...	...	...	5
1844, . .	...	...	...	...	...	2	...	...	...	...	...	...	2
1845, . .	...	...	...	...	1	1	1	2	...	...	...	...	5
1846, . .	...	...	...	...	1	6	2	3	...	...	...	...	12
1847, . .	...	...	...	...	1	1	2	...	1	...	...	...	5
1848, . .	...	...	...	...	...	3	...	...	...	...	...	...	3
1849, . .	...	...	...	...	...	2	4	5	...	...	...	...	11
1850, . .	...	...	...	...	1	...	1	...	...	...	1	...	3
1851, . .	...	...	...	1	...	...	...	1	...	...	...	...	2
1852, . .	1	...	...	...	...	7	3	2	...	...	...	1	14
1853, . .	2	...	...	1	1	3	...	...	...	...	...	...	7
1854, . .	1	1	...	...	6	...	1	...	...	...	...	...	9
1855, . .	...	...	...	...	...	2	3	1	...	...	...	...	6
1856, . .	...	...	...	...	1	...	1	1	1	...	...	...	4
1857, . .	...	...	...	1	1	...	2	2	1	...	...	...	7
1858, . .	...	...	...	...	...	4	1	1	...	...	...	...	6
1859, . .	...	...	...	...	...	2	...	...	...	...	...	...	2
1860, . .	...	...	...	...	2	3	1	...	...	2	...	...	8
1861, . .	...	...	...	...	...	...	5	...	...	1	...	...	6
1862, . .	1	...	...	...	1	...	...	...	2	...	...	...	4
1863, . .	...	1	...	1	...	1	1	...	...	1	...	...	5
1864, . .	...	...	...	...	1	3	...	...	...	...	...	...	4
1865, . .	...	...	...	...	...	...	2	...	...	...	...	...	2
1866, . .	...	...	...	...	...	...	2	2	...	...	...	...	4
1867, . .	...	...	...	...	3	2	2	1	2	...	...	...	10
1868, . .	...	1	...	1	2	2	...	3	2	...	1	...	12
1869, . .	...	...	...	1	...	1	1	...	1	...	...	...	4
1870, . .	...	...	...	...	...	1	1	1	...	...	...	...	3
1871, . .	...	...	...	1	...	3	4	1	...	...	...	...	9
1872, . .	...	...	...	1	4	2	4	3	4	2	...	...	20
1873, . .	...	...	1	...	...	1	4	2	1	2	...	...	11
1874, . .	...	...	...	...	1	3	5	4	...	...	...	1	14
1875, . .	...	...	...	...	2	...	2	1	1	...	...	...	6
1876, . .	...	...	...	...	1	2	2	...	1	...	1	...	7
1877, . .	...	...	...	...	...	3	3	4	...	...	...	...	10
1878, . .	...	...	...	...	2	1	1	3	...	...	...	1	8
1879, . .	...	...	...	...	...	4	1	3	1	...	...	...	9
1880, . .	...	...	1	...	...	5	5	1	...	...	...	...	12

TABLE XXXVI.—*continued.*

Year.	Jan.	Feb.	Mar.	April.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.	Year.
1881, . .	...	...	1	1	2	...	2	1	...	...	1	...	8
1882, . .	...	...	...	...	2	3	7	2	...	1	...	...	15
1883, . .	...	...	3	...	1	1	2	2	2	2	...	...	13
1884, . .	...	...	...	2	1	1	6	3	2	...	...	...	15
1885, . .	...	...	...	...	3	...	...	...	...	...	...	1	4
1886, . .	2	...	...	...	1	2	1	1	1	3	...	...	11
1887, . .	...	...	...	...	...	...	3	2	2	...	...	...	7
1888, . .	...	...	...	...	1	2	2	...	...	...	...	...	5
1889, . .	...	...	...	...	1	3	2	3	...	...	...	...	9
1890, . .	...	...	1	2	...	1	1	1	1	...	...	...	7
1891, . .	...	...	...	...	...	...	3	2	...	...	...	...	5
1892, . .	1	1	...	...	...	1	...	1	...	1	...	...	5
1893, . .	...	...	...	...	3	...	8	2	...	...	...	...	13
1894, . .	1	2	1	1	1	2	5	2	...	...	...	...	15
1895, . .	...	...	...	...	4	3	3	5	1	...	...	...	16
1896, . .	...	...	...	1	...	2	...	1	...	...	...	...	4
Totals, . 1770–1896, .	15	10	11	31	103	170	233	154	59	18	7	7	818
Means,	0·12	0·08	0·09	0·24	0·81	1·34	1·83	1·21	0·46	0·14	0·05	0·05	6·44

Decennial Means in Days.

1771-80, .	0·1	0·1	...	...	0·3	**1·2**	1·1	0·7	0·5	0·1	0·1	...	4·2
1781-90, .	...	...	...	0·2	0·5	0·7	**2·0**	0·9	0·6	...	...	...	4·9
1791-1800, .	...	...	...	0·3	0·5	1·0	**1·1**	0·6	0·4	...	0·1	...	4·0
1801-10, .	...	...	...	0·2	0·7	0·3	1·7	**1·8**	0·2	...	...	...	4·9
1811-20, .	0·1	0·2	...	...	1·3	1·2	**1·6**	1·1	0·2	...	...	...	5·7
1821-30, .	0·3	...	0·1	0·5	0·8	1·6	**2·4**	1·1	0·6	...	0·1	0·2	7·7
1831-40, .	...	0·1	...	0·4	0·7	1·4	**1·9**	1·5	0·4	0·2	...	0·1	6·7
1841-50, .	0·1	...	0·2	...	0·7	**1·8**	1·3	1·2	0·3	...	0·1	...	5·7
1851-60, .	0·4	0·1	...	0·3	1·1	**2·1**	1·2	0·8	0·2	0·2	...	0·1	6·5
1861-70, .	0·1	0·2	...	0·3	0·7	1·0	**1·4**	0·7	0·7	0·2	0·1	...	5·4
1871-80, .	...	...	0·2	0·2	1·0	2·4	**3·1**	2·2	0·8	0·4	0·1	0·2	10·6
1881-90, .	0·2	...	0·5	0·5	1·2	1·3	**2·6**	1·5	0·8	0·6	0·1	0·1	9·4

Table XXXVII.

Diurnal Distribution of Thunderstorms.

	Number in Hour ending.																							
	A.M.												P.M.											
	1	2	3	4	5	6	7	8	9	10	11	Noon.	1	2	3	4	5	6	7	8	9	10	11	Mid-night.
January, . .	...	...	1	...	...	...	1	...	...	...	1	2	...	...	..	...	...	...	...	...	1	...	...	1
February, . .	...	...	...	...	...	...	...	...	2	...	...	...	...	...	...	1	...	1	...	1	...	...	...	...
March, . . .	...	...	...	...	...	...	...	1	...	...	...	1	...	3	3	1	1	...	1	1	...	...	...	...
April, . . .	...	...	...	...	...	...	...	...	...	...	1	1	...	2	4	3	2	1	...	2	...	...	...	...
May, . . .	...	...	...	...	...	...	...	...	...	...	1	5	1	8	7	2	1	...	3	4	3	...	...	...
June, . . .	1	1	...	...	...	1	1	...	...	2	5	6	6	9	7	2	4	3	3	...	1	..	...	1
July, . . .	2	...	2	2	1	1	1	...	...	3	2	8	3	14	11	11	4	5	5	2	1	3	..	...
August, . .	...	...	...	...	...	1	...	...	2	1	1	5	7	6	6	6	8	3	2	1	3	1	...	3
September, . .	1	...	...	...	...	...	...	...	...	2	...	1	1	5	3	1	1	...	...	...	...	1	...	...
October, . .	...	...	...	...	...	...	...	...	1	...	...	1	...	1	...	1	...	1	1	1	...	...	...	...
November, . .	...	...	...	...	...	...	...	1	1	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
December, . .	...	...	...	1	...	...	...	...	...	...	...	1	...	...	...	1	...	...	...	...	1	...	1	...
Year, . . .	4	1	3	3	1	3	3	2	6	8	11	31	18	48	41	29	21	14	15	12	10	5	1	5

Lightning without Thunder.

January, . .	...	...	...	...	1	1	...	...	...	...	...	...	...	...	...	...	...	...	2	...	1	1	...	...
February, . .	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	2	...	...	...
March, . . .	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	2	...	...
April, . . .	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	..	...	...	1	...	...	..	...
May, . . .	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	1	...	...	...	1	3	...
June, . . .	...	...	...	...	...	...	..	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	..	1
July, . . .	...	...	...	...	...	...	1	...	...	...	...	...	...	...	...	...	...	...	..	...	1	2	4	...
August, . .	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	1	...	1	1	...	...
September, . .	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	2	1	1	1	2	...
October, . .	...	...	...	...	...	...	...	...	...	..	...	...	...	...	...	...	...	1	...	1	...	..	...	...
November, . .	...	...	...	...	...	...	...	...	...	...	...	...	...	2	1	...	...	...	1	...	1	...	2	...
December, . .	...	...	...	...	...	...	...	...	...	..	...	...	...	...	...	...	...	...	...	1	...	...	2	1
Year, . . .	...	...	...	...	1	1	1	...	...	...	...	...	...	2	1	...	...	1	6	4	7	8	13	2

Table XXXVIII.

Days with Snow.

	Jan.	Feb.	Mar.	Apr.	May.	Oct.	Nov.	Dec.	Year.	Winter.		Jan.	Feb.	Mar.	Apr.	May.	Oct.	Nov.	Dec.	Year.	Winter.
1770	5	2	11	4	2	...	...	5	29	24	1820	7	2	5	1	...	...	...	7	22	18
1771	7	1	8	3	...	..	1	...	20	35	1821	5	2	2	1	1	...	4	3	18	20
1772	12	14	7	1	...	...	...	...	34	10	1822	2	2	7	2	...	...	...	...	13	29
1773	3	3	...	2	2	...	1	3	14	20	1823	15	8	4	2	...	...	...	4	33	15
1774	10	4	2	...	...	...	4	4	24	22	1824	...	1	8	2	...	2	6	4	23	25
1775	7	3	4	...	...	...	2	...	16	22	1825	2	3	5	2	1	...	4	7	24	26
1776	11	3	4	2	...	...	2	4	26	28	1826	6	4	2	3	...	...	3	1	19	22
1777	2	8	7	5	...	...	1	4	27	14	1827	..	8	7	3	...	...	3	...	21	21
1778	2	5	2	...	...	...	...	1	10	9	1828	9	3	5	1	...	...	2	...	20	17
1779	2	...	1	2	3	...	3	5	16	29	1829	8	4	...	3	...	...	1	5	21	31
1780	8	6	2	5	...	...	6	1	28	15	1830	10	8	5	2	...	...	...	1	26	16
1781	5	...	2	...	1	1	2	1	12	43	1831	3	9	3	...	...	...	3	...	18	7
1782	11	6	15	3	4	3	3	2	47	33	1832	3	...	1	...	...	...	...	3	7	15
1783	7	6	10	1	1	...	1	6	32	27	1833	3	1	6	2	...	...	2	2	16	12
1784	5	7	6	2	...	1	...	5	26	27	1834	1	3	3	1	...	...	...	...	8	9
1785	7	9	4	..	1	...	...	5	26	32	1835	3	2	2	2	...	...	...	1	10	14
1786	4	12	10	1	...	...	...	5	32	10	1836	2	3	7	1	...	2	2	4	21	49
1787	4	...	1	...	...	...	...	8	13	18	1837	8	9	12	10	2	...	...	1	42	42
1788	...	7	2	1	...	...	1	6	17	28	1838	13	12	8	5	3	2	2	1	46	32
1789	9	2	8	2	...	...	...	...	21	9	1839	7	6	10	2	2	...	1	2	30	17
1790	4	1	...	4	...	...	1	3	13	9	1840	6	6	2	...	...	...	...	1	15	18
1791	5	...	...	...	...	...	...	5	10	18	1841	11	6	...	...	...	...	3	2	22	23
1792	4	4	5	...	...	...	1	3	17	17	1842	11	4	3	...	...	2	3	2	25	25
1793	2	1	7	3	...	...	...	...	13	5	1843	4	9	1	4	...	1	...	...	19	24
1794	5	...	...	...	...	...	1	4	10	36	1844	3	13	7	...	...	...	...	5	28	19
1795	10	14	6	...	1	...	4	...	35	7	1845	2	3	9	...	...	...	...	2	16	6
1796	1	...	2	...	...	...	...	5	8	9	1846	...	1	3	...	...	...	1	6	11	13
1797	1	1	2	...	...	...	3	2	9	15	1847	2	2	1	1	...	...	...	1	7	7
1798	1	6	3	...	...	...	...	3	13	31	1848	4	2	...	...	...	...	1	...	7	13
1799	3	13	8	4	...	...	...	5	33	27	1849	4	2	2	4	...	...	1	4	17	17
1800	13	6	3	...	...	...	2	4	28	30	1850	8	1	2	...	1	...	...	...	12	2
1801	4	7	8	5	...	...	3	8	35	33	1851	1	...	...	...	1	...	2	...	4	7
1802	4	7	4	2	5	...	1	1	24	17	1852	4	1	...	...	...	1	...	...	6	22
1803	5	5	3	1	1	...	2	7	24	35	1853	3	9	7	...	2	...	...	2	23	10
1804	5	4	11	6	..	...	1	6	33	17	1854	6	2	...	...	...	...	1	4	13	36
1805	2	4	1	2	1	...	...	4	14	25	1855	5	14	8	1	3	...	...	2	33	4
1806	10	5	3	3	...	...	1	...	22	21	1856	1	1	...	...	...	...	...	1	3	12
1807	2	6	8	4	...	...	7	2	29	34	1857	5	3	3	...	...	...	...	...	11	13
1808	9	3	6	7	...	1	...	7	33	34	1858	1	2	9	1	...	...	1	...	14	14
1809	10	7	3	5	1	...	...	4	30	19	1859	2	2	2	7	...	...	...	4	17	20
1810	5	4	5	...	1	...	3	2	20	20	1860	6	7	2	1	...	..	2	5	23	19
1811	8	4	...	3	...	...	...	5	20	35	1861	3	4	3	...	2	...	5	1	18	17
1812	5	3	16	4	2	...	1	1	32	14	1862	1	2	7	1	...	...	4	1	16	11
1813	2	2	3	5	...	...	2	1	15	35	1863	1	2	2	1	...	...	...	2	8	19
1814	10	9	13	...	...	...	4	9	45	30	1864	1	11	5	...	...	...	...	4	21	24
1815	12	...	4	1	...	...	2	7	26	40	1865	9	10	1	...	...	...	...	1	21	8
1816	5	8	10	6	2	...	5	4	40	23	1866	1	2	3	...	1	...	...	2	9	30
1817	4	3	5	2	...	2	4	6	26	46	1867	11	4	13	...	...	...	...	3	31	13
1818	9	9	11	5	...	...	...	1	35	23	1868	5	2	2	1	...	...	1	...	11	15
1819	8	7	6	1	...	3	3	9	37	30	1869	...	3	11	...	...	...	1	...	15	20

TABLE XXXVIII.—*continued.*

	Jan.	Feb.	Mar.	Apr.	May.	Oct.	Nov.	Dec.	Year.	Winter.
1870	2	12	5	...	...	...	4	9	32	27
1871	4	3	3	4	...	...	2	1	17	12
1872	1	...	7	1	...	...	...	2	11	11
1873	2	4	3	...	...	1	...	2	12	9
1874	3	...	2	1	...	...	2	13	21	26
1875	5	4	2	...	..	...	4	5	20	37
1876	5	9	9	4	1	...	3	5	36	18
1877	4	2	3	1	...	...	...	3	13	14
1878	3	1	6	1	...	1	...	7	19	34
1879	5	8	9	3	1	...	2	3	31	8
1880	3	...	...	...	...	2	3	4	12	31
1881	7	7	8	...	...	...	1	2	25	7
1882	...	1	3	...	...	...	2	8	14	32
1883	3	3	14	...	2	...	5	...	27	15
1884	5	1	3	...	1	...	1	2	13	13
1885	1	3	4	...	2	...	...	4	14	37
1886	13	11	8	1	...	...	...	4	37	18
1887	7	..	5	2	...	...	1	4	19	27
1888	2	6	10	4	...	1	1	1	25	12
1889	...	6	3	...	...	...	...	4	13	14
1890	...	4	3	3	...	1	2	5	18	20
1891	...	...	9	2	1	...	...	5	17	34
1892	6	8	10	5	...	1	1	2	33	16
1893	5	5	1	1	...	...	2	3	17	19
1894	8	3	1	...	2	...	...	1	15	28
1895	11	7	3	...	...	4	..	5	30	16
1896	1	1	5	...	...	1	...	2	10	...
Totals.										
1770–1896	623	570	611	207	57	33	173	390	2664	...
Means.	4·9	4·5	4·8	1·6	·4	·3	1·4	3·1	21·0	...

Decennial Means.

1770–79	**6·4**	4·7	5·7	2·0	·5	...	2·0	2·2	21·5
1780–89	5·6	5·0	**5·8**	1·4	·7	·5	·8	4·1	23·9
1790–99	**4·5**	**4·5**	3·6	·7	·1	...	1·1	3·1	17·6
1800–09	**5·6**	5·2	5·2	3·5	·9	·1	1·8	4·1	26·4
1810–19	7·0	4·7	**7·3**	2·8	·4	·5	2·1	5·0	29·8
1820–29	**5·7**	4·3	4·5	2·1	·2	·2	2·3	2·5	21·8
1830–39	4·9	5·1	**5·4**	2·3	·7	·4	1·0	1·5	21·3
1840–49	**4·9**	4·3	2·8	·9	·1	·3	·9	2·2	16·4
1850–59	3·4	**4·1**	3·1	1·0	·6	·1	·6	1·8	14·7
1860–69	3·4	**5·2**	**5·2**	·3	·3	...	1·5	2·3	18·2
1870–79	3·5	3·1	4·4	1·5	·2	·4	1·6	**4·5**	19·2
1880–89	3·8	4·2	**6·1**	1·0	·5	·2	1·3	3·4	20·5

Table XXXIX.

Showing Date of First and Last Snow by Winters.

Year, Winter of.	Earliest Snow.	Latest Snow.	Year, Winter of.	Earliest Snow.	Latest Snow.
1770-71, . .	December 11	April 16	1834-35, . .	January 11, 1835	April 16
1771-72, . .	November 6	,, 19	1835-36, . .	December 6	,, 2
1772-73, . .	January 9, 1773	May 5	1836-37, . .	October 27	May 10
1773-74, . .	November 22	March 5	1837-38, . .	December 6	,, 17
1774-75, . .	,, 24	,, 29	1838-39, . .	October 12	,, 14
1775-76, . .	,, 12	April 13	1839-40, . .	November 28	March 24
1776-77, . .	,, 17	,, 25	1840-41, . .	December 15	February 10
1777-78, . .	,, 27	March 24	1841-42, . .	November 13	March 26
1778-79, . .	December 30	May 3	1842-43, . .	October 26	April 12
1779-80, . .	November 17	April 7	1843-44, . .	,, 17	March 24
1780-81, . .	,, 6	May 8	1844-45, . .	December 9	,, 17
1781-82, . .	October 30	,, 5	1845-46, . .	,, 3	,, 21
1782-83, . .	,, 20	,, 6	1846-47, . .	November 28	April 1
1783-84, . .	November 13	April 29	1847-48, . .	December 29	February 24
1784-85, . .	October 30	May 17	1848-49, . .	November 8	April 18
1785-86, . .	December 1	April 30	1849-50, . .	,, 6	May 4
1786-87, . .	,, 17	March 5	1850-51, . .	January 31, 1851	,, 3
1787-88, . .	,, 7	April 4	1851-52, . .	November 19	February 3
1788-89, . .	November 26	,, 2	1852-53, . .	October 8	May 10
1789-90, . .	January 1, 1790	,, 13	1853-54, . .	December 13	January 17
1790-91, . .	November 30	March 13	1854-55, . .	November 24	May 10
1791-92, . .	December 3	,, 31	1855-56, . .	December 5	February 19
1792-93, . .	November 20	April 18	1856-57, . .	,, 29	March 24
1793-94, . .	January 23, 1794	January 30	1857-58, . .	January 31, 1858	April 3
1794-95, . .	November 30	May 8	1858-59, . .	November 19	,, 21
1795-96, . .	,, 18	March 25	1859-60, . .	December 13	,, 9
1796-97, . .	December 4	,, 7	1860-61, . .	November 18	May 8
1797-98, . .	November 19	,, 30	1861-62, . .	,, 15	April 14
1798-99, . .	December 25	April 8	1862-63, . .	,, 9	,, 8
1799-00, . .	,, 18	March 12	1863-64, . .	December 3	March 26
1800-01, . .	November 4	April 12	1864-65, . .	,, 16	,, 8
1801-02, . .	,, 21	May 19	1865-66, . .	,, 30	May 2
1802-03, . .	,, 12	,, 2	1866-67, . .	,, 6	March 22
1803-04, . .	,, 14	April 24	1867-68, . .	,, 2	April 9
1804-05, . .	,, 27	May 1	1868-69, . .	November 6	March 27
1805-06, . .	December 1	April 16	1869-70, . .	,, 28	,, 26
1806-07, . .	November 29	,, 16	1870-71, . .	,, 10	April 20
1807-08, . .	,, 13	,, 22	1871-72, . .	,, 16	,, 21
1808-09, . .	October 14	May 30	1872-73, . .	December 13	March 12
1809-10, . .	December 10	,, 6	1873-74, . .	October 22	April 4
1810-11, . .	November 6	April 9	1874-75, . .	November 11	March 13
1811-12, . .	December 2	May 7	1875-76, . .	,, 25	May 1
1812-13, . .	November 18	April 28	1876-77, . .	,, 8	April 10
1813-14, . .	,, 17	March 23	1877-78, . .	December 7	,, 1
1814-15, . .	,, 9	April 14	1878-79, . .	October 29	May 1
1815-16, . .	,, 16	May 11	1879-80, . .	November 22	January 17
1816-17, . .	,, 8	April 16	1880-81, . .	October 26	March 29
1817-18, . .	October 1	,, 11	1881-82, . .	November 1	,, 21
1818-19, . .	December 21	,, 21	1882-83, . .	,, 8	May 8
1819-20, . .	October 22	,, 7	1883-84, . .	,, 9	,, 1
1820-21, . .	December 3	,, 26	1884-85, . .	,, 30	,, 8
1821-22, . .	November 3	,, 11	1885-86, . .	December 9	April 10
1822-23, . .	January 1, 1823	,, 22	1886-87, . .	,, 1	,, 6
1823-24, . .	December 11	,, 10	1887-88, . .	November 14	,, 22
1824-25, . .	October 13	May 27	1888-89, . .	October 4	March 21
1825-26, . .	November 21	April 28	1889-90, . .	December 11	April 13
1826-27, . .	,, 6	,, 25	1890-91, . .	October 26	May 16
1827-28, . .	,, 22	,, 5	1891-92, . .	December 10	April 28
1828-29, . .	,, 10	,, 30	1892-93, . .	October 23	,, 16
1829-30, . .	,, 25	,, 3	1893-94, . .	November 13	May 20
1830-31, . .	December 29	March 25	1894-95, . .	December 29	March 5
1831-32, . .	November 15	,, 24	1895-96, . .	October 24	,, 27
1832-33, . .	December 14	April 16	1896-97, . .	,, 10	April 14
1833-34, . .	November 8	,, 28			

TABLE XXXIX.—*continued.*

Decennial Values.

Winter.	Earliest Snow.		Latest Snow.	
	Date.	Difference from Mean Days.	Date.	Difference from Mean Days.
1770-71 to 1779-80, ...	November 30	**6**	April 11	**1**
1780-81 ,, 1789-90, ...	,, 21	*3*	,, 21	**11**
1790-91 ,, 1799-00, ...	December 7	**13**	March 25	*16*
1800-01 ,, 1809-10, ...	November 17	*7*	April 30	**20**
1810-11 ,, 1819-20, ...	,, 12	*12*	,, 18	**8**
1820-21 ,, 1829-30, ...	,, 21	*3*	,, 22	**12**
1830-31 ,, 1839-40, ...	,, 28	**4**	,, 17	**7**
1840-41 ,, 1849-50, ...	,, 18	*6*	March 25	*16*
1850-51 ,, 1859-60, ...	December 11	**17**	,, 28	*13*
1860-61 ,, 1869-70, ...	November 28	**4**	April 6	*4*
1870-71 ,, 1879-80, ...	,, 16	*8*	,, 1	*9*
1880-81 ,, 1889-90, ...	,, 14	*10*	,, 14	**4**
Mean date, ...	,, 24	...	,, 10	...

NOTE.—Black—days later than average. Italic—days earlier than average.

TABLE XL.

Days with Hail.

Year.	Jan.	Feb.	Mar.	April	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.	Year.
1770, . .	1	1	2	1	1	...	...	...	...	...	...	...	6
1771, . .	1	...	...	...	...	...	...	...	...	...	1	...	2
1772, . .	...	...	2	1	...	...	1	...	...	...	...	...	4
1773, . .	...	1	...	2	2	...	1	...	...	...	...	1	7
1774, . .	...	1	...	3	...	1	...	...	...	...	2	1	8
1775, . .	1	1	4	1	1	...	...	...	...	...	...	...	8
1776, . .	...	2	...	2	1	2	...	...	...	...	...	...	7
1777, . .	...	...	1	...	2	...	...	...	1	...	...	...	4
1778, . .	...	...	...	4	...	...	...	...	...	1	...	...	5
1779, . .	...	1	...	2	1	1	...	...	...	...	...	...	5
1780, . .	...	...	1	5	...	...	...	...	1	...	...	...	7
1781, . .	...	...	...	...	1	1	1	...	...	2	...	1	6
1782, . .	...	3	...	3	6	...	...	...	...	...	1	1	14
1783, . .	...	2	4	...	...	...	...	...	2	2	...	2	12
1784, . .	1	3	3	4	...	...	1	...	...	2	2	7	23
1785, . .	...	...	3	1	...	...	...	...	...	...	...	3	7
1786, . .	1	1	2	1	...	...	1	...	1	1	...	...	8
1787, . .	...	...	...	1	2	2	1	...	...	...	...	...	6
1788, . .	...	...	...	2	...	...	...	...	...	...	...	2	4
1789, . .	...	...	3	2	...	...	...	...	...	...	...	...	5
1790, . .	...	1	1	8	...	1	1	...	1	...	2	2	17
1791, . .	...	3	...	...	2	...	...	...	...	1	3	1	10
1792, . .	...	1	...	...	5	...	...	...	1	...	...	...	7
1793, . .	1	1	3	2	1	1	...	1	...	1	1	...	12
1794, . .	...	1	2	1	2	...	...	...	1	...	...	...	7
1795, . .	...	...	2	3	4	1	...	...	...	...	4	...	14
1796, . .	...	...	3	1	1	1	1	...	...	2	3	1	13
1797, . .	...	...	...	...	3	...	...	...	1	...	...	...	4
1798, . .	...	...	1	...	...	...	...	...	...	...	1	...	2
1799, . .	...	...	1	2	...	1	...	...	...	2	1	...	7
1800, . .	...	1	2	...	1	...	1	...	...	...	...	...	5

TABLE XL.—*continued.*

Year.	Jan.	Feb.	Mar.	April.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.	Year.
1801, . .	...	1	1	3	1	...	...	...	...	1	4	3	14
1802, . .	1	...	2	6	3	1	...	...	1	...	...	...	14
1803, . .	...	2	...	9	3	1	...	...	1	1	1	1	19
1804, . .	...	...	2	4	1	...	...	...	...	...	1	1	9
1805, . .	...	...	2	4	7	1	...	...	...	...	...	1	15
1806, . .	...	1	2	3	1	...	...	...	...	...	1	1	9
1807, . .	3	...	4	6	...	...	1	...	2	...	1	1	18
1808, . .	7	...	...	3	2	...	...	...	1	...	1	...	14
1809, . .	1	2	3	4	2	...	1	2	...	...	...	...	15
1810, . .	1	...	2	7	2	...	...	1	...	...	2	...	15
1811, . .	1	...	4	3	...	...	...	...	...	...	3	...	11
1812, . .	...	1	4	5	1	...	1	...	...	1	2	...	15
1813, . .	...	4	2	1	1	...	...	...	...	...	...	...	8
1814, . .	...	...	1	2	1	1	...	...	...	1	...	...	6
1815, . .	...	1	3	2	3	...	...	...	...	...	1	...	10
1816, . .	...	2	...	4	5	1	...	...	2	...	...	1	15
1817, . .	1	1	2	1	1	1	...	...	...	1	...	2	10
1818, . .	...	...	4	3	...	1	...	...	...	...	...	...	8
1819, . .	1	1	3	2	4	...	...	...	...	4	1	3	19
1820, . .	1	1	2	3	1	...	...	2	1	...	...	...	11
1821, . .	1	...	3	6	9	2	...	...	...	...	4	1	26
1822, . .	2	1	6	5	2	1	...	...	1	...	...	1	19
1823, . .	...	2	1	6	3	2	...	...	1	...	2	1	18
1824, . .	...	4	9	4	2	...	...	2	2	2	4	3	32
1825, . .	1	2	3	...	3	...	...	1	...	1	2	2	15
1826, . .	2	1	7	7	1	...	...	1	...	...	1	...	20
1827, . .	4	1	2	1	...	1	...	...	1	...	...	1	11
1828, . .	1	...	7	2	...	...	...	...	...	...	1	1	12
1829, . .	2	3	1	4	2	2	1	...	...	1	1	3	20
1830, . .	1	...	1	2	...	2	...	...	...	...	...	1	7
1831, . .	...	...	1	...	1	...	...	...	...	...	...	...	2
1832, . .	...	...	...	1	2	...	...	...	...	...	...	...	3
1833, . .	...	...	7	5	...	...	...	...	...	...	2	...	14
1834, . .	1	2	5	1	...	...	...	...	...	...	...	...	9
1835, . .	1	3	2	5	1	...	...	...	...	...	...	2	14
1836, . .	...	...	4	7	...	...	...	...	1	...	...	3	15
1837, . .	2	1	1	4	4	...	...	...	...	1	2	...	15
1838, . .	...	...	1	1	...	...	...	...	...	2	...	...	4
1839, . .	1	...	3	2	2	...	1	...	...	...	...	...	9
1840, . .	1	...	2	1	1	...	1	1	...	...	...	...	7
1841, . .	...	1	...	4	1	...	...	...	...	1	1	3	11
1842, . .	...	...	5	1	...	...	...	1	...	1	1	...	9
1843, . .	1	5	3	3	...	...	1	...	...	1	...	1	15
1844, . .	...	...	...	...	1	...	...	...	...	1	...	...	2
1845, . .	...	...	3	...	1	...	...	...	...	...	...	1	5
1846, . .	...	1	...	1	1	...	...	...	...	...	...	...	3
1847, . .	1	...	2	2	1	...	...	...	...	...	...	...	6
1848, . .	...	...	...	1	...	...	...	...	...	...	...	...	1
1849, . .	...	1	...	3	...	...	...	...	...	1	1	...	6
1850, . .	...	...	...	...	...	1	...	...	...	1	1	...	3
1851, . .	...	...	...	2	2	...	...	...	...	...	...	...	4
1852, . .	1	...	...	1	5	1	...	...	...	1	1	1	11
1853, . .	...	...	...	4	2	...	...	...	...	...	...	1	7
1854, . .	...	2	...	...	4	1	...	...	...	1	1	...	9
1855, . .	...	...	2	...	...	...	...	1	...	...	...	...	3
1856, . .	...	...	1	1	...	...	...	...	...	...	...	...	2
1857, . .	2	...	...	1	...	...	...	...	...	...	1	...	4
1858, . .	...	...	...	...	...	...	...	...	...	1	...	...	1
1859, . .	1	...	2	...	...	...	...	...	1	...	1	1	6
1860, . .	...	...	1	1	2	...	...	...	...	1	2	1	8

TABLE XL.—*continued.*

Year.	Jan.	Feb.	Mar.	April.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.	Year.
1861, . .	...	1	...	1	1	...	1	...	...	...	2	...	6
1862, . .	...	1	...	1	...	...	...	...	...	1	...	1	4
1863, . .	...	1	1	3	1	...	1	...	...	1	...	...	8
1864, . .	...	1	1	2	...	...	1	...	...	...	...	1	6
1865, . .	1	...	1	...	...	...	...	1	...	...	...	...	3
1866, . .	1	1	...	2	3	...	...	...	1	...	...	1	9
1867, . .	1	...	...	1	...	...	...	...	...	...	...	...	2
1868, . .	1	...	...	2	1	...	...	...	...	...	1	...	5
1869, . .	...	...	1	...	1	...	...	...	...	...	...	...	2
1870, . .	...	...	...	1	...	...	...	1	...	...	...	...	2
1871, . .	1	...	...	1	1	...	1	1	1	1	...	...	7
1872, . .	...	...	...	1	...	...	...	1	...	1	...	...	3
1873, . .	...	...	...	2	...	...	1	...	1	1	3	1	9
1874, . .	...	...	2	5	1	1	...	1	...	1	...	2	13
1875, . .	...	1	...	1	...	1	...	...	...	...	3	1	7
1876, . .	...	1	...	2	2	...	...	...	2	...	2	3	12
1877, . .	...	...	1	...	...	1	...	1	...	2	1	...	6
1878, . .	1	...	1	1	3	2	...	...	...	1	...	1	10
1879, . .	...	...	2	...	2	...	...	...	...	...	...	...	4
1880, . .	...	...	...	...	1	2	...	1	2	3	1	...	10
1881, . .	1	3	2	1	1	2	2	1	...	2	...	1	16
1882, . .	1	...	3	...	3	1	2	...	...	...	1	...	11
1883, . .	...	2	5	1	1	1	...	...	...	3	...	2	15
1884, . .	...	1	1	3	4	...	...	1	1	...	...	2	13
1885, . .	2	1	1	...	5	...	...	...	...	2	...	1	12
1886, . .	...	...	...	2	1	...	...	...	1	...	...	...	4
1887, . .	...	...	1	...	1	...	1	...	...	...	...	2	5
1888, . .	...	1	2	1	...	...	...	...	...	3	1	...	8
1889, . .	...	3	...	4	...	...	2	2	3	2	...	...	16
1890, . .	...	...	2	7	2	...	2	1	1	1	...	1	17
1891, . .	...	1	3	1	4	...	3	2	...	...	...	1	15
1892, . .	4	4	4	4	...	2	...	1	1	1	...	...	21
1893, . .	2	1	...	1	...	...	...	1	1	...	...	1	7
1894, . .	6	3	1	1	7	1	2	...	...	1	...	...	22
1895, . .	1	...	3	3	2	4	1	...	...	2	2	...	18
1896, . .	...	...	3	3	...	...	...	1	...	2	...	...	9
Totals, .	72	98	209	274	176	50	36	30	39	71	85	86	1226
Means, 1770–1896,	0·6	0·8	1·6	2·2	1·4	0·4	0·3	0·2	0·3	0·6	0·7	0·7	9·7

Decennial Means.

1771–80, .	0·2	0·6	0·8	**2·0**	0·7	0·4	0·2	...	0·2	0·1	0·3	0·2	5·7
1781–90, .	0·2	1·0	1·6	**2·2**	0·9	0·4	0·5	...	0·4	0·7	0·5	0·8	9·2
1791–00, .	0·1	0·7	1·4	0·9	**1·9**	0·4	0·2	0·1	0·3	0·6	1·3	0·2	8·1
1801–10, .	1·3	0·6	1·8	**4·9**	2·2	0·3	0·2	0·3	0·5	0·2	1·1	0·8	14·2
1811–20, .	0·4	1·1	2·5	**2·6**	1·7	0·4	0·1	0·2	0·3	0·7	0·7	0·6	11·3
1821–30, .	1·4	1·4	**4·0**	3·7	2·2	1·0	0·1	0·4	0·5	0·4	1·5	1·4	18·0
1831–40, .	0·6	0·6	2·6	**2·7**	1·1	...	0·2	0·1	0·1	0·3	0·4	0·5	9·2
1841–50, .	0·2	0·8	1·3	**1·5**	0·5	0·1	0·1	0·1	...	0·6	0·4	0·5	6·1
1851–60, .	0·4	0·2	0·6	1·0	**1·5**	0·2	...	0·1	0·1	0·4	0·6	0·4	5·5
1861–70, .	0·4	0·5	0·4	**1·3**	0·7	...	0·3	0·2	0·1	0·2	0·3	0·3	4·7
1871–80, .	0·2	0·2	0·6	**1·3**	1·0	0·7	0·2	0·5	0·6	1·0	1·0	0·8	8·1
1881–90, .	0·4	1·1	1·7	**1·9**	1·8	0·4	0·9	0·5	0·6	1·3	0·2	0·9	11·7

TABLE XLI.

Days with Gales.

Year.	Jan.	Feb.	Mar.	April.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.	Year.
1770, . .	5	8	6	3	1	4	2	3	2	3	6	10	53
1771, . .	9	1	3	1	9	...	4	1	1	7	4	3	43
1772, . .	...	...	2	...	2	6	3	2	3	4	8	5	35
1773, . .	11	5	1	2	...	...	...	1	5	...	2	2	29
1774, . .	1	11	1	5	1	1	4	3	...	...	4	6	37
1775, . .	7	11	5	2	6	...	...	...	...	4	1	4	40
1776, . .	1	10	6	3	1	1	1	...	...	2	7	1	33
1777, . .	3	1	1	1	...	...	1	2	2	3	3	1	18
1778, . .	3	2	4	2	3	...	...	3	...	...	2	6	25
1779, . .	6	5	7	5	2	...	1	...	3	1	1	3	34
1780, . .	1	3	6	2	...	2	2	...	1	3	1	1	22
1781, . .	2	2	2	3	...	2	2	1	3	1	1	3	22
1782, . .	5	1	9	...	...	...	...	1	...	3	2	2	23
1783, . .	2	3	3	...	3	...	4	...	3	5	...	2	25
1784, . .	5	3	...	...	1	...	1	2	1	1	2	2	18
1785, . .	4	...	...	...	...	...	...	...	2	1	2	3	12
1786, . .	7	3	...	1	1	...	1	...	...	1	3	2	19
1787, . .	...	3	4	1	...	...	...	...	3	...	2	2	15
1788, . .	...	...	...	1	1	...	1	...	5	2	5	...	15
1789, . .	1	4	...	...	...	...	...	...	1	...	2	4	12
1790, . .	3	4	1	1	1	3	1	4	...	6	5	1	30
1791, . .	13	2	1	...	...	...	...	...	1	1	3	5	26
1792, . .	1	4	...	7	1	2	...	...	1	4	3	9	32
1793, . .	1	4	2	2	...	1	...	1	2	3	2	3	21
1794, . .	6	3	4	2	...	...	...	...	1	1	7	2	26
1795, . .	1	2	2	1	1	...	...	3	1	4	7	3	25
1796, . .	6	1	1	...	1	...	...	1	2	2	2	2	18
1797, . .	5	3	...	...	1	...	...	...	1	4	2	1	17
1798, . .	7	7	2	2	...	...	...	...	1	...	1	...	20
1799, . .	2	4	...	10	1	...	1	2	1	3	5	4	33
1800, . .	5	...	1	5	1	1	5	4	...	10	6	2	40
1801, . .	5	4	5	3	2	...	...	2	...	...	6	5	32
1802, . .	3	2	4	2	2	1	...	...	1	3	1	3	22
1803, . .	1	9	1	2	2	...	...	...	...	3	2	4	24
1804, . .	2	...	1	...	2	...	...	...	2	...	...	1	8
1805, . .	7	5	3	...	...	...	1	2	2	1	2	1	24
1806, . .	5	5	1	...	1	3	...	2	...	4	2	5	28
1807, . .	1	1	1	2	2	3	...	1	2	4	3	3	23
1808, . .	8	1	2	3	2	1	...	...	1	6	4	2	30
1809, . .	4	6	...	4	...	2	3	1	1	1	...	8	30
1810, . .	...	2	2	1	4	2	1	...	1	1	...	4	18
1811, . .	7	1	4	5	3	5	2	3	...	4	3	5	42
1812, . .	5	3	5	...	5	...	1	...	1	3	5	6	34
1813, . .	6	9	4	1	...	1	...	3	7	6	7	1	45
1814, . .	1	5	1	2	2	2	1	6	4	3	4	6	37
1815, . .	5	5	9	7	...	2	1	2	7	4	3	12	57
1816, . .	6	8	5	5	...	7	1	6	5	3	9	12	67
1817, . .	7	8	12	1	1	3	2	2	5	2	8	5	56
1818, . .	14	8	16	10	2	3	...	2	2	4	2	9	72
1819, . .	10	1	3	...	3	...	...	1	2	7	1	3	31
1820, . .	8	6	2	2	3	...	2	6	6	4	2	3	44

TABLE XLI.—*continued.*

Year.	Jan.	Feb.	Mar.	April.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.	Year.
1821, . .	6	1	4	...	1	...	...	1	6	3	8	6	36
1822, . .	4	8	10	1	2	1	...	1	2	6	4	3	42
1823, . .	1	5	4	1	1	1	2	1	3	2	4	5	30
1824, . .	4	3	1	2	...	2	1	...	...	5	5	9	32
1825, . .	9	4	5	2	...	...	...	2	1	4	8	1	36
1826, . .	5	8	7	3	...	...	1	2	6	1	2	1	36
1827, . .	12	2	12	6	2	3	1	1	1	3	...	8	51
1828, . .	8	5	5	4	...	...	...	...	4	1	7	12	46
1829, . .	2	2	...	...	...	...	...	3	3	6	4	4	24
1830, . .	3	3	10	...	2	...	...	2	4	2	7	...	33
1831, . .	...	3	5	...	2	1	...	5	...	3	2	4	25
1832, . .	1	4	5	...	...	...	2	1	5	3	3	4	28
1833, . .	1	5	1	3	3	...	...	7	2	5	8	12	47
1834, . .	11	3	9	...	1	2	2	2	...	10	...	...	40
1835, . .	1	3	5	5	...	...	...	2	...	2	3	...	21
1836, . .	4	5	1	6	...	1	2	...	...	3	5	8	35
1837, . .	7	7	4	...	2	1	1	1	4	3	4	4	38
1838, . .	1	4	3	9	...	3	3	1	2	11	6	4	47
1839, . .	11	10	6	2	5	...	4	3	7	2	1	5	56
1840, . .	12	4	2	5	2	4	...	3	7	1	3	...	43
1841, . .	7	2	6	6	2	5	...	...	2	7	6	10	53
1842, . .	4	9	8	...	2	3	1	1	3	2	2	12	47
1843, . .	8	4	1	2	1	1	...	...	2	3	5	8	35
1844, . .	3	2	4	...	...	1	...	...	1	1	5	...	17
1845, . .	4	1	4	1	3	1	2	...	1	8	2	9	36
1846, . .	2	5	4	...	...	...	...	...	...	1	1	...	13
1847, . .	2	1	3	5	...	...	...	2	7	4	5	5	34
1848, . .	1	6	0	2	1	1	...	1	1	1	3	6	23
1849, . .	11	9	3	2	...	...	1	3	1	4	1	1	36
1850, . .	1	9	4	...	...	3	2	4	...	...	...	...	23
1851, . .	4	4	4	...	4	4	2	...	2	5	3	2	34
1852, . .	11	4	...	1	...	...	...	...	...	1	1	5	23
1853, . .	5	1	...	1	...	...	3	...	3	4	3	...	20
1854, . .	6	3	2	...	1	1	1	...	...	2	1	8	25
1855, . .	1	...	2	2	...	...	...	3	...	5	...	4	17
1856, . .	...	2	...	...	...	...	...	...	1	...	...	2	5
1857, . .	2	1	4	...	...	...	1	...	...	1	...	7	16
1858, . .	2	2	1	...	...	...	...	...	...	1	...	1	7
1859, . .	8	2	2	1	...	...	...	1	1	...	3	1	19
1860, . .	...	...	3	...	1	2	...	1	...	3	...	...	10
1861, . .	3	5	5	...	...	...	...	1	1	1	3	3	22
1862, . .	5	...	...	1	...	...	2	...	1	5	...	8	22
1863, . .	11	5	2	2	1	...	...	...	3	1	1	4	30
1864, . .	2	5	2	...	1	...	...	...	6	5	2	...	23
1865, . .	1	1	1	3	1	...	...	...	...	...	...	5	12
1866, . .	...	...	...	...	...	2	...	...	...	...	1	...	3
1867, . .	2	3	...	6	...	...	...	...	4	5	...	6	26
1868, . .	6	9	4	3	1	1	1	2	1	2	2	4	36
1869, . .	4	5	...	...	...	...	3	...	4	2	2	3	23
1870, . .	...	2	1	...	2	1	...	...	...	1	...	2	9
1871, . .	2	2	2	...	1	...	...	2	...	1	1	5	16
1872, . .	6	2	1	3	1	...	...	...	3	2	5	4	27
1873, . .	7	...	...	2	...	3	...	...	...	1	3	4	20
1874, . .	3	1	4	3	...	1	...	1	...	1	1	2	17
1875, . .	4	2	1	1	...	...	...	2	...	3	1	5	19
1876, . .	5	2	3	...	...	...	...	2	1	...	...	3	16
1877, . .	7	2	1	2	6	2	...	...	3	6	4	2	35
1878, . .	5	3	3	...	...	...	...	1	2	3	3	1	21
1879, . .	2	3	6	...	...	1	3	1	3	1	4	6	30
1880, . .	4	7	2	4	3	...	2	...	2	3	7	4	38

TABLE XLI.—*continued.*

Year.	Jan.	Feb.	Mar.	April.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.	Year.
1881, . .	1	4	7	1	2	6	4	7	2	6	11	6	57
1882, . .	6	2	9	2	4	1	2	3	1	2	3	8	43
1883, . .	8	8	4	5	5	2	1	2	2	2	10	4	53
1884, . .	10	9	7	...	4	3	2	...	8	9	8	9	69
1885, . .	7	11	4	3	2	2	3	2	8	3	2	8	55
1886, . .	3	...	5	1	...	1	...	...	4	4	2	2	22
1887, . .	8	2	2	...	1	...	...	3	1	2	2	3	24
1888, . .	2	1	6	1	2	1	...	1	...	3	9	2	28
1889, . .	8	5	2	...	...	...	...	1	1	1	2	5	25
1890, . .	12	3	4	3	1	3	1	2	4	3	3	...	39
1891, . .	7	2	6	...	1	1	...	...	7	2	1	4	31
1892, . .	3	1	...	1	...	1	1	...	...	1	3	2	13
1893, . .	...	2	4	...	1	...	...	1	...	3	4	5	20
1894, . .	10	7	4	...	...	...	...	...	...	...	1	3	25
1895, . .	...	...	1	2	...	...	1	...	1	...	3	3	11
1896, . .	1	...	2	...	...	...	...	2	1	1	...	1	8
Totals, 1770–1896,	578	471	414	233	152	131	107	163	251	356	394	499	3749
Means, . .	4·5	3·7	3·3	1·8	1·2	1·0	0·8	1·3	2·0	2·8	3·1	3·9	29·4

Decennial Means.

1771–80, .	4·2	**4·9**	3·6	2·3	2·4	*1·0*	1·6	1·2	1·5	2·4	3·3	3·2	31·7
1781–90, .	**2·9**	2·3	1·9	0·7	0·7	*0·5*	1·0	0·8	1·8	2·0	2·4	2·1	19·1
1791–00, .	**4·7**	3·0	1·3	2·9	0·6	*0·4*	0·6	1·1	1·1	3·2	3·8	3·1	25·8
1801–10, .	**3·6**	3·5	2·0	1·7	1·7	1·2	*0·5*	0·8	1·0	2·3	2·0	**3·6**	23·9
1811–20, .	**6·9**	5·4	6·1	3·3	1·9	2·3	*1·0*	3·1	3·9	4·0	4·4	6·2	48·5
1821–30, .	5·4	4·1	**5·8**	1·9	0·8	0·7	*0·5*	1·3	3·0	3·3	4·9	4·9	36·6
1831–40, .	**4·9**	4·8	4·1	3·0	1·5	*1·2*	1·4	2·5	2·7	4·3	3·5	4·1	36·0
1841–50, .	4·3	4·8	3·7	1·8	0·9	1·5	*0·6*	1·1	1·8	3·1	3·0	**5·1**	31·7
1851–60, .	**3·9**	1·9	1·8	*0·5*	0·6	0·7	0·7	*0·5*	0·7	2·2	1·1	3·0	17·6
1861–70, .	3·4	**3·5**	1·5	1·5	0·6	0·4	0·6	*0·3*	2·0	2·2	1·1	**3·5**	20·6
1871–80, .	**4·5**	2·4	2·3	1·5	1·1	0·7	*0·5*	0·9	1·4	2·1	2·9	3·6	23·9
1881–90, .	**6·5**	4·5	5·0	1·6	2·1	1·9	*1·3*	2·1	3·1	3·5	5·2	4·7	41·5

Table XLII.

Days with Mist or Fog.

Year.	Jan.	Feb.	Mar.	April.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.	Year.
1770, . .	1	...	...	...	4	2	4	5	3	...	...	...	19
1771, . .	...	1	...	1	2	2	...	...	2	1	...	...	9
1772, . .	...	...	2	...	...	...	2	...	4	5	...	...	13
1773, . .	...	...	...	...	4	1	1	...	1	...	...	1	8
1774, . .	1	...	8	3	5	...	...	3	1	...	4	...	25
1775, . .	1	...	...	3	...	6	1	...	3	1	3	2	20
1776, . .	1	...	...	1	...	2	2	...	...	...	...	...	6
1777, . .	...	...	1	...	1	...	...	...	2	...	...	...	4
1778, . .	...	...	2	1	2	...	...	1	2	...	3	1	12
1779, . .	2	...	3	...	...	...	...	4	...	2	...	...	11
1780, . .	2	...	2	1	1	...	1	13	1	2	1	1	25
1781, . .	...	...	...	1	...	4	2	2	...	...	...	1	10
1782, . .	...	...	...	...	1	...	...	...	...	2	...	...	3
1783, . .	3	...	...	...	1	4	4	5	...	1	1	...	19
1784, . .	...	...	...	...	...	...	...	...	...	...	...	...	...
1785, . .	2	...	...	...	1	...	...	...	1	3	2	2	11
1786, . .	4	...	1	3	...	...	...	...	...	7	...	...	15
1787, . .	1	1	...	...	...	...	...	...	2	1	1	2	8
1788, . .	...	1	1	...	...	...	...	...	...	1	...	...	3
1789, . .	1	...	...	...	...	6	1	...	1	1	...	...	10
1790, . .	...	...	...	...	...	...	...	...	...	...	...	...	...
1791, . .	...	...	1	3	...	3	1	...	...	...	...	...	8
1792, . .	6	2	...	...	...	...	...	...	...	1	...	...	9
1793, . .	1	1	1	...	...	...	...	...	1	...	4	3	11
1794, . .	...	2	1	...	...	...	...	...	...	...	...	1	4
1795, . .	...	...	...	1	...	...	...	...	...	...	...	...	1
1796, . .	...	...	...	2	...	...	...	...	...	...	1	...	3
1797, . .	5	...	...	...	...	1	...	...	...	...	...	1	7
1798, . .	...	...	...	...	...	1	...	...	...	...	...	...	1
1799, . .	...	...	...	...	1	...	1	1	4	4	4	...	15
1800, . .	2	...	5	...	3	2	2	3	5	1	...	1	24
1801, . .	1	...	...	2	...	...	...	...	...	...	...	...	3
1802, . .	...	...	...	...	...	1	1	...	...	...	...	...	2
1803, . .	...	...	...	...	...	1	1	...	...	...	...	...	2
1804, . .	...	...	1	...	4	...	...	...	...	...	...	1	6
1805, . .	1	1	...	...	3	2	1	1	...	2	1	1	13
1806, . .	...	...	3	1	7	...	4	1	1	...	...	1	18
1807, . .	2	...	2	1	4	9	...	6	...	...	...	6	30
1808, . .	...	5	4	...	2	8	6	...	10	1	2	...	38
1809, . .	...	...	6	...	2	2	8	2	1	...	...	1	22
1810, . .	2	...	3	1	...	3	...	...	5	2	1	3	20
1811, . .	...	...	...	1	4	3	1	...	5	1	...	...	15
1812, . .	2	1	...	...	...	...	...	2	...	...	3	3	11
1813, . .	...	...	1	4	3	1	1	...	...	...	...	...	10
1814, . .	...	...	2	...	...	...	...	...	1	2	...	...	5
1815, . .	...	2	...	1	4	6	...	1	1	...	...	...	15
1816, . .	...	...	...	7	2	...	...	...	...	...	...	...	9
1817, . .	...	...	...	...	...	5	...	...	2	2	...	...	9
1818, . .	2	...	1	1	1	2	...	...	...	1	...	...	8
1819, . .	...	...	...	1	...	...	4	1	...	...	...	...	6
1820, . .	3	2	...	1	2	3	3	...	...	...	...	1	15

TABLE XLII.—*continued.*

Year.	Jan.	Feb.	Mar.	April.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.	Year.
1821, . .	...	2	1	2	2	...	...	2	...	...	...	...	9
1822, . .	...	...	...	1	...	3	3	2	3	2	...	7	21
1823, . .	2	...	...	...	4	...	...	2	1	2	4	2	17
1824, . .	...	...	...	...	3	6	...	3	1	5	...	1	19
1825, . .	...	...	2	2	5	1	3	...	4	...	...	3	20
1826, . .	2	...	...	2	2	2	...	...	2	3	4	6	23
1827, . .	4	...	...	1	1	1	1	...	5	6	2	3	24
1828, . .	...	2	1	4	...	2	...	1	2	3	2	...	17
1829, . .	7	6	1	...	5	2	1	...	...	1	2	2	27
1830, . .	...	...	...	1	1	...	4	3	1	1	1	3	15
1831, . .	1	...	4	4	4	...	1	...	...	...	...	...	14
1832, . .	2	2	...	4	1	6	1	...	...	2	1	...	19
1833, . .	3	...	1	2	1	2	3	1	3	3	1	...	20
1834, . .	...	1	2	...	1	...	5	...	3	...	1	1	14
1835, . .	3	...	1	1	1	4	1	2	...	...	1	2	16
1836, . .	1	3	...	...	2	...	...	...	1	3	...	...	10
1837, . .	1	...	...	...	...	...	...	2	...	...	...	4	7
1838, . .	...	...	...	2	1	3	1	...	1	1	...	...	9
1839, . .	...	...	...	...	2	...	...	1	...	1	1	...	5
1840, . .	...	1	...	1	7	...	1	1	...	3	...	...	14
1841, . .	3	...	2	...	1	5	2	1	5	1	2	1	23
1842, . .	2	1	1	10	6	5	1	1	...	...	1	2	30
1843, . .	...	...	1	2	9	6	...	4	5	1	2	...	30
1844, . .	...	...	1	1	6	2	...	3	4	...	2	1	20
1845, . .	2	...	...	8	...	1	3	...	2	1	2	...	19
1846, . .	1	...	...	2	1	1	2	8	8	4	4	...	31
1847, . .	1	...	2	...	3	1	1	...	1	3	3	1	16
1848, . .	...	2	...	2	3	8	...	...	3	...	...	...	18
1849, . .	...	...	5	...	6	...	4	4	8	...	4	1	32
1850, . .	...	...	1	3	5	...	3	2	3	...	2	2	21
1851, . .	...	...	...	1	...	...	2	1	...	1	1	5	11
1852, . .	2	...	5	2	3	...	...	2	...	1	1	...	16
1853, . .	7	4	7	...	...	2	...	...	...	...	1	2	23
1854, . .	...	...	...	...	...	...	...	1	2	...	1	...	4
1855, . .	10	1	2	...	4	1	3	...	3	4	2	...	30
1856, . .	2	1	1	...	2	...	...	5	2	1	4	...	18
1857, . .	...	3	4	...	...	4	...	2	2	4	1	1	21
1858, . .	...	2	2	3	...	3	...	...	2	1	9	4	26
1859, . .	...	1	...	2	3	...	1	...	2	3	5	3	20
1860, . .	4	3	...	...	...	2	1	...	...	4	4	10	28
1861, . .	4	3	1	1	...	2	3	...	...	5	4	8	31
1862, . .	2	1	3	...	4	1	...	...	2	1	2	...	16
1863, . .	2	1	2	2	...	...	...	1	3	2	1	1	15
1864, . .	2	2	3	1	...	...	2	...	...	4	2	...	16
1865, . .	...	2	...	4	6	2	1	1	1	2	...	1	20
1866, . .	...	...	...	1	2	6	...	3	3	4	...	1	20
1867, . .	2	...	...	...	6	...	1	1	...	1	...	...	11
1868, . .	...	...	...	...	...	...	1	1	1	2	3	1	9
1869, . .	2	1	...	...	...	...	...	1	4	2	1	4	15
1870, . .	5	1	4	2	...	2	3	6	2	5	3	...	33
1871, . .	3	1	1	4	...	6	...	2	6	3	3	2	31
1872, . .	6	1	1	...	2	5	7	5	1	1	2	4	35
1873, . .	...	2	4	...	...	1	...	5	...	3	2	2	19
1874, . .	...	...	...	2	1	...	1	...	...	4	1	...	9
1875, . .	3	2	1	2	2	4	1	3	3	...	2	1	24
1876, . .	1	1	2	3	1	...	1	4	1	3	3	4	24
1877, . .	2	...	3	1	3	4	1	4	...	1	...	2	21
1878, . .	1	2	...	9	1	...	3	2	...	...	5	3	26
1879, . .	1	3	4	1	1	3	2	4	...	3	1	1	24
1880, . .	5	...	2	1	...	4	6	3	...	5	1	1	28

TABLE XLII.—*continued.*

Year.	Jan.	Feb.	Mar.	April.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.	Year.
1881, . .	3	...	1	2	2	1	...	2	1	2	3	1	18
1882, . .	3	2	...	1	...	1	...	...	...	2	4	2	15
1883, . .	1	1	2	1	...	2	1	...	2	3	1	2	16
1884, . .	...	...	1	3	1	...	...	1	2	2	...	1	11
1885, . .	...	...	...	...	...	...	2	...	...	...	2	3	7
1886, . .	2	3	...	...	...	2	...	1	1	3	...	...	12
1887, . .	5	1	2	1	1	3	1	...	...	...	1	1	16
1888, . .	2	...	...	...	...	2	1	...	4	1	...	...	10
1889, . .	1	...	...	...	1	...	2	...	1	2	...	2	9
1890, . .	1	...	...	...	4	1	...	1	...	...	...	1	8
1891, . .	...	...	...	...	2	6	1	...	...	...	1	...	10
1892, . .	...	3	...	1	1	...	1	1	...	...	...	...	7
1893, . .	...	...	1	3	...	2	1	...	...	...	...	...	7
1894, . .	...	...	3	3	...	1	...	...	...	...	1	3	11
1895, . .	1	3	1	2	4	2	2	...	4	1	1	3	24
1896, . .	2	...	...	...	1	5	...	...	4	...	2	...	14
Totals, 1770–1896,	168	90	140	155	205	221	145	157	184	176	154	155	1950
Means, . .	1·3	0·7	1·1	1·2	1·6	1·7	1·1	1·2	1·5	1·4	1·2	1·2	15·2

Decennial Means.

1771–80, .	0·7	*0·1*	1·8	1·0	1·5	1·1	0·7	**2·1**	1·6	1·1	1·1	0·5	13·3
1781–90, .	1·1	*0·2*	*0·2*	0·4	0·3	1·4	0·7	0·7	0·4	**1·6**	0·4	0·5	7·9
1791–00, .	**1·4**	0·5	0·8	0·6	*0·4*	0·7	*0·4*	*0·4*	1·0	0·6	0·9	0·6	8·3
1801–10, .	0·6	0·5	1·9	0·5	2·2	**2·6**	2·1	1·0	1·7	0·5	*0·4*	1·3	15·4
1811–20, .	0·7	0·5	0·4	1·6	1·6	**2·0**	0·9	0·4	0·9	0·6	*0·3*	0·4	10·3
1821–30, .	1·5	1·0	*0·5*	1·3	**2·3**	1·7	1·2	1·3	1·9	**2·3**	1·5	2·7	19·2
1831–40, .	1·1	0·7	0·8	1·4	**2·0**	1·5	1·3	0·7	0·8	1·3	*0·5*	0·7	12·8
1841–50, .	0·9	*0·3*	1·3	2·8	**4·0**	2·9	1·6	2·3	3·9	1·0	2·2	0·8	24·0
1851–60, .	2·5	1·5	2·1	0·8	1·2	1·2	*0·7*	1·1	1·3	1·9	**2·9**	2·5	19·7
1861–70, .	1·9	*1·1*	1·3	*1·1*	1·8	1·3	*1·1*	1·4	1·6	**2·8**	1·6	1·6	18·6
1871–80, .	2·2	1·2	1·8	2·3	*1·1*	2·7	2·2	**3·2**	*1·1*	2·3	2·0	2·0	24·1
1881–90, .	**1·8**	0·7	*0·6*	0·8	0·9	1·2	0·7	0·5	1·1	1·5	1·1	1·3	12·2

TABLE XLIII.

Showing the number of Auroras observed in Edinburgh from 1773 *to* 1781 *and from* 1800 *to* 1896.

NOTE.—During the greater part of the time it is probable that only the brighter displays of this meteor were recorded.

Year.	Jan.	Feb.	Mar.	April.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.	Year.
1773, . .	...	...	...	...	...	...	1	...	1	...	...	...	2
1774, . .	...	...	2	...	...	...	...	1	...	...	2	1	6
1775, . .	...	1	...	...	...	...	...	...	...	...	...	...	1
1776, . .	...	...	1	2	...	...	...	...	...	...	...	...	3
1777, . .	...	...	...	...	...	...	...	...	...	...	...	...	...
1778, . .	...	1	1	1	...	...	...	...	...	3	...	...	6
1779, . .	...	4	2	3	...	...	1	...	...	...	...	...	10
1780, . .	...	1	...	...	...	...	...	...	...	...	...	...	1
1781, . .	...	...	...	1	...	...	...	...	...	...	3	1	5
1800, . .	...	...	...	...	...	...	...	...	...	...	1	...	1
1801, . .	...	...	...	...	...	...	...	...	...	...	...	...	...
1802, . .	...	...	...	...	...	...	...	...	...	...	...	...	...
1803, . .	...	...	...	...	...	...	...	...	...	...	...	...	...
1804, . .	...	...	...	...	...	...	...	...	1	...	...	...	1
1805, . .	...	...	...	...	...	...	...	...	1	...	...	...	1
1806, . .	...	...	...	...	...	...	...	...	...	...	...	...	...
1807, . .	...	...	...	...	...	...	...	...	...	...	...	...	...
1808, . .	...	...	...	...	...	...	...	...	...	...	...	...	...
1809, . .	...	...	...	...	...	...	...	...	...	...	...	...	...
1810, . .	...	...	...	...	...	...	...	...	...	...	...	...	...
1811, . .	...	...	...	...	...	...	...	...	...	...	...	...	...
1812, . .	...	...	...	...	...	...	...	...	...	...	...	...	...
1813, . .	...	...	...	...	...	...	...	...	...	1	...	...	1
1814, . .	...	...	...	...	...	...	...	...	...	...	...	...	...
1815, . .	...	...	...	...	...	...	...	...	...	...	...	...	...
1816, . .	...	...	...	...	...	...	...	...	...	...	...	...	...
1817, . .	1	1	...	...	...	...	...	...	...	...	...	...	2
1818, . .	...	...	...	...	...	...	...	...	1	1	...	1	3
1819, . .	...	3	1	...	...	...	...	...	...	...	...	...	4
1820, . .	...	1	...	...	...	...	...	...	...	...	...	...	1
1821, . .	...	...	...	...	...	...	...	...	...	...	...	...	...
1822, . .	...	...	1	...	...	...	...	...	...	...	2	...	3
1823, . .	...	...	...	...	...	...	...	...	...	...	...	...	...
1824, . .	...	...	...	...	...	...	...	...	1	...	...	...	1
1825, . .	1	...	1	...	...	...	...	1	1	1	4	...	9
1826, . .	3	1	1	...	...	...	...	...	...	...	...	...	5
1827, . .	1	...	...	...	...	...	...	...	3	...	...	1	5
1828, . .	...	...	...	...	...	...	...	...	2	...	...	...	2
1829, . .	...	...	...	...	...	...	...	...	...	...	...	...	...
1830, . .	...	1	...	1	...	...	...	4	1	2	1	1	11
1831, . .	1	...	4	...	...	...	...	...	...	...	...	...	5
1832, . .	...	...	...	...	...	...	...	...	...	1	...	...	1
1833, . .	...	...	1	...	...	...	...	...	1	1	...	...	3
1834, . .	...	...	...	...	...	...	...	...	...	1	...	1	2
1835, . .	...	1	...	...	...	...	...	...	...	...	1	...	2
1836, . .	...	...	...	...	...	...	...	...	...	1	...	...	1
1837, . .	...	...	...	2	...	...	...	...	...	2	...	...	4
1838, . .	...	...	...	...	...	...	...	...	1	...	...	...	1
1839, . .	3	...	...	1	...	...	...	1	2	2	...	...	9
1840, . .	3	...	...	...	...	...	...	1	...	...	2	...	6

TABLE XLIII.—*continued.*

Year.	Jan.	Feb.	Mar.	April.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.	Year.
1841, . .	2	...	1	3	...	...	...	...	...	...	...	1	7
1842, . .	...	3	...	1	...	...	...	...	1	...	...	...	5
1843, . .	...	...	2	1	...	...	...	1	...	...	...	...	4
1844, . .	...	...	...	...	...	...	...	...	...	...	...	...	...
1845, . .	1	1	...	...	...	...	...	...	1	...	...	1	4
1846, . .	...	1	...	...	...	...	...	...	2	...	1	...	4
1847, . .	...	...	...	...	...	...	...	...	1	1	...	...	2
1848, . .	3	...	...	...	...	...	...	...	...	3	3	...	9
1849, . .	...	2	1	...	...	...	...	...	...	...	...	1	4
1850, . .	...	1	...	...	...	...	...	...	...	...	...	...	1
1851, . .	...	...	...	...	...	...	...	...	...	...	...	...	...
1852, . .	...	2	...	...	...	...	...	...	...	...	...	...	2
1853, . .	...	...	...	...	...	...	...	...	...	...	...	...	...
1854, . .	...	...	...	...	...	...	...	...	...	...	...	...	...
1855, . .	...	...	...	...	...	...	...	...	...	...	...	...	...
1856, . .	...	...	...	...	...	...	...	...	...	...	...	...	...
1857, . .	...	...	...	...	...	...	...	...	...	...	...	...	...
1858, . .	...	...	...	...	...	...	...	...	...	...	...	...	...
1859, . .	...	1	1	...	...	...	...	1	1	1	...	1	6
1860, . .	...	1	2	...	...	...	...	...	1	...	...	...	4
1861, . .	1	...	...	...	...	...	...	...	...	...	...	...	1
1862, . .	...	...	...	...	...	...	...	...	...	...	1	1	2
1863, . .	...	1	2	1	1	...	...	...	...	...	1	1	7
1864, . .	...	...	...	...	...	...	...	...	...	...	...	...	...
1865, . .	...	...	...	...	...	...	...	...	...	...	...	...	...
1866, . .	...	...	...	...	...	...	...	...	...	...	1	...	1
1867, . .	1	1	...	...	...	...	...	...	1	2	1	...	6
1868, . .	...	...	...	1	...	...	...	...	...	2	...	...	3
1869, . .	...	...	...	...	1	...	...	...	...	2	...	2	5
1870, . .	2	...	2	...	1	...	...	4	5	4	...	1	19
1871, . .	...	1	6	4	1	...	...	2	4	2	1	...	21
1872, . .	...	1	...	1	...	...	...	...	2	3	...	1	8
1873, . .	4	3	...	2	...	...	...	...	1	1	1	...	12
1874, . .	...	2	1	1	...	...	...	...	1	3	2	...	10
1875, . .	2	...	...	...	...	...	...	...	1	1	...	...	4
1876, . .	...	...	...	...	...	...	...	...	...	...	...	...	...
1877, . .	1	...	...	...	...	...	...	...	...	...	1	...	2
1878, . .	...	...	...	...	...	...	...	...	1	...	...	...	1
1879, . .	...	...	...	...	...	...	...	...	...	...	...	...	...
1880, . .	...	...	2	...	3	1	...	3	...	1	2	...	12
1881, . .	1	...	...	...	...	...	...	...	...	...	1	...	2
1882, . .	...	2	...	...	...	...	...	...	...	...	3	...	5
1883, . .	...	1	1	...	...	...	...	...	...	...	...	...	2
1884, . .	...	...	...	1	1	...	...	...	1	...	...	...	3
1885, . .	...	1	2	...	1	...	...	...	1	...	...	...	5
1886, . .	1	...	1	...	1	...	1	1	...	...	1	...	6
1887, . .	...	...	...	...	...	...	...	...	...	...	...	...	...
1888, . .	...	...	...	...	...	...	...	...	...	1	...	...	1
1889, . .	...	...	...	...	...	...	...	...	...	...	...	...	...
1890, . .	...	...	...	...	...	...	...	...	...	...	...	...	...
1891, . .	...	...	2	2	...	...	...	...	...	2	1	...	7
1892, . .	...	...	3	1	1	...	...	...	2	1	1	...	9
1893, . .	...	...	...	...	...	...	...	1	...	...	1	...	2
1894, . .	...	1	2	...	1	...	3	...	...	...	1	...	8
1895, . .	...	1	1	3	...	...	...	...	...	...	1	1	7
1896, . .	...	...	...	...	...	...	...	...	...	1	...	...	1
Total, .	32	42	47	33	12	1	6	21	43	47	41	17	342
Max. and	4	4	6	4	3	1	3	4	5	4	4	2	21
Year, .	1873	1779	1871	1871	1880	1880	1894	1830,1870	1870	1870	1825	1869	1871

Table XLIV.

Showing the number of Days on which Lightning without Thunder was observed from 1807 *to* 1835, *and from* 1868 *to* 1896.

Year.	Jan.	Feb.	Mar.	April.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.	Year.
1807, . .	...	...	...	...	1	...	1	...	...	...	...	...	2
1808, . .	...	...	...	...	...	...	...	...	1	...	...	...	1
1809, . .	...	...	...	...	...	...	...	...	...	...	...	1	1
1810, . .	...	...	...	...	...	...	...	1	1	...	...	...	2
1811, . .	...	...	...	...	...	...	...	...	...	...	...	1	1
1812, . .	...	...	...	...	...	...	...	1	...	...	...	...	1
1813, . .	...	...	...	...	...	...	1	...	...	...	...	...	1
1814, . .	...	...	...	...	...	...	...	...	...	...	...	...	...
1815, . .	...	...	...	...	...	...	...	...	1	...	...	...	1
1816, . .	...	...	...	...	...	...	1	...	...	...	...	...	1
1817, . .	...	...	...	...	...	...	...	...	...	...	...	...	...
1818, . .	...	3	...	1	...	...	2	...	...	...	...	...	6
1819, . .	1	...	...	...	...	...	...	...	...	1	...	1	3
1820, . .	...	...	...	...	...	...	...	...	1	...	...	...	1
1821, . .	...	...	...	—	...	...	...	...	...	...	...	...	...
1822, . .	...	...	...	...	...	...	...	...	...	...	...	...	...
1823, . .	...	...	...	...	...	...	...	...	...	...	...	...	...
1824, . .	...	...	...	...	...	...	...	...	1	...	...	...	1
1825, . .	...	...	...	...	...	...	...	...	...	...	...	...	...
1826, . .	...	...	...	...	...	...	...	...	...	...	...	...	...
1827, . .	...	...	...	...	...	...	...	...	...	...	...	...	...
1828, . .	...	...	...	...	1	...	...	...	...	...	...	1	2
1829, . .	...	...	...	...	...	...	...	...	...	...	...	...	...
1830, . .	...	...	...	...	...	...	...	...	...	...	...	...	...
1831, . .	...	...	...	...	...	...	...	...	1	...	...	...	1
1832, . .	...	...	...	...	...	...	...	...	...	...	...	...	...
1833, . .	...	...	...	...	1	1	...	...	...	...	...	1	3
1834, . .	1	...	...	...	...	...	...	...	1	...	...	...	2
1835, . .	...	...	...	...	...	...	...	...	1	...	...	...	1
1868, . .	1	...	...	...	...	...	...	...	...	...	...	1	2
1869, . .	...	...	1	1	...	...	2	...	1	...	...	...	5
1870, . .	...	...	...	...	...	...	...	...	...	...	...	...	...
1871, . .	...	...	...	...	...	...	...	...	...	...	...	...	...
1872, . .	...	...	...	...	...	...	...	...	1	...	...	...	1
1873, . .	...	...	...	...	...	...	...	...	...	...	...	...	..
1874, . .	...	...	...	...	...	...	...	...	...	...	...	...	...
1875, . .	1	...	...	...	...	...	...	2	...	1	...	...	4
1876, . .	...	...	...	...	...	...	...	...	1	2	1	...	4
1877, . .	...	...	...	...	...	1	1	1	...	1	...	...	4
1878, . .	...	...	...	...	1	...	...	...	...	...	...	...	1
1879, . .	...	...	...	...	...	...	1	...	...	...	...	...	1
1880, . .	...	1	...	...	...	...	...	...	...	...	...	...	1
1881, . .	...	...	...	...	...	...	...	...	1	...	...	...	1
1882, . .	1	...	...	...	...	...	...	...	...	1	...	...	2
1883, . .	...	...	...	...	...	...	...	1	...	...	2	...	3
1884, . .	2	...	...	...	...	...	...	...	3	...	1	...	6
1885, . .	...	...	...	...	1	...	...	...	1	...	...	...	2
1886, . .	1	...	...	...	...	...	1	...	...	...	...	...	2
1887, . .	...	...	...	...	...	...	...	...	...	...	...	...	...
1888, . .	...	...	...	...	...	...	...	...	...	...	...	...	...
1889, . .	...	...	...	...	...	...	...	...	...	...	...	...	...
1890, . .	1	...	...	...	...	...	2	...	...	...	1	...	4
1891, . .	...	...	...	1	...	...	...	...	...	...	...	1	2
1892, . .	...	...	...	...	...	...	...	...	...	...	...	1	1
1893, . .	...	...	...	...	1	...	...	1	...	1	...	...	3
1894, . .	...	1	1	...	...	...	...	...	...	...	...	...	2
1895, . .	...	...	...	...	1	...	...	...	1	...	...	...	2
1896, . .	...	...	...	1	...	...	...	...	...	...	...	1	2

Table XLV.

Bright Sunshine for Hour ending Greenwich Time for Six Years ending July 1896.

	A.M.							Noon.	P.M.							
	5	6	7	8	9	10	11		1	2	3	4	5	6	7	8
	Hrs.	Hrs.	Hrs.	Hrs.	Hrs.	Hrs.	Hrs.	Hrs.	Hrs.	Hrs.	Hrs.	Hrs.	Hrs.	Hrs.	Hrs.	Hrs.
January, .	...	...	...	...	...	0·8	6·4	7·7	**8·4**	7·4	4·7	0·6	...	...	...	...
February, .	...	...	...	...	1·2	4·3	8·9	**9·8**	9·6	**9·8**	9·5	5·3	1·6	...	...	...
March, . .	...	...	0·4	3·7	9·2	12·3	13·8	14·0	**14·3**	13·4	11·9	12·1	8·0	1·8	...	...
April, . .	...	1·0	3·7	7·2	10·2	12·1	13·1	**14·9**	14·3	13·8	14·1	13·9	11·6	8·5	1·5	...
May, . .	1·1	6·1	8·8	9·6	10·1	12·4	13·4	14·5	14·9	**15·6**	14·6	14·1	13·7	12·9	9·3	1·6
June, . .	3·1	7·1	9·2*	9·9	10·3	11·2	10·4	12·2	12·6	12·3	**12·7**	12·4	12·4	11·9	10·2	3·7
July, . .	1·9	5·3	6·9	8·5	9·2	9·6	9·5	10·7	11·0	11·3	*11·4*	10·5	10·2	9·5	7·0	1·7
August, .	0·1	1·4	5·3	8·3	11·0	12·3	12·0	12·8	11·9	**13·2**	12·7	11·8	10·0	7·9	3·5	0·1
September, .	...	...	1·0	5·2	9·6	11·6	12·4	12·8	**13·3**	13·2	12·3	12·5	10·0	4·3	0·4	...
October, .	...	...	...	0·8	5·0	10·5	12·0	**13·2**	13·0	13·1	10·8	7·4	2·3	...	...	...
November, .	...	...	...	...	...	3·3	8·4	**9·7**	9·3	7·5	6·1	1·1	0·1	...	...	...
December, .	...	...	...	...	...	0·4	3·2	6·0	**7·3**	5·8	2·3	...	...	...	...	...
Spring, .	1·1	7·1	12·9	20·5	29·5	36·8	40·3	43·4	**43·5**	42·8	40·6	40·1	33·3	23·2	10·8	1·6
Summer, .	5·1	13·8	21·4	26·7	30·5	33·1	31·9	35·7	35·5	**36·8**	**36·8**	34·7	32·6	29·3	20·7	5·5
Autumn, .	...	...	1·0	6·0	14·6	25·4	32·8	**35·7**	35·6	33·8	29·2	21·0	12·4	4·3	0·4	...
Winter, .	...	...	...	...	1·2	5·5	18·5	23·5	**25·3**	23·0	16·5	5·9	1·6	...	...	...
Year, .	6·2	20·9	35·3	53·2	74·8	100·8	123·5	138·3	**139·9**	136·4	123·1	101·7	79·9	56·8	31·9	7·1

TABLE XLVI.

Showing the Number of Sunless and Sunny Days in Edinburgh for the Six Years ending July 1896.

Per cent of possible duration.

Month.	Sunless.	1–10	11–20	21–30	31–40	41–50	51–60	61–70	71–80	+80	Max.
											%
January,	83	33	8	19	13	15	12	2	1	...	72
February,	41	39	19	15	15	17	8	11	4	1	82
March,	30	25	20	24	23	13	18	13	15	5	85
April,	22	26	20	13	27	26	19	13	12	2	84
May,	18	30	17	21	21	27	19	9	14	10	87
June,	28	33	17	17	14	26	13	15	8	9	87
July,	22	37	29	24	26	20	10	13	5	...	78
August,	11	38	30	27	22	15	24	13	5	1	82
September,	21	39	19	16	15	17	20	18	13	2	81
October,	29	40	14	21	24	16	24	10	8	...	80
November,	70	29	12	19	12	19	5	10	4	...	72
December,	106	26	11	13	8	9	5	5	3	...	73
Total	481	395	216	229	220	220	177	132	92	30	
Per cent.,	22	18	10	11	10	10	8	6	4	1	

Seasonal Percentages.

	Sunless.	1–10	11–20	21–30	31–40	41–50	51–60	61–70	71–80	+80
Spring,	13	15	10	10	13	12	10	6	8	3
Summer,	11	20	14	12	11	11	9	7	3	2
Autumn,	22	20	8	10	9	10	9	7	4	1
Winter,	42	18	7	9	7	8	5	3	1	...

Departure from Mean of Year.

	Sunless.	1–10	11–20	21–30	31–40	41–50	51–60	61–70	71–80	+80
Spring,	*9*	*3*	...	*1*	**3**	**2**	**2**	...	**4**	**2**
Summer,	*11*	**2**	**4**	**1**	**1**	**1**	**1**	**1**	*1*	**1**
Autumn,	...	**2**	*2*	*1*	*1*	...	**1**	**1**	...	...
Winter,	**20**	...	*3*	*2*	*3*	*2*	*3*	*3*	*3*	*1*

NOTE.—The heavy type indicates an excess, and the italic type a defect.

Table XLVII.

Mean Temperature at 9 A.M.

Year.	Jan.	Feb.	Mar.	April.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.	Year.
	°	°	°	°	°	°	°	°	°	°	°	°	°
1731, . .	...	...	...	...	...	54·5	59·9	55·8	53·5	47·8	39·2	34·4	...
1732, . .	34·1	42·0	41·7	44·0	49·2	58·2	57·6	54·4	49·5	44·7	36·7	35·5	45·6
1733, . .	39·2	40·0	38·9	47·6	52·8	60·9	61·0	55·2	50·0	43·7	43·5	43·3	48·0
1734, . .	35·6	41·8	44·8	50·0	49·7	58·1	61·8	57·9	50·0	46·8	36·9	36·1	47·5
1735, . .	36·3	38·7	39·3	47·7	50·6	57·7	59·0	58·0	49·8	43·9	42·2	38·2	46·8
1736, . .	36·0	32·8	42·0	47·1	50·3	...	...	...	...	...	...	...	...
Means, .	36·2	39·1	41·3	47·2	50·4	57·9	59·9	56·2	50·6	45·4	39·7	37·5	46·8

Table XLVIII.

Mean Temperature at 9 A.M. *Brought to Mean of Max. and Min.*

Year.	Jan.	Feb.	Mar.	April.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.	Year.
	°	°	°	°	°	°	°	°	°	°	°	°	°
1731, . .	...	...	...	...	...	54·3	59·9	56·0	53·8	48·1	39·6	34·7	...
1732, . .	34·7	42·8	42·4	44·0	48·8	58·0	57·6	54·6	49·8	45·0	37·1	35·8	45·9
1733, . .	39·8	40·8	39·6	47·6	52·4	60·7	61·0	55·4	50·3	44·0	43·9	43·6	48·3
1734, . .	36·2	42·6	45·5	50·0	49·3	57·9	61·8	58·1	50·3	47·1	37·3	36·4	47·7
1735, . .	36·9	39·5	40·0	47·7	50·2	57·5	59 0	58·2	50·1	44·2	42·6	38·5	47·0
1736, . .	36·6	33·6	42·7	47·1	49·9	...	...	...	...	...	...	...	...
Means, 5 Years, .	36·8	39·9	42·0	47·2	50·0	57·7	59·9	56·5	50·9	45·7	40·1	37·8	47·0

Table XLIX.

Rainfall.—Inches.

Year.	Jan.	Feb.	Mar.	April.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.	Year.
	°	°	°	°	°	°	°	°	°	°	°	°	°
1731, . .	...	...	...	...	...	2·06	1·54	1·86	2·12	1·48	1·42	3·12	?
1732, . .	1·28	2·41	0·79	3·11	4·62	1·20	3·20	1·62	?	2·52	0·42	3·62	?
1733, . .	1·37	2·52	2·64	0·82	0·08	2·14	0·64	2·68	1·84	1·08	0·33	3·63	19·76
1734, . .	0·59	0·60	2·12	1·01	3·31	2·21	0·71	1·28	1·27	1·32	1·61	2·33	18·27
1735, . .	3·00	3·51	5·38	1·63	0·72	...	...	...	...	...	...	...	...

TABLE L.

Mean Variability of Temperature at 9 A.M.

Year.	Jan.	Feb.	Mar.	April.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.	Year.
	°	°	°	°	°	°	°	°	°	°	°	°	°
1731, . .	...	...	...	...	...	3·7	2·7	2·9	3·9	5·3	3·6	4·8	...
1732, . .	3·2	4·5	3·4	3·1	2·8	2·5	3·5	2·7	3·3	2·9	3·8	3·6	3·28
1733, . .	3·1	3·7	3·6	3·0	3·0	3·1	3·6	3·8	2·3	3·6	4·4	3·8	3·42
1734, . .	3·5	3·2	3·9	3·6	3·4	3·5	3·3	2·7	3·0	3·1	3·6	3·9	3·39
1735, . .	3·3	3·0	2·6	3·2	3·1	3·8	3·1	3·8	3·4	3·9	3·8	3·7	3·39
1736, . .	3·7	3·1	2·8	4·2	3·4	...	...	...	...	...	...	...	...
Means, .	3·4	3·5	3·3	3·4	3·1	3·3	3·2	3·2	3·2	3·8	3·8	4·0	3·4

TABLE LI.

Mean Humidity.

Year.	Jan.	Feb.	Mar.	April.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.	Year.
	°	°	°	°	°	°	°	°	°	°	°	°	°
1731, . .	...	...	...	...	...	2·37	1·77	2·35	1·80	2·06	2·41	2·34	?
1732, . .	2·62	2·37	1·94	2·27	1·77	1·64	1·90	1·82	1·84	2·45	2·49	2·64	2·15
1733, . .	2·14	2·17	2·55	2·10	1·66	1·51	1·71	1·76	2·00	1·96	2·08	2·15	1·98
1734, . .	2·20	2·02	1·75	1·81	1·60	1·98	1·85	2·02	1·96	2·35	2·24	2·47	2·02
1735, . .	2·38	2·27	2·56	2·45	1·66	1·76	2·00	1·81	1·96	2·40	2·59	2·77	2·22
1736, . .	2·54	2·46	2·39	2·11	1·81	...	...	...	...	...	...	...	?
Means, .	2·38	2·26	2·24	2·15	1·70	1·85	1·85	1·95	1·91	2·24	2·36	2·47	2·11

TABLE LII.

Thermal Windrose, June 1731 *to May* 1736.

	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Monthly Range.
	°	°	°	°	°	°	°	°	°
January, ...	32·3	35·8	34·8	34·2	35·9	38·5	35·7	26·5	12·0
February, ...	31·7	34·8	33·9	35·9	41·4	42·4	39·1	35·0	10·7
March, ...	34·2	37·5	40·6	39·4	44·2	44·1	43·6	38·3	10·0
April, ...	45·1	44·5	44·5	50·7	50·8	50·8	47·8	42·5	8·3
May, ...	47·4	49·6	51·0	50·2	54·7	54·3	50·7	47·8	7·3
June, ...	55·5	54·6	58·9	59·0	65·3	63·9	55·8	54·9	10·7
July, ...	59·0	56·1	60·0	62·2	63·2	62·1	59·6	59·2	7·1
August, ...	56·3	54·5	57·3	60·3	53·8	58·4	54·9	54·4	5·9
September, ...	44·3	49·5	50·5	50·1	54·6	52·2	50·3	48·8	10·3
October, ...	43·4	45·0	45·8	45·4	45·3	46·4	43·0	39·0	7·4
November, ...	37·7	39·5	38·3	39·9	43·1	40·4	40·5	32·6	10·5
December, ...	33·0	34·6	37·4	35·4	37·7	41·5	36·3	36·3	8·5
Range, ...	27·3	21·5	26·1	28·0	29·4	25·4	23·9	32·7	
Spring, ...	43·3	46·1	46·3	44·7	48·8	48·8	47·2	43·0	5·8
Summer, ...	57·5	55·0	58·0	60·6	61·7	60·4	57·9	57·2	6·7
Autumn, ...	41·5	45·0	46·8	44·6	45·6	46·6	44·2	40·3	5·3
Winter, ...	32·4	34·9	35·5	35·1	37·0	40·8	36·8	33·8	8·4

Table LIII.

Thermal Windrose, 1770–1776—7 *Years.*

	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.	Mean.	Range.
January, . .	32·2	31·8	32·6	31·8	36·5	**37·6**	34·1	30·3	*27·1*	33·2	10·5
February, . .	33·4	34·0	35·5	37·0	36·5	**39·0**	33·4	35·4	*31·7*	36·3	7·3
March, . .	35·3	38·6	36·3	36·9	**41·3**	41·2	38·5	*34·1*	36·0	38·3	7·2
April, . .	40·4	43·3	42·5	42·8	**46·6**	45·3	45·3	*39·4*	45·0	44·1	7·2
May, . . .	47·6	46·6	50·1	52·0	**52·8**	51·5	50·7	*45·3*	48·3	49·3	7·5
June, . . .	*53·6*	55·3	55·9	**58·7**	57·4	56 3	55·6	54·2	...	55·7	5·1
July, . . .	56·7	*56·2*	58·2	56·6	**60·6**	59·6	58·0	57·4	62·0	58·4	4·4
August, . .	57·2	56·8	57·8	58·3	**60·0**	58·2	57·8	*56·4*	58·0	57·5	3·6
September, . .	51·7	52·9	53·8	**53·9**	53·7	51·3	*50·8*	51·9	...	52·3	3·1
October, . .	42·1	48·8	47·0	44·7	**49·1**	48·3	45·6	*40·5*	43·5	46·8	8·6
November, . .	37·8	40·9	40·9	39·3	41·4	40·8	39·1	*36·5*	**44·0**	39·7	7·5
December, . .	36·1	38·1	37·7	38·4	39 0	**40·2**	*35·7*	37·3	37·0	38·5	4·5
Range, . .	24·5	25·0	25·6	26·9	24·1	22·0	24·6	27·1	...	...	...
Spring, . .	42·4	44·1	43·8	40·1	45·7	**46·4**	44·8	*38·9*	44·1	43·9	7·5
Summer, . .	*55·7*	55·8	57·2	57·9	**59·4**	58·3	57·2	56·1	60·0	57·2	4·3
Autumn, . .	44·3	**49·8**	48·7	45·5	47·6	46·2	44·8	*42·6*	43·7	46·3	5·0
Winter, . .	33·4	35·4	35·1	35·0	37·7	**39·2**	34·4	33·7	*29·5*	36·0	9·7
Year, . .	44·0	**48·1**	46·0	43·1	*47·9*	45·9	46·5	*42·4*	37·9	...	...

Table LIV.

Mean Humidity with Different Winds, 1731–1736. *Scale* 0·5 *to* 5·0.

	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.
January, ...	2·47	3·65	2·53	2·67	2·36	2·27	2·36	1·66
February, ...	2·53	2·57	2·30	2·28	2·12	2·14	2·23	2·11
March, ...	2·40	2·78	2·60	2·38	2·19	2·09	2·05	1·91
April, ...	2·02	2·41	2·68	2·10	2·10	1·89	1·97	1·74
May, ...	1·52	1·85	1·96	1·63	1·74	1·61	1·55	1·38
June, ...	2·03	2·39	1·86	1·64	1·51	1·54	1·57	1·79
July, ...	1·95	2·43	2·07	1·93	1·68	1·70	1·71	1·58
August, ...	2·68	2·30	2·23	1·92	1·82	1·82	1·88	1·64
September, ...	1·88	2·45	2·10	2·04	1·76	1·92	1·89	1·70
October, ...	2·26	2·91	2·50	2·40	2·14	2·29	2·14	2·05
November, ...	2·52	2·82	2·30	2·44	2·47	2·25	2·42	2·13
December, ...	2·66	2·40	2·97	2·65	2·51	2·35	2·41	2·39
Range, ...	1·14	1·06	1·11	1·04	1·00	0·81	0·87	1·01
Spring, ...	1·89	2·17	2·35	2·18	2·07	1·91	1·88	1·67
Summer, ...	2·12	2·38	2·04	1·85	1·65	1·73	1·74	1·66
Autumn, ...	2·22	2·73	2·31	2·33	2·18	2·10	2·13	1·96
Winter, ...	2·58	2·69	2·62	2·53	2·39	2·24	2·34	2·11
Year, ...	2·19	2·36	2·26	2·26	2·19	2·06	2·02	1·80

Table LV.

Showing the Departure of Temperature from the Normal, smoothed by continuous Five Year Groups.

Year.	Jan.	Feb.	Mar.	April.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.	Year.
	°	°	°	°	°	°	°	°	°	°	°	°	°
1766	*1·7*	*1·4*	*1·1*	**0·3**	**0·3**	*1·5*	*0·2*	**0·7**	*1·6*	*0·6*	*0·6*	*0·8*	*0·7*
1767	*1·9*	*1·6*	*0·7*	**0·6**	*0·0*	*1·8*	*0·2*	**0·4**	*1·0*	*0·8*	*0·2*	**0·1**	*0·6*
1768	*1·9*	*0·0*	*1·6*	*0·0*	*0·8*	*1·8*	*0·4*	**0·7**	*0·4*	*1·3*	*0·0*	**0·5**	*0·6*
1769	*2·0*	**0·7**	*1·9*	*0·8*	*0·1*	*1·8*	*0·7*	**0·1**	*0·5*	*1·2*	*0·2*	**1·3**	*0·6*
1770	*1·8*	*1·0*	*2·1*	*1·2*	*0·1*	*1·2*	*0·4*	*0·4*	*1·2*	*0·5*	*0·3*	**1·4**	*0·7*
1771	*0·8*	*1·4*	*1·6*	*1·4*	*1·0*	*1·3*	*0·8*	*0·5*	*1·1*	*0·7*	*0·5*	**0·9**	*0·8*
1772	*1·8*	*1·3*	*2·0*	*1·8*	*2·0*	*1·4*	*1·5*	*0·4*	*1·5*	*0·1*	*0·7*	**0·3**	*1·2*
1773	*2·1*	*1·6*	*1·2*	*0·7*	*1·0*	*1·0*	*0·9*	*0·6*	*1·8*	**0·2**	*0·7*	**0·6**	*0·9*
1774	*3·1*	*1·9*	*0·0*	**0 5**	*1·0*	*1·0*	*0·5*	*0·5*	*1·7*	**0·3**	*0·9*	*0·1*	*0·8*
1775	*2·5*	*1·4*	**0·4**	**0·3**	*0·5*	*1·5*	*0·6*	*0·1*	*0·8*	**0·3**	*0·8*	*0·3*	*0·6*
1776	*2·6*	*0·7*	*0·2*	*0·0*	**0·5**	*0·5*	**0·4**	*0·0*	*0·8*	*0·4*	*0·5*	**1·1**	*0·2*
1777	*1·1*	**1·4**	**1·5**	**0·7**	**1·6**	**0·4**	**2·1**	**1·4**	*0·0*	*0·4*	*0·1*	**0·2**	**0·7**
1778	*3·1*	**0·4**	**2·4**	*0·4*	**1·7**	**0·5**	**2·0**	**2·5**	**0·8**	*0·4*	*0·0*	**0·3**	**0·7**
1779	*1·7*	**1·2**	**2·9**	*0·2*	**2·2**	**1·8**	**2·4**	**2·9**	**1·0**	*0·3*	**0·5**	**0·9**	**2·0**
1780	*0·9*	**1·1**	**2·4**	*0·5*	**1·4**	**2·5**	**2·9**	**2·3**	**0·2**	*1·2*	*1·0*	**0·1**	**0·8**
1781	*1·0*	**0·9**	**1·9**	**0·3**	**0·7**	**1·6**	**3·3**	**2·2**	**0·6**	*0·3*	*0·9*	*1·0*	**0·8**
1782	*2·1*	*1·5*	*0·4*	*0·8*	**1·6**	**0·6**	**2·0**	**0·7**	**0·4**	*0·8*	*1·2*	*0·8*	**0·1**
1783	*0·2*	*2·0*	*2·5*	**0·6**	**1·1**	**1·4**	**1·5**	*1·1*	*0·3*	*0·8*	*0·3*	*1·5*	*0·3*
1784	*0·2*	*2·7*	*4·3*	**0·1**	**0·7**	**0·9**	**0·7**	*1·1*	*0·6*	*1·7*	*1·2*	*2·5*	*0·9*
1785	*0·1*	*0·8*	*3·4*	**0·8**	**1·2**	**0·3**	**0·7**	*0·3*	*0·1*	*0·9*	*0·7*	*2·2*	*0·4*
1786	**0·2**	*1·2*	*3·0*	**0·9**	**1·2**	**0·9**	**0·1**	*0·2*	*0·0*	*0·8*	*0·4*	*3·2*	*0·4*
1787	**0·7**	*0·1*	*3·1*	**1·4**	**0·8**	**1·5**	**0·6**	**0·8**	**0·1**	*0·6*	*0·2*	*1·2*	**0·1**
1788	**1·0**	**2·3**	*1·3*	**0·1**	**1·2**	**1·2**	**0·7**	**1·6**	*0·2*	*0·0*	*0·8*	*0·9*	**0·5**
1789	**1·5**	**2·7**	**0·4**	**0·6**	**1·7**	**1·0**	**1·2**	**1·5**	**0·5**	**0·6**	*0·4*	*1·6*	**0·9**
1790	**0·5**	**2·0**	*0·3*	**1·8**	**1·5**	**1·0**	**0·8**	**1·6**	*0·0*	**0·2**	**0·9**	*1·4*	**0·8**
1791	**0·2**	**2·5**	*0·4*	*0·0*	**1·3**	**0·3**	**0·8**	**1·4**	*0·3*	**1·0**	**0·6**	**0·2**	**0·7**
1792	**0·9**	**3·1**	**1·5**	**0·6**	**0·8**	**0·6**	**0·7**	**0·3**	*0·9*	**0·9**	**0·6**	*0·5*	**0·8**
1793	*1·0*	**0·5**	**0·3**	**1·1**	**0·1**	*0·5*	**0·5**	**0·6**	*0·0*	**1·3**	**0·2**	**0·5**	**0·3**
1794	*0·0*	**0·7**	*0·6*	**1·4**	*0·5*	*0·8*	**0·3**	**0·9**	*0·0*	**0·9**	*0·2*	**0·3**	**0·2**
1795	**1·2**	**1·5**	*0·9*	**0·4**	**0·2**	*0·7*	**0·6**	**0·4**	**0·6**	**0·6**	*1·4*	**0·8**	**0·3**
1796	**1·4**	**1 3**	*0·3*	**2·3**	**1·0**	**0·7**	**0·9**	**0·7**	**1·0**	**0·1**	*1·8*	*0·2*	**0·6**
1797	**1·2**	*0 1*	*1·4*	**1·1**	**0·5**	**0·3**	**0·4**	**0·6**	**1·5**	*0·5*	*1·9*	*1·2*	**0·1**
1798	**2·3**	**0·8**	*1·2*	**1·5**	**0·9**	**0·6**	**1·2**	**0·5**	**1·1**	*1·1*	*1·5*	*2·5*	**0·3**
1799	**1·3**	**0·7**	*0·5*	**0·9**	**1·5**	**0·9**	**1·4**	**0·9**	**1·3**	*0·2*	*1·3*	*2·1*	**0·5**
1800	**0·6**	*0·5*	**0·1**	**1·3**	**0·9**	**1·2**	**0·5**	**1·3**	**1·6**	**0·8**	*0·7*	*2·5*	**0·4**
1801	*0·0*	*0·7*	**0·3**	**0·7**	**0·2**	**0·2**	**0·9**	**1·2**	**1·1**	**0·6**	*0·7*	*2·0*	**0·2**
1802	**0·6**	*0·7*	**0·4**	**1·1**	**1·4**	**1·0**	**1·1**	**1·8**	**0·4**	**1·5**	*0·5*	*1·7*	**0·6**
1803	**1·0**	*0·2*	**1·2**	**0·9**	**0·7**	**0·7**	**0·7**	**1·7**	**0·7**	**1·2**	*0·1*	*1·6*	**0·6**
1804	**0·2**	*0·8*	**0·6**	*0·0*	**0·3**	**0·5**	**0·4**	**1·4**	**0·5**	**1·2**	**0·5**	*0·5*	**0·4**
1805	**0·1**	*1·2*	*0·0*	*0·4*	**0·1**	**0·4**	**1·3**	**1·3**	*0·9*	**1·4**	*1·0*	*1·0*	*0·0*
1806	*0·0*	*1·4*	*0·9*	*1·4*	**0·9**	**0·6**	**1·3**	**1·6**	*0·5*	**0·7**	*0·8*	*1·6*	*0·3*
1807	*0·8*	*1·2*	*0·1*	*1·9*	**0·5**	*0·3*	**0·9**	**1·4**	*0·2*	**1·0**	*1·1*	*1·6*	*0·3*
1808	*0·9*	*1·7*	*1·3*	*2·3*	*0·1*	**0·1**	**0·5**	**1·1**	*0·3*	**1·4**	*1·7*	*1·8*	*0·6*
1809	*1·1*	*1·5*	*0·5*	*1·8*	**0·4**	*0·5*	**0·9**	**0·7**	*0·5*	**1·8**	*1·5*	*2·6*	*0·5*
1810	*1·0*	*0·8*	*1·0*	*2·6*	**0·6**	*0·2*	**0·1**	**0·1**	**0·6**	**1·2**	*0·4*	*2·6*	*0·5*

Note.—The heavy type indicates an excess, and the italic type a defect.

TABLE LV.—*continued.*

Year.	Jan.	Feb.	Mar.	April.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.	Year.
	°	°	°	°	°	°	°	°	°	°	°	°	°
1811	*1·1*	*0·0*	**0·2**	*2·0*	*0·4*	*0·3*	*0·6*	*0·4*	**0·4**	**1·3**	*0·9*	*2·2*	*0·4*
1812	*3·0*	*0·7*	*0·7*	*0·4*	*1·2*	*0·8*	*0·2*	*0·5*	**0·7**	**0·2**	*1·0*	*2·2*	*0·8*
1813	*3·7*	**0·4**	**0·3**	*0·3*	**0·1**	*0·7*	*0·0*	*0·5*	**0·3**	*0·0*	*1·3*	*2·6*	*0·6*
1814	*3·3*	*0·0*	*1·2*	*0·8*	*0·7*	*1·1*	*0·7*	*0·7*	*0·5*	*1·0*	*2·4*	*2·7*	*1·2*
1815	*2·9*	**0·2**	*0·6*	*0·1*	*1·4*	*1·2*	*0·7*	*1·4*	*0·4*	*2·1*	*1·4*	*2·6*	*1·2*
1816	*2·5*	*0·6*	*1·8*	*0·9*	*1·2*	*0·6*	*0·5*	*1·6*	*0·5*	*0·4*	**0·4**	*2·3*	*1·0*
1817	*0·3*	*0·4*	*0·9*	*1·5*	*0·6*	*0·2*	*0·5*	*0·5*	*0·6*	*0·2*	**0·1**	*2·9*	*0·7*
1818	*0·9*	*0·7*	*1·1*	*1·2*	*0·9*	*0·4*	*0·3*	*0·8*	*0·9*	*1·0*	**0·9**	*1·9*	*0·7*
1819	*0·2*	**0·2**	**0·2**	**0·4**	*1·0*	*0·5*	*0·0*	*0·1*	**0·4**	*0·1*	**1·8**	*0·5*	**0·1**
1820	*0·1*	**0·2**	**1·0**	**0·6**	**0·4**	**0·4**	**0·2**	**0·4**	*0·3*	**1·0**	**1·7**	*0·6*	**0·5**
1821	*1·3*	*0·0*	**1·7**	**0·9**	**0·6**	*0·7*	*0·5*	**0·3**	*0·5*	*0·5*	**1·2**	*0·9*	**0·1**
1822	*1·3*	**0·5**	**1·2**	**1·0**	**0·5**	*0·2*	*0·4*	*0·8*	*0·3*	*0·6*	**1·9**	**0·1**	**0·2**
1823	**0·8**	**0·3**	**1·3**	**0·9**	**0·5**	*0·0*	*0·0*	*0·1*	**0·6**	**0·6**	**1·3**	**0·1**	**0·6**
1824	*0·7*	**0·7**	**1·1**	**0·5**	**1·4**	**1·7**	**0·9**	**0·4**	**0·1**	**0·5**	**0·4**	**0·1**	**0·6**
1825	*1·4*	*0·7*	**0·3**	**0·4**	**1·0**	**1·1**	**1·0**	**0·1**	**1·0**	**1·0**	**0·2**	**1·3**	**0·5**
1826	**0·3**	**0·5**	**0·8**	**1·0**	**1·0**	**1·8**	**1·3**	**0·3**	**1·5**	**1·7**	**0·2**	**2·5**	**1·1**
1827	*1·3*	**0·4**	**0·8**	**0·3**	**1·3**	**1·8**	**0·6**	*0·3*	**0·7**	**1·5**	*0·0*	**2·0**	**0·7**
1828	*2·2*	*0·2*	**1·4**	**0·3**	**1·1**	**0·8**	*0·2*	*1·8*	*0·3*	**1·4**	**0·8**	**1·3**	**0·3**
1829	*1·6*	*0·6*	**1·5**	*0·1*	**0·5**	**0·2**	*0·7*	*2·0*	*0·1*	**1·9**	**1·1**	**1·5**	**0·2**
1830	*0·9*	**0·5**	**1·8**	**0·1**	**0·1**	**0·1**	*0·8*	*1·6*	*0·3*	**1·9**	**0·8**	**1·1**	**0·3**
1831	*1·8*	**0·4**	**1·1**	*0·1*	**1·0**	*0·1*	*0·6*	*2·1*	*0·6*	**2·0**	**0·2**	**0·5**	*0·0*
1832	*0·0*	**0·7**	**1·7**	**0·6**	**1·2**	*0·0*	*0·0*	*1·2*	**0·1**	**2·5**	**0·9**	**1·8**	**0·7**
1833	**0·8**	**1·5**	**1·0**	**0·2**	**1·0**	**0·4**	*0·0*	**0·1**	**0·1**	**1·9**	**0·9**	**2·4**	**0·9**
1834	**1·4**	**1·2**	**0·5**	*0·3*	**1·4**	*0·0*	*0·7*	*1·1*	*1·0*	**0·4**	**0·9**	**1·9**	**0·4**
1835	**0·6**	**0·8**	*0·9*	*1·7*	**1·3**	*0·0*	*0·3*	*1·3*	*1·4*	**0·3**	**0·6**	**1·9**	*0·0*
1836	*0·2*	*1·1*	*0·9*	*2·3*	*0·7*	*0·2*	*0·3*	*0·8*	*1·3*	*0·0*	*0·2*	**2·0**	*0·5*
1837	*1·4*	*1·6*	*1·8*	*2·6*	*1·3*	*0·4*	*0·4*	*1·2*	*1·4*	*0·3*	*0·4*	**1·2**	*0·9*
1838	*1·1*	*2·1*	*1·8*	*1·5*	*1·5*	*0·2*	*0·7*	*1·2*	*1·6*	*0·1*	*0·5*	**0·7**	*0·9*
1839	*2·1*	*1·9*	*0·4*	*1·4*	*1·4*	*0·7*	*0·7*	*0·7*	*0·7*	*0·2*	*0·7*	**0·7**	*0·8*
1840	*2·1*	*1·7*	**1·1**	*0·0*	*0·5*	*0·5*	*1·3*	**0·3**	*0·0*	*0·9*	*0·6*	**1·6**	*0·3*
1841	*0·3*	*0·8*	**1·7**	**0·9**	*0·3*	*0·9*	*1·4*	**0·5**	**0·7**	*1·4*	**0·7**	**3·1**	**0·3**
1842	**0·8**	*1·1*	**2·4**	**2·1**	*0·3*	*1·1*	*1·6*	**0·3**	**0·7**	*1·5*	**0·8**	**2·1**	**0·3**
1843	**0·3**	*1·5*	**1·5**	**1·5**	*0·3*	*0·8*	*1·9*	*0·3*	**1·2**	*0·9*	**1·2**	**2·4**	**0·2**
1844	**2·0**	*0·1*	**0·6**	**1·4**	*0·1*	**0·9**	*1·3*	**0·2**	**2·2**	*0·2*	**2·4**	**1·6**	**0·8**
1845	**2·3**	*1·0*	**0·8**	**0·8**	*0·3*	**0·7**	*0·3*	*0·4*	**1·5**	**0·5**	**3·3**	**0·4**	**0·7**
1846	**1·1**	**0·2**	**0·6**	**0·5**	**1·3**	**1·2**	*0·2*	*1·2*	**0·8**	**0·9**	**2·5**	*1·1*	**0·6**
1847	**0·3**	**1·4**	**0·9**	*0·8*	**1·8**	**0·8**	*0·3*	*0·9*	**0·7**	**0·5**	**2·2**	*0·3*	**0·6**
1848	*0·8*	**2·7**	**2·0**	*0·6*	**1·8**	**1·1**	**0·6**	*0·7*	**0·4**	*0·4*	**2·1**	*0·1*	**0·7**
1849	*1·1*	**1·9**	**1·6**	*0·8*	**1·3**	*0·2*	*0·0*	*1·4*	*1·0*	**0·1**	**0·6**	**1·2**	**0·2**
1850	*0·5*	**2·8**	**1·2**	**0·3**	**1·3**	*0·4*	**0·4**	*0·9*	*0·5*	*0·5*	**0·1**	**1·7**	**0·5**
1851	**0·5**	**2·3**	**0·5**	**0·6**	**0·1**	*0·0*	**0·4**	*0·1*	*0·6*	*0·2*	**0·7**	**1·0**	**0·4**
1852	**0·4**	**1·0**	**1·0**	**1·3**	**0·1**	**1·0**	**1·0**	**0·5**	**0·3**	**0·4**	**0·7**	**1·6**	**0·9**
1853	**1·8**	*1·2*	*0·0*	*1·1*	*0·3*	**0·7**	**1·5**	**1·1**	**0·7**	**0·9**	**0·4**	**1·1**	**0·7**
1854	**1·0**	*1·1*	**0·1**	**1·5**	*0·8*	**1·4**	**2·2**	**1·5**	**0·7**	**1·1**	**1·5**	**1·1**	**0·9**
1855	**0·8**	*0·9*	**0·2**	**0·8**	*0·7*	**1·6**	**1·5**	**1·7**	**1·3**	**2·1**	**2·1**	**1·9**	**1·1**
1856	**1·1**	*0·5*	**0·8**	**0·2**	*0·4*	**1·7**	**0·8**	**1·5**	**1·5**	**1·4**	**1·3**	**2·4**	**1·1**
1857	**1·7**	*0·5*	**0·4**	*0·9*	*0·6*	**1·1**	**0·6**	**0·8**	**0·7**	**0·9**	**0·7**	**1·1**	**0·6**
1858	**1·1**	**0·1**	**0·6**	*1·8*	**0·2**	*0·1*	*0·2*	*0·2*	**0·1**	**0·7**	**0·1**	**0·3**	**0·2**
1859	**1·0**	*0·4*	**0·7**	*2·2*	**0·6**	*0·8*	*0·6*	*0·2*	**0·2**	**0·3**	*0·8*	*0·3*	*0·2*
1860	**1·1**	*0·5*	*0·0*	*2·2*	**0·5**	*1·3*	*2·1*	*1·1*	*0·6*	*0·5*	*2·6*	*1·2*	*0·8*

NOTE.—The heavy type indicates an excess, and the italic type a defect.

TABLE LV.—*continued.*

Year.	Jan.	Feb.	Mar.	April.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.	Year.
	°	°	°	°	°	°	°	°	°	°	°	°	°
1861	**0·7**	**0·5**	**0·4**	*1·9*	**0·2**	*1·9*	*1·6*	*1·2*	*1·6*	*0·1*	*1·7*	*0·8*	*0·7*
1862	*0·1*	*0·8*	*0·8*	*0·7*	**0·4**	*1·8*	*1·9*	*1·7*	*1·5*	*0·1*	*1·2*	**0·4**	*0·8*
1863	*0·0*	*0·9*	*1·1*	**0·3**	**0·4**	*0·7*	*1·6*	*1·4*	*0·2*	*0·2*	*0·6*	**2·4**	*0·3*
1864	**0·8**	*1·1*	*2·0*	**0·1**	*0·0*	*0·3*	*1·3*	*1·8*	*0·2*	*0·2*	**0·2**	**3·3**	*0·2*
1865	*0·3*	*0·6*	*2·1*	**0·3**	*0·5*	**0·4**	*1·0*	*1·1*	**0·1**	*0·3*	**1·2**	**2·9**	*0·1*
1866	*0·4*	*0·3*	*1·9*	**0·7**	**0·2**	**0·7**	*0·2*	*0·5*	**1·1**	*0·5*	**0·3**	**2·9**	*0·2*
1867	**0·6**	**1·5**	*1·7*	**1·0**	*0·9*	**0·8**	**0·4**	*0·1*	**1·4**	*0·0*	**0·4**	**2·4**	**0·5**
1868	**0·7**	**1·8**	*1·2*	**1·6**	*0·5*	**0·9**	**0·8**	**0·4**	**0·8**	**0·2**	**0·2**	**0·8**	**0·6**
1869	*0·2*	**2·7**	*0·0*	**1·6**	*0·0*	*0·0*	**0·8**	**1·1**	**0·7**	*0·3*	*0·6*	*0·0*	**0·6**
1870	**0·9**	**2·2**	**0·9**	**1·1**	*0·3*	**0·2**	**1·4**	**0·7**	**0·2**	*0·6*	*0·7*	*0·2*	**0·6**
1871	**1·4**	**0·8**	*0·1*	**0·9**	*1·2*	*0·3*	**1·0**	**0·3**	*0·2*	*0·6*	*0·1*	*0·0*	**0·2**
1872	**1·5**	**0·1**	**1·2**	**0·8**	*1·0*	*0·1*	**1·0**	**0·2**	*0·4*	*0·6*	*0·2*	*1·1*	**0·2**
1873	**2·4**	**0·5**	**1·5**	**0·4**	*1·1*	*0·5*	**0·2**	**0·2**	*0·5*	*0·5*	*0·2*	*0·1*	**0·3**
1874	**3·4**	*0·3*	**0·6**	**0·7**	*1·2*	*0·0*	**0·5**	*0·3*	*0·5*	**0·1**	**0·3**	**0·4**	**0·4**
1875	**3·6**	*0·2*	*0·0*	**0·3**	*1·2*	*0·1*	**0·2**	*0·6*	*0·7*	**0·5**	**0·7**	**0·5**	**0·3**
1876	**3·1**	**1·1**	**0·4**	**0·4**	*0·8*	**0·3**	**0·4**	*0·4*	*0·2*	**1·2**	*0·2*	*1·7*	**0·4**
1877	**1·2**	**0·3**	*1·0*	*1·0*	*0·9*	*0·4*	*0·7*	*0·5*	*0·6*	**0·7**	*0·5*	*1·1*	*0·3*
1878	**0·5**	**1·4**	*0·8*	*1·1*	*1·4*	*0·4*	*0·6*	*0·0*	*0·4*	**0·1**	*0·6*	*1·5*	*0·4*
1879	*1·8*	**1·2**	*0·9*	*1·4*	*0·9*	*0·5*	*0·8*	*0·5*	*0·2*	*1·1*	**0·5**	*1·8*	*0·8*
1880	*1·2*	**1·6**	**0·3**	*0·9*	*0·2*	*0·9*	*0·8*	*0·1*	*0·1*	*0·9*	*0·2*	*3·1*	*0·5*
1881	*1·1*	**1·1**	*0·6*	*1·0*	*0·7*	*1·5*	*1·7*	*0·4*	*0·3*	*1·2*	**0·5**	*1·1*	*0·6*
1882	**0·9**	**2·4**	**0·4**	*0·2*	*0·0*	*1·0*	*1·2*	*0·0*	**0·3**	*0·6*	**0·9**	*0·5*	**0·2**
1883	**0·9**	**2·0**	**0·2**	*0·6*	*0·6*	*1·2*	*0·7*	*1·2*	*0·3*	*0·6*	**1·2**	*0·2*	*0·0*
1884	**2·0**	**1·9**	**0·1**	*0·7*	*1·3*	*1·3*	*0·7*	*0·6*	*0·3*	**0·6**	**0·9**	*0·9*	**0·1**
1885	**1·5**	**1·2**	*1·4*	*0·9*	*1·5*	*0·4*	*0·0*	*0·4*	*0·1*	*0·1*	**0·9**	*0·3*	*0·0*
1886	**1·5**	**0·1**	*1·4*	*1·5*	*1·2*	*0·9*	*0·2*	*0·8*	*0·3*	*0·0*	**1·2**	*0·2*	*0·2*
1887	**1·2**	*0·5*	*1·8*	*1·8*	*0·7*	*0·3*	**0·1**	*1·0*	*0·7*	*0·5*	**1·7**	**0·1**	*0·3*
1888	**2·1**	*1·1*	*1·3*	*1·6*	**0·2**	*0·2*	**0·6**	*0·5*	**0·4**	**0·6**	**1·8**	*0·6*	*0·0*
1889	**2·5**	**0·5**	*1·3*	*1·6*	**0·2**	*0·1*	**0·5**	*0·8*	**0·9**	**0·1**	**1·1**	**0·3**	**0·2**
1890	**2·0**	**0·1**	*1·2*	*1·4*	**0·5**	*0·9*	*1·6*	*1·0*	**0·7**	*0·2*	**1·8**	*0·1*	*0·1*
1891	**1·7**	**0·9**	**0·2**	*0·4*	**1·3**	**0·5**	*0·9*	**0·2**	**1·0**	*0·0*	**1·2**	*0·0*	**0·5**
1892	**1·3**	**1·5**	**1·0**	**0·5**	**0·3**	*0·1*	*1·0*	**0·1**	**0·8**	**0·1**	**1·6**	**0·2**	**0·6**
1893	*0·6*	**0·1**	**0·6**	**0·8**	**0·7**	**0·2**	*0·8*	**0·6**	**0·9**	*1·0*	**2·0**	**0·7**	**0·6**
1894	**0·2**	*0·2*	**1·5**	**2·0**	**2·0**	**0·5**	*0·9*	**0·6**	**0·3**	*1·9*	**2·1**	**0·6**	**0·6**

NOTE.—The heavy type indicates an excess, and the italic type a defect.

Table LVI.

Showing the Smoothed Percentage Excess or Defect of West Winds from Average of One Hundred and Thirty-Three Years. The Values have been smoothed by continuous Five Year Groups.

Year.	Jan.	Feb.	Mar.	April.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.	Year.
	%	%	%	%	%	%	%	%	%	%	%	%	%
1766	*6*	*9*	*7*	*4*	**4**	*5*	*10*	*5*	**7**	**15**	**3**	*19*	*3·3*
1767	*6*	*6*	*5*	*5*	*2*	*4*	*10*	*5*	**3**	**9**	**9**	*10*	*2·8*
1768	**2**	*1*	*6*	*5*	*5*	*1*	*15*	**3**	*2*	**7**	**4**	*4*	*2·0*
1769	*0*	*4*	*8*	*0*	*4*	*2*	*9*	**4**	*6*	**10**	**5**	**2**	*1·3*
1770	*3*	*5*	*14*	**5**	*8*	**6**	*7*	**10**	*7*	**2**	**3**	**7**	*2·7*
1771	**1**	*0*	*5*	**12**	*12*	**5**	*2*	**11**	*3*	**5**	**5**	**2**	**0·7**
1772	*0*	**3**	**4**	**14**	*15*	**7**	**3**	**12**	*7*	**9**	**1**	*2*	**0·6**
1773	*8*	**2**	*6*	**21**	*3*	*2*	**7**	**12**	*5*	**9**	*2*	*1*	**1·1**
1774	*16*	**8**	*0*	**22**	*2*	**3**	**4**	**8**	*5*	**10**	*4*	*0*	**1·0**
1775	*14*	**8**	**6**	**19**	**5**	*1*	**3**	**2**	*2*	**7**	**2**	*3*	**3·2**
1776	*15*	**10**	*0*	**14**	**10**	**5**	**7**	**6**	*5*	**8**	*3*	**6**	**3·1**
1777	*11*	**11**	**5**	**20**	**17**	*3*	**3**	**3**	**2**	*1*	*2*	**9**	**4·5**
1778	*16*	**6**	**6**	**10**	**17**	**7**	**5**	*7*	**5**	*0*	**6**	**1**	**3·3**
1779	*11*	**2**	**4**	**3**	**13**	*0*	**4**	*9*	**8**	**1**	**6**	*2*	**2·2**
1780	*6*	*3*	**4**	*3*	**10**	**3**	**6**	*9*	**3**	**6**	*7*	*5*	**0·2**
1781	*3*	*15*	**3**	*3*	**9**	*2*	**7**	*13*	**4**	**15**	*6*	*12*	*0·4*
1782	*4*	*26*	*3*	*7*	**11**	**2**	**6**	*9*	**2**	**11**	*2*	*25*	*2·5*
1783	*1*	*22*	*16*	**1**	**11**	**1**	**7**	*9*	**1**	**11**	*1*	*30*	*3·3*
1784	*1*	*27*	*18*	*1*	**19**	**3**	**16**	*4*	**7**	*0*	*13*	*30*	*4·1*
1785	*3*	*17*	*14*	**8**	**19**	*2*	**14**	*1*	**6**	**1**	*12*	*34*	*2·5*
1786	*4*	*19*	*21*	**13**	**17**	*7*	**17**	*2*	**4**	*3*	*11*	*36*	*3·5*
1787	*11*	*11*	*24*	**13**	**12**	*9*	**16**	**2**	**1**	*7*	*15*	*22*	*4·1*
1788	*5*	**3**	*15*	*0*	**2**	*7*	**17**	**5**	**6**	*13*	*23*	*14*	*3·3*
1789	*0*	**6**	*7*	*1*	*1*	*7*	**15**	**5**	*1*	*15*	*15*	*11*	*2·5*
1790	*7*	*0*	*12*	*0*	**2**	*9*	**15**	*5*	*1*	*20*	*7*	*3*	*3·3*
1791	*8*	**8**	*4*	*11*	**6**	**1**	**13**	*5*	**1**	*20*	*17*	**4**	*2·5*
1792	**4**	**5**	**6**	*10*	**11**	**3**	**12**	*5*	*1*	*12*	*18*	**2**	*0·3*
1793	*6*	*9*	**1**	*4*	**17**	*7*	**6**	*4*	*5*	*11*	*13*	**4**	*2·7*
1794	*5*	*9*	*7*	**5**	**14**	**1**	**5**	**1**	*3*	**1**	*13*	**5**	*0·6*
1795	**1**	*1*	*9*	**2**	**14**	**2**	**5**	**14**	**2**	**9**	*13*	**7**	**2·4**
1796	**7**	**1**	*5*	**5**	**13**	**1**	**10**	**15**	**3**	**9**	*6*	**2**	**4·4**
1797	**3**	*3*	*12*	*1*	**10**	**7**	**5**	**15**	**4**	**7**	*0*	*11*	**2·1**
1798	**3**	*3*	*12*	**4**	**8**	**12**	**12**	**12**	**4**	**12**	**7**	*15*	**3·8**
1799	**3**	*3*	*7*	**3**	**10**	**11**	**7**	**6**	*1*	**7**	**10**	*17*	**2·4**
1800	**3**	*5*	**3**	**12**	**7**	**15**	**2**	**2**	*3*	**10**	**6**	*20*	**2·9**
1801	*8*	*5*	**6**	**17**	**10**	**13**	*3*	**1**	*3*	**6**	**5**	*19*	**1·9**
1802	*11*	*3*	**6**	**20**	**15**	**13**	*4*	**3**	*0*	**3**	*4*	*13*	**2·6**
1803	*7*	**6**	**14**	**13**	**11**	**9**	*8*	**9**	**4**	*7*	*7*	*11*	**2·5**
1804	*9*	**11**	**6**	**8**	**5**	**9**	*10*	**14**	**13**	*12*	*7*	*6*	**2·0**
1805	*6*	**9**	*6*	**1**	**2**	**8**	*4*	**14**	**12**	*14*	*2*	*7*	**0·7**
1806	**3**	**6**	*20*	*2*	*3*	**4**	*13*	**10**	**7**	*8*	*4*	*7*	*2·2*
1807	*5*	**6**	*21*	*3*	*8*	**2**	*10*	**2**	**3**	*9*	*1*	**1**	*3·5*
1808	*2*	**7**	*27*	*0*	*4*	**4**	*10*	*1*	*0*	*1*	*10*	**2**	*3·4*
1809	*4*	**8**	*21*	*0*	*4*	**2**	*5*	*1*	*8*	**1**	*9*	**2**	*3·6*
1810	*4*	**5**	*19*	*6*	*2*	**2**	*8*	*11*	*9*	*2*	*13*	*4*	*6·1*

Note.—The heavy type indicates an excess, and the italic type a defect.

TABLE LVI.—*continued.*

Year.	Jan.	Feb.	Mar.	April.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.	Year.
	°/₀	°/₀	°/₀	°/₀	°/₀	°/₀	°/₀	°/₀	°/₀	°/₀	°/₀	°/₀	°/₀
1811	*4*	**8**	*7*	*8*	*6*	*5*	*3*	*8*	*10*	*14*	*7*	*1*	*5·5*
1812	**3**	**9**	*7*	*0*	*12*	*11*	**3**	*3*	*9*	*8*	*1*	*7*	*3·5*
1813	**3**	**7**	**2**	*4*	*11*	*14*	**3**	**1**	*8*	*12*	**8**	*8*	*2·7*
1814	**2**	**10**	*1*	*7*	*8*	*15*	*1*	*5*	*6*	*15*	**4**	*11*	*4·9*
1815	**3**	**11**	**6**	*1*	*6*	*17*	*6*	*0*	*6*	*31*	**11**	*4*	*2·5*
1816	**4**	**11**	**8**	*7*	*9*	*6*	*3*	*7*	*6*	*16*	*0*	*4*	*3·1*
1817	**9**	**12**	**15**	*11*	*4*	**4**	*12*	*11*	**1**	*20*	*2*	*2*	*1·8*
1818	**9**	**13**	**10**	*5*	*6*	**6**	*16*	*12*	**3**	*18*	*3*	*10*	*2·3*
1819	**10**	**9**	**13**	**2**	*8*	*4*	*12*	*10*	**8**	*9*	*2*	*6*	**0·2**
1820	**8**	**8**	**14**	*2*	*11*	*5*	*14*	*5*	**6**	*5*	*1*	*7*	**0·1**
1821	*5*	**2**	**12**	**8**	*4*	*6*	*14*	**7**	**12**	*4*	**9**	*3*	**1·2**
1822	*2*	*3*	**12**	**9**	*8*	*9*	*9*	**6**	**7**	*4*	**12**	**4**	**1·4**
1823	**2**	**2**	**10**	**6**	*14*	*4*	*7*	**4**	**6**	*0*	**13**	**8**	**2·4**
1824	**3**	**8**	**7**	**9**	*16*	**9**	*4*	**9**	**1**	*2*	**15**	**7**	**3·9**
1825	*3*	*3*	**5**	**8**	*7*	**14**	**2**	**6**	**4**	*2*	**14**	**12**	**3·5**
1826	**3**	*1*	**6**	**9**	*17*	**16**	*6*	*8*	*2*	**2**	**8**	**9**	**1·5**
1827	*9*	**4**	**4**	**6**	*13*	**12**	*13*	*8*	*3*	**2**	**4**	**1**	*1·5*
1828	*6*	*0*	**3**	**8**	*9*	**8**	*8*	*14*	*0*	**3**	**4**	*2*	*2·1*
1829	*20*	*4*	**9**	*2*	*8*	**5**	*10*	*17*	*4*	**5**	**5**	**1**	*3·5*
1830	*18*	**7**	**10**	*6*	*10*	*2*	*14*	*16*	**2**	**15**	**5**	*0*	*2·6*
1831	*14*	**9**	*0*	*1*	*5*	*4*	*5*	*7*	**5**	**10**	**13**	**3**	**0·1**
1832	*5*	**10**	**11**	*1*	*2*	**2**	*7*	*3*	*0*	**17**	**13**	**9**	**3·6**
1833	**2**	**11**	**10**	*4*	*2*	*0*	*7*	*1*	*5*	**15**	**9**	**14**	**3·6**
1834	**1**	**10**	**10**	*1*	*3*	**4**	*5*	*1*	*2*	**9**	**5**	**8**	**3·8**
1835	**5**	**8**	**2**	**2**	*1*	**10**	*5*	*4*	*12*	**9**	**5**	**4**	**2·4**
1836	**2**	**4**	**8**	*2*	*9*	**8**	*5*	*5*	*13*	**13**	*4*	**4**	**0·2**
1837	**2**	**5**	*0*	**1**	*13*	*1*	**3**	*6*	*5*	**5**	*9*	*3*	*1·6*
1838	**3**	*4*	*5*	*0*	*16*	**3**	**5**	*1*	*3*	**4**	*7*	*2*	*1·6*
1839	*1*	*9*	*4*	**1**	*8*	*3*	**1**	**1**	*5*	**1**	*6*	**3**	*2·3*
1840	**1**	*9*	**4**	*3*	*8*	*8*	**1**	**6**	*4*	*1*	*15*	**9**	*2·2*
1841	**10**	*11*	**3**	*0*	*7*	*14*	**3**	**8**	*3*	*2*	*6*	**11**	*0·7*
1842	**11**	*15*	**9**	**6**	*11*	*10*	**1**	**9**	*11*	*0*	*4*	**4**	*0·9*
1843	**10**	*3*	**11**	**2**	*12*	*7*	*3*	**6**	*12*	**1**	*3*	**7**	*1·0*
1844	**11**	*3*	**9**	*4*	*12*	*2*	**1**	*1*	*11*	**2**	*6*	**3**	*1·0*
1845	**4**	*6*	*1*	**1**	*13*	*0*	**2**	*1*	*4*	*6*	**5**	*2*	*1·8*
1846	*1*	**6**	*1*	*4*	*6*	**2**	**6**	*3*	*3*	*12*	**5**	*7*	*1·2*
1847	*4*	**12**	*2*	*14*	*4*	**4**	**7**	*3*	*6*	*10*	**11**	*3*	*1·1*
1848	*10*	**17**	**1**	*15*	**2**	**4**	**8**	**5**	*9*	*10*	**14**	*2*	**0·7**
1849	*7*	**16**	*0*	*9*	**4**	**6**	**7**	**9**	*7*	*1*	**18**	**4**	**3·1**
1850	**4**	**21**	**3**	*14*	**3**	**5**	**1**	**6**	*10*	**2**	**14**	**4**	**3·1**
1851	**3**	**7**	*0*	*7*	*6*	**10**	**1**	**8**	*9*	**6**	**12**	*0*	**2·2**
1852	**1**	**6**	**4**	*4*	**4**	**8**	**1**	**10**	**2**	**8**	**8**	**11**	**5·1**
1853	**1**	*8*	**4**	**6**	**1**	**7**	*2*	**9**	**5**	**7**	**2**	**7**	**4·4**
1854	*3*	*10*	*1*	**9**	*1*	*3*	*5*	**11**	**1**	**3**	*0*	**4**	**0·5**
1855	*8*	*8*	*1*	**12**	*2*	**1**	**7**	**6**	*0*	**3**	*4*	**9**	**1·6**
1856	*7*	*5*	**2**	**9**	**2**	**5**	**2**	**5**	**3**	**6**	*4*	**19**	**0·4**
1857	*1*	*3*	**2**	**8**	*10*	**4**	*0*	**4**	**4**	**1**	**2**	**13**	**1·9**
1858	**3**	**8**	**5**	*1*	*4*	**1**	*0*	**2**	**7**	**3**	*2*	**7**	**2·2**
1859	**8**	**9**	**14**	*6*	*5*	*2*	**1**	**7**	**16**	*0*	*5*	**7**	**4·5**
1860	**12**	**3**	**8**	*3*	*1*	**1**	**2**	**12**	**19**	**11**	**2**	**3**	**5·9**

NOTE.—The heavy type indicates an excess, and the italic type a defect.

TABLE LVI.—*continued.*

Year.	Jan.	Feb.	Mar.	April.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.	Year.
	°/₀	°/₀	°/₀	°/₀	°/₀	°/₀	°/₀	°/₀	°/₀	°/₀	°/₀	° /	°/₀
1861	**15**	**12**	**9**	*1*	*5*	*4*	*3*	**10**	**15**	**8**	*2*	*2*	**7·2**
1862	**13**	**5**	**1**	*1*	**10**	*1*	*3*	**7**	**14**	**4**	*7*	*1*	**4·6**
1863	**11**	*5*	*3*	**6**	**6**	*3*	**6**	*0*	**15**	*7*	*1*	*3*	**3·0**
1864	**11**	*7*	*13*	*3*	**6**	*5*	*1*	*7*	*4*	*11*	*4*	*5*	*1·3*
1865	*3*	*2*	*11*	**6**	*3*	*2*	*11*	*2*	*3*	*9*	*4*	*7*	*0·0*
1866	*3*	*1*	*5*	*3*	*0*	*5*	*14*	*6*	*5*	*5*	*2*	*2*	*1·3*
1867	*7*	*6*	*4*	*5*	*6*	*1*	*12*	*1*	*5*	*0*	*7*	*6*	*0·2*
1868	*6*	*8*	*9*	**10**	*3*	*3*	*11*	*0*	*2*	*4*	**13**	*3*	*1·4*
1869	*8*	*6*	*2*	*9*	*4*	*2*	*2*	*5*	*5*	*3*	*3*	*0*	*1·0*
1870	*5*	*2*	*3*	*5*	*4*	*2*	*2*	*1*	*7*	*1*	*1*	*5*	*0·2*
1871	*1*	*6*	*10*	*1*	*2*	*2*	*8*	*0*	*1*	*1*	*2*	*1*	*0·6*
1872	*5*	*10*	*5*	*1*	*1*	*2*	*9*	*1*	*1*	*5*	*5*	*2*	*0·2*
1873	*7*	*11*	*5*	*3*	*2*	*3*	*6*	*1*	*4*	*3*	*13*	*5*	*1·1*
1874	*8*	*11*	*8*	*2*	*1*	*4*	*5*	*5*	*3*	*2*	*13*	*2*	*2·2*
1875	*8*	*6*	*4*	*5*	*3*	*3*	**11**	*7*	*5*	*0*	*7*	*1*	*1·3*
1876	**10**	*1*	*8*	*5*	*5*	*3*	*3*	*17*	*4*	*4*	*7*	*2*	*4·6*
1877	*2*	*6*	*1*	*13*	*0*	*6*	*2*	*19*	*6*	*6*	*7*	*5*	*4·6*
1878	*6*	*3*	*0*	*14*	*4*	*12*	*2*	*23*	*0*	*3*	*2*	*4*	*3·7*
1879	*4*	*3*	*1*	*19*	*2*	*12*	*1*	*22*	*1*	*4*	**10**	**14**	*1·9*
1880	*7*	*4*	*4*	*16*	*0*	*12*	*2*	*13*	*5*	*9*	*7*	**10**	*0·4*
1881	*4*	*2*	*5*	*13*	*4*	*7*	*6*	*0*	*1*	*6*	**12**	**10**	*0·7*
1882	**11**	*8*	*0*	*13*	*8*	*4*	**13**	*3*	*1*	*5*	**14**	*8*	*4·0*
1883	*5*	*6*	*7*	*12*	**12**	*1*	**22**	*2*	*1*	*7*	*9*	*8*	*5·0*
1884	*4*	*6*	*4*	*12*	**10**	*0*	**16**	*7*	*4*	*5*	*8*	*6*	*4·2*
1885	*3*	*4*	*4*	*5*	**10**	*1*	**16**	*1*	*5*	*1*	*6*	**11**	*5·3*
1886	*6*	*0*	*6*	*4*	**10**	*6*	**12**	*2*	*6*	*3*	*2*	**13**	*3·4*
1887	*4*	*0*	*4*	*0*	*2*	*9*	*2*	*4*	*2*	*3*	*1*	**14**	*1·2*
1888	**10**	*6*	*4*	*0*	*4*	*7*	*2*	*4*	*1*	*5*	*4*	*5*	*1·3*
1889	**13**	*0*	*6*	*1*	*6*	*11*	*1*	*0*	*5*	**12**	*0*	*8*	*2·0*
1890	**12**	*4*	*0*	*5*	*4*	*12*	*8*	*3*	*7*	*6*	*0*	*2*	*0·3*
1891	*8*	*2*	*5*	*7*	*7*	*6*	*6*	*0*	**10**	*6*	*2*	*2*	*0·4*
1892	*7*	*3*	*7*	*12*	*1*	*5*	*4*	*0*	*5*	*5*	*5*	*3*	*0·9*
1893	*3*	*1*	*3*	*11*	*1*	*8*	*5*	*4*	*6*	*1*	*1*	*6*	*1·0*
1894	*2*	*2*	*9*	*1*	*1*	*6*	*7*	*4*	*1*	*6*	*0*	*7*	*1·0*

NOTE.—The heavy type indicates an excess, and the italic type a defect.

Table LVII.

Showing the Departure of Pressure from the Normal, Smoothed by continuous Five Year Groups.

Year.	Jan.	Feb.	Mar.	April.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.	Year.
					Hundredths of an Inch.								Thousandths of an Inch.
1772	*6*	*5*	**2**	*5*	**2**	*4*	**4**	*1*	*12*	*15*	*6*	*2*	*40*
1773	*11*	*11*	**1**	**1**	**6**	**2**	**1**	*6*	*11*	*12*	**1**	**5**	*29*
1774	*6*	*26*	*1*	**2**	**11**	*3*	*0*	*5*	*13*	*5*	*3*	**11**	*32*
1775	*5*	*22*	**2**	**5**	**4**	*7*	*1*	*3*	*9*	**3**	**4**	**11**	*15*
1776	*4*	*25*	*5*	**5**	**2**	*6*	*5*	*2*	*0*	**4**	**2**	**12**	*18*
1777	**5**	*20*	*5*	**4**	*5*	*1*	*2*	**4**	**2**	**3**	*2*	**4**	*10*
1778	**10**	*17*	*5*	*4*	*11*	*3*	*2*	**11**	**2**	**7**	*2*	**10**	*1*
1779	**11**	*8*	**1**	*8*	*10*	*1*	**1**	**11**	**2**	**10**	*3*	**8**	**12**
1780	**7**	*3*	*2*	*12*	*12*	**2**	**2**	**13**	*1*	**16**	*0*	**10**	**17**
1781	**1**	*4*	*3*	*3*	*8*	*0*	**4**	**10**	*7*	**19**	**6**	**15**	**25**
1782	*3*	*6*	*4*	*4*	*2*	*6*	**1**	**6**	*5*	**23**	**7**	**19**	**22**
1783	*6*	**1**	**7**	**6**	**1**	*2*	**1**	*1*	*6*	**21**	*1*	**13**	**31**
1784	*14*	**6**	**5**	**10**	*3*	**1**	**3**	*0*	*10*	**20**	**7**	**9**	**29**
1785	*4*	*0*	**6**	**13**	**2**	*1*	*0*	*2*	*10*	**12**	**4**	**5**	**21**
1786	**7**	**2**	**5**	**10**	**3**	**1**	*1*	*1*	*8*	**18**	**4**	**5**	**41**
1787	**1**	*7*	**3**	**9**	*0*	**3**	*2*	*1*	*10*	**7**	**2**	*3*	**4**
1788	**4**	*8*	**2**	**4**	*0*	**1**	*4*	*2*	*8*	**6**	**4**	*9*	*8*
1789	*4*	*10*	**2**	*0*	**1**	*3*	*10*	**1**	*0*	*1*	*4*	*8*	*13*
1790	*12*	*5*	*0*	*4*	*1*	*1*	*10*	**1**	*4*	*0*	*3*	*12*	*22*
1791	*13*	*6*	**2**	*5*	**1**	*5*	*10*	*0*	*2*	*6*	*3*	*17*	*33*
1792	*8*	*2*	**4**	*2*	**3**	**1**	*4*	*0*	*0*	*4*	*3*	*10*	*2*
1793	*4*	*6*	*4*	*6*	**7**	**1**	**1**	**3**	**5**	*10*	*2*	*7*	**1**
1794	**2**	*6*	*1*	*0*	**5**	*0*	*1*	**4**	**2**	*9*	**3**	**1**	*0*
1795	**7**	*1*	**5**	*0*	**5**	**1**	**1**	*0*	**3**	*8*	**4**	*0*	**12**
1796	**5**	**5**	**8**	*2*	**4**	**4**	*6*	**2**	*3*	*9*	*4*	**5**	**5**
1797	**5**	**6**	**8**	*4*	**1**	**3**	*10*	*3*	*5*	*10*	*3*	**8**	*6*
1798	*9*	**8**	**10**	*7*	*5*	**3**	*9*	*1*	*12*	*6*	*9*	**4**	*28*
1799	*3*	**6**	**2**	*9*	*3*	**7**	*6*	**2**	*12*	*7*	*12*	*5*	*36*
1800	*6*	*4*	**4**	*9*	**1**	**4**	*10*	**6**	*6*	*9*	*14*	*3*	*38*
1801	*4*	*9*	**5**	*10*	*5*	**2**	**2**	**5**	**2**	*4*	*15*	*11*	*34*
1802	*11*	*0*	*0*	*9*	*6*	**2**	**4**	**10**	**8**	*5*	*9*	*13*	*24*
1803	*8*	*3*	*0*	*2*	*3*	**2**	**1**	**7**	**12**	**1**	**9**	*12*	*1*
1804	*11*	*3*	**2**	**1**	*1*	**3**	*0*	*0*	**13**	**4**	**1**	*15*	*5*
1805	*12*	*3*	**5**	**2**	*4*	**8**	**2**	*1*	**9**	**7**	*6*	*10*	*4*
1806	*16*	**4**	**8**	**2**	*3*	**8**	**1**	*3*	**4**	*1*	**2**	*8*	*1*
1807	*15*	*7*	**16**	**5**	*0*	**6**	**3**	*5*	*2*	**7**	**1**	*19*	*7*
1808	*6*	*4*	**13**	**4**	**1**	**8**	*0*	*7*	*0*	**6**	*12*	*18*	*13*
1809	**4**	*10*	**18**	**3**	*3*	**5**	**5**	*4*	**2**	**1**	*5*	*13*	*4*
1810	**1**	*12*	**13**	**2**	*3*	**4**	**7**	*1*	**6**	*7*	**2**	*8*	*1*

Note.—The heavy type indicates an excess, and the italic type a defect.

TABLE LVII.—*continued.*

Year.	Jan.	Feb.	Mar.	April.	May.	June.	July.	Aug	Sept.	Oct.	Nov.	Dec.	Year.
					Hundredths		of an	Inch.					Thousandths of an Inch.
1811	8	*22*	9	*0*	*5*	5	4	2	9	*5*	*1*	*7*	*3*
1812	10	*14*	4	*1*	*2*	7	4	5	15	*11*	*7*	*2*	9
1813	7	*17*	1	1	*4*	4	9	5	13	*13*	2	*2*	6
1814	1	*9*	*5*	2	*3*	4	2	6	12	*9*	*1*	*1*	*10*
1815	*3*	*5*	*8*	8	*6*	1	*3*	*1*	8	4	*2*	*11*	*20*
1816	*12*	*4*	*19*	7	*1*	*1*	1	*1*	3	5	2	*7*	*27*
1817	*13*	*12*	*18*	7	*4*	*5*	2	3	*0*	8	4	*4*	*32*
1818	*12*	*4*	*11*	6	*6*	*4*	*0*	2	2	5	3	2	*19*
1819	*3*	5	*17*	3	*5*	3	5	2	1	4	*0*	*5*	*7*
1820	5	8	*15*	*3*	2	9	6	5	1	*5*	*6*	5	9
1821	10	3	*9*	*3*	*1*	7	1	*0*	3	*8*	*1*	*5*	*3*
1822	15	8	*10*	*2*	*0*	10	*0*	*3*	2	*10*	*8*	*8*	*6*
1823	19	7	*6*	*1*	5	9	4	*2*	*0*	*8*	*13*	*18*	*4*
1824	21	*4*	*6*	5	9	10	4	*3*	2	9	*10*	*7*	20
1825	15	*0*	1	4	2	5	8	2	2	*6*	1	*16*	13
1826	16	7	3	1	3	5	5	2	2	*0*	*4*	*12*	21
1827	15	9	7	*7*	4	6	*0*	*0*	*2*	5	7	*0*	35
1828	14	1	5	*13*	*0*	3	*7*	*1*	*6*	11	9	3	15
1829	9	*1*	1	*16*	*2*	*5*	*6*	*1*	*8*	8	6	*1*	*12*
1830	12	*3*	6	*14*	2	*5*	*4*	*6*	*6*	10	2	2	*2*
1831	19	*10*	8	*15*	5	*8*	3	*3*	*7*	5	1	*5*	*7*
1832	11	*10*	7	*1*	3	*11*	9	*3*	*0*	4	*2*	*5*	3
1833	10	*14*	5	9	4	*6*	11	1	*1*	*4*	1	4	20
1834	9	*6*	*4*	10	10	*8*	9	2	*1*	*1*	1	5	19
1835	10	*9*	1	7	10	*7*	5	4	*2*	*1*	*1*	6	15
1836	7	*1*	*5*	6	8	*5*	4	1	*2*	2	*4*	18	22
1837	12	*5*	*11*	2	8	*4*	*0*	3	*12*	6	*8*	7	*3*
1838	2	1	*1*	1	8	*8*	*3*	1	*10*	11	*16*	8	*9*
1839	2	1	5	*0*	*1*	*5*	*4*	*3*	*12*	7	*16*	4	*19*
1840	2	2	*1*	6	*3*	*3*	*4*	*2*	*12*	9	*15*	4	*16*
1841	*9*	1	4	7	*6*	*2*	*5*	*0*	*8*	3	*13*	9	*16*
1842	*6*	*2*	3	7	*1*	*3*	*4*	*3*	2	*6*	*11*	18	*6*
1843	*2*	*2*	*4*	3	*1*	*4*	*2*	*5*	6	*9*	*10*	7	*18*
1844	*5*	*3*	*6*	2	1	*3*	*2*	*2*	11	*10*	*6*	13	*8*
1845	*8*	*2*	*0*	*8*	*0*	*5*	1	*2*	9	*11*	*3*	9	*17*
1846	*1*	*9*	*7*	*7*	3	*8*	*0*	*5*	4	*8*	*1*	1	*32*
1847	*5*	1	*1*	*14*	*3*	*6*	*1*	*2*	4	*4*	*2*	*1*	*30*
1848	*0*	*4*	2	*19*	*5*	*4*	*0*	*2*	7	*5*	*0*	4	*23*
1849	*2*	*0*	3	*16*	*2*	*6*	*0*	*1*	12	*0*	1	10	*2*
1850	*10*	*2*	7	*7*	*1*	*11*	*1*	*5*	15	*1*	*6*	5	*14*
1851	*18*	5	14	*7*	*1*	*9*	*4*	*2*	14	*5*	*4*	9	*5*
1852	*20*	4	15	*1*	*5*	*12*	*3*	*2*	14	*6*	*3*	2	*13*
1853	*17*	8	4	6	*5*	*11*	*4*	*1*	13	*11*	5	1	*10*
1854	*19*	5	12	3	*8*	*11*	*2*	*3*	5	*5*	9	*10*	*22*
1855	*13*	5	4	*1*	*6*	*3*	*4*	4	2	*5*	19	2	4
1856	*1*	11	3	*0*	*10*	1	*1*	5	5	1	19	*4*	24
1857	6	5	*4*	*6*	*4*	4	3	4	*2*	*0*	21	*4*	20
1858	*7*	7	*4*	*5*	*4*	*2*	6	*2*	*4*	*5*	18	*5*	2
1859	4	5	*15*	3	*0*	*1*	*0*	*4*	*5*	2	8	3	2
1860	5	8	*16*	2	*3*	*6*	*2*	*9*	*3*	*1*	4	*1*	*17*

NOTE.—The heavy type indicates an excess, and the italic type a defect.

TABLE LVII.—*continued.*

Year.	Jan.	Feb.	Mar.	April.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.	Year.
					Hundredths of an Inch.								Thousandths of an Inch.
1861	*7*	**9**	*17*	**4**	*0*	*11*	**2**	*10*	*8*	*5*	**2**	**2**	*32*
1862	*4*	**13**	*19*	**10**	**1**	*11*	**1**	*5*	*5*	**2**	**2**	**9**	*5*
1863	*3*	**12**	*13*	**14**	**2**	*0*	*1*	**1**	*0*	*1*	*1*	**16**	**23**
1864	*11*	**10**	*9*	**10**	**2**	*1*	**6**	*0*	*2*	**2**	**6**	**10**	**20**
1865	*11*	**7**	*5*	**5**	**5**	**4**	**8**	*1*	*4*	**1**	**11**	**12**	**30**
1866	*8*	**1**	*5*	**5**	**3**	**8**	**6**	*1*	*0*	**5**	**13**	*0*	**24**
1867	*13*	*3*	*1*	**1**	*0*	**10**	**5**	*2*	*3*	**4**	**13**	*4*	**4**
1868	*7*	*3*	**2**	*1*	*0*	**7**	**7**	**2**	*7*	**5**	**10**	*5*	**7**
1869	*5*	**2**	**6**	*4*	**2**	**9**	**3**	**6**	*0*	**1**	**13**	*1*	**34**
1870	*10*	*1*	**2**	**1**	*1*	**5**	**5**	**9**	*3*	*1*	*0*	*10*	*5*
1871	*12*	**5**	**4**	**6**	*0*	**3**	**2**	**10**	*3*	*2*	*1*	**5**	**10**
1872	*14*	**9**	**7**	**3**	**3**	**5**	**1**	**2**	*2*	*8*	*0*	**5**	**9**
1873	*17*	**13**	**9**	**3**	**2**	*0*	**1**	**1**	*2*	*5*	**3**	**5**	**12**
1874	*9*	**10**	*2*	**4**	**4**	*1*	**6**	*0*	*6*	*6*	*0*	*5*	*4*
1875	*5*	**10**	*3*	**2**	**4**	**1**	**3**	*4*	**1**	*2*	*2*	**2**	**7**
1876	**6**	**10**	*0*	*2*	*0*	**2**	**7**	*5*	**1**	*6*	*3*	*5*	**6**
1877	**12**	**3**	*2*	*2*	**1**	*6*	**3**	*4*	**3**	**3**	**4**	*2*	**15**
1878	**22**	*8*	*4*	*6*	**5**	*4*	*0*	*3*	**2**	**7**	**3**	*2*	**10**
1879	**17**	*5*	**3**	*3*	**2**	*5*	*3*	*5*	**7**	**10**	*1*	**6**	**20**
1880	**27**	**1**	**5**	*4*	**5**	*7*	*4*	*5*	**2**	**10**	**2**	**1**	**24**
1881	**21**	*5*	**5**	*1*	**9**	*6*	*9*	*3*	*0*	**15**	*3*	**8**	**27**
1882	**16**	*0*	**2**	**1**	**6**	*1*	*5*	**1**	**2**	**12**	*5*	*2*	**23**
1883	**7**	*2*	**1**	*0*	*2*	**1**	*2*	*0*	*2*	**7**	*4*	**4**	**6**
1884	*1*	**3**	**3**	*3*	*5*	**2**	*2*	**2**	*3*	**2**	*1*	*3*	*6*
1885	*8*	**6**	**8**	**1**	*5*	**9**	**3**	**4**	*3*	**6**	**1**	*1*	**17**
1886	*0*	**10**	**1**	*2*	*5*	**9**	**5**	**5**	**5**	**8**	**2**	*4*	**25**
1887	**5**	**11**	**3**	*4*	*6*	**10**	**1**	**1**	**6**	**2**	**2**	**1**	**26**
1888	**1**	**26**	*4*	*4*	*3*	**7**	*3*	*2*	**12**	**7**	*1*	**2**	**32**
1889	**10**	**31**	*6*	*0*	*5*	**11**	*7*	*7*	**9**	**3**	*1*	**7**	**38**
1890	**8**	**23**	*6*	*0*	*7*	**6**	*9*	*9*	**8**	*3*	**4**	**11**	**21**
1891	**6**	**13**	**2**	**7**	*5*	**6**	*9*	*9*	*0*	*7*	**10**	**10**	**19**
1892	*2*	**12**	*0*	**9**	*3*	**4**	*7*	*7*	**5**	*2*	**4**	**7**	**16**
1893	**1**	**10**	*1*	**10**	**3**	**7**	*7*	*8*	**5**	*2*	**5**	*2*	**16**
1894	**6**	**7**	*4*	**10**	**10**	**4**	*3*	*1*	**2**	*5*	**11**	*3*	**32**

NOTE.—The heavy type indicates an excess, and the italic type a defect.

Table LVIII.

Showing the Smoothed Percentage Excess or Defect of Rainfall from the Normal.
The Averages have been Smoothed by continuous Five Year Groups.

Year.	Jan.	Feb.	Mar.	April.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.	Year.
	°/₀	°/₀	°/₀	°/₀	°/₀	°/₀	°/₀	°/₀	°/₀	°/₀	°/₀	°/₀	°/₀
1768	*13*	**33**	*10*	*5*	**13**	**8**	**2**	*27*	**8**	*11*	**66**	**9**	**4**
1769	*6*	**37**	*28*	*19*	*11*	*18*	**6**	*14*	**2**	**9**	**76**	**1**	**3**
1770	*7*	**26**	*33*	*10*	*9*	**7**	*2*	*7*	**18**	**18**	**93**	**6**	**9**
1771	*0*	*15*	*25*	**22**	**1**	*13*	*28*	*6*	**19**	**13**	**90**	**4**	**4**
1772	**7**	*19*	*33*	**17**	**14**	**2**	*27*	**4**	**19**	**13**	**79**	*2*	**6**
1773	**46**	**5**	*23*	*2*	**4**	*12*	**2**	**3**	**23**	**45**	**53**	*27*	**11**
1774	**69**	**19**	*11*	**9**	*4*	**6**	**11**	*3*	**31**	**15**	**44**	*17*	**14**
1775	**48**	**43**	*10*	**29**	*6*	**1**	*1*	*5*	**11**	**45**	**20**	*26*	**12**
1776	**48**	**36**	*0*	**7**	*2*	**15**	**28**	*1*	*1*	**71**	**52**	*9*	**19**
1777	**34**	**32**	*8*	**19**	*7*	*2*	**53**	*28*	**16**	**95**	**21**	**2**	**20**
1778	*5*	**5**	*11*	**55**	**11**	**5**	**21**	*41*	**22**	**75**	**2**	**1**	**12**
1779	*20*	**9**	*30*	**59**	**26**	**3**	**11**	*13*	**9**	**69**	**14**	**24**	**14**
1780	**36**	*2*	*0*	**46**	**64**	*4*	**9**	*4*	**41**	**36**	*2*	**33**	**20**
1781	**44**	**34**	*7*	**21**	**57**	*5*	*4*	**3**	**64**	**23**	*8*	**4**	**17**
1782	**36**	**22**	**19**	**12**	**39**	*7*	*31*	**6**	**15**	*5*	*3*	*6*	*4*
1783	**43**	**39**	**9**	*27*	**16**	*15*	*21*	**15**	**63**	*8*	*22*	*0*	**12**
1784	**51**	**21**	**19**	*41*	**19**	*33*	**7**	*17*	**64**	**14**	**6**	*27*	**7**
1785	*23*	**9**	*5*	*48*	**11**	*19*	**30**	*31*	**31**	**9**	**8**	**36**	**2**
1786	*42*	*10*	*5*	*27*	**4**	*20*	**22**	*42*	**27**	*23*	*15*	**37**	*7*
1787	*21*	*6*	*20*	*37*	**3**	*27*	**31**	*42*	**42**	*2*	**14**	**46**	**3**
1788	*16*	*13*	*13*	*9*	**19**	*10*	**27**	*30*	*22*	*5*	*2*	**58**	*0*
1789	*19*	**0**	*12*	**25**	**14**	**12**	*3*	*19*	*21*	*4*	**7**	**54**	**3**
1790	*3*	**2**	*3*	**28**	*1*	**25**	*6*	*10*	**1**	**11**	**20**	**16**	**7**
1791	**2**	**3**	**15**	**21**	**1**	**19**	*17*	*1*	*21*	**19**	**29**	**27**	**8**
1792	*11*	**16**	**11**	**36**	**8**	**18**	*17*	**2**	*12*	**20**	**23**	**27**	**8**
1793	*2*	**41**	**17**	**41**	*6*	**28**	*15*	**2**	*25*	**41**	**40**	**37**	**15**
1794	**8**	**31**	**13**	**18**	*10*	**14**	*5*	*19*	*18*	**21**	**23**	**37**	**8**
1795	**4**	**22**	*9*	**17**	*23*	*10*	*1*	*8*	*21*	**13**	**12**	**11**	*0*
1796	**6**	**2**	*28*	**21**	*18*	*0*	**6**	*5*	*7*	**10**	**12**	**2**	**1**
1797	**1**	*5*	*35*	**21**	*4*	*2*	**8**	**23**	*0*	*2*	*10*	*22*	*0*
1798	**6**	*45*	*35*	**8**	**10**	*33*	*7*	**9**	**12**	*14*	*40*	*29*	*12*
1799	*9*	*44*	*30*	**2**	**14**	*41*	**11**	**11**	**15**	*12*	*42*	*19*	*10*
1800	*15*	*30*	*37*	*8*	**5**	*41*	**4**	*8*	**10**	*19*	*35*	*21*	*14*
1801	*25*	*18*	*47*	*13*	*0*	*51*	*5*	*15*	**7**	*20*	*33*	*24*	*19*
1802	**3**	*30*	*20*	*14*	*17*	*47*	*10*	*27*	*20*	*17*	*32*	*17*	*21*
1803	*23*	*17*	*31*	*33*	*33*	*39*	*2*	*16*	*19*	*32*	*37*	*30*	*25*
1804	*18*	*20*	*32*	*31*	*30*	*39*	*20*	*3*	*32*	*30*	*6*	*34*	*23*
1805	*15*	*36*	*23*	*12*	*21*	*54*	*41*	*0*	*16*	*20*	*5*	*31*	*23*
1806	*15*	*29*	*25*	**9**	*13*	*43*	*10*	**20**	*11*	*12*	*18*	*15*	*13*
1807	*25*	**1**	*56*	**9**	*7*	*27*	*7*	**32**	**7**	*21*	*23*	*4*	*9*
1808	*17*	*2*	*23*	**20**	**1**	*22*	**20**	**34**	*5*	*22*	**11**	**8**	*1*
1809	*27*	**23**	*12*	**32**	**12**	**10**	**10**	**30**	**1**	*10*	**4**	**26**	**11**
1810	*19*	**60**	**12**	**20**	**16**	**26**	**11**	**36**	*25*	*17*	**18**	**22**	**11**

Note.—The heavy type indicates an excess, and the italic type a defect.

TABLE LVIII.—*continued.*

Year.	Jan.	Feb.	Mar.	April.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.	Year.
	°/₀	°/₀	°/₀	°/₀	°/₀	°/₀	°/₀	°/₀	°/₀	°/₀	°/₀	°/₀	°/₀
1811	*18*	**61**	**4**	**8**	**30**	**15**	*8*	**8**	*35*	*10*	**24**	**6**	**4**
1812	*37*	**31**	**25**	**20**	**13**	*0*	*6*	*16*	*48*	*8*	**44**	**4**	*1*
1813	*37*	**32**	**13**	**12**	**29**	**3**	*18*	*29*	*42*	**4**	**11**	*7*	*5*
1814	*32*	**6**	**9**	**6**	**13**	*13*	*0*	*28*	*32*	*7*	*13*	*18*	*11*
1815	*29*	*19*	*20*	*6*	**16**	**11**	**18**	*14*	*34*	*17*	*23*	**6**	*9*
1816	*9*	*36*	*1*	*0*	**12**	**13**	**28**	*16*	*23*	*25*	*10*	**19**	*4*
1817	**18**	*22*	*11*	**3**	**31**	**20**	**20**	*18*	*23*	*6*	*21*	**18**	*0*
1818	**8**	*25*	*26*	*2*	**43**	**26**	**13**	*8*	*29*	*8*	*14*	**25**	*0*
1819	**12**	*31*	*8*	**15**	**40**	**14**	*13*	*14*	*40*	*12*	**13**	**31**	*1*
1820	**22**	*19*	**27**	**31**	**33**	*19*	*8*	*35*	*36*	*5*	**8**	**12**	*4*
1821	**16**	**17**	**13**	**21**	**28**	*25*	*6*	*12*	*43*	**3**	*8*	**29**	*2*
1822	*11*	**16**	**19**	*12*	**11**	*22*	*6*	*15*	*41*	**11**	*9*	**37**	*3*
1823	*3*	**10**	**10**	*0*	**1**	*34*	*15*	*21*	*27*	**7**	*29*	**33**	*3*
1824	*22*	**24**	*4*	*14*	*3*	*37*	*8*	*13*	*32*	**7**	**1**	**17**	*10*
1825	*27*	**14**	**12**	**3**	*10*	*35*	*24*	*0*	*33*	**27**	*9*	**29**	*6*
1826	*32*	*21*	**19**	*1*	*15*	*36*	*22*	*3*	*29*	**10**	**15**	**10**	*9*
1827	*16*	*23*	**6**	**36**	*14*	*36*	*2*	**35**	*27*	*7*	*1*	*13*	*5*
1828	*19*	*16*	**24**	**47**	*27*	*32*	**42**	**69**	*21*	*23*	*5*	*10*	**4**
1829	*18*	**10**	**32**	**49**	*33*	*21*	**45**	**84**	*17*	*17*	**11**	*9*	**11**
1830	*35*	**8**	*16*	**28**	*32*	*9*	**37**	**75**	*19*	*13*	**11**	*16*	**4**
1831	*47*	**26**	*12*	**27**	*43*	**16**	**15**	**59**	*19*	*11*	*16*	*1*	**1**
1832	*39*	**18**	**15**	*11*	*46*	**10**	**6**	**19**	**3**	*21*	*26*	**1**	*6*
1833	*30*	**36**	**12**	*31*	*44*	*4*	*31*	*15*	**17**	*6*	*29*	*4*	*12*
1834	*4*	**5**	**36**	*31*	*46*	**6**	*2*	*26*	**28**	*10*	*29*	*7*	*6*
1835	**3**	**13**	**36**	*26*	*44*	**6**	**23**	*22*	**34**	*37*	*20*	**1**	*3*
1836	**22**	*2*	**53**	*20*	*23*	**21**	**29**	*9*	**47**	*34*	*0*	*26*	**4**
1837	**6**	**5**	**51**	*22*	*23*	**44**	**31**	*5*	**36**	*25*	**3**	*25*	**6**
1838	**33**	*6*	**27**	*29*	*4*	**60**	**46**	*5*	**12**	*26*	*1*	*36*	**6**
1839	**5**	*6*	*15*	*34*	**2**	**51**	**27**	**4**	**10**	*3*	*7*	*40*	**1**
1840	**2**	*18*	**15**	*53*	**1**	**36**	**6**	*16*	**8**	*11*	*10*	*39*	*6*
1841	*5*	*16*	**7**	*52*	**3**	**7**	**14**	*27*	*17*	**9**	*17*	*43*	*12*
1842	*11*	*13*	**20**	*51*	*1*	*4*	**8**	*25*	*20*	*3*	**2**	*54*	*14*
1843	*30*	*24*	**36**	*49*	*18*	*1*	*6*	*14*	*25*	**29**	*4*	*42*	*13*
1844	*16*	*25*	**41**	*26*	*17*	**18**	*4*	*4*	*19*	**22**	*8*	*53*	*9*
1845	*21*	*29*	*5*	*12*	**16**	**25**	*5*	*8*	*21*	**42**	*8*	*25*	*4*
1846	*25*	**17**	**4**	*22*	*8*	**61**	*21*	*4*	*16*	**44**	*6*	*12*	**1**
1847	*9*	**8**	*14*	*6*	**7**	**58**	*20*	*2*	*22*	**51**	*10*	*2*	**2**
1848	*11*	**34**	*34*	*0*	**16**	**40**	*21*	*11*	*28*	**13**	*2*	*10*	*3*
1849	*8*	**22**	*3*	*11*	**9**	**27**	*29*	*17*	*37*	*7*	*9*	*10*	*9*
1850	**19**	**37**	**3**	*21*	*19*	**37**	*25*	**8**	*29*	*18*	**5**	**4**	*3*
1851	**25**	*6*	*27*	*28*	*14*	**45**	*17*	**17**	*26*	*28*	*8*	**2**	*6*
1852	**26**	*10*	*28*	*45*	*6*	**51**	*22*	**10**	*36*	*30*	*12*	**5**	*7*
1853	**17**	*29*	*16*	*49*	*19*	**64**	*6*	**15**	*47*	*19*	*22*	**5**	*7*
1854	**13**	*9*	*57*	*51*	**8**	**71**	*13*	**9**	*17*	*21*	*17*	**23**	*3*
1855	*4*	*28*	*38*	*33*	**5**	**82**	*17*	*5*	**2**	*29*	*26*	*20*	*8*
1856	*8*	*34*	*23*	*30*	**11**	**42**	*7*	*13*	**4**	*22*	*19*	*22*	*9*
1857	*16*	*25*	**3**	**1**	*12*	**32**	**3**	*17*	**11**	*7*	*22*	*17*	*5*
1858	**18**	*20*	**12**	**1**	*25*	**42**	*16*	*20*	**33**	*5*	*10*	**39**	**5**
1859	**1**	*30*	**39**	*1*	*38*	**39**	*6*	*19*	**30**	**7**	**11**	**23**	**4**
1860	**24**	*25*	**73**	*12*	*17*	**29**	**4**	*9*	**9**	**25**	**8**	**32**	**11**

NOTE.—The heavy type indicates an excess, and the italic type a defect.

TABLE LVIII.—*continued.*

Year.	Jan.	Feb.	Mar.	April.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.	Year.
	°/₀	°/₀	°/₀	°/₀	°/₀	°/₀	°/₀	°/₀	°/₀	°/₀	°/₀	°/₀	°/₀
1861	**44**	*22*	**62**	**6**	*18*	**37**	*20*	*0*	**14**	**10**	**11**	**37**	**12**
1862	**33**	*14*	**64**	*15*	**2**	**29**	*27*	**1**	**28**	**37**	**3**	**33**	**13**
1863	**16**	*13*	**54**	*18*	**21**	*0*	*13*	**7**	**7**	**46**	*7*	*22*	**6**
1864	**33**	**9**	**48**	*19*	**29**	*13*	*14*	**1**	*7*	**37**	*18*	*10*	**6**
1865	**51**	**21**	**2**	*2*	**29**	*13*	**7**	*7*	*11*	**22**	*28*	*23*	**4**
1866	**43**	**51**	**18**	**15**	**31**	*42*	**5**	*1*	*7*	**22**	*32*	*9*	**6**
1867	**59**	**37**	*12*	**13**	**37**	*37*	*5*	*1*	**1**	*20*	*35*	*10*	**1**
1868	**53**	**84**	*10*	**14**	**13**	*20*	*16*	*16*	**11**	*37*	*42*	*1*	*0*
1869	**40**	**71**	*20*	**55**	**6**	*14*	*16*	*18*	**8**	*28*	*41*	*6*	*0*
1870	**20**	**75**	**6**	**42**	**3**	*11*	*35*	*13*	**42**	*13*	*18*	**2**	**6**
1871	**17**	**67**	**2**	**3**	**9**	*5*	*17*	*11*	**51**	*6*	*9*	*17*	**6**
1872	**6**	**44**	**14**	**1**	**1**	*6*	**1**	**18**	**31**	**1**	**5**	*14*	**10**
1873	**17**	*10*	**11**	**5**	*5*	*8*	**13**	**15**	**38**	**6**	**40**	*20*	**10**
1874	**12**	**2**	**38**	*10*	*3*	*2*	**2**	**23**	**49**	**5**	**46**	**26**	**17**
1875	**28**	*0*	**18**	**6**	*16*	*13*	**9**	**59**	**13**	*2*	**33**	**20**	**14**
1876	**31**	*10*	**4**	**22**	*12*	*2*	*6*	**55**	*0*	*12*	**36**	**27**	**10**
1877	**28**	**3**	**12**	**39**	*11*	**31**	**12**	**38**	*1*	*23*	**26**	**20**	**14**
1878	**6**	**6**	**20**	**71**	*11*	**27**	**14**	**33**	*1*	*17*	**13**	**29**	**14**
1879	**10**	*1*	*1*	**41**	*5*	**19**	**26**	**49**	*5*	*19*	**3**	*15*	**9**
1880	*28*	*5*	**7**	**35**	*0*	**28**	**21**	*1*	*1*	*18*	**10**	**17**	**4**
1881	*27*	**1**	**14**	**35**	*23*	**22**	**45**	*6*	*5*	*16*	*3*	**6**	**2**
1882	**2**	*7*	**5**	**16**	*13*	*21*	**36**	*7*	*1*	*15*	*8*	**19**	*1*
1883	**10**	*4*	**4**	*1*	**1**	*32*	**17**	**7**	*4*	*30*	*24*	*2*	**6**
1884	**33**	*23*	**6**	**1**	**26**	*34*	**15**	*28*	*12*	*17*	*31*	**2**	*8*
1885	**26**	*30*	*4*	*18*	**17**	*57*	**3**	*26*	**8**	*28*	*27*	*32*	*16*
1886	**19**	*26*	**24**	*20*	**17**	*51*	**13**	*36*	*6*	*35*	*3*	*33*	*14*
1887	*18*	*29*	**10**	**2**	**8**	*44*	**8**	*16*	*18*	*17*	*8*	*49*	*16*
1888	**6**	*39*	**8**	*13*	**2**	*22*	**18**	*10*	*20*	*8*	**18**	*40*	*9*
1889	*23*	*52*	**21**	*25*	*25*	*31*	**17**	**16**	*7*	*22*	**16**	*19*	*10*
1890	*20*	*41*	**15**	*25*	*15*	*8*	**9**	**37**	*33*	*7*	*0*	*20*	*8*
1891	*32*	*30*	*22*	*22*	*9*	*14*	*14*	**44**	*26*	**7**	*24*	*9*	*11*
1892	*16*	**36**	*14*	*34*	**1**	*0*	*25*	**33**	*30*	**4**	*20*	*5*	*7*
1893	*37*	**28**	**6**	*31*	*5*	*1*	*8*	**40**	*39*	**9**	*37*	**1**	*7*
1894	*37*	**36**	*12*	*22*	*16*	**25**	**2**	**22**	*43*	*22*	*44*	*17*	*3*

NOTE.—The heavy type indicates an excess, and the italic type a defect.

Table LIX.

Showing the Smoothed Departure from the Average of the Non-Instrumental Phenomena.

Year.	Snow. Days.	Thunder-storms. Days.	Hail. Days.	Gales. Days.	Fog. Days.	Year.	Snow. Days.	Thunder-storms. Days.	Hail. Days.	Gales. Days.	Fog. Days.
1772	3	*3·0*	*4·2*	10	*0*	1811	2	*0·8*	3·2	5	1
1773	1	*2·6*	*3·8*	8	*0*	1812	5	*2·2*	1·4	6	*3*
1774	2	*2·0*	*2·8*	6	*1*	1813	7	*2·0*	0·4	14	*4*
1775	*0*	*2·8*	*2·8*	2	*2*	1814	11	*1·6*	1·2	19	*5*
1776	*0*	*2·0*	*3·2*	2	*2*	1815	10	*0·4*	0·2	23	*5*
1777	*2*	*1·0*	*3·8*	1	*4*	1816	13	*0·2*	0·2	26	*6*
1778	*0*	*1·8*	*3·8*	*3*	*3*	1817	12	*0·0*	2·8	26	*6*
1779	*2*	*1·8*	*4·0*	*5*	*3*	1818	12	0·6	3·0	24	*6*
1780	2	*1·8*	*2·0*	*4*	*3*	1819	7	0·4	5·2	19	*6*
						1820	4	1·0	7·0	16	*3*
1781	6	*1·2*	*0·8*	*4*	*1*						
1782	8	*2·4*	0·8	*7*	*4*	1821	4	0·4	9·0	8	*2*
1783	8	*2·0*	2·8	*9*	*6*	1822	1	0·6	11·6	8	1
1784	12	*2·8*	3·2	*10*	*5*	1823	1	1·0	12·4	6	2
1785	5	*2·2*	1·6	*11*	*4*	1824	1	1·0	11·2	6	5
1786	2	*3·0*	*0·0*	*13*	*8*	1825	3	*0·0*	9·6	8	6
1787	1	*1·2*	*3·6*	*14*	*6*	1826	*0*	1·4	8·4	11	6
1788	*2*	*1·0*	*1·6*	*11*	*8*	1827	*0*	2·4	7·0	10	7
1789	*6*	*1·0*	*1·2*	*9*	*9*	1828	1	1·6	4·4	9	6
1790	*5*	*1·4*	*1·0*	*6*	*9*	1829	*0*	2·8	0·8	7	4
						1830	*3*	2·4	*0·8*	2	3
1791	*6*	*1·6*	0·4	*5*	*7*						
1792	*8*	*2·6*	0·8	*2*	*9*	1831	*3*	1·8	*0·4*	2	4
1793	*4*	*3·0*	0·2	*3*	*8*	1832	*6*	2·0	*2·6*	6	1
1794	*4*	*3·2*	0·8	*5*	*9*	1833	*9*	0·6	*1·2*	3	2
1795	*6*	*3·0*	0·4	*8*	*10*	1834	*9*	*0·6*	1·4	5	1
1796	*6*	*1·6*	*1·6*	*8*	*12*	1835	*2*	0·8	3·8	7	*2*
1797	*1*	*1·8*	*1·6*	*6*	*10*	1836	4	*0·0*	1·8	7	*4*
1798	*3*	*1·6*	*3·4*	*3*	*5*	1837	9	1·0	1·8	10	*6*
1799	3	*1·6*	*3·2*	*2*	*5*	1838	10	*0·0*	0·4	15	*6*
1800	6	*2·0*	*0·8*	1	*6*	1839	10	*0·2*	*0·5*	18	*3*
						1840	7	*1·6*	*1·6*	20	1
1801	8	*3·4*	2·2	1	*6*	1841	1	*1·4*	0·6	18	5
1802	8	*3·4*	2·6	*4*	*8*	1842	1	*2·2*	*0·8*	10	8
1803	5	*3·4*	4·6	*7*	*10*	1843	1	*1·8*	*1·2*	9	9
1804	2	*2·6*	3·8	*8*	*7*	1844	*1*	*0·4*	*2·8*	1	11
1805	3	*2·2*	4·6	*8*	*3*	1845	*5*	*0·6*	*3·4*	*2*	8
1806	5	*0·6*	3·4	*6*	6	1846	*7*	*1·0*	*6·2*	*4*	6
1807	5	0·4	4·6	*2*	9	1847	*9*	0·8	*5·4*	3	8
1808	6	0·4	4·6	*3*	11	1848	*10*	0·4	*5·8*	1	9
1809	5	0·4	5·0	*0*	10	1849	*11*	*1·6*	*5·6*	5	5
1810	6	0·2	4·4	2	6	1850	*12*	0·2	*4·6*	3	5

Note.—The heavy type indicates an excess, and the italic type a defect.

TABLE LIX.—*continued.*

Year.	Snow. Days.	Thunder-storms. Days.	Hail. Days.	Gales. Days.	Fog. Days.	Year.	Snow. Days.	Thunder-storms. Days.	Hail. Days.	Gales. Days.	Fog. Days.
1851	*9*	**1·0**	*3·4*	**2**	**6**	1873	*5*	**5·6**	*1·8*	*9*	**9**
1852	*9*	**0·6**	*2·8*	*4*	*0*	1874	*1*	**5·2**	*0·8*	*9*	**7**
1853	*5*	**1·2**	*2·8*	*5*	**2**	1875	*1*	**3·4**	*0·2*	*8*	**4**
1854	*5*	**1·6**	*3·2*	*11*	**3**	1876	**1**	**2·6**	*0·0*	*7*	**6**
1855	*4*	**0·2**	*4·6*	*12*	**4**	1877	**3**	**1·6**	*1·8*	*5*	**9**
1856	*6*	*0·0*	*5·8*	*15*	**5**	1878	**1**	**2·8**	*1·2*	*1*	**10**
1857	*5*	*1·4*	*6·4*	*16*	**8**	1879	*1*	**3·0**	*0·4*	**7**	**8**
1858	*7*	*1·0*	*5·2*	*18*	**8**	1880	*1*	**4·0**	**0·6**	**9**	**7**
1859	*4*	*0·6*	*4·6*	*14*	**10**						
1860	*3*	*1·2*	*4·6*	*13*	**9**	1881	**1**	**5·0**	**1·6**	**15**	**5**
						1882	*3*	**6·2**	**3·4**	**23**	**3**
1861	*5*	*1·4*	*3·2*	*8*	**7**	1883	*2*	**5·6**	**3·8**	**25**	*2*
1862	*4*	*1·0*	*3·2*	*8*	**6**	1884	*0*	**5·2**	**1·4**	**19**	*3*
1863	*4*	*2·2*	*4·2*	*7*	**5**	1885	**1**	**3·6**	**0·2**	**15**	*3*
1864	*6*	*2·6*	*3·6*	*11*	**2**	1886	**1**	**2·0**	*1·2*	**11**	*4*
1865	*3*	*1·4*	*4·0*	*10*	**1**	1887	**1**	**0·8**	*0·6*	**2**	*4*
1866	*2*	*0·0*	*4·6*	*9*	*0*	1888	**1**	**1·4**	**0·4**	*1*	*4*
1867	*4*	*0·0*	*5·4*	*9*	*0*	1889	*3*	**0·2**	**2·6**	*0*	*4*
1868	*1*	**0·2**	*5·6*	*10*	**3**	1890	*0*	*0·2*	**5·8**	*2*	*6*
1869	*0*	**1·2**	*6·0*	*7*	**5**						
1870	*4*	**3·2**	*5·8*	*7*	**10**	1891	*1*	**1·4**	**5·6**	*3*	*7*
						1892	*1*	**2·6**	**6·8**	*3*	*6*
1871	*4*	**3·0**	*5·0*	*10*	**12**	1893	**1**	**4·4**	**7·0**	*9*	*3*
1872	*2*	**5·0**	*2·8*	*11*	**10**	1894	*0*	**4·2**	**5·8**	*14*	*2*

NOTE.—The heavy type indicates an excess, and the italic type a defect.

TABLE LX.

Showing the Number of Times the Shade Minimum fell to or below Freezing Point in each Month during 81 *Years.*

Year.	Jan.	Feb.	Mar.	April.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.	Year.
1803, . .	16	7	8	...	...	...	...	...	...	...	8	9	48
1804, . .	7	15	11	6	...	...	...	...	...	...	2	11	52
1805, . .	14	8	3	2	1	...	...	...	...	5	4	10	47
1806, . .	17	7	10	6	...	...	...	...	...	2	2	4	48
1807, . .	15	19	23	10	...	...	...	...	...	...	21	17	105
1808, . .	17	15	18	7	...	...	...	...	1	4	7	11	80
1809, . .	22	8	4	12	1	...	...	...	...	...	7	14	68
1810, . .	10	15	23	5	3	...	...	...	...	2	7	13	78
1811, . .	18	11	10	5	...	...	...	...	...	...	...	18	62
1812, . .	16	7	17	5	...	...	...	...	...	...	7	15	67
1813, . .	16	3	3	3	...	...	...	...	...	8	14	12	59
1814, . .	29	20	18	...	...	2	...	...	...	2	10	18	99
1815, . .	26	2	9	3	...	...	...	...	...	...	15	20	75
1816, . .	21	18	16	10	3	...	...	...	...	...	16	22	106
1817, . .	13	11	15	6	1	...	...	...	...	5	3	17	71
1818, . .	17	19	22	11	...	...	...	...	...	...	...	5	74
1819, . .	11	10	2	...	1	...	...	...	...	5	13	21	63
1820, . .	18	7	12	...	...	...	...	...	...	...	2	13	52
1821, . .	9	9	2	...	...	...	...	...	...	1	...	7	28
1822, . .	10	5	5	...	...	...	...	...	...	...	1	12	33
1823, . .	23	20	8	1	...	...	...	...	...	1	...	9	62
1824, . .	9	10	16	12	6	...	...	...	3	8	13	12	89
1825, . .	9	12	16	8	3	...	...	...	...	5	15	14	82
1826, . .	21	4	10	7	4	...	...	...	...	3	14	9	72
1827, . .	18	20	13	6	2	...	...	...	...	1	8	4	72
1828, . .	10	13	12	6	...	...	...	...	...	4	3	2	50
1829, . .	22	13	14	8	...	...	...	...	...	6	10	18	91
1830, . .	23	17	10	4	...	...	...	...	...	3	7	17	81
1831, . .	17	12	10	1	4	...	...	...	...	...	11	5	60
1840, . .	15	18	17	3	1	...	...	...	...	1	10	21	86
1841, . .	21	14	2	2	1	...	...	...	...	4	15	10	69
1842, . .	23	9	8	4	...	...	...	...	...	7	7	6	64
1843, . .	19	15	11	5	...	...	...	...	1	12	14	2	79
1844, . .	17	23	22	1	3	...	...	...	1	5	5	25	102
1845, . .	19	22	21	8	...	...	...	...	1	2	6	19	98
1846, . .	6	8	12	12	1	...	...	...	...	2	5	26	72
1847, . .	20	19	9	9	1	...	...	...	1	...	7	15	81
1848, . .	25	12	14	14	...	...	...	...	...	5	11	11	92
1849, . .	16	7	4	8	...	...	...	...	...	7	10	15	67
1850, . .	26	7	9	4	5	...	...	...	...	1	5	4	61
1851, . .	6	8	7	7	1	1	...	...	...	...	17	10	57
1857, . .	16	5	6	4	...	...	...	...	...	...	4	...	35
1858, . .	7	17	11	4	...	...	...	...	...	2	10	8	59
1859, . .	10	9	4	14	...	...	...	...	...	9	12	18	76
1860, . .	22	21	12	9	...	...	...	...	...	...	7	15	86
1861, . .	17	9	6	3	2	...	...	...	...	...	11	17	65
1862, . .	8	5	12	4	1	...	...	...	...	1	19	1	51
1863, . .	9	8	7	1	...	...	...	...	...	1	4	10	40
1864, . .	18	20	15	3	1	...	...	...	...	1	9	11	78
1865, . .	20	19	24	3	...	...	...	...	...	4	9	1	80
1866, . .	10	14	16	3	2	...	...	...	...	1	7	8	61
1867, . .	21	5	19	...	1	...	...	...	...	1	5	10	62
1868, . .	13	6	4	3	1	...	...	...	...	1	13	6	47
1869, . .	11	7	23	5	5	...	...	...	...	6	12	15	84
1870, . .	18	19	15	1	...	...	...	...	...	1	12	16	82

TABLE LX.—*continued.*

Year.	Jan.	Feb.	Mar.	April.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.	Year.
1871, . .	19	4	4	3	2	...	...	...	...	5	19	19	75
1872, . .	18	9	9	6	...	...	...	...	...	1	2	9	54
1873, . .	8	12	3	...	...	...	...	...	...	2	9	9	43
1874, . .	10	16	4	...	...	...	...	...	...	...	6	21	57
1875, . .	8	7	7	...	...	...	...	...	...	1	7	9	39
1876, . .	9	9	9	5	...	...	...	...	...	...	6	7	45
1877, . .	5	3	16	9	5	...	...	...	...	2	6	5	51
1878, . .	14	5	14	4	...	...	...	...	...	3	13	26	79
1879, . .	28	19	16	9	2	...	...	...	...	5	11	18	108
1880, . .	16	1	6	1	...	...	...	...	...	3	11	16	54
1881, . .	25	15	16	8	...	...	...	...	...	4	1	14	83
1882, . .	4	4	3	3	...	...	...	...	...	2	8	18	42
1883, . .	6	3	20	...	1	...	...	...	...	...	3	5	38
1884, . .	8	4	7	6	...	...	...	...	...	1	7	12	45
1885, . .	14	8	11	3	1	...	...	...	1	5	5	10	58
1886, . .	19	19	19	2	2	...	...	...	...	...	...	3	64
1887, . .	11	10	15	9	2	...	...	...	...	2	5	17	71
1888, . .	9	17	20	8	...	...	...	...	...	...	2	6	62
1889, . .	6	13	9	...	...	...	...	...	...	...	5	12	45
1890, . .	6	15	4	4	...	...	...	...	...	2	7	12	50
1891, . .	18	6	16	4	1	...	...	...	...	...	5	6	56
1892, . .	15	11	21	7	...	...	...	...	...	4	4	19	81
1893, . .	11	7	6	...	...	...	...	...	...	2	7	5	38
1894, . .	13	7	3	...	1	...	...	...	...	3	1	9	37
1895, . .	26	22	5	4	...	...	...	...	...	3	4	8	72
1896, . .	7	7	4	...	...	...	...	...	...	4	4	12	38
Decennial Means.													
1811–20, .	**18·5**	10·8	12·4	4·3	0·5	0·2	...	...	...	2·0	8·0	16·1	72·8
1821–30, .	**15·4**	12·3	10·6	5·2	1·5	...	...	...	0·3	3·2	7·1	10·4	66·0
1841–50, .	**19·2**	13 6	11·2	6·7	1·1	...	...	...	0·4	4·5	8·5	13·3	78·5
1861–70, .	**14·5**	11·2	14·1	2 6	1 3	...	...	...	...	1·7	10·1	9 5	65·0
1871–80, .	13·5	8·5	8·8	3·7	0 9	...	...	...	...	2·2	9·0	**13·9**	60·5
1881–90, .	10·8	10·8	**12·4**	4·3	0·6	...	...	...	0·1	1·6	4·3	10·9	55·8
Means, 81 years, .	**15·1**	11·3	11·3	4·6	0·9	0·0	0·0	0·0	0·1	2·4	7·7	11·9	65·3

TABLE LXI.

Showing the Number of Times the Shade Minimum fell to or below 32° in each Winter, with Date of First and Last Frost.

Winter.	Days with Frost.	First Frost.	Last Frost.	Winter.	Days with Frost.	First Frost.	Last Frost.
1802–3, . .	43	November 8	March 15	1857–58, . .	43	November 25	April 14
1803–4, . .	56	,, 13	April 22	1858–59, . .	57	October 29	,, 24
1804–5, . .	41	,, 23	May 1	1859–60, . .	103	,, 21	,, 28
1805–6, . .	59	October 18	April 15	1860–61, . .	59	November 3	May 9
1806–7, . .	75	,, 22	,, 21	1861–62, . .	58	,, 1	,, 16
1807–8, . .	95	November 7	,, 21	1862–63, . .	46	October 30	April 7
1808–9, . .	70	September 27	May 2	1863–64, . .	72	,, 6	May 31
1809–10, . .	77	November 1	,, 17	1864–65, . .	87	,, 21	April 30
1810–11, . .	66	October 28	April 10	1865–66, . .	59	,, 23	May 3
1811–12, . .	63	December 4	,, 22	1866–67, . .	62	,, 26	,, 14
1812–13, . .	47	November 5	,, 3	1867–68, . .	43	,, 4	,, 6
1813–14, . .	103	October 13	June 8	1868–69, . .	71	,, 20	,, 29
1814–15, . .	70	,, 8	April 24	1869–70, . .	86	,, 17	April 9
1815–16, . .	103	November 2	May 11	1870–71, . .	61	,, 15	May 17
1816–17, . .	84	,, 6	,, 18	1871–72, . .	85	,, 4	April 23
1817–18, . .	94	October 3	April 18	1872–73, . .	35	,, 5	March 15
1818–19, . .	29	December 9	May 28	1873–74, . .	50	,, 9	,, 12
1819–20, . .	76	October 20	March 25	1874–75, . .	47	November 1	,, 20
1820–21, . .	35	November 14	,, 22	1875–76, . .	49	October 12	April 23
1821–22, . .	28	,, 4	,, 24	1876–77, . .	51	November 9	May 8
1822–23, . .	65	,, 29	April 19	1877–78, . .	50	October 17	April 6
1823–24, . .	63	October 29	May 22	1878–79, . .	116	,, 29	May 10
1824–25, . .	84	September 27	,, 31	1879–80, . .	58	,, 15	April 30
1825–26, . .	80	October 21	,, 12	1880–81, . .	94	,, 19	,, 10
1826–27, . .	85	,, 6	,, 12	1881–82, . .	33	,, 16	,, 15
1827–28, . .	54	,, 29	April 8	1882–83, . .	58	,, 26	May 6
1828–29, . .	66	,, 18	,, 30	1883–84, . .	33	November 7	April 27
1829–30, . .	88	,, 7	,, 4	1884–85, . .	57	October 11	May 12
1830–31, . .	71	,, 17	May 14	1885–86, . .	82	September 27	,, 27
1840–41, . .	72	,, 25	,, 3	1886–87, . .	50	October 12	,, 5
1841–42, . .	73	,, 21	April 12	1887–88, . .	78	,, 12	April 26
1842–43, . .	70	,, 13	,, 27	1888–89, . .	36	November 27	March 27
1843–44, . .	95	September 29	May 19	1889–90, . .	46	,, 17	April 20
1844–45, . .	106	,, 22	April 25	1890–91, . .	66	October 27	May 17
1845–46, . .	67	,, 23	May 15	1891–92, . .	65	November 23	April 19
1846–47, . .	91	October 26	,, 17	1892–93, . .	51	October 18	March 28
1847–48, . .	88	September 27	April 30	1893–94, . .	38	,, 30	May 21
1848–49, . .	62	October 18	,, 21	1894–95, . .	70	,, 19	April 14
1849–50, . .	83	,, 2	May 15	1895–96, . .	33	,, 24	March 31
1850–51, . .	40	,, 8	June 4	1896–97, . .	51	,, 11	April 8

Table LXII.

Showing the Number of Times Frost was Registered on each Day of the Year during Eighty-one Years.

	Jan.	Feb.	Mar.	April.	May.	June.	Sept.	Oct.	Nov.	Dec.		Jan.	Feb.	Mar.	April.	May.	June.	Sept.	Oct.	Nov.	Dec.
1, . .	38	33	37	20	3	...	...	...	12	27	19, . .	30	34	26	16	3	...	...	8	27	29
2, . .	40	38	36	16	5	...	...	1	11	29	20, . .	44	38	23	12	1	...	...	8	18	28
3, . .	41	32	31	17	2	...	...	1	9	29	21, . .	48	34	29	9	2	...	...	9	17	35
4, . .	39	33	33	18	3	1	...	3	7	25	22, . .	46	29	34	8	1	...	1	6	20	40
5, . .	38	28	34	10	4	...	...	3	16	24	23, . .	46	30	32	8	...	...	1	6	25	40
6, . .	42	32	32	14	6	...	...	4	18	29	24, . .	36	31	28	7	...	...	...	5	22	35
7, . .	45	33	29	11	2	1	...	4	19	23	25, . .	31	32	22	11	1	...	...	11	21	36
8, . .	41	31	32	9	5	1	...	3	12	28	26, . .	42	36	25	3	...	...	...	15	25	39
9, . .	40	41	36	15	4	...	...	4	20	28	27, . .	39	38	21	8	1	...	4	12	30	42
10, . .	39	31	37	16	3	...	...	3	22	30	28, . .	34	34	21	5	2	...	1	10	26	37
11, . .	45	33	37	18	3	...	...	6	16	29	29, . .	39	(5)	20	7	1	...	2	13	28	39
12, . .	43	33	36	19	3	...	...	5	25	27	30, . .	35	...	20	6	...	...	...	11	26	41
13, . .	38	35	33	19	2	...	...	5	22	29	31, . .	31	...	20	...	1	...	...	10	...	35
14, . .	40	26	30	19	4	...	...	2	22	30											
15, . .	41	30	31	12	2	...	...	7	24	24											
16, . .	40	26	34	14	2	...	...	4	21	24											
17, . .	39	28	29	13	4	...	...	5	29	26	Totals, .	1220	916	917	373	72	3	9	194	624	966
18, . .	32	32	29	13	2	...	...	10	34	29											

TABLE LXIII.

Showing the Number of Times the Minimum Temperature in Shade fell to or below 20° during Eighty-one Years.

Year.	Jan.	Feb.	Mar.	April.	Oct.	Nov.	Dec.	Year.	Year.	Jan.	Feb.	Mar.	April.	Oct.	Nov.	Dec.	Year.
1803	1	...	...	...	...	...	...	1	1857	1	...	...	...	...	...	...	1
1804	1	1	...	...	...	...	...	2	1858	...	...	...	...	...	...	...	...
1805	...	...	...	...	...	...	...	...	1859	...	...	...	...	...	...	...	...
1806	...	...	...	...	...	...	...	...	1860	...	1	...	...	...	...	5	6
1807	...	1	...	...	...	2	...	3									
1808	2	...	...	...	...	...	...	2	1861	2	...	...	...	...	...	...	2
1809	4	...	...	...	...	...	...	4	1862	...	...	1	...	...	...	...	1
1810	...	6	...	...	...	...	...	6	1863	...	...	...	...	...	...	...	...
									1864	2	1	1	...	...	...	...	4
1811	4	...	...	...	...	...	...	4	1865	1	2	...	...	...	...	...	3
1812	1	...	...	...	...	...	3	4	1866	1	...	...	...	...	...	...	1
1813	...	...	...	...	...	2	...	2	1867	5	...	...	...	...	...	...	5
1814	14	1	...	...	...	...	1	16	1868	...	...	...	...	...	...	...	...
1815	3	...	...	...	...	1	3	7	1869	...	...	...	...	...	...	3	3
1816	...	2	...	...	...	...	1	3	1870	...	2	...	...	...	...	1	3
1817	...	...	...	...	...	...	...	...									
1818	...	...	1	...	...	...	...	1	1871	...	...	...	...	...	...	...	...
1819	...	...	...	...	...	...	3	3	1872	...	...	...	...	...	...	...	...
1820	5	...	...	...	...	...	...	5	1873	...	...	...	...	...	...	...	...
									1874	...	1	...	...	...	...	4	5
1821	1	...	...	...	...	...	...	1	1875	1	...	...	...	...	...	...	1
1822	...	...	...	...	...	...	...	...	1876	...	...	...	...	...	...	...	...
1823	...	2	...	...	...	...	...	2	1877	...	...	...	...	...	...	3	3
1824	...	...	...	...	1	...	1	2	1878	1	...	...	...	...	...	2	3
1825	...	...	...	...	...	1	...	1	1879	3	...	1	...	...	...	...	4
1826	6	...	...	...	...	...	...	6	1880	...	...	...	...	...	...	...	...
1827	3	2	1	...	...	...	...	6									
1828	1	...	...	...	...	...	...	1	1881	13	...	1	...	...	...	5	19
1829	6	...	...	...	...	...	...	6	1882	...	...	...	...	...	...	...	...
1830	1	3	...	...	...	...	3	7	1883	...	...	...	...	...	...	...	...
									1884	...	...	...	...	...	...	1	1
1831	2	1	...	1	...	...	...	4	1885	...	...	...	...	...	...	...	...
1840	...	...	...	...	...	...	...	...	1886	2	...	1	...	...	...	...	3
									1887	...	1	...	...	...	...	...	1
1841	4	...	...	...	...	...	...	4	1888	...	1	...	...	...	...	...	1
1842	2	...	...	...	...	1	...	3	1889	...	...	...	...	...	...	...	...
1843	1	3	...	...	...	...	...	4	1890	...	...	...	...	...	...	...	...
1844	...	4	...	...	...	...	2	6									
1845	3	5	...	...	...	...	...	8	1891	...	...	...	...	...	...	...	...
1846	...	...	4	...	...	...	4	8	1892	...	1	...	...	...	...	3	4
1847	...	4	1	...	...	...	...	5	1893	2	...	...	...	...	...	...	2
1848	4	...	...	...	...	...	...	4	1894	2	...	...	...	...	...	...	2
1849	4	...	...	...	...	...	...	4	1895	3	9	...	...	...	1	...	13
1850	5	...	...	...	...	...	...	5	1896	...	...	...	...	...	...	...	...
1851	...	...	...	...	...	...	...	...	Totals,	117	54	10	1	1	8	48	239

Decennial Totals.

1811–20	27	3	1	...	...	3	11	45	1861–70	11	5	2	...	...	...	4	22
1821–30	18	7	1	...	1	1	4	32	1871–80	5	1	1	...	...	...	9	16
1841–50	23	16	5	...	...	1	6	51	1881–90	15	2	2	...	...	...	6	25

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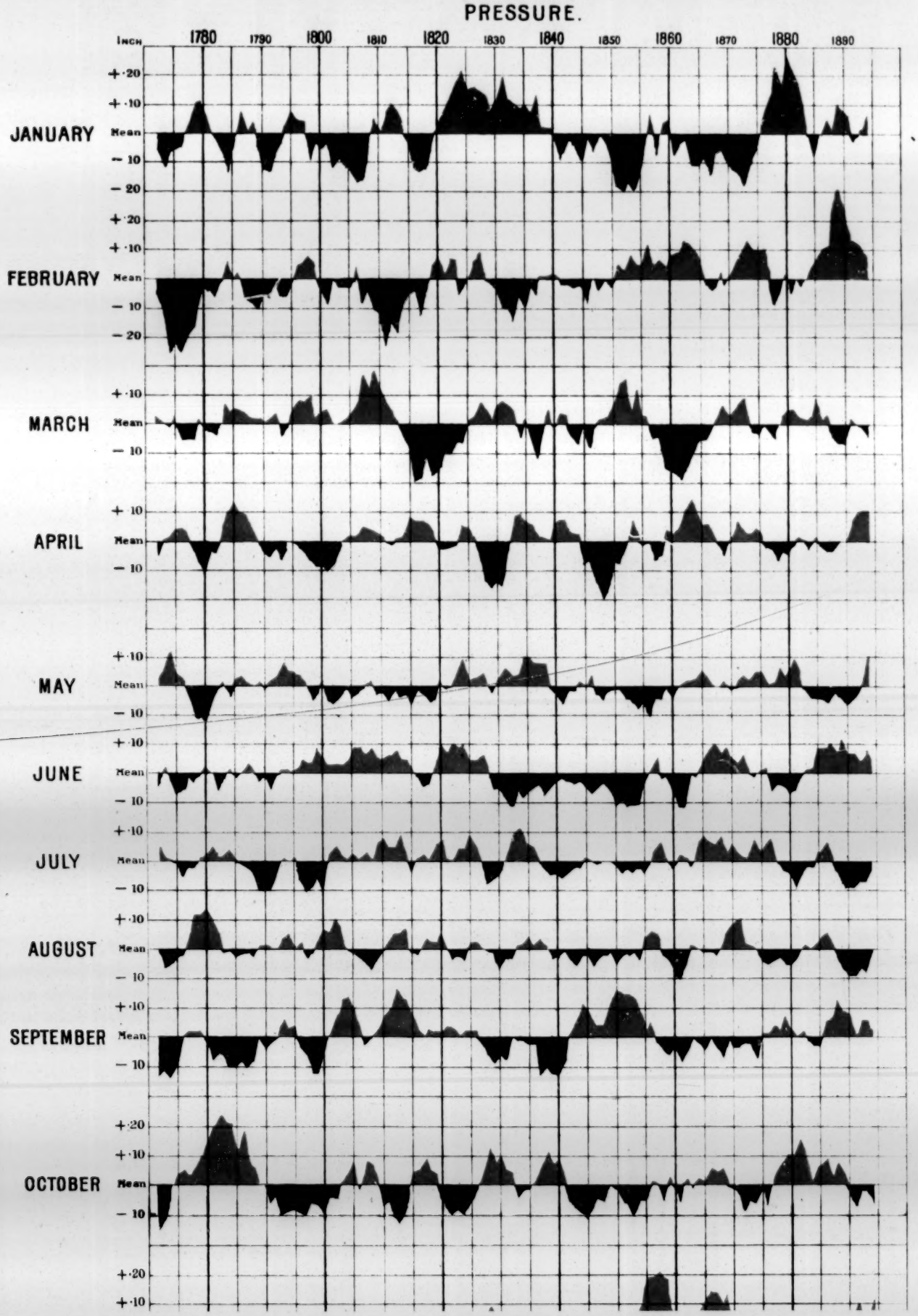
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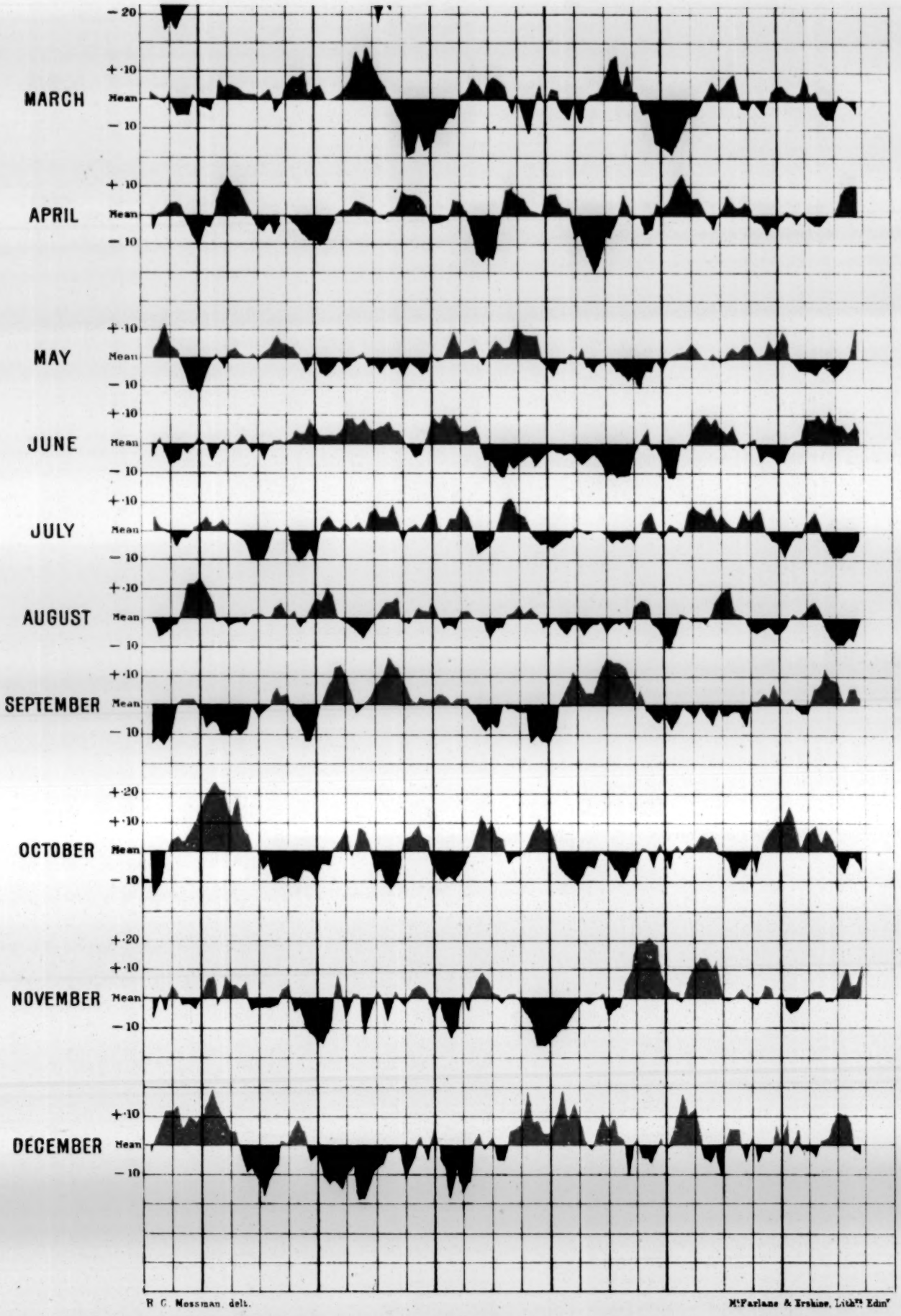
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NOTE.— THE RED INDICATES AN EXCESS AND THE BLUE A DEFECT.





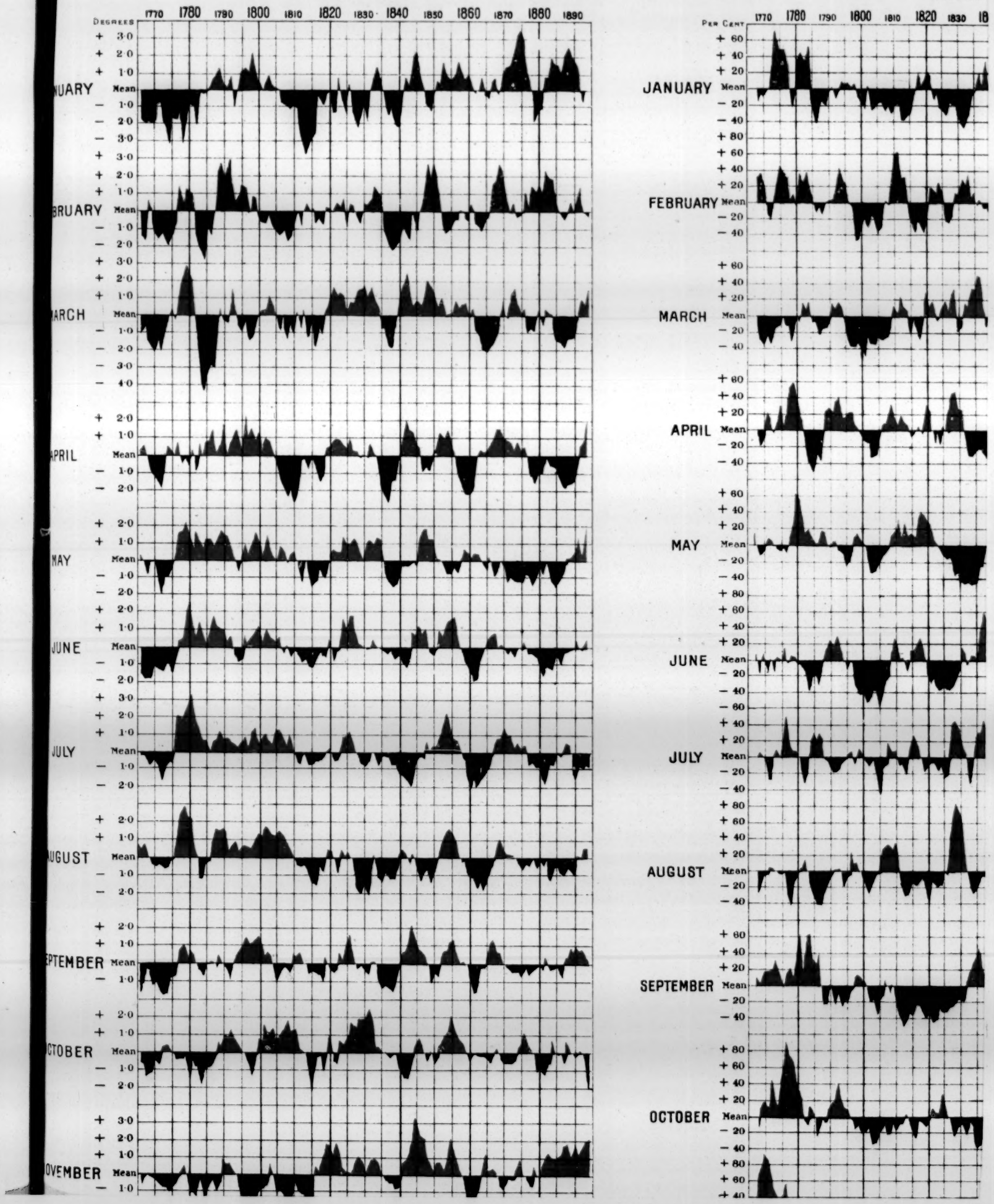
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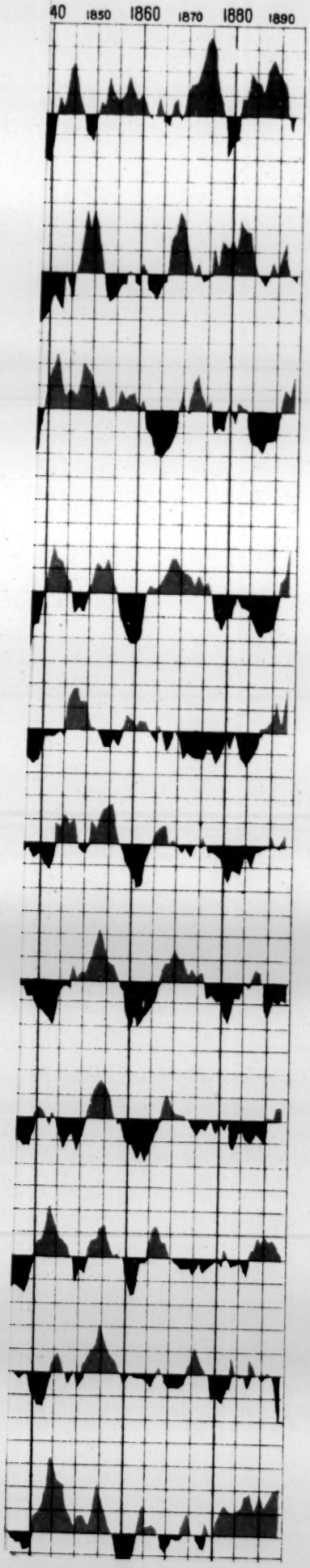
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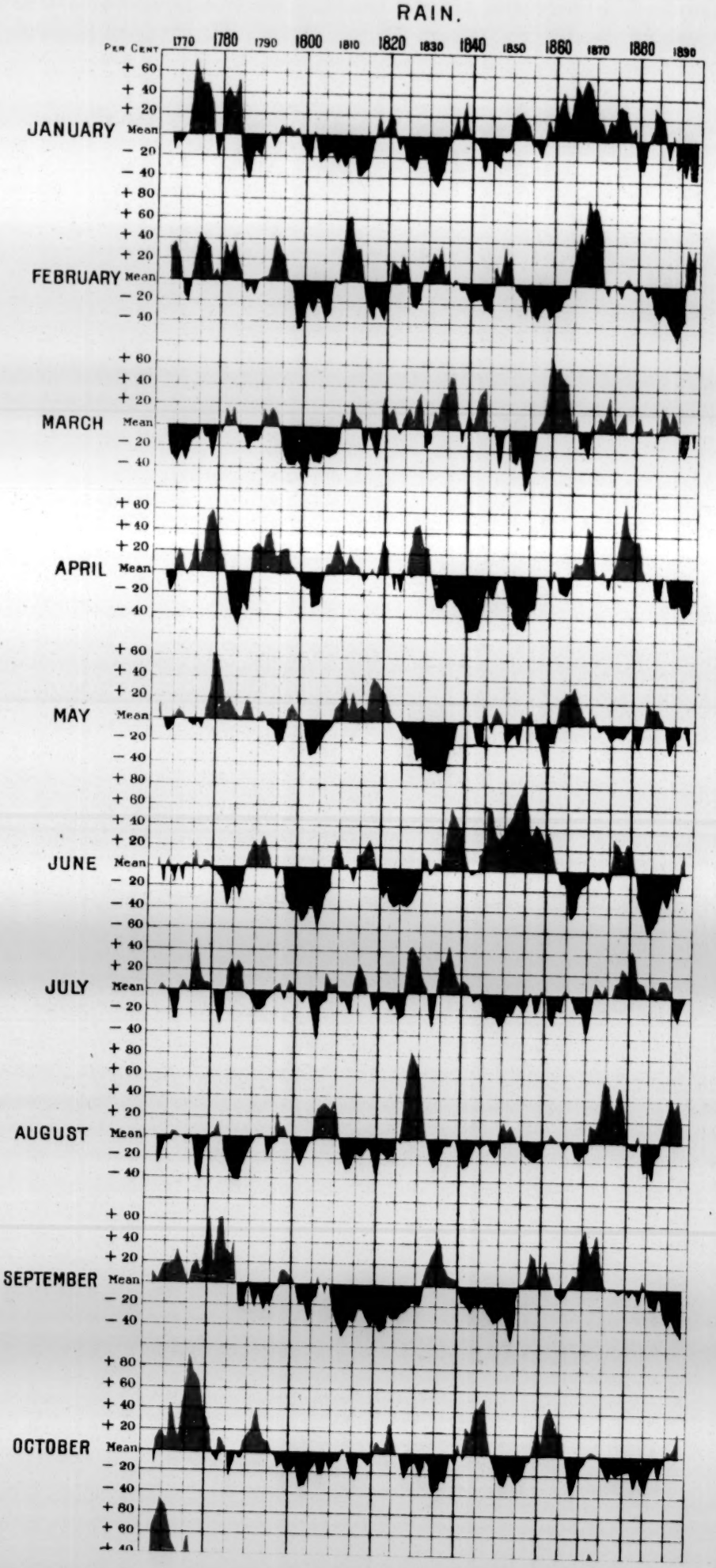
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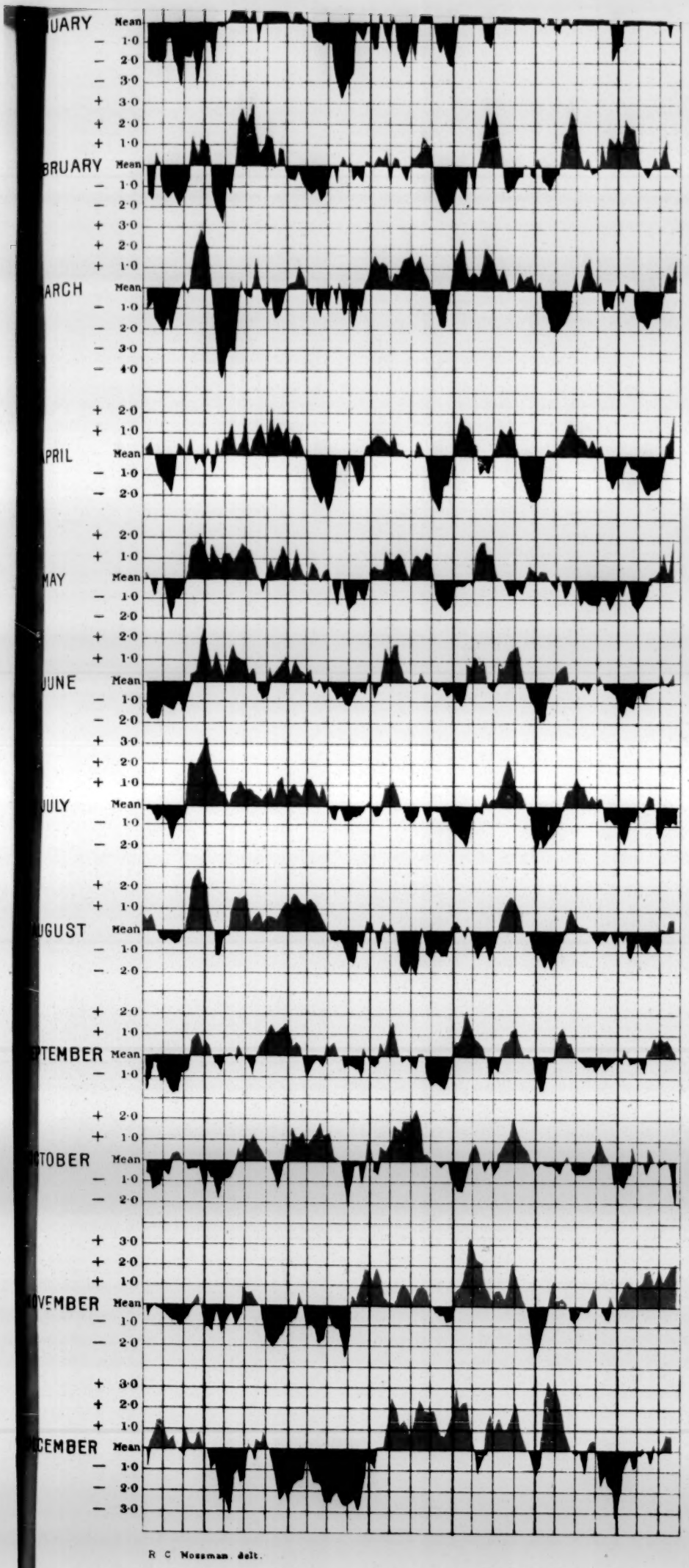
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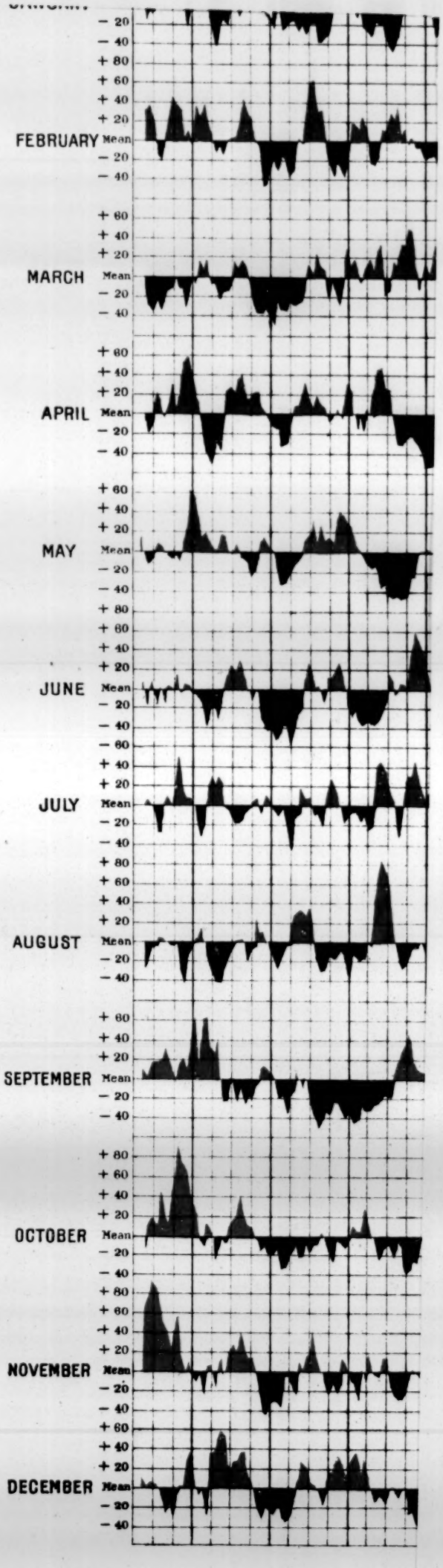
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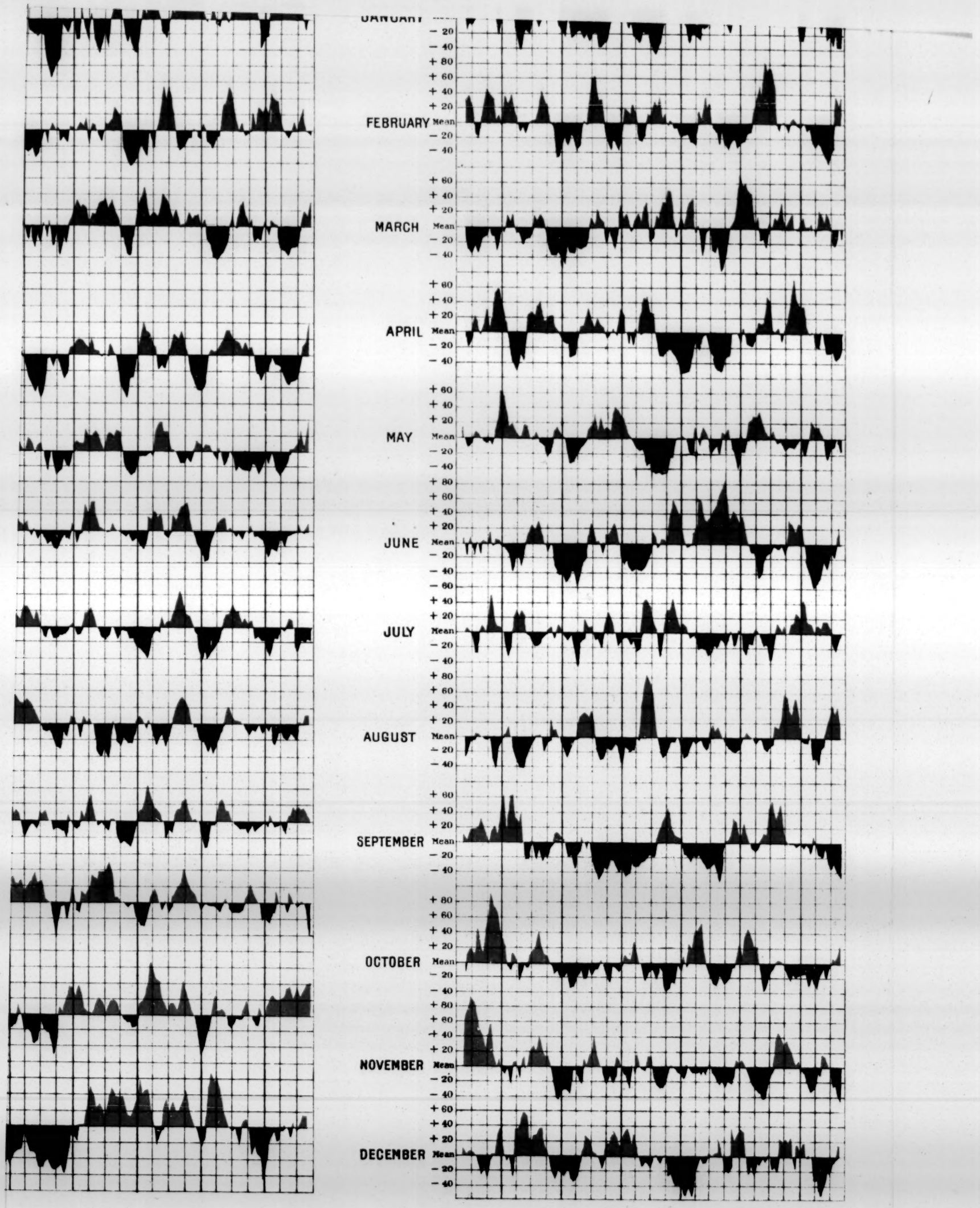
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NOTE.— THE VALUES HAVE BEEN SMOOTHED BY CONTINUOUS FIVE YEAR GROUPS



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PLATE III.— SHOWING THE DEPARTURE F

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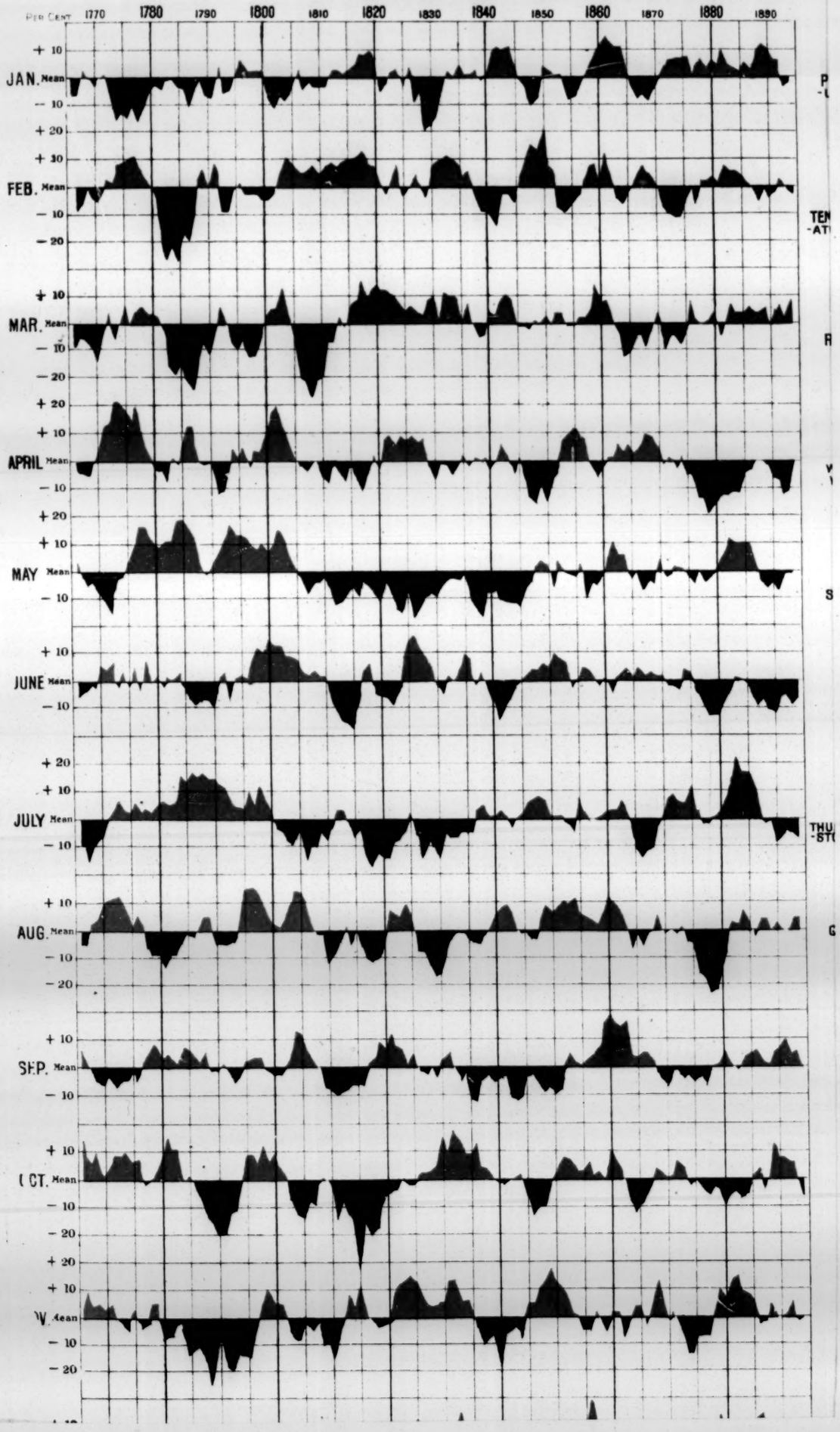
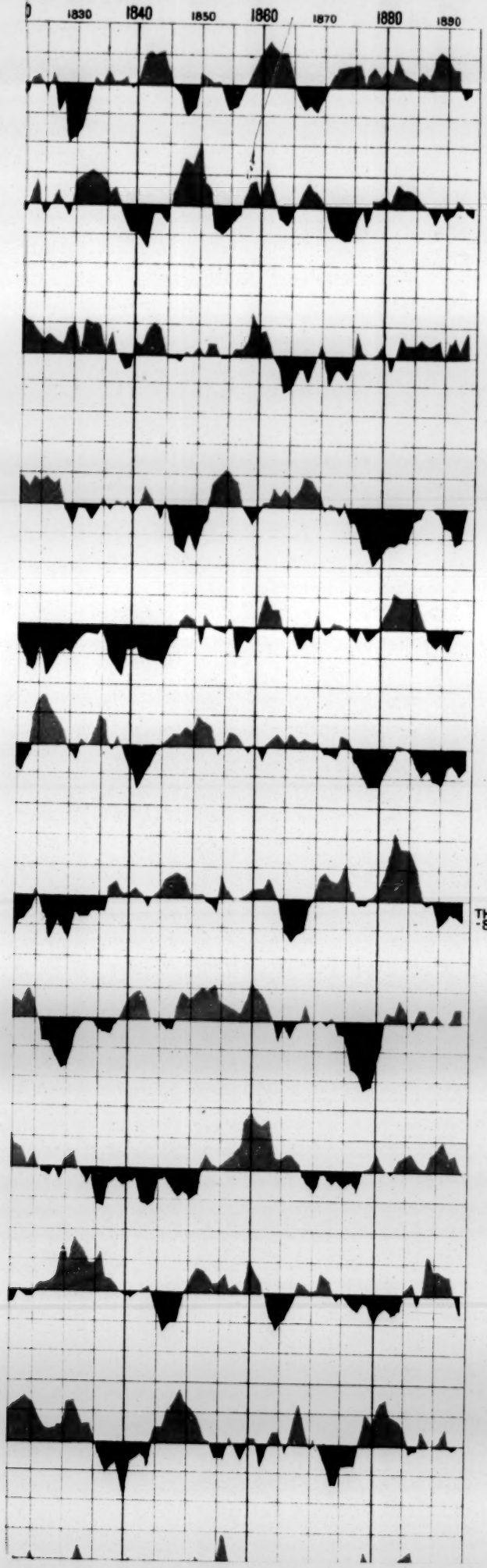


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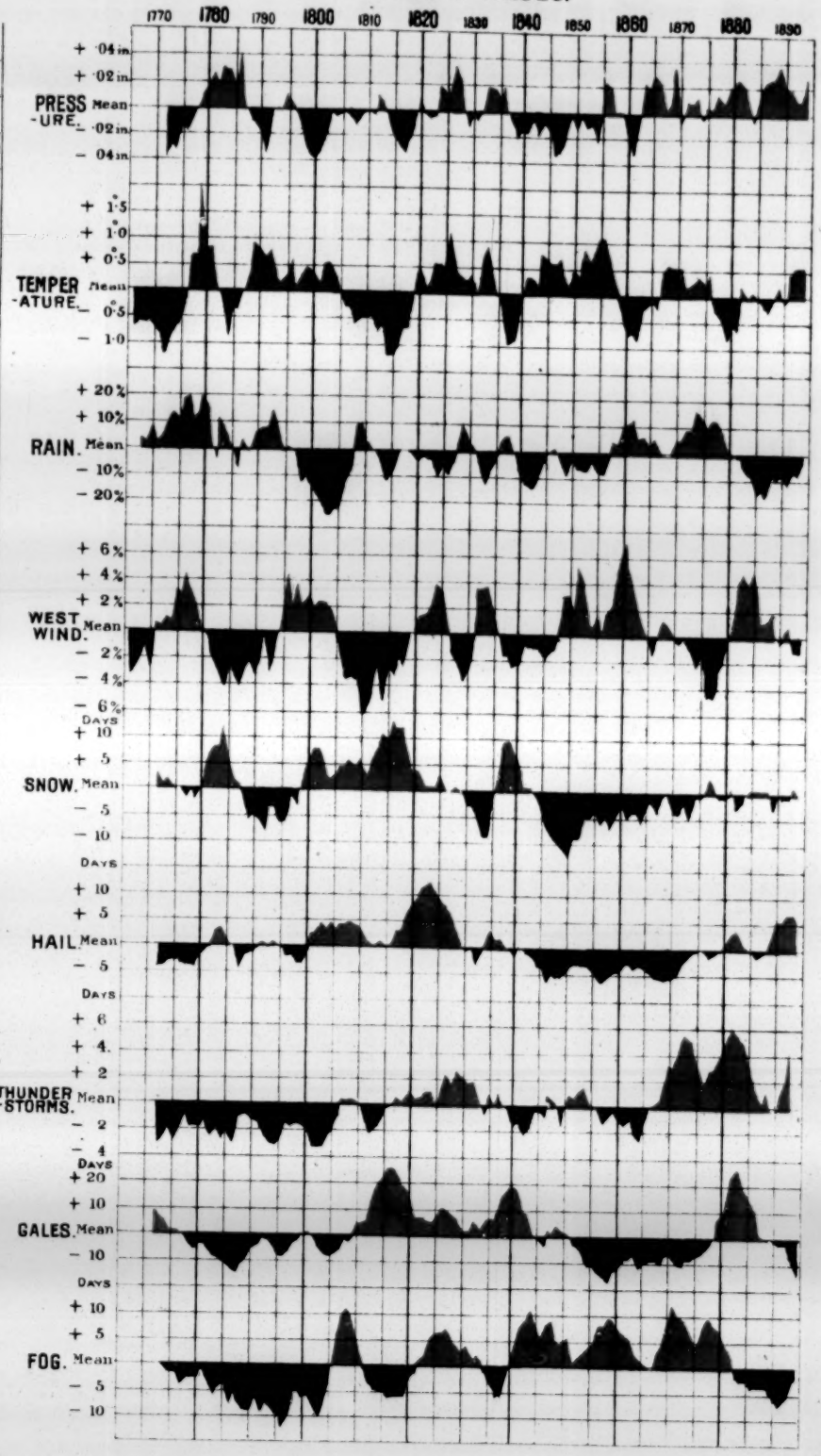
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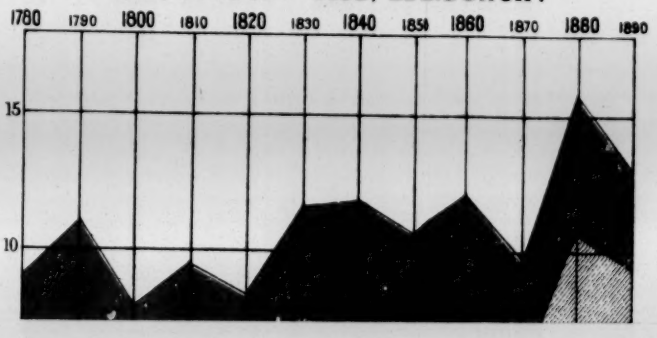
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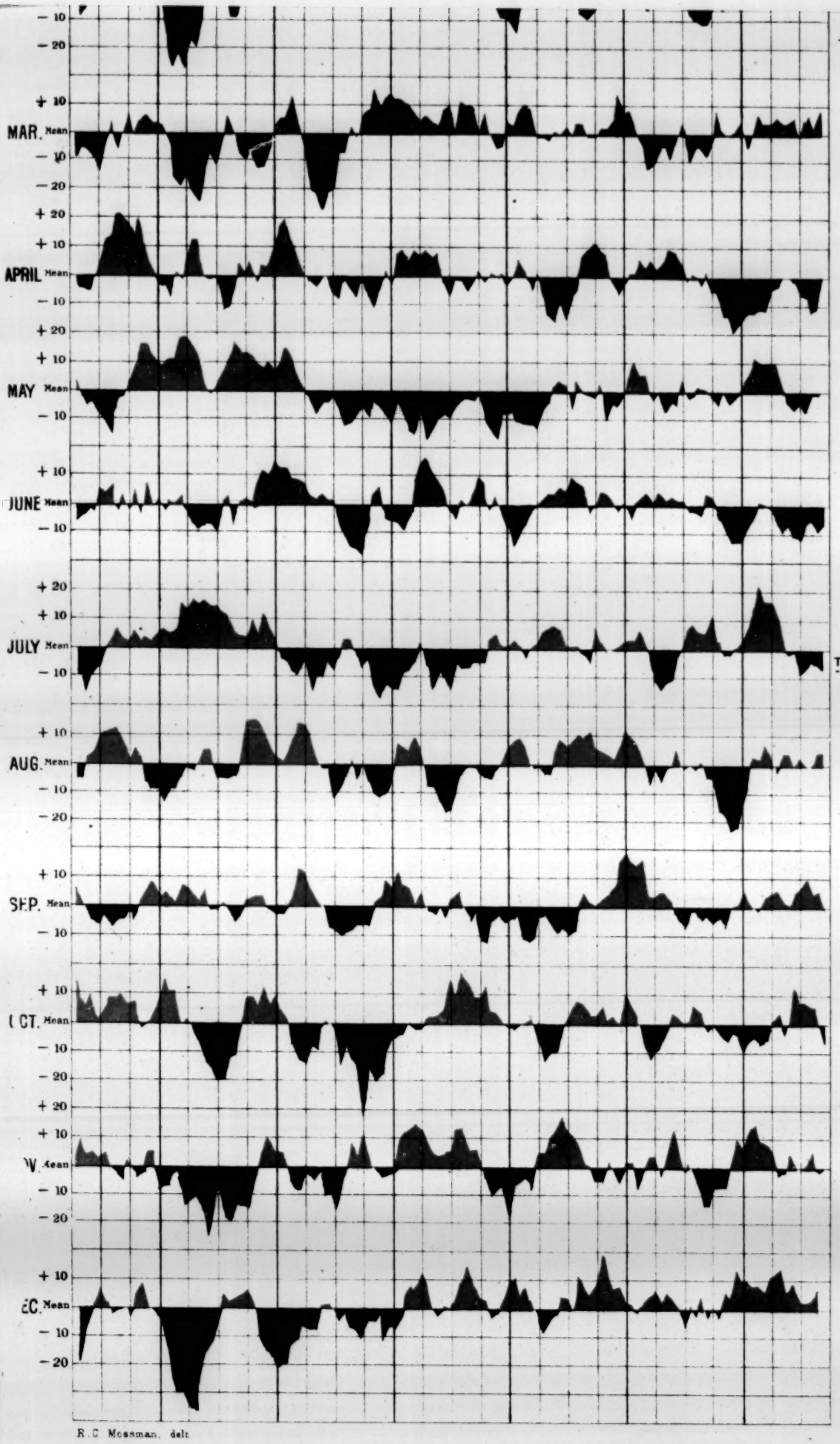
1770 1780 1790 1800 1810 1820 1830 1840 1850 1860 1870 1880 1890



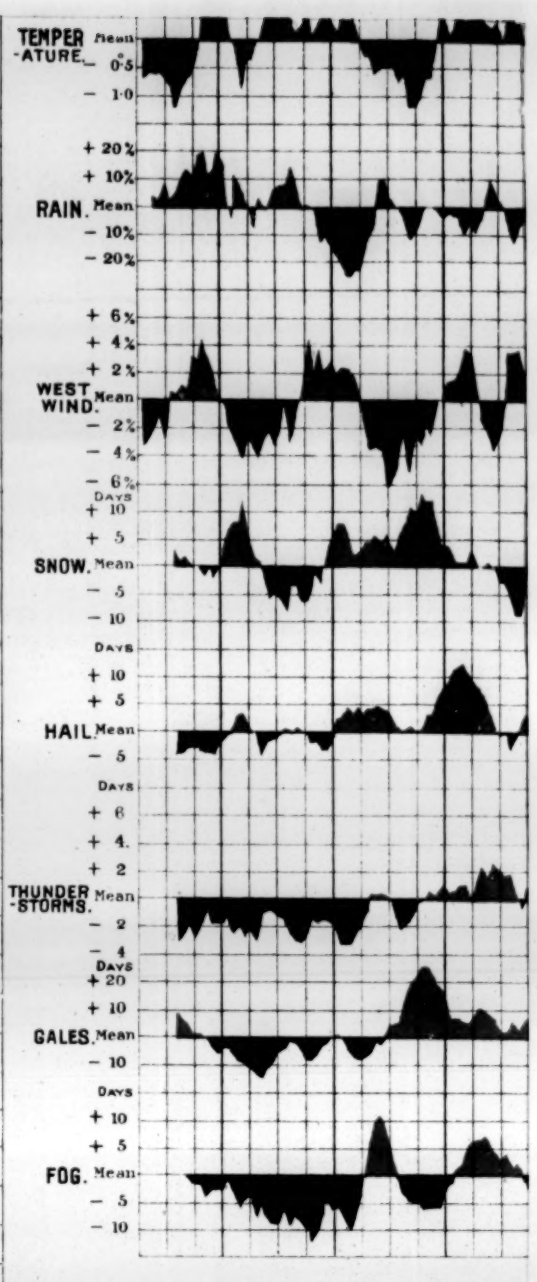
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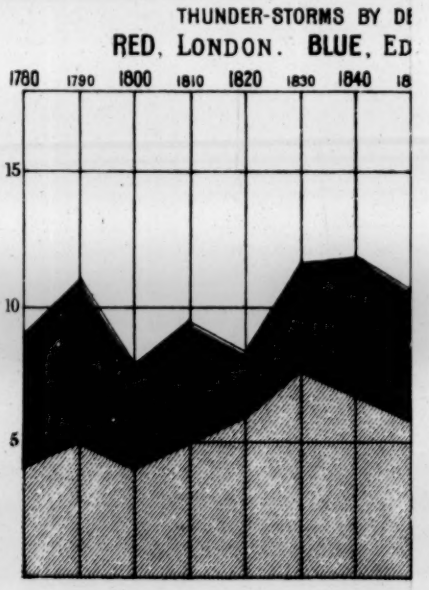
RED, LONDON. BLUE, EDINBURGH.



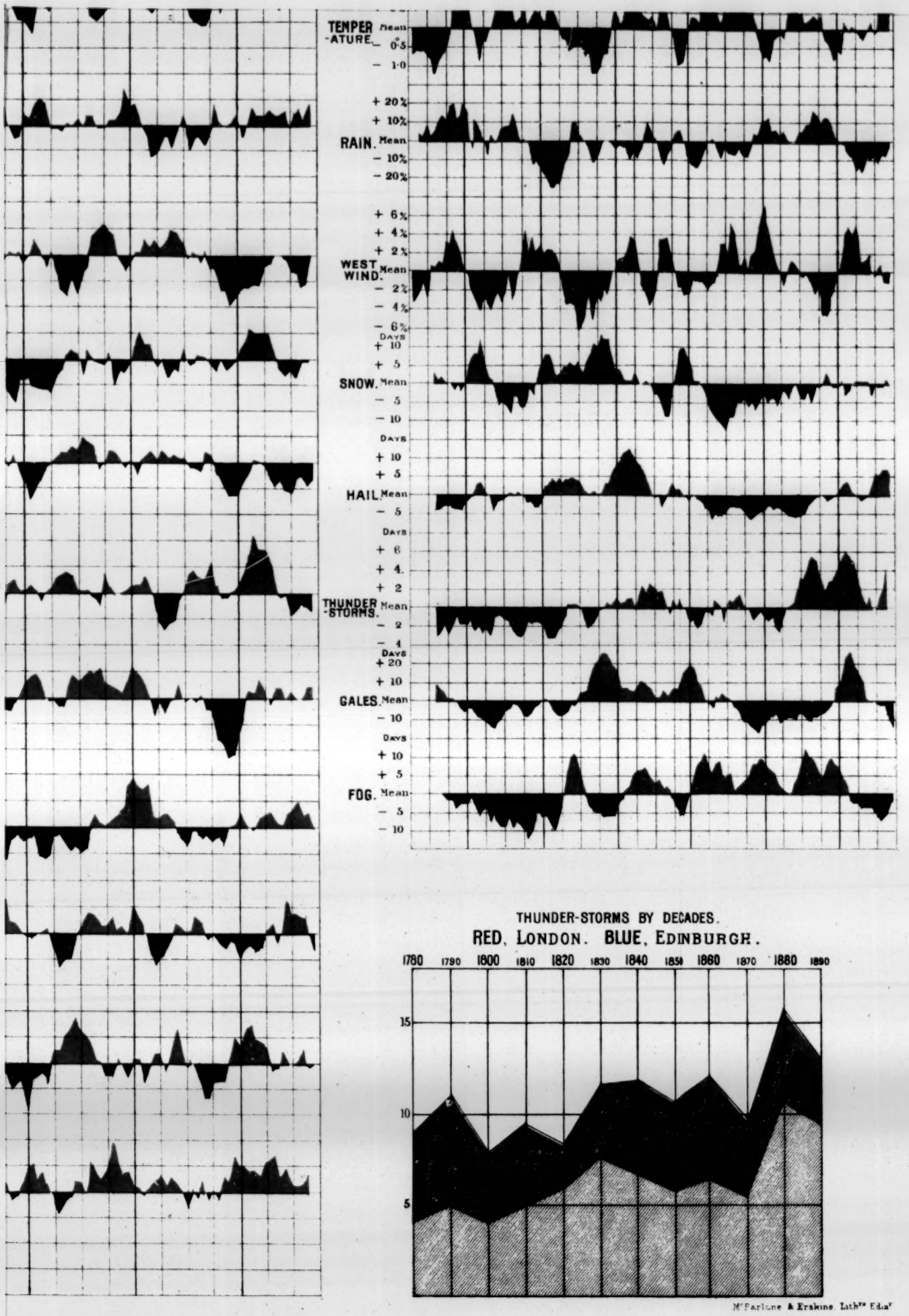


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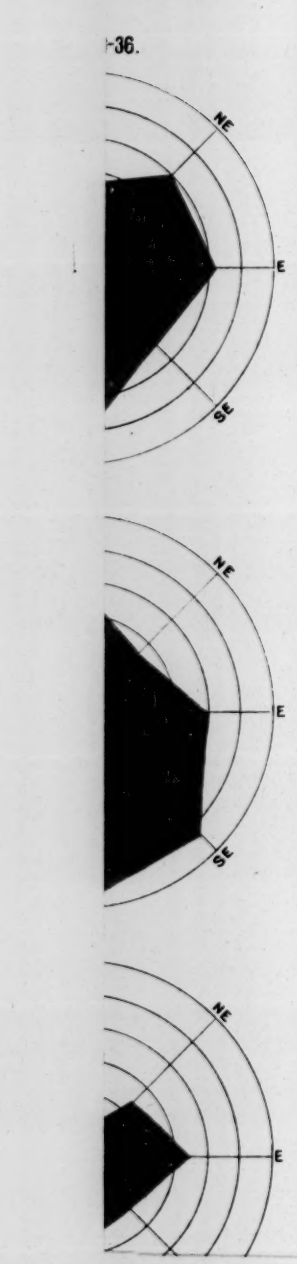
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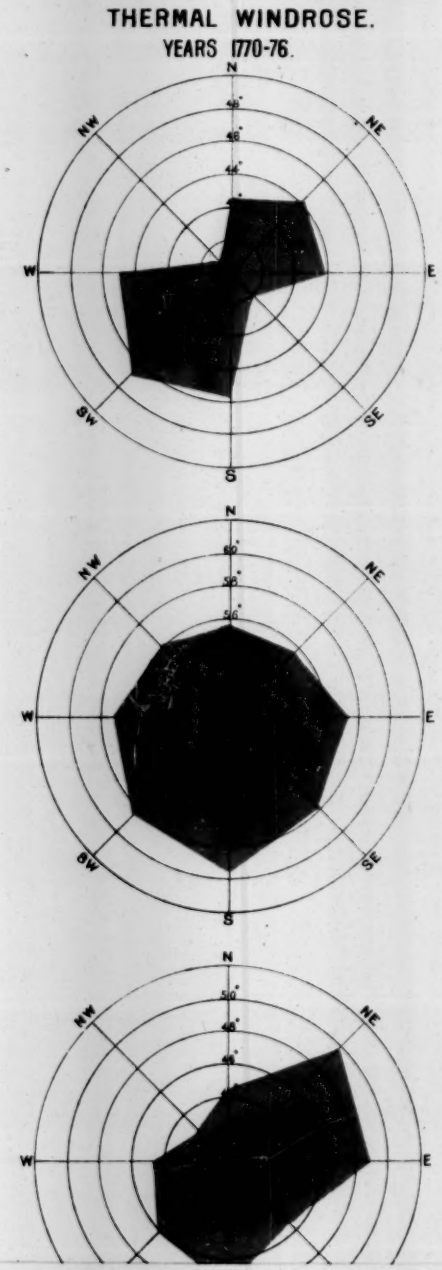
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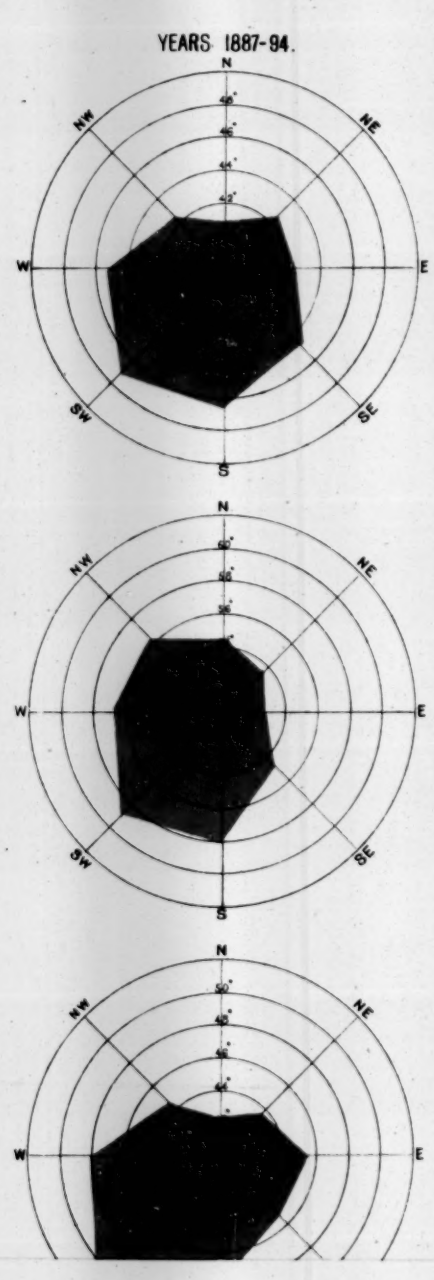
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Plate IV. — Showing the Thermal and Hygrometric Windroses in Edinburgh.

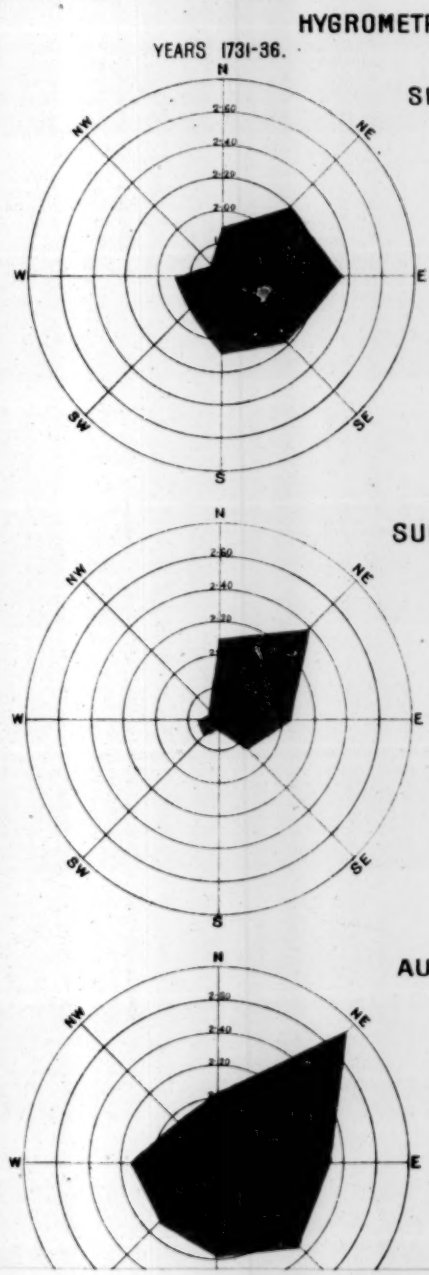


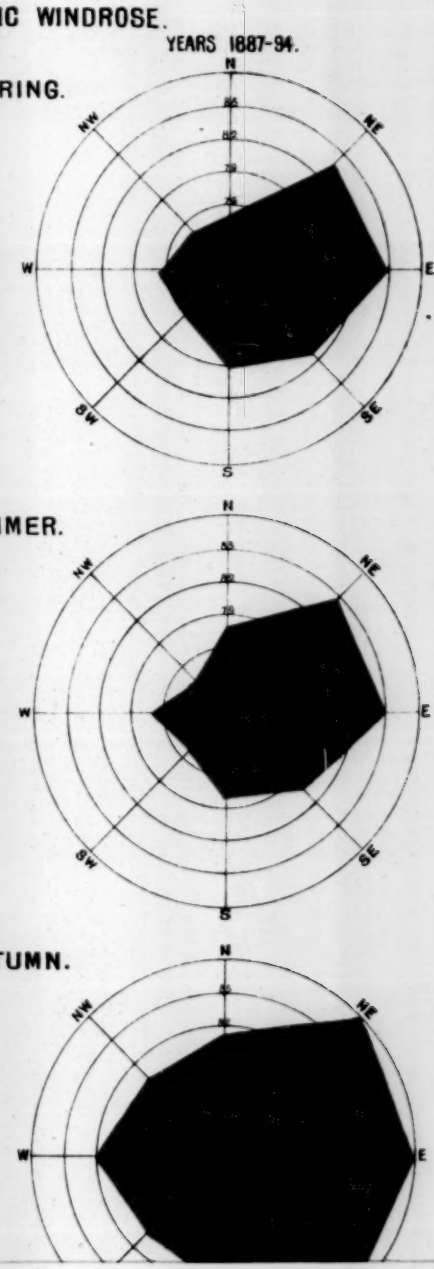
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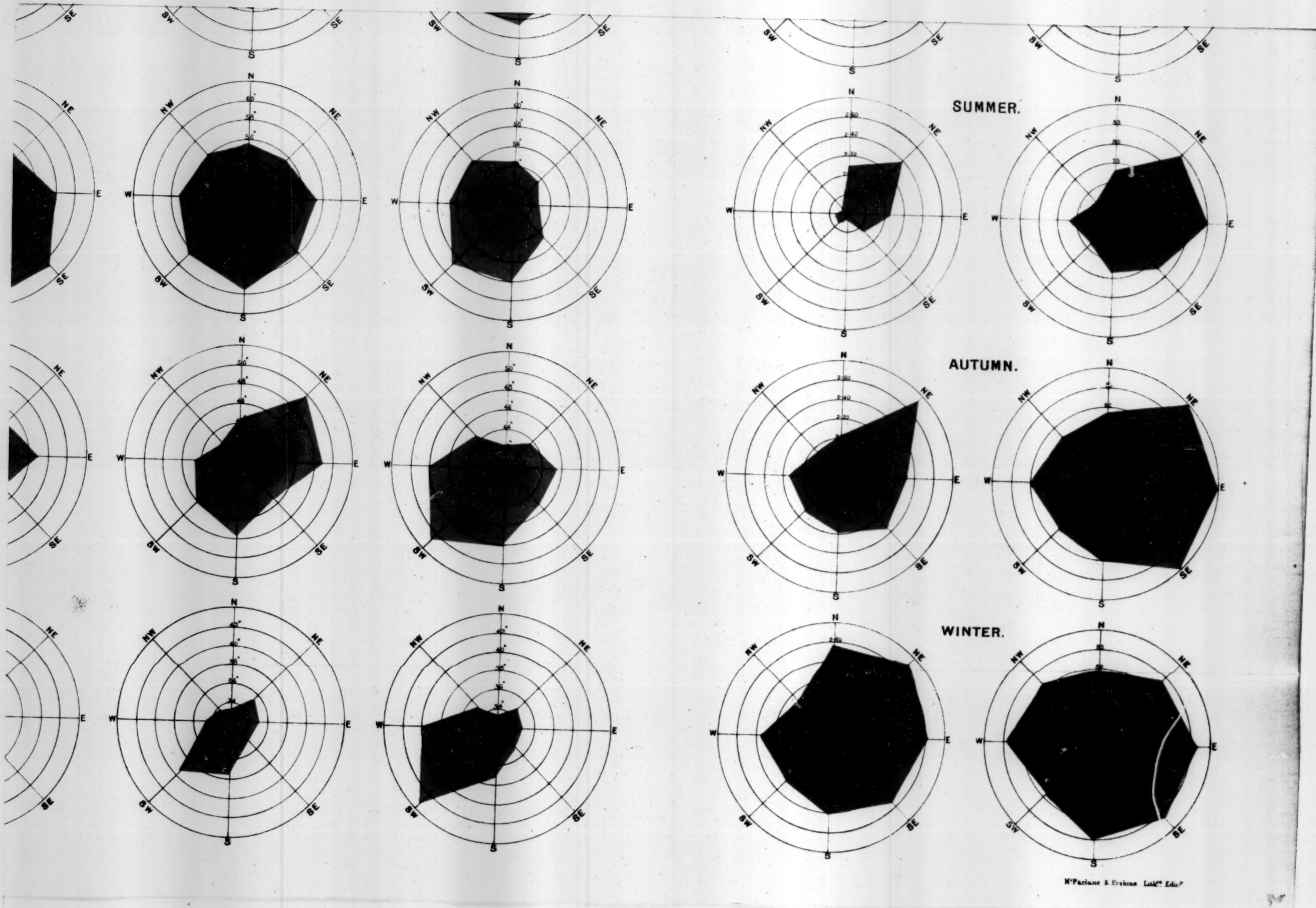




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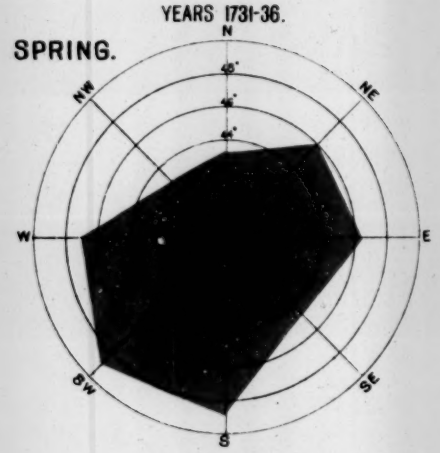
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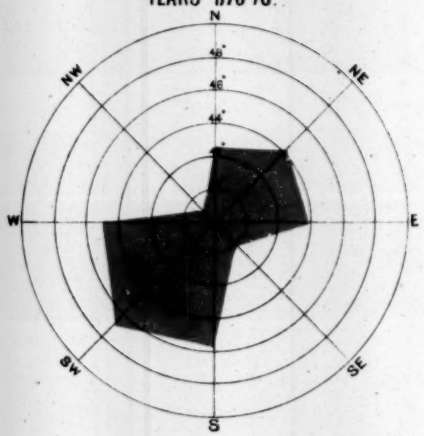
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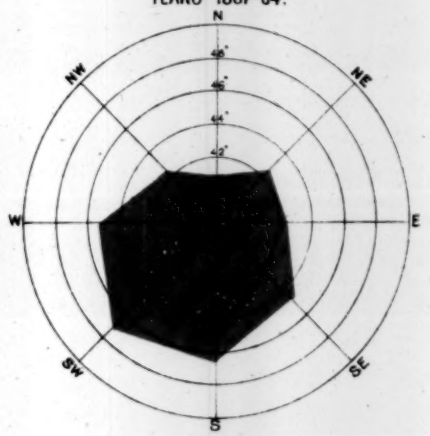
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YEARS 1770-76.

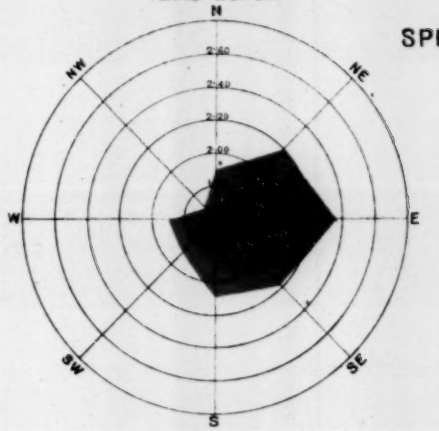


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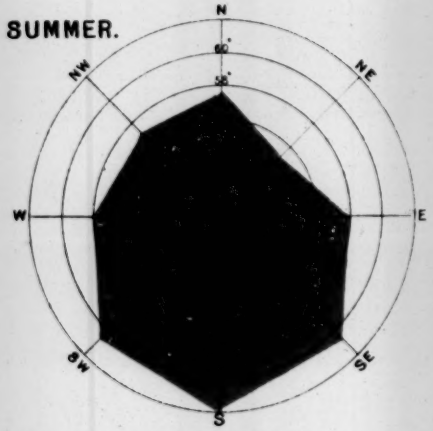


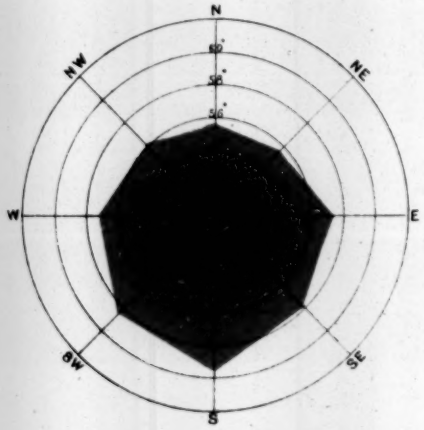
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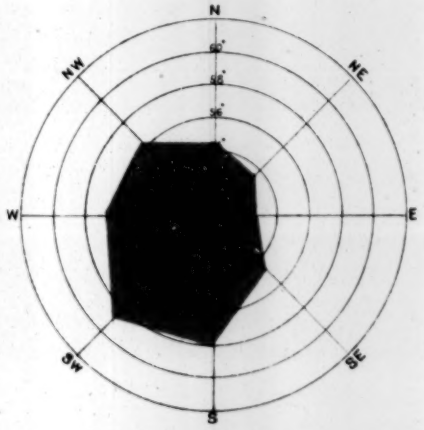
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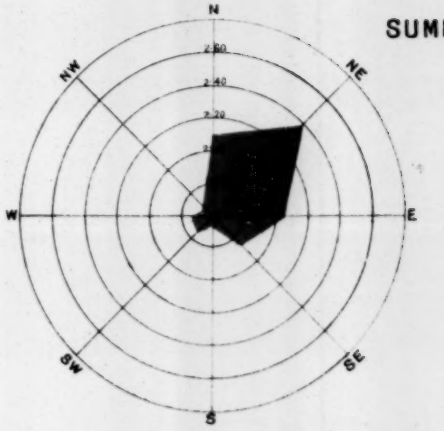
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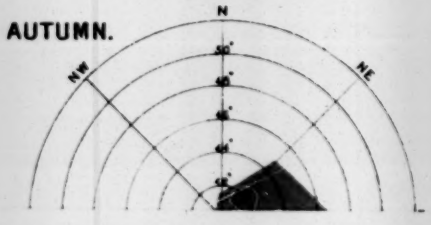


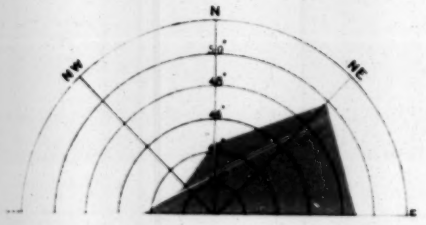


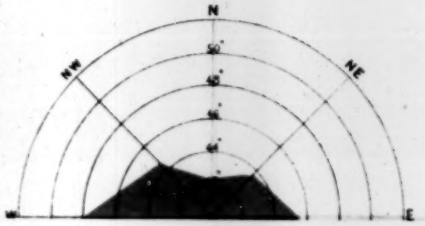
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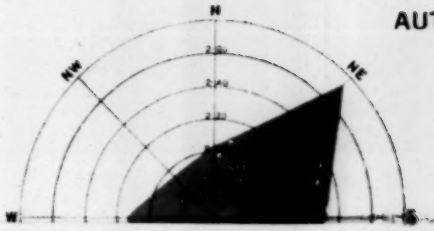
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VII.—*The Automorphic Linear Transformation of a Quadric.*

By Thomas Muir, LL.D.

(Read April 5, 1897.)

(1) It is well known that Cayley effected the transformation of

$$x_1^2+x_2^2+x_3^2+\ldots+x_n^2 \quad \text{into} \quad \xi_1^2+\xi_2^2+\xi_3^2+\ldots\xi^2$$

by introducing an intermediary set of variables

$$\theta_1, \theta_2, \theta_3, \ldots, \theta_n$$

connected with each of the other sets by means of a linear substitution of a peculiar type. The substitutions in fact were

$$\left.\begin{aligned} l_{11}\theta_1+l_{12}\theta_2+l_{13}\theta_3+\ldots &= x_1 \\ l_{21}\theta_1+l_{22}\theta_2+l_{23}\theta_3+\ldots &= x_2 \\ l_{31}\theta_1+l_{32}\theta_2+l_{33}\theta_3+\ldots &= x_3 \\ \ldots\ldots\ldots\ldots & \end{aligned}\right\}$$

and

$$\left.\begin{aligned} l_{11}\theta_1+l_{21}\theta_2+l_{31}\theta_3+\ldots &= \xi_1 \\ l_{12}\theta_1+l_{22}\theta_2+l_{32}\theta_3+\ldots &= \xi_2 \\ l_{13}\theta_1+l_{23}\theta_2+l_{33}\theta_3+\ldots &= \xi_3 \\ \ldots\ldots\ldots\ldots & \end{aligned}\right\},$$

where the determinant of the first substitution is unit-axial and skew, and the determinant of the second substitution is got from the preceding determinant by changing rows into columns, and where, therefore, the number of arbitrary quantities introduced by the two substitutions is only $\frac{1}{2}n(n-1)$. Cayley did not in any way indicate how he was led to the substitutions. It has to be carefully noted, however, that when they had been divined or devised, the essential difficulty of the problem had been overcome; all that remained was the simple algebraical process of eliminating $\theta_1, \theta_2, \theta_3, \ldots$ and so finding the expression for each of the x's in terms of the ξ's. It is true that this process does not take a simple form in the original memoir.* In later days, however, he would doubtless have dismissed the matter in a couple of lines. For, writing the two sets of substitutions in the form

$$\begin{pmatrix} l_{11} & l_{12} & l_{13} & \ldots \\ l_{21} & l_{22} & l_{23} & \ldots \\ l_{31} & l_{32} & l_{33} & \ldots \\ \ldots & \ldots & \ldots & \ldots \end{pmatrix} (\theta_1, \theta_2, \theta_3, \ldots)=(x_1, x_2, x_3, \ldots),$$

$$\begin{pmatrix} l_{11} & l_{21} & l_{31} & \ldots \\ l_{12} & l_{22} & l_{32} & \ldots \\ l_{13} & l_{23} & l_{33} & \ldots \\ \ldots & \ldots & \ldots & \ldots \end{pmatrix} (\theta_1, \theta_2, \theta_3, \ldots)=(\xi_1, \xi_2, \xi_3, \ldots),$$

* Cayley, A., "Sur quelques propriétés des déterminants gauches," *Crelle's Journ.*, xxxii. pp. 119–123; or *Collected Math. Papers*, i. pp. 332–336.

we have from the second

$$(\theta_1, \theta_2, \theta_3, \dots) = \begin{pmatrix} l_{11} & l_{21} & l_{31} & \dots \\ l_{12} & l_{22} & l_{32} & \dots \\ l_{13} & l_{23} & l_{33} & \dots \\ \dots & \dots & \dots & \dots \end{pmatrix}^{-1} (\xi_1, \xi_2, \xi_3, \dots),$$

and therefore by substituting in the first

$$\begin{pmatrix} l_{11} & l_{12} & l_{13} & \dots \\ l_{21} & l_{22} & l_{23} & \dots \\ l_{31} & l_{32} & l_{33} & \dots \\ \dots & \dots & \dots & \dots \end{pmatrix} \begin{pmatrix} l_{11} & l_{21} & l_{31} & \dots \\ l_{12} & l_{22} & l_{32} & \dots \\ l_{13} & l_{23} & l_{33} & \dots \\ \dots & \dots & \dots & \dots \end{pmatrix}^{-1} (\xi_1, \xi_2, \xi_3, \dots) = (x_1, x_2, x_3, \dots),$$

which is all that was required. Indeed, it may be seriously questioned whether up to the present date this be not the best way of formulating the theorem for the construction of an orthogonal substitution. For example, in the case of *two* variables, we have

$$(x_1, x_2) = \begin{pmatrix} 1 & \lambda \\ -\lambda & 1 \end{pmatrix} \begin{pmatrix} 1 & -\lambda \\ \lambda & 1 \end{pmatrix}^{-1} (\xi_1, \xi_2),$$

$$= \begin{pmatrix} 1 & \lambda \\ -\lambda & 1 \end{pmatrix} \begin{pmatrix} \frac{1}{1+\lambda^2} & \frac{\lambda}{1+\lambda^2} \\ \frac{-\lambda}{1+\lambda^2} & \frac{1}{1+\lambda^2} \end{pmatrix} (\xi_1, \xi_2),$$

$$= \begin{pmatrix} \frac{1-\lambda^2}{1+\lambda^2} & \frac{2\lambda}{1+\lambda^2} \\ \frac{-2\lambda}{1+\lambda^2} & \frac{1-\lambda^2}{1+\lambda^2} \end{pmatrix} (\xi_1, \xi_2).$$

(2) The next step in advance was taken by HERMITE,* but he also made use of an intermediary set of variables, his mode of defining them being that each one of the set was to be the arithmetic mean of the corresponding members of the two given sets—that is to say, θ_r was defined by the equation

$$\theta_r = \tfrac{1}{2}(x_r + \xi_r).$$

Strictly speaking there was not much new in this, for it was in entire correspondence with CAYLEY'S intermediary equations, and very probably was derived from them. Thus, from the first equations of the two sets we have by addition

$$2l_{11}\theta_1 + (l_{12}+l_{21})\theta_2 + (l_{13}+l_{31})\theta_3 + \dots = x_1 + \xi_1,$$

i.e.

$$2\theta_1 = x_1 + \xi_1.$$

His after-procedure, however, was different from CAYLEY'S, and apparently lent itself more readily to generalisation, both in his own hands and in CAYLEY'S.

(3) An entirely new departure was taken by VELTMANN.† He dispensed with intermediary variables, laying down at once the connecting equations

* HERMITE, CH., "Sur la théorie des formes quadratiques ternaires indéfinies," *Crelle's Journ.*, xlvii. pp. 307–312. See also xlvii. pp. 313–342; and *Camb. and Dub. Math. Journ.*, ix. pp. 63–67.

† VELTMANN, W., "Die orthogonale Substitution," *Zeitschr. für Math. u. Phys.*, xvi. pp. 523–525.

$$\begin{aligned} l_{11}x_1+l_{12}x_2+l_{13}x_3+\ldots &= l_{11}\xi_1+l_{21}\xi_2+l_{31}\xi_3+\ldots \\ l_{21}x_1+l_{22}x_2+l_{23}x_3+\ldots &= l_{12}\xi_1+l_{22}\xi_2+l_{32}\xi_3+\ldots \\ l_{31}x_1+l_{32}x_2+l_{33}x_3+\ldots &= l_{13}\xi_1+l_{23}\xi_2+l_{33}\xi_3+\ldots \\ \ldots\ldots\ldots\

But the left-hand member of this may be partitioned into

$$\begin{array}{cccc|c} x_1 & x_2 & x_3 & \dots & \\ \hline a & h & g & \dots & x_1 \\ h & b & f & \dots & x_2 \\ g & f & c & \dots & x_3 \\ . & . & . & . & . \end{array} \quad \text{and} \quad \begin{array}{cccc|c} x_1 & x_2 & x_3 & \dots & \\ \hline . & l_{12} & l_{13} & \dots & x_1 \\ l_{21} & . & l_{23} & \dots & x_2 \\ l_{31} & l_{32} & . & \dots & x_3 \\ . & . & . & . & . \end{array},$$

the latter of which evidently vanishes. Consequently if we partition the right-hand member in the same way we deduce

$$\begin{array}{cccc|c} x_1 & x_2 & x_3 & \dots & \\ \hline a & h & g & \dots & x_1 \\ h & b & f & \dots & x_2 \\ g & f & c & \dots & x_3 \\ . & . & . & . & . \end{array} = \begin{array}{cccc|c} \xi_1 & \xi_2 & \xi_3 & \dots & \\ \hline a & h & g & \dots & x_1 \\ h & b & f & \dots & x_2 \\ g & f & c & \dots & x_3 \\ . & . & . & . & . \end{array} + \begin{array}{cccc|c} \xi_1 & \xi_2 & \xi_3 & \dots & \\ \hline . & l_{12} & l_{13} & \dots & x_1 \\ l_{21} & . & l_{23} & \dots & x_2 \\ l_{31} & l_{32} & . & \dots & x_3 \\ . & . & . & . & . \end{array}.$$

Had we used the multipliers $\xi_1, \xi_2, \xi_3, \dots$ the same process would clearly have given us

$$\begin{array}{cccc|c} \xi_1 & \xi_2 & \xi_3 & \dots & \\ \hline a & h & g & \dots & \xi_1 \\ h & b & f & \dots & \xi_2 \\ g & f & c & \dots & \xi_3 \\ . & . & . & . & . \end{array} = \begin{array}{cccc|c} x_1 & x_2 & x_3 & \dots & \\ \hline a & h & g & \dots & \xi_1 \\ h & b & f & \dots & \xi_2 \\ g & f & c & \dots & \xi^3 \\ . & . & . & . & . \end{array} + \begin{array}{cccc|c} x_1 & x_2 & x_3 & \dots & \\ \hline . & l_{21} & l_{31} & \dots & \xi_1 \\ l_{12} & . & l_{32} & \dots & \xi_2 \\ l_{13} & l_{23} & . & \dots & \xi_3 \\ . & . & . & . & . \end{array}.$$

We are thus led to two equations whose right-hand members are seen to be equal: hence the left-hand members are also equal—which is what was to be proved.

(5.) CAYLEY's solution of the problem of automorphic linear transformation of a quadric * differs so seriously from this that a thorough investigation of the discrepancy between the two seemed imperatively called for.

Writing the substitution of the preceding paragraph in CAYLEY's manner we have

$$\begin{vmatrix} a & h+l_{12} & g+l_{13} & \dots \\ h+l_{21} & b & f+l_{23} & \dots \\ g+l_{31} & f+l_{32} & c & \dots \\ . & . & . & . \end{vmatrix} (x_1, x_2, x_3, \dots) = \begin{vmatrix} a & h_1+l_{21} & g+l_{31} & \dots \\ h+l_{12} & b & f+l_{32} & \dots \\ g+l_{13} & f+l_{23} & c & \dots \\ . & . & . & . \end{vmatrix} (\xi_1, \xi_2, \xi_3, \dots),$$

and consequently

$$(x_1, x_2, x_3, \dots) = \begin{vmatrix} a & h+l_{12} & g+l_{13} & \dots \\ h+l_{21} & b & f+l_{23} & \dots \\ g+l_{31} & f+l_{32} & c & \dots \\ . & . & . & . \end{vmatrix}^{-1} \begin{vmatrix} a & h+l_{21} & g+l_{31} & \dots \\ h+l_{12} & b & f+l_{32} & \dots \\ g+l_{13} & f+l_{23} & c & \dots \\ . & . & . & . \end{vmatrix} (\xi_1, \xi_2, \xi_3, \dots).$$

Now if, for shortness' sake, we put this in the form

$$(x_1, x_2, x_3 \dots) = (\Delta)^{-1}(\Delta')(\xi_1, \xi_2, \xi_3, \dots),$$

CAYLEY's corresponding result will be found to be

$$(x_1, x_2, x_3, \dots) = (\mathrm{D})^{-1}(\Delta')(\Delta)^{-1}(\mathrm{D})(\xi_1, \xi_2, \xi_3, \dots),$$

* *Crelle's Journ.*, l. pp. 288–299; or *Collected Math. Papers*, ii. pp. 192–201.

where D is the matrix corresponding to the discriminant of the quadric. The latter result is evidently much the more complicated, but the difference between the two is far greater than the outward appearance of the expressions

$$(\Delta)^{-1}(\Delta') \quad \text{and} \quad (D)^{-1}(\Delta')(\Delta)^{-1}(D)$$

might at first suggest. Even in the case of only two variables—the transformation of $ax^2+2hxy+by^2$—the repeated multiplications implied by $(D)^{-1}(\Delta')(\Delta)^{-1}(D)$ involve a surprising amount of labour.

It was soon ascertained that considerable simplification of CAYLEY's coefficients was possible, and at last it became apparent that in every case they could be reduced to the comparatively simple forms given by the other formula. The consequence, of course, is that when D and Δ have the forms

$$\begin{pmatrix} a & h & g & \dots \\ h & b & f & \dots \\ g & f & c & \dots \\ \dots & \dots & \dots & \dots \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} a & h+l_{12} & g+l_{13} & \dots \\ h+l_{21} & b & f+l_{23} & \dots \\ g+l_{31} & f+l_{32} & c & \dots \\ \dots & \dots & \dots & \dots \end{pmatrix}$$

we can assert the curious theorem in matrix multiplication that

$$(D)^{-1}(\Delta')(\Delta)^{-1}(D) = (\Delta^{-1})(\Delta').$$

(6.) On account of the lengthy expressions involved, the proof is not by any means easily set down if the calculus of matrices be not utilised.* Confining ourselves to matrices of the 3rd order and writing, ν, $-\mu$, λ for l_{12}, l_{13}, l_{23}, the identity to be established is

$$\begin{pmatrix} a & h & g \\ h & b & f \\ g & f & c \end{pmatrix}^{-1}\begin{pmatrix} a & h-\nu & g+\mu \\ h+\nu & b & f-\lambda \\ g-\mu & f+\lambda & c \end{pmatrix}\begin{pmatrix} a & h+\nu & g-\mu \\ h-\nu & b & f+\lambda \\ g+\mu & f-\lambda & c \end{pmatrix}^{-1}\begin{pmatrix} a & h & g \\ h & b & f \\ g & f & c \end{pmatrix}$$

$$= \begin{pmatrix} a & h+\nu & g-\mu \\ h-\nu & b & f+\lambda \\ g+\mu & f-\lambda & c \end{pmatrix}^{-1}\begin{pmatrix} a & h-\nu & g+\mu \\ h+\nu & b & f-\lambda \\ g-\mu & f+\lambda & c \end{pmatrix}.$$

Now if we denote the complementary minors of a, h, ... in D by $A, -H, \dots$, and the complementary minors of a, $h+\nu$, ... in Δ by $\mathrm{cof}(a), -\mathrm{cof}(h+\nu), \dots$ we know from CAYLEY that

* With the aid of this calculus, however, the proof is very simple, and will be seen to hinge entirely on the fact that $\Delta+\Delta'=2D$. At its fullest extent it stands as follows:—

$$\begin{aligned} D^{-1}\Delta'\Delta^{-1}D &= D^{-1}(2D-\Delta)\Delta^{-1}D, \\ &= (2-D^{-1}\Delta)\Delta^{-1}D, \\ &= 2\Delta^{-1}D-1, \\ &= \Delta^{-1}(2D-\Delta), \\ &= \Delta^{-1}\Delta'. \end{aligned}$$

Here Δ' is the matrix got from Δ by changing rows into columns: but this relationship is not a necessity for the existence of the identity, which will hold if Δ, D, Δ' be any three matrices whatever fulfilling the condition $\Delta+\Delta'=2D$.

$$\begin{pmatrix} a & h & g \\ h & b & f \\ g & f & c \end{pmatrix}^{-1} = \begin{pmatrix} \frac{A}{D} & \frac{H}{D} & \frac{G}{D} \\ \frac{H}{D} & \frac{B}{D} & \frac{F}{D} \\ \frac{G}{D} & \frac{F}{D} & \frac{C}{D} \end{pmatrix},$$

and

$$\begin{pmatrix} a & h+\nu & g-\mu \\ h-\nu & b & f+\lambda \\ g+\mu & f-\lambda & c \end{pmatrix}^{-1} = \begin{pmatrix} \frac{\text{cof}(a)}{\Delta} & \frac{\text{cof}(h-\nu)}{\Delta} & \frac{\text{cof}(g+\mu)}{\Delta} \\ \frac{\text{cof}(h+\nu)}{\Delta} & \frac{\text{cof}(b)}{\Delta} & \frac{\text{cof}(f-\lambda)}{\Delta} \\ \frac{\text{cof}(g-\mu)}{\Delta} & \frac{\text{cof}(f+\lambda)}{\Delta} & \frac{\text{cof}(c)}{\Delta} \end{pmatrix}.$$

Consequently the product of the four matrices on the left side of the identity is a matrix whose (1,1) element is *

$$\begin{array}{ccc|ccc} \frac{A}{D} & \frac{H}{D} & \frac{G}{D} & a & h & g \\ \hline a & h+\nu & g-\mu & \frac{\text{cof}(a)}{\Delta} & \frac{\text{cof}(h-\nu)}{\Delta} & \frac{\text{cof}(g+\mu)}{\Delta} \\ h-\nu & b & f+\lambda & \frac{\text{cof}(h+\nu)}{\Delta} & \frac{\text{cof}(b)}{\Delta} & \frac{\text{cof}(f-\lambda)}{\Delta} \\ g+\mu & f-\lambda & c & \frac{\text{cof}(g-\mu)}{\Delta} & \frac{\text{cof}(f+\lambda)}{\Delta} & \frac{\text{cof}(c)}{\Delta} \end{array},$$

and the product of the two matrices on the right is a matrix whose (1,1) element is

$$\frac{\frac{\text{cof}(a)}{\Delta}, \quad \frac{\text{cof}(h-\nu)}{\Delta}, \quad \frac{\text{cof}(g+\mu)}{\Delta}}{a \quad , \quad h+\nu \quad , \quad g-\mu},$$

or, as CAYLEY would have written it,

$$\left(\frac{\text{cof}(a)}{\Delta}, \quad \frac{\text{cof}(h-\nu)}{\Delta}, \quad \frac{\text{cof}(g+\mu)}{\Delta}\right)\left(a, \quad h+\nu, \quad g-\mu\right).$$

We have thus to prove the identity of these two elements, and eight other identities like it.

Multiplying both sides by $D\Delta$ we see that this is the same as to prove that

$$\begin{array}{ccc|ccc} A & H & G & a & h & g \\ \hline a & h+\nu & g-\mu & \text{cof}(a) & \text{cof}(h-\nu) & \text{cof}(g+\mu) \\ h-\nu & b & f+\lambda & \text{cof}(h+\nu) & b & \text{cof}(f-\lambda) \\ g+\mu & f-\lambda & c & \text{cof}(g-\mu) & \text{cof}(f+\lambda) & c \end{array} = D \cdot \frac{\text{cof}(a), \quad \text{cof}(h-\nu), \quad \text{cof}(g+\mu)}{a \quad , \quad h+\nu \quad , \quad g-u}.$$

But the left-hand member of this can be partitioned into

$$\begin{array}{ccc|ccc} A & H & G & a & h & g \\ \hline a & h & g & \text{cof}(a) & \ldots & \ldots \\ h & b & f & \ldots & \ldots & \ldots \\ g & f & c & \ldots & \ldots & \ldots \end{array} + \begin{array}{ccc|ccc} A & H & G & a & h & g \\ \hline . & \nu & -\mu & \text{cof}(a) & \ldots & \ldots \\ -\nu & . & \lambda & \ldots & \ldots & \ldots \\ \mu & -\lambda & . & \ldots & \ldots & \ldots \end{array},$$

* *Trans. Roy. Soc. Edin.*, xxxii. pp. 461–482.

and by reason of the identities

$$\frac{\mathrm{A}\ \mathrm{H}\ \mathrm{G}}{a\ \ h\ \ g} = \mathrm{D}\,,$$

$$\frac{\mathrm{A}\ \mathrm{H}\ \mathrm{G}}{h\ \ b\ \ f} = 0\,,$$

$$\frac{\mathrm{A}\ \mathrm{H}\ \mathrm{G}}{g\ \ f\ \ c} = 0\,,$$

the first of these parts

$$= \mathrm{D}\cdot\frac{a\ ,\ h\ ,\ g}{\mathrm{cof}\,(a),\ \mathrm{cof}\,(h-\nu),\ \mathrm{cof}\,(g+\mu)},$$

$$= \mathrm{D}\cdot\frac{a\ ,\ h+\nu\ ,\ g-\mu}{\mathrm{cof}\,(a),\ \mathrm{cof}\,(h-\nu),\ \mathrm{cof}\,(g+\mu)} + \mathrm{D}\cdot\frac{.\ ,\ -\nu\ ,\ \mu}{\mathrm{cof}\,(a),\ \mathrm{cof}\,(h-\nu),\ \mathrm{cof}\,(g+\mu)}.$$

Consequently by removing the expression which is common to both sides we have only to prove that

$$\mathrm{D}\cdot\{-\nu\,\mathrm{cof}\,(h-\nu)+\mu\,\mathrm{cof}\,(g+\mu)\} + \begin{array}{ccc|c} \mathrm{A} & \mathrm{H} & \mathrm{G} & a\ \ h\ \ g \\ \hline . & \nu & -\mu & \mathrm{cof}\,a, \ldots . \\ -\nu & . & \lambda & \ldots . \\ \mu & -\lambda & . & \ldots . \end{array} = 0\,.$$

Now, as will be seen from a later paragraph,

$$\mathrm{cof}\,(a) = \mathrm{A}+\lambda^2,$$

$$\mathrm{cof}\,(h-\nu) = \mathrm{H}+\lambda\mu-\frac{\lambda\ ,\ \mu\ ,\ \nu}{g\ ,\ f\ ,\ c},$$

. . . . . .

The first part of the vanishing expression is thus

$$= \mathrm{D}\left(-\nu\mathrm{H}-\lambda\mu\nu+\nu\frac{\lambda\ ,\ \mu\ ,\ \nu}{g\ ,\ f\ ,\ c}+\mu\mathrm{G}+\mu\frac{\lambda\ ,\ \mu\ ,\ \nu}{h\ ,\ b\ ,\ f}+\lambda\mu\nu\right),$$

$$= \mathrm{D}\left(-\nu\mathrm{H}+\mu\mathrm{G}+\begin{array}{ccc|c} \lambda & \mu & \nu & \\ \hline a & h & g & . \\ h & b & f & \mu \\ g & f & c & \nu \end{array}\right);$$

and the second part

$$= \begin{array}{ccc|ccc} \mathrm{A} & \mathrm{H} & \mathrm{G} & a & h & g \\ \hline . & \nu & -\mu & \mathrm{A}+\lambda^2 & \mathrm{H}+\mu\lambda-\dfrac{\lambda\ \mu\ \nu}{g\ f\ c} & \mathrm{G}+\nu\lambda+\dfrac{\lambda\ \mu\ \nu}{h\ b\ f} \\ -\nu & . & \lambda & \mathrm{H}+\lambda\mu+\dfrac{\lambda\ \mu\ \nu}{g\ f\ c} & \mathrm{B}+\mu^2 & \mathrm{F}+\nu\mu-\dfrac{\lambda\ \mu\ \nu}{a\ h} \\ \mu & -\lambda & . & \mathrm{G}+\lambda\nu-\dfrac{\lambda\ \mu\ \nu}{h\ b\ f} & \mathrm{F}+\mu\nu+\dfrac{\lambda\ \mu\ \nu}{a\ h\ g} & \mathrm{C}+\nu^2 \end{array}.$$

By reason, however, of the trinomial elements in the second square array this can be partitioned into

$$\begin{array}{ccc|ccc} A & H & G & a & h & g \\ \hline . & \nu & -\mu & A & H & G \\ -\nu & . & \lambda & H & B & F \\ \mu & -\lambda & . & G & F & C \end{array} + \begin{array}{ccc|ccc} A & H & G & a & h & g \\ \hline . & \nu & -\mu & \lambda^2 & \mu\lambda & \nu\lambda \\ -\nu & . & \lambda & \lambda\mu & \mu^2 & \nu\mu \\ \mu & -\lambda & . & \lambda\nu & \mu\nu & \nu^2 \end{array}$$

$$+ \begin{array}{ccc|ccc} A & H & G & a & h & g \\ \hline . & \nu & -\mu & . & -\dfrac{\lambda\ \mu\ \nu}{g\ f\ c} & \dfrac{\lambda\ \mu\ \nu}{h\ b\ f} \\ -\nu & . & \lambda & \dfrac{\lambda\ \mu\ \nu}{g\ f\ c} & . & -\dfrac{\lambda\ \mu\ \nu}{a\ h\ g} \\ \mu & -\lambda & . & -\dfrac{\lambda\ \mu\ \nu}{h\ b\ f} & \dfrac{\lambda\ \mu\ \nu}{a\ h\ g} & . \end{array};$$

and taking these three terms in order we have the first

$$= \frac{A\ H\ G}{.\ \ \nu\ -\mu} \cdot \frac{a\ h\ g}{A\ H\ G} + 0 + 0,$$

$$= (\nu H - \mu G)D;$$

the second

$$= \begin{array}{ccc|ccc} A & H & G & \lambda a & \mu h & \nu g \\ \hline . & \nu & -\mu & \lambda & \lambda & \lambda \\ -\nu & . & \lambda & \mu & \mu & \mu \\ \mu & -\lambda & . & \nu & \nu & \nu \end{array},$$

$$= \begin{array}{ccc|c} A & H & G & \\ \hline . & . & . & \lambda a \\ . & . & . & \mu h \\ . & . & . & \nu g \end{array} = 0;$$

and the third, if ω be used for $a\lambda^2 + b\mu^2 + c\nu^2 + 2f\mu\nu + 2g\nu\lambda + 2h\lambda\mu$,

$$= \begin{array}{ccc|c} A & H & G & \\ \hline -\omega + \lambda\dfrac{\lambda\ \mu\ \nu}{a\ h\ g} & \lambda\dfrac{\lambda\ \mu\ \nu}{h\ b\ f} & \lambda\dfrac{\lambda\ \mu\ \nu}{g\ f\ c} & a \\ \mu\dfrac{\lambda\ \mu\ \nu}{a\ h\ g} & -\omega + \mu\dfrac{\lambda\ \mu\ \nu}{h\ b\ f} & \mu\dfrac{\lambda\ \mu\ \nu}{g\ f\ c} & h \\ \nu\dfrac{\lambda\ \mu\ \nu}{a\ h\ g} & \nu\dfrac{\lambda\ \mu\ \nu}{h\ b\ f} & -\omega + \nu\dfrac{\lambda\ \mu\ \nu}{g\ f\ c} & g \end{array},$$

$$= -\omega D + \frac{A\frac{\lambda\ \mu\ \nu}{a\ h\ g}\ \ H\frac{\lambda\ \mu\ \nu}{h\ b\ f}\ \ G\frac{\lambda\ \mu\ \nu}{g\ f\ c}}{\begin{array}{ccc|c} 1 & 1 & 1 & \lambda a \\ 1 & 1 & 1 & \mu h \\ 1 & 1 & 1 & \nu g \end{array}},$$

$$= -\omega D + \frac{\begin{array}{ccc} A & H & G \end{array}}{\begin{array}{ccc|c} a & h & g & \lambda \\ h & b & f & \mu \\ g & f & c & \nu \end{array}} (\lambda a + \mu h + \nu g),$$

$$= -\omega D + \lambda D(\lambda a + \mu h + \nu g),$$

$$= -\mathrm{D}\left\{\begin{array}{c|c} \begin{matrix}\lambda & \mu & \nu \\ a & h & g \\ h & b & f \\ g & f & c\end{matrix} & \begin{matrix}-\lambda(\lambda a+\mu h+\nu g) \\ \lambda \\ \mu \\ \nu\end{matrix}\end{array}\right\},$$

$$= -\mathrm{D}.\begin{array}{c|c} \begin{matrix}\lambda & \mu & \nu \\ a & h & g \\ h & b & f \\ g & f & c\end{matrix} & \begin{matrix}\cdot \\ \lambda \\ \mu \\ \nu\end{matrix}\end{array}.$$

The sum of the three parts is thus

$$\mathrm{D}\left(\nu\mathrm{H}-\mu\mathrm{G}-\begin{array}{c|c} \begin{matrix}\lambda & \mu & \nu \\ a & h & g \\ h & b & f \\ g & f & c\end{matrix} & \begin{matrix}\cdot \\ \lambda \\ \mu \\ \nu\end{matrix}\end{array}\right),$$

an expression differing merely in sign from that to which it has to be added. The desired vanishing result has thus been reached.*

(7.) It will be remembered that in the case of an orthogonal substitution, viz., the case where $f=g=h=\ldots=0$ and $a=b=c=\ldots=1$, CAYLEY was enabled to give a simple rule for the formation of the coefficients of the substitution, the original wording of the rule being†

"*Les co-efficients propres à la transformation de co-ordonnées rectangulaires peuvent être exprimés rationnellement au moyen de quantités arbitraires* l_{rs} *soumises au conditions*

$$l_{rs}=-l_{sr}\ [r\neq s]\ ;\quad l_{rr}=1\ .$$

Pour développer les formules, il faut d'abord former le déterminant Δ *de ce système, puis le système inverse* L_{rs}, . . . *et écrire*

$$\Delta a_{rs}=2\mathrm{L}_{rs}\ [r\neq s]\ ;\quad \Delta a_{rr}=2\mathrm{L}_{rr}-\Delta$$

ce qui donne le système cherché."

Such a rule was a practical impossibility when the problem for the general quadric came to be solved, because of the very complicated character of the results to which both he and HERMITE were led. It will now be seen that as a consequence of the simplification above effected this impossibility disappears.

Taking once more, for shortness' sake, the quadric with only three variables, viz.

* Since this was written I have ascertained that the simplification here given of CAYLEY'S solution was known to FROBENIUS, whose paper of the year 1877, "Ueber lineare Substitutionen und bilineare Formen" (*Crelle's Journ.* lxxxiv. pp. 1–63) is a carefully written and methodically arranged exposition of the theory of matrices with applications. It would appear not to have received due attention from subsequent writers.

The simplification is also explicitly referred to in one of a series of valuable papers by Dr HENRY TABER in the *Proc. Lond. Math. Soc.* (1890–93).

† *Crelle's Journ.*, xxxii. pp. 119–123 ; or *Collected Math. Papers*, i. pp. 332–336.

$$\begin{array}{ccc|c} x & y & z & \\ \hline a & h & g & x \\ h & b & f & y \\ g & f & c & z \end{array},$$

we know that the requisite automorphic substitution in its *implicit* form is

$$\left.\begin{array}{rcl} ax+(h+\nu)y+(g-\mu)z &=& a\xi+(h-\nu)\eta+(g+\mu)\zeta \\ (h-\nu)x+by+(f+\lambda)z &=& (h+\nu)\xi+b\eta+(f-\lambda)\zeta \\ (g+\mu)x+(f-\lambda)y+cz &=& (g-\mu)\xi+(f+\lambda)\eta+c\zeta \end{array}\right\}.$$

Consequently, if we denote the determinant of the coefficients of x, y, z by Δ, we have on solving for x

$$\Delta . x = \begin{vmatrix} a\xi+(h-\nu)\eta+(g+\mu)\zeta & h+\nu & g-\mu \\ (h+\nu)\xi+b\eta+(f-\lambda)\zeta & b & f+\lambda \\ (g-\mu)\xi+(f+\lambda)\eta+c\zeta & f-\lambda & c \end{vmatrix},$$

$$= \begin{vmatrix} a\xi+2h\eta+2g\zeta & h+\nu & g-\mu \\ (h+\nu)\xi+2b\eta+2f\zeta & b & f+\lambda \\ (g-\mu)\xi+2f\eta+2c\zeta & f-\lambda & c \end{vmatrix},$$

$$= \begin{vmatrix} (2a-a)\xi+2h\eta+2g\zeta & h+\nu & g-\mu \\ (2h-\overline{h-\nu})\xi+2b\eta+2f\zeta & b & f+\lambda \\ (2g-\overline{g+\mu})\xi+2f\eta+2c\zeta & f-\lambda & c \end{vmatrix},$$

$$= 2\begin{vmatrix} a & h+\nu & g-\mu \\ h & b & f+\lambda \\ g & f-\lambda & c \end{vmatrix}\xi - \Delta\xi + 2\begin{vmatrix} a & h+\nu & g-\mu \\ b & b & f+\lambda \\ f & f-\lambda & c \end{vmatrix}\eta + 2\begin{vmatrix} g & h+\nu & g-\mu \\ f & b & f+\lambda \\ c & f-\lambda & c \end{vmatrix}.$$

Now all the determinants here have their last two columns identical with the corresponding columns of Δ. It follows, therefore, that if the determinant adjugate to Δ be

$$\begin{vmatrix} L_{11} & L_{12} & L_{13} \\ L_{21} & L_{22} & L_{23} \\ L_{31} & L_{32} & L_{33} \end{vmatrix},$$

the result which we have just reached may be written :—

$$\Delta . x = \left(2\frac{L_{11}L_{21}L_{31}}{a\ h\ g}-\Delta\right)\xi + \left(2\frac{L_{11}L_{21}L_{31}}{h\ b\ f}\right)\eta + \left(2\frac{L_{11}L_{21}L_{31}}{g\ f\ c}\right)\zeta .$$

Similarly

$$\Delta . y = \left(2\frac{L_{12}L_{22}L_{32}}{a\ h\ g}\right)\xi + \left(2\frac{L_{12}L_{22}L_{32}}{h\ b\ f}-\Delta\right)\eta + \left(2\frac{L_{12}L_{22}L_{32}}{g\ f\ c}\right)\zeta ,$$

and

$$\Delta . z = \left(2\frac{L_{13}L_{23}L_{33}}{a\ h\ g}\right)\xi + \left(2\frac{L_{13}L_{23}L_{33}}{h\ b\ f}\right)\eta + \left(2\frac{L_{13}L_{23}L_{33}}{g\ f\ c}-\Delta\right)\zeta .$$

We have thus the following rule for the formation of the coefficients a_{rs} of the linear substitution which transforms the quadric

$$ax^2+by^2+cz^2+\ldots.+2fyz+2gzx+2hxy+\ldots.$$

into itself :—

Denote the discriminant by D, *and form the determinant* R *adjugate to*

$$\begin{vmatrix} a & h+\nu & g-\mu & \dots \\ h-\nu & b & f+\lambda & \dots \\ g+\mu & f-\lambda & c & \dots \\ \dots & \dots & \dots & \dots \end{vmatrix} \quad or \quad \Delta,$$

then

$$\Delta \,.\, a_{rs} = 2(r^{th}\ col.\ of\ \mathrm{R})(s^{th}\ row\ of\ \mathrm{D}) \quad when\ r \neq s,$$

and

$$\Delta \,.\, a_{rr} = 2(r^{th}\ col.\ of\ \mathrm{R})(r^{th}\ row\ of\ \mathrm{D}) - \Delta.$$

This is manifestly just as simple as CAYLEY's rule for the formation of an orthogonal substitution, into which rule it is seen to pass when $a=b=c=\dots=1$ and $f=g=h=\dots=0$.

(8) The most elegant proof of the rule in all its generality is got by using the theory of matrices as follows :—

Taking the substitution in its implicit form, and solving for $x, y, z, \dots$ we have

$$(x,y,z,\dots) = \begin{pmatrix} a & h+\nu & g-\mu & \dots \\ h-\nu & b & f+\lambda & \dots \\ g+\mu & f-\lambda & c & \dots \\ \dots & \dots & \dots & \dots \end{pmatrix}^{-1} \begin{pmatrix} a & h-\nu & g+\mu & \dots \\ h+\nu & b & f-\lambda & \dots \\ g-\mu & f+\lambda & c & \dots \\ \dots & \dots & \dots & \dots \end{pmatrix} (\xi,\eta,\zeta,\dots).$$

Now the first matrix here is equal to

$$\begin{pmatrix} \frac{L_{11}}{\Delta} & \frac{L_{21}}{\Delta} & \frac{L_{31}}{\Delta} & \dots \\ \frac{L_{12}}{\Delta} & \frac{L_{22}}{\Delta} & \frac{L_{32}}{\Delta} & \dots \\ \frac{L_{13}}{\Delta} & \frac{L_{23}}{\Delta} & \frac{L_{33}}{\Delta} & \dots \\ \dots & \dots & \dots & \dots \end{pmatrix},$$

and the second can be expressed as the difference of two, viz., as

$$\begin{pmatrix} 2a & 2h & 2g & \dots \\ 2h & 2b & 2f & \dots \\ 2g & 2f & 2c & \dots \\ \dots & \dots & \dots & \dots \end{pmatrix} - \begin{pmatrix} a & h+\nu & g-\mu & \dots \\ h-\nu & b & f+\lambda & \dots \\ g+\mu & f-\lambda & c & \dots \\ \dots & \dots & \dots & \dots \end{pmatrix}.$$

On performing the required multiplication we thus have

$$\begin{pmatrix} 2\frac{L_{11}\ L_{21}\ L_{31}\ \dots}{a\ h\ g\ \dots} & 2\frac{L_{11}\ L_{21}\ L_{31}\ \dots}{h\ b\ f\ \dots} & 2\frac{L_{11}\ L_{21}\ L_{31}\ \dots}{g\ f\ c\ \dots} & \dots\dots \\ 2\frac{L_{12}\ L_{22}\ L_{32}\ \dots}{a\ h\ g\ \dots} & 2\frac{L_{12}\ L_{22}\ L_{32}\ \dots}{h\ b\ f\ \dots} & 2\frac{L_{12}\ L_{22}\ L_{32}\ \dots}{g\ f\ c\ \dots} & \dots\dots \\ 2\frac{L_{13}\ L_{23}\ L_{33}\ \dots}{a\ h\ g\ \dots} & 2\frac{L_{13}\ L_{23}\ L_{33}\ \dots}{h\ b\ f\ \dots} & 2\frac{L_{13}\ L_{23}\ L_{33}\ \dots}{g\ f\ c\ \dots} & \dots\dots \\ \dots & \dots & \dots & \dots \end{pmatrix} - \begin{pmatrix} 1 & . & . & \dots \\ . & 1 & . & \dots \\ . & . & 1 & \dots \\ \dots & \dots & \dots & \dots \end{pmatrix};$$

and this difference being expressed as one matrix we obtain the matrix of the substitution. A glance suffices to show that it is identical with the matrix

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} & \dots \\ a_{21} & a_{22} & a_{23} & \dots \\ a_{31} & a_{32} & a_{33} & \dots \\ \dots & \dots & \dots & \dots \end{pmatrix}$$

as specified in the preceding paragraph.*

(9) It will have been observed that in the construction of the substitution use is made of the peculiar matrix

$$\begin{pmatrix} a & h+\nu & g-\mu & \dots \\ h-\nu & b & f+\lambda & \dots \\ g+\mu & f-\lambda & c & \dots \\ \dots & \dots & \dots & \dots \end{pmatrix}.$$

In fact this matrix is the material out of which the substitution is built up, just as the matrix of a unit-axial skew determinant is the material for the formation of an orthogonal substitution. And as in dealing with the problem of orthogonal substitution CAYLEY was thus led to study the properties of skew determinants, we cannot do better than follow his example and examine into the nature of the determinant of the analogous matrix employed in the more general problem which we have just been dealing with.

This more general determinant is seen to be obtainable by the superposition, so to speak, of the zero-axial skew determinant

$$\begin{vmatrix} . & \nu & -\mu & \dots \\ -\nu & . & \lambda & \dots \\ \mu & -\lambda & . & \dots \\ \dots & \dots & \dots & \dots \end{vmatrix}$$

upon the axisymmetric determinant

$$\begin{vmatrix} a & h & g & \dots \\ h & b & f & \dots \\ g & f & c & \dots \\ \dots & \dots & \dots & \dots \end{vmatrix}.$$

And as by changing the signs of all the Greek letters the value of the determinant is unaltered, it follows that in the final development of the determinant there can be no terms involving the product of an *odd* number of these letters. Further, as each term of the final development of the determinant must be of the nth degree in the letters which it contains, it follows that the said terms may be classified as follows:—

1. terms of the nth degree in Italic letters and of the 0th . . . . . Greek . . . ,
2. terms of the $(n-2)^{\text{th}}$ degree in Italic letters and of the 2nd . . . . . Greek letters ,

. . . . . .

or, for shortness' sake, let us say terms of the degree $n+0$, $\overline{n-2}+2$, $\overline{n-4}+4$, . . .

* See also the third line of the proof that $D^{-1}\Delta'\Delta^{-1}D=\Delta^{-1}\Delta'$ in the footnote to § 6.

Bearing this in mind it is not difficult to arrive at the final development in any given case. Thus, for the determinant of the 3rd order, we look first for terms of the degree $3+0$, and then for terms of the degree $1+2$. The aggregate of the former is readily seen to be

$$\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix};$$

and, each of the latter being composed of an Italic letter with a Greek cofactor, the aggregate is almost as easily seen to be

$$\begin{array}{ccc|c} \lambda & \mu & \nu & \\ \hline a & h & g & \lambda \\ h & b & f & \mu \\ g & f & c & \nu. \end{array}$$

We have thus

$$\begin{vmatrix} a & h+\nu & g-\mu \\ h-\nu & b & f+\lambda \\ g+\mu & f-\lambda & c \end{vmatrix} = \begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} + \begin{array}{ccc|c} \lambda & \mu & \nu & \\ \hline a & h & g & \lambda' \\ h & b & f & \mu \\ g & f & c & \nu \end{array}$$

an identity which may also be viewed as giving an expression for the sum of a ternary quadric and its discriminant.

Next taking the determinant of the 4th order we first look for terms of the degree $4+0$, then for terms of the degree $2+2$, and lastly for terms of the degree $0+4$. The aggregate of the first kind lies to hand as readily as before: those of the second kind are most easily obtained by seeking for the coefficients of $\rho^2, \rho\sigma, \sigma^2, \ldots\ldots$, which are seen to be

$$\begin{vmatrix} a & h \\ h & b \end{vmatrix}, \quad \begin{vmatrix} a & g \\ h & f \end{vmatrix}, \quad \begin{vmatrix} a & g \\ g & c \end{vmatrix}, \ldots;$$

and the aggregate of those of the last kind by dropping out all the Italic letters from the determinant which thus becomes zero-axial skew with the Pfaffian equivalent

$$\begin{vmatrix} \nu & -\mu & \tau \\ & \lambda & -\sigma \\ & & \rho \end{vmatrix}^2 \quad \text{or} \quad \begin{vmatrix} \lambda & \mu & \nu \\ & & \rho & \sigma \\ & & & \tau \end{vmatrix}^2.$$

When we come to determinants of the 5th order it is seen that there are still only three kinds of terms, the degrees of which are $5+0$, $3+2$, $1+4$: and in obtaining the aggregates no new consideration is necessary.

In the first four cases the expansions therefore stand thus :—

$$\begin{vmatrix} a & h+\nu \\ h-\nu & b \end{vmatrix} = \begin{vmatrix} a & h \\ h & b \end{vmatrix} + \nu^2;$$

$$\begin{vmatrix} a & h+\nu & g-\mu \\ h-\nu & b & f+\lambda \\ g+\mu & f-\lambda & c \end{vmatrix} = \begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} + \begin{matrix} \lambda & \mu & \nu \\ \hline a & h & g \\ h & b & f \\ g & f & c \end{matrix} \begin{matrix} \lambda \\ \mu \\ \nu \end{matrix};$$

$$\begin{vmatrix} a & h+\nu & g-\mu & r+\tau \\ h-\nu & b & f+\lambda & q-\sigma \\ g+\mu & f-\lambda & c & p+\rho \\ r-\tau & q+\sigma & p-\rho & d \end{vmatrix} = \begin{vmatrix} a & h & g & r \\ h & b & f & q \\ g & f & c & p \\ r & q & p & d \end{vmatrix} + \mathrm{B} + \mathrm{C},$$

where

$$\mathrm{B} = \begin{array}{c|cccccc} & \rho & \sigma & \lambda & \tau & \mu & \nu \\ \hline \rho & \begin{vmatrix} a & h \\ h & b \end{vmatrix}, & \begin{vmatrix} a & g \\ h & f \end{vmatrix}, & \ldots & \ldots & \ldots & \ldots \\ \sigma & \begin{vmatrix} a & h \\ g & f \end{vmatrix}, & \ldots & \ldots & \ldots & \ldots & \ldots \\ \lambda & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ \tau & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ \mu & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ \nu & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \end{array}$$

and

$$\mathrm{C} = \begin{vmatrix} \lambda & \mu & \nu \\ \rho & \sigma & \\ & \tau & \end{vmatrix}^2;$$

$$\begin{vmatrix} a & h+\nu & g-\mu & r+\tau & n-\psi \\ h-\nu & b & f+\lambda & q-\sigma & m+\chi \\ g+\mu & f-\lambda & c & p+\rho & l-\phi \\ r-\tau & q+\sigma & p-\rho & d & k+\theta \\ n+\psi & m-\chi & l+\phi & k-\theta & e \end{vmatrix} = \begin{vmatrix} a & h & g & r & n \\ h & b & f & q & m \\ g & f & c & p & l \\ r & q & p & d & k \\ n & m & l & k & e \end{vmatrix} + \mathrm{D} + \mathrm{E},$$

where

$$\mathrm{D} = \begin{array}{c|cccccccccc} & \theta & \phi & \rho & \chi & \sigma & \lambda & \psi & \tau & \mu & \nu \\ \hline \theta & \begin{vmatrix} 123 \\ 123 \end{vmatrix} & \begin{vmatrix} 123 \\ 124 \end{vmatrix} & \begin{vmatrix} 123 \\ 125 \end{vmatrix} & \begin{vmatrix} 123 \\ 134 \end{vmatrix} & \begin{vmatrix} 123 \\ 135 \end{vmatrix} & \begin{vmatrix} 123 \\ 145 \end{vmatrix} & \begin{vmatrix} 123 \\ 234 \end{vmatrix} & \begin{vmatrix} 123 \\ 235 \end{vmatrix} & \begin{vmatrix} 123 \\ 245 \end{vmatrix} & \begin{vmatrix} 123 \\ 345 \end{vmatrix} \\ \phi & \begin{vmatrix} 124 \\ 123 \end{vmatrix} & \begin{vmatrix} 124 \\ 124 \end{vmatrix} & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ \rho & \begin{vmatrix} 125 \\ 123 \end{vmatrix} & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ \chi & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ \cdot & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \end{array}$$

$\left|\begin{matrix}123\\123\end{matrix}\right|$ being used to denote the minor obtained by deleting the 4th and 5th rows and 4th and 5th columns of the determinant preceding D, and where

$$E=a\begin{vmatrix}\lambda & \sigma & \chi\\ \rho & \phi & \\ & \theta & \end{vmatrix}^2 + 2h\begin{vmatrix}\lambda & \sigma & \chi\\ & \rho & \phi\\ & & \theta\end{vmatrix}\cdot\begin{vmatrix}\mu & \tau & \psi\\ & \rho & \phi\\ & & \theta\end{vmatrix} + \ldots$$

(10) The law of formation of the bipartites B and D, when properly looked at, is very simple. The square array in each case consists of the secondary minors of the axisymmetric determinant which immediately precedes it,—in other words, the elements of the square array of B are exactly the elements of the second compound of the determinant which precedes it, and the corresponding elements of D are the elements of the third compound. The bordering elements, of D say, are the elements of the quasi-Pfaffian

$$\begin{vmatrix}\nu & \mu & \tau & \psi\\ & \lambda & \sigma & \chi\\ & & \rho & \phi\\ & & & \theta\end{vmatrix}$$

taken in a backward order, viz., in the order θ; ϕ, ρ; χ, σ, λ; ψ, τ, μ, ν.

The law of formation of E is equally simple. Each term is the product of three factors, the first being an element of the preceding axisymmetric determinant, and the two others being primary minors of the quasi-Pfaffian just referred to. The Pfaffian factors which are to accompany any element are determined by the position of that element in the determinant-array, the law being that when the element belongs to the place (r,s) we delete the rth frame-line of the quasi-Pfaffian to obtain the one factor and the sth frame-line to obtain the other. Consequently, if we denote the quasi-Pfaffian by ff and the minor of it obtained by deleting the rth frame-line by $ff^{(r}$, we may write each term of E in the form

$$(r,s).ff^{(r}ff^{(s}.$$

We are thus further led to see that E also is a bipartite having the beautifully simple form

$ff^{(1}$	$ff^{(2}$	$ff^{(3}$	$ff^{(4}$	$ff^{(5}$	
a	h	g	r	n	$ff^{(1}$
h	b	f	q	m	$ff^{(2}$
g	f	c	p	l	$ff^{(3}$
r	q	p	d	k	$ff^{(4}$
n	m	l	k	e	$ff^{(5}$,

and it is suggested to inquire whether there be not a mode of writing D and E, and perhaps even the term preceding D, which will put in evidence the fact that they are members of a series advancing according to a definite law.

Each term, it will be observed, is constructed from two distinct sources, viz., the axisymmetric determinant and the quasi-Pfaffian. From the first source we have

in the first term	the determinant itself,
in the second term, viz., D	the array of its secondary minors,
in the third term, viz., E	the array of its quaternary minors,

the last of these being merely the elements of the determinant by reason of the fact that the order of the determinant is the 5th. Now there is no difficulty in writing all these in the same way if we employ the so-called umbral notation throughout. They are in fact

$$\left|\begin{matrix}1&2&3&4&5\\1&2&3&4&5\end{matrix}\right|,$$

$$\left|\begin{matrix}1&2&3\\1&2&3\end{matrix}\right|,\left|\begin{matrix}1&2&3\\1&2&4\end{matrix}\right|,\ .\ .\ .\ .\ .\ .\quad(10^2\text{ in number})$$

$$\left|\begin{matrix}1\\1\end{matrix}\right|,\left|\begin{matrix}1\\2\end{matrix}\right|,\ .\ .\ .\ .\ .\ .\ .\ .\quad(5^2\text{ in number})$$

Again, from the second source we have

in the third term, viz., E	the primary minors of the Pfaffian,
in the second term, viz., D	the tertiary minors . . . . . ,
in the first term	nothing,

the last involving no anomaly because the quasi-Pfaffian we are dealing with has only *five* frame-lines. Now, if we adopt for this quasi-Pfaffian an umbral notation analogous to the preceding, viz.,

$$\|\,1\ 2\ 3\ 4\ 5\,|$$

it will be found possible to represent its minors in a fashion closely resembling that for the minors of the determinant. In fact, its primary minors will be

$$\|\,2\ 3\ 4\ 5\,|,\quad\|\,1\ 3\ 4\ 5\,|,\ .\ .\ .\quad(5\text{ in number})$$

and its tertiary minors

$$\|\,4\ 5\,|,\quad\|\,3\ 5\,|,\quad\|\,3\ 4\,|,\ .\ .\ .\ .\quad(10\text{ in number})$$

The three terms of our development thus are

$$\left|\begin{matrix}1&2&3&4&5\\1&2&3&4&5\end{matrix}\right|,$$

$$\begin{array}{l|l}\|\,4\ 5\,|,\quad\|\,3\ 5\,|,\quad\|\,3\ 4\,| & \\ \hline \left|\begin{matrix}1&2&3\\1&2&3\end{matrix}\right|,\left|\begin{matrix}1&2&3\\1&2&4\end{matrix}\right|,\left|\begin{matrix}1&2&3\\1&2&5\end{matrix}\right|,\ .\ .\ .\ .\ .\ . & \|\,4\ 5\,|\\ \left|\begin{matrix}1&2&4\\1&2&3\end{matrix}\right|,\ .\ .\ .\ .\ .\ .\ .\ .\ .\ .\ .\ .\ .\ .\ . & \|\,3\ 5\,|\\ .\ .\ .\ .\ .\ .\ .\ .\ .\ .\ .\ .\ .\ .\ .\ .\ .\ . & .\end{array}$$

$\parallel 2\,3\,4\,5 \mid$	$\parallel 1\,3\,4\,5 \mid$	
$\left\lvert \begin{matrix} 1 \\ 1 \end{matrix} \right\rvert$	$\left\lvert \begin{matrix} 1 \\ 2 \end{matrix} \right\rvert$	$\parallel 2\,3\,4\,5 \mid$
$\left\lvert \begin{matrix} 2 \\ 1 \end{matrix} \right\rvert$		$\parallel 1\,3\,4\,5 \mid$
. . . . .		.

where, as evidence of the appropriateness of the notation, it is most important to observe that if any element of a square array be taken it will be found to be such that its row-numbers taken along with the numbers of the bordering element in the same row with it form the full set 1 2 3 4 5, and its column-numbers taken along with the numbers of the bordering element in the same column with it form the set 1 2 3 4 5 also. Thus the element $\left\lvert \begin{matrix} 1\,2\,3 \\ 1\,2\,4 \end{matrix} \right\rvert$ in the square array of D has $\parallel 4\,5 \mid$ for the bordering element in the same row with it, and $\parallel 3\,5 \mid$ for the bordering element in the same column with it; and $\left\lvert \begin{matrix} 2 \\ 1 \end{matrix} \right\rvert$ in E in like manner has the bordering elements $\parallel 1\,3\,4\,5 \mid$ and $\parallel 2\,3\,4\,5 \mid$.

(11) In the verification of the foregoing developments it is very interesting to observe the necessity which arises for using a linear relation like KRONECKER's between co-ordinate minors of an axisymmetric determinant. In the case of the 4th order the term B, and in the case of the 5th order the term D are dependent upon it for certain of their details.

Let us consider the mode in which the details of the term D are ascertained, these being the Italic-letter coefficients of θ^2, $\theta\phi$, . . . in the development of the determinant

$$\left\lvert \begin{matrix} a & h+\nu & g-\mu & r+\tau & n-\psi \\ h-\nu & b & f+\lambda & q-\sigma & m+\chi \\ g+\mu & f-\lambda & c & p+\rho & l-\phi \\ r-\tau & q+\sigma & p-\rho & d & k+\theta \\ n+\psi & m-\chi & l+\phi & k-\theta & e \end{matrix} \right\rvert .$$

The case of such a product as $\theta\phi$ presents no difficulty. We have only got to delete the 4th and 5th rows and the 3rd and 5th columns, and from the resulting minor

$$\left\lvert \begin{matrix} a & h+\nu & r+\tau \\ h-\nu & b & q-\sigma \\ g+\mu & f-\lambda & p+\rho \end{matrix} \right\rvert$$

strike out all the Greek letters. But when we come to the case of $\theta\lambda$, or $\theta\mu$, or $\theta\nu$ the process is not so simple. Fixing the attention on the θ in the place (4, 5) we see that it can go along with not only the λ in the place (2, 3) but also with the λ in the place (3, 2). In the former case its Italic-letter cofactor is

$$\left\lvert \begin{matrix} a & h & r \\ g & f & p \\ n & m & k \end{matrix} \right\rvert , \quad \text{or} \quad \left\lvert \begin{matrix} 1\,3\,5 \\ 1\,2\,4 \end{matrix} \right\rvert ,$$

and in the latter

$$-\begin{vmatrix} a & g & r \\ h & f & q \\ n & l & k \end{vmatrix} \text{ or } -\begin{vmatrix} 1 & 2 & 5 \\ 1 & 3 & 4 \end{vmatrix}.$$

But there exists the identity

$$\begin{vmatrix} 1\,3\,5 \\ 1\,2\,4 \end{vmatrix} - \begin{vmatrix} 1\,2\,5 \\ 1\,3\,4 \end{vmatrix} - \begin{vmatrix} 1\,4\,5 \\ 1\,2\,3 \end{vmatrix} = 0; \qquad (K')$$

consequently the cofactor of $\theta\lambda$ is

$$\begin{vmatrix} 1\,4\,5 \\ 1\,2\,3 \end{vmatrix} \text{ or } \begin{vmatrix} 1\,2\,3 \\ 1\,4\,5 \end{vmatrix},$$

as desired for the sake of symmetry in the square array of D.

The identity (K′) is noteworthy in that the terms of it are third-order minors of an axisymmetric determinant of the *fifth* order, and not of the sixth order as is the case in KRONECKER's theorem. The best mode of establishing it is to make it dependent upon the latter. Thus, taking the axisymmetric determinant of the fourth order

$$\begin{vmatrix} 2\,3\,4\,5 \\ 2\,3\,4\,5 \end{vmatrix}_{rs=sr},$$

we have from KRONECKER

$$\begin{vmatrix} 2\,3 \\ 4\,5 \end{vmatrix} - \begin{vmatrix} 2\,4 \\ 3\,5 \end{vmatrix} + \begin{vmatrix} 2\,5 \\ 3\,4 \end{vmatrix} = 0: \qquad (K)$$

and this is simply what (K′) becomes if $\begin{matrix} 1 \\ 1 \end{matrix}$ be struck out of each term of it. In other words (K′) is the Extensional * of (K). The idea of applying the Law of Extensible Minors to all KRONECKER's identities is thus suggested. For example, knowing from KRONECKER that

$$\begin{vmatrix} 1\,2\,3 \\ 4\,5\,6 \end{vmatrix} - \begin{vmatrix} 1\,2\,4 \\ 3\,5\,6 \end{vmatrix} + \begin{vmatrix} 1\,2\,5 \\ 3\,4\,6 \end{vmatrix} - \begin{vmatrix} 1\,2\,6 \\ 3\,4\,5 \end{vmatrix} = 0$$

if the terms be minors of the axisymmetric determinant

$$\begin{vmatrix} 1\,2\,3\,4\,5\,6 \\ 1\,2\,3\,4\,5\,6 \end{vmatrix}_{rs=sr},$$

we can at once vouch for the identity

$$\begin{vmatrix} 1\,2\,3\,7\,8\,\ldots \\ 4\,5\,6\,7\,8\,\ldots \end{vmatrix} - \begin{vmatrix} 1\,2\,4\,7\,8\,\ldots \\ 3\,5\,6\,7\,8\,\ldots \end{vmatrix} + \begin{vmatrix} 1\,2\,5\,7\,8\,\ldots \\ 3\,4\,6\,7\,8\,\ldots \end{vmatrix} - \begin{vmatrix} 1\,2\,6\,7\,8\,\ldots \\ 3\,4\,5\,7\,8\,\ldots \end{vmatrix} = 0$$

in connection with the axisymmetric determinant

$$\begin{vmatrix} 1\,2\,3\,4\,5\,6\,7\,8\,\ldots \\ 1\,2\,3\,4\,5\,6\,7\,8\,\ldots \end{vmatrix}_{rs=sr}.$$

* MUIR's *Theory of Determinants*, p. 213, § 179; or *Trans. Roy. Soc. Edin.*, xxx. p. 2.

(12) The procedure followed in § 9 is equally effective in finding the like development of the primary minors of the determinant. For example, let us consider in the determinant of the 4th order the cofactor of the element (13), viz. :—

$$\begin{vmatrix} h-\nu & b & q-\sigma \\ g+\mu & f-\lambda & p+\rho \\ r-\tau & q+\sigma & d \end{vmatrix}.$$

Here the aggregate of terms of the degree $3+0$ is

$$\begin{vmatrix} h & b & q \\ g & f & p \\ r & q & d \end{vmatrix}.$$

The aggregate of terms of the degree $2+1$

$$= \begin{vmatrix} h & b & -\sigma \\ g & f & \rho \\ r & q & . \end{vmatrix} + \begin{vmatrix} h & . & q \\ g & -\lambda & p \\ r & \sigma & d \end{vmatrix} + \begin{vmatrix} -\nu & b & q \\ \mu & f & p \\ -\tau & q & d \end{vmatrix},$$

$$= -\rho \begin{vmatrix} h & r \\ b & q \end{vmatrix} - \sigma \begin{vmatrix} h & r \\ f & p \end{vmatrix} - \lambda \begin{vmatrix} h & r \\ q & d \end{vmatrix} - \tau \begin{vmatrix} b & q \\ f & p \end{vmatrix} - \mu \begin{vmatrix} b & q \\ q & d \end{vmatrix} - \nu \begin{vmatrix} f & p \\ q & d \end{vmatrix};$$

and as the coefficients of the Greek letters here are determinants of the 2nd order formable from the 2nd and 4th rows of D, this may be written

$$- \begin{vmatrix} h & b & f & q \\ r & q & p & d \end{vmatrix} (\rho, \sigma, \lambda, \tau, \mu, \nu).$$

The aggregate of terms of the degree $1+2$

$$= \begin{vmatrix} h & . & -\sigma \\ g & -\lambda & \rho \\ r & \sigma & . \end{vmatrix} + \begin{vmatrix} -\nu & b & -\sigma \\ \mu & f & \rho \\ -\tau & q & . \end{vmatrix} + \begin{vmatrix} -\nu & . & q \\ \mu & -\lambda & p \\ -\tau & \sigma & d \end{vmatrix},$$

$$= \begin{array}{ccc|c} -\sigma & -\tau & \nu & \\ \hline h & b & q & \rho' \\ g & f & p & \sigma \\ r & q & d & \lambda \end{array}$$

where the borders ρ, σ, λ and $-\sigma$, $-\tau$, ν are got from the Pfaffian

$$\begin{vmatrix} \nu & -\mu & \tau \\ & \lambda & -\sigma \\ & & \rho \end{vmatrix}$$

by deleting the 1st and 3rd frame-lines respectively and changing one sign.

Lastly, the aggregate of terms of the degree $0+3$

$$= \begin{vmatrix} -\nu & . & -\sigma \\ \mu & -\lambda & \rho \\ -\tau & \sigma & . \end{vmatrix},$$

which being a bordered zero-axial skew determinant is expressible in the form

$$\sigma \left| \begin{matrix} \nu & -\mu & \tau \\ & \lambda & -\sigma \\ & & \rho \end{matrix} \right|,$$

where, be it observed, σ is the Greek letter in the place (4, 2).

We thus have

cofactor of (1,3), *i.e.* $g-\mu$, in Δ

$$= G - \left| \begin{matrix} 2^{\text{nd}} \text{ row of D} \\ 4^{\text{th}} \text{ row of D} \end{matrix} \right|^{(\rho,\sigma,\lambda,\tau,\mu,\nu)} + \frac{-\sigma,\ -\tau,\ \nu}{G} \left| \begin{matrix} \rho \\ \sigma \\ \lambda \end{matrix} \right| + \sigma \left| \begin{matrix} \nu & -\mu & \tau \\ & \lambda & -\sigma \\ & & \rho \end{matrix} \right|.$$

Similarly

cofactor of (1,4), *i.e.* $r+\tau$, in Δ

$$= R + \left| \begin{matrix} 2^{\text{nd}} \text{ row of D} \\ 3^{\text{rd}} \text{ row of D} \end{matrix} \right|^{(\rho,\sigma,\lambda,\tau,\mu,\nu)} - \frac{\lambda\ \ \mu\ \ \nu}{R} \left| \begin{matrix} \rho \\ \sigma \\ \lambda \end{matrix} \right| + \lambda \left| \begin{matrix} \nu & -\mu & \tau \\ & \lambda & -\sigma \\ & & \rho \end{matrix} \right|.$$

A reference to the original determinant Δ shows that the cofactor of (3, 1) differs from the cofactor of (1, 3) simply in having all the Greek letters of the opposite sign. Consequently

cofactor of (3,1), *i.e.* $g+\mu$, in Δ

$$= G + \left| \begin{matrix} 2^{\text{nd}} \text{ row of D} \\ 4^{\text{th}} \text{ row of D} \end{matrix} \right|^{(\rho,\sigma,\lambda,\tau,\mu,\nu)} + \frac{-\sigma,\ -\tau,\ \nu}{G} \left| \begin{matrix} \rho \\ \sigma \\ \lambda \end{matrix} \right| - \sigma \left| \begin{matrix} \nu & -\mu & \tau \\ & \lambda & -\sigma \\ & & \rho \end{matrix} \right|.$$

(13) The relation between the cofactor of (r, s) and the cofactor of (s, r) just referred to shows that the adjugate of Δ is a determinant of the same form as Δ, and therefore has a development similar to that of Δ as given in § 9. But the adjugate of Δ is equal to the 3rd power of Δ; consequently we may equate the one development to the 3rd power of the other,—a process from which one or two curious identities arise.

A similar result follows from the verification of the identity

$$\Delta = (1,1).\text{cofactor of } (1,1) + (1,2).\text{cofactor of } (1,2) + \ldots$$

by substituting for Δ, cofactor of (1,1), cofactor of (1,2), . . . their developments obtained in §§ 9, 12.

Thus, recurring for shortness' sake to the 3rd order, we have the identity

$$\frac{\begin{matrix} X & Y & Z \end{matrix}}{\begin{matrix} A & H & G \\ H & B & F \\ G & F & C \end{matrix}} \left| \begin{matrix} X \\ Y \\ Z \end{matrix} \right| = \frac{\begin{matrix} x & y & z \end{matrix}}{\begin{matrix} a & h & g \\ h & b & f \\ g & f & c \end{matrix}} \left| \begin{matrix} x \\ y \\ z \end{matrix} \right| \cdot \left| \begin{matrix} a & h & g \\ h & b & f \\ g & f & c \end{matrix} \right|$$

$$\text{if} \quad (X, Y, Z) = \begin{pmatrix} a & h & g \\ h & b & f \\ g & f & c \end{pmatrix} (x, y, z),$$

—in other words, we have a condensed expression for the product of a ternary quadric and its discriminant. For, on performing the substitution, the left-hand member

$$= \begin{array}{ccc|ccc} x & y & z & & & \\ \hline a & h & g & A & H & G \\ h & b & f & H & B & F \\ g & f & c & G & F & C \\ \hline & & & a & h & g \\ & & & h & b & f \\ & & & g & f & c \end{array} \begin{array}{|c} \\ \\ \\ \\ x \\ y \\ z, \end{array}$$

and therefore

$$= \begin{array}{ccc} x & y & z \\ \hline & & \\ & \Omega & \\ & & \end{array} \begin{array}{|c} x \\ y \\ z, \end{array}$$

where Ω is the product of the preceding three square arrays in reverse order. But by actual multiplication

$$\Omega = \begin{pmatrix} \Delta & . & . \\ . & \Delta & . \\ . & . & \Delta \end{pmatrix} \begin{pmatrix} a & h & g \\ h & b & f \\ g & f & c \end{pmatrix},$$

$$= \begin{vmatrix} a\Delta & h\Delta & g\Delta \\ h\Delta & b\Delta & f\Delta \\ g\Delta & f\Delta & c\Delta \end{vmatrix},$$

—which proves the theorem.

Again, we have the identity

$$\begin{array}{cccccc} \rho & & \sigma & & \lambda & \\ \hline \begin{vmatrix} a & h \\ h & b \end{vmatrix} & \begin{vmatrix} a & g \\ h & f \end{vmatrix} & \begin{vmatrix} a & r \\ h & q \end{vmatrix} & \begin{vmatrix} h & g \\ b & f \end{vmatrix} & \begin{vmatrix} h & r \\ b & q \end{vmatrix} & \begin{vmatrix} g & r \\ f & q \end{vmatrix} \\ \begin{vmatrix} a & h \\ g & f \end{vmatrix} & \begin{vmatrix} a & g \\ g & c \end{vmatrix} & \cdots & & & \\ \begin{vmatrix} a & h \\ r & q \end{vmatrix} & \cdots & & & & \\ \cdots & & & & & \\ \cdots & & & & & \\ \cdots & & & & & \end{array} \begin{array}{|c} \rho \\ \sigma \\ \lambda \\ \tau \\ \mu \\ \nu \end{array}$$

$$= \begin{array}{cccc|c} a & g & h & r & \rho \\ \hline & & & & g \\ & & & & b \\ & & \delta & & f \\ & & & & q \end{array} + \begin{array}{cccc|c} a & g & h & r & \sigma \\ \hline & & & & h \\ & & & & f \\ & & \delta & & c \\ & & & & p \end{array} + \begin{array}{cccc|c} a & g & h & r & \lambda \\ \hline & & & & r \\ & & & & q \\ & & \delta & & p \\ & & & & d \end{array}$$

$$+\frac{\begin{vmatrix} g & b & f & q \\ & & & \end{vmatrix}\tau}{\delta\,\Big|\begin{matrix} h \\ f \\ c \\ p \end{matrix}} + \frac{\begin{vmatrix} g & b & f & q \\ & & & \end{vmatrix}\mu}{\delta\,\Big|\begin{matrix} r \\ q \\ p \\ d \end{matrix}} + \frac{\begin{vmatrix} h & f & c & p \\ & & & \end{vmatrix}\nu}{\delta\,\Big|\begin{matrix} r \\ q \\ p \\ d \end{matrix}},$$

where δ is the zero-axial skew array

$$\begin{matrix} . & -\rho & -\sigma & -\lambda \\ \rho & . & -\tau & -\mu \\ \sigma & \tau & . & -\nu \\ \lambda & \mu & \nu & . \end{matrix};$$

and connected with it the identity

$$\begin{array}{c|cccc|} & \rho & \sigma & \lambda & \\ \hline \rho & \begin{vmatrix} a & h \\ h & b \end{vmatrix}, & \begin{vmatrix} a & g \\ h & f \end{vmatrix}, & \begin{vmatrix} a & r \\ h & q \end{vmatrix} & \\ \sigma & \begin{vmatrix} a & h \\ g & f \end{vmatrix}, & \begin{vmatrix} a & g \\ g & c \end{vmatrix}, . . . & & \\ \lambda & \begin{vmatrix} a & h \\ r & q \end{vmatrix}, ... & & & \\ \tau & . . . . . & & & \\ \mu & . . . . . & & & \\ \nu & . . . . . & & & \end{array} = \frac{\begin{matrix} a & g & h & r \end{matrix}\Big|\begin{matrix} \rho & \sigma & \lambda \end{matrix}}{\begin{matrix} . & -\rho & -\sigma & -\lambda \\ \rho & . & -\tau & -\mu \\ \sigma & \tau & . & -\nu \\ \lambda & \mu & \nu & . \end{matrix}\Big|\begin{matrix} g & h & r \\ b & f & q \\ f & c & p \\ q & p & d \end{matrix}},$$

and a similar identity got from the two by subtraction.

VIII.—*A Contribution to the Comparative Anatomy of the Mammalian Organ of Jacobson.* By R. Broom, M.D., B.Sc. Communicated by Sir Wm. Turner. (With Two Plates.)

(Read 7th June 1897.)

Since the Organ of Jacobson was first described in 1811, a large amount of study has been given to its structure, development, and morphology; and as a result of these investigations, we have now a very good idea of the distribution of the organ in the animal kingdom, of its relations in many typical forms, and of its minute anatomy in a few representative types. With the exception, however, of Seydel's work among the Amphibia, very little has been done to the study of the comparative anatomy of the organ, and it is hoped that the present contribution will assist towards a clearer understanding of its comparative anatomy in the Mammalia.

The organ is present in the large majority of mammals, and is generally fairly well developed. The most important investigations into the general anatomy of the organ and its relations have been those of Gratiolet, Balogh, Klein and Herzfeld. Gratiolet has apparently examined the organ and its relations in a considerable number of the higher mammals, but, unfortunately, I have been unable to see his paper. Balogh has made a very careful study of the relations and minute anatomy of the organ in the sheep, and has shown the complicated arrangement of the cartilages in connection with the organ and its duct and with the naso-palatine canal. To Klein we are indebted for a very careful study of the organ and its relations in the guinea-pig, the rabbit and the dog. Though Herzfeld has also added considerably to our knowledge by the examination of some interesting new groups, his most important contribution to the subject has been in connection with the comparative anatomy of the organ. He has apparently been the first to recognise that, according to the various relations of the organ and its duct found in different mammals, it was possible to arrange the animals examined into a few not altogether unnatural groups. Owing to observations having up to that time been made in an insufficient number of forms, he has, unfortunately, over-estimated the importance of certain points, and thus to a large extent has rendered his grouping unsatisfactory.

The present communication deals with the results of an extended examination into the general relations of the mammalian organ — its distribution, varying degree of development, and extent of variation in allied forms. In all the forms I have studied, I have made the examination by a series of vertical transverse sections, which seems the most satisfactory method, and gives uniformity in the results.

The following is the list of forms I have examined :—

MONOTREMATA.

Ornithorhynchus anatinus ; adult.
Echidna aculeata ; adult.

MARSUPIALIA.

Dasyurus viverrinus ; mammary fœtal, and $\frac{2}{3}$ grown.
Dasyurus maculatus ; adult.
Phascologale penicillata ; mammary fœtal.
Didelphys murinus ; mammary fœtal.
Didelphys marsupialis ; mammary fœtal.
Perameles nasuta ; mammary fœtal, and adult.
Petaurus breviceps ; almost adult.
Petauroides volans ; adult.
Pseudochirus peregrinus ; mammary fœtal, and adult.
Trichosurus vulpecula ; intra uterine, mammary fœtal (3 stages), and adult.
Phascolarctus cinereus ; $\frac{2}{3}$ grown, and adult.
Phascolomys mitchelli ; $\frac{1}{2}$ grown.
Macropus ualabatus ; large mammary fœtal.
Macropus sp. ? ; early mammary fœtal.
Æpyprymnus rufescens ; mammary fœtal.

EDENTATA.

Dasypus villosus ; adult.

RODENTIA.

Lepus europæus ; young.
Lepus cuniculus ; fœtal (3 stages).
Mus musculus ; adult.
Hydromys chrysogaster ; adult.

UNGULATA.

Sus scrofa ; fœtal.
Bos taurus ; fœtal (2 stages).
Equus caballus ; fœtal.

CETACEA.

Delphinaptera leucas ; fœtal.

INSECTIVORA.

Erinaceus europæus ; fœtal and adult.
Talpa europæa ; early fœtal.

CARNIVORA.

Felis domestica ; young.

CHIROPTERA.

Miniopterus schreibersii ; adult.
Nyctophilus sp. ? ; adult.
Pteropus poliocephalus ; adult.

PRIMATES.

Homo sapiens ; early fœtal.

GENERAL OBSERVATIONS.

In its typical form the mammalian organ of Jacobson is a specialised portion of the nasal mucous membrane, situated in the anterior part of the base of the nasal septum, and forming a tubular process, blind posteriorly, but opening in front in the region of the naso-palatine canal. The inner wall of the tubular organ is lined with highly specialised neuro-epithelium, while the outer wall is composed of ciliated columnar epithelium, considerably resembling that of the nasal passage. Into the organ there open a large number of mucous glands, which are situated chiefly at its posterior part. Along the outer wall, which generally bulges into the lumen of the organ, there run one or more blood-vessels, frequently forming a sort of plexus. On its inner and under sides, at least, the organ is supported by a curved cartilaginous plate—the recurrent cartilage—which is itself supported by a curved bony plate, the palatine process of the premaxillary.

While, as a rule, the organ is fairly well developed, in a number of animals it is quite absent, and in others only a rudiment is present. It is difficult at present to account for the remarkable differences in the degree of development of the organ in different mammals, as absolutely nothing is known for certain as to its special function; still, from comparative observations, a few interesting general conclusions can be arrived at. In the first place, the organ is, as a rule, better developed in the lower forms than in the higher. Thus, in both the Monotremes the organ is exceedingly well developed, and in all the Marsupials yet examined it is at least fairly well developed, while among the Primates, it is absent, according to HERZFELD, in Cercopithecus and Inuus—the only old world monkeys examined—and in man it is quite rudimentary. Another conclusion that may be safely arrived at is, that in large animals the organ is, relatively, considerably less developed than in the smaller sorts. For example, in the two species of Dasyurus, *D. maculatus* and *D. viverrinus*, though the former is about twice the size of the latter, the organs are absolutely about the same size in each; the cartilaginous capsule of the organ, however, is much larger, and the extra space is filled up by a great development of glandular tissue. In all forms in which the organ is developed it receives the secretion from numerous mucous glands, which lie chiefly towards the posterior part of the organ, and in most small animals, *e.g.*, the mouse, where it fills the greater part of cartilaginous capsule, only at that part. In the larger animals, *e.g.*, rabbit, dog, etc., where the organ only occupies a small portion

of the space enclosed by the cartilage, it receives the ducts of glands throughout its whole length.

While it will thus be noticed that there may be a very considerable difference in the degree of development of the organ in even closely allied forms, I have been led to a much more important conclusion, viz., that though the degree of development may vary greatly, there is throughout large groups a very marked constancy of the type, followed by the organ in its general relations and connections. For example, in all the Marsupials that have been examined, with one exception, which will be referred to later, the organ opens into the upper end of the naso-palatine canal, near the point where the canal opens out into the nasal cavity, while there is never more than a rudimentary cartilaginous support given off to support the naso-palatine canal either from Jacobson's cartilage or from the nasal floor cartilage. While this holds good for practically all Marsupials, of no Eutherian that has yet been examined can the same be asserted. The two known Monotremes have in their organs a number of features in common, but which differ from those of any other animal. And the same can be said of the different Rodents which have been examined. Again, in all the higher Eutheria in which Jacobson's organ is well developed, there is a very complex development of the nasal floor cartilage, giving rise to a cartilaginous support for the naso-palatine canal as well as to a posterior nasal floor cartilage, while Jacobson's cartilage is produced downwards and forwards to form a support to Jacobson's duct; and so unvarying is this type, that even in forms in which the organ of Jacobson is completely aborted, as in Pteropus, the cartilages still retain the same general arrangement. From the small tendency to variation in the organ and its cartilages, we have in them a factor of considerable value in the classification of the Eutherian orders, probably of more value than either dentition or placentation.

In the following pages the relations of the organ in the principal orders will be considered, and the affinities and significance of the various arrangements dwelt upon.

Monotremata.

The differences of the organ in the two Monotremes are such that it will be more convenient to consider the two separately.

Ornithorhynchus.—The presence of the organ in the Platypus was recognised by Sir W. Turner in 1885, but was first described in detail by Symington in 1891. Since then further details have been supplied by Wilson, Wilson and Martin, W. N. Parker, Elliot Smith, and myself.

It is unfortunate that in Ornithorhynchus—the most primitive mammal at present existing—there is a most remarkable degree of specialisation of the structures in the anterior part of the snout, and that this specialisation is in a direction entirely dissimilar to that found in any other known vertebrate. This peculiar development in the snout to some extent affects the organ of Jacobson and its cartilages, and renders it difficult to

say whether certain of the peculiarities presented are due to the specialisation, or are retained primitive characters.

The condition of the cartilages of the snout may be briefly described as follows. In the hinder part of the nasal region the nasal septum is a short, thick, simple cartilage, which gives off above two well marked alinasals, as in higher mammals. On passing forward, when we reach the plane passing through the anterior part of the prevomer or dumb-bell shaped bone, the nasal septum becomes divided into a very slender upper part supporting the alinasals, and a thick lower part. This lower part is here in close contact with the inner parts of the well developed nasal floor cartilages, while a little in front it becomes united with these, and forms a well developed flat cartilage, stretching right across from the one premaxillary bone to the other. About 3 mm. in front of the plane of the anterior nares the large flat plate of cartilage becomes thickened and arrested in the middle line, but laterally it passes out to the margins of the beak, and sending developments in front and behind forms the support of the beak throughout its whole extent. Though in none of the higher vertebrates is there any cartilaginous development similar to the large rostral cartilage of Ornithorhynchus, there is reason to believe that it is a very great development of the prenasal element.

With regard to the nasal floor cartilages, which on passing back are separated off from the median or septal part of the main cartilage, there is little or no doubt but they are the homologues of the nasal floor cartilages of the higher mammals, though in some respects they differ from those of any higher form. In the typical mammalian condition we have the inner part of the nasal floor cartilage quite simple and curving up by the base of the septum ; in Ornithorhynchus alone among mammals the nasal floor cartilage at its inner part is thickened and excavated to receive the anterior part of the organ of Jacobson.

The organ itself differs from that of any other known mammal in being made up of an anterior and a posterior part with the opening near the middle, as was shown by SYMINGTON. In fig. 1, Plate I., we have represented a transverse vertical section through the anterior part of the organ. The large organ (J.O.) is seen completely surrounded by the nasal floor cartilage, and almost completely divided into an upper and a lower part by a large turbinal plate (t.J.c.). The inner part of the nasal floor cartilage rests on the anterior part of the prevomer or dumb-bell shaped bone (Pvo), and at the outer side of the nasal passage the cartilage is seen united with the alinasal (a.n.). In this section is also seen the hinder part of the nasal valve (val.).

On approaching the region of the naso-palatine canal and the opening of the organ, the turbinal plate becomes considerably thickened, while the cartilages surrounding the two organs become separated by the body of the prevomer. On reaching the plane passing through the opening of the organ, as seen in fig. 2, the cartilage is found to be open below, and the outer part of the cartilage surrounding the organ is seen to be free from the nasal floor cartilage. The organ is here made up of an upper part lying above the turbinal plate and an inner part. Inferiorly, the inner part may be said to open

directly into the mouth, as the little pocket into which it opens in common with the naso-palatine canal is frequently so shallow that the ducts practically open independently into the mouth.

Behind the plane represented by fig. 2 the organ becomes much less expanded laterally, while the outer part of the cartilage of the organ becomes united with the lower part of the inner portion, forming a complete capsule to the organ; and the organ becomes almost cylindrical instead of flat as in the anterior part. Fig. 3 illustrates a section across the posterior part of the organ. The turbinal plate, which in the anterior and middle region is moderately flat, here passes upwards and inwards, and then downwards, being to a considerable extent folded on itself. Below the prevomer, which here attains its maximum development, is seen a large thin plate of cartilage (o.n.f.c.) stretching across from one side to the other, but distinct at the sides from the alinasals. This cartilage is a backward continuation of the part of the nasal floor cartilage on the outer side of the naso-palatine canal, which, on passing backwards, becomes distinct from the alinasal, and sweeping inwards below the prevomer unites with the cartilage of the opposite side, and forms the large plate which supports the back part of the dumb-bell.

It is exceedingly difficult, as already remarked, to pick out which characters of the organ are inherited from reptilian ancestors and which are specialisations peculiar to the genus or order. Unfortunately the order of reptiles, which probably contained the ancestors of the Monotremes—the Theromora—is long since extinct, and the only living reptilian order in which the organ is known to be well developed—the Squamata—is only but distantly related to the Monotremes. Among lizards the organ of Jacobson is usually very well developed, but there are great differences in the structure and relations of the organ in different groups. In the Varanidæ and in snakes a most complex and highly specialised arrangement is found; in the Scincoidæ a different mode of specialisation is met with; while in the Agamidæ and Geckonidæ, on the other hand, though the organ is well developed, it is comparatively simple both in structure and its relations, and it is in these latter simple Lacertilian types that we recognise some affinities to the organ of the Monotremes. In the Gecko we find the opening of the organ situated inferiorly, almost exactly as in Ornithorhynchus, while we have an even more remarkable resemblance in the presence of a large cartilaginous turbinal process invaginating the organ from the outer side. It would thus seem probable that the mode of opening of the organ and the well-marked turbinal in Ornithorhynchus are reptilian heirlooms. The anterior extension of the organ would seem at first sight to be also a reptilian character, seeing that the organ extends slightly in front of the duct in lizards, but it is possible that the specialisation of the beak may be sufficient to account for this peculiarity, and until the very early development is known it had better be regarded as a doubtful reptilian character. Another reptilian character is to be noticed in the prevomer or dumb-bell shaped bone being quite distinct from the premaxillary, and not united with it as a palatine process, as is usually the case in mammals.

Echidna.—In Echidna the organ is considerably simpler than in Ornithorhynchus, and it is uncomplicated by any remarkable cartilaginous developments. W. N. PARKER has studied the structure of the organ and its relations in the young animal, while the adult condition has been described by myself.

The most striking point of difference of the organ in Echidna from that of Ornithorhynchus is the absence of any anterior development. The nasal floor cartilage is comparatively simple, and, in the adult, but feebly developed, and quite distinct from the alinasal. Immediately in front of the region of the naso-palatine canal, the nasal floor cartilage is a small cartilaginous plate which abuts against the base of the nasal septum by its thickened inner edge, and becomes rapidly thinned away on passing outwards, only forming a support to the inner third of the nasal floor. On passing a little backwards the thickened inner part becomes divided off from the slender outer part by the naso-palatine canal passing upwards (figs. 4 and 5). At first the inner part is irregularly square shaped, but a little farther back it becomes hollowed out on the outer side for the reception of the duct of Jacobson's organ, as it opens into the naso-palatine canal (fig. 5). On passing still farther back this inner cartilage becomes very distinctly "C"-shaped, with the anterior end of Jacobson's organ in its concavity. The cartilage is about uniform in thickness, but at the outer end of the upper part of the "C" there is a very distinct thickening. A very little beyond this plane the lower part of the "C" joins with the thickened outer rim, and Jacobson's organ becomes completely enclosed in cartilage. The naso-palatine canal receives the duct of Jacobson anterior to its opening into the nasal cavity, so that the organ only communicates with the nasal cavity indirectly by the upper part of the canal. Fig. 6 represents a section through the posterior wall of the naso-palatine canal. To the outer side of the cartilage of Jacobson the thickened portion will be observed (rud.t.); this is found, on examining the succeeding sections, to be continuous with the turbinal, and is, no doubt, the remnant of a turbinal which once extended to the front of the organ as in Ornithorhynchus. The organ itself at this plane is found on section to have the ordinary mammalian kidney shape. The outer part of the nasal floor cartilage is still seen as a small inconspicuous fragment. A short distance behind the plane of fig. 5 the organ is met with in its full development, and assumes an appearance which it retains to near its posterior end. On section it is found to be nearly circular with the outer wall so completely invaginated as to leave very little lumen in the organ. It is completely surrounded by cartilage, and the invaginated wall is supported by a flat turbinal plate of cartilage passing inwards from the outer wall of the capsule. The capsules of each side rest on a flat cartilaginous plate, which passes right across from the one side to the other, and completes what would otherwise be a little gap in the lower wall of the cartilage of Jacobson. This cartilaginous plate is exactly comparable with the similar plate in Ornithorhynchus, and is developed from the small outer portions of the nasal-floor cartilage, which become greatly enlarged, and pass inwards.

On comparing the structure of the organ and its relations in Echidna with that in

Ornithorhynchus, it will be observed that in the posterior parts of the organ there is a close agreement between the two forms. In Echidna, the whole organ being posterior to the duct, this region is naturally developed to a greater extent than in Ornithorhynchus, where the organ extends in front of the duct as well as behind, but otherwise the only points of difference at present worthy of note are the absence of the prevomer in Echidna, and the different degrees of development of the turbinal plates—in Ornithorhynchus large and curved; in Echidna small and flat. The anterior part of the organ in Echidna differs remarkably from that in Ornithorhynchus, and yet it is not difficult to imagine an intermediate or ancestral form from which both would be derived—the Echidna type by simplification and the Ornithorhynchus type by specialisation. In this hypothetical ancestral form the organ probably extended little, if at all, in front of the duct opening, but had a well-developed turbinal which extended right to the front of the organ. If in Echidna the turbinal were carried forward to the front of the organ, and the part of the organ in the neighbourhood of the naso-palatine canal, instead of being reduced to a duct, were well developed, there would be as great a resemblance to the Ornithorhynchus condition even in this region as is to be seen in the posterior part. The well-developed anterior part of the organ in Ornithorhynchus is but a continuation forwards of the various structures met with in the region of the duct rendered possible by the great development of the nasal floor cartilage in the anterior region.

Marsupialia.

The marsupial organ of Jacobson was apparently first described in 1891 by Symington, who studied the condition in pouch specimens of the kangaroo and rock wallaby. At that time the organ in Echidna was undescribed, and Symington, comparing the Marsupial organ with that in Ornithorhynchus, concluded that its affinities were more with the higher Eutherian type than with that of the Monotreme. With the exception of a short note by C. Röse on the organ in the wombat and opossum, and a few incidental references in one or two of my own papers, nothing further had been written on the subject till last year, when I communicated a paper "On the Comparative Anatomy of the Organ of Jacobson in Marsupials" to the Linnean Society of New South Wales. This paper, which will appear in the *Proceedings* of the Society, contains a description of the organ in representative genera of the principal groups, and of the changes met with at different stages of development. From my study of the organ in the various marsupials, it was seen that in all the diverse forms the organ conformed more or less to one main type. Of this main type, however, there are two minor varieties—the simpler one found in the Polyprotodonts, the more complex in the Diprotodonts. In the present paper I will take Dasyurus as the typical representative of the former group, and Petaurus of the latter.

Dasyurus.—In Dasyurus we meet with the simplest form of the marsupial organ. As the structure of the organ and its relations have been described in some detail in the

above-mentioned paper, a brief account will here suffice. The nasal floor cartilage is well developed and simple in its structure. Fig. 8 represents a transverse section just in front of the naso-palatine canal. Here the nasal floor cartilage is seen as a curved plate of cartilage resting on the premaxillary bone, and with its inner end curved upwards by the side of the base of the septum, and then slightly outwards, forming a support to the inferior septal ridge. The naso-palatine canal on passing upwards passes slightly backwards, and in fig. 9 it is seen cut across as it lies between the premaxillary and its palatine process, and dividing the nasal floor cartilage into an inner and an outer part. The inner part, which becomes Jacobson's cartilage, is very irregularly "C"-shaped, there being an inner curved portion for the reception of the organ and a well-marked little process passing out into the septal ridge. In fig. 10 the canal is found first opening into the anterior end of Jacobson's organ, and then communicating with the nasal cavity.

If the three sections of Dasyurus figured be compared with the corresponding figures representing the condition in Echidna, the wonderful agreement will be at once evident—the simple condition of the nasal floor cartilage, its division into two parts by the naso-palatine canal, and the mode of communication of the canal with the anterior end of Jacobson's organ and with the nasal cavity. Even the details of structure of Jacobson's cartilage are exceedingly instructive. The outer end of the upper part of Jacobson's cartilage is in fig. 10 seen detached from the inner plate, forming a bar along the concavity of the organ. A very short distance, however, behind the plane of fig. 10 the lower end of the inner plate of Jacobson's cartilage passes outwards and unites with the bar. This interesting little bar of cartilage, which is present in nearly all Marsupials, I have elsewhere called the "outer bar of Jacobson's cartilage." Its chief interest lies in the fact that there is not the slightest doubt but it is exactly homologous with the rudimentary turbinal found in the anterior part of the cartilage in Echidna, and that it is thus the remnant of an ancestral turbinal. The posterior and main part of the body of the organ lies in an open curved plate of cartilage, which only supports the organ on its inner and under sides. We thus have in this primitive marsupial type a series of structures which are in all details easily traceable to the monotreme condition. In Ornithorhynchus the ancestral monotreme type is complicated by excessive cartilaginous developments, while in Echidna the primitive condition is obscured by the rudimentary condition of the cartilage in the anterior region. In Dasyurus, on the other hand, we probably have a nearer approach to the proto-mammalian proportions than in either of the existing monotremes, but, unfortunately, the organ has become much more feebly developed, and the cartilages are so reduced that at first sight one fails to observe the traces of their former grand developments.

Didelphys agrees very closely with Dasyurus as regards its organ and cartilages; Perameles, on the other hand, while agreeing fairly well with these more normal polyprotodonts, shows some slight peculiarities, which have been considered in my above-mentioned paper.

Petaurus.—Petaurus I have chosen for consideration, as it shows the diprotodont marsupial characteristics in their most typical form. The main part of the organ agrees very closely with that in Dasyurus, as will be seen by comparing fig. 15 with fig. 11. In front, however, there are some remarkable differences. The nasal region, generally at its anterior part, is somewhat more expanded laterally than in Dasyurus, and the inferior septal ridges are much more developed. It is probably in connection with this broadening out of the base of the septum that the inner part of the nasal floor cartilage assumes its characteristic development. In front of the naso-palatine canal the nasal floor cartilage, as seen in fig. 12, closely resembles the condition in Dasyurus (fig. 8). In fig. 13, in the plane just behind the point where the palatine process divides off from the premaxillary, the nasal floor cartilage is seen in its characteristic form. Its outer part is very much reduced, but the inner part, which becomes Jacobson's cartilage, is much better developed than in Dasyurus. By comparing this figure with fig. 9, the following points of difference will be noticed. Instead of the cartilage being composed of a vertical curved plate, with a short process passing to the inferior septal ridge from the upper end, we have here a moderately flat plate passing markedly outwards, and resting on the palatine process of the premaxillary, while from near its middle there passes upwards and outwards a well-marked process, which passes into and supports the well-developed inferior septal ridge. The interpretation of the morphology of the different structures here seems at first sight rather difficult, but an examination of the succeeding planes gives a satisfactory solution of the problem. In fig. 14 we have represented a transverse section through the point where Jacobson's organ opens into the naso-palatine canal and into the nasal cavity. It will be observed that there is a much more marked connection between the organ and the nasal cavity than in Dasyurus. In the ridge we find a portion of cartilage which we have no difficulty in recognising to be the outer bar of Jacobson's cartilage, and we are thus driven to the conclusion that the ridge support seen in fig. 13 is the homologue of the short plate which passes out to the little ridge in Dasyurus. This being so, we see that two of the peculiarities of the Petaurus arrangement are (1) a great development of the ridge process, and (2) its arising from near the middle of the inner plate instead of from its upper end, which probably means that in Petaurus the upper part of the cartilage is developed to a much greater degree than in Dasyurus. A third peculiarity, and probably the most important, is that the inner plate of the cartilage at its lower end passes down on the outside of the palatine process of the premaxillary, and forms the inner wall of part of the naso-palatine canal. In almost all other respects the details of the anatomy in Petaurus agree with those in Dasyurus. One feature worthy of note is, that in the diprotodonts the organ almost always has a well-marked vascular plexus, while in the polyprotodonts the plexus is more or less rudimentary. In my paper above referred to I have pointed out the peculiarities of the different diprotodonts, and here need only call attention to one, viz., Æpyprymnus. In this rat-kangaroo we have a peculiar arrangement found in no other marsupial yet examined, namely, that the organ opens directly into the nasal cavity

slightly in front of the point where the naso-palatine canal ends. In every other respect the characters agree perfectly with the Marsupial type, and the peculiarity is probably due to the lengthening of the front of the snout in connection with the well-developed front incisors. It will be noted later that the opening of Jacobson's organ into the nasal floor in front of the naso-palatine canal is one of the most noticeable characteristics of the rodents, and it is interesting here to notice a parallel development in an animal which to a considerable degree approximates to the rodent type of dentition.

Edentata.

With the exception of Parker's Monograph on the development of the skull in the Edentata, practically nothing has been published on the organ of Jacobson in this order, and Parker's figures, though showing the presence of the organ in different forms, do not enable us to form any idea of the more delicate relations. Symington, who has made sections of the snout of the Peba armadillo, and of a fœtal 3-toed sloth, but has not yet published his results, kindly informs me that he finds the organ well developed in the armadillo, and that in the sloth it is rudimentary, and opens into the nasal cavity. Through the kindness of Mr F. E. Beddard, F.R.S., I recently obtained the head of an adult hairy armadillo (*Dasypus villosus*), and have since made a study of the organ of Jacobson and its relations in that form. In this species the organ is moderately well developed, and though it is possible that in such a varied group as the Edentata there may be some considerable variations in the relations of the organ, *Dasypus villosus* may provisionally be taken as the type of the order; and, judging by the structure in this form, it seems probable to me that the other genera will not depart very greatly from the Dasypus type.

Dasypus.—In a short paper recently published I described the condition of the nasal floor cartilage in its anterior region, and more especially the remarkable little nasal floor bone which is associated with it. The cartilage differs in some respects from that of any of the lower mammals, and also from the majority of the higher forms. In most mammals the nasal floor cartilage arises as two lateral plates from the base of the nasal septum: here, in front, they appear to rise by a splitting up of the lower third of the nasal septum. In the plane passing through the anterior part of the papilla the cartilages are quite below the base of the septum, and do not form any floor to nasal cavity, the floor being formed by the little nasal floor bone.

Fig. 16 represents a transverse section near the middle of the papilla. Here the nasal floor cartilage has almost its normal development, for though the outer and inner parts appear detached by the posterior part of the nasal floor bone, they are quite united round behind the bone. The nasal floor cartilage and bone both rest on the peculiarly flattened out premaxillary. The inner part of the nasal floor cartilage is very large, and is seen curving upwards and outwards almost exactly as in Dasyurus; there is here, however, practically no inferior septal ridge, the large glandular ridge being apparently the homologue of the upper of the two ridges in Echidna, and not of the lower, which

corresponds to the inferior septal ridge of marsupials. A very short distance behind the plane of fig. 16 the inner part of the nasal floor cartilage, or Jacobson's cartilage, becomes much thinner, and the upper part curves outwards and downwards, thereby forming a cavity for the reception of Jacobson's duct, which in fig. 17 we find lying in the concavity thus formed, and opening into the floor of the nasal cavity. In this latter figure we find the outer part of Jacobson's cartilage detached as a little flat plate which, though different in shape, is, there is little doubt, the homologue of the outer bar of Jacobson's cartilage in marsupials. The duct of the organ is seen to have a plentiful supply of vascular tissue, especially on its upper side. A little behind this plane the outer bar becomes united with the lower part of the inner plate, and the organ rests in a sort of "V"-shaped trough. For a short distance Jacobson's cartilage still retains its connection with the outer nasal floor cartilage, then becoming detached the "V" becomes gradually rounded into the normal "C" shape, and the outer nasal floor cartilage becomes more and more reduced. Fig. 18 represents a section just beyond the point where the naso-palatine canal opens into the nasal cavity, and it will be observed that there is no connection whatever between the canal and Jacobson's organ. The two cartilages of Jacobson are separated by the vomer, and rest on the palatine processes of the premaxillary. The organ has the usual mammalian kidney-shape, and in structure does not differ apparently from the marsupial organ. Along the concavity of the organ there is a rudimentary plexus composed of one large and three or four small blood-vessels.

From the consideration of the above-mentioned details, it will be seen that there is little to distinguish the Edentate organ from that of the Marsupial. The most striking difference is the opening of the organ into the nasal cavity much in advance of the naso-palatine canal, but this is only an extreme exaggeration of the condition met with in the rat-kangaroo, *Æpyprymnus*.

Rodentia.

In no order of mammals has the organ and its relations been studied so thoroughly as in the Rodentia, and yet in no order is the difficulty of interpreting its affinities so great. Klein has fully described the details of the anatomy in the rabbit and guinea-pig, while Herzfeld has examined the organ in the rat. I have myself examined the organ in the hare, at birth, two stages of fœtal rabbit, the mouse, and the Australian water-rat, Hydromys.

Though the organ is probably better developed throughout this order than in any other of the Eutheria, there are certain peculiarities both in the organ itself and in its relations which distinguish it from that of any other mammal. At present I will leave out of consideration the minute structure of the organ, the well-developed plexus, and the glandular connections, and confine myself to the study of the relations of the organ and its cartilages.

The most striking characteristic of the organ is that it apparently always opens on to the floor of the nasal cavity in its anterior part, and well in advance of the naso-palatine canal, with which it has no connection. This peculiarity has been found in all the rodents yet examined, and from the examination of Dr BEARD'S sections of the embryo rabbit of 13 mm. described in his recent paper "On Certain Problems in Vertebrate Embryology" I find that the characteristic peculiarity is already distinctly marked even at this early period.

Lepus.—To illustrate the Rodent type of organ I have taken a fœtal rabbit (head length 19 mm.), as in adult animals the great development of the premaxillaries to some extent mar the cartilaginous arrangements. In front the nasal floor cartilage presents no remarkable peculiarity, differing little from the simple marsupial type, except in that the development is confined chiefly to the inner part, which curves up close to the base of the feeble nasal septum. In fig. 36 we see the much compressed nasal cavity passing down and curving into the outer concavity of the nasal floor cartilage, or, as it may be here called, Jacobson's cartilage. Almost immediately beyond this plane the duct of Jacobson is found passing off from the nasal cavity, and occupying the concavity of the cartilage. In fig. 37 we find Jacobson's organ quite separated from the nasal cavity, and almost surrounded by cartilage. It will be noticed that there is a rudimentary outer bar, on the one side united above, on the other below, resembling more the condition in Dasypus than in Marsupials. On the palate the extreme anterior part of the naso-palatine canal is cut across. Fig. 38 represents a section near the posterior part of the naso-palatine canal. Jacobson's organ and cartilage are found in their normal form, and the cartilages are supported by the palatine processes of the premaxillary. Below and on the outer side of the naso-palatine canal is seen a small cartilaginous element, the explanation of the significance of which is the most troublesome problem in the snout of the rodent. The little cartilaginous process passes forwards almost to the front of the canal, supporting it on its outer side. Behind, it supports chiefly the lower wall of the canal, and when the canal opens into the nasal cavity the cartilage forms a true nasal floor cartilage. As it passes still further back it curves inwards and upwards, and forms an inner wall to the lower part of the nasal cavity. Throughout its whole length it is quite unconnected with either Jacobson's cartilage or the alinasal. In the lower mammals the only cartilage with which homology can be claimed is the outer nasal floor cartilage of the Monotremes. In the higher Eutheria, however, we have a somewhat similar cartilaginous development complicated in front by the presence of an anterior process of Jacobson's cartilage, which is absent in the rodents. By a comparison with the simple higher Eutherian type, as found in Miniopterus, it will be seen that the peculiar process of cartilage is a much modified outer nasal floor cartilage.

From the consideration of the above features it will be noticed that in the rodent we have a number of lower mammalian characters together with what would seem to be a higher Eutherian feature. The well-developed condition of the organ, with its large

vascular plexus and its numerous glands, all point to an affinity with the lower mammals —Monotremes and Marsupials, and the structure of Jacobson's cartilage with its rudimentary outer bar gives strong support to this affinity. The mode of opening of the organ is similar to that in Dasypus, and is but an extreme degree of the condition in Æpyprymnus. The well-developed naso-palatine canal, and the process of the outer nasal floor cartilage passing forward with it, reveals a character which seems to remove the rodent from its lower relatives, and suggests an affinity with the higher Eutheria. Taking the various points into consideration, one of two conclusions seems to be possible—(1) Either the rodents are an aberrant group sprung off from the main Eutherian stem somewhat earlier than the development of the common ancestors of the higher Eutheria; or (2) they are a modified and specialised branch of the higher Eutheria. From the primitive characters of the organ and its cartilage found in the rodents and in none of the higher Eutheria, and from the fact that in no known higher Eutherian has a condition similar to that of the rodent arisen by secondary development, the first of the two conclusions, viz., that the rodents are a specialised offshoot from the early ancestors of the higher Eutherians, seems much the more probable.

Higher Eutheria.

In a few typical members of the higher Eutheria the organ has been carefully studied, but though much has been done in the way of describing the details of the anatomy practically no attempt has been made to indicate the significance of the various details. Not taking into consideration the Anthropoidea, in which the organ is generally rudimentary or absent, I have studied the organ in the following orders:—Chiroptera, Insectivora, Carnivora, and Ungulata (Artiodactyla and Perissodactyla). Notwithstanding the great differences in the general structure of the members of these different orders, the organs of Jacobson are formed on a common plan, and the differences are very slight. In the bat, when the organ is developed, we have the same type as in the pig, while the organ in the ox scarcely differs in one detail from that in the cat.

The affinities are such as to lead irresistibly to the conclusion that, in spite of the great outward differences in structure and the differences in habits and dentition, we have the various groups connected by ties of a common ancestry. And, furthermore, not only do the common ties indicate a close relationship, but they distinguish at once the higher Eutheria from the lower mammals. The simplest form of the higher Eutherian type that I have met with is that found in the Chiroptera; it will, therefore, be convenient to consider first the structure in its simple form as seen in Miniopterus.

Chiroptera.—Until recently it was believed that there was no organ of Jacobson in the Chiroptera, but in 1895 Messrs Duval and Garnault discovered a moderately developed organ in *Vesperugo pipistrellus*, and in the same year I found a very well-developed organ in *Miniopterus schreibersii*. Though the organ is thus seen to be

occasionally present, it must be admitted to be more normally absent: it is interesting, however, as showing the value of the general anatomical arrangement of parts as a factor in classification, that even when the organ is quite absent (Pteropus, Nyctophilus) the cartilages still follow the same general arrangement as in Miniopterus where the organ is so well developed.

Though Miniopterus gives us the simplest form of the Eutherian type, and though it has certain affinities with the Marsupial arrangement, it is further removed from the Marsupial type than is the Marsupial from the Monotreme.

If a transverse vertical section be made through the posterior part of the papilla an appearance will be presented like that shown in fig. 19. The nasal septum is rather slender, and from a little below its base on each side there passes out a thin nasal floor cartilage, which is present here as a moderately flat plate, curving slightly upwards at its inner end. In the middle line, a considerable distance below the base of the septum is a small oval cartilage—a development of the prenasal. By the side of the papilla is seen the naso-palatine canal passing upwards and inwards, and at its upper part curving first outwards, then inwards. The upper portion is considerably dilated, and represents really the anterior prolongation of the lower part of the nasal cavity. Round this dilated portion there is a curved cartilaginous support which surrounds it on its outer, upper, and inner sides. On tracing the curved cartilage forwards the inner part becomes lost, only a small portion of cartilage remaining on the outer side of the canal at its upper end. If we trace the curved cartilage backwards we find a most remarkable development. Fig. 20 represents a section a very short distance behind fig. 19, and here the naso-palatine canal is found freely opening into the nasal cavity. What in fig. 19 represented the nasal floor cartilage is here divided into two parts, while the outer part of the curved roof of the naso-palatine canal is likewise separated from the inner, and, furthermore, the inner part of the nasal floor cartilage is found united with the inner part of the curved roof of the canal, forming one piece, which is really Jacobson's cartilage. Jacobson's cartilage thus consists of an upper moderately flat portion and a lower portion, which is markedly concave, and which has in its concavity the anterior continuation of the duct of Jacobson's organ. The outer part of the nasal floor cartilage proper becomes lost shortly behind this plane, but the outer part of the cartilage which supported the canal (*o.n.f.c.*) becomes well developed. In fig. 21 we have a section a little way behind the naso-palatine canal. Jacobson's duct is seen almost surrounded by the well-developed Jacobson's cartilage, while what was the outer canal cartilage becomes a very well developed nasal floor cartilage. On the inner side of each cartilage of Jacobson is seen the anterior part of the prevomer. In this region Jacobson's duct is a pure duct lined with squamous epithelium. Fig. 22 represents a section through the most developed part of the organ. The organ is here almost surrounded by a cartilaginous capsule; while the nasal floor cartilage forms a large flat plate which, to some extent, passes below Jacobson's cartilage. The broad posterior part of the prevomer is seen stretching from the one cartilage of Jacobson to the other.

In tracing the affinities between this type and that of the Marsupials we have at first a slight difficulty. With regard to the main body of the organ and its cartilaginous capsule the agreement between the two is close, but as we pass to the front the differences become more marked. The organ has a fairly long distinct duct, and even after it opens into the upper part of the naso-palatine canal it preserves for a short distance its individuality as a distinct groove. One of the main characteristics of the Miniopterus type, and of the higher Eutherian type generally, is due to this anterior extension of the duct of Jacobson being supported by an anterior process from the cartilage of Jacobson. Then, again, the outer part of the nasal floor cartilage is much better developed than in the Marsupials, and in some respects it has more resemblance to the Monotreme type, especially where the posterior part becomes well developed and passes in below the organ of Jacobson. The great development, however, of the outer part of the nasal floor cartilage gives rise in front to a special process passing forward on the outer side of the naso-palatine canal. The anterior process of Jacobson's cartilage, and that from the outer nasal floor cartilage, unite in front by their upper edges, the united cartilages forming the support to the upper part of the naso-palatine canal. Though there is no similar development in Marsupials, there is frequently present in Diprotodonts a downward process of Jacobson's cartilage by the side of the naso-palatine canal, which is apparently a rudimentary homologue of the anterior process in Miniopterus and the higher Eutheria. Different modifications of this downward development of Jacobson's cartilage are found in Perameles, Trichosurus, Phascolarctus, Macropus, and Phascolomys. The posterior and anterior developments of the outer nasal floor cartilage are no doubt homologous with the outer nasal floor cartilage and its anterior process in the rabbit, but whether the rodent condition represents a degeneration from the elaborate arrangement found in the higher Eutheria, or a pure parallel development, it is difficult to decide definitely, though the latter alternative appears the more likely.

Lemuridæ.—I have not had an opportunity of examining personally any member of this group, but fortunately HERZFELD has made an examination of the organ and its cartilages in Lemur, and so far as he has figured his sections the type followed differs in no essentials from that of Miniopterus.

Insectivora.—The organ in members of the Insectivora has been examined by HARVEY and HERZFELD, while PARKER in his monograph on the development of the skull gives numerous figures of sections through the organ. PARKER's figures, however, are on too small a scale to give more than rough indications of the arrangements. HARVEY has noted the general features in the hedgehog, and HERZFELD has figured the organ in the mole, though this latter animal is a much less satisfactory insectivorous type than the former.

Taking the hedgehog as the insectivorous type, we find in it an organ which differs but little from that of the bat, except that in a few points there are indications of a more advanced stage of organisation. As in the majority of the higher Eutheria the naso-palatine canal is long, and passes very obliquely upwards and backwards, and the great

length of the canal is one of the points in which we have an advance on the Chiropterus condition. The other main points of difference are due to a less degree of development of the organ in the hedgehog, and a greater of the bony tissues. Though the naso-palatine canal passes up very obliquely, it is supported by cartilaginous walls almost exactly as in Miniopterus, and unquestionably of the same nature. Fig. 23, which represents a section of the snout of the hedgehog cut slightly obliquely, shows on the one side the naso-palatine canal almost surrounded by cartilage. The upper and inner corner of the canal is really the continuation of Jacobson's duct. If this figure be compared with fig. 19 the close agreement of the two will be at once manifest. In the hedgehog the nasal floor cartilage is represented only by the inner part above, and by the feebly developed outer wall of the naso-palatine canal. In the other side of fig. 23 the section passes through the point where the naso-palatine canal opens into the nasal cavity, and Jacobson's duct is likewise seen opening into the nasal cavity at this point. Jacobson's cartilage is here represented in two portions—a lower small concave portion which lodges the duct of Jacobson, and a small upper plate which is the continuation of the nasal floor cartilage proper. This section may be compared with fig. 20 of Miniopterus. Almost immediately beyond this plane the two portions of Jacobson's cartilage unite, and we soon get on section an appearance like fig. 24, with the organ well protected by a large cartilage. This section may be seen to agree fairly closely with fig. 21 from Miniopterus, the outer nasal floor cartilage being unrepresented in the hedgehog.

Carnivora.—As a result of KLEIN'S work we have a very complete account of the organ and its relations in one of the members of this order, the dog. Though the arrangement of the cartilages in the dog is quite characteristic of the carnivorous type, their development indicates some degree of degeneration, and in the cat we have a much better representative of the order, as the cartilages here attain their full development.

As in the hedgehog we have an advance upon the Miniopterus condition, so in the cat we have a further stage in the specialisation of the same type; and the hedgehog condition stands almost intermediate between the primitive bat condition and the more specialised condition of the cat.

In the cat we have a very long naso-palatine canal supported by cartilaginous walls as in the hedgehog. The mutual relations of the canal and its cartilaginous walls are well shown by KLEIN in his paper on the organ in the dog, and the condition in the cat is essentially similar. In both the bat and hedgehog the organ was seen to open into the nasal cavity as well as into the naso-palatine canal; in the cat the duct of the organ opens into the canal well in advance of the posterior end of the canal, and thus only communicates with the nasal cavity indirectly by means of the naso-palatine canal. In fig. 25 the naso-palatine canal is supported by cartilage almost exactly as in the hedgehog; the nasal floor cartilage is, however, much better developed. Fig. 26 represents Jacobson's duct separating off from the canal, and already the inner part of the cartilage —the anterior process of Jacobson's cartilage—is seen distinct from the outer portion or the process from the outer nasal floor cartilage. In fig. 27 we see the whole four

elements which are derived from the cornual cartilages all distinct. The section is made in a plane a little behind the point where the naso-palatine canal opens into the nasal cavity, and the palatine process has just become detached from the premaxillary bone. The nasal floor cartilage proper is seen divided into an inner and an outer part, and at the outer and lower corner of the naso-palatine canal is the remains of the outer canal cartilage. Though in this type this little cartilage which supports the outer wall of the canal is quite unconnected with the outer part of the nasal floor cartilage above, a comparison with the condition in Miniopterus, and especially with higher Ungulate types, leaves little doubt but that it is morphologically a part of the outer nasal floor cartilage. Beyond the plane of fig. 27 the inner part of the nasal floor cartilage proper unites with the lower part to form Jacobson's cartilage, which now presents an appearance similar to that in the hedgehog.

Ungulata.—Notwithstanding the numerous points in the anatomy of the Ungulates, which would seem to mark them off from the rest of the Eutheria as a well-defined group, it is remarkable that in the relations of the organ of Jacobson and its cartilages there is the closest agreement even in small points of detail with the condition found in other higher Eutherians such as the Carnivora and Insectivora. The agreement is more marked than between the Polyprotodont and Diprotodont Marsupials, and one is forced to the conclusion that there is a more intimate relationship between the Ungulates and some other of the so-called orders of the Eutheria than is at present believed.

The different groups of Ungulates seem to be related to each other in much the same way as are the Chiroptera, Insectivora, and Carnivora; and in the types which have been examined so far we find evidences of a parallel development, the pig representing the simpler condition and the cow the more specialised.

Sus.—The general anatomy of the snout of the pig has been fairly well illustrated by Parker, but he has not entered upon the details of the anatomy of the structures related to the organ of Jacobson or the naso-palatine canal. I have myself examined the snout of a fœtal pig (head length 19 mm.), which will be found to illustrate fairly well the Ungulate type in its simplest form. On the whole there is a marked agreement with the condition in Miniopterus, except that the outer nasal floor cartilage and the anterior process which it normally gives rise to is likewise undeveloped. In fig. 28 we have a section through the point of entrance of the naso-palatine canal. By the side of the base of the septum is the inner part of the nasal floor cartilage. Above the upper part of the naso-palatine canal, and towards its inner side, is the well marked anterior process of Jacobson's cartilage, and between this and the nasal floor cartilage and a little internally is situated the palatine process of the premaxillary. The close agreement with Miniopterus will be evident on comparing this figure with fig. 19. In fig. 29 we have the condition of parts a little further back. Here Jacobson's cartilage is complete, the anterior process being in contact with the upper part or nasal floor cartilage exactly as is seen in Miniopterus (fig. 20). On the one side the duct of Jacobson has just become separated off from the naso-palatine canal, and on the other

side the canal is opening into the nasal cavity. From this section it will thus be seen that Jacobson's duct opens into the naso-palatine canal just as the canal is opening into the nasal cavity. In this it agrees with Miniopterus and the hedgehog, and also with most Marsupials. In fig. 30 we have a section through the body of the organ, and showing the normal relations of the organ, the cartilage, and the palatine process.

Bos.—In the Ruminants the condition has long ago been carefully studied, and the structure and relations of the cartilages, etc., were fully described by BALOGH in 1860, though his illustrations unfortunately are exceedingly diagrammatic. Of the ruminant type I have examined two stages of fœtal calf and a young goat, but the peculiarities of the arrangements are perhaps best shown in the larger of the fœtal calves—one about six inches in length. As in the pig, the condition of parts closely agrees with that in Miniopterus, so in the Ruminants we have an arrangement as closely corresponding with that of the Carnivora. In the cat we have a long naso-palatine canal, and the duct of Jacobson opening into the canal much in front of the union of the canal and the nasal cavity. In the Ruminant a precisely similar condition is found; but the agreement of the cartilaginous structures is even more remarkable. Fig. 33 represents a section through the anterior part of the naso-palatine canal, and even in this plane the duct of Jacobson is already seen to be split off from the canal. The nasal floor cartilage is very well developed, and still attached to the nasal septum. Around the naso-palatine canal and Jacobson's duct is a curved plate of cartilage in which there is no difficulty in recognising the two elements—the inner, which embraces Jacobson's duct, being the anterior process of Jacobson's cartilage, the outer, the anterior process of the outer nasal floor cartilage. Between these anterior cartilaginous processes and the nasal floor cartilage lies the delicate palatine process of the premaxillary. In fig. 34 we find the nasal floor cartilage almost united with the anterior process of Jacobson's cartilage, and farther out the outer nasal floor cartilage separated from the inner is seen united with its anterior process. Fig. 35 shows the appearances just anterior to the opening of the naso-palatine canal into the nasal cavity. Jacobson's cartilage has now attained its normal form, and has as its support on the inner side the premaxillary palatine process. The lower part of the outer nasal floor cartilage again forms a sort of nasal floor, as is found in Miniopterus, and in the rabbit. The close agreement between the condition of parts in the calf with those in the cat will be well seen by comparing figures 33 and 35 with 25 and 27 respectively.

Equus.—In the horse we have an aberrant modification of the Ungulate type brought about probably by the great development of the premaxillaries. As I have elsewhere described the peculiarity in detail, I will only here briefly mention the main features.

In the horse the naso-palatine canal does not open into the mouth, but ends blindly after passing forward for a short distance. The organ of Jacobson, which is normally formed, opens into the canal which carries the secretion back into the nasal cavity. In connection with the rudimentary canal and the well-developed premaxillaries, the cartilages are modified considerably anteriorly. The nasal floor cartilage is much com-

pressed laterally, and instead of an inner and an outer process being sent forward to support the ducts, we have these processes rudimentary, and retaining their attachment to the nasal floor cartilage throughout their whole extent. Fig. 31 shows Jacobson's duct and the naso-palatine canal distinct, while the cartilaginous supports are seen as outgrowths from the laterally compressed nasal floor cartilage. In fig. 38 the structures are all seen in their usual relations.

Cetacea.

Through the kindness of Professor D'Arcy Thompson, I have been enabled to make an examination of the snout of a young fœtal Beluga. It has long been known that the organ of Jacobson is absent in the whale tribe, but I was anxious to see if the arrangement of the cartilages would give any evidence of the affinities of the group. My work for the most part confirms Kükenthal's recent researches. Before conclusive results can be obtained, however, younger embryos than any yet studied will have to be examined.

The peculiarities of the Cetacean are due to the nasal openings being shifted from their normal situation in the anterior part of the snout to the upper region of the head. The palatal region does not depart much from the normal type, there being even a small papilla in the anterior part; there is, however, no trace of a naso-palatine canal by the side of the papilla. Fig. 39 represents a curved section cut so as to approximate to the transverse in both the region of the nasal cavities and the snout. Above are seen the two nasal cavities separated by the cartilaginous nasal septum, which passes right down to near the palatal region where it rests on the vomer. On each side of the nasal septum is seen a peculiarly developed cartilaginous plate. At its upper part it forms a floor to the nasal cavity, but its chief part is closely placed against the nasal septum which it supports down almost to its lower end. This cartilage is, with little doubt, the true nasal floor cartilage, and its peculiar development is evidently the result of the shifting of the anterior nares. It passes well forward in advance of the region of the nasal cavities still resting by the side of the nasal septum, and ends about midway between the nasal passage and the papilla.

The only mammal in which I have met with a nasal floor cartilage at all comparable with that in the whale is the horse, where, owing to a sort of rostrum being formed by the well-developed premaxillaries, the nasal floor cartilage becomes laterally compressed somewhat as in the Cetacean. Though the evidence afforded by the condition of the cartilages is too slight to lead to any conclusions, so far as it goes it suggests affinities with the higher Eutheria.

Conclusion.

From the careful examination of the relations of the organ in the various groups of mammals it will be observed that not only is there a close agreement in the details of the anatomy in allied forms, but that the anatomical details of the structures related to the organ are so little affected by variations in the habits of the animals that, even in allied orders, evidences of affinity are here manifested when lost in most of the other characters. It will also be seen that in the organ and its cartilages we have a steady evolution which has apparently been only but slightly influenced by the great changes in external structure.

In the Prototheria we have an organ in a highly-developed condition, well supplied with glandular tissue, and having a large vascular plexus along its outer side. In the Marsupialia, though there are numerous little modifications—specialisations and degenerations—when these are examined it is found that they all point back to the Prototherian type, and leave little doubt but that in the Marsupial organ we have only a degenerate and slightly specialised variety of the type found in the Monotremes. In the Edentata, so far as known, the organ might be regarded as a more degenerate and slightly aberrant variety of that seen in Marsupials. In Jacobson's organ and its relations we thus have a feature which reveals an affinity between the Monotremes, the Marsupials, and this the lowest order of the Eutheria, notwithstanding the great differences manifested in their modes of development. In the Rodentia we have a well developed organ whose cartilage bears some resemblance to that both in the Marsupials and in the Edentata, with the additional feature which has not been observed in either of these groups—a posterior nasal floor cartilage which is continued forwards as a supporting cartilage to the naso-palatine canal. As this cartilage is found in the higher Eutheria, *e.g.*, Miniopterus, we see a certain affinity with this higher group. On the whole, however, the agreement is more with the lower than with the higher forms.

The examination of the organ in the higher Eutheria also reveals some striking relationships. As a rule, the organ itself is more or less rudimentary, the plexus absent, and the glandular tissue much reduced. In the cartilages, however, it has been seen that there is almost invariably a peculiar and characteristic development by which any higher Eutherian in which the organ is developed, and the majority of those even in which it is absent, can be at once distinguished from any of the lower mammals. In the complex development of the nasal floor cartilage we have, apparently, a thoroughly reliable character by which the higher Eutheria can be divided off from the lower into a distinct group by themselves. For this group I would propose the name *Cœnorhinata*, while for those Eutheria which have the primitive arrangement of the cartilages of the nasal floor the distinguishing name *Archæorhinata* might be given. In the former group would be included the following orders:—Primates, Carnivora, Insectivora,

Chiroptera, and Ungulata; in the latter, the Edentata, and probably the Rodentia. There should be no difficulty in placing the Sirenia in its proper group, as in it there is known to be a well-developed organ of Jacobson. The position of the Cetacea will have to be decided by other characters.

In the Marsupialia or Metatheria there is no doubt we have a most satisfactory sub-class, but there seems reason from the present investigation to divide it into two sub-orders, the Polyprotodontia and the Diprotodontia. The position of the Bandicoots has frequently been a matter of doubt, and there are unquestionably some Eutherian characters to be met with in the group, as the presence of an allantoic placenta, as discovered by J. P. Hill, the ossified patella, and a character which I have observed in no other marsupial, the intercommunication of the two nasal cavities behind the region of Jacobson's organ. Notwithstanding these advanced characters, there is little doubt but that the Peramelidæ are rightly placed with the other Polyprotodonts, as has been done by Thomas.

The Rodents, as has already been shown, have the organ so specialised that it is a little difficult to decide whether we have an advancement of the early condition, or a specialisation of the later; the evidence, however, mostly points to the former conclusion, and at present we may tentatively regard the Rodentia as belonging to the Archæorhinata. This being so, we may classify the Mammalia thus :—

Class—Mammalia.				
Sub-class I.	Protheria.		Order Monotremata.	
Sub-class II.	Metatheria.		Order Marsupialia.	S. O. Polyprodontia.
				S. O. Diprotodontia.
Sub-class III.	Eutheria.	Group 1. Archæorhinata.		Order Edentata.
				Order Rodentia (?).
		Group 2. Cænorhinata.		Order Chiroptera.
				Order Insectivora.
				Order Carnivora.
				Order Primates.
				Order Ungulata.
				Order Sirenia.
				Order Cetacea.

A further subdivision of the orders in the group Cænorhinata might be made, the Chiroptera, Insectivora, Carnivora and Primates being apparently more nearly allied to each other than to the Ungulata.

In conclusion, I must express my thanks to the following gentlemen for their kindness in assisting me with specimens and in other ways:—Sir William Turner, Sir William Flower, Mr F. E. Beddard, Dr Elliot Smith, Mr A. G. Hamilton, Professor Wilson, Professor D'Arcy Thompson, and Dr Beard.

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GENERAL.—*a.J.c.*, anterior process of Jacobson's cartilage; *a.n.f.c.*, anterior process of the nasal floor cartilage; *g.*, glands; *i.t.*, inferior turbinal bone; *J.c.*, Jacobson's cartilage; *J.d.*, Jacobson's duct; *J.o.*, Jacobson's organ; *l.d.*, lachrymal duct; *mx.*, maxillary bone; *n.f.b.*, nasal floor bone; *n.f.c.*, nasal floor cartilage; *n.p.c.*, naso-palatine canal; *n.s.*, nasal septum; *o.b.J.c.*, outer bar of Jacobson's cartilage; *o.n.f.c.*, outer part of the nasal floor cartilage; *p.c.*, papillary cartilage; *p.n.*, prenasal cartilage; *Pmx.*, premaxillary bone; *p.Pmx.*, palatine process of the premaxillary; *Pvo.*, prevomer; *rud.t.*, rudimentary turbinal cartilage; *u.c.c.*, united canal cartilages; *i.e.*, anterior processes of Jacobson's cartilage and of the outer nasal floor cartilage; *t.p.*, turbinal plate; *v.*, blood-vessel; *val.*, nasal valve; *vo.*, vomer; *v.s.*, vascular space.

The parts shaded in lines are bones; whilst the dotted structures are cartilages.

PLATE I.

Fig. 1–3. Transverse sections of the organ of Jacobson in *Ornithorhynchus anatinus*, × 5.
Fig. 4–7. ,, ,, ,, *Echidna aculeata*, × 20.
Fig. 8–11. ,, ,, ,, *Dasyurus viverrinus* (mammary fœtal specimen), × 20.
Fig. 12–15. ,, ,, ,, *Petaurus breviceps*, × 20.
Fig. 16–18. ,, ,, ,, *Dasypus villosus*, × 5.

PLATE II.

Fig. 19–22. Transverse sections of the organ of Jacobson in *Miniopterus schreibersii*, × 25.
Fig. 23–24. ,, ,, ,, *Erinaceus europæus*, × 10.
Fig. 25–27. ,, ,, ,, *Felis domestica* (Young), × 10.
Fig. 28–30. ,, ,, ,, *Sus scrofa* (fœtal), × 20.
Fig. 31–32. ,, ,, ,, *Equus caballus* (fœtal), × 7.
Fig. 33–35. ,, ,, ,, *Bos taurus* (fœtal), × 10.
Fig. 36–38. ,, ,, ,, *Lepus cuniculus* (fœtal), × 20.
Fig. 39. Transverse section of snout of *Delphinaptera leucas* (fœtal), × $1\frac{1}{2}$.

Dr R. Broom on Mammalian Organ of Jacobson — Plate I.

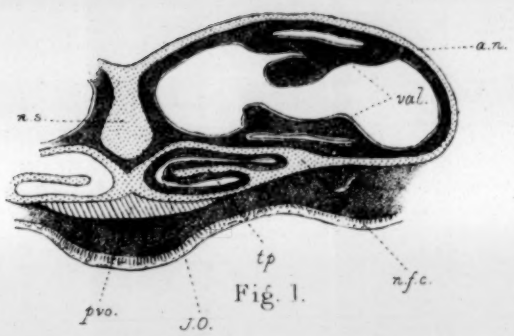


Fig. 1.

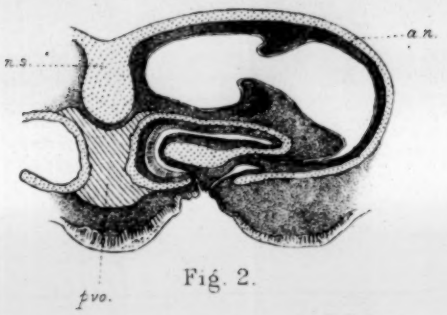


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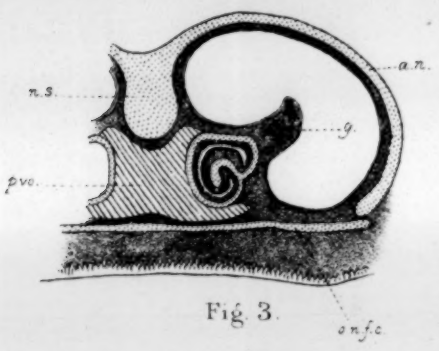


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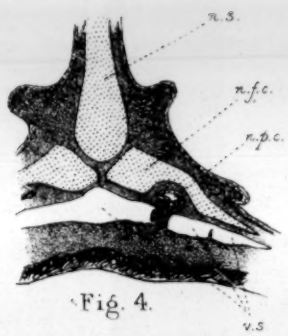


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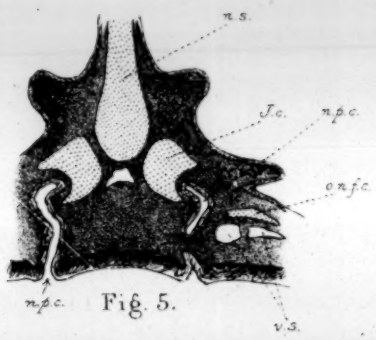


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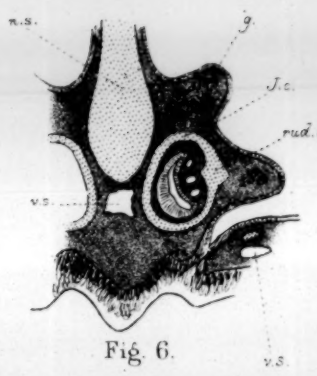


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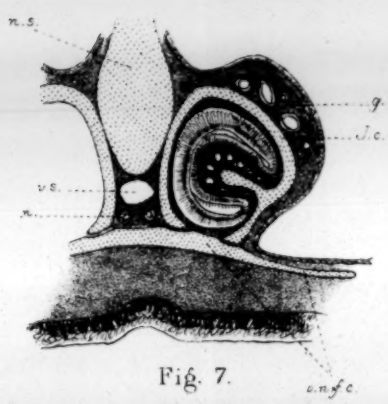


Fig. 7.

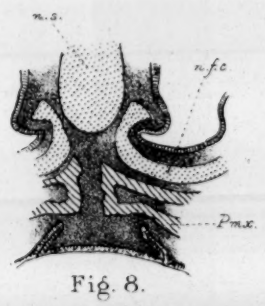


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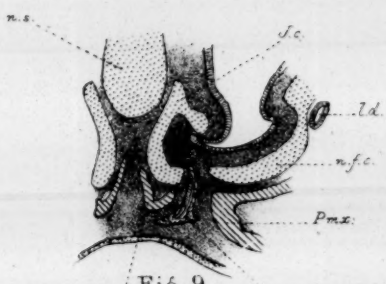


Fig. 9.

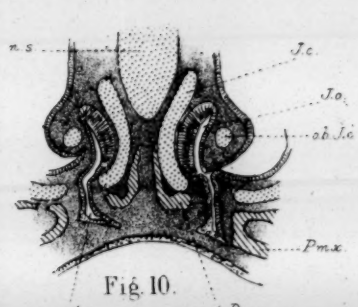


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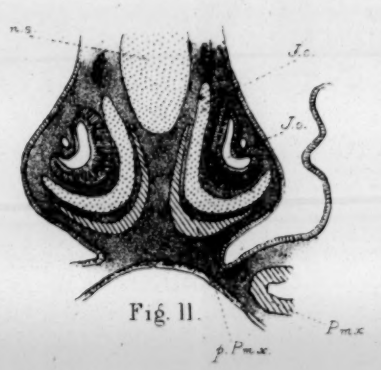


Fig. 11.

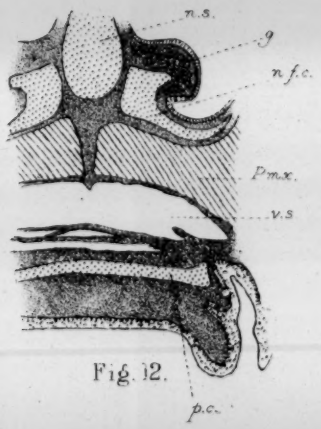


Fig. 12.

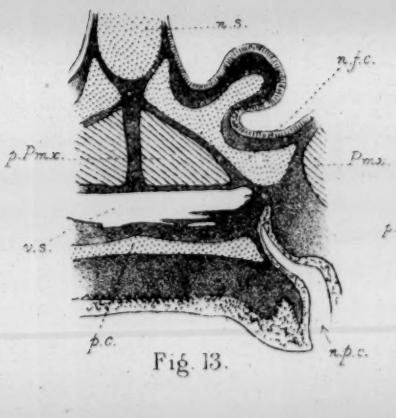


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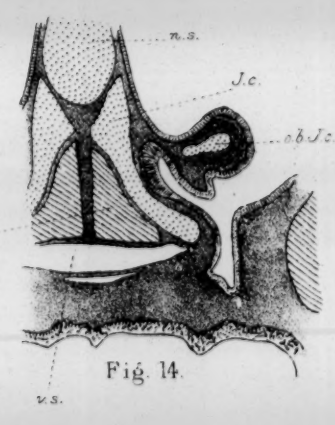


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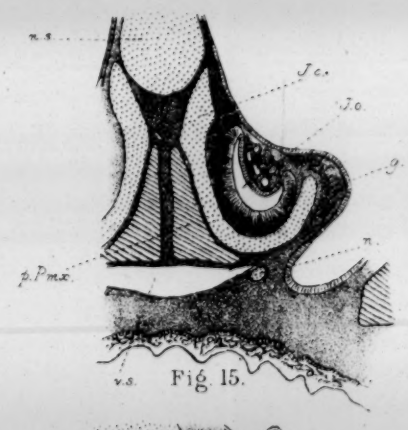


Fig. 15.

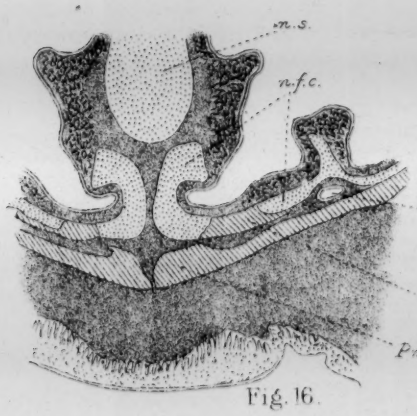


Fig. 16.

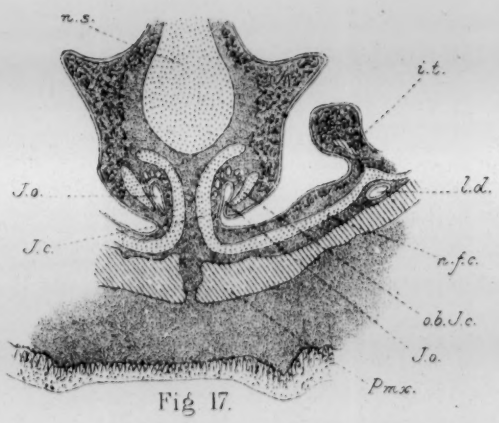


Fig. 17.

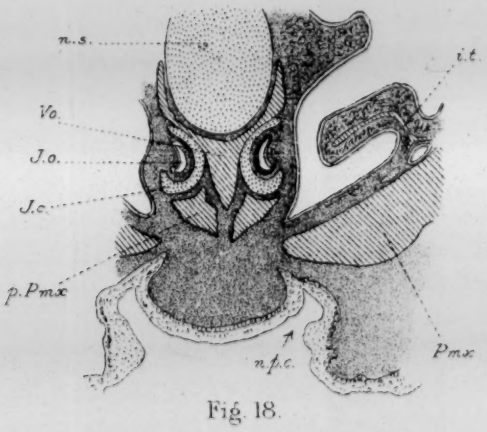


Fig. 18.

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ORNITHORHYNCHUS, ECHIDNA, DASYURUS,
PETAURUS, DASYPUS.

Dr R. Broom on Mammalian Organ of Jacobson.— Plate II.

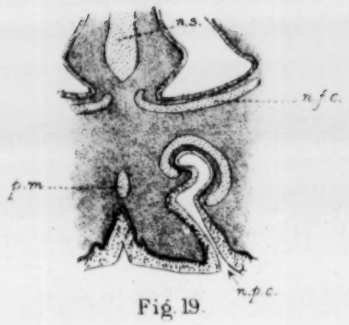


Fig. 19.

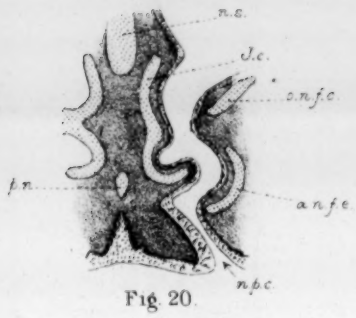


Fig. 20.

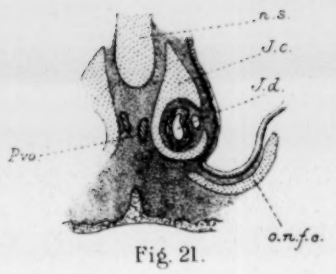


Fig. 21.

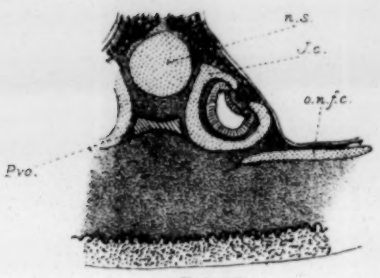


Fig. 22.

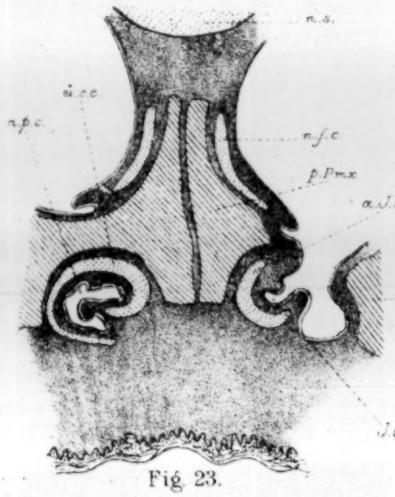


Fig. 23.

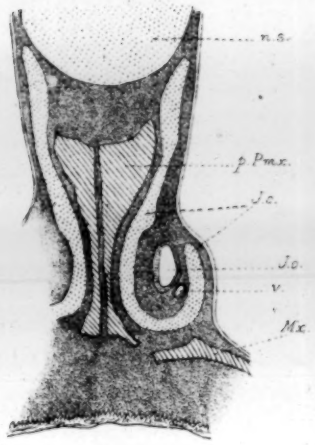


Fig. 24.



Fig. 25.



Fig. 26.

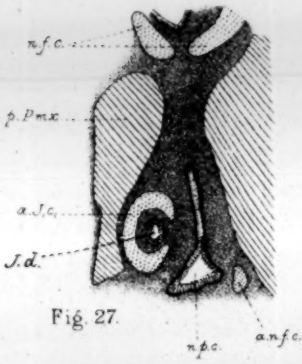


Fig. 27.

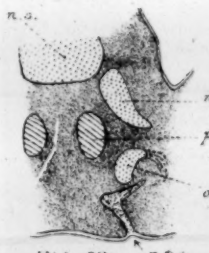


Fig. 28.

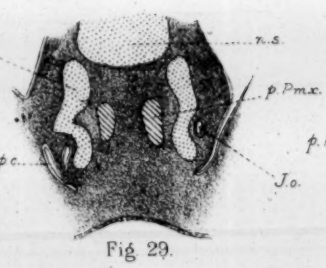


Fig. 29.

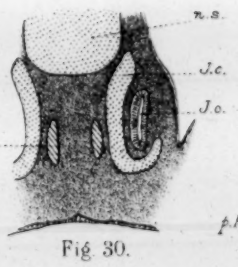


Fig. 30.

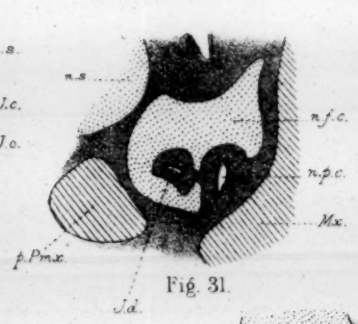


Fig. 31.

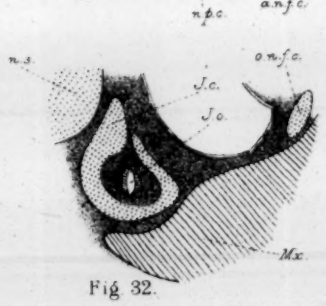


Fig. 32.

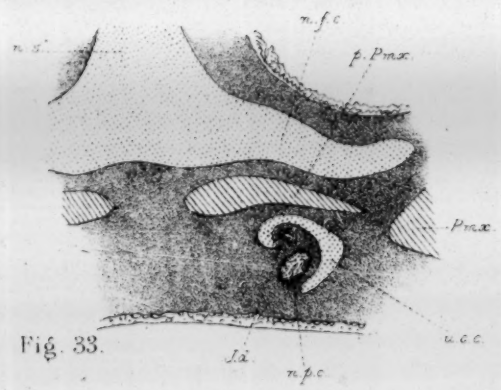


Fig. 33.

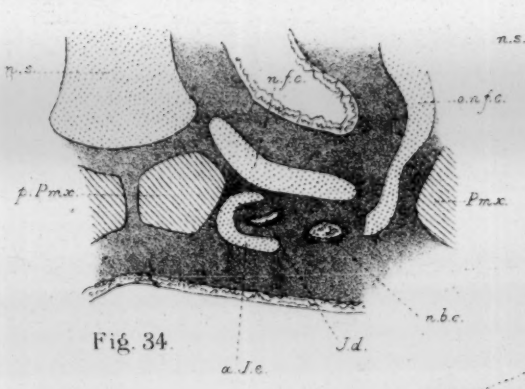


Fig. 34.

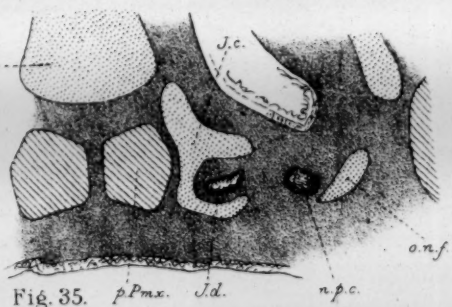


Fig. 35.



Fig. 36.

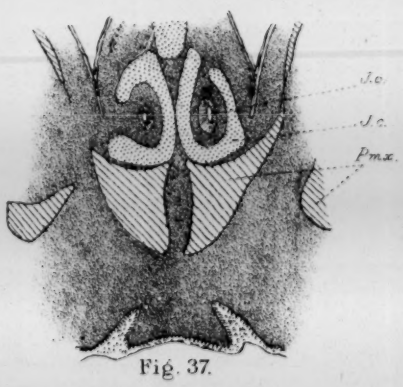


Fig. 37.

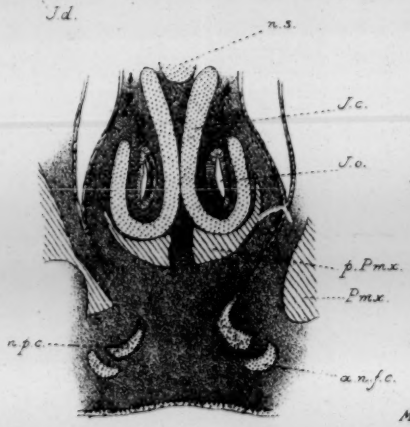


Fig. 38.

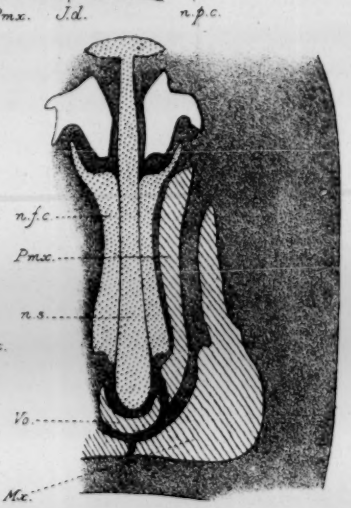


Fig. 39.

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MINIOPTERUS, ERINACEUS, FELIS, SUS, EQUUS, BOS,
LEPUS, DELPHINAPTERA.

IX.—*On the Definite Integral* $\frac{2}{\sqrt{\pi}}\int_0^t \epsilon^{-z^2}dt$, *with Extended Tables of Values.*

By Jas. Burgess, C.I.E., LL.D.

(Read July 15, 1895.)

1. The integral $\int \epsilon^{-t^2}dt$ occurs so frequently in various branches of research that, as far back as 1783, Laplace suggested that it would be useful to tabulate its values for successive ranges of integration.* It is employed in investigations on the theories of refraction, conduction of heat, of errors of observation, of probabilities, etc. These are familiar to physicists and need not be dwelt upon.†

The Integral.—Previous Tables.

2. The important formula or result—

$$\int_0^\infty \epsilon^{-t^2}dt = \tfrac{1}{2}\sqrt{\pi}. \qquad (1)$$

appears to have been discovered about 1730 by Euler,‡ who expressed it in the form—

$$\int_0^1 \left(\log_\epsilon \frac{1}{x}\right)^{-\frac{1}{2}} dx = -\sqrt{\pi};\S$$

for, putting $x = \epsilon^{-t^2}$, we have $\left(\log_\epsilon \frac{1}{x}\right)^{-\frac{1}{2}} dx = -2\epsilon^{-t^2}dt$.

3. Since

$$\int_0^\infty \epsilon^{-t^2}dt = \int_0^\infty \epsilon^{-t^2}dt + \int_t^\infty \epsilon^{-t^2}dt, \qquad (2)$$

* *Histoire de l'Acad. Roy. des Sciences*, 1783, p. 434; conf. Todhunter, *Hist. of the Theory of Probabilities*, p. 486.

† Conf. Glaisher, in *Phil. Mag.*, vol. xlii, (1871), pp. 429–31.

‡ Gauss ascribed this integration to Laplace; Oriani (in Zach's *Monatliche Corresp.* for March 1810, Bd. xxi, S. 280 f.) pointed out Euler's prior claim, but Gauss did not correct his statement, *Theoria Motus Corp. Cœl.*, art. 177, p. 212, and *Werke*, Bd. vii, Ss. 233, 280, 289; Davis's transl. of *Theor. Mot.*, pp. 258, 259. Legendre (*Exercices de Calcul Intégral* (1811), tom. i, p. 301) asserts Euler's discovery, and refers to his paper, "Evolutio formulæ integralis $\int x^{f-1}dx(\mathrm{l}.x)^{\frac{m}{n}}$," in *Novi Commentarii Acad. Scient. Imp. Petropol.*, tom. xvi, (for 1771) p. 111. Conf. *ib.*, p. 101; and *Comment. Acad. Scient. Petrop.*, tom. v, (for 1730–1731) p. 44; also Euler's letter to Goldbach of 8th Jan. 1730, in Fuss, *Correpond. Math. et Phys.*, tom. i, p. 13.

§ This is the form used by Legendre in his "Traité des Intégrales Eulériennes" in *Fonctions Elliptiques*, etc. tom. ii, pp. 365, 517–524.

the integral may be taken as separated into two parts—

(1) $\int_0^t \epsilon^{-t^2} dt$, which Mr J. W. L. GLAISHER calls the *Error-function complement*, and indicates by 'Erfc.' And—

(2) $\int^\infty \epsilon_{-t^2} dt$, which Mr GLAISHER proposes to call the *Error-function*, denoting it by 'Erf.'* Mr R. PENDLEBURY accepts Mr GLAISHER's name for the second, and writes the first as 'erf,'—which might lead to mistakes.† For convenience of reference, we may indicate the second by G.

And we shall put for the multiple of the first function here dealt with—

$$\mathrm{H} = \frac{2}{\sqrt{\pi}} \int_0^t \epsilon^{-t^2} dt . \qquad (3)$$

Whence, from (2)— $\mathrm{H} = 1 - \frac{2}{\sqrt{\pi}} \mathrm{G}$; and $\mathrm{G} = \frac{\sqrt{\pi}}{2} (1 - \mathrm{H})$. (4)

And since $\frac{1}{2}\sqrt{\pi} = 0{\cdot}886\,226\,925\,452\,758\,013\,649\,083\,741\,670$,

and its reciprocal— $\frac{2}{\sqrt{\pi}} = 1{\cdot}128\,379\,167\,095\,512\,573\,896\,158\,903\,120$,

also $\log \frac{2}{\sqrt{\pi}} = 0{\cdot}052\,455\,059\,316\,914\,268\,038\,104\,750\,579$,

it is comparatively easy to derive the value of G from that of H, or the converse.

4. In 1789, M. KRAMP, in his *Analyse des Réfractions*, was the first to tabulate G from $t = 0{\cdot}00$ to $t = 3{\cdot}00$, for every hundredth of a unit, together with the logarithmic values and differences. To these he added a third table of the logarithmic values of $\epsilon^{t^2}\mathrm{G} = \epsilon^{t^2} \int_t^\infty \epsilon^{-t^2} dt$, which is useful in connection with the theory of refraction. KRAMP apparently computed the earlier part of his table by the usual formula (8) given below; but it converges so slowly for values of $t > 1$, that KRAMP employed a difference formula —to be referred to later—in order to fill up and complete his table. For the lower values of t his results are carried to eight places, and are generally quite accurate; from $t = 2$ to $t = 3$ the values are carried to eleven places, and for the last he gives $\mathrm{G} = {\cdot}00001957729$ in the table and ${\cdot}00001957669$ in the text,‡—the true value being ${\cdot}00001957719\ 3236779$.

BESSEL, in discussing the theory of refraction in his *Fundamenta Astronomiæ* (1818), pp. 36, 37, next gave two tables,§ the first of log. $\epsilon^{t^2} \int_t^\infty \epsilon^{-t^2} dt$ from $t = 0$ to $t = 1{\cdot}00$,

* *Philos. Mag.*, vol. xlii., 4th ser. (1871), pp. 296, 297, 421.

† *Ibid.*, p. 437. If either is to be called "Error-function," it would seem to apply rather to H than to G.

‡ Twice, pp. 134, 135.

§ In March 1816 appeared GAUSS' *Bestimmung der Genauigkeit der Beobachtungen*, in which he employs several of the constants dependent on values of H.—*Werke*, Bd. iv, Ss. 110, 111, 116.

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agreeing in the main with KRAMP's third table, but differing occasionally in the last, or 7th, figure. This may have been due to some recomputation in places where the third differences were irregular. His second table is a continuation of the first, employing as arguments $\log_{10} x$, from 0 to 1 at intervals of ·01, with first and second differences. This is equivalent to a short table of $\log_{10}(\epsilon^{t^2}\mathrm{G})$ from $t=1$ to $t=10$, arranged at intervals in a geometrical proportion of which the ratio is—

$$t \times \log^{-1} \cdot 01 = t \times 1{\cdot}023\ 292\ 992\ 281.$$

It is not explained how this table was computed.

The next table of the kind appeared in LEGENDRE's "Intégrales Eulériennes" (1826),* giving 130 values of 2G, computed to ten decimal places, and arranged in two parts. The first contains the values from $t=0{\cdot}00$ to $t=0{\cdot}50$, computed by the usual series, and by halving the values of the integrals we can readily verify or correct the early part of KRAMP's first Table. The second part is adapted to EULER's form of the integral, viz.—

$$\int \left(\log_\epsilon \frac{1}{x}\right)^{-\frac{1}{2}} dx: \qquad t = \left(\log_\epsilon \frac{1}{x}\right)^{\frac{1}{2}},$$

and is arranged with x as argument, from $x=0{\cdot}80$ (that is, $t=0{\cdot}472\ 380\ 727\ 077$) to $x=0{\cdot}00$ or $t=\infty$. But, though when $x=0$, t is infinite,—in the previous entry, $x=0{\cdot}01$ makes $t=\sqrt{\log_\epsilon 100}=2{\cdot}145\ 966\ 026\ 289$,—so that this table does not really cover the extent of KRAMP's. It was computed by quadratures, and the process is laborious and effected by means of logarithmic tables extended to twelve decimal places.†

In his "Theory of Probabilities"‡ (1837), DE MORGAN reproduced KRAMP's table of this integral (G) without revision. Mr GLAISHER, in the *Philosophical Magazine* for December 1871,§ has further extended it from $t=3{\cdot}0$ to $t=4{\cdot}5$ at intervals of $0{\cdot}01$, to eleven places for the first fifty values, thirteen for the next, and fourteen for the last fifty. It would be easy enough to compute it in the way indicated below for any higher values of the argument (§ 25).

5. But it is with the other integral that this paper is concerned, viz.—

$$\mathrm{H} = \frac{2}{\sqrt{\pi}}\int_0^t \epsilon^{-t^2} dt = 1 - \frac{2}{\sqrt{\pi}}\int_t^\infty \epsilon^{-t^2} dt.$$

A table of this was first published by ENCKE, in a paper on the Method of Least Squares, in the *Berliner Astronomisches Jahrbuch* for 1834,‖ giving the values of the integral, for the arguments $t=0$ to $t=2{\cdot}00$ at intervals of $0{\cdot}01$, computed to seven decimal places, with first and second differences. This table, the author says, was derived

* In his *Traité des Fonctions Elliptiques et des Integrales Euleriennes*, tom. ii, pp. 520, 521.

† *Op. cit.*, tom. ii, pp. 517-524. The method explained below (§ 12) is different.

‡ In *Encyclopædia Metropolitana*, vol. ii, pp. 359–458. He also gave a short abstract of it in his *Differential and Integral Calculus* (1842), p. 657.

§ Vol. xlii, 4th ser., p. 436.

‖ The paper is continued through the vols. for 1834 (Ss. 249–312), 1835 (253–320), and 1836 (253–308). The Table is in the *Jahrbuch* for 1834, Ss. 305–308.

immediately from the table for the integral $\int e^{-t}dt$ in BESSEL'S *Fundamenta Astronomiæ*.* There seems to be a mistake here, for the table could be derived directly only from KRAMP'S Table I.

DE MORGAN reproduced this table also in his "Theory of Probabilities" (*Encyclop. Metrop.*, 1837), and again in his *Essay on Probabilities* (1838), but there he extended it to $t = 3{\cdot}00$ from KRAMP'S data. Again, GALLOWAY, in his "Treatise on Probability" (1839), prepared for the 7th edition of the *Encyclopædia Britannica*, printed ENCKE'S Table, also continued to the same point.

6. Further, and in dependence upon this integral, ENCKE gave a table† of the values of

$$\frac{2}{\sqrt{\pi}}\int_0^{\rho t} \epsilon^{-t^2}dt = \mathrm{K}, \qquad (5)$$

ρ being the numerical value of t when $\mathrm{H} = \frac{1}{2}$, giving 0·5 for the value of the integral K when the argument is $\mathrm{T}(=\rho t) = 1$. His table gives the values of K to five decimal places only with the argument T, at intervals of 0·01 from $\mathrm{T}=0$ to $\mathrm{T}=3{\cdot}40$ and at intervals of 0·1 from $\mathrm{T}=3{\cdot}4$ to $\mathrm{T}=5$. It was computed from the previous table by direct interpolation, and was also reprinted by DE MORGAN both in his *Theory* and his *Essay*.

Here it may be noted that this second table is so readily derived from a table of the values of H, when these are determined with precision, that there seems little reason for computing it. For if we multiply the arguments in such a table by $1/\rho = 2{\cdot}096\ 716\ 165$, or approximately by $\frac{65}{31}$ or $\frac{629}{300}$, we have at once a table of the values of K, only with arguments at intervals that are inconvenient on account of the fractions. But since the arguments required in practical applications nearly always lie between two consecutive tabular arguments, and interpolation has to be made at any rate, we may as well perform the operation on the values in a table of H as in one of K. This is done by multiplying the argument (T) for K by $\rho = 0{\cdot}476\ 936$, or, approximately by $\frac{31}{65}$, and taking the corresponding value from the table for H. Thus, if for the argument for K we have $\mathrm{T} = 3{\cdot}72$, then $3{\cdot}72 \times \rho = 1{\cdot}7742 = t$, for which our table gives $\mathrm{H} = 0{\cdot}987\ 8960$: and ENCKE'S table, by interpolation, for arg. 3·72, gives $\mathrm{K} = 0{\cdot}98790$.

But, we might also compute the first part of ENCKE'S table from the formula—

$$\begin{aligned}\mathrm{K} = {} & 0{\cdot}538\ 164\ 958\ 101\ 235\mathrm{T} - {\cdot}040\ 805\ 140\ 181\ 145\mathrm{T}^3 + {\cdot}002\ 784\ 561\ 677\ 8354\mathrm{T}^5 \\ & - {\cdot}000\ 150\ 809\ 348\ 77027\mathrm{T}^7 + {\cdot}000\ 006\ 670\ 286\ 943\ 3025\mathrm{T}^9 - {\cdot}000\ 000\ 248\ 189\ 408\mathrm{T}^{11} \\ & + {\cdot}000\ 000\ 007\ 964\ 597\ 724\mathrm{T}^{13} - {\cdot}000\ 000\ 000\ 224\ 304\ 823\mathrm{T}^{15} + {\cdot}000\ 000\ 000\ 005\ 627\ 456\mathrm{T}^{17} \\ & - {\cdot}000\ 000\ 000\ 000\ 127\ 2874\mathrm{T}^{19} + \text{etc.} \end{aligned} \qquad (6)$$

This will give values correct to fourteen decimal places, as far as $\mathrm{T} = 1$, and seven

* *Berl. Astronom. Jahrbuch* für 1834, S. 269. Mr J. W. L. GLAISHER (*Phil. Mag.* (1871), vol. xlii, p. 434) remarks that, if ENCKE'S table were derived from BESSEL'S, it must have been "by interpolation from his second table." But he overlooks the fact that BESSEL'S Table II. is only a continuation of Table I., giving the logarithmic values of the multiple of the integral by ϵ^{t^2} from $t=1$ to $t=10$, with logarithms of t for argument.

† *Berl. Astron. Jahrb.*, 1834, Ss. 309-312.

terms only will give correct results up to that point to nine places; but at T = 2 (K = ·822 656 449) the whole ten terms will be required to give eight figures correctly. When T^2 consists of only two figures, the computation is easy, if we begin with the term having the highest power of T. For the larger values of T, however, if not for all, it is easier to derive the values of K by interpolation from those of H.

7. It was a suspicion of some errors in the last figures of a few of the values in these two tables in DE MORGAN'S *Essay*, and in some values in AIRY'S *Theory of Errors of Observations* (1861),* that led me to recompute the table of H. It was begun during a holiday in the hot season of 1862 at an Indian hill sanatorium, where I had very few books, and rather as an amusement to occupy the middle hours of the day, than with any idea of publication.

Commencing on a more extensive scale than ENCKE'S table, in fact computing for intervals of 0·001, the values were worked out to about twelve places, but only nine were preserved, together with first and second differences. To this I added the values† of $\frac{2}{\sqrt{\pi}}\epsilon^{-t^2}$, partly as a check on the working, with differences. The work was at that time advanced from $t = 0$ to $t = 1{\cdot}250$, after which it was entirely laid aside for more than thirty years. The computation of the portion carrying the argument to $t = 3$ is exceedingly laborious, even with the intervals doubled after $t = 1{\cdot}5$. But the values have been given to fifteen decimal places from computations generally made to three or four figures more, and might have been depended on as accurate even beyond the sixteenth place.

This table, then, as recomputed, besides enabling us to construct ENCKE'S second table of K to seven or more decimal places, affords also the means of reconstructing or verifying and extending KRAMP'S Table I. (for G) by means of the expression (4). Several important constants also have been computed to a degree of accuracy perhaps beyond any practical requirement.‡

The Formulæ.

8. The formulæ available for computation, as pointed out by LAPLACE,§ are primarily three,—(8), (10) and (11), with the continued fraction (13), which he supplied to facilitate calculation where the series become very slowly convergent.

(1) In the integral $\int \epsilon^{-t^2}dt$, if we develope ϵ^{-t^2}, we get—

$$\int dt\left(1 - t^2 + \frac{t^4}{1.2} - \frac{t^6}{1.2.3} + \text{etc.}\right) = t - \frac{t^3}{3} + \frac{1}{1.2}\cdot\frac{t^5}{5} - \frac{1}{3!}\cdot\frac{t^7}{7} + \text{etc.}, \qquad (7)$$

* *Op. cit.*, pp. 16, 20, 22–24.

† I began by using the value of $\frac{2}{\sqrt{\pi}}$ given in SHORTREDE'S *Logarithmic Tables* (1858), p. 602, viz., 1·283 791 670 946 99 which is correct only to the tenth place, and therefore could not affect any of the results up to the eleventh place. This was examined later, and the true value of the constant found to be 1·283 791 670 955 126. SHORTREDE'S logarithm of $\frac{2}{\sqrt{\pi}}$ is correct. His value of sin 1^0 is also in error after the tenth decimal.

‡ In the small table given by AIRY, *Theory of Errors*, p. 24, six of the constants dependent on ρ are in error in the 5th and 6th places, three of them in the 4th.

§ *Théorie Analytique des Probabilités*, 2e. ed. (1814), p. 103, and *Mécanique Céleste*, liv. x, c. i, sec. 5.

and taking the integral from $t=0$ to $t=t$, we have

$$\int_0^t dt\epsilon^{-t^2}=t-\frac{t^3}{3}+\frac{t^5}{1.2.5}-\frac{t^7}{3!\,7}+\frac{t^9}{4!\,9}-\text{ etc.}=\tfrac{1}{2}\sqrt{\pi}.\mathrm{H}=\tfrac{1}{2}\sqrt{\pi}-\mathrm{G}. \tag{8}$$

That is— $$\frac{2}{\sqrt{\pi}}\int_0^t \epsilon^{-t^2}dt=\frac{2}{\sqrt{\pi}}\left(t-\frac{t^3}{3}+\frac{t^5}{1.2.5}-\frac{t^7}{3!\,7}+\frac{t^9}{4!\,9}-\frac{t^{11}}{5!\,11}+\text{etc.}\right)=\mathrm{H}. \tag{9}$$

(2) Integration by parts shows at once that—

$$\int t^n dt\epsilon^{-t^2}=\frac{}{n+1}.t^{n+1}\epsilon^{-t^2}+\frac{2}{n+1}\int t^{n+2}dt\epsilon^{-t^2}.$$

And putting successively $n=0$, $n=2$, $n=4$, $n=6$, etc., we get by repeated substitutions—

$$\int dt\epsilon^{-t^2}=t\epsilon^{-t^2}+2\int t^2dt\epsilon^{-t^2}=\epsilon^{-t^2}\left(t+\frac{2}{3}t^3\right)+\frac{2^2}{3}\int t^4dt\epsilon^{-t^2}=\epsilon^{-t^2}\left(t+\frac{2t^3}{3}+\frac{2^2t^5}{3.5}\right)+\frac{2^3}{3.5}\int t^6dt\epsilon^{-t^2},\text{ etc.},$$

which vanishes when $t=0$, and when $t=t$, we have—

$$\int_0^t dt\epsilon^{-t^2}=\epsilon^{-t^2}t\left\{1+\frac{2t^2}{1.3}+\frac{(2t^3)^2}{1.3.5}+\frac{(2t^7)^3}{1.3.5.7}+\text{ etc.}\right\}=\tfrac{1}{2}\sqrt{\pi}.\mathrm{H}. \tag{10}$$

(3) By a process similar to the last we find that—

$$\int t^{-n}dt\epsilon^{-t^2}=-\tfrac{1}{2}t^{-n-1}\epsilon^{-t^2}-\tfrac{1}{2}(n+1)\int t^{-n-2}dt\epsilon^{-t^2},\text{ etc.}$$

Hence $$\int dt\epsilon^{-t^2}=\mathrm{C}-\frac{\epsilon^{-t^2}}{2t}\left(1-\frac{1}{2t^2}+\frac{1.3}{(2t^2)^2}-\frac{1.3.5}{(2t^2)^3}+\text{etc.}\right).$$

Putting $t=\tau$, the constant quantity is eliminated by making the integral vanish, and we have—

$$\int_t^\tau dt\epsilon^{-t^2}=\frac{\epsilon^{-t^2}}{2t}\left(1-\frac{1}{2t^2}+\frac{1.3}{(2t^2)^2}-\frac{1.3.5}{(2t^2)^3}+\text{etc.}\right)-\frac{\epsilon^{-t^2}}{2\tau}\left(1-\frac{1}{2\tau^2}+\frac{1.3}{(2\tau^2)^2}-\text{etc.}\right).$$

Then putting $\tau=\infty$, we have the series—

$$\int_t^\infty dt\epsilon^{-t^2}=\frac{\epsilon^{-t^2}}{2t}\left(1-\frac{1}{2t^2}+\frac{1.3}{(2t^2)^2}-\frac{1.3.5}{(2t^2)^3}+\text{etc.}\right)=\mathrm{G}, \tag{11}$$

and $$\int_0^t dt\epsilon^{-t^2}=\tfrac{1}{2}\sqrt{\pi}-\frac{\epsilon^{-t^2}}{2t}\left(1-\frac{1}{2t^2}+\frac{1.3}{(2t^2)^2}-\frac{1.3.5}{(2t^2)^2}+\text{etc.}\right)^*=\tfrac{1}{2}\sqrt{\pi}.\mathrm{H} \tag{12}$$

The series (8) and (10) are convergent, but when t exceeds 2, the convergence becomes very slow. The first (8) and third (11) are alternately greater and less than the integral, so that if we add to any number of their terms the half of the following term, the error

* Conf. Hymers' *Integ. Calc.*, pp. 123, 151.

will be less than that half. But the third series (11) is not convergent, the numerators of the successive fractions soon exceeding any value of $2t^2$ that is likely to be used. To meet this case, we have LAPLACE'S continued fraction,* into which the series is converted, and which becomes more convergent the higher the value of t. And this can be used for either G or H.

Laplace's Continued Fraction.

9. When $t>1{\cdot}5$ it becomes very laborious to compute values of H, and LAPLACE gave the series for $\int_t^\infty dt.\epsilon^{-t^2} = \frac{\epsilon^{-t^2}}{2t}\left\{1-\frac{1}{2t^2}+\frac{1.3}{2^2t^4}-\frac{1.3.5}{2^3t^6}+\text{etc.}\right\}$, the form of a continued fraction, putting $q=\frac{1}{2t^2}$,—

$$\text{G or } \int_t^\infty \epsilon^{-t^2}dt = \frac{\epsilon^{-t^2}}{2t}\cdot\cfrac{1}{1+\cfrac{q}{1+\cfrac{2q}{1+\cfrac{3q}{1+\cfrac{4q}{1+\cfrac{}{1+\text{etc.},}}}}}} \tag{13}$$

and this gives a series of common fractions alternately greater and less than the integral. Mr GLAISHER has used this in computing his table of the values of the other function, G, from $t=3$ to $t=4{\cdot}5$. And for higher values of t the approximation of the successive fractions is increasingly rapid. But at any stage the degree of approximation can be estimated only by reducing two consecutive fractions to decimals. To attain a nearly correct value too, with values of t under 3, the computation of a long series of fractions of the form—

$$\frac{1}{1},\ \frac{1}{1+q},\ \frac{1+2q}{1+3q},\ \frac{1+5q}{1+6q+3q^2},\ \frac{1+9q+8q^2}{1+10q+15q^2},\ \frac{1+14q+33q^2}{1+15q+45q^2+15q^3},\ \text{etc.},$$

becomes tedious. This is obviated to a considerable extent, by determining once for all the coefficients α', b', c', etc., and α, b, c, etc., in the following expressions for the numerator and denominator of the fraction when it involves high powers of q. Thus we get two consecutive fractions of the form—(when n is even)—

$$\left.\begin{aligned} \mathrm{L}_{n-1} &= \frac{1+a'q+b'q^2+c'q^3+\ldots+l'q^{\frac{1}{2}n-1}}{1+aq+bq^2+cq^3+\ldots+lq^{\frac{1}{2}n-1}} \\ \text{and}\quad \mathrm{L}_n &= \frac{1+(a'+n)q+\ldots+l''q^{\frac{1}{2}n-1}}{1+(a+n)q+\ldots+mq^{\frac{n}{2}}} \end{aligned}\right\} \tag{14}$$

and the numerator and denominator for L_{n+1} are found by multiplying those of L_{n-1} by nq and adding those of L_n.

* See LAPLACE'S *Méc. Cél., ut sup.*, and *Theor. Anal. des Probab.*, p. 104; DE MORGAN, "Theory of Probabilities, § 63; and *Diff. and Integ. Calc.*, p. 591.

$$\text{Thus, putting } L_n=\frac{N_n}{D_n}, \qquad L_{n+1}=\frac{nqN_{n-1}+N_n}{nqD_{n-1}+D_n}. \tag{15}$$

When n is an even number the fraction L_n is less than the true value, and when odd, it is in excess by a quantity $c<\frac{1}{2}(L_n\sim L_{n+1})$.

The larger t is, the more rapidly the fraction approaches its limit, and consequently a lower value of n in L_n will give a sufficiently close approximation.

The following values of the coefficients of q in L_n can be made to serve in nearly all cases when $t>1\cdot5$:—

$$L_{13}=\frac{1+77q+2070q^2+23814q^3+114\,765q^4+187\,425q^5+46080q^6}{1+78q+2145q^2+25740q^3+135\,135q^4+270\,270q^5+135\,135q^6}-$$

$$L_{14}=\frac{1+90q+2915q^2+42300q^3+278\,019q^4+729\,330q^5+509\,985q^6}{1+91q+3003q^2+45045q^3+315\,315q^4+945\,945q^5+945\,945q^6+135\,135q^7}+$$

$$L_{15}=\frac{1+104q+3993q^2+71280q^3+611\,415q^4+2336\,040q^5+3133\,935q^6+645\,120q^7}{1+105q+4095q^2+75075q^3+675\,675q^4+2837\,835q^5+4729\,725q^6+2027\,025q^7}-$$

$$L_{17}=\frac{1+135q+7007q^2+178\,893q^3+2386\,395q^4+16288\,965q^5+51450\,525q^6+58437\,855q^7+10321\,920q^8}{1+136q+7140q^2+185\,640q^3+2552\,550q^4+18378\,360q^5+64324\,260q^6+91891\,800q^7+34459\,425q^8}-$$

$$L_{18}=\frac{1+152q+9030q^2+269\,724q^3+4341\,480q^4+37469\,520q^5+162\,058\,050q^6+297\,693\,900q^7+151\,335\,135q^8}{1+153q+9180q^2+278\,460q^3+4594\,590q^4+41351\,310q^5+192\,972\,780q^6+413\,513\,100q^7+310\,134\,825q^8+34459\,425q^9}+$$

$$L_{20}=\frac{1+189q+14348q^2+567\,420q^3+12686\,310q^4+162\,912\,750q^5+1167\,180\,300q^6+4302\,906\,300q^7+6859\,400\,625q^8+3061\,162\,125q^9}{1+190q+14535q^2+581\,400q^3+13226\,850q^4+174\,594\,420q^5+1309\,458\,150q^6+5237\,832\,600q^7+9820\,936\,125q^8+6547\,290\,750q^9+654\,729\,075q^{10}}+$$

$$L_{21}=\frac{1+209q+17748q^2+796\,620q^3+20603\,310q^4+314\,143\,830q^5+2775\,672\,900q^6+13408\,094\,700q^7+31335\,467\,625q^8+27125\,492\,625q^9+3715\,891\,200q^{10}}{1+210q+17955q^2+813\,960q^4+21366\,450q^4+333\,316\,620q^5+3055\,402\,350q^6+15713\,497\,800q^7+41247\,931\,725q^8+45831\,035\,250q^9+13749\,310\,575q^{10}}-$$

$$L_{23}=\frac{1+252q+26315q^2+1488\,384q^3+50044\,770q^4+1033\,829\,160q^5+13108\,004\,910q^6+98983\,684\,800q^7+416\,674\,583\,325q^8+860\,553\,193\,500q^9+664\,761\,133\,575q^{10}+81749\,606\,400q^{11}}{1+253q+26565q^2+1514\,205q^3+51482\,970q^4+1081\,142\,370q^5+14054\,850\,810q^6+110\,430\,970\,650q^7+496\,939\,367\,925q^8+1159\,525\,191\,825q^9+1159\,525\,191\,825q^{10}+316\,234\,143\,225q^{11}}-$$

$$L_{24}=\frac{1+275q+31605q^2+1987\,875q^3+75297\,114q^4+1781\,769\,150q^5+26460\,800\,730q^6+241\,511\,019\,750q^7+1288\,808\,846\,325q^8+3659\,572\,691\,775q^9+4601\,737\,965\,825q^{10}+1645\,756\,410\,375q^{11}}{1+276q+31878q^2+2018\,940q^3+77224\,455q^4+1853\,386\,920q^5+28109\,701\,620q^6+265\,034\,329\,560q^7+1490\,818\,103\,775q^8+4638\,100\,767\,300q^9+6957\,151\,150\,950q^{10}+3794\,809\,718\,700q^{11}+316\,234\,143\,225q^{12}}+$$

The multiplier q being always a proper fraction, we begin by dividing the last coefficient by $2t^2$, add the next preceding and divide again, and so on to the first coefficient of q, adding unity to the last quotient. If, for example, we take $t=1\cdot75$, $q=\frac{8}{49}=\frac{7}{49}+\frac{1}{49}$,—which is easily manipulated—and we find, on dividing down the coefficients in the terms for L_{23}—

$$L_{23}=\frac{1007439\cdot089305}{1139733\cdot366404}=0\cdot883\,925\,239\,886,$$

and

$$L_{24}=\frac{2535470\cdot688789}{2868422\cdot115642}=0\cdot883\,925\,233\,655.$$

These agree to the eighth decimal place, the first being too large and the second too small but nearer the true value,—which is 0·883 925 236 007 66.

For $t=1\cdot75$, the value of ϵ^{-t^2} is 0·052 774 995 930 150 374 66, and since (4)—

$$H = 1 - \frac{2}{\sqrt{\pi}} \frac{\epsilon^{-t^2}}{2t} \cdot L = 1 - \frac{\epsilon^{-t^2} L}{t\sqrt{\pi}}, \qquad (16)$$

with L_{23} we have $H = 0{\cdot}986\,671\,671\,161+$
and with L_{24} we have $H = 0{\cdot}986\,671\,671\,255-$
the true value being, $H = 0{\cdot}986\,671\,671\,219.$

Hence this degree of approximation, being to the tenth place in decimals, would be practically sufficient for all purposes. And for higher values of t, the results are still more close, and even a lower order of the fraction L would suffice. For $t=3$, L_{23} comes out ·951 813 839 1839 +, which is correct to the last—the 13th—figure.

10. When the values of L_{n-1} and L_n are not sufficiently accordant, either from t being small or n not sufficiently high, we may readily compute L_{n+1}. Then if $L_{n-1} - L_n = a$, and $L_n - L_{n+1} = b$, we may find a correction $\frac{a^2}{a-b}$, $\frac{ab}{a-b}$, or $\frac{b^2}{a-b}$ (regard being had to the signs of a and b, one of which is always negative), and—

$L_{n-1} + \frac{a^2}{a-b}$, or $L_n + \frac{ab}{a-b}$, or $L_{n+1} + \frac{b^2}{a-b}$,—which will be equal or very nearly so,—will give a closer approximation to the value of L than before. It will be greater or less than the true value, according as L_{n-1} and L_{n+1} are both greater or both less than L_n.*

11. By means of equation (9) we may compute any values of H up to a certain point with considerable facility, but with $t>1$ it becomes rapidly more difficult. We may, however, use it for such values of t as 2, 2·5, and even 3, though the work is lengthy; and for purposes of verification this has been done in the following table. For extreme accuracy the continued fraction is scarcely less laborious, till we reach $t=3$. Up to $t=1{\cdot}25$ the values were determined for moderate and equal intervals by means of (9), and the intermediate values inserted by interpolation, using the highest order of differences that could by any chance affect the results.

12. We might, however, make use of the method of *quadratures*. For H may be regarded as the area of a curve of which the equation is $y = \frac{2}{\sqrt{\pi}}\epsilon^{-t^2}$. Hence the value of $\frac{2}{\sqrt{\pi}}\epsilon^{-t^2}$ represents the rate of increment of that area at t; and the area between any two ordinates is the difference of the values of H between the two corresponding values of t. And if the intervals between the ordinates are so small as to enable us to find the area with sufficient accuracy, we may compute values of H,—or rather of the differences of H between two values of t,—with great precision. If, for example, we take the ordinates, given in the first part of the table, from $t=1{\cdot}160$ to $t=1{\cdot}170$ inclusive, the area is found by SIMPSON'S rule† to be ·002 904 196 086 +, and adding this to the value of H for $t=1{\cdot}160$ (from the second part of the table), the sum is the value of H when $t=1{\cdot}170$, viz., 0·902 000 398 966,—which is correct to the last figure.

Or, generally, if V_0, V_1, V_2, . . . V_n, be the values of the successive ordinates whose

* In the example above of $t=1{\cdot}75$, L_{25} will be 0·883 925 237 509, and $a=-6231$, $b=+3854$, whence the corrections are, −3850, +2381, and −1473, respectively, each giving ·883 925 236 036.

† T. SIMPSON'S *Mathematical Dissertations* (1743), pp. 109 f. This rule gives a very close approximation. Conf. HYMERS' *Int. Calc.*, p. 181; HUTTON'S *Mensuration*, p. 374.

distance apart is $\theta = t_1 - t_0$, $\Delta_1 = V_1 - V_0$, $\Delta'_1 = V_n - V_{n-1}$, and Δ_2, Δ'_2, the first and last of the second differences, and so on; then between V_0 and V_n the area is—

$$\theta\Big\{\Big(\tfrac{1}{2}V_0 + V_1 + V_2 + \ldots + \tfrac{1}{2}V_n\Big) - \frac{1}{12}(\Delta'_1 - \Delta_1) - \frac{1}{24}(\Delta'_2 + \Delta_2) - \frac{19}{720}(\Delta'_3 - \Delta_3) - \frac{3}{160}(\Delta'_4 + \Delta_4)$$
$$- \frac{863}{60\,480}(\Delta'_5 - \Delta_5) - \frac{275}{24\,192}(\Delta'_6 + \Delta_6) - \frac{33\,953}{3628\,800}(\Delta'_7 - \Delta_7) - \frac{8183}{1030\,800}(\Delta'_8 + \Delta_8)$$
$$- \frac{3250\,433}{479\,001\,600}(\Delta'_9 - \Delta_9) \ldots\Big\} \ast \qquad (17)$$

of which expression the first three terms will generally be sufficient. Taking the same example, we have—

$\Delta_1 =$	$-681\,145$	$\Delta_2 =$	$+994$
$\Delta'_1 =$	$-672\,183$	$\Delta'_2 =$	$+997$
$\Delta'_1 - \Delta_1 =$	$+8962$	$\Delta'_2 + \Delta_2 =$	$+1991$

$\frac{1}{2}V_0 + V_1 + \ldots + V_9 + \frac{1}{2}V_{10} =$	·290 419 6916
$-\frac{1}{12}(\Delta_1 - \Delta_1) =$	$-747-$
$-\frac{1}{24}(\Delta'_1 + \Delta_2) =$	$-\ 83-$
Sum, . .	·290 419 6086+

and $\theta = \cdot 01$; hence the area is ·002 904 196 086+, as before.

For a single interval, as between V_0 and V_1, by putting Δ^0_2 for the second difference, derived from V_{-1} and V_1, and Δ_2 the next in succession, derived from V_0 and V_2; Δ^0_4 the fourth difference, in line with V_0, and Δ_4, for the next below, etc., we have the area expressed by—

$$\theta(V_0 + V_1) - \frac{\theta}{24}(\Delta^0_2 + \Delta_2) - \frac{11\theta}{1440}(\Delta^0_4 + \Delta_4) - \frac{191\theta}{120\,960}(\Delta^0_6 + \Delta_6) - \frac{2497\theta}{7257\,600}(\Delta^0_8 + \Delta_8) \ldots \qquad (18)$$

Taking the values of V at 1·130, 1·140, . . . and 1·180, we find for 1·160, $\Delta^0_2 = +99\,373$, and $\Delta_2 = +99\,759$, also $\Delta^0_4 = +70$ and $\Delta_4 = +66$. Then—

$\frac{1}{2}(\cdot 293\,811\,239 + \cdot 287\,044\,575) =$	·290 427 907
$-\frac{\cdot 1}{24}(99\,373 + 99\,759) =$	$-8297+$
$-\frac{11}{1440}(70 + 66) =$	$-\ \ 1$
Sum, as before, nearly . .	·290 419 609−.

Interpolation.

13. The method of interpolation employed is familiar, but the process may be explained by which the transference is made from the differences found from the computed values, to the differences required for those to be interpolated.† I have not met with it in any text book at my command, and I think the formation of these differences indicates that too much stress may be laid on the common warning that most reliance is to be placed on results which lie nearest the middle of the series of values

* Conf. DE MORGAN'S *Diff. and Integ. Calc.*, pp. 262, 313–318; WOOLHOUSE, *Assurance Mag.*, vol. xi (1864), p. 309. By this method the computation might have been abridged in some portions, had I noticed its advantages earlier.

† Mr W. T. B. WOOLHOUSE, in a paper "On Interpolation, Summation, and the Adjustment of Numerical Tables," in *The Assurance Magazine*, 1863-65 (vol. xi, pp. 61–88, 301–332, and vol. xii, pp. 136–176), has developed a formula with necessary tables for interpolating terms in the *middle* interval of a series. The treatment is interesting, and the formulæ are rapidly convergent, but not altogether convenient for computing a lengthy table.

from which the differences used are derived.* It appears that if the intervals between a series of values be sufficiently small and their number so large that the last difference is practically zero, then the results will usually be about equally correct along the whole series,—for the first interpolated value is affected by the last difference.

14. In the computation of the values of any function to be tabulated with equi-different arguments, the two usual formulæ are—

$$V_n = V_0 + an + bn^2 + cn^3 + dn^4 + en^5 + fn^6 + gn^7 + \text{etc.} \quad (19)$$

and $$V_n = V_0 + n\Delta + \frac{n.n-1}{1.2}\Delta_2 + \frac{n.n-1.n-2}{1.2.3}\Delta_3 + \frac{n.n-1.n-2.n-3}{4!}\Delta_4 + \text{etc.} \quad (20)$$

By the first each value has to be computed separately; by the second, if we determine the values of Δ', Δ'_2, Δ'_3, etc., for the intervals to be adopted, the process is reduced to one of continuous addition and subtraction, according as the signs of the differences require. Now the conversion of the one formula into the other is readily effected by means of the numerical values of $\Delta^n 0^m$.† The following table, rearranged and extended to $\Delta^{12}0^{12}$, will suffice for all purposes :—

	0^1	0^2	0^3	0^4	0^5	0^6	0^7	0^8	0^9	0^{10}	0^{11}	0^{12}
Δ^1	1	1	1	1	1	1	1	1	1	1	1	1
Δ^2		2	6	14	30	62	126	254	510	1022	2046	4094
Δ^3			6	36	150	540	1806	5796	18150	55980	171006	519156
Δ^4				24	240	1560	8400	40824	186480	818520	3498000	14676024
Δ^5					120	1800	16800	126000	834120	5103000	29607600	165528000
Δ^6						720	15120	191520	1905120	16435440	129230640	953029440
Δ^7							5040	141120	2328480	29635200	322494480	3162075840
Δ^8								40320	1451520	30240000	479001600	6411968640
Δ^9									362880	16329600	419126400	8083152000
Δ^{10}										3628800	199584000	6187104000
Δ^{11}											39916800	2634508800
Δ^{12}												479001600

* Conf., *e.g.*, DE MORGAN'S *Diff. and Integ. Calc.*, pp. 544, 545; and WOOLHOUSE in *Assur. Mag.*, vol. xi, p. 73, note.

† HERSCHEL, *Examp. of Calculus of Finite Differences*, p. 9. His table extends to $\Delta^{10}0^{10}$ (conf. DE MORGAN, *Diff. and Int. Calc.*, p. 253.) This table is readily computed by the formula—

$$\Delta^{n+1}0^{m+1} = (n+1)(\Delta^n 0^m + \Delta^{n+1}0^m). \quad (21)$$

That is, the sum of the quantities in the two lines for Δ^n and Δ^{n+1}, in the preceding column for 0^m, multiplied by

We have here the coefficients in the following values—

$$\left.\begin{aligned}
\Delta_1 &= a+b+c+d+e+f+g+h+i+k+ \text{ etc.}\\
\Delta_2 &= 2b+6c+14d+30e+62f+126g+254h+510i+1022k+ \text{ etc.}\\
\Delta_3 &= 6c+36d+150e+540f+1806g+5796h+18150i+55980k, \text{ etc.}\\
\Delta_4 &= 24d+240e+1560f+8400g+40824h+186\,480i+818\,520k, \text{ etc.}\\
\Delta_5 &= 120e+1800f+16800g+126\,000h+834\,120i+5103\,000k, \text{ etc.}\\
\Delta_6 &= 720f+15120g+191\,520h+1905\,120i+16\,435\,440k, \text{ etc.}\\
\Delta_7 &= 5040g+141\,120h+2328\,480i+29\,635\,200k, \text{ etc.}\\
\Delta_8 &= 40320h+1451\,520i+30\,240\,000k, \text{ etc.}\\
\Delta_9 &= 362\,880i+16\,329\,600k, \text{ etc.}\\
\Delta_{10} &= 3628\,800k, \text{ etc.}
\end{aligned}\right\} \quad (22)$$

If we write A, B, C, etc., for the first terms of each value in the above, and reverse the arrangement, we have—

$$\left.\begin{aligned}
\Delta_{10} &= K+ \text{ etc.}\\
\Delta_9 &= I+\frac{9}{2}K, \text{ etc.}\\
\Delta_8 &= H+4I+\frac{25}{3}K, \text{ etc.}\\
\Delta_7 &= G+\frac{7}{2}H+\frac{77}{12}I+\frac{49}{6}K,\\
\Delta_6 &= F+3G+\frac{19}{4}H+\frac{21}{4}I+\frac{1087}{240}K,\\
\Delta_5 &= E+\frac{5}{2}F+\frac{10}{3}G+\frac{25}{8}H+\frac{331}{144}I+\frac{45}{32}K,\\
\Delta_4 &= D+2E+\frac{13}{6}F+\frac{5}{3}G+\frac{81}{80}H+\frac{37}{72}I+\frac{6821}{30240}K,\\
\Delta_3 &= C+\frac{3}{2}D+\frac{5}{4}E+\frac{3}{4}F+\frac{43}{120}G+\frac{23}{160}H+\frac{605}{12096}I+\frac{311}{20160}K,\\
\Delta_2 &= B+C+\frac{7}{12}D+\frac{1}{4}E+\frac{31}{360}F+\frac{1}{40}G+\frac{127}{20160}H+\frac{17}{12096}I+\frac{73}{259200}K,\\
\Delta_1 &= A+\frac{1}{2}B+\frac{1}{6}C+\frac{1}{24}D+\frac{E}{5!}+\frac{F}{6!}+\frac{G}{7!}+\frac{H}{8!}+\frac{I}{9!}+\frac{K}{10!}.
\end{aligned}\right\} \quad (23)$$

15. These equations readily give us the values of A, B, $C \ldots K$; and now, if n denote any subdivision of the intervals for which Δ, Δ_2, Δ_3, etc., represent the successive differences, and Δ', Δ'_2. Δ'_3, etc., represent the differences for these smaller intervals in the value of the argument,—then we have—

$$n^{10}\Delta'_{10} = K+ \text{ etc.} \qquad n^9\Delta'_9 = I+\frac{9K}{2n}+, \text{ etc.} \qquad n^8\Delta'_8 = H+\frac{4I}{n}+\frac{25K}{3n^2}; \text{ and so on.} \quad (24)$$

the index of Δ in the second line, gives the value in the 0^{m+1} column : thus $\Delta^3 0^7 + \Delta^4 0^7 = 1806+8400 = 10206$, and $10206 \times 4 = 40824 = \Delta^4 0^8$. The formula is derived from that for $\Delta^n 0^m$ in HERSCHEL'S Appendix to LACROIX'S *Differ. and Integ. Calculus*, (1816), p. 478.

And, by means of (23) we may thus obtain the values of Δ'_1, Δ'_2, Δ'_3, etc. Or, by transposition, we have

$$\left.\begin{aligned}
n^{10}\Delta'_{10} &= \Delta_{10} = K + \text{ etc.} \\
n^9\,\Delta'_9 &= \Delta_9 - \frac{9(n-1)}{2n}K, \\
n^8\,\Delta'_8 &= \Delta_8 - \frac{4(n-1)}{n}I - \frac{25(n^2-1)}{3n^2}K, \\
n^7\Delta'_7 &= \Delta_7 - \frac{7(n-1)}{2n}H - \frac{77(n^2-1)}{12n^2}I - \frac{49(n^3-1)}{6n^3}K, \\
n^6\Delta'_6 &= \Delta_6 - \frac{3(n-1)}{n}G - \frac{19(n^2-1)}{4n^2}H - \frac{21(n^3-1)}{4n^3}I - \frac{1087(n^4-1)}{240n^4}K, \text{ etc., etc.}
\end{aligned}\right\} \quad (25)$$

In actual calculation, it is convenient to compute and arrange the quantities in equation (23) thus,—the sum of the quantities in each column being equal to the value of Δ at the top of it:—

Δ_{10}	Δ_9	Δ_8	Δ_7	Δ_6	Δ_5	Δ_4	Δ_3	Δ_2	Δ_1
$K.$	$\frac{9}{2}K.$	$\frac{25}{3}K.$	$\frac{49}{6}K.$	$\frac{1087}{240}K.$	$\frac{45}{32}K.$	$\frac{6821}{3.7.12.120}K.$	$\frac{311}{2.7.12.120}K.$	$\frac{73}{2.9.120.120}K$	$\frac{1}{10!}.K=k.$
	$I.$	$4I.$	$\frac{77}{12}I.$	$\frac{21}{4}I.$	$\frac{331}{144}I.$	$\frac{37}{72}I.$	$\frac{605}{7.12.12.12}I.$	$\frac{17}{7.12.12.12}I.$	$\frac{1}{9!}I=i.$
		$H.$	$\frac{7}{2}H.$	$\frac{19}{4}H.$	$\frac{25}{8}H.$	$\frac{81}{80}H.$	$\frac{23}{160}H.$	$\frac{127}{2.7.12.120}H.$	$\frac{1}{8!}H=h.$
			$G.$	$3G.$	$\frac{10}{3}G.$	$\frac{5}{3}G.$	$\frac{43}{120}G.$	$\frac{1}{40}G.$	$\frac{1}{7!}G=g.$
				$F.$	$\frac{5}{2}F.$	$\frac{13}{6}F.$	$\frac{3}{4}F.$	$\frac{31}{360}F.$	$\frac{1}{6!}F=f.$
					$E.$	$2E.$	$\frac{5}{4}E.$	$\frac{1}{4}E.$	$\frac{1}{5!}E=e.$
						$D.$	$\frac{3}{2}D.$	$\frac{7}{12}D.$	$\frac{1}{4!}D=d.$
							$C.$	$C.$	$\frac{1}{6}C=c.$
								$B.$	$\frac{1}{2}B=b.$
									$A=a.$

After $K=\Delta_{10}$, the values of I, H, G, etc., are successively found by subtracting the sum of the quantities in the proper column from the value of Δ_n above it. If the values (a, b, c, etc.) in the last column are determined with extreme accuracy, they afford a ready means of verification of the whole operation, since

$$V_n - V_0 = an + bn^2 + cn^3 + dn^4 + \ldots + kn^{10}.$$

Then, by equation (24), we readily deduce the values of Δ'_1, Δ'_2, etc., from the above by dividing successively upwards each quantity in the column, except the lowest, by n, n^2, n^3, etc., adding the quotients to the value of A, B, or C, etc., and lastly dividing the sum by the coefficient of Δ'. When $n=10$, this can be done by mere inspection. Thus, for example—

$$10^4\Delta'_4 = D+\frac{2E}{10}+\frac{13F}{6.10^2}+\frac{5G}{3.10^3}+\cdots+\frac{6821K}{30240.10^6}.$$

And the quantities in the same horizontal lines may be computed by the fractional coefficients; or, k, i, h, etc., being first found directly from K, I, H, etc., we may use the integral coefficients in eq. (22).

16. Again, if in a series of values of a function, the first differences before and after any value V be Δ_{-1} and Δ; $\Delta^2=\Delta_{-1}-\Delta$; the third differences, before and after Δ^2, be Δ^3_{-1} and Δ^3; $\Delta^4=\Delta^3_{-1}-\Delta^3$; the fifth differences, before and after Δ^4, be $\Delta^5_{-1}-\Delta^6$; and so on,—

$$\left.\begin{aligned}
\text{Putting } \Delta^1_0&=\tfrac{1}{2}(\Delta_{-1}+\Delta)=\Delta-\tfrac{1}{2}\Delta^2,\\
\Delta^3_0&=\tfrac{1}{2}(\Delta^3_{-1}+\Delta^3)=\Delta^3-\tfrac{1}{2}\Delta^4,\\
\Delta^5_0&=\tfrac{1}{2}(\Delta^5_{-1}+\Delta^5)=\Delta^5-\tfrac{1}{2}\Delta^6,\\
\Delta^7_0&=\tfrac{1}{2}(\Delta^7_{-1}+\Delta^7)=\Delta^7-\tfrac{1}{2}\Delta^8,\\
\Delta^9_0&=\tfrac{1}{2}(\Delta^9_{-1}+\Delta^9)=\Delta^9-\tfrac{1}{2}\Delta^{10}.
\end{aligned}\right\} \qquad (26)$$

Then, as before, expressing the values of Δ_0, Δ^2, Δ^3_0, etc., in terms of the coefficients a, b, c . . . in the formula (19), we have—

$$\left.\begin{aligned}
\Delta^1_0&=a+c+e+g+i, & \Delta^2&=2(b+d+f+h+k),\\
\Delta^3_0&=3!\,(c+5e+21g+85i), & \Delta^4&=4!\,(d+5f+21h+85k),\\
\Delta^5_0&=5!\,(e+14g+147i), & \Delta^6&=6!\,(f+14h+147k),\\
\Delta^7_0&=7!\,(g+30i), & \Delta^8&=8!\,(h+30k),\\
\Delta^9_0&=9!\,.i, & \text{and } \Delta^{10}&=10!\,.k.
\end{aligned}\right\} \qquad (27)$$

From these we deduce—

$$\left.\begin{aligned}
a&=\Delta^1_0-\frac{\Delta^3_0}{3!}+\frac{4\Delta^5_0}{5!}-\frac{36\Delta^7_0}{7!}+\frac{576\Delta^9_0}{9!}, & b&=\tfrac{1}{2}\Delta^2-\frac{\Delta^4}{4!}+\frac{4\Delta^6}{6!}-\frac{36\Delta^8}{8!}+\frac{576\Delta^{10}}{10!},\\
c&=\frac{\Delta^3_0}{3!}-\frac{\Delta^5_0}{4!}+\frac{7\Delta^7_0}{6!}-\frac{820\Delta^9_0}{9!}, & d&=\frac{\Delta^4}{4!}-\frac{5\Delta^6}{6!}+\frac{49\Delta^8}{8!}-\frac{82\Delta^{10}}{9!},\\
e&=\frac{\Delta^5_0}{5!}-\frac{2\Delta^7_0}{6!}+\frac{273\Delta^9_0}{9!}, & f&=\frac{\Delta^6}{6!}-\frac{14\Delta^8}{8!}+\frac{273\Delta^{10}}{10!},\\
g&=\frac{\Delta^7_0}{7!}-\frac{30\Delta^9_0}{9!}, \qquad h=\frac{\Delta^8}{8!}-\frac{3\Delta^{10}}{9!}, & i&=\frac{\Delta^9_0}{9!}, \quad \text{and } k=\frac{\Delta^{10}}{10!}.
\end{aligned}\right\} \qquad (28)$$

Substituting these values in the general form of the function (19), and simplifying, we have—

$$V_n=V+n(\Delta^1_0+\tfrac{n}{2}\Delta^2)+\frac{n(n^2-1)}{3!}(\Delta^3_0+\tfrac{n}{4}\Delta^4)+\frac{n(n^2-1)(n^2-4)}{5!}(\Delta^5_0+\tfrac{n}{6}\Delta^6)+\text{etc.}^* \qquad (29)$$

* This is only an altered mode of writing the formula given in DE MORGAN'S *Diff. and Integ. Calculus*, p. 546; conf. WOOLHOUSE, *Assur. Mag.*, vol. xi, (1863), p. 68.

and replacing Δ^1_0, Δ^3_0, etc., by the second equivalents from (26) we have finally—

$$V_n = V + n(\Delta + \frac{n-1}{2}\Delta^2) + \frac{n(n^2-1)}{3!}.(\Delta^3 + \frac{n-2}{4}\Delta^4) + \frac{n(n^2-1)(n^2-4)}{5!}.(\Delta^5 + \frac{n-3}{6}\Delta^6)$$
$$+ \frac{n(n^2-1)(n^2-4)(n^2-9)}{7!}.(\Delta^7 + \frac{n-4}{8}\Delta_8) + \text{etc.} \qquad (30)$$

Either of these formulæ, which converge rapidly, may be used for interpolating terms in a series of values already found, especially if we form tables of the values of each term for the various coefficients of Δ^2, Δ^3, Δ^4, etc. Thus, to insert values at intervals of 0·1 between V and V_{10}, we have—

$$\left.\begin{aligned}
V_1 &= V + \frac{\Delta}{10} - \cdot045.\Delta^2 - \cdot0165.\Delta^3 + \cdot007\,8375.\Delta^4 + \cdot003\,291\,75.\Delta^5 - \cdot001\,591\,0125.\Delta^6 - \text{etc.}\\
V_2 &= V_1 + \frac{\Delta}{10} - \cdot035.\Delta^2 - \cdot0155.\Delta^3 + \cdot006\,5625.\Delta^4 + \cdot003\,044\,25.\Delta^5 - \cdot001\,365\,7875.\Delta^6 - \text{etc.}\\
V_3 &= V_2 + \frac{\Delta}{10} - \cdot025.\Delta^2 - \cdot0135.\Delta^3 + \cdot004\,9375.\Delta^4 + \cdot002\,559\,25.\Delta^5 - \cdot001\,046\,06\,25.\Delta^6 - \text{etc.}\\
V_4 &= V_3 + \frac{\Delta}{10} - \cdot015.\Delta^2 - \cdot0105.\Delta^3 + \cdot003\,0625.\Delta^4 + \cdot001\,856\,75.\Delta^5 - \cdot000\,656\,3375.\Delta^6 - \text{etc.}\\
V_5 &= V_4 + \frac{\Delta}{10} - \cdot005.\Delta^2 - \cdot0065.\Delta^3 + \cdot001\,0375.\Delta^4 + \cdot000\,966\,75.\Delta^5 - \cdot000\,222\,9125.\Delta^6 - \text{etc.}\\
V_6 &= V_5 + \frac{\Delta}{10} + \cdot005.\Delta^2 - \cdot0015.\Delta^3 - \cdot001\,0375.\Delta^4 - \cdot000\,070\,75.\Delta^5 + \cdot000\,222\,9125.\Delta^6 + \text{etc.}\\
V_7 &= V_6 + \frac{\Delta}{10} + \cdot015.\Delta^2 + \cdot0045.\Delta^3 - \cdot003\,0625.\Delta^4 - \cdot001\,205\,75.\Delta^5 + \cdot000\,656\,3375.\Delta^6 + \text{etc.}\\
V_8 &= V_7 + \frac{\Delta}{10} + \cdot025.\Delta^2 + \cdot0115.\Delta^3 - \cdot004\,9375.\Delta^4 - \cdot002\,378\,25.\Delta^5 + \cdot001\,046\,0625.\Delta^6 + \text{etc.}\\
V_9 &= V_8 + \frac{\Delta}{10} + \cdot035.\Delta^2 + \cdot0195.\Delta^3 - \cdot006\,5625.\Delta^4 - \cdot003\,518\,25.\Delta^5 + \cdot001\,365\,7875.\Delta^6 + \text{etc.}\\
V_{10} &= V_9 + \frac{\Delta}{10} + \cdot045.\Delta^2 + \cdot0285.\Delta^3 - \cdot007\,8375.\Delta^4 - \cdot004\,545\,75.\Delta^5 + \cdot001\,591\,0125.\Delta^6 + \text{etc.}
\end{aligned}\right\} \quad (31)$$

If the interval n be $\frac{1}{5}$, we have—

$$\left.\begin{aligned}
V_1 &= V + 0{\cdot}2\Delta - \cdot08.\Delta^2 - \cdot032.\Delta^3 + \cdot0144.\Delta^4 + \cdot006336.\Delta^5 - \cdot0029568.\Delta^6 - \text{etc.}\\
V_2 &= V_1 + 0{\cdot}2\Delta - \cdot04.\Delta^2 - \cdot024.\Delta^3 + \cdot008.\ \Delta^4 + \cdot004416\,\Delta^5 - \cdot0017024.\Delta^6 - \text{etc.}\\
V_3 &= V_2 + 0{\cdot}2\Delta \qquad - \cdot008.\Delta^3 \qquad + \cdot000896.\Delta^5 \qquad - \text{etc.}\\
V_4 &= V_3 + 0{\cdot}2\Delta + \cdot04.\Delta^2 + \cdot016.\Delta^3 - \cdot008.\ \Delta^4 - \cdot003584.\Delta^5 + \cdot0017024\,\Delta^6 + \text{etc.}\\
V_5 &= V_4 + 0{\cdot}2\Delta + \cdot08.\Delta^2 + \cdot048.\Delta^3 - \cdot0144.\Delta^4 - \cdot008064.\Delta^5 + \cdot0029568.\Delta^6 + \text{etc.}
\end{aligned}\right\} \quad (32)$$

The series converges so rapidly that it is seldom necessary to go beyond the fourth or fifth differences, and the last result in each case is a check on the accuracy of the work. But, as it requires fresh arrangements for each short series of interpolated values, it is not so satisfactory for computing a lengthy table as the method above explained, though a larger number of differences is required to compensate for the more rapid convergence. For isolated values, however (30), is most convenient. We may proceed by successively correcting the differences in a retrograde order, correcting the highest employed, if necessary, to its mean value, by adding half the next above it. Thus, if five orders of difference are to be used, make $\Delta^5_c = \Delta^5 + \frac{1}{2}\Delta^6$. Then—

$$\Delta_c^4=\Delta^4+\frac{2+n}{5}\Delta_c^5,\quad \Delta_c^3=\Delta^3-\frac{2-n}{4}\Delta_c^4,\quad \Delta_c^2=\Delta^2+\frac{1+n}{3}\Delta_c^3,\quad \Delta_c=\Delta-\frac{1-n}{2}\Delta_c^2,\text{ and } V_n=V+n\Delta_c.$$

To bisect an interval, $n=\frac{1}{2}$, and—

$$V_{\frac{1}{2}}=V+\frac{\Delta}{2}-\frac{\Delta^2}{8}-\frac{\Delta^3}{16}+\frac{3\Delta^4}{128}+\frac{3\Delta^5}{256}-\frac{5\Delta^6}{1024}-\frac{5\Delta^7}{2048}+\frac{35\Delta^8}{32768}+\text{etc.} \tag{33}$$

Or, $\Delta_c^4=\Delta^4+\frac{1}{2}(\Delta^5+\frac{1}{2}\Delta^6)$, $\Delta_c^3=\Delta^3-\frac{3}{8}\Delta_c^4$, $\Delta_c^2=\Delta^2+\frac{1}{2}\Delta_c^3$, $\Delta_c=\Delta-\frac{1}{4}\Delta_c^2$, and $V_{\frac{1}{2}}=V+\frac{1}{2}\Delta_c$. (34)

Thus if it be required to find the value of H corresponding to $t=1{\cdot}575$, we take the differences following and on line with 1·574 in the table, and proceed thus :—

Δ^4	Δ^3	Δ^2	Δ	H
−18656	+5985 292	−1 192 978 427	+188 870 390 940	·973 983 952 882 675
$-13=\frac{1}{2}\Delta^5=$	+7 001	+2 996 146	+ 297 470 570	+94 583 930 755
$-18669\times-\frac{3}{8}=$	$+5992\,293\times\frac{1}{2}=$	$-1\,189\,882\,281\times-\frac{1}{4}=$	$+189\,167\,861\,510\times\frac{1}{2}=$	·974 078 536 813 430.

This value, H = ·974 078 536 813 430, is correct to the last figure, and $\frac{1}{2}\Delta^5=-13$, is so small that it might have been neglected without affecting the result.

After determining the values of H for moderate intervals, the differences for the smaller intervals of ·001 or ·002 were determined by means of the formulæ (22) to (25), and the table thus filled up throughout.

The Difference Formula.

17. The difficulty of computation, due to the slowness of convergence of the series for values of t above 1·0, led KRAMP, who computed the table so often reprinted, to adopt a difference-formula* obtained from the general series by means of TAYLOR's theorem, viz.—

$$\Delta\int_t^\infty \epsilon^{-t^2}dt=-r\epsilon^{-t^2}\left(1-rt+\frac{2t^2-1}{3}r^2-\frac{2t^3-3t}{6}r^3+\text{etc.}\right), \tag{35}$$

where $\Delta t=r=0{\cdot}01$. This implies the separate computation of the values of the differences for each entry in the table. When r is small, three terms of the series may be sufficient, and M. KRAMP says he used no more. Mr J. W. L. GLAISHER, in computing the values of the same function from $t=3$ to $t=4{\cdot}50$, tells us that he computed separate tables of log. ϵ^{-t^2} and of log. $\left(r-tr^2+\frac{2t^2-1}{3}r^3-\frac{2t^3-3t}{6}r^4\right)$, and then built up his table by the successive differences.† This requires for his table about a hundred and fifty computations of the values of (35), and an error in one would have been perpetuated

* *Analyse des Refractions astronomiques et terrestres* (Strasbourg, 1799), p. 135.

† *Philos. Mag.*, xlii, (1871), p. 434. Conf. DE MORGAN, *ut cit.*, § 117. Mr GLAISHER remarks (p. 432) that "KRAMP does not state what value he started from in applying the differences, or what means of verification he adopted. In all cases where a table is constructed by means of differences, the last value should be calculated independently, and then the agreement of the two values would verify all the preceding portion of the table." And he adds that KRAMP's value for $t=3$ is in error in the tenth and eleventh figures, so that probably a portion of his table is incorrect in the last two figures (see § 4 above).

through the rest, if he had not checked his work by means of LAPLACE's continued fraction.

18. But the formula may be applied with great effect in this way: r may be taken as negative as well as positive, so that from a value H, corresponding to t, we can derive both the values at $t-r$ and $t+r$; and by developing the formula more fully, we may use it with much larger values than $r=0{\cdot}01$. Putting $x=-2t$, the general term is—

$$\frac{1}{n+1}\left\{\frac{x^n}{n!}-\frac{x^{n-2}}{(n-2)!}+\frac{x^{n-4}}{1.2.(n-4)!}-\frac{x^{n-6}}{3!(n-6)!}+\frac{x^{n-8}}{4!(n-8)!}-\text{etc.}\right\}$$

Expanding and adapting to the integral $\frac{2}{\sqrt{\pi}}\int_0^t \epsilon^{-t^2}dt$,—

$$\left.\begin{aligned}\Delta H=\frac{2\epsilon^{-t^2}}{\sqrt{\pi}}r\Big\{&1-rt+\frac{2t^2-1}{3}r^2-\frac{2t^3-3t}{6}r^3+\frac{4t^4-12t^2+3}{2.3.5}r^4-\frac{4t^5-20t^3+15t}{3.5.6}r^5\\&+\frac{8t^6-60t^4+90t^2-15}{3.5.7.3!}r^6-\frac{8t^7-84t^5+210t^3-105t}{3.5.7.4!}r^7+\frac{16t^8-224t^6+840t^4-840t^2+105}{3.5.7.9.4!}r^8\\&-\frac{16t^9-288t^7+1512t^5-2520t^3+945t}{3.5.7.9.5!}r^9+\frac{32t^{10}-720t^8+5040t^6-12600t^4+9450t^2-945}{3.5.7.9.11.5!}r^{10}\\&-\frac{32t^{11}-880t^9+7920t^7-27720t^5+34650t^3-10395t}{3.5.7.9.11.6!}r^{11}\\&+\frac{64t^{12}-2112t^{10}+23760t^8-110\,880t^6+207\,900t^4-124\,740t^2+10395}{3.5.7.9.11.13.6!}r^{12}\\&-\frac{64t^{13}-2496t^{11}+34320t^9-205\,920t^7+540\,540t^5-540\,540t^3+135\,135t}{3.5.7.9.11.13.7!}r^{13}+\text{etc.}\Big\}\end{aligned}\right\}(36)$$

For any portion of the table then, say from $t=1{\cdot}9$ to $t=3$, we may compute the coefficients of the powers of r for t at the values 2·0, 2·2, 2·4, 2·6, 2·8, and 3; and by means of the first we find the differences from $t=1{\cdot}90$ to $t=2{\cdot}10$, by the second series from $t=2{\cdot}10$ to 2·30, and so on. If, also, we know the values for $t=2$ and $t=3$ (which I have computed separately, both by the general series and by LAPLACE's fraction), we can fill up the table,—first, for all values of t differing by 0·01; and, secondly, by forming from these values the differences in the series $H_0+n\Delta'+\frac{n.n-1}{2}\Delta'_2+$ etc., for 1/5, 1/10, or any other subdivision of the interval, we may complete the table from $t=1{\cdot}900$ to $t=3{\cdot}100$. This sufficiently explains the method of computation for the portion of the table beyond $t=1{\cdot}000$.

19. Since the computation of these coefficients of the powers of r is also required for the other branch of the integral—G, they may be preserved here.

For $t=1$, $\epsilon^{-1}=0{\cdot}367\,879\,441\,171\,442\,321\,595\,524$,

$$\begin{aligned}\Delta G=-r\epsilon^{-t^2}\Big(&1-r+\frac{1}{3}r^2+\frac{1}{6}r^3-\frac{1}{6}r^4+\frac{1}{90}r^5+\dot{0}36\,507\dot{9}r^6-\cdot011\,507\,936\dot{r}^7-\cdot004\,541\,446\,208\,112\,875r^8\\&+\cdot002\,954\,144\,620\,811\,287r^9+\cdot000\,206\,028\,539\,362r^{10}-\cdot000\,481\,935\,7597r^{11}\\&+\cdot000\,045\,088\,6562r^{12}+\cdot000\,057\,110\,732r^{13},\text{ etc.}\Big).\end{aligned}$$

Also $\frac{2}{\sqrt{\pi}}\epsilon^{-1}=0{\cdot}415\,107\,497\,420\,594\,703\,340\,268=E$, and $H=0{\cdot}842\,700\,792\,949\,714\,869\,34$.

* If we make $t=0$ in this series, r then becomes t, and we have the series in (9) from which it is derived.

And, for our purpose, multiplying the above coefficients by $\frac{2}{\sqrt{\pi}}\epsilon^{-t^2}$, we have—

$$\begin{aligned}\Delta H = Er &- 0{\cdot}415\,107\,497\,420\,594\,703\,340\,27r^2 + 0{\cdot}138\,369\,165\,806\,864\,901\,113r^3\\ &+ 0{\cdot}069\,184\,582\,903\,432\,450\,557r^4 - 0{\cdot}069\,184\,582\,903\,432\,4506r^5\\ &+ 0{\cdot}004\,612\,305\,526\,895\,496\,70r^6 + 0{\cdot}015\,154\,718\,159\,799\,49r^7 - 0{\cdot}004\,777\,030\,724\,284\,62r^8\\ &- 0{\cdot}001\,885\,188\,370\,120r^9 + 0{\cdot}001\,226\,287\,580\,563r^{10} + 0{\cdot}000\,085\,523\,9914r^{11}\\ &- 0{\cdot}000\,200\,055\,147r^{12} + 0{\cdot}000\,018\,716\,6392r^{13} + 0{\cdot}000\,023\,707\,93r^{14}.\end{aligned}$$

These values would be sufficient to compute to seventeen or eighteen places all values from $t = 0{\cdot}90$ to $t = 1{\cdot}10$, making r negative for values below $1{\cdot}0$; and, taken to r^8, they would give accurate results to ten or eleven decimal places.

For $t = 1{\cdot}1$, $\epsilon^{-t^2} = 0{\cdot}298\,197\,279\,429\,887\,378\,618\,226$.

$$\begin{aligned}\Delta G = -r\epsilon^{-t^2}\{1 &- 1{\cdot}1r + {\cdot}473\dot{3}r^2 + {\cdot}106\dot{3}r^3 - {\cdot}188\,786\dot{4}r^4 + {\cdot}040\,866\dot{2}r^5 + {\cdot}032\,105\,5365\tfrac{5}{63}r^6\\ &- {\cdot}017\,586\,07\tfrac{1}{63}r^7 - {\cdot}001\,943\,926\,059\,964\,7266r^8 + {\cdot}003\,554\,076\,205\,855\,38r^9\\ &- {\cdot}000\,392\,718\,249\,540\,48r^{10} - {\cdot}000\,466\,498\,049\,0775r^{11} + {\cdot}000\,134\,329\,166\,57r^{12}\\ &+ {\cdot}000\,040\,407\,3572r^{13}, \text{ etc.}\}.\end{aligned}$$

and $\frac{2}{\sqrt{\pi}}\epsilon^{-t^2} = 0{\cdot}336\,479\,597\,793\,244\,144\,101\,453$.

For $t = 1{\cdot}25 = \frac{5}{4}$. $\epsilon^{-t^2} = 0{\cdot}209\,610\,951\,665\,850\,449\,333\,13$;

$$\begin{aligned}\Delta G = -r\epsilon^{-t^2}\{1 &- 1{\cdot}25r + {\cdot}708\dot{3}r^2 - {\cdot}026\,41\dot{6}r^3 - {\cdot}199\,4791\dot{6}r^4 + {\cdot}090\,060\,763\dot{8}r^5\\ &+ {\cdot}015\,330\,481\,1\dot{5}0\,7936\dot{6}r^6 - {\cdot}024\,089\,510\,478\,670\,635r^7 + {\cdot}003\,710\,603\,798\,0875r^8\\ &+ {\cdot}003\,354\,928\,691\,130\,68r^9 - {\cdot}001\,369\,673\,505\,853r^{10} - {\cdot}000\,222\,973\,881\,906r^{11}\\ &+ {\cdot}000\,236\,037\,9076r^{12} - {\cdot}000\,012\,746\,614r^{13}, \text{ etc.}\}.\end{aligned}$$

$\frac{2}{\sqrt{\pi}}\epsilon^{-t^2} = {\cdot}236\,521\,122\,447\,290\,787\,220\,015 = \mathrm{E}$; $\mathrm{H} = 0{\cdot}922\,900\,128\,256\,458\,230\,14$;

$$\begin{aligned}\text{and } \Delta H = Er &- {\cdot}295\,651\,403\,059\,113\,348\,402\,50r^2 + {\cdot}167\,535\,795\,066\,830\,974\,28r^3\\ &- {\cdot}006\,159\,404\,230\,398\,197\,58r^4 - {\cdot}047\,181\,036\,404\,850\,1935r^5\\ &+ {\cdot}021\,301\,272\,963\,460\,433r^6 + {\cdot}003\,625\,982\,609\,442\,75r^7\\ &- {\cdot}005\,697\,678\,057\,620\,95r^8 + {\cdot}000\,877\,636\,175\,281r^9 + {\cdot}000\,793\,511\,499\,76r^{10}\\ &- {\cdot}000\,323\,956\,7150r^{11} - {\cdot}000\,052\,738\,033r^{12} + {\cdot}000\,055\,827\,95r^{13} - {\cdot}000\,003\,0148r^{14}\end{aligned}$$

$t = 1{\cdot}4$: $\epsilon^{-t^2} = 0{\cdot}140\,858\,420\,921\,044\,996\,147\,971$;

$$\begin{aligned}\Delta G = -r\epsilon^{-t^2}\{1 &- 1{\cdot}4r + {\cdot}973\dot{3}r^2 - {\cdot}2146\dot{6}r^3 - {\cdot}171\,786\dot{6}r^4 + {\cdot}137\,4115\dot{5}r^5 - {\cdot}014\,063\,0\dot{3}4\,9206\dot{6}r^6\\ &- {\cdot}024\,523\,27\dot{1}r^7 + {\cdot}010\,363\,941\,013\,58r^8 + {\cdot}001\,457\,789\,123\,950\,06r^9\\ &- {\cdot}002\,066\,991\,235\,5915r^{10} + {\cdot}000\,261\,420\,814\,979r^{11} + {\cdot}000\,235\,192\,7423r^{12}\\ &- {\cdot}000\,081\,536\,3055r^{13}\}\end{aligned}$$

$\frac{2}{\sqrt{\pi}}\epsilon^{-t^2} = 0{\cdot}158\,941\,707\,677\,277\,875\,860\,084 = \mathrm{E}$; $\mathrm{H} = 0{\cdot}952\,285\,119\,762\,648\,810\,5165$;

$$\begin{aligned}\Delta H = Er &- {\cdot}222\,518\,390\,748\,189\,026\,2041r^2 + {\cdot}154\,703\,262\,139\,217\,132\,504r^3\\ &- {\cdot}034\,119\,486\,581\,388\,984\,02r^4 - {\cdot}027\,304\,066\,156\,187\,3087r^5\\ &+ {\cdot}021\,840\,427\,294\,591\,140r^6 - {\cdot}002\,235\,202\,785\,41091r^7 - {\cdot}003\,897\,770\,588\,2329r^8\\ &+ {\cdot}001\,647\,262\,502\,391r^9 + {\cdot}000\,231\,703\,492\,79r^{10} - {\cdot}000\,328\,531\,1167r^{11}\\ &+ {\cdot}000\,041\,550\,671r^{12} + {\cdot}000\,037\,381\,94r^{13} - {\cdot}000\,012\,9579r^{14}\end{aligned}$$

For $t = 1{\cdot}5$ $\epsilon^{-t^2} = {\cdot}105\,399\,224\,561\,864\,336\,783\,218$;

$$\begin{aligned}\Delta G = -r\epsilon^{-t^2}\{1 &- 1{\cdot}5r + 1{\cdot}1\dot{6}r^2 - 0{\cdot}375r^3 - {\cdot}125r^4 + 0{\cdot}1625r^5 - 0{\cdot}039\,88\dot{0}\,952\dot{3}r^6 - 0{\cdot}019\,866\,07\dot{1}\,4285\dot{7}r^7\\ &+ 0{\cdot}014\,376\,653\,439r^8 - 0{\cdot}000\,781\,25r^9 - 0{\cdot}002\,139\,475\,108r^{10} + 0{\cdot}000\,653\,239\,989r^{11}\\ &+ 0{\cdot}000\,150\,973\,16r^{12} - 0{\cdot}000\,473\,971\,71r^{13}\}.\end{aligned}$$

$\frac{2}{\sqrt{\pi}}\epsilon^{-t^2} = 0{\cdot}118\,930\,289\,223\,629\,371\,531\,02 = \mathrm{E}$; $\mathrm{H} = 0{\cdot}966\,105\,146\,475\,310\,727\,067$;

and $\Delta\mathrm{H} = \mathrm{E}r - 0{\cdot}178\,395\,433\,835\,444\,057\,297r^2 + 0{\cdot}138\,752\,004\,094\,234\,2668r^3$
$- 0{\cdot}044\,598\,858\,458\,861\,0143r^4 - 0{\cdot}014\,866\,286\,152\,953\,671r^5$
$+ 0{\cdot}019\,326\,171\,998\,839\,77r^6 - 0{\cdot}004\,743\,053\,201\,180\,46r^7 - 0{\cdot}002\,362\,677\,620\,737r^8$
$+ 0{\cdot}001\,709\,819\,551\,58r^9 - 0{\cdot}000\,092\,914\,288\,46r^{10} - 0{\cdot}000\,254\,448\,393r^{11}$
$+ 0{\cdot}000\,077\,690\,02r^{12} + 0{\cdot}000\,017\,955\,28r^{13} - 0{\cdot}000\,014\,092r^{14}$.

For $t = 1{\cdot}6$, $\epsilon^{-t^2} = {\cdot}077\,304\,740\,443\,299\,745\,990\,466$;

$\Delta\mathrm{G} = -r\epsilon^{-t^2}\{1 - 1{\cdot}6r + 1{\cdot}37\dot{3}r^2 - 0{\cdot}565\dot{3}r^3 - 0{\cdot}050\,18\dot{6}r^4 + 0{\cdot}177\,521\dot{7}r^5 - 0{\cdot}069\,203\,6\dot{0}6\,349\dot{2}r^6$
$- 0{\cdot}010\,358\,93\dot{8}\,412\,6\dot{9}r^7 + 0{\cdot}017\,139\,434\,892\,416r^8 - 0{\cdot}003\,643\,030\,1144r^9$
$- 0{\cdot}001\,744\,844\,218r^{10} + 0{\cdot}001\,017\,260\,52r^{11} - 0{\cdot}000\,004\,336r^{12}$
$- 0{\cdot}000\,133\,153\,85r^{13} + \}$.

$\frac{2}{\sqrt{\pi}}\epsilon^{-t^2} = 0{\cdot}087\,229\,058\,633\,945\,352\,846\,147 = \mathrm{E}$; $\mathrm{H} = 0{\cdot}976\,348\,383\,344\,644\,007\,77$

$\Delta\mathrm{H} = \mathrm{E}r - 0{\cdot}139\,566\,493\,814\,312\,564\,553\,836r^2 + 0{\cdot}119\,794\,573\,857\,284\,951\,242r^3$
$- 0{\cdot}049\,313\,494\,481\,057\,106\,14r^4 - 0{\cdot}004\,377\,735\,689\,308\,937\,44r^5$
$+ 0{\cdot}015\,485\,057\,562\,579\,995r^6 - 0{\cdot}006\,036\,565\,435\,915\,39r^7$
$- 0{\cdot}000\,903\,600\,446\,1867r^8 + 0{\cdot}001\,495\,056\,771\,183r^9 - 0{\cdot}000\,317\,778\,087\,45r^{10}$
$- 0{\cdot}000\,152\,201\,1186r^{11} + 0{\cdot}000\,088\,734\,678r^{12} - 0{\cdot}000\,000\,378\,2405r^{13}$
$- 0{\cdot}000\,011\,614\,885r^{14}$.

For $t = 1{\cdot}8$, $\epsilon^{-t^2} = {\cdot}039\,163\,895\,098\,987\,073\,739\,770\,994$

$\Delta\mathrm{G} = -r\epsilon^{-t^2}\{1 - 1{\cdot}8r + 1{\cdot}82\dot{6}r^2 - 1{\cdot}044r^3 + 0{\cdot}203\,68r^4 + 0{\cdot}156\,192r^5 - 0{\cdot}128\,822\,5\dot{5}2\,380\dot{9}r^6$
$+ 0{\cdot}024\,500\,434\tfrac{2}{7}r^7 + 0{\cdot}015\,248\,655\,\dot{9}15\,34\dot{3}r^8 - 0{\cdot}009\,845\,148\,89\tfrac{1}{7}r^9$
$+ 0{\cdot}000\,726\,814\,123\,775r^{10} + 0{\cdot}001\,273\,644\,989r^{11} - 0{\cdot}000\,455\,201\,117r^{12}$
$- 0{\cdot}000\,509\,014\,6956r^{13} + \text{etc.}\}$.

$\frac{2}{\sqrt{\pi}}\epsilon^{-t^2} = 0{\cdot}044\,191\,723\,332\,011\,061\,234\,953\,87 = \mathrm{E}$; $\mathrm{H} = 0{\cdot}989\,090\,501\,635\,730\,714\,19$;

$\Delta\mathrm{H} = \mathrm{E}r - 0{\cdot}079\,545\,101\,997\,619\,910\,222\,917r^2 + 0{\cdot}080\,723\,547\,953\,140\,205\,189r^3$
$- 0{\cdot}046\,136\,159\,158\,619\,547\,93r^4 + 0{\cdot}009\,000\,970\,208\,264\,013r^5$
$+ 0{\cdot}006\,902\,393\,650\,673\,472r^6 - 0{\cdot}005\,692\,890\,593\,742\,55r^7 + 0{\cdot}001\,082\,716\,413\,4684r^8$
$+ 0{\cdot}000\,673\,864\,383\,396r^9 - 0{\cdot}000\,435\,074\,095\,97r^{10} + 0{\cdot}000\,032\,119\,1687r^{11}$
$+ 0{\cdot}000\,056\,284\,567r^{12} - 0{\cdot}000\,020\,116\,12r^{13} - 0{\cdot}000\,002\,2494r^{14} + \text{etc.}$

For $t = 2$, $\epsilon^{-t^2} = 0{\cdot}018\,315\,638\,888\,734\,180\,293\,718$,

$\Delta\mathrm{G} = -r\epsilon^{-t^2}\{1 - 2r + 2{\cdot}\dot{3}r^2 - 1{\cdot}\dot{6}r^3 + 0{\cdot}6\dot{3}r^4 + 0{\cdot}0\dot{2}r^5 - 0{\cdot}1\dot{6}3\,492\dot{0}r^6 + 0{\cdot}076\,\dot{9}84\,12\dot{6}r^7$
$- 0{\cdot}002\,425\,044\,091\,71r^8 - 0{\cdot}012\dot{}716\,049\,38\dot{}r^9 + 0{\cdot}005\,020.843\,354r^{10}$
$+ 0{\cdot}000\,253\,059\,6975r^{11} - 0{\cdot}000\,785\,932\,1748r^{12} + 0{\cdot}000\,191\,191\,54r^{13} - \text{etc.}\}$.

$\frac{2}{\sqrt{\pi}}\epsilon^{-t^2} = 0{\cdot}020\,666\,985\,354\,092\,053\,857\,069 = \mathrm{E}$; $\mathrm{H} = 0{\cdot}995\,322\,265\,018\,952\,734\,1517$;

$\Delta\mathrm{H} = \mathrm{E}r - 0{\cdot}041\,333\,970\,708\,184\,107\,714\,14r^2 + 0{\cdot}048\,222\,965\,826\,214\,792\,3332r^3$
$- 0{\cdot}034\,444\,975\,590\,153\,423\,095r^4 + 0{\cdot}013\,089\,090\,724\,258\,300\,78r^5$
$+ 0{\cdot}000\,459\,266\,341\,202\,0456r^6 - 0{\cdot}003\,378\,888\,081\,700\,764r^7$
$+ 0{\cdot}001\,591\,029\,824\,878\,52r^8 - 0{\cdot}000\,050\,118\,350\,7264r^9 - 0{\cdot}000\,262\,802\,406\,3545r^{10}$
$+ 0{\cdot}000\,103\,765\,696\,06r^{11} + 0{\cdot}000\,005\,229\,9811r^{12} - 0{\cdot}000\,016\,242\,849r^{13}$
$+ 0{\cdot}000\,003\,95114r^{14} - \text{etc.}$

For $t = 2{\cdot}2$, $\epsilon^{-t^2} = {\cdot}007\,907\,054\,051\,593\,440\,493\,635\,645$,

$$\Delta G = -r\epsilon^{-t^2}\{1 - 2{\cdot}2r + 2{\cdot}893\dot{r}^2 - 2{\cdot}4493\dot{r}^3 + 1{\cdot}287\,413\dot{r}^4 - {\cdot}290\,9475\dot{r}^5 - {\cdot}123\,645\,663\,4920\dot{r}^6$$
$$+ {\cdot}130\,351\,019\,682\,53\dot{r}^7 - {\cdot}039\,684\,952\,832\,4515r^8 - {\cdot}005\,712\,135\,363\,95r^9$$
$$+ {\cdot}008\,778\,755\,518\,163r^{10} - {\cdot}002\,353\,401\,968\,184r^{11} - {\cdot}000\,441\,493\,573\,588r^{12}$$
$$+ {\cdot}000\,449\,093\,84r^{13} + \}.$$

$$\frac{2}{\sqrt{\pi}}\epsilon^{-t^2} = {\cdot}008\,922\,155\,064\,916\,204\,491\,2763 = \mathrm{E};$$

$$\Delta H = \mathrm{E}r - {\cdot}019\,628\,741\,142\,815\,649\,8808r^2 + {\cdot}025\,814\,768\,654\,490\,8850r^3$$
$$- {\cdot}021\,853\,331\,805\,668\,0902r^4 + {\cdot}011\,486\,501\,392\,640\,65r^5 - {\cdot}002\,595\,879\,206\,4449r^6$$
$$- {\cdot}001\,103\,185\,782\,780r^7 + {\cdot}001\,163\,012\,010\,47r^8 - {\cdot}000\,354\,075\,302\,915r^9$$
$$- {\cdot}000\,050\,964\,5574r^{10} + {\cdot}000\,078\,325\,418r^{11} - {\cdot}000\,020\,997\,4173r^{12}$$
$$- {\cdot}000\,003\,939\,07r^{13} + {\cdot}000\,004\,0068r^{14} + \text{etc.}$$

For $t = 2{\cdot}4$, $\epsilon^{-t^2} = 0{\cdot}003\,151\,111\,598\,444\,440\,557\,819\,11$,

$$\Delta G = -r\epsilon^{-t^2}\{1 - 2{\cdot}4r + 3{\cdot}506\dot{r}^2 - 3{\cdot}408r^3 + 2{\cdot}21968r^4 - {\cdot}866\,944r^5 + {\cdot}065\,980\,647\,6190\dot{r}^6$$
$$+ {\cdot}146\,185\,32\tfrac{4}{7}r^7 - {\cdot}090\,795\,077\,417\,989\dot{r}^8 + {\cdot}017\,593\,1348\tfrac{1}{7}r^9$$
$$+ {\cdot}007\,180\,372\,023\,41r^{10} - {\cdot}005\,537\,775\,295\,94r^{11} + {\cdot}001\,032\,100\,464\,94r^{12}$$
$$+ {\cdot}000\,376\,393\,066r^{13}\}.$$

$$\frac{2}{\sqrt{\pi}}\epsilon^{-t^2} = 0{\cdot}003\,555\,648\,680\,877\,747\,112 = \mathrm{E},$$

$$\Delta H = \mathrm{E}r - {\cdot}008\,533\,556\,834\,106\,5928r^2 + {\cdot}012\,468\,474\,707\,611\,2995r^3$$
$$- {\cdot}012\,117\,650\,704\,431\,3617r^4 + {\cdot}007\,892\,402\,268\,970\,72r^5$$
$$- {\cdot}003\,082\,548\,289\,9949r^6 + {\cdot}000\,234\,604\,002\,670r^7 + {\cdot}000\,519\,783\,660\,54r^8$$
$$- {\cdot}000\,322\,835\,397\,25r^9 + {\cdot}000\,062\,555\,0066r^{10} + {\cdot}000\,025\,530\,8798r^{11}$$
$$- {\cdot}000\,019\,690\,38r^{12} + {\cdot}000\,003\,669\,787r^{13} + {\cdot}000\,001\,338\,32r^{14} - \text{etc.}$$

For $t = 2{\cdot}5$, $\epsilon^{-t^2} = {\cdot}0{\cdot}001\,930\,454\,136\,227\,709\,242\,213\,515$;

$$\Delta G = -r\epsilon^{-t^2}\{1 - 2{\cdot}5r + 3{\cdot}83\dot{r}^2 - 3{\cdot}9583\dot{r}^3 + 2{\cdot}8083\dot{r}^4 - 1{\cdot}284\,72\dot{r}^5 + 0{\cdot}249\,007\,9365\dot{r}^6$$
$$+ {\cdot}119\,667\,658\,7301\dot{r}^7 - {\cdot}114\,900\,242\,504\,41r^8 + {\cdot}036\,175\,870\,8113r^9$$
$$+ {\cdot}002\,358\,280\,2229r^{10} - {\cdot}006\,463\,809\,307r^{11} + {\cdot}003\,355\,4256r^{12}$$
$$+ {\cdot}000\,231\,605\,1738r^{13} - \}.$$

and $\frac{2}{\sqrt{\pi}}\epsilon^{-t^2} = 0{\cdot}002\,178\,284\,230\,352\,709\,720\,3867 = \mathrm{E}$; $\mathrm{H} = 0{\cdot}999\,593\,047\,982\,555\,0361$;

$$\Delta H = \mathrm{E}r - {\cdot}005\,445\,710\,575\,881\,774\,35r^2 + {\cdot}008\,350\,089\,549\,685\,3876r^3$$
$$- {\cdot}008\,622\,375\,078\,479\,476r^4 + {\cdot}006\,117\,348\,213\,573\,86r^5 - {\cdot}002\,798\,490\,157\,0504r^6$$
$$+ {\cdot}000\,542\,410\,061\,328r^7 + {\cdot}000\,260\,670\,173\,895r^8 - {\cdot}000\,250\,285\,386\,31r^9$$
$$+ {\cdot}000\,078\,801\,328\,91r^{10} + {\cdot}000\,005\,137\,0046r^{11} - {\cdot}000\,014\,080\,014r^{12} + {\cdot}000\,007\,309\,07r^{13}$$
$$+ {\cdot}000\,000\,5044r^{14} - \text{etc.}$$

For $t = 2{\cdot}6$, $\epsilon^{-t^2} = 0{\cdot}001\,159\,229\,173\,904\,591\,150\,012$;

$$\Delta G = -r\epsilon^{-t^2}\{1 - 2{\cdot}6r + 4{\cdot}173\dot{r}^2 - 4{\cdot}5586\dot{r}^3 + 3{\cdot}489\,013\dot{r}^4 - 1{\cdot}808\,1671\dot{r}^5 + 0{\cdot}512\,492\,393\,6507\dot{r}^6$$
$$+ {\cdot}054\,344\,325\dot{5}\,079\,36r^7 - {\cdot}131\,050\,242\,144\,62r^8 + {\cdot}058\,484\,912\,567\,76r^9$$
$$- {\cdot}006\,202\,829\,135\,64r^{10} - {\cdot}010\,449\,018\,028\,06r^{11} + {\cdot}005\,053\,337\,26r^{12}$$
$$+ {\cdot}002\,601\,512\,3865r^{13} - \}.$$

and $\frac{2}{\sqrt{\pi}}\epsilon^{-t^2} = 0{\cdot}001\,308\,050\,049\,723\,251\,542\,496 = \mathrm{E}$;

$$\Delta H = \mathrm{E}r - {\cdot}003\,400\,930\,129\,280\,4540r^2 + {\cdot}005\,458\,928\,874\,178\,370r^3 - {\cdot}005\,962\,964\,160\,005\,063r^4$$
$$+ {\cdot}004\,563\,804\,064\,151\,75r^5 - {\cdot}002\,365\,173\,079\,5968r^6 + {\cdot}000\,670\,365\,700\,9977r^7$$
$$+ {\cdot}000\,071\,085\,097\,12r^8 - {\cdot}000\,171\,420\,2756r^9 + {\cdot}000\,076\,501\,193r^{10} - {\cdot}000\,008\,113\,61r^{11}$$
$$- {\cdot}000\,013\,6678r^{12} + {\cdot}000\,006\,610r^{13} + {\cdot}000\,003\,4029r^{14} - \text{etc.}$$

For $t = 2{\cdot}8$, $\epsilon^{-t^2} = 0{\cdot}000\ 393\ 669\ 040\ 655\ 078\ 210\ 9805$;

$$\Delta G = -r\epsilon^{-t^2}\{1 - 2{\cdot}8r + 4{\cdot}89\dot{3}r^2 - 5{\cdot}917\dot{3}r^3 + 5{\cdot}159\ 413\dot{3}r^4 - 3{\cdot}237\ 4968\dot{r}^5 + 1{\cdot}361\ 565\ 76\dot{5}\ 0793\dot{r}^6$$
$$- 0{\cdot}259\ 346\ 702\dot{r}^7 - {\cdot}103\ 377\ 617\ 382\ 716\ 05r^8 + {\cdot}103\ 997\ 546\ 129\ 383r^9$$
$$- {\cdot}036\ 027\ 867\ 912\ 33r^{10} + {\cdot}001\ 055\ 801r^{11} + {\cdot}004\ 625\ 1919r^{12} - {\cdot}001\ 989\ 645r^{13}, \text{ etc.}\}.$$

and $\frac{2}{\sqrt{\pi}}\epsilon^{-t^2} = 0{\cdot}000\ 444\ 207\ 944\ 205\ 666\ 629\ 3623 = \mathrm{E}$;

$$\Delta H = \mathrm{E}r - 0{\cdot}001\ 243\ 782\ 243\ 775\ 866\ 56r^2 + {\cdot}002\ 173\ 657\ 540\ 313\ 0620r^3$$
$$- {\cdot}002\ 628\ 526\ 475\ 179\ 665r^4 + {\cdot}002\ 291\ 852\ 390\ 107\ 31r^5 - {\cdot}001\ 438\ 121\ 837\ 3856r^6$$
$$+ {\cdot}000\ 604\ 818\ 329\ 407r^7 - {\cdot}000\ 115\ 203\ 865\ 431r^8 - {\cdot}000\ 045\ 921\ 158\ 89r^9$$
$$+ {\cdot}000\ 046\ 196\ 536\ 17r^{10} - {\cdot}000\ 016\ 003\ 8651r^{11} + {\cdot}000\ 000\ 468\ 995r^{12} + {\cdot}000\ 002\ 054\ 511r^{13}$$
$$- {\cdot}000\ 000\ 088\ 38r^{14} + \text{ etc.}$$

Lastly, for $t = 3$, $\epsilon^{-t^2} = 0{\cdot}000\ 123\ 409\ 804\ 086\ 679\ 549\ 4976$;

$$\Delta G = -r\epsilon^{-t^2}\{1 - 3r + 5{\cdot}\dot{6}\ r^2 - 7{\cdot}5\ r^3 + 7{\cdot}3r^4 - 5{\cdot}3r^5 + 2{\cdot}80\dot{4}\ 7619\dot{r}^6 - 0{\cdot}967\ 857\tfrac{1}{7}r^7 + {\cdot}099\ 8\dot{6}7\ 724\dot{r}^8$$
$$+ {\cdot}112\tfrac{1}{7}r^9 - {\cdot}077\ \dot{5}10\ 82\dot{2}r^{10} + {\cdot}021\ 7\dot{6}4\ 069\dot{2}r^{11} + {\cdot}000\ 8\dot{8}6\ 058\dot{3}r^{12}$$
$$- {\cdot}003\ 249\ 626\ 464\ 0122r^{13} + \text{ etc.}\}.$$

and $\frac{2}{\sqrt{\pi}}\epsilon^{-t^2} = 0{\cdot}000\ 139\ 253\ 051\ 946\ 747\ 853\ 89 = \mathrm{E}$; $\mathrm{H} = 0{\cdot}999\ 977\ 909\ 503\ 001\ 4145$;

$$\Delta H = \mathrm{E}r - {\cdot}000\ 417\ 759\ 155\ 840\ 243\ 561\ 67r^2 + {\cdot}000\ 789\ 100\ 627\ 698\ 237\ 8386r^3$$
$$- {\cdot}001\ 044\ 397\ 889\ 600\ 608\ 904r^4 + {\cdot}001\ 016\ 547\ 279\ 211\ 259\ 33r^5$$
$$- {\cdot}000\ 738\ 041\ 175\ 317\ 7636r^6 + {\cdot}000\ 390\ 571\ 655\ 222\ 069r^7 - {\cdot}000\ 134\ 777\ 060\ 991\ 32r^8$$
$$+ {\cdot}000\ 013\ 906\ 885\ 4788r^9 + {\cdot}000\ 015\ 616\ 235\ 111r^{10} - {\cdot}000\ 010\ 793\ 618\ 59r^{11}$$
$$+ {\cdot}000\ 003\ 030\ 713\ 07r^{12} + {\cdot}000\ 000\ 123\ 386r^{13} - {\cdot}000\ 000\ 452\ 53r^{14} + \text{ etc.}$$

These data will enable anyone to verify the table, and also to recompute to the like degree of accuracy KRAMP's first Table of the values of G. Any value of G may also be found for verification by multiplying $1 - \mathrm{H}$ by $\frac{1}{2}\sqrt{\pi}$.

The constant ρ and its derivatives.

20. The value of $t = \rho$, in the solution of the equation—

$$t - \frac{t^3}{3} + \frac{1}{1.2}\frac{t^5}{5} + \frac{1}{1.2.3}\frac{t^7}{7} - \text{ etc.} = \frac{\sqrt{\pi}}{4}; \text{ or } \frac{2}{\sqrt{\pi}}\left(t - \frac{t^3}{3} + \frac{t^5}{2\,5} - \frac{t^7}{3!7} + \right) = \frac{1}{2} \qquad (37)$$

is of importance, as it enters into the coefficients of various formulæ. BESSEL employed the value 0·476 9364, ENCKE, followed by DE MORGAN, uses 0·476 9360, and AIRY gives 0·476 948.* To obtain this value with extreme accuracy, we may proceed thus: Since $0{\cdot}475 = \frac{1}{2}\left(1 - \frac{1}{2\,0}\right)$ and $0{\cdot}475^2 = \frac{1}{5} + \frac{1}{5.8} + \frac{1}{40.40}$, the computation of the series for this value is comparatively easy, and gives—for $t = {\cdot}475$—

* It seems strange that the late Astronomer Royal, so late as 1861, should have adopted a value differing from that so generally recognised as correct at least to six decimal figures; he gives its reciprocal also as 2·096 665 (*Theory of Errors*, pp. 23, 24). LAPLACE (*Theorie Anal. des Probabilités*, 2e ed., p. 238), in one of the very few examples he gives, makes $t^2 = {\cdot}210\ 2497$, which would give $\rho = {\cdot}45853$. M. POISSON, also (*Connaissance des Temps*, 1832, Add. p. 20), gives ·47414 for the value of ρ, and ·67336 for that of $\rho\sqrt{\pi}$, and again (*Rech. sur la Prob. des Jugements*, p. 208), he has ·4765 and ·6739 for the same quantities. GAUSS (*Werke*, Bd. iv, S. 110) gave the value as ·476 9363, which is correct to the nearest figure in the seventh place. Lastly, O. BYRNE (*Dual Arithmetic*, p. 200) finds 0·476 936 2744, which errs only in the last two decimal figures.

$$\frac{2}{\sqrt{\pi}}\int^{t} \epsilon^{-t^2}dt = 0{\cdot}498\,258\,053\,711\,787\,564\,127\,43 = \mathrm{H}.$$

For the same value of t, we have—

$$\frac{2}{\sqrt{\pi}}\,\epsilon^{-t^2} = 0{\cdot}900\,466\,098\,615\,398\,685\,314\,176 = e\,,$$

and KRAMP'S formula (36) for a difference r, becomes—

$$\begin{aligned}\tfrac{1}{2}-\mathrm{H} &= er(1-0{\cdot}475r-0{\cdot}182\,916\dot{r}^2+0{\cdot}201\,776\,0416\dot{r}^3+0{\cdot}016\,537\,552r^4\\ &\quad -0{\cdot}056\,42539r^5+0{\cdot}00372r^6+\text{etc.})\\ &= 0{\cdot}900\,466\,098\,615\,398\,685\,314\,176r-0{\cdot}427\,721\,396\,842\,314\,8755r^2\\ &\quad -0{\cdot}164\,710\,257\,205\,0667r^3+0{\cdot}181\,692\,485\,0336r^4+0{\cdot}014\,891\,505r^5\\ &\quad -0{\cdot}050\,809r^6+0{\cdot}00335r^7+\text{etc.}\end{aligned}$$

Using the first three terms, and taking $r = {\cdot}001\,936$ as a first approximation, we obtain $\mathrm{H}'-\mathrm{H} = +{\cdot}001\,741\,699\,03$, etc. But $\frac{1}{2}-\mathrm{H} = {\cdot}001\,741\,946\,29$, and the difference of these is $\frac{1}{2}-\mathrm{H}' = +{\cdot}000\,000\,247\,26$. The value of $\frac{2}{\sqrt{\pi}}\epsilon^{-t^2}$ for $t = 0{\cdot}476\,936$ is $0{\cdot}898\,80814$. Hence the correction is $\frac{{\cdot}002\,4726}{8988} = +0{\cdot}000\,000\,275$, and the new value of ρ is $0{\cdot}476\,936 + {\cdot}000\,000\,275 = {\cdot}476\,936\,275$; and from this,—taking in the higher powers of r,—we readily arrive at the value, correct to the twenty-fourth place of decimals, viz. :—

$$\rho = 0{\cdot}476\,936\,276\,204\,469\,873\,383\,506.$$

Otherwise, we may form a difference-formula for the computation of this and other values of t corresponding to definite values of H. Thus let H be the tabular or computed value corresponding to t, and H′ the value for which $t' = t + \Delta t$ is sought. Put $h = \frac{1}{2}\sqrt{\pi}(\mathrm{H}'-\mathrm{H})\epsilon^{-t^2}$. Then—

$$\Delta t = h\left(1+ht+\frac{4t^2+1}{3}h^2+\frac{12t^3+7t}{6}h^3+\frac{96t^4+92t^2+7}{2.3.5}h^4+\frac{480t^5+652t^3+127t}{3.5.6}h^5+\text{etc.}\right). \qquad (38)$$

Using the above value of H for $t = {\cdot}475$, we find $h = {\cdot}001\,934\,494\,025\,806\,1229$, and this series becomes—

$$\Delta t = h(1+{\cdot}475h+{\cdot}634\,16\dot{h}^2+{\cdot}768\,510\,416\dot{h}^3+1{\cdot}088\,151\,25h^4+1{\cdot}575\,641\,961\,805\dot{h}^5+\ldots),$$

and this gives at once the value of ρ correctly to seventeen figures. When h is very small, the first three terms of (38) will usually be sufficient to determine the values of t corresponding to $\mathrm{H} = 0{\cdot}1$, $0{\cdot}2$, $0{\cdot}3$, etc., as given below, § 23.

21. The following table contains the values of the factors dependent on this constant, ρ, together with some others used in Probabilities,* with their logarithms, computed to a degree of accuracy far beyond what can be required.

* These constants will be met with, among other places, in BESSEL'S *Fundamenta Astron.*, p. 18 ; and *Ueber d. Bahn des Olberschen Kometen*, in *Abh. d. Math. Kl. d. Königl. Preuss. Akad.*, 1812–13, S. 142 ; DE MORGAN'S *Theory of Probab.*, §§ 68, 100, 116, 150, 152, etc. ; ENCKE, in *Berl. Ast. Jahrb.*, 1834, Ss. 270, 293, 298 ; GAUSS, *Werke*, Bd. iv, S. 6 ; AIRY, *Theory of Errors*, pp. 23, 24 ; POISSON, *Rech. sur la Probab. des Jugements*, p. 176, etc.

TABLE.

	Constants.	Values of Constants.	Logarithms.
1	ρ	0·476 936 276 204 469 873 383 51	$\bar{1}$·678 460 356 521 217 913 230 78
2	$\frac{1}{\rho}$	2·096 716 165 015 061 071 615 78	0·321 539 643 478 782 086 769 22
3	ρ^2	0·227 468 211 559 786 375 973 25	$\bar{1}$·356 920 713 042 435 826 461 56
4	$\rho\sqrt{2}$	0·674 489 750 196 035 151 103 81	$\bar{1}$·828 975 354 353 208 510 837 65
5	$2\rho\sqrt{\pi}$	1·690 695 078 790 009 806 981 30	0·228 065 288 532 266 035 620 15
6	$\frac{2}{\sqrt{\pi}}\rho$	0·538 164 958 101 235 048 729 82	$\bar{1}$·730 915 415 838 132 181 268 88
7	$\frac{2}{\sqrt{\pi}}\rho^2$	0·256 670 391 159 638 137 627 19	$\bar{1}$·409 375 772 459 350 094 499 66
8	$\frac{1}{\rho\sqrt{\pi}}$	1·182 945 419 957 695 955 821 42	0·072 964 707 131 715 159 593 59
9	$\frac{1}{2\rho\sqrt{\pi}}$	0·591 472 709 978 847 977 910 71	$\bar{1}$·771 934 711 467 733 964 379 85
10	$\rho\sqrt{\pi}$	0·845 347 539 395 004 903 490 65	$\bar{1}$·927 035 292 868 284 840 406 41
11	$\frac{1}{\rho\sqrt{2}}$	1·482 602 218 505 601 860 540 58	0·171 024 645 646 791 489 162 35
12	$\rho\sqrt[6]{\pi}$	0·577 189 827 811 086 284 473 01	$\bar{1}$·761 318 668 636 906 888 955 99
13	$\rho\sqrt[10]{\frac{\pi}{4}}$	0·465 553 230 574 244 418 753 06	$\bar{1}$·667 969 344 657 835 059 623 16
14	$\rho\sqrt{\pi-2}$	0·509 584 182 684 138 078 029 73	$\bar{1}$·707 215 939 186 776 502 110 25
15	$\rho\sqrt{\frac{15\pi-8}{36}}$	0·497 198 854 778 314 121 494 65	$\bar{1}$·696 530 119 639 696 588 914 00
16	$\rho\sqrt{\frac{945\pi-128}{1600}}$	0·635 508 087 011 832 529 750 44	$\bar{1}$·803 121 081 439 621 574 379 57
17	$\rho\sqrt{\frac{4}{3}}$	0·550 718 574 905 896 772 795 56	$\bar{1}$·740 929 724 825 367 889 797 00
18	$\rho\sqrt[4]{\frac{4}{3}}$	0·512 501 381 805 211 150 143 34	$\bar{1}$·709 695 040 673 292 901 513 89
19	$\rho\sqrt{\frac{113}{45}}$	0·755 776 391 184 821 580 506 05	$\bar{1}$·878 393 321 375 255 934 940 23
20	$\rho\sqrt[6]{\frac{8}{15}}$	0·429 497 009 734 013 564 961 27	$\bar{1}$·632 960 144 510 594 970 490 77
21	ϵ^{ρ^2}	1·255 417 531 354 680 356 016 89	0·098 788 189 088 816 702 448 09
22	$\epsilon^{\rho^2}\sqrt{\pi}$	2·225 169 637 943 592 189 588 00	0·347 363 125 435 823 629 623 72
23	$\epsilon^{-\rho^2}$	0·796 547 742 105 315 688 192 06	$\bar{1}$·901 211 810 911 183 297 551 91
24	$\frac{2}{\sqrt{\pi}}\epsilon^{-\rho^2}$	0·898 807 877 788 607 267 593 84	$\bar{1}$·953 666 870 228 097 565 590 02
25	$\frac{\epsilon}{2\sqrt{2}}$	0·961 057 757 039 779 206 215 42	$\bar{1}$·982 749 488 407 280 034 830 52
26	$\frac{\epsilon}{\sqrt{2\pi}}$	1·084 437 551 419 227 546 611 58	0·035 204 547 724 194 302 868 63
27	$\sqrt{\frac{\pi}{2}}$	1·253 314 137 315 500 215 207 88	0·098 059 938 515 076 329 568 76
28	$\sqrt{\frac{2}{\pi}}$	0·797 884 560 802 865 355 879 89	$\bar{1}$·901 940 061 484 923 670 431 24
29	$\frac{1}{\sqrt{\pi}}$	0·564 189 583 547 756 286 948 08	$\bar{1}$·751 425 063 652 933 072 824 37
30	$\frac{2}{\sqrt{\pi}}$	1·128 379 167 095 512 573 896 16	0·052 455 059 316 914 268 038 10
31	$\frac{1}{2}\sqrt{\pi}$	0·886 226 925 452 758 013 649 08	$\bar{1}$·947 544 940 683 085 731 961 90

22. In the theory of Errors of Observations, we may state the proportions of the different constants for 'modulus,' 'mean error,' 'error of mean square,' and 'probable error,' as in the adjoining table.* And the ordinary relations of 'mean' or average error, A (double the mean risk); weight of an observation, or square of the number of observations divided by twice the sum of the squares of the errors, W; modulus, M; the error of mean square, S; and probable error, E,—are expressed by the equations,—

	Modulus.	Mean Error.	Error of Mean Square.	Probable Error.
In terms of modulus . .	1	$\frac{1}{\sqrt{\pi}}$	$\frac{1}{\sqrt{2}}$	ρ
In terms of mean error . .	$\sqrt{\pi}$	1	$\sqrt{\frac{\pi}{2}}$	$\rho\sqrt{\pi}$
In terms of error of mean square	$\sqrt{2}$	$\sqrt{\frac{2}{\pi}}$	1	$\rho\sqrt{2}$
In terms of probable error .	$\frac{1}{\rho}$	$\frac{1}{\rho\sqrt{\pi}}$	$\frac{1}{\rho\sqrt{2}}$	1

$$\left.\begin{aligned} &M = A\sqrt{\pi} = S\sqrt{2} = \frac{E}{\rho} = \frac{1}{\sqrt{W}}. \qquad A = \frac{M}{\sqrt{\pi}} = S\sqrt{\frac{2}{\pi}} = \frac{E}{\rho\sqrt{\pi}} = \frac{1}{\sqrt{\pi W}}. \\ &E = M\rho = A\rho\sqrt{\pi} = S\rho\sqrt{2} = \frac{\rho}{W}. \qquad S = \frac{M}{\sqrt{2}} = A\sqrt{\frac{\pi}{2}} = \frac{E}{\rho\sqrt{2}} = \frac{1}{\sqrt{2W}}, \\ &\text{and } W = \frac{1}{M^2} = \frac{1}{\pi A^2} = \frac{1}{2S^2} = \frac{\rho^2}{E^2}. \end{aligned}\right\} (39)$$

The values of the constants are found in the table above; but for approximations, that are occasionally useful, the following may be given :—

For $\sqrt{\pi}$ we may use $\frac{296}{167} = 1{\cdot}772\,455$, or roughly $\frac{709}{400}$;

for $\sqrt{\frac{2}{\pi}}$ „ „ $\frac{679}{851} = 0{\cdot}797\,8848$, or $\frac{379}{475} = {\cdot}797\,895$, or $\frac{75}{94} = {\cdot}797\,87$, or $\frac{399}{500}$;

for $\sqrt{2}$ „ „ $\frac{577}{408} = 1{\cdot}414\,2156$, or $\frac{239}{169} = 1{\cdot}414\,201$, or $\frac{99}{70} = 1{\cdot}414\,286$;

for ρ „ „ $\frac{455}{954} = 0{\cdot}476\,939$, or $\frac{300}{629} = {\cdot}476\,9475$, or $\frac{31}{65} = {\cdot}476\,923$;

for $\rho\sqrt{2}$ „ „ $\frac{661}{980} = 0{\cdot}674\,4898$, or $\frac{201}{298} = {\cdot}674\,497$, or $\frac{29}{43} = {\cdot}674\,45$;

for $\rho\sqrt{\pi}$ „ „ $\frac{645}{763} = 0{\cdot}845\,3472$, or $\frac{82}{97} = {\cdot}845\,36$, or $\frac{71}{84} = {\cdot}845\,24$;

and for ρ^2 „ „ $\frac{53}{233} = 0{\cdot}227\,468$, or $\frac{48}{211} = {\cdot}227\,488$, or $\frac{5}{22} = {\cdot}227\,27$;

Whence,— $M = \frac{296}{167}A = \frac{239}{169}S$, or $\frac{99}{70}S = \frac{629}{300}E$, or $\frac{65}{31}E = \frac{1}{\sqrt{W}}$.

* Conf. AIRY's *Theory of Errors*, p. 24; Galloway's *Treat. on Probability*, §§ 145–148, pp. 194–197; De Morgan's *Essay*. p. 139.

$$A=\frac{167}{296}M=\frac{679}{851}S, \text{ or } \frac{75}{94}S=\frac{763}{645}E, \text{ or } \frac{97}{82}E=\frac{167}{296\sqrt{W}}.$$

$$E=\frac{455}{954}M, \text{ or } \frac{31}{65}M=\frac{645}{763}A, \text{ or } \frac{82}{97}A=\frac{661}{980}S=\frac{300}{629\sqrt{W}} \text{ or } \frac{31}{65\sqrt{W}}.$$

$$S=\frac{408}{577}M, \text{ or } \frac{70}{99}M=\frac{851}{679}A, \text{ or } \frac{94}{75}A=\frac{298}{201}E=\frac{169}{239\sqrt{W}}.$$

$$\text{and } W=\frac{1}{M^2}=\frac{113}{355A^2} \text{ or } \frac{7}{22A^2}=\frac{1}{2S^2}=\frac{53}{233E^2} \text{ or } \frac{5}{22E^2}.$$

23. Besides ρ, other values of t corresponding to certain definite values of H may occasionally be required,* and the extent of the table now given will enable us to determine them with a high degree of accuracy by simple interpolation †; thus :—

For H = 0·1,	t = 0·088 885 991	.. log $\bar{2}$·948 832 9230	... 0·186 367 523.ρ
0·2,	0·179 143 455	$\bar{1}$·253 200 9459	0·375 512 978.ρ
0·3,	0·272 462 716	$\bar{1}$·435 303 8936	0·571 272 788.ρ
0·4,	0·370 807 149	$\bar{1}$·569 148 0986	0·777 377 028.ρ
0·5,	0·476 936 276	$\bar{1}$·678 460 3565	1·000 000 000.ρ
0·6,	0·595 116 079	$\bar{1}$·744 601 6843	1·247 789 503.ρ
0·7,	0·732 869 079	$\bar{1}$·865 026 3985	1·536 618 445.ρ
0·8,	0·906 193 802	$\bar{1}$·957 221 0875	1·900 031 193.ρ
0·9,	1·163 087 153	0·065 612 2587	2·438 663 635.ρ
1·0,	∞	∞	∞

Construction of the Table.

24. In both divisions of the general integral the factor ϵ^{-t^2} forms a multiplier. Assistance in obtaining the values of this factor might have been derived from the extensive tables of ϵ^{-x} by Prof. F. W. NEWMAN and Mr GLAISHER,‡ had they been in existence when the following table was begun. But the interpolation for values of ϵ^{-t^2}, by means of the formula—

$$\epsilon^{-x\pm h}=\epsilon^{-x}\left\{1\pm\frac{h}{1}+\frac{h^2}{1.2}\pm\frac{h^3}{1.2.3}+\text{etc.}\right\} \tag{40}$$

is somewhat laborious, since h in this case has the form of $2xh+h^2$. As the factor in the function H is the multiple $\frac{2}{\sqrt{\pi}}\epsilon^{-t^2}$, it is occasionally convenient to find its value logarithmically, and also as part of the computation of the value of the function, the former proving a check on the working for the latter.§ In the first part of the

* GAUSS, *Bestimm. d. Genauigkeit d. Beobacht.*, § 2 ; *Werke*, Bd. iv, S. 110.

† Or, the difference formula (38), given above, § 20, may be used to find these values.

‡ *Trans. Camb. Phil. Soc.*, vol. xiii, (1883), pp. 145-272.

§ If we compute in succession, as is naturally the easiest method, the terms of the expression—

$$\frac{2}{\sqrt{\pi}}(1+t-t^2-t^3+\frac{t^4}{2}+\frac{t^5}{2}-\frac{t^6}{3!}-\frac{t^7}{3!}+\frac{t^8}{4!}+\frac{t^9}{4!}-\text{etc.})$$

the sum of the 1st, 3rd, 5th, 7th, etc., terms will give the value of $\frac{2}{\sqrt{\pi}}\epsilon^{-t^2}$, whilst the sum of the quotients of the 2nd, 4th, 6th, 8th, etc., terms, divided respectively by 1, 3, 5, 7, etc., will give the value of H.

tables the values of this factor are given for every value of t, and, at larger intervals, from $t=1{\cdot}25$ to $t=6{\cdot}0$ (on p. 295). The following values were also computed with extreme accuracy :—

x.	ϵ^{-x^2}.	$\frac{2}{\sqrt{\pi}}\epsilon^{-x^2}$.
0	1·000	1·128 379 167 095 512 573 896 158 903
$\frac{1}{2}$	·606 530 659 712 633 423 603 799 534 990	0·684 396 560 624 433 066 358 502 37
1	·367 879 441 171 442 321 595 523 770 161	0·415 107 497 420 594 703 340 268 249
2	·135 335 283 236 612 691 893 999 494 972	0·152 709 514 177 164 314 421 873 367
3	·049 787 068 367 863 942 979 342 415 650	0·056 178 690 737 057 656 594 924 613
4	·018 315 638 888 734 180 293 718 021 273	0·020 666 985 354 092 053 857 068 941
5	·006 737 946 999 085 467 096 636 048 423	0·007 602 959 022 761 767 784 966 646
6	·002 478 752 176 666 358 423 045 167 431	0·002 796 972 316 542 974 354 763 250
7	·000 911 881 965 554 516 208 003 136 084	0·001 028 948 612 781 823 885 494 178
8	·000 335 462 627 902 511 838 821 389 126	0·000 378 529 040 664 308 164 933 856
9	·000 123 409 804 086 679 549 497 636 691	0·000 139 253 051 946 747 853 890 418
10	·000 045 399 929 762 484 851 535 591 516	0·000 051 228 334 931 587 428 772 169

25. The first part of the table contains the values of H from $t=0$ to $t=1{\cdot}250$, at intervals of ·001 to nine places of decimals, together with the first and second differences, and the corresponding values of $\frac{2}{\sqrt{\pi}}\epsilon^{-t^2}$.* These values were computed in 1862, by using the general series for intervals of ·02, and interpolating for the intermediate values with six or more orders of differences. The second part contains the values from $t=1{\cdot}00$ to $t=3{\cdot}00$, computed recently, to fifteen decimal figures,—(1) from $t=1{\cdot}000$ to $t=1{\cdot}500$ at intervals of ·001, and (2) from $t=1{\cdot}5$ to $t=3{\cdot}0$ at intervals of ·002, with four orders of differences. In the last column of this portion of the table are given the corresponding values of $\log \frac{2}{\sqrt{\pi}}\epsilon^{-t^2}+10$, to sixteen decimal places. And (3) lastly, values of H and G from 3·0 to 6·0 are appended,† computed by means of LAPLACE's fraction (§ 9),—the values of L (16) being preserved. These would enable us to extend the general table still farther if required.

* The differences of these values have been omitted from want of room on the page. The differences given throughout the tables are stated to the nearest figure in the last place, being taken from the computations.

† Mr J. W. L. GLAISHER's table (referred to above, § 4) of the values of G from $t=3{\cdot}00$ to 4·50 (*Phil. Mag.*, 4th ser., 1871, vol. xlii. p. 436) is computed for differences of 0·01 and to seven significant figures, that is from eleven to fourteen decimal places; the appended table gives the values computed to fifteen places. But the values of L would enable us to carry them to a much larger number of figures.

TABLE OF THE VALUES OF $H = \frac{2}{\sqrt{\pi}}\int_0^t \epsilon^{-t^2} dt$. (1) FROM $t=0$ TO $t=1{\cdot}250$.

$t = 0{\cdot}0$] [to ·099

t	H	Δ +	Δ_2 −	$\frac{2}{\sqrt{\pi}}\epsilon^{-t^2}$	t	H	Δ +	Δ_2 −	$\frac{2}{\sqrt{\pi}}\epsilon^{-t^2}$
0·000	·000 000 000		00	1·128 379 167	0·050	·056 371 978		113	1·125 561 742
1	001 128 379	1 128 379	02	378 039	51	057 497 483	1 125 505	15	448 066
2	002 256 755	377	05	374 654	52	058 622 873	390	17	332 151
3	003 385 127	372	07	369 012	53	059 748 146	273	19	213 998
4	004 513 493	365	09	361 113	54	060 873 300	154	22	093 606
0·005	·005 641 849	1 128 356	11	1·128 350 958	0·055	·061 998 209	1 125 032	124	1·124 970 978
6	006 770 194	345	14	338 546	56	063 123 242	1 124 909	26	846 113
7	007 898 525	331	16	323 878	57	064 248 024	783	28	719 012
8	009 026 841	316	18	306 953	58	065 372 679	655	30	589 677
9	010 155 138	298	20	287 772	59	066 497 203	524	33	458 108
0·010	·011 283 416	1 128 277	23	1·128 266 335	0·060	·067 621 594	1 124 391	135	1·124 324 305
11	012 411 670	255	25	242 641	61	068 745 851	256	37	188 270
12	013 539 900	230	27	216 692	62	069 869 970	119	39	050 004
13	014 668 103	203	29	188 487	63	070 993 950	1 123 980	42	1·123 909 506
14	015 796 276	173	32	158 026	64	072 117 788	838	44	766 779
0·015	·017 924 418	1 128 142	34	1·128 125 310	0·065	·073 241 483	1 123 694	146	1·123 621 822
16	018 052 526	108	36	090 339	66	074 365 031	548	48	474 637
17	019 180 598	072	38	053 126	67	075 488 431	400	51	325 225
18	020 308 632	034	41	013 631	68	076 611 681	250	53	173 586
19	021 436 625	1 127 993	43	1·127 971 896	69	077 734 778	097	55	019 722
0·020	·022 564 575	1 127 950	45	1·127 927 906	0·070	·078 857 720	1 122 942	157	1·122 863 633
21	023 692 480	905	47	881 662	71	079 980 504	785	59	705 321
22	024 820 337	858	50	833 164	72	081 103 130	625	62	544 785
23	025 948 145	808	52	782 412	73	082 225 593	464	64	382 028
24	027 075 901	756	54	729 408	74	083 347 893	300	66	217 050
0·025	·028 203 603	1 127 702	56	1·127 674 150	0·075	·084 470 027	1 122 134	168	1·122 049 852
26	029 331 249	646	59	616 641	76	085 591 992	1 121 965	71	1·121 880 435
27	030 458 836	587	61	556 878	77	086 713 787	795	73	708 801
28	031 586 362	526	63	494 865	78	087 835 409	622	75	534 949
29	032 713 825	463	65	430 599	79	088 956 856	447	77	358 882
0·030	·033 841 222	1 127 398	68	1·127 364 083	0·080	·090 078 126	1 121 270	179	1·121 180 600
31	034 968 552	330	70	295 316	81	091 199 216	091	82	000 105
32	036 095 812	260	72	224 298	82	092 320 125	1 120 909	84	1·120 817 397
33	037 223 000	188	74	151 031	83	093 440 850	725	86	632 477
34	038 350 114	113	77	075 514	84	094 561 390	539	88	445 347
0·035	·039 477 150	1 127 037	79	1·126 997 749	0·085	·095 681 740	1 120 351	190	1·120 256 008
36	040 604 108	1 126 958	81	917 735	86	096 801 901	160	93	064 460
37	041 730 985	877	83	835 473	87	097 921 869	1 119 968	95	1·119 870 706
38	042 857 778	793	86	750 963	88	099 041 641	773	97	674 746
39	043 984 486	708	88	664 207	89	100 161 217	576	99	476 581
0·040	·045 111 106	1 126 620	90	1·126 575 204	0·090	·101 280 594	1 119 377	201	1·119 276 213
41	046 237 636	530	92	483 955	91	102 399 769	175	04	073 642
42	047 364 073	437	95	390 461	92	103 518 740	1 118 971	06	1·118 868 870
43	048 490 416	343	97	294 722	93	104 637 506	766	08	661 899
44	049 616 662	246	99	196 738	94	105 756 064	557	10	452 728
0·045	·050 742 809	1 126 147	101	1·126 096 511	0·095	·106 874 411	1 118 347	212	1·118 241 361
46	051 868 854	045	104	1·125 994 041	96	107 992 546	135	15	027 797
47	052 994 796	1 125 942	106	889 329	97	109 110 466	1 117 920	17	1·117 812 039
48	054 120 632	836	108	782 374	98	110 228 169	703	19	594 086
49	055 246 360	728	110	673 179	99	111 345 653	484	21	373 942
		1 125 618					1 117 263		

TABLE OF THE VALUES OF $H = \frac{2}{\sqrt{\pi}}\int_0^t \epsilon^{-t^2} dt$. (1) FROM $t=0$ TO $t=1{\cdot}250$.

t	H	Δ +	Δ_2 −	$\frac{2}{\sqrt{\pi}}\epsilon^{-t^2}$	t	H	Δ +	Δ_2 −	$\frac{2}{\sqrt{\pi}}\epsilon^{-t^2}$
0·100	·112 462 916		223	1·117 151 607	0·150	·167 995 971		331	1·103 274 127
		1 117 040					1 103 108		
1	113 579 956	6 814	26	6 927 082	51	169 099 080	2 775	33	2 942 091
2	114 696 769	6 586	28	6 700 369	52	170 201 855	2 440	35	2 607 950
3	115 813 356	6 356	30	6 471 469	53	171 304 295	2 103	37	2 271 706
4	116 929 712		32	6 240 383	54	172 406 398		39	1 933 361
		1 116 124					1 101 763		
0·105	·118 045 836	5 890	234	1·116 007 113	0·155	·173 508 161	1 422	341	1·081 592 916
6	119 161 725	5 653	37	5 771 660	56	174 609 583	1 078	44	1 250 374
7	120 277 378	5 414	39	5 534 026	57	175 710 661	0 733	46	1·080 905 736
8	121 392 792	5 173	41	5 294 212	58	176 811 394	0 385	48	0 559 006
9	122 507 966		43	5 052 220	59	177 911 778		50	0 210 184
		1 114 930					1 100 035		
0·010	·123 622 896	4 685	245	1·114 808 050	0·160	·179 011 813	1 099 683	352	1·099 850 273
11	124 737 581	4 438	47	4 561 705	61	180 111 496	9 329	54	9 506 274
12	125 852 019	4 188	50	4 313 185	62	181 210 825	8 973	56	9 151 191
13	126 966 207	3 936	52	4 062 493	63	182 309 798	8 615	58	8 794 025
14	128 080 143		54	3 809 629	64	183 408 412		60	8 434 778
		1 113 682					1 098 254		
0·015	·129 193 825	3 426	256	1·113 554 596	0·165	·184 506 667	7 892	362	1·098 073 453
16	130 307 252	3 168	58	3 297 395	66	185 604 559	7 527	64	7 710 051
17	131 420 420	2 907	60	3 038 027	67	186 702 086	7 161	67	7 344 574
18	132 533 327	2 645	63	2 776 493	68	187 799 247	6 792	69	6 977 025
19	133 645 972		65	2 512 797	69	188 896 039		71	6 607 406
		1 112 380					1 096 422		
0·120	·134 758 352	2 113	267	1·112 246 938	0·170	·189 992 461	6 049	373	1·096 235 719
21	135 870 465	1 844	69	1 978 919	71	191 088 510	5 674	75	5 861 967
22	136 982 309	1 573	71	1 708 741	72	192 184 184	5 297	77	5 486 150
23	138 093 882	1 299	73	1 436 405	73	193 279 482	4 918	79	5 108 273
24	139 205 181		76	1 161 914	74	194 374 400		81	4 728 336
		1 111 024					1 094 538		
0·125	·140 316 205	0 746	278	1·110 885 270	0·175	·195 468 938	4 154	383	1·094 346 343
26	141 426 951	0 466	80	0 606 472	76	196 563 092	3 769	85	3 962 294
27	142 537 417	0 184	82	0 325 524	77	197 656 862	3 382	87	3 576 194
28	143 647 601		84	0 042 428	78	198 750 244	2 993	89	3 188 043
29	144 757 501	1 109 900	86	1·109 757 183	79	199 843 237		91	2 797 845
		1 109 614					1 092 602		
0·130	·145 867 115	9 325	288	1·109 469 793	0·180	·200 935 839	2 209	393	1·092 405 601
31	146 976 440	9 035	91	9 180 260	81	202 028 048	1 813	95	2 011 314
32	148 085 475	8 742	93	8 888 584	82	203 119 861	1 416	97	1 614 985
33	149 194 216	8 447	95	8 594 767	83	204 211 277	1 017	99	1 216 619
34	150 302 663		97	8 298 812	84	205 302 294		401	0 816 216
		1 108 150					1 090 615		
0·135	·151 410 813	1 107 851	299	1·108 000 719	0·185	·206 392 909	0212	403	1·090 413 779
36	152 518 664	7 549	301	7 700 492	86	207 483 120	1 089 806	05	0 009 310
37	153 626 214	7 246	03	7 398 131	87	208 572 927	9 399	08	1·089 602 812
38	154 733 460	6 941	06	7 093 638	88	209 662 325	8 989	10	9 194 388
39	155 840 400		08	6 787 016	89	210 751 315		12	8 783 739
		1 106 633					1 088 578		
0·140	·156 947 033	6 323	310	1·106 478 265	0·190	·211 839 892	8 164	414	1·088 371 168
41	158 053 356	6 011	12	6 167 389	91	212 928 056	7 748	16	7 956 578
42	159 159 367	5 697	14	5 854 388	92	214 015 805	7 331	18	7 539 970
43	160 265 064	5 381	16	5 539 264	93	215 103 135	6 911	20	7 121 348
44	161 370 445		18	5 222 020	94	216 190 047		22	6 700 713
		1 105 062					1 086 490		
0·145	·162 475 507	4 742	320	1·104 902 657	0·195	·217 276 536	6 066	424	1·086 278 069
46	163 580 250	4 420	23	4 581 177	96	218 362 602	5 640	26	5 853 417
47	164 684 669	4 095	25	4 257 582	97	219 448 242	5 213	28	5 426 761
48	165 788 764	3 768	27	3 931 874	98	220 533 455	4 783	30	4 998 102
49	166 892 532	1 103 439	29	3 604 055	99	221 618 238	1 084 351	32	4 567 443

TABLE OF THE VALUES OF $H = \frac{2}{\sqrt{\pi}}\int_0^t \epsilon^{-t^2} dt$. (1) FROM $t=0$ TO $t=1{\cdot}250$.

t	H	Δ +	Δ_2 −	$\frac{2}{\sqrt{\pi}}\epsilon^{-t^2}$	t	H	Δ +	Δ_2 −	$\frac{2}{\sqrt{\pi}}\epsilon^{-t^2}$
0·200	·222 702 589			1·084 134 787	0·250	·276 326 390			1·060 014 129
		1 083 918	434				1 059 749	530	
1	223 786 507	3 482	36	3 700 136	51	277 386 139	9 217	32	1·059 483 195
2	224 869 989	3 044	38	3 263 493	52	278 445 356	8 683	34	8 950 409
3	225 953 033	2 605	40	2 824 860	53	279 504 039	8 148	36	8 415 774
4	227 035 638		42	2 384 240	54	280 562 187		37	7 879 294
		1 082 163					1 057 610		
0·205	·228 117 801	1 720	444	1·081 941 635	0·255	·281 619 797	7 071	539	1·057 340 970
6	229 199 520	1 274	46	1 497 049	56	282 676 868	6 530	41	6 800 807
7	230 280 794	0 826	48	1 050 483	57	283 733 398	5 987	43	6 258 807
8	231 361 621	0 377	50	0 601 940	58	284 789 385	5 442	45	5 714 974
9	232 441 998		52	0 151 423	59	285 844 827		47	5 169 310
		1 079 925					1 054 896		
0·210	·233 521 923	9 472	453	1·079 698 934	0·260	·286 899 723	4 347	548	1·054 621 819
11	234 601 395	9 016	55	9 244 477	61	287 954 071	3 797	50	4 072 505
12	235 680 411	8 559	57	8 788 053	62	289 007 868	3 245	52	3 521 369
13	236 758 970	8 100	59	8 329 665	63	290 061 113	2 691	54	2 968 415
14	237 837 070		61	7 869 317	64	291 113 804		56	2 413 647
		1 077 638					1 052 136		
0·215	·238 914 708	7 175	463	1·077 407 010	0·265	·292 165 939	1 578	557	1·051 857 067
16	239 991 883	6 710	65	6 942 748	66	293 217 517	1 019	59	1 298 680
17	241 068 593	6 243	67	6 476 532	67	294 268 536	0 458	61	0 738 487
18	242 144 836	5 773	69	6 008 367	68	295 318 994	1 049 895	63	0 176 492
19	243 220 609		71	5 538 254	69	296 368 888		65	1·049 612 699
		1 075 302					1 049 330		
0·220	·244 295 912	4 829	473	1·075 066 196	0·270	·297 418 219	8 764	566	1·049 047 110
21	245 370 741	4 354	75	4 592 197	71	298 466 982	8 195	68	8 479 729
22	246 445 095	3 877	77	4 116 258	72	299 515 177	7 625	70	7 910 559
23	247 518 973	3 399	79	3 638 382	73	300 562 803	7 053	72	7 339 603
24	248 592 371		81	3 158 573	74	301 609 856		74	6 766 865
		1 072 918					1 046 480		
0·225	·249 665 289	2 435	483	1·072 676 833	0·275	·302 656 336	5 904	575	1·046 192 348
26	250 737 724	1 951	85	2 193 165	76	303 702 240	5 327	77	5 616 055
27	251 809 675	1 464	87	1 707 571	77	304 747 567	4 748	79	5 037 989
28	252 881 139	0 975	88	1 220 055	78	305 792 316	4 168	81	4 458 154
29	253 952 114		90	0 730 620	79	306 836 483		82	3 876 552
		1 070 485					1 043 585		
0·230	·255 022 600	1 069 993	492	1·070 239 267	0·280	·307 880 068	3 001	584	1·043 293 188
31	256 092 592	9 499	94	1 069 746 001	81	308 923 069	2 415	86	2 708 065
32	257 162 091	9 002	96	9 250 823	82	309 965 484	1 827	88	2 121 186
33	258 231 093	8 504	98	8 753 737	83	311 007 311	1 238	90	1 532 554
34	259 299 598		500	8 254 745	84	312 048 548		91	0 942 172
		1 068 004					1 040 646		
0·235	·260 367 602	7 503	502	1·067 753 851	0·285	·313 089 194	0 053	593	1·040 350 044
36	261 435 105	6 999	04	7 251 058	86	314 129 248	1 039 459	95	1·039 756 174
37	262 502 104	6 493	06	6 746 367	87	315 168 706	8 862	96	9 160 564
38	263 568 597	5 986	08	6 239 783	88	316 207 568	8 264	98	8 563 219
39	264 634 583		09	5 731 308	89	317 245 832		600	7 964 141
		1 065 476					1 037 664		
0·240	·265 700 059	4 965	511	1·065 220 945	0·290	·318 283 496	7 062	602	1·037 363 333
41	266 765 024	4 452	13	4 708 697	91	319 220 558	6 459	03	6 760 800
42	267 829 476	3 937	15	4 194 567	92	320 357 017	5 854	05	6 156 545
43	268 893 412	3 420	17	3 678 557	93	321 392 871	5 247	07	5 550 571
44	269 956 832		19	3 160 672	94	322 428 117		09	4 942 881
		1 062 901					1 034 638		
0·245	·271 019 733	2 380	521	1·062 640 914	0·295	·323 462 756	4 028	610	1·034 333 479
46	272 082 113	1 858	23	2 119 285	96	324 496 784	3 416	12	3 722 369
47	273 143 971	1 333	24	1 595 789	97	325 530 200	2 802	14	3 109 553
48	274 205 304	0 807	26	1 070 429	98	326 563 002	2 187	15	2 495 036
49	275 266 111	1 060 279	28	0 543 208	99	327 595 189	1 031 570	17	1 878 820

TABLE OF THE VALUES OF $H = \frac{2}{\sqrt{\pi}}\int_0^t \epsilon^{-t^2} dt$. (1) FROM $t=0$ TO $t=1{\cdot}250$.

t	H	Δ +	Δ_2 −	$\frac{2}{\sqrt{\pi}}\epsilon^{-t^2}$	t	H	Δ +	Δ_2 −	$\frac{2}{\sqrt{\pi}}\epsilon^{-t^2}$
0·300	·328 626 759	1 030 951	619	1·031 260 910	0·350	·379 382 054	997 934	699	0·998 283 712
1	329 657 711	0 331	20	0 641 308	51	380 379 988	7 234	700	7 584 160
2	330 688 042	1 029 709	22	0 020 019	52	381 377 221	6 532	02	6 883 105
3	331 717 750	9 085	24	1·029 397 045	53	382 373 753	5 829	03	6 180 550
4	332 746 835	1 028 459	25	8 772 391	54	383 369 582	995 124	05	5 476 500
0·305	·333 775 294	7 832	627	1·028 146 059	0·355	·384 364 706	4 418	706	0·994 770 957
6	334 803 127	7 203	29	7 518 054	56	385 359 123	3 710	08	4 063 926
7	335 830 330	6 573	31	6 888 378	57	386 352 833	3 001	09	3 355 411
8	336 856 903	5 941	32	6 257 036	58	387 345 834	2 290	11	2 645 416
9	337 882 843	1 025 307	34	5 624 031	59	388 338 124	991 578	12	1 933 944
0·310	·338 908 150	4 671	635	1·024 989 366	0·360	·389 329 701	0 864	714	0·991 221 000
11	339 932 822	4 034	37	4 353 045	61	390 320 565	0 149	15	0 506 587
12	340 956 856	3 395	39	3 715 072	62	391 310 714	989 432	17	0·989 790 710
13	341 980 251	2 755	40	3 075 450	63	392 300 146	8 714	18	9 073 372
14	343 003 006	1 022 113	42	2 434 182	64	393 288 860	987 995	20	8 354 577
0·315	·344 025 119	1 469	644	1·021 791 274	0·365	·394 276 855	7 274	721	0·987 634 329
16	345 046 588	0 824	45	1 146 727	66	395 264 128	6 551	22	6 912 632
17	346 067 412	0 177	47	0 500 545	67	396 250 689	5 827	24	6 189 490
18	347 087 589	1 019 528	49	1·019 852 733	68	397 236 507	5 102	25	5 464 907
19	348 107 117	1 018 878	50	9 203 294	69	398 221 609	984 375	27	4 738 887
0·320	·349 125 995	8 226	652	1·018 552 231	0·370	·399 205 984	3 647	728	0·984 011 434
21	350 144 221	7 573	53	7 899 548	71	400 189 631	2 918	30	3 282 551
22	351 161 793	6 917	55	7 245 249	72	401 172 549	2 186	31	2 552 244
23	352 178 711	6 261	57	6 589 338	73	402 154 735	1 454	32	1 820 515
24	353 194 971	1 015 602	58	5 931 817	74	403 136 189	980 720	34	1 087 369
0·325	·354 210 574	4 942	660	1·015 272 691	0·375	·404 116 909	979 985	735	0·980 352 810
26	355 225 516	4 281	62	4 611 964	76	405 096 894	9 248	37	0·979 616 841
27	356 239 797	3 618	63	3 949 638	77	406 076 143	8 510	38	8 879 467
28	357 253 415	2 953	65	3 285 719	78	407 054 653	7 771	39	8 140 692
29	358 266 368	1 012 287	66	2 620 209	79	408 032 424	977 030	41	7 400 520
0·330	·359 278 655	1 619	668	1·011 953 112	0·380	·409 009 453	6 288	742	0·976 658 954
31	360 290 274	0 949	69	1 284 432	81	409 985 741	5 544	44	5 915 999
32	361 301 223	0 278	71	0 614 173	82	410 961 285	4 799	45	5 171 660
33	362 311 502	1 009 606	73	1·009 942 337	83	411 936 084	4 053	46	4 425 939
34	363 321 107	1 008 932	74	9 268 931	84	412 910 136	973 305	48	3 678 840
0·335	·364 330 039	8 256	676	1·008 593 955	0·385	·413 883 441	2 556	749	0·972 930 369
36	365 338 295	7 578	77	7 917 416	86	414 855 997	1 805	51	2 180 529
37	366 345 873	6 900	79	7 239 316	87	415 827 802	1 053	52	1 429 324
38	367 352 773	6 219	80	6 559 659	88	416 798 855	0 300	53	0 676 758
39	368 358 992	1 005 537	82	5 878 448	89	417 769 155	969 545	55	0·969 922 835
0·340	·369 364 529	4 854	684	1·005 195 689	0·390	·418 738 700	8 789	756	0·969 167 559
41	370 369 383	4 169	85	4 511 383	91	419 707 489	8 032	57	8 410 935
42	371 373 552	3 482	87	3 825 536	92	420 675 521	7 273	59	7 652 966
43	372 377 034	2 794	88	3 138 151	93	421 642 795	6 513	60	6 893 656
44	373 379 827	1 002 104	90	2 449 232	94	422 609 308	965 752	61	6 133 010
0·345	·374 381 932	1 413	691	1·001 758 782	0·395	·423 575 060	4 989	763	0·965 371 032
46	375 383 344	0 720	93	1 066 806	96	424 540 050	4 226	64	4 607 725
47	376 384 065	0 026	94	0 373 307	97	425 504 275	3 460	65	3 843 095
48	377 384 091	999 330	96	0·999 678 289	98	426 467 736	2 694	67	3 077 144
49	378 383 421	998 633	97	8 981 756	99	427 430 429	961 926	68	2 309 877

TABLE OF THE VALUES OF $H = \frac{2}{\sqrt{\pi}}\int_0^t \epsilon^{-t^2} dt$. (1) FROM $t=0$ TO $t=1{\cdot}250$.

t	H	Δ +	Δ_2 −	$\frac{2}{\sqrt{\pi}}\epsilon^{-t^2}$	t	H	Δ +	Δ_2 −	$\frac{2}{\sqrt{\pi}}\epsilon^{-t^2}$
0·400	·428 392 355	961 156	769	0·961 541 299	0·450	·475 481 720	921 117	829	0·921 532 013
1	429 353 512	0 386	71	0 771 413	51	476 402 837	0 287	30	0 702 087
2	430 313 897	959 614	72	0 000 223	52	477 323 124	919 455	32	0·919 871 068
3	431 273 512	8 841	73	0·959 227 734	53	478 242 579	8 622	33	9 038 961
4	432 232 352	958 067	74	8 453 949	54	479 161 201	917 789	34	8 205 771
0·405	·433 190 419	7 291	776	0·957 678 873	0·455	·480 078 990	6 954	835	0·917 371 501
6	434 147 710	6 514	77	6 902 511	56	480 995 944	6 118	36	6 536 156
7	435 104 224	5 736	78	6 124 865	57	481 912 062	5 281	37	5 699 740
8	436 059 959	4 956	80	5 345 941	58	482 827 343	4 443	38	4 862 258
9	437 014 915	954 175	81	4 565 742	59	483 741 786	913 604	39	4 023 714
0·410	·437 969 090	3 393	782	0·953 784 273	0·460	·484 655 390	2 764	840	0·913 184 112
11	438 922 483	2 610	83	3 001 537	61	485 568 154	1 923	41	2 343 457
12	439 875 093	1 825	85	2 217 540	62	486 480 077	1 080	42	1 501 752
13	440 826 918	1 039	86	1 432 284	63	487 391 157	0 237	43	0 659 003
14	441 777 957	950 252	87	0 645 775	64	488 301 394	909 393	44	0·909 815 213
0·415	·442 728 209	949 464	788	0·949 858 016	0·465	·489 210 787	8 548	845	0·908 970 387
16	443 677 673	8 674	90	9 069 012	66	490 119 335	7 701	46	8 124 530
17	444 626 347	7 883	91	8 278 767	67	491 027 036	6 854	47	7 277 645
18	445 574 230	7 091	92	7 487 284	68	491 933 890	6 005	48	6 429 737
19	446 521 321	946 298	93	6 694 569	69	492 839 895	905 156	49	5 580 810
0·420	·447 467 618	5 503	795	0·945 900 626	0·470	·493 745 051	4 305	850	0·904 730 869
21	448 413 122	4 707	96	5 105 458	71	494 649 356	3 454	51	3 879 917
22	449 357 829	3 910	97	4 309 069	72	495 552 810	2 602	52	3 027 960
23	450 301 739	3 112	98	3 511 465	73	496 455 412	1 748	53	2 175 002
24	451 244 851	942 313	99	2 712 649	74	497 357 160	900 894	54	1 321 047
0·425	·452 187 164	1 512	801	0·941 912 626	0·475	·498 258 054	0 038	855	0·900 466 099
26	453 128 676	0 710	02	1 111 399	76	499 158 092	899 182	56	0·899 610 162
27	454 069 387	939 907	03	0 308 974	77	500 057 274	8 324	57	8 753 242
28	455 009 294	9 103	04	0·939 505 353	78	500 955 598	7 466	58	7 895 343
29	455 948 397	939 383	05	8 700 542	79	501 853 064	896 607	59	7 036 468
0·430	·456 886 695	7 491	807	0·937 894 544	0·480	·502 749 671	5 746	860	0·896 176 622
31	457 824 186	6 683	08	7 087 365	81	503 645 417	4 885	61	5 315 810
32	458 760 869	5 874	09	6 279 007	82	504 540 302	4 023	62	4 454 036
33	459 696 743	5 064	10	5 469 476	83	505 434 325	3 160	63	3 591 304
34	460 631 807	934 253	11	4 658 775	84	506 327 484	892 295	64	2 727 619
0·435	461 566 060	3 440	812	0·933 846 910	0·485	·507 219 780	1 430	865	0·891 862 985
36	462 499 501	2 627	14	3 033 883	86	508 111 210	0 564	66	0 997 406
37	463 432 128	1 812	15	2 219 700	87	509 001 774	889 697	67	0 130 888
38	464 363 940	0 996	16	1 404 365	88	509 891 471	8 829	68	0·889 263 433
39	465 294 936	930 179	17	0 587 881	89	510 780 301	887 960	69	8 395 047
0·440	·466 225 115	929 361	818	0·929 770 254	0·490	·511 668 261	7 091	870	0·887 525 734
41	467 154 476	8 542	19	8 951 487	91	512 555 352	6 220	71	6 655 498
42	468 083 018	7 721	20	8 131 585	92	513 441 572	5 348	72	5 784 344
43	469 010 739	6 900	22	7 310 552	93	514 326 920	4 476	73	4 912 276
44	469 937 639	926 077	23	6 488 392	94	515 211 396	883 602	73	4 039 298
0·445	·470 863 715	5 253	824	0·925 665 110	0·495	·516 094 999	2 728	874	0·883 165 416
46	471 788 968	4 428	25	4 840 709	96	516 977 727	1 853	75	2 290 632
47	472 713 396	3 602	26	4 015 195	97	517 859 580	0 977	76	1 414 952
48	473 636 998	2 775	27	3 188 572	98	518 740 556	0 100	77	0 538 381
49	474 559 773	921 947	28	2 360 843	99	519 620 656	879 222	78	0·879 660 921

TABLE OF THE VALUES OF $H = \frac{2}{\sqrt{\pi}}\int_0^t \epsilon^{-t^2} dt$. (1) FROM $t=0$ TO $t=1{\cdot}250$.

t	H	Δ +	Δ_2 −	$\frac{2}{\sqrt{\pi}}\epsilon^{-t^2}$	t	H	Δ +	Δ_2 −	$\frac{2}{\sqrt{\pi}}\epsilon^{-t^2}$
0·500	·520 499 878	878 343	879	0·878 782 579	0·550	·563 323 366	833 378	917	0·833 836 647
01	521 378 221	7 463	80	7 903 358	51	564 156 744	2 460	18	2 919 098
02	522 255 684	6 583	81	7 023 262	52	564 989 204	1 542	19	2 000 895
03	523 132 267	5 701	81	6 142 296	53	565 820 746	0 622	19	1 082 042
04	524 007 969	874 819	82	5 260 465	54	566 651 368	829 703	20	0 162 543
0·505	·524 882 788	3 936	883	0·874 377 773	0·555	·567 481 071	8 782	920	0·829 242 403
06	525 756 724	3 052	84	3 494 224	56	568 309 853	7 861	21	8 321 626
07	526 629 776	2 167	85	2 609 822	57	569 137 714	6 939	22	7 400 217
08	527 501 943	1 282	86	1 724 572	58	569 964 653	6 017	22	6 478 180
09	528 373 225	870 395	87	0 838 479	59	570 790 670	825 094	23	5 555 519
0·510	·529 243 620	869 508	887	0·869 951 547	0·560	·571 615 764	4 170	924	0·824 632 239
11	530 113 128	8 620	88	9 063 779	61	572 439 934	3 246	24	3 708 345
12	530 981 747	7 731	89	8 175 182	62	573 263 180	2 321	25	2 783 839
13	531 849 478	6 841	90	7 285 758	63	574 085 502	1 396	25	1 858 728
14	532 716 318	865 950	91	6 395 513	64	574 906 898	820 470	26	0 933 015
0·515	·533 582 268	5 059	891	0·865 504 450	0·565	·575 727 367	819 543	927	0·820 006 705
16	534 447 327	4 166	92	4 612 575	66	576 546 911	8 616	27	0·819 079 801
17	535 311 493	3 273	93	3 719 892	67	577 365 527	7 688	28	8 152 309
18	536 174 767	2 379	94	2 826 404	68	578 183 215	6 760	28	7 224 233
19	537 037 146	861 485	95	1 932 117	69	578 999 975	815 831	29	6 295 577
0·520	·537 898 630	0 589	895	0·861 037 034	0·570	·579 815 806	4 901	930	0·815 366 346
21	538 759 220	859 693	96	0 141 161	71	580 630 708	3 971	30	4 436 544
22	539 618 913	8 796	97	0·859 244 502	72	581 444 679	3 041	31	3 506 174
23	540 477 708	7 898	98	8 347 060	73	582 257 720	2 110	31	2 575 243
24	541 335 606	856 999	99	7 448 841	74	583 069 829	811 178	32	1 643 753
0·525	·542 192 606	6 100	899	0·856 549 849	0·575	·583 881 007	0 245	932	0·810 711 710
26	543 048 706	5 200	900	5 650 088	76	584 691 253	809 313	33	0·809 779 118
27	543 903 906	4 299	01	4 749 562	77	585 500 565	8 379	33	8 845 981
28	544 758 205	3 397	02	3 848 277	78	586 308 944	7 445	34	7 912 303
29	545 611 602	852 495	02	2 946 236	79	587 116 390	806 511	34	6 978 089
0·530	546 464 097	1 592	903	0·852 043 444	0·580	·587 922 900	5 576	935	0·806 043 343
31	547 315 689	0 688	04	1 139 906	81	588 728 476	4 640	36	5 108 070
32	548 166 376	849 783	05	0 235 625	82	589 533 116	3 704	36	4 172 273
33	549 016 160	8 878	05	0·849 330 606	83	590 336 821	2 768	37	3 235 958
34	549 865 037	847 972	06	8 424 853	84	591 139 588	801 831	37	2 299 129
0·535	·550 713 009	7 065	907	0·847 518 372	0·585	·591 941 419	0 893	938	0·801 361 789
36	551 560 074	6 157	08	6 611 165	86	592 742 311	799 955	38	0 423 943
37	552 406 231	5 249	08	5 703 239	77	593 542 266	9 016	39	0·799 485 597
38	553 251 480	4 340	09	4 794 596	78	594 341 283	8 077	39	8 546 753
39	554 095 820	843 430	10	3 885 242	79	595 139 360	797 138	40	7 607 416
0·540	·554 939 250	2 520	910	0·842 975 181	0·590	·595 936 497	6 197	940	0·796 667 591
41	555 781 770	1 609	11	2 064 417	91	596 732 695	5 257	41	5 727 282
42	556 623 379	0 697	12	1 152 955	92	597 527 952	4316	41	4 786 493
43	557 464 076	839 784	13	0 240 799	93	598 322 268	3 374	42	3 845 229
44	558 303 860	838 871	13	0·839 327 954	94	599 115 642	792 432	42	2 903 494
0·545	·559 142 732	7 957	914	0·838 414 423	0·595	·599 908 074	1 490	942	0·791 961 292
46	559 980 689	7 043	15	7 500 212	96	600 699 564	0 547	43	1 018 627
47	560 817 732	6 128	15	6 585 324	97	601 490 111	789 604	43	0 075 505
48	561 653 859	5 212	16	5 669 765	98	602 279 715	8 660	44	0·789 131 928
49	562 489 071	834 295	17	4 753 538	99	603 068 375	787 716	44	0·788 187 903

TABLE OF THE VALUES OF $H = \frac{2}{\sqrt{\pi}}\int_0^t \epsilon^{-t^2} dt$. (1) FROM $t=0$ TO $t=1{\cdot}250$.

[$t = {\cdot}600$] [to ${\cdot}699$

t	H	Δ +	Δ_2 −	$\frac{2}{\sqrt{\pi}}\epsilon^{-t^2}$	t	H	Δ +	Δ_2 −	$\frac{2}{\sqrt{\pi}}\epsilon^{-t^2}$
0·600	·603 856 091	786 771	945	0·787 243 432	0·650	·642 029 327	739 066	961	0·739 546 763
01	4 642 862	5 826	45	6 298 520	51	2 768 393	8 104	62	8 585 239
02	5 428 688	4 880	46	5 353 172	52	3 506 498	7 143	62	7 623 489
03	6 213 568	3 934	46	4 407 391	53	4 243 640	6 180	62	6 661 518
04	6 997 502	782 988	46	3 461 182	54	4 979 821	735 218	62	5 699 330
0·605	·607 780 490	2 041	947	0·782 514 550	0·655	·645 715 039	4 256	963	0·734 736 930
06	8 562 531	1 094	47	1 567 499	56	6 449 295	3 293	63	3 774 321
07	9 343 625	0 146	48	0 620 032	57	7 182 587	2 330	63	2 811 507
08	·610 123 771	779 198	48	0 779 672 155	58	7 914 917	1 367	63	1 848 494
09	0 902 969	778 250	48	8 723 871	59	8 646 284	730 404	63	0 885 284
0·610	·611 681 219	7 301	949	0·777 775 185	0·660	·649 376 688	729 440	963	0·729 921 881
11	2 458 520	6 351	49	6 826 101	61	·650 106 128	8 476	64	8 958 291
12	3 234 871	5 402	50	5 876 623	62	0 834 604	7 513	64	7 994 517
13	4 010 273	4 452	50	4 926 756	63	1 562 117	6 549	64	7 030 563
14	4 784 724	773 501	50	3 976 504	64	2 288 666	725 584	64	6 066 433
0·615	·615 558 226	2 550	951	0·773 025 871	0·665	·653 014 250	4 620	964	0·725 102 132
16	6 330 776	1 599	51	2 074 862	66	3 738 870	3 655	65	4 137 66 3
17	7 102 375	0 648	52	1 123 480	67	4 462 525	2 691	65	3 173 031
18	7 873 023	769 696	52	0 171 731	68	5 185 216	1 726	65	2 208 239
19	8 642 718	768 743	52	0·759 219 617	69	5 906 942	720 761	65	1 243 292
0·620	·619 411 462	7 791	953	0·768 267 144	0·670	·656 627 702	719 796	965	0·720 278 193
21	·620 179 253	6 838	53	7 314 316	71	7 347 498	8 830	65	0·719 312 947
22	0 946 090	5 884	53	6 361 137	72	8 066 328	7 865	65	8 347 558
23	1 711 975	4 931	54	5 407 611	73	8 784 193	6 899	66	7 382 030
24	2 476 906	763 977	54	4 453 742	74	9 501 092	715 933	66	6 416 367
0·625	·623 240 882	3 022	954	0·763 499 536	0·675	·660 217 026	4 968	966	0·715 450 573
26	4 003 904	2 068	55	2 544 995	76	0 931 993	4 002	66	4 484 652
27	4 765 972	1 113	55	1 590 125	77	1 645 995	3 036	66	3 518 608
28	5 527 085	0 157	55	0 634 928	78	2 359 030	2 069	66	2 552 445
29	6 287 242	759 202	56	0·759 679 411	79	3 071 100	711 103	66	1 586 167
0·630	·627 046 443	8 246	956	0·758 723 576	0·680	·663 782 203	0 137	966	0·710 619 778
31	7 804 689	7 289	56	7 767 429	81	4 492 339	709 170	67	0·709 653 283
32	8 561 978	6 333	57	6 810 973	82	5 201 509	8 203	67	8 686 684
33	9 318 311	5 376	57	5 854 212	83	5 909 713	7 237	67	7 719 987
34	·630 073 686	754 418	57	4 897 151	84	6 616 949	706 270	67	6 753 195
0·635	·630 828 105	3 461	958	0·753 939 794	0·685	·667 323 219	5 303	967	0·705 786 311
36	1 581 566	2 503	58	2 982 146	86	8 028 522	4 336	67	4 819 341
37	2 334 069	1 545	58	2 024 209	87	8 732 858	3 369	67	3 852 288
38	3 085 614	0 587	58	1 065 989	88	9 436 226	2 402	67	2 885 157
39	3 836 201	749 628	59	0 107 490	89	·670 138 628	701 434	67	1 917 950
0·640	·634 585 829	8 669	959	0·749 148 716	0·690	·670 840 062	0 467	967	0·700 950 672
41	5 334 498	7 710	59	8 189 671	91	1 540 529	699 500	67	0·699 983 327
42	6 082 208	6 751	59	7 230 359	92	2 240 029	8 532	67	9 015 919
43	6 828 959	5 791	60	6 270 785	93	2 938 561	7 565	67	8 048 453
44	7 574 750	744 831	60	5 310 952	94	3 636 126	696 597	68	7 080 930
0·645	·638 319 581	3 871	960	0·744 350 865	0·695	·674 332 723	5 630	968	0·696 113 357
46	9 063 452	2 910	60	3 390 528	96	5 028 352	4 662	68	5 145 737
47	9 806 362	1 950	61	2 429 945	97	5 723 014	3 694	68	4 178 073
48	·640 548 311	0 989	61	1 469 121	98	6 416 709	2 727	68	3 210 370
49	1 289 300	740 027	61	0 508 059	99	7 109 435	691 759	68	2 242 631

TABLE OF THE VALUES OF $H=\frac{2}{\sqrt{\pi}}\int_0^t \epsilon^{-t^2}dt$. (1) FROM $t=0$ TO $t=1{\cdot}250$.

[t = ·700] [to ·799

t	H	Δ +	Δ_2 −	$\frac{2}{\sqrt{\pi}}\epsilon^{-t^2}$	t	H	Δ +	Δ_2 −	$\frac{2}{\sqrt{\pi}}\epsilon^{-t^2}$
0·700	·677 801 194			0·691 274 860	0·750	·711 155 634			0·642 931 069
		690 791	968				642 449	964	
01	8 491 985	689 823	68	0 307 062	51	1 798 083	1 485	64	1 966 754
02	9 181 808	8 855	68	0·689 339 241	52	2 439 567	0 521	64	1 002 602
03	9 870 663	7 887	68	8 371 399	53	3 080 088	639 557	64	0 038 619
04	·680 558 551	686 920	68	7 403 542	54	3 719 645	638 593	64	0·639 074 807
0·705	·681 245 470		968	0·686 435 672	0·755	·714 358 237	7 629	964	0·638 111 171
06	1 931 422	5 952	68	5 467 794	56	4 995 867	6 666	63	7 147 713
07	2 616 406	4 984	68	4 499 912	57	5 632 533	5 703	63	6 184 437
08	3 300 422	4 016	68	3 532 030	58	6 268 236	4 740	63	5 221 347
09	3 983 470	3 048	68	2 564 151	59	6 902 976	633 777	63	4 258 447
0·710	·684 665 550	682 080	968	0·681 596 279	0·760	·717 536 753	2 814	963	0·633 295 740
11	5 346 663	1 112	68	0 628 419	61	8 169 567	1 852	62	2 333 229
12	6 026 807	0 144	68	0·679 660 573	62	8 801 419	0 890	62	1 370 919
13	6 705 984	679 177	68	8 692 747	63	9 432 309	629 928	62	0 408 812
14	7 384 193	8 209	68	7 724 943	64	·720 062 237	628 966	62	0·629 446 912
0·715	·688 061 434	677 241	968	0·676 757 166	0·765	·720 691 203	8 004	962	0·628 485 223
16	8 737 707	6 273	68	5 789 419	66	1 319 208	7 043	61	7 523 749
17	9 413 012	5 306	68	4 821 706	67	1 946 251	6 082	61	6 562 492
18	·690 087 350	4 338	68	3 854 031	68	2 572 333	5 121	61	5 601 456
19	0 760 721	3 370	68	2 886 398	69	3 197 454	624 160	61	4 640 645
0·720	·691 433 123	672 403	968	0·671 918 811	0·770	·723 821 614	3 200	960	0·623 680 063
21	2 104 558	1 435	68	0 951 274	71	4 444 814	2 240	60	2 719 712
22	2 775 026	0 468	67	0·669 983 789	72	5 067 053	1 280	60	1 759 596
23	3 444 526	669 500	67	9 016 362	73	5 688 333	0 320	60	0 799 719
24	4 113 058	8 533	67	8 048 995	74	6 308 653	619 360	60	0·619 840 085
0·725	·694 780 624	667 565	967	0·667 081 693	0·775	·726 928 013	8 401	959	0·618 880 695
26	5 447 222	6 598	67	6 114 459	76	7 546 414	7 442	59	7 921 556
27	6 112 853	5 631	67	5 147 298	77	8 163 857	6 483	59	6 962 668
28	6 777 516	4 664	67	4 180 212	78	8 780 340	5 525	59	6 004 037
29	7 441 213	3 697	67	3 213 206	79	9 395 865	614 567	58	5 045 665
0·730	·698 103 943	662 730	966	0·662 246 284	0·780	·730 010 431	3 609	958	0·614 087 556
31	8 765 706	1 763	67	1 279 448	81	0 624 040	2 651	58	3 129 713
32	9 426 502	0 796	67	0 312 704	82	1 236 691	1 693	57	2 172 139
33	·700 086 331	659 829	67	0·659 346 054	83	1 848 384	0 736	57	1 214 839
34	0 745 194	8 863	67	8 379 503	84	2 459 121	609 779	57	0 257 816
0·735	·701 403 090	657 896	966	0·657 413 053	0·785	·733 068 900	8 823	957	0·609 301 072
36	2 060 020	6 930	66	6 446 709	86	3 677 723	7 867	56	8 344 611
37	2 715 984	5 964	66	5 480 475	87	4 285 589	6 910	56	7 388 438
38	3 370 981	4 997	66	4 514 354	88	4 892 500	5 955	56	6 432 554
39	4 025 012	4 031	66	3 548 350	89	5 498 455	604 999	55	5 476 963
0·740	·704 678 078	653 065	966	0·652 582 466	0·790	·736 103 454	4 044	955	0·604 521 670
41	5 330 177	2 100	66	1 616 707	91	6 707 498	3 089	55	3 566 676
42	5 981 311	1 134	66	0 651 076	92	7 310 587	2 135	55	2 611 986
43	6 631 480	0 168	65	0·649 685 576	93	7 912 722	1 181	54	1 657 602
44	7 280 683	649 203	65	8 720 211	94	8 513 903	600 227	54	0 703 529
0·745	·707 928 920	648 238	965	0·647 754 986	0·795	·739 114 129	599 273	954	0·599 749 769
46	8 576 193	7 272	65	6 789 903	96	9 713 402	8 320	53	8 796 326
47	9 222 500	6 307	65	5 824 966	97	·740 311 722	7 367	53	7 843 203
48	9 867 843	5 343	65	4 860 179	98	0 909 089	6 414	53	6 890 403
49	·710 512 220	4 378	65	3 895 546	99	1 505 503	595 462	52	5 937 930
		643 413							

TABLE OF THE VALUES OF $H = \frac{2}{\sqrt{\pi}}\int_0^t \epsilon^{-t^2} dt$. (1) FROM $t=0$ TO $t=1{\cdot}250$.

t	H	Δ +	Δ_2 −	$\frac{2}{\sqrt{\pi}}\epsilon^{-t^2}$	t	H	Δ +	Δ_2 −	$\frac{2}{\sqrt{\pi}}\epsilon^{-t^2}$
0·800	·742 100 965			0·594 985 786	0·850	·770 668 058			0·547 869 717
		594 510	952				547 404	931	
01	2 695 475	3 558	52	4 033 976	51	1 215 462	6 473	31	6 938 583
02	3 289 033	2 607	51	3 082 502	52	1 761 935	5 543	30	6 007 939
03	3 881 640	1 656	51	2 131 369	53	2 307 478	4 613	30	5 077 789
04	4 473 296		51	1 180 578	54	2 852 091		29	0·544 148 135
		590 705					543 684		
0·805	·745 064 001		950	0·590 230 133	0·855	·773 395 774		929	0·543 218 980
		589 755					2 755		
06	5 653 756	8 805	50	0·589 280 038	56	3 938 529	1 826	28	2 290 327
07	6 242 561	7 856	50	8 330 295	57	4 480 355	0 898	28	1 362 179
08	6 830 417	6 906	49	7 380 909	58	5 021 253	539 971	27	0 434 538
09	7 417 323		49	6 431 881	59	5 561 224		27	0·539 507 408
		585 958					539 044		
0·810	·748 003 281		948	0·585 483 216	0·860	·776 100 268		926	0·538 580 792
		5 009					8 118		
11	8 588 290	4 061	48	4 534 917	61	6 638 386	7 192	26	7 654 691
12	9 172 351	3 113	48	3 586 986	62	7 175 578	6 267	25	6 729 110
13	9 755 464	2 166	47	2 639 427	63	7 711 844	5 342	25	5 804 050
14	·750 337 630		47	1 692 243	64	8 247 186		24	4 879 515
		581 219					534 417		
0·815	·750 918 848		947	0·580 745 438	0·865	·778 781 604		924	0·533 955 508
		0 272					3 494		
16	1 499 121	579 326	46	0·579 799 014	66	9 315 097	2 571	23	3 032 030
17	2 078 446	8 380	46	8 852 975	67	9 847 668	1 648	23	2 109 086
18	2 656 827	7 435	45	7 907 324	68	·780 379 316	0 726	22	1 186 677
19	3 234 261		45	6 962 064	69	0 910 041		22	0 264 806
		576 490					529 804		
0·820	·753 810 751		945	0·576 017 197	0·870	·781 439 845		921	0·529 343 477
		5 545					8 883		
21	4 386 296	4 601	44	5 072 728	71	1 968 729	7 963	21	8 422 692
22	4 960 896	3 657	44	4 128 659	72	2 496 691	7 043	20	7 502 454
23	5 534 553	2 713	43	3 184 994	73	3 023 734	6 123	19	6 582 764
24	6 107 267		43	2 241 736	74	3 549 857		19	5 663 627
		571 770					525 204		
0·825	·756 679 037		943	0·571 298 886	0·875	·784 075 061		918	0·524 745 045
		0 828					4 286		
26	7 249 864	569 885	42	0 356 450	76	4 599 347	3 368	18	3 827 021
27	7 819 750	8 944	42	0·569 414 430	77	5 122 715	2 451	17	2 909 556
28	8 388 693	8 002	41	8 472 828	78	5 645 166	1 534	17	1 992 655
29	8 956 696		41	7 531 649	79	6 166 701		16	1 076 319
		567 061					520 618		
0·830	·759 523 757		941	0·566 590 894	0·880	·786 687 319		915	0·520 160 551
		6 121					519 703		
31	·760 089 878	5 181	40	5 650 568	81	7 207 022	8 788	15	0·519 245 355
32	0 655 058	4 241	40	4 710 673	82	7 725 810	7 874	14	8 330 732
33	1 219 299	3 302	39	3 771 212	83	8 243 684	6 960	14	7 416 685
34	1 782 601		39	2 832 188	84	8 760 644		13	6 503 217
		562 363					516 047		
0·835	·762 344 964		938	0·561 893 605	0·885	·789 276 690		913	0·515 590 330
		1 424					5 134		
36	2 906 388	0 487	38	0 955 465	86	9 791 825	4 222	12	4 678 028
37	3 466 875	559 549	37	0 017 771	87	·790 306 047	3 311	11	3 766 312
38	4 026 424	8 612	37	0·559 080 526	88	0 819 357	2 400	11	2 855 186
39	4 585 036		37	8 143 734	89	1 331 757		10	1 944 651
		557 676					511 490		
0·840	·765 142 711		936	0·557 207 397	0·890	·791 843 247		910	0·511 034 712
		6 739					0 580		
41	5 699 451	5 804	36	6 271 518	91	2 353 827	509 671	09	0 125 369
42	6 255 255	4 869	35	5 336 100	92	2 863 498	8 763	08	0·509 216 626
43	6 810 123	3 934	35	4 401 147	93	3 372 260	7 855	08	8 308 485
44	7 364 057		34	3 466 661	94	3 880 115		07	7 400 949
		553 000					506 947		
0·845	·767 917 057		934	0·552 532 644	0·895	·794 387 062		907	0·506 494 020
		2 066					6 041		
46	8 469 123	1 133	33	1 599 101	96	4 893 103	5 135	06	5 587 701
47	9 020 255	0 200	33	0 666 034	97	5 398 238	4 229	05	4 681 994
48	9 570 455	549 267	32	0·549 733 446	98	5 902 467	3 325	05	3 776 903
49	·770 119 722	548 335	32	8 801 339	99	6 405 792	502 420	04	2 872 428

TABLE OF THE VALUES OF $H = \frac{2}{\sqrt{\pi}}\int_0^t \epsilon^{-t^2} dt$. (1) FROM $t=0$ TO $t=1{\cdot}250$.

$t = {\cdot}900$] [${\cdot}999$

t	H	Δ +	Δ_2 −	$\frac{2}{\sqrt{\omega}}\epsilon^{-t^2}$	t	H	Δ +	Δ −	$\frac{2}{\sqrt{\pi}}\epsilon^{-t^2}$
0·900	·796 908 212	501 517	904	0·501 968 574	0·950	·820 890 807	457 185	869	0·457 619 255
01	7 409 729	0 614	03	1 065 342	51	1 347 992	6 316	69	6 750 147
02	7 910 343	499 712	02	0 162 736	52	1 804 308	5 448	68	5 881 778
03	8 410 055	8 810	02	0·499 260 756	53	2 259 756	4 581	67	5 014 150
04	8 908 865	497 909	01	8 359 407	54	2 714 336	453 714	67	4 147 265
0·905	·799 406 774	7 009	900	0·497 458 689	0·955	·823 168 050	2 848	866	0·453 281 124
06	9 903 783	6 109	00	6 558 607	56	3 620 899	1 983	65	2 415 731
07	·800 399 891	5 210	899	5 659 162	57	4 072 882	1 119	64	1 551 087
08	0 895 101	4 311	98	4 760 356	58	4 524 001	0 256	64	0 687 194
09	1 389 412	493 413	98	3 862 193	59	4 974 257	449 393	63	0·449 824 055
0·910	·801 882 826	2 516	897	0·492 964 674	0·960	·825 423 650	8 531	862	0·448 961 670
11	2 375 342	1 620	97	2 067 802	61	5 872 180	7 670	61	8 100 042
12	2 866 962	0 724	96	1 171 580	62	6 319 850	6 809	60	7 239 174
13	3 357 685	489 828	95	0 276 009	63	6 766 659	5 949	60	6 379 067
14	3 847 514	488 934	95	0·489 381 093	64	7 212 608	445 090	59	5 519 723
0·915	·804 336 448	8 040	894	0·488 486 833	0·965	·827 657 699	4 232	858	0·444 661 143
16	4 824 488	7 147	93	7 593 232	66	8 101 931	3 375	57	3 803 331
17	5 311 634	6 254	93	6 700 292	67	8 545 306	2 518	57	2 946 288
18	5 797 889	5 362	92	5 808 016	68	8 987 824	1 662	56	2 090 016
19	6 283 251	484 471	91	4 916 406	69	9 429 486	440 807	55	1 234 516
0·920	·806 767 722	3 580	891	0·484 025 464	0·970	·829 870 293	439 953	854	0·440 379 791
21	7 251 302	2 690	90	3 135 193	71	·830 310 246	9 099	54	0·439 525 843
22	7 733 992	1 801	89	2 245 595	72	0 749 345	8 246	53	8 672 674
23	8 215 793	0 912	89	1 356 672	73	1 187 591	7 394	52	7 820 284
24	8 696 706	480 025	88	0 468 427	74	1 624 986	436 543	51	6 968 678
0·925	·809 176 730	479 137	887	0·479 580 861	0·975	·832 061 529	5 693	850	0·436 117 855
26	9 655 868	8 251	87	8 693 978	76	2 497 222	4 843	50	5 267 819
27	·810 134 119	7 365	86	7 807 779	77	2 932 065	3 994	49	4 418 570
28	0 611 483	6 480	85	6 922 268	78	3 366 059	3 146	48	3 570 111
29	1 087 963	475 595	84	6 037 445	79	3 799 205	432 299	47	2 722 444
0·930	·811 563 559	4 712	884	0·475 153 313	0·980	·834 231 504	1 453	846	0·431 875 571
31	2 038 270	3 828	83	4 269 875	81	4 662 957	0 607	46	1 029 493
32	2 512 099	2 946	82	3 387 133	82	5 093 564	429 762	45	0 184 212
33	2 985 045	2 064	82	2 505 089	83	5 523 326	8 918	44	0·429 339 730
34	3 457 109	471 183	81	1 623 745	84	5 952 243	428 075	43	8 496 049
0·935	813 928 292	0 303	880	0·470 743 103	0·985	·836 380 318	7 232	842	0·427 653 170
36	4 398 595	469 423	80	0·469 863 166	86	6 807 550	6 390	42	6 811 096
37	4 868 019	8 545	79	8 983 936	87	7 233 940	5 541	41	5 969 828
38	5 336 564	7 666	78	8 105 415	88	7 659 490	4 709	40	5 129 367
39	5 804 230	466 789	77	7 227 605	89	8 084 199	423 870	39	4 289 717
0·940	816 271 019	5 912	877	0·466 350 509	0·990	·838 508 070	3 032	838	0·423 450 878
41	6 736 931	5 036	76	5 474 128	91	8 931 101	2 194	38	2 612 852
42	7 201 967	4 161	75	4 598 465	92	9 353 296	1 357	37	1 775 641
43	7 666 128	3 286	75	3 723 522	93	9 774 653	0 521	36	0 939 247
44	8 129 415	462 412	74	2 849 301	94	·840 195 174	419 686	35	0 103 671
0·945	·818 591 827	1 539	873	0·461 975 804	0·995	·840 614 861	8 852	834	0·419 268 915
46	9 053 367	0 667	72	1 103 033	96	1 033 712	8 018	34	8 434 981
47	9 514 034	459 795	72	0 230 991	97	1 451 731	7 186	33	7 601 871
48	9 973 829	8 924	71	0·459 359 679	98	1 868 916	6 354	32	6 769 586
49	·820 432 753	458 054	70	8 489 099	99	2 285 270	415 523	31	5 938 127

TABLE OF THE VALUES OF $H = \frac{2}{\sqrt{\pi}}\int_0^t \epsilon^{-t^2} dt$. (1) FROM $t=0$ TO $t=1{\cdot}250$.

t	H	Δ +	Δ_2 −	$\frac{2}{\sqrt{\pi}}\epsilon^{-t^2}$	t	H	Δ +	Δ_2 −	$\frac{2}{\sqrt{\pi}}\epsilon^{-t^2}$
1·000	·842 700 793	414 693	830	0·415 107 497	1·050	·862 436 106	374 274	787	0·374 666 957
01	3 115 485	3 863	29	4 277 698	51	2 810 380	3 488	86	3 880 608
02	3 529 349	3 035	29	3 448 730	52	3 183 868	2 703	85	3 095 163
03	3 942 383	2 207	28	2 620 596	53	3 556 570	1 919	84	2 310 624
04	4 354 590	411 380	27	1 793 297	54	3 928 489	371 136	83	1 526 991
1·005	·844 765 970	0 554	826	0·410 966 835	1·055	·864 299 625	0 353	782	0·370 744 267
06	5 176 524	409 729	25	0 141 211	56	4 669 978	369 572	81	0·369 962 451
07	5 586 253	8 904	24	0·409 316 427	57	5 039 550	8 791	80	9 181 546
08	5 995 157	8 081	24	8 492 485	58	5 408 341	8 012	80	8 401 552
09	6 403 238	407 258	23	7 669 386	59	5 776 353	367 233	79	7 622 471
1·010	·846 810 496	6 436	822	0·406 847 132	1·060	·866 143 587	6 456	778	0·366 844 303
11	7 216 933	5 615	21	6 025 724	61	6 510 042	5 679	77	6 067 051
12	7 622 548	4 795	20	5 205 164	62	6 875 721	4 903	76	5 290 715
13	8 027 343	3 976	19	4 385 454	63	7 240 624	4 128	75	4 515 297
14	8 431 319	403 158	18	3 566 595	64	7 604 752	363 354	74	3 740 797
1·015	·848 834 477	2 340	818	0·402 748 588	1·065	·867 968 106	2 581	773	0·362 967 216
16	9 236 817	1 523	17	1 931 436	66	8 330 687	1 809	72	2 194 557
17	9 638 340	0 707	16	1 115 139	67	8 692 495	1 037	71	1 422 819
18	·850 039 047	399 892	15	0 299 700	68	9 053 533	0 267	70	0 652 004
19	0 438 940	399 078	14	0·399 485 119	69	9 413 800	359 498	69	0·359 882 114
1·020	·850 838 018	8 265	813	0·398 671 399	1·070	·869 773 297	8 729	769	0·359 113 149
21	1 236 283	7 452	12	7 858 541	71	870 132 026	7 961	68	8 345 110
22	1 633 735	6 641	12	7 046 545	72	0 489 988	7 195	67	7 577 999
23	2 030 376	5 830	11	6 235 415	73	0 847 183	6 429	66	6 811 816
24	2 426 206	395 020	10	5 425 151	74	1 203 612	355 664	65	6 046 563
1·025	·852 821 227	4 211	809	0·394 615 754	1·075	·871 559 276	4 900	764	0·355 282 240
26	3 215 438	3 403	08	3 807 226	76	1 914 176	4 138	63	4 518 850
27	3 608 841	2 596	07	2 999 570	77	2 268 314	3 376	62	3 756 392
28	4 001 437	1 790	06	2 192 785	78	2 621 690	2 614	61	2 994 867
29	4 393 227	390 984	05	1 386 873	79	2 974 304	351 854	60	2 234 278
1·030	·854 784 211	0 180	805	0·390 581 837	1·080	·873 326 158	1 095	759	0·351 474 625
31	5 174 391	389 376	04	0·389 777 677	81	3 677 254	0 337	58	0 715 908
32	5 563 767	8 573	03	8 974 394	82	4 027 591	349 580	57	0·349 958 129
33	5 952 340	7 771	02	8 171 991	83	4 377 170	8 823	56	9 201 289
34	6 340 111	386 970	01	7 370 468	84	4 725 993	348 068	55	8 445 390
1·035	·856 727 081	6 170	800	0·386 569 827	1·085	·875 074 061	7 313	754	0·347 690 431
36	7 113 251	5 371	799	5 770 069	86	5 421 375	6 560	54	6 936 413
37	7 498 622	4 572	98	4 971 196	87	5 767 934	5 807	53	6 183 339
38	7 883 194	3 775	98	4 173 209	88	6 113 742	5 056	52	5 431 209
39	8 266 969	382 978	97	3 376 110	89	6 458 797	344 305	51	4 680 023
1·040	·858 649 947	2 182	796	0·382 579 899	1·090	·876 803 102	3 555	750	0·343 929 783
41	9 032 129	1 387	95	1 784 578	91	7 146 657	2 806	49	3 180 489
42	9 413 516	0 593	94	0 990 148	92	7 489 463	2 058	48	2 432 143
43	9 794 109	379 800	93	0 196 611	93	7 831 522	1 311	47	1 684 746
44	·860 173 909	79 008	92	0·379 403 968	94	8 172 833	340 565	46	0 938 298
1·045	·860 552 918	8 217	791	0·378 612 221	1·095	·878 513 399	339 820	745	0·340 192 800
46	0 931 134	7 426	90	7 821 370	96	8 853 219	9 076	44	0·339 448 254
47	1 308 561	6 637	90	7 031 416	97	9 192 295	8 333	43	8 704 659
48	1 685 197	5 848	89	6 242 362	98	9 530 629	7 591	42	7 962 018
49	2 061 046	375 061	88	5 454 209	99	9 868 220	336 850	41	7 220 331

TABLE OF THE VALUES OF $H = \frac{2}{\sqrt{\pi}}\int_0^t \epsilon^{-t^2} dt$. (1) FROM $t=0$ TO $t=1{\cdot}250$.

t	H	Δ +	Δ_2 −	$\frac{2}{\sqrt{\pi}}\epsilon^{-t^2}$	t	H	Δ +	Δ_2 −	$\frac{2}{\sqrt{\pi}}\epsilon^{-t^2}$
1·100	·880 205 070				1·150	·896 123 843			
		336 110	740	0·336 479 598			300 332	692	0·300 677 276
01	0 541 179	5 370	39	5 739 821	51	6 424 175	299 641	91	0·299 986 213
02	0 876 550	4 632	38	5 001 000	52	6 723 816	8 952	90	9 296 140
03	1 211 182	3 895	37	4 263 136	53	7 022 767	8 263	89	8 607 056
04	1 545 076	333 158	36	3 526 231	54	7 321 030	297 575	88	7 918 964
1·105	·881 878 234	2 423	735	0·332 790 285	1·155	·897 618 605	6 889	687	0·297 231 863
06	2 210 657	1 688	35	2 055 298	56	7 915 494	6 203	86	6 545 753
07	2 542 345	0 955	34	1 321 273	57	8 211 697	5 518	85	5 860 635
08	2 873 300	0 222	33	0 588 208	58	8 507 216	4 835	84	5 176 510
09	3 203 522	329 490	32	0·329 856 106	59	8 802 051	294 152	83	4 493 378
1 110	·883 533 012	8 760	731	0·329 124 967	1·160	·899 096 203	3 471	682	0·293 811 239
11	3 861 772	8 030	30	8 394 791	61	9 389 673	2 790	81	3 130 094
12	4 189 802	7 301	29	7 665 581	62	9 682 463	2 110	80	2 449 943
13	4 517 104	6 574	28	6 937 335	63	9 974 574	1 432	79	1 770 787
14	4 843 677	325 847	27	6 210 056	64	·900 266 005	290 754	78	1 092 625
1·115	·885 169 524	5 121	726	0·325 483 743	1·165	·900 556 759	0 077	677	0·290 415 459
16	5 494 645	4 396	25	4 758 398	66	0 846 837	289 402	76	0·289 739 289
17	5 819 041	3 672	24	4 034 022	67	1 136 238	8 727	75	9 064 116
18	6 142 713	2 949	23	3 310 615	68	1 424 965	8 053	74	8 389 938
19	6 465 663	322 227	22	2 588 177	69	1 713 018	287 381	73	7 716 758
1·120	·886 787 890	1 506	721	0·321 866 710	1·170	·902 000 399	6 709	672	0·287 044 575
21	7 109 397	0 786	20	1 146 215	71	2 287 108	6 038	71	6 373 389
22	7 430 183	0 067	19	0 426 691	72	2 573 146	5 369	70	5 703 202
23	7 750 250	319 349	18	0·319 708 140	73	2 858 515	4 700	69	5 034 013
24	8 069 600	318 632	17	8 990 562	74	3 143 214	284 032	68	4 365 822
1·125	·888 388 232	7 916	716	0·318 273 959	1·175	·903 427 247	3 365	667	0·283 698 631
26	8 706 148	7 201	15	7 558 330	76	3 710 612	2 700	66	3 032 439
27	9 023 349	6 487	14	6 843 676	77	3 993 312	2 035	65	2 367 247
28	9 339 835	5 774	13	6 129 998	78	4 275 347	1 371	64	1 703 054
29	9 655 609	315 061	12	5 417 298	79	4 556 718	280 709	63	1 039 862
1·130	·889 970 670	4 350	711	0·314 705 574	1·180	·904 837 427	0 047	662	0·280 377 670
31	·890 285 020	3 640	10	3 994 829	81	5 117 474	279 386	61	0·279 716 479
32	0 598 660	2 931	09	3 285 062	82	5 396 860	8 727	60	9 056 290
33	0 911 591	2 222	08	2 576 274	83	5 675 587	8 068	59	8 397 101
34	1 223 813	311 515	07	1 868 466	84	5 953 655	277 410	58	7 738 915
1·135	·891 535 328	0 809	706	0·311 161 639	1·185	·906 231 065	6 754	657	0·277 081 730
36	1 846 137	0 103	05	0 455 793	86	6 507 819	6 098	56	6 425 547
37	2 156 240	309 399	04	0·309 750 928	87	6 783 916	5 443	55	5 770 367
38	2 465 639	8 696	03	9 047 046	88	7 059 360	4 790	54	5 116 190
39	2 774 335	307 993	02	8 344 146	89	7 334 149	274 137	53	4 463 015
1·140	·893 082 328	7 292	701	0·307 642 230	1·190	·907 608 286	3 485	652	0·273 810 844
41	3 389 619	6 591	00	6 941 298	91	7 881 771	2 835	51	3 159 676
42	3 696 211	5 892	699	6 241 350	92	8 154 606	2 185	50	2 509 511
43	4 002 102	5 193	98	5 542 387	93	8 426 791	1 536	49	1 860 350
44	4 307 296	304 496	97	4 844 410	94	8 698 327	270 889	48	1 212 194
1·145	·894 611 791	3 799	696	0·304 147 420	1·195	·908 969 215	0 242	647	0·270 565 041
46	4 915 591	3 104	96	3 451 415	96	9 239 457	269 596	46	0·269 918 893
47	5 218 695	2 409	95	2 756 398	97	9 509 053	8 952	45	9 273 749
48	5 521 104	1 716	94	2 062 369	98	9 778 005	8 308	44	8 629 610
49	5 822 820	301 023	93	1 369 328	99	·910 046 313	267 665	43	7 986 476

TABLE OF THE VALUES OF $H = \frac{2}{\sqrt{\pi}}\int_0^t \epsilon^{-t^2} dt$. (1) FROM $t=0$ TO $t=1{\cdot}250$.

1·200] 1·249] [1·25 [6·0

t	H	Δ_1 +	Δ_2 −	$\frac{2}{\sqrt{\pi}}\epsilon^{-t^2}$	t	$\frac{2}{\sqrt{\pi}}\epsilon^{-t^2}$	t	$\frac{2}{\sqrt{\pi}}\epsilon^{-t^2}$
1·200	·910 313 978	267 024	642	0·267 344 347	1·25	·236 521 122 447 291	2·00	·020 666 985 354 092
1	0 581 002	6 383	1	6 703 223	1·26	·230 658 328 140 766	2·02	·019 070 402 324 130
2	0 847 385	5 743	0	6 063 105	1·27	·224 895 874 843 793	2·04	·017 583 087 747 389
3	1 113 128	5 105	639	5 423 992	1·28	·219 233 531 734 808	2·06	·016 198 805 688 967
4	1 378 233	264 467	8	4 785 885	1·29	·213 671 014 478 186	2·08	·014 911 571 415 508
1·205	·911 642 701	3 831	637	0·264 148 783	1·30	·208 207 986 796 070	2·10	·013 715 649 999 807
6	1 906 531	3 195	6	3 512 687	1·31	·202 844 062 051 747	2·12	·012 605 554 077 274
7	2 169 726	2 560	5	2 877 598	1·32	·197 578 804 842 403	2·15	·011 089 930 302 398
8	2 432 287	1 927	4	2 243 514	1·33	·192 411 732 599 137	2·17	·010 171 986 461 662
9	2 694 214	261 294	3	1 610 437	1·34	·187 342 317 192 141	2·20	·008 922 155 064 916
1·210	·912 955 508	0 663	632	0·260 978 366	1·35	·182 369 986 539 023	2·25	·007 142 319 022 018
11	3 216 171	0 032	1	0 347 302	1·36	·177 494 126 214 278	2·30	·005 689 017 242 525
12	3 476 203	259 403	0	0·259 717 244	1·37	·172 714 081 057 979	2·35	·004 508 829 189 593
13	3 735 606	8 774	629	9 088 193	1·38	·168 029 156 781 793	2·40	·003 555 648 680 878
14	3 994 380	258 147	8	8 460 148	1·39	·163 438 621 570 507	2·45	·002 789 988 619 011
1·215	·914 252 526	7 520	627	0·257 833 110	1·40	·158 941 707 677 278	2·50	·002 178 284 230 353
16	4 510 046	6 894	6	7 207 079	1·41	·154 537 613 010 895	2·55	·001 692 213 637 679
17	4 766 941	6 270	5	6 582 055	1·42	·150 225 502 713 389	2·60	·001 308 050 049 723
18	5 023 211	5 646	4	5 958 038	1·43	·146 004 510 726 399	2·65	·001 006 055 779 156
19	5 278 857	255 024	3	5 335 028	1·44	·141 873 741 344 739	2·70	·000 769 924 759 855
1·220	·915 533 881	4 402	621	0·254 713 024	1·45	·137 832 270 755 693	2·75	·000 586 277 247 094
21	5 788 283	3 782	0	4 092 028	1·46	·133 879 148 562 625	2·80	·000 444 207 944 206
22	6 042 065	3 162	619	3 472 039	1·47	·130 013 399 291 529	2·85	·000 334 886 877 468
23	6 295 228	2 544	8	2 853 057	1·48	·126 234 023 879 239	2·90	·000 251 210 892 521
24	6 547 772	251 927	7	2 235 082	1·49	·122 540 001 142 057	2·95	·000 187 502 615 679
1·225	·916 799 698	1 310	616	0·251 618 114	1·50	·118 930 289 223 629	3·00	·000 139 253 051 983
26	7 051 008	0 695	5	1 002 153	1·52	·111 959 535 587 539	3·1	·000 075 663 266 797
27	7 301 703	0 080	4	0 387 199	1·54	·105 313 068 275 229	3·2	·000 040 297 635 533
28	7 551 783	249 467	3	0·249 773 253	1·56	·098 981 950 627 349	3·3	·000 021 037 210 443
29	7 801 250	248 854	2	9 160 313	1·58	·092 957 046 104 774	3·4	·000 010 764 921 037
1·230	·918 050 104	8 243	611	0·248 548 381	1·60	·087 229 058 633 945	3·5	*·(5)5 399 426 777 385
31	8 298 347	7 632	0	7 937 455	1·62	·081 788 571 130 589	3·6	·(5)2 654 596 844 717
32	8 545 979	7 023	609	7 327 536	1·64	·076 626 082 133 553	3·7	·(5)1 279 274 084 534
33	8 793 002	6 415	8	6 718 625	1·66	·071 732 040 494 964	3·8	·(5)0 604 286 289 322
34	9 039 417	245 807	7	6 110 720	1·68	·067 096 878 086 755	3·9	·(5)0 279 792 448 958
1·235	·919 285 224	5 201	606	0·245 503 822	1·70	·062 711 040 496 868	4·0	·(5)0 126 982 346 719
36	9 530 425	4 595	5	4 897 931	1·72	·058 565 015 701 018	4·1	·(5)0 056 489 121 206
37	9 775 020	3 991	4	4 293 047	1·74	·054 649 360 707 757	4·2	·(5)0 024 632 040 987
38	·920 019 011	3 388	3	3 689 169	1·76	·050 954 726 185 724	4·3	·(5)0 010 528 102 122
39	0 262 399	242 785	2	3 086 298	1·78	·047 471 879 092 242	4·4	·(8)4 410 764 694 683
1·240	·920 505 184	2 184	601	0·242 484 434	1·80	·044 191 723 332 011	4·5	·(8)1 811 305 895 909
41	0 747 368	1 584	0	1 883 575	1·82	·041 105 318 483 320	4·6	·(8)0 729 094 500 238
42	0 988 952	0 984	599	1 283 723	1·84	·038 203 896 637 112	4·7	·(8)0 287 666 940 281
43	1 229 936	0 386	8	0 684 878	1·86	·035 478 877 401 325	4·8	·(8)0 111 252 606 898
44	1 470 322	239 789	7	0 087 038	1·88	·032 921 881 129 160	4·9	·(8)0 042 173 976 220
1·245	·921 710 110	9 192	596	0·239 490 205	1·90	·030 524 740 435 448	5·0	·(8)0 015 670 866 531
46	1 949 303	8 597	5	8 894 377	1·92	·028 279 510 069 901	5·1	·(11)5 707 627 016 929
47	2 187 900	8 003	4	8 299 555	1·94	·026 178 475 220 036	5·3	·(11)0 713 055 054 375
48	2 425 902	7 409	3	7 705 739	1·96	·024 214 158 319 685	5·5	·(11)0 082 233 160 452
49	2 663 311	236 817	592	7 112 928	1·98	·022 379 324 441 554	6·0	·(14)0 261 730 123 925

* The figures in parentheses indicate the number of ciphers between the decimal point and the figures that follow.

TABLE OF THE VALUES OF $H=\frac{2}{\sqrt{\pi}}\int_0^t \epsilon^{-t^2}dt$. (2) FROM $t=1{\cdot}000$ TO $t=3{\cdot}000$.

t	H	Δ_1 +	Δ_2 −	Δ_3 +	Δ_4 +	$\log\frac{2}{\sqrt{\pi}}\epsilon^{-t^2}+10.$
1·000	0·842 700 792 949 715	415 522 743 218	830 214 856	829 383	1660	9·618 160 577 413 6624
1	3 115 485 478 076	4 692 528 361	829 383 813	831 043	1652	7 291 554 155 3740
2	3 529 348 622 624	3 863 144 548	8 551 118	2 695	1644	6 421 662 308 1218
3	3 942 383 216 054	3 034 593 430	7 716 779	4 339	1636	5 550 901 871 9058
4	4 354 590 092 706	2 206 876 651	6 880 804	5 975	1627	4 679 272 846 7260
1·005	0·844 765 970 088 552	411 379 995 847	826 043 202	837 602	1619	9·613 806 775 232 5823
6	5 176 524 041 197	0 553 952 644	5 203 982	9 221	1611	2 933 409 029 4749
7	5 586 252 789 859	409 728 748 663	4 363 150	840 832	1602	2 059 174 237 4037
8	5 995 157 175 372	8 904 385 513	3 520 716	2 434	1594	1 184 070 856 3686
9	6 403 238 040 168	8 080 864 796	2 676 688	4 028	1586	0 308 098 886 3697
1·010	0·846 810 496 228 277	407 258 188 108	821 831 074	845 614	1578	9·609 431 258 327 4071
11	7 216 932 585 311	6 436 357 034	0 983 883	7 192	1569	8 553 549 179 4806
12	7 622 547 958 462	5 615 273 152	0 135 122	8 761	1561	7 674 971 442 5903
13	8 027 343 196 492	4 795 238 030	819 284 800	850 322	1553	6 795 525 116 7362
14	8 431 319 149 722	3 975 953 230	8 432 925	1 875	1545	5 915 210 201 9184
1·015	0·848 834 476 670 026	403 157 520 304	817 579 506	853 419	1536	9·605 034 026 698 1367
16	9 236 816 610 825	2 339 940 798	6 724 550	4 956	1528	4 151 974 605 3911
17	9 638 339 827 073	1 523 216 248	5 868 067	6 484	1520	3 269 053 923 6818
18	0·850 039 047 175 254	0 707 348 181	5 010 063	8 003	1512	2 385 264 653 0087
19	0 438 939 513 372	399 892 338 118	4 150 548	9 515	1503	1 500 606 793 3718
1·020	0·850 838 017 700 942	399 078 187 570	813 289 530	861 018	1495	9·600 615 080 344 7711
21	1 236 282 598 982	8 264 898 040	2 427 016	2 514	1487	9·599 728 685 307 2065
22	1 633 735 070 006	7 452 471 024	1 563 016	4 001	1479	8 841 421 680 6782
23	2 030 375 978 015	6 640 908 008	0 697 536	5 479	1471	7 953 289 465 1860
24	2 426 206 188 487	5 830 210 472	809 830 586	6 950	1462	7 064 288 660 7301
1·025	0·852 821 226 568 372	395 020 379 886	808 962 174	868 412	1454	9·596 174 419 267 3103
26	3 215 437 986 084	4 211 417 711	8 092 308	9 866	1446	5 283 681 284 9267
27	3 608 841 311 487	3 403 325 403	7 220 996	871 312	1438	4 392 074 713 5794
28	4 001 437 415 894	2 596 104 407	6 348 246	2 750	1430	3 499 599 553 2682
29	4 393 227 172 054	1 789 756 161	5 474 067	4 179	1421	2 606 255 803 9932
1·030	0·854 784 211 454 148	390 984 282 094	804 598 466	875 601	1413	9·591 712 043 465 7544
31	5 174 391 137 776	0 179 683 628	3 721 452	7 014	1405	0 816 962 538 5518
32	5 563 767 099 952	389 375 962 176	2 843 033	8 419	1397	9·589 921 013 022 3854
33	5 952 340 219 095	8 573 119 143	1 963 217	9 816	1389	9 024 194 917 2552
34	6 340 111 375 020	7 771 155 925	1 082 013	881 205	1381	8 126 508 223 1612
1·035	0·856 727 081 448 933	386 970 073 913	800 199 427	882 585	1372	9·587 227 952 940 1033
36	7 113 251 323 418	6 169 874 485	799 315 470	3 958	1364	6 328 528 068 0817
37	7 498 621 882 434	5 370 559 015	8 430 148	5 322	1356	5 428 236 607 0963
38	7 883 194 011 301	4 572 128 867	7 543 470	6 678	1348	4 527 075 557 1470
39	8 266 968 596 698	3 774 585 397	6 655 444	8 026	1340	3 625 045 918 2340
1·040	0·858 649 946 526 651	382 977 929 953	795 766 078	889 366	1332	9·582 722 147 690 3571
41	9 032 128 690 527	2 182 163 875	4 875 380	890 698	1324	1 818 380 873 5164
42	9 413 515 979 022	1 387 288 495	3 983 359	2 021	1316	0 913 745 467 7120
43	9 794 109 284 158	0 593 305 136	3 090 022	3 337	1307	0 008 241 472 9437
44	0·860 173 909 499 272	379 800 215 114	2 195 377	4 645	1299	9·579 101 868 889 2116
1·045	0·860 552 917 519 009	379 008 019 737	791 299 433	895 944	1291	9·578 194 627 716 5157
46	0 931 134 239 312	8 216 720 303	0 402 198	7 235	1283	7 286 517 954 8560
47	1 308 560 557 417	7 426 318 105	789 503 680	8 518	1275	6 377 539 604 2325
48	1 685 197 371 843	6 636 814 426	8 603 886	9 794	1267	5 467 692 664 6452
49	2 061 045 582 382	5 848 210 540	7 702 825	901 061	1259	4 556 977 136 0941
		375 060 507 714		902 320		

TABLE OF THE VALUES OF $H = \frac{2}{\sqrt{\pi}}\int_0^t \epsilon^{-t^2} dt$. (2) FROM $t = 1{\cdot}000$ TO $t = 3{\cdot}000$.

1·050] [1·099

t	H	Δ_1 +	Δ_2 −	Δ_3 +	Δ_4 +	$\log \frac{2}{\sqrt{\pi}}\epsilon^{-t^2} + 10.$
1·050	0·862 436 106 090 097	374 273 707 209	786 800 505	903 571	1251	9·573 645 393 018 5791
51	2 810 379 797 306	3 487 810 274	5 896 935	4 814	1243	2 732 940 312 1004
52	3 183 867 607 580	2 702 818 154	4 992 121	6 049	1235	1 819 619 016 6579
53	3 556 570 425 734	1 918 732 082	4 086 072	7 276	1227	0 905 429 132 2515
54	3 928 489 157 815	371 135 553 286	3 178 796	908 495	1219	9·569 990 370 658 8814
1·055	0·864 299 624 711 101	0 353 282 984	782 270 302	9 705	1211	9·569 074 443 596 5474
56	4 669 977 994 085	369 571 922 388	1 360 596	910 908	1203	8 157 647 945 2496
57	5 039 549 916 473	8 791 472 700	0 449 688	2 103	1195	7 239 983 704 9881
58	5 408 341 389 172	8 011 935 115	779 537 585	3 290	1187	6 321 450 875 7627
59	5 776 353 324 288	367 233 310 821	8 624 295	914 469	1179	5 402 049 457 5735
1·060	0·866 143 586 635 108	6 455 600 995	777 709 826	5 640	1171	9·564 481 779 450 4205
61	6 510 042 236 103	5 678 806 809	6 794 186	6 803	1163	3 560 640 854 3037
62	6 875 721 042 912	4 902 929 426	5 877 383	7 958	1155	2 638 633 669 2231
63	7 240 623 972 338	4 127 970 001	4 959 425	9 105	1147	1 715 757 895 1787
64	7 604 751 942 340	363 353 929 681	4 040 320	920 244	1139	0 792 013 532 1705
1·065	0·867 968 105 872 021	2 580 809 605	773 120 076	1 375	1131	9·559 867 400 580 1985
66	8 330 686 681 626	1 808 610 904	2 198 701	2 498	1123	8 941 919 039 2626
67	8 692 495 292 529	1 037 334 701	1 276 203	3 614	1115	8 015 568 909 3630
68	9 053 532 627 231	0 266 982 112	0 352 589	4 721	1107	7 088 350 190 4996
69	9 413 799 609 343	359 497 554 244	769 427 868	925 821	1099	6 160 262 882 6723
1·070	0·869 773 297 163 587	8 729 052 196	768 502 048	6 912	1092	9·555 231 306 985 8813
71	0·870 132 026 215 783	7 961 477 061	7 575 135	7 996	1084	4 301 482 500 1264
72	0 489 987 692 844	7 194 829 921	6 647 140	9 072	1076	3 370 789 425 4077
73	0 847 182 522 766	6 429 111 853	5 718 068	930 140	1068	2 439 227 761 7252
74	1 203 611 634 619	355 664 323 925	4 787 929	931 200	1060	1 506 797 509 0790
1·075	0·871 559 275 958 544	4 900 467 196	763 856 729	2 252	1052	9·550 573 498 667 4689
76	1 914 176 425 740	4 137 542 719	2 924 477	3 296	1044	9·549 639 331 236 8950
77	2 268 313 968 459	3 375 551 538	1 991 181	4 333	1037	8 704 295 217 3573
78	2 621 689 519 997	2 614 494 690	1 056 848	5 361	1029	7 768 390 608 8558
79	2 974 304 014 687	351 854 373 203	0 121 487	936 382	1021	6 831 617 411 3905
1·080	0·873 326 158 387 890	1 095 188 099	759 185 105	7 395	1013	9·545 893 975 624 9613
81	3 677 253 575 988	0 336 940 389	8 247 709	8 401	1005	4 955 465 249 5684
82	4 027 590 516 377	349 579 631 081	7 309 309	9 398	997	4 016 086 285 2117
83	4 377 170 147 458	8 823 261 170	6 369 911	940 388	990	3 075 838 731 8911
84	4 725 993 408 628	348 067 831 647	5 429 523	941 370	982	2 134 722 589 6068
1·085	0·875 074 061 240 276	7 313 343 494	754 488 153	2 344	974	9·541 192 737 858 3586
86	5 421 374 583 770	6 559 797 685	3 545 809	3 310	966	0 249 884 538 1467
87	5 767 934 381 455	5 807 195 185	2 602 499	4 269	959	9·539 306 162 628 9709
88	6 113 741 576 640	5 055 536 955	1 658 231	5 220	951	8 361 572 130 8313
89	6 458 797 113 595	344 304 823 943	0 713 011	946 163	943	7 416 113 043 7280
1·090	0·876 803 101 937 538	3 555 057 095	749 766 848	7 098	935	9·536 469 785 367 6608
91	7 146 656 994 633	2 806 237 345	8 819 750	8 026	928	5 522 589 102 6298
92	7 489 463 231 978	2 058 365 620	7 871 725	8 946	920	4 574 524 248 6350
93	7 831 521 597 598	1 311 442 841	6 922 779	9 858	912	3 625 590 805 6764
94	8 172 833 040 440	340 565 469 921	5 972 921	950 763	905	2 675 788 773 7540
1·095	0·878 513 398 510 361	339 820 447 763	745 022 158	1 660	897	9·531 725 118 152 8677
96	8 853 218 958 123	9 076 377 265	4 070 498	2 549	889	0 773 578 943 0177
97	9 192 295 335 388	8 333 259 315	3 117 949	3 431	882	9·529 821 171 144 2039
98	9 530 628 594 703	7 591 094 796	2 164 519	4 304	874	8 867 894 756 4262
99	9 868 219 689 500	336 849 884 582	1 210 214	955 171	866	7 913 749 779 6848

TABLE OF THE VALUES OF $H=\frac{2}{\sqrt{\pi}}\int_0^t \epsilon^{-t^2}dt$. (2) FROM $t=1{\cdot}000$ TO $t=3{\cdot}000$.

t	H	Δ_1 +	Δ_2 −	Δ_3 +	Δ_4 +	$\log\frac{2}{\sqrt{\pi}}\epsilon^{-t^2}+10.$
1·100	0·880 205 069 574 082	336 109 629 538	740 255 044	956 030	859	9·526 958 736 213 9796
1	0 541 179 203 620	5 370 330 524	739 299 014	6 881	851	6 002 854 059 3105
2	0 876 549 534 144	4 631 988 391	8 342 134	7 724	843	5 046 103 315 6776
3	1 211 181 522 535	3 894 603 981	7 384 409	8 560	836	4 088 483 983 0810
4	1 545 076 126 517	333 158 178 132	6 425 850	959 388	828	3 129 996 061 5205
1·105	0·881 878 234 304 648	2 422 711 671	735 466 461	960 209	821	9·522 170 639 550 9962
6	2 210 657 016 319	1 688 205 418	4 506 252	1 022	813	1 210 414 451 5081
7	2 542 345 221 737	0 954 660 188	3 545 230	1 828	806	0 249 320 763 0562
8	2 873 299 881 925	0 222 076 785	2 583 403	2 626	798	9·519 287 358 485 6405
9	3 203 521 958 710	329 490 456 008	1 620 777	963 416	791	8 324 527 619 2610
1·110	0·883 533 012 414 718	8 759 798 647	730 657 361	4 199	783	9·517 360 828 163 9177
11	3 861 772 213 365	8 030 105 486	729 693 162	4 975	775	6 396 260 119 6106
12	4 189 802 318 851	7 301 377 299	8 728 187	5 743	768	5 430 823 486 3396
13	4 517 103 696 150	6 573 614 854	7 762 444	6 503	760	4 464 518 264 1049
14	4 843 677 311 004	325 846 818 913	6 795 941	967 256	753	3 497 344 452 9064
1·115	0 885 169 524 129 917	5 120 990 227	725 828 685	8 002	746	9·512 529 302 052 7440
16	5 494 645 120 144	4 396 129 543	4 860 684	8 740	738	1 560 391 063 6179
17	5 819 041 249 687	3 672 237 599	3 891 944	9 470	731	0 590 611 485 5279
18	6 142 713 487 286	2 949 315 125	2 922 474	970 193	723	9·509 619 963 318 4741
19	6 465 662 802 411	322 227 362 844	1 952 281	970 909	716	8 648 446 562 4566
1·120	0·886 787 890 165 255	1 506 381 472	720 981 372	1 617	708	9·507 676 061 217 4752
21	7 109 396 546 726	0 786 371 717	0 009 755	2 318	701	6 702 807 283 5300
22	7 430 182 918 443	0 067 334 280	719 037 437	3 012	693	5 728 684 760 2099
23	7 750 250 252 724	319 349 269 856	8 064 425	3 698	686	4 753 693 648 7482
24	8 069 599 522 579	318 632 179 128	7 090 727	974 376	679	3 777 833 947 9116
1·125	0·888 388 231 701 708	7 916 062 778	716 116 351	5 048	671	9·502 801 105 658 1112
26	8 706 147 764 485	7 200 921 475	5 141 303	5 712	664	1 823 508 779 3470
27	9 023 348 685 960	6 486 755 884	4 165 591	6 369	657	0 845 043 311 6189
28	9 339 835 441 844	5 773 566 661	3 189 222	7 018	649	9·499 865 709 254 9271
29	9 655 609 008 506	315 061 354 457	2 212 205	977 660	642	8 885 506 609 2715
1·130	0·889 970 670 362 962	4 350 119 912	711 234 545	8 295	635	9·497 904 435 374 6520
31	0·890 285 020 482 874	3 639 863 662	0 256 250	8 922	627	6 922 495 551 0688
32	0 598 660 346 536	2 930 586 334	709 277 328	9 542	620	5 939 687 138 5217
33	0 911 590 932 870	2 222 288 547	8 297 786	980 155	613	4 956 010 137 0108
34	1 223 813 221 417	311 514 970 916	7 317 631	980 761	606	3 971 464 546 5362
1·135	0·891 535 328 192 333	0 808 634 046	706 336 871	1 359	598	9·492 986 050 367 0977
36	1 846 136 826 379	0 103 278 534	5 355 512	1 950	591	1 999 767 598 6954
37	2 156 240 104 913	309 398 904 972	4 373 562	2 534	584	1 012 616 241 3293
38	2 465 639 009 885	8 695 513 944	3 391 028	3 111	577	0 024 596 294 9994
39	2 774 334 523 830	307 993 106 027	2 407 917	983 680	569	·9489 035 707 759 7057
1·140	0·893 082 327 629 857	7 291 681 790	701 424 237	4 242	562	9·488 045 950 635 4482
41	3 389 619 311 646	6 591 241 795	0 439 995	4 797	555	7 055 324 922 2269
42	3 696 210 553 441	5 891 786 597	699 455 198	5 345	548	6 063 830 620 0418
43	4 002 102 340 038	5 193 316 744	8 469 853	5 886	541	5 071 467 728 8928
44	4 307 295 656 782	304 495 832 778	7 483 967	986 420	534	4 078 236 248 7801
1·145	0·894 611 791 489 560	3 799 335 231	696 497 547	6 946	526	9·483 084 136 179 7035
46	4 915 590 824 791	3 103 824 630	5 510 601	7 465	519	2 089 167 521 6632
47	5 218 694 649 421	2 409 301 495	4 523 135	7 978	512	1 093 330 274 6590
48	5 521 103 950 916	1 715 766 337	3 535 158	8 483	505	0 096 624 438 6911
49	5 822 819 717 253	301 023 219 662	2 546 675	988 981	498	9·479 099 050 013 7593

TABLE OF THE VALUES OF $H = \frac{2}{\sqrt{\pi}}\int_0^t \epsilon^{-t^2} dt$. (2) FROM $t = 1{\cdot}000$ TO $t = 3{\cdot}000$.

t	H	Δ_1 +	Δ_2 −	Δ_3 +	Δ_4 +	$\log \frac{2}{\sqrt{\pi}}\epsilon^{-t^2} + 10.$
1·150	0·896 123 842 936 915	300 331 661 969	691 557 694	989 472	491	9·478 100 606 999 8637
51	6 424 174 598 884	299 641 093 747	0 568 222	9 956	484	7 101 295 397 0043
52	6 723 815 692 630	8 951 515 481	689 578 266	990 433	477	6 101 115 205 1812
53	7 022 767 208 111	8 262 927 648	8 587 833	0 903	470	5 100 066 424 3942
54	7 321 030 135 759	297 575 330 718	7 596 930	991 366	463	4 098 049 054 6434
1·155	0·897 618 605 466 477	6 888 725 153	686 605 565	1 821	456	9·473 095 363 095 9287
56	7 915 494 191 630	6 203 111 409	5 613 743	2 270	449	2 091 708 548 2503
57	8 211 697 303 039	5 518 489 936	4 621 473	2 712	442	1 087 185 411 6081
58	8 507 215 792 976	4 834 861 175	3 628 761	3 147	435	0 081 793 686 0021
59	8 802 050 654 151	294 152 225 561	2 635 614	993 575	428	9·469 075 533 371 4322
1·160	0·899 096 202 879 712	3 470 583 522	681 642 039	3 996	421	9·468 068 404 467 8986
61	9 389 673 463 234	2 789 935 479	0 648 043	4 410	414	7 060 406 975 4012
62	9 682 463 398 713	2 110 281 845	679 653 633	4 817	407	6 051 540 893 9399
63	9 974 573 680 558	1 431 623 029	8 658 816	5 217	400	5 041 806 223 5148
64	0·900 266 005 303 587	290 753 959 430	7 663 599	995 611	393	4 031 202 964 1260
1·165	0·900 556 759 263 017	0 077 291 442	676 667 988	5 997	386	9·463 019 731 115 7733
66	0 846 836 554 459	289 401 619 450	5 671 991	6 377	380	2 007 390 678 4568
67	1 136 238 173 909	8 726 943 836	4 675 615	6 749	373	0 994 181 652 1765
68	1 424 965 117 745	8 053 264 970	3 678 865	7 115	366	9·459 980 104 036 9325
69	1 713 018 382 715	287 380 583 220	2 681 750	997 474	359	8 965 157 832 7245
1·170	0·902 000 398 965 936	6 708 898 945	671 684 276	7 827	352	9·457 949 343 039 5528
71	2 287 107 864 880	6 038 212 496	0 686 449	8 172	345	6 932 659 657 4173
72	2 573 146 077 376	5 368 524 218	669 688 277	8 511	339	5 915 107 686 3180
73	2 858 514 601 595	4 699 834 452	8 689 767	8 842	332	4 896 687 126 2549
74	3 143 214 436 046	284 032 143 528	7 690 924	999 168	325	3 877 397 977 2280
1·175	0·903 427 246 579 574	3 365 451 771	666 691 757	9 486	318	9·452 857 240 239 2372
76	3 710 612 031 345	2 699 759 501	5 692 271	9 798	312	1 836 213 912 2827
77	3 993 311 790 846	2 035 067 027	4 692 473	1000 102	305	0 814 318 996 3643
78	4 275 346 857 873	1 371 374 657	3 692 371	0 401	298	9·449 791 555 491 4822
79	4 556 718 232 530	280 708 682 687	2 691 970	1000 692	291	8 767 923 397 6362
1·180	0·904 837 426 915 217	0 046 991 409	661 691 278	0 977	285	9·447 743 422 714 8264
81	5 117 473 906 625	279 386 301 107	0 690 301	1 255	278	6 718 053 443 0528
82	5 396 860 207 733	8 726 612 061	659 689 046	1 526	271	5 691 815 582 3155
83	5 675 586 819 794	8 067 924 542	8 687 520	1 791	265	4 664 709 132 6143
84	5 953 654 744 336	277 410 238 813	7 685 728	1002 049	258	3 636 734 093 9493
1·185	0·906 231 064 983 149	6 753 555 134	656 683 679	2 301	252	9·442 607 890 466 3205
86	6 507 818 538 283	6 097 873 756	5 681 378	2 546	245	1 578 178 249 7279
87	6 783 916 412 039	5 443 194 924	4 678 832	2 784	238	0 547 597 444 1714
88	7 059 359 606 963	4 789 518 877	3 676 048	3 016	232	9·439 516 148 049 6512
89	7 334 149 125 840	274 136 845 845	2 673 031	1003 241	225	8 483 830 066 1672
1·190	0·907 608 285 971 685	3 485 176 055	651 669 790	3 460	219	9·437 450 643 493 7194
91	7 881 771 147 741	2 834 509 726	0 666 330	3 672	212	6 416 588 332 3077
92	8 154 605 657 466	2 184 847 068	649 662 657	3 878	206	5 381 664 581 9323
93	8 426 790 504 535	1 536 188 289	8 658 779	4 077	199	4 345 872 242 5930
94	8 698 326 692 824	270 888 533 587	7 654 702	1004 270	193	3 309 211 314 2899
1·195	0·908 969 215 226 411	0 241 883 155	646 650 432	4 456	186	9·432 271 681 797 0231
96	9 239 457 109 565	269 596 237 178	5 645 976	4 636	180	1 233 283 690 7924
97	9 509 053 346 743	8 951 595 837	4 641 341	4 809	173	0 194 016 995 5979
98	9 778 004 942 581	8 307 959 305	3 636 532	4 976	167	9·429 153 881 711 4396
99	0·910 046 312 901 886	267 665 327 749	2 631 556	1005 136	160	8 112 877 838 3175

TABLE OF THE VALUES OF $H=\frac{2}{\sqrt{\pi}}\int_0^t \epsilon^{-t^2}dt$. (2) FROM $t=1{\cdot}000$ TO $t=3{\cdot}000$.

1·200] [1·249

t	H	Δ_1 +	Δ_2 −	Δ_3 +	Δ_4 + −	$\log\frac{2}{\sqrt{\pi}}\epsilon^{-t^2}+10.$
1·200	0·910 313 978 229 635	267 023 701 329	641 626 420	1005 290	+154	9·427 071 005 376 2316
1	0 581 001 930 965	6 383 080 199	0 621 130	438	148	6 028 264 325 1819
2	0 847 385 011 164	5 743 464 507	639 615 692	579	141	4 984 654 685 1684
3	1 113 128 475 671	5 104 854 394	8 610 113	714	135	3 940 176 456 1911
4	1 378 233 330 066	264 467 249 995	7 604 399	1005 842	129	2 894 829 638 2500
1·205	0·911 642 700 580 061	3 830 651 438	636 598 557	964	122	9·421 848 614 231 3450
6	1 906 531 231 498	3 195 058 845	5 592 593	1006 080	116	0 801 530 235 4763
7	2 169 726 290 344	2 560 472 333	4 586 512	190	110	9·419 753 577 650 6437
8	2 432 286 762 677	1 926 892 011	3 580 322	293	103	8 704 756 476 8474
9	2 694 213 654 688	261 294 317 982	2 574 029	1006 390	97	7 655 066 714 0872
1·210	0·912 955 507 972 669	0 662 750 342	631 567 639	481	91	9·416 604 508 362 3633
11	0·913 216 170 723 012	0 032 189 184	0 561 158	565	84	5 553 081 421 6755
12	476 202 912 196	259 402 634 591	629 554 593	643	78	4 500 785 892 0239
13	735 605 546 786	8 774 086 641	8 547 950	715	72	3 447 621 773 4085
14	994 379 633 427	258 146 545 406	7 541 234	1006 781	66	2 393 589 065 8293
1·215	0·914 252 526 178 834	7 520 010 953	626 534 453	841	60	9·411 338 687 769 2863
16	510 046 189 787	6 894 483 341	5 527 612	894	53	0 282 917 883 7795
17	766 940 673 128	6 269 962 622	4 520 718	941	47	9·409 226 279 409 3089
18	0·915 023 210 635 750	5 646 448 845	3 513 777	983	41	8 168 772 345 8745
19	278 857 084 596	255 023 942 051	2 506 794	1007 017	35	7 110 396 693 4763
1·220	0·915 533 881 026 647	4 402 442 274	621 499 777	046	29	9·406 051 152 452 1142
21	788 283 468 921	3 781 949 543	0 492 731	069	23	4 991 039 621 7884
22	0·916 042 065 418 464	3 162 463 882	619 485 662	086	17	3 930 058 202 4988
23	295 227 882 346	2 543 985 306	8 478 576	096	11	2 868 208 194 2453
24	547 771 867 652	251 926 513 826	7 471 480	1007 101	+5	1 805 489 597 0281
1·225	0·916 799 698 381 479	1 310 049 447	616 464 379	099	−2	9·400 741 902 410 8470
26	0·917 051 008 430 926	0 694 592 168	5 457 280	092	8	9·399 677 446 635 7021
27	301 703 023 094	0 080 141 980	4 450 188	078	14	8 612 122 271 5934
28	551 783 165 073	249 466 698 870	3 443 110	058	20	7 545 929 318 5210
29	801 249 863 943	248 854 262 818	2 436 051	1007 033	26	6 478 867 776 4847
1·230	0·918 050 104 126 761	8 242 833 800	611 429 019	001	32	9·395 410 937 645 4846
31	298 346 960 561	7 632 411 782	0 422 017	1006 964	38	4 342 138 925 5207
32	545 979 372 343	7 022 996 729	609 415 053	920	43	3 272 471 616 5930
33	793 002 369 072	6 414 588 596	8 408 133	871	49	2 201 935 718 7015
34	0·919 039 416 957 668	245 807 187 334	7 401 262	1006 816	55	1 130 531 231 8461
1·235	0·919 285 224 145 002	5 200 792 888	606 394 446	755	61	9·390 058 258 156 0270
36	530 424 937 890	4 595 405 196	5 387 692	687	67	9·388 985 116 491 2441
37	775 020 343 087	3 991 024 192	4 381 004	615	73	7 911 106 237 4973
38	0·920 019 011 367 279	3 387 649 802	3 374 390	536	79	6 836 227 394 7868
39	262 399 017 081	242 785 281 949	2 367 854	1006 451	85	5 760 479 963 1124
1·240	0·920 505 184 299 030	2 183 920 546	601 361 403	361	90	9·384 683 863 942 4743
41	747 368 219 576	1 583 565 504	0 355 042	264	96	3 606 379 332 8723
42	988 951 785 080	0 984 216 727	599 348 777	162	102	2 528 026 134 3065
43	0·921 229 936 001 806	0 385 874 112	8 342 615	055	108	1 448 804 346 7769
44	470 321 875 918	239 788 537 551	7 336 560	1005 941	114	0 368 713 970 2835
1·245	0·921 710 110 413 469	9 192 206 932	596 330 619	822	119	9·379 287 755 004 8264
46	949 302 620 401	8 596 882 134	5 324 798	697	125	8 205 927 450 4054
47	0·922 187 899 502 535	8 002 563 033	4 319 101	566	131	7 123 231 307 0205
48	425 902 065 568	7 409 249 498	3 313 535	429	136	6 039 666 574 6719
49	663 311 315 066	236 816 941 392	2 308 106	1005 287	142	4 955 233 253 3595

TABLE OF THE VALUES OF $H = \frac{2}{\sqrt{\pi}}\int_0^t \epsilon^{-t^2} dt$. (2) FROM $t = 1{\cdot}000$ TO $t = 3{\cdot}000$.

1·250]

[1·299

t	H	Δ_1 +	Δ_2 −	Δ_3 +	Δ_4 −	log. $\frac{2}{\sqrt{\pi}}\epsilon^{-t^2} + 10$.
1·250	0·922 900 128 256 458					9·373 869 931 343 0829
51	0·923 136 353 895 032	236 225 638 574	591 302 818	1005 139	148	
52	371 989 235 927	5 635 340 895	0 297 679	1004 986	153	2 783 760 843 8433
53	607 035 284 129	5 046 048 202	589 292 693	827	159	1 696 721 755 6394
54	841 493 044 464	4 457 760 336	8 287 866	662	165	0 608 814 078 4718
1·255	0·924 075 363 521 596	233 870 477 132	7 283 204	1004 492	170	9·369 520 037 812 3403
56	308 647 720 015	3 284 198 419	586 278 712	316	176	9·368 430 392 957 2451
57	541 346 644 038	2 698 924 023	5 274 396	134	182	7 339 879 513 1860
58	773 461 297 799	2 114 653 761	4 270 262	1003 947	187	6 248 497 480 1631
59	0·925 004 992 685 244	1 531 387 446	3 266 315	755	193	5 156 246 858 1764
1·260	0·925 235 941 810 130	230 949 124 885	2 262 560	1003 556	198	4 063 127 647 2260
61	466 309 676 011	0 367 865 881	581 259 004	353	204	9·362 969 139 847 3117
62	696 097 286 241	229 787 610 230	0 255 651	144	209	1 874 283 458 4336
63	925 305 643 964	9 208 357 723	579 252 508	1002 929	215	0 778 558 480 5917
64	0·926 153 935 752 108	8 630 108 144	8 249 579	709	220	9·359 681 964 913 7860
1·265	0·926 381 988 613 383	228 052 861 275	7 246 870	1002 483	226	8 584 502 758 0164
66	609 465 230 271	7 476 616 888	576 244 386	252	231	9·357 486 172 013 2831
67	836 366 605 025	6 901 374 754	5 242 134	016	236	6 386 972 679 5860
68	0·927 062 693 739 660	6 327 134 635	4 240 118	1001 774	242	5 286 904 756 9250
69	288 447 635 951	5 753 896 291	3 238 344	527	247	4 185 968 245 3003
1·270	0·927 513 629 295 425	225 181 659 473	2 236 818	1001 274	253	3 084 163 144 7118
71	738 239 719 354	4 610 423 930	571 235 544	016	258	9·351 981 489 455 1594
72	962 279 908 757	4 040 189 402	0 234 527	1000 753	263	0 877 947 176 6432
73	0·928 185 750 864 384	3 470 955 628	569 233 775	484	269	9·349 773 536 309 1633
74	408 653 586 721	2 902 722 337	8 233 290	210	274	8 668 256 852 7195
1·275	0·928 630 989 075 979	222 335 489 257	7 233 080	999 931	279	7 562 108 807 3119
76	852 758 332 086	1 769 256 108	566 233 149	9 646	285	9·346 455 092 172 9405
77	0·929 073 962 354 692	1 204 022 605	5 233 503	9 357	290	5 347 206 949 6053
78	294 602 143 151	0 639 788 459	4 234 146	9 062	295	4 238 453 137 3063
79	514 678 696 526	0 076 553 375	3 235 084	8 761	300	3 128 830 736 0435
1·280	0·929 734 193 013 578	219 514 317 052	2 236 323	998 456	305	2 018 339 745 8169
81	953 146 092 763	8 953 079 185	561 237 867	8 145	311	9·340 906 980 166 6265
82	0·930 171 538 932 226	8 392 839 463	0 239 722	7 829	316	9·339 794 751 998 4722
83	389 372 529 796	7 833 597 570	559 241 893	7 508	321	8 681 655 241 3542
84	606 647 882 982	7 275 353 186	8 244 384	7 182	326	7 567 689 895 2724
1·285	0·930 823 365 988 965	216 718 105 983	7 247 202	996 851	331	6 452 855 960 2267
86	0·931 039 527 844 596	6 161 855 631	556 250 352	6 514	336	9·335 337 153 436 2173
87	255 134 446 390	5 606 601 794	5 253 838	6 173	342	4 220 582 323 2440
88	470 186 790 519	5 052 344 129	4 257 665	5 826	347	3 103 142 621 3069
89	684 685 872 809	4 499 082 290	3 261 839	5 474	352	1 984 834 330 4061
1·290	0·931 898 632 688 734	213 946 815 925	2 266 365	995 117	357	0 865 657 450 5414
91	0·932 112 028 233 411	3 395 544 677	551 271 248	4 756	362	9·329 745 611 981 7129
92	324 873 501 596	2 845 268 185	0 276 492	4 389	367	8 624 697 923 9206
93	537 169 487 678	2 295 986 082	549 282 103	4 017	372	7 502 915 277 1645
94	748 917 185 674	1 747 697 995	8 288 087	3 640	377	6 380 264 041 4446
1·295	0·932 960 117 589 223	211 200 403 549	7 294 447	993 258	382	5 256 744 216 7609
96	0·933 170 771 691 583	0 654 102 360	546 301 189	2 871	387	9·324 132 355 803 1134
97	380 880 485 625	0 108 794 042	5 308 318	2 479	392	3 007 098 800 5020
98	590 444 963 829	209 564 478 204	4 315 838	2 083	397	1 880 973 208 9269
99	799 466 118 278	9 021 154 449	3 323 755	1 681	402	0 753 979 028 3880
		208 478 822 374	2 332 074	991 275	407	9·319 626 116 258 8852

TABLE OF THE VALUES OF $H=\frac{2}{\sqrt{\pi}}\int_0^t \epsilon^{-t^2}dt$. (2) FROM $t=1{\cdot}000$ TO $t=3{\cdot}000$.

1·300] [1·349

t	H	Δ_1 +	Δ_2 −	Δ_3 +	Δ_4 −	log. $\frac{2}{\sqrt{\pi}}\epsilon^{-t^2}+10$.
1·300	0·934 007 944 940 652	207 937 481 574	541 340 800	990 863	411	9·318 497 384 900 4187
1	215 882 422 227	7 397 131 638	0 349 937	0 447	416	7 367 784 952 9883
2	423 279 553 864	6 857 772 147	539 359 490	0 026	421	6 237 316 416 5942
3	630 137 326 012	6 319 402 683	8 369 464	989 600	426	5 105 979 291 2362
4	836 456 728 695	205 782 022 818	7 379 865	989 169	431	3 973 773 576 9144
1·305	0·935 042 238 751 513	5 245 632 123	536 390 696	8 733	436	9·312 840 699 273 6288
6	247 484 383 636	4 710 230 160	5 401 963	8 293	440	1 706 756 381 3794
7	452 194 613 796	4 175 816 490	4 413 670	7 848	445	0 571 944 900 1662
8	656 370 430 256	3 642 390 669	3 425 822	7 398	450	9·309 436 264 829 9892
9	860 012 820 955	203 109 952 245	2 438 424	986 943	455	8 299 716 170 8484
1·310	0·936 063 122 773 200	2 578 500 764	531 451 481	6 484	459	9·307 162 298 922 7438
11	265 701 273 963	2 048 035 767	0 464 997	6 020	464	6 024 013 085 6754
12	467 749 309 730	1 518 556 789	529 478 977	5 551	469	4 884 858 659 6432
13	669 267 866 519	0 990 063 363	8 493 426	5 077	473	3 744 835 644 6471
14	870 257 929 882	200 462 555 014	7 508 349	984 599	478	2 603 944 040 6873
1·315	0·937 070 720 484 896	199 936 031 264	526 523 750	4 117	483	9·301 462 183 847 7636
16	270 656 516 160	9 410 491 631	5 539 633	3 629	487	0 319 555 065 8762
17	470 067 007 791	8 885 935 627	4 556 004	3 137	492	9·299 176 057 695 0249
18	668 952 943 418	8 362 362 760	3 572 867	2 641	497	8 031 691 735 2098
19	867 315 306 178	197 839 772 534	2 590 226	982 139	501	6 886 457 186 4310
1·320	0·938 065 155 078 711	7 318 164 447	521 608 087	1 634	506	9·295 740 354 048 6883
21	262 473 243 158	6 797 537 994	0 626 453	1 123	510	4 593 382 321 9818
22	459 270 781 152	6 277 892 664	519 645 330	0 608	515	3 445 542 006 3115
23	655 548 673 815	5 759 227 942	8 664 722	0 089	519	2 296 833 101 6774
24	851 307 901 757	195 241 543 310	7 684 632	979 565	524	1 147 255 608 0795
1·325	0·939 046 549 445 067	4 724 838 243	516 705 067	9 037	528	9·289 996 809 525 5178
26	241 274 283 310	4 209 112 213	5 726 030	8 504	533	8 845 494 853 9923
27	435 483 395 522	3 694 364 687	4 747 526	7 967	537	7 693 311 593 5029
28	629 177 760 209	3 180 595 128	3 769 559	7 425	542	6 540 259 744 0498
29	822 358 355 336	192 667 802 994	2 792 134	976 879	546	5 386 339 305 6329
1·330	0·940 015 026 158 330	2 155 987 739	511 815 255	6 329	551	9·284 231 550 278 2521
31	207 182 146 070	1 645 148 813	0 838 926	5 774	555	3 075 892 661 9076
32	398 827 294 883	1 135 285 661	509 863 152	5 214	559	1 919 366 456 5992
33	589 962 580 544	0 626 397 723	8 887 938	4 651	564	0 761 971 662 3270
34	780 588 978 267	190 118 484 437	7 913 287	974 083	568	9·279 603 708 279 0911
1·335	0·940 970 707 462 704	189 611 545 233	506 939 204	3 511	572	9·278 444 576 306 8913
36	0·941 160 319 007 937	9 105 579 539	5 965 693	2 934	577	7 284 575 745 7277
37	349 424 587 476	8 600 586 780	4 992 759	2 353	581	6 123 706 595 6003
38	538 025 174 256	8 096 566 374	4 020 406	1 768	585	4 961 968 856 5091
39	726 121 740 630	187 593 517 736	3 048 638	971 179	589	3 799 362 528 4541
1·340	0·941 913 715 258 365	7 091 440 276	502 077 460	0 585	594	9·272 635 887 611 4353
41	0·942 100 806 698 641	6 590 333 402	1 106 875	969 987	598	1 471 544 105 4527
42	287 397 032 043	6 090 196 514	0 136 988	9 385	602	0 306 332 010 5062
43	473 487 228 557	5 591 029 011	499 167 503	8 779	606	9·269 140 251 326 5960
44	659 078 257 568	185 092 830 288	8 198 724	968 168	610	7 973 302 053 7220
1·345	0·942 844 171 087 856	4 595 599 732	497 230 556	7 554	615	9·266 805 484 191 8841
46	0·943 028 766 687 588	4 099 336 730	6 263 002	6 935	619	5 636 797 741 0825
47	212 866 024 318	3 604 040 663	5 296 067	6 312	623	4 467 242 701 3170
48	396 470 064 980	3 109 710 907	4 329 755	5 685	627	3 296 819 072 5878
49	579 579 775 887	182 616 346 837	3 364 070	965 054	631	2 125 526 854 8947

TABLE OF THE VALUES OF $H = \frac{2}{\sqrt{\pi}}\int_0^t \epsilon^{-t^2} dt$. (2) FROM $t = 1{\cdot}000$ TO $t = 3{\cdot}000$.

t	H	Δ_1 +	Δ_2 −	Δ_3 +	Δ_4 −	log. $\frac{2}{\sqrt{\pi}}\epsilon^{-t^2}$ + 10.
1·350	0·943 762 196 122 724	182 123 947 820	492 399 017	964 419	635	9·260 953 366 048 2378
51	944 320 070 544	1 632 513 222	1 434 598	3 779	639	9·259 780 336 652 6171
52	0·944 125 952 583 767	1 142 042 404	0 470 819	3 136	643	8 606 438 668 0326
53	307 094 626 170	0 652 534 721	489 507 683	2 489	647	7 431 672 094 4843
54	487 747 160 891	180 163 989 527	8 545 194	961 837	651	6 256 036 931 9722
1·355	0·944 667 911 150 418	179 676 406 171	487 583 357	1 182	655	9·255 079 533 180 4963
56	847 587 556 589	9 189 783 996	6 622 175	0 523	659	3 902 160 840 0566
57	·945 026 777 340 585	8 704 122 344	5 661 652	959 859	663	2 723 919 910 6531
58	205 481 462 930	8 219 420 552	4 701 793	9 192	667	1 544 810 392 2858
59	383 700 883 482	177 735 677 951	3 742 600	−958 521	671	0 364 832 284 9546
1·360	0·945 561 436 561 433	7 252 893 872	482 784 080	7 846	675	9·249 183 985 588 6597
61	738 689 455 305	6 771 067 638	1 826 234	7 167	679	8 002 270 303 4009
62	915 460 522 943	6 290 198 571	0 869 067	6 484	683	6 819 686 429 1784
63	0·946 091 750 721 514	5 810 285 988	479 912 583	5 797	687	5 636 233 965 9920
64	267 561 007 502	175 331 329 201	8 956 786	955 106	691	4 451 912 913 8419
1·365	0·946 442 892 336 703	4 853 327 522	478 001 680	4 412	694	9·243 266 723 272 7279
66	617 745 664 224	4 376 280 254	7 047 268	3 714	698	2 080 665 042 6501
67	792 121 944 478	3 900 186 699	6 093 554	3 011	702	0 893 738 223 6085
68	966 022 131 177	3 425 046 156	5 140 543	2 306	706	9·239 705 942 815 6031
69	0·947 139 447 177 333	172 950 857 919	474 188 237	951 596	710	8 517 278 818 6339
1·370	0·947 312 398 035 252	2 477 621 277	473 236 642	0 882	713	9·237 327 746 232 7009
71	484 875 656 529	2 005 335 518	2 285 759	0 165	717	6 137 345 057 8041
72	656 880 992 047	1 533 999 924	1 335 594	949 444	721	4 946 075 293 9435
73	828 414 991 971	1 063 613 774	0 386 150	8 720	725	3 753 936 941 1191
74	999 478 605 746	170 594 176 345	469 437 430	947 991	728	2 560 929 999 3308
1·375	0·948 170 072 782 090	0 125 686 906	468 489 439	7 259	732	9·231 367 054 468 5788
76	340 198 468 996	169 658 144 727	7 542 179	6 524	736	0 172 310 348 8629
77	509 856 613 723	9 191 549 071	6 595 656	5 784	739	9·228 976 697 640 1833
78	679 048 162 794	8 725 899 199	5 649 871	5 041	743	7 780 216 342 5398
79	847 774 061 993	168 261 194 369	4 704 830	944 295	747	6 582 866 455 9326
1·380	0·949 016 035 256 363	7 797 433 834	463 760 535	3 545	750	9·225 384 647 980 3615
81	183 832 690 197	7 334 616 844	2 816 991	2 791	754	4 185 560 915 8266
82	351 167 307 040	6 872 742 644	1 874 200	2 034	757	2 985 605 262 3279
83	518 040 049 684	6 411 810 478	0 932 166	1 273	761	1 784 781 019 8654
84	684 451 860 162	165 951 819 584	459 990 894	940 508	764	0 583 088 188 4391
1·385	0·949 850 403 679 746	5 492 769 198	459 050 385	939 740	768	9·219 380 526 768 0490
86	0·950 015 896 448 944	5 034 658 553	8 110 645	8 969	771	8 177 096 758 6951
87	180 931 107 498	4 577 486 877	7 171 676	8 194	775	6 972 798 160 3774
88	345 508 594 375	4 121 253 394	6 233 482	7 415	778	5 767 630 973 0959
89	509 629 847 769	163 665 957 327	5 296 067	936 634	782	4 561 595 196 8505
1·390	0·950 673 295 805 096	3 211 597 894	454 359 433	5 848	785	9·213 354 690 831 6414
91	836 507 402 991	2 758 174 309	3 423 585	5 060	789	2 146 917 877 4685
92	999 265 577 299	2 305 685 783	2 488 526	4 267	792	0 938 276 334 3317
93	0·951 161 571 263 083	1 854 131 525	1 554 258	3 472	796	9·209 728 766 202 2312
94	323 425 394 608	161 403 510 739	0 620 786	932 673	799	8 518 387 481 1668
1·395	0·951 484 828 905 347	0 953 822 625	449 688 113	1 871	802	9·207 307 140 171 1386
96	645 782 727 972	0 505 066 383	8 756 243	1 065	806	6 095 024 272 1467
97	806 287 794 355	0 057 241 205	7 825 178	0 256	809	4 882 039 784 1909
98	966 345 035 560	159 610 346 283	6 894 922	929 444	812	3 668 186 707 2713
99	0·952 125 955 381 843	159 164 380 805	5 965 478	928 628	816	2 453 465 041 3879

TABLE OF THE VALUES OF $H=\frac{2}{\sqrt{\pi}}\int_0^t \epsilon^{-t^2}dt$. (2) FROM $t=1{\cdot}000$ TO $t=3{\cdot}000$.

1·400] [1·449

t	H	Δ_1 +	Δ_2 −	Δ_3 +	Δ_4 −	log. $\frac{2}{\sqrt{\pi}}\epsilon^{-t^2}+10.$
1·400	0·952 285 119 762 649	158 719 343 956	445 036 850	927 809	819	9·201 237 874 786 5407
1	443 839 106 604	8 275 234 915	4 109 040	6 987	822	0 021 415 942 7297
2	602 114 341 520	7 832 052 862	3 182 053	6 162	825	9·198 804 088 509 9549
3	759 946 394 382	7 389 796 971	2 255 891	5 333	829	7 585 892 488 2162
4	917 336 191 352	156 948 466 412	1 330 558	924 501	832	6 366 827 877 5138
1·405	0·953 074 284 657 765	6 508 060 355	440 406 067	3 666	835	9·195 146 894 677 8476
6	230 792 718 120	6 068 577 965	439 482 391	2 828	838	3 926 092 889 2175
7	386 861 296 085	5 630 018 402	8 559 563	1 987	841	2 704 422 511 6237
8	542 491 314 487	5 192 380 826	7 637 576	1 142	845	1 481 883 545 0660
9	697 683 695 313	154 755 664 392	6 716 434	920 294	848	0 258 475 989 5446
1·410	0·953 852 439 359 705	4 319 868 253	435 796 140	919 443	851	9·189 034 199 845 0593
11	0·954 006 759 227 958	3 884 991 557	4 876 696	8 590	854	7 809 055 111 6102
12	160 644 219 515	3 451 033 450	3 958 107	7 732	857	6 583 041 789 1974
13	314 095 252 965	3 017 993 076	3 040 374	6 872	860	5 356 159 877 8207
14	467 113 246 041	152 585 869 574	2 123 502	916 009	863	4 128 409 377 4802
1·415	0·954 619 699 115 615	2 154 662 081	431 207 493	5 143	866	9·182 899 790 288 1759
16	771 853 777 696	1 724 369 732	0 292 350	4 274	869	1 670 302 609 9078
17	923 578 147 428	1 294 991 656	429 378 076	3 401	872	0 439 946 342 6759
18	0·955 074 873 139 084	0 866 526 981	8 464 675	2 526	875	9·179 208 721 486 4801
19	225 739 666 064	150 438 974 832	7 552 149	911 648	878	7 976 628 041 3206
1·420	0·955 376 178 640 896	0 012 334 331	426 640 501	0 766	881	9·176 743 666 007 1973
21	526 190 975 227	149 586 604 596	5 729 735	909 882	884	5 509 835 384 1101
22	675 777 579 823	9 161 784 744	4 819 852	8 995	887	4 275 136 172 0592
23	824 939 364 567	8 737 873 886	3 910 857	8 105	890	3 039 568 371 0444
24	973 677 238 453	148 314 871 134	3 002 752	907 212	893	1 803 131 981 0659
1·425	0·956 121 992 109 586	7 892 775 593	422 095 541	6 316	896	9·170 565 827 002 1235
26	269 884 885 179	7 471 586 369	1 189 224	5 417	899	9·169 327 653 434 2174
27	417 356 471 548	7 051 302 561	0 283 807	4 516	902	8 088 611 277 3474
28	564 407 774 109	6 631 923 270	419 379 292	3 611	904	6 848 700 531 5136
29	711 039 697 379	146 213 447 590	8 475 680	902 704	907	5 607 921 196 7160
1·430	0·956 857 253 144 969	5 795 874 613	417 572 977	1 794	910	9·164 366 273 272 9546
31	0·957 003 049 019 582	5 379 203 430	6 671 183	0 881	913	3 123 756 760 2294
32	148 428 223 012	4 963 433 128	5 770 302	899 965	916	1 880 371 658 5404
33	293 391 656 140	4 548 562 791	4 870 337	9 047	919	0 636 117 967 8876
34	437 940 218 931	144 134 591 501	3 971 291	898 125	921	9·159 390 995 688 2710
1·435	0·957 582 074 810 432	3 721 518 335	413 073 165	7 201	924	9·158 145 004 819 6905
36	725 796 328 767	3 309 342 371	2 175 964	6 275	927	6 898 145 362 1463
37	869 105 671 139	2 898 062 682	1 279 689	5 345	929	5 650 417 315 6382
38	0·958 012 003 733 821	2 487 678 338	0 384 344	4 413	932	4 401 820 680 1664
39	154 491 412 158	142 078 188 406	409 489 931	893 478	935	3 152 355 455 7307
1·440	0·958 296 569 600 565	1 669 591 953	408 596 453	2 541	937	9·151 902 021 642 3313
41	438 239 192 518	1 261 888 041	7 703 913	1 601	940	0 650 819 239 9680
42	579 501 080 559	0 855 075 729	6 812 312	0 658	943	9·149 398 748 248 6409
43	720 356 156 287	0 449 154 074	5 921 654	889 712	945	8 145 808 668 3501
44	860 805 310 362	140 044 122 133	5 031 942	888 764	948	6 892 000 499 0954
1·445	0·959 000 849 432 494	139 639 978 955	404 143 177	7 814	951	9·145 637 323 740 8769
46	140 489 411 449	9 236 723 592	3 255 364	6 861	953	4 381 778 393 6946
47	279 726 135 041	8 834 355 089	2 368 503	5 905	956	3 125 364 457 5485
48	418 560 490 129	8 432 872 491	1 482 598	4 947	958	1 868 081 932 4385
49	556 993 362 620	138 032 274 839	0 597 651	883 986	961	0 609 930 818 3648

TABLE OF THE VALUES OF $H = \frac{2}{\sqrt{\pi}}\int_0^t \epsilon^{-t^2} dt$. (2) FROM $t = 1{\cdot}000$ TO $t = 3{\cdot}000$.

t	H	Δ_1 +	Δ_2 −	Δ_3 +	Δ_4 −	$\log \frac{2}{\sqrt{\pi}}\epsilon^{-t^2} + 10$.
1·450	0·959 695 025 637 459	137 632 561 174	399 713 665	883 023	963	9·139 350 911 115 3273
51	832 658 198 633	7 233 730 531	8 830 643	2 057	966	8 091 022 823 3260
52	969 891 929 164	6 835 781 945	7 948 586	1 088	968	6 830 265 942 3608
53	0·960 106 727 711 109	6 438 714 447	7 067 498	0 118	971	5 568 640 472 4319
54	243 166 425 556	136 042 527 067	6 187 380	879 144	973	4 306 146 413 5391
1·455	0·960 379 208 952 623	5 647 218 832	395 308 235	8 169	976	9·133 042 783 765 6826
56	514 856 171 455	5 252 788 765	4 430 067	7 191	978	1 778 552 528 8622
57	650 108 960 220	4 859 235 889	3 552 876	6 210	980	0 513 452 703 0780
58	784 968 196 109	4 466 559 224	2 676 666	5 227	983	9·129 247 484 288 3300
59	919 434 755 333	134 074 757 785	1 801 438	874 242	985	7 980 647 284 6183
1·460	0·961 053 509 513 118	3 683 830 589	390 927 196	3 254	988	9·126 712 941 691 9427
61	187 193 343 707	3 293 776 648	0 053 942	2 264	990	5 444 367 510 3033
62	320 487 120 355	2 904 594 971	389 181 677	1 272	992	4 174 924 739 7001
63	453 391 715 326	2 516 284 566	8 310 405	0 278	995	2 904 613 380 1331
64	585 907 999 891	132 128 844 438	7 440 127	869 281	997	1 633 433 431 6022
1·465	0·961 718 036 844 329	1 742 273 591	386 570 847	8 281	999	9·120 361 384 894 1076
66	849 779 117 921	1 356 571 026	5 702 566	7 280	1002	9·119 088 467 767 6492
67	981 135 688 946	0 971 735 740	4 835 286	6 276	4	7 814 682 052 2269
68	0·962 112 107 424 686	0 587 766 730	3 969 010	5 270	6	6 540 027 747 8409
69	242 695 191 416	130 204 662 990	3 103 740	864 262	8	5 264 504 854 4911
1·470	0·962 372 899 854 406	129 822 423 512	382 239 478	3 251	1010	9·113 988 113 372 1774
71	502 722 277 918	9 441 047 285	1 376 227	2 239	13	2 710 853 300 8999
72	632 163 325 203	9 060 533 297	0 513 988	1 224	15	1 432 724 640 6587
73	761 223 858 499	8 680 880 532	379 652 765	0 207	17	0 153 727 391 4536
74	889 904 739 031	128 302 087 974	8 792 558	859 187	19	9·108 873 861 553 2847
1·475	0·963 018 206 827 005	7 924 154 603	377 933 371	8 166	1021	9·107 593 127 126 1520
76	146 130 981 608	7 547 079 398	7 075 205	7 143	3	6 311 524 110 0555
77	273 678 061 007	7 170 861 336	6 218 062	6 117	6	5 029 052 504 9952
78	400 848 922 343	6 795 499 391	5 361 945	5 089	8	3 745 712 310 9711
79	527 644 421 734	126 420 992 535	4 506 856	854 059	1030	2 461 503 527 9832
1·480	0·963 654 065 414 269	6 047 339 738	373 652 797	3 028	1032	9·101 176 426 156 0315
81	780 112 754 007	5 674 539 969	2 799 769	1 994	4	9·099 890 480 195 1159
82	905 787 293 976	5 302 592 193	1 947 776	0 958	6	8 603 665 645 2366
83	0·964 031 089 886 170	4 931 495 376	1 096 818	849 920	8	7 315 982 506 3935
84	156 021 381 545	124 561 248 477	0 246 898	848 880	1040	6 027 430 778 5865
1·485	0·964 280 582 630 022	4 191 850 458	369 398 019	7 838	1042	9·094 738 010 461 8158
86	404 774 480 481	3 823 300 277	8 550 181	6 794	4	3 447 721 556 0812
87	528 597 780 758	3 455 596 889	7 703 388	5 748	6	2 156 564 061 3828
88	652 053 377 647	3 088 739 249	6 857 640	4 700	8	0 864 537 977 7207
89	775 142 116 896	122 722 726 308	6 012 941	843 650	1050	9·089 571 643 305 0947
1·490	0·964 897 864 843 204	2 357 557 017	365 169 291	2 598	1052	9·088 277 880 043 5049
91	0·965 020 222 400 221	1 993 230 324	4 326 693	1 544	4	6 983 248 192 9513
92	142 215 630 545	1 629 745 175	3 485 149	0 488	6	5 687 747 753 4339
93	263 845 375 720	1 267 100 514	2 644 661	839 431	8	4 391 378 724 9527
94	385 112 476 234	120 905 295 284	1 805 230	838 371	9	3 094 141 107 5077
1·495	0·965 506 017 771 519	0 544 328 426	360 966 858	7 310	1061	9·081 796 034 [illegible]01 0989
96	626 562 099 945	0 184 198 878	0 129 548	6 247	3	0 497 060 105 7262
97	746 746 298 822	119 824 905 577	359 293 301	5 182	5	9·079 197 216 721 3898
98	866 571 204 399	466 447 458	8 458 119	4 115	7	7 896 504 748 0896
99	986 037 651 857	119 108 823 454	7 624 004	833 047	9	6 594 924 185 8255

TABLE OF THE VALUES OF $H=\frac{2}{\sqrt{\pi}}\int_0^t \epsilon^{-t^2}dt$. (2) FROM $t=1{\cdot}000$ TO $t=3{\cdot}000$.

1·500] [1·598

t	H	Δ_1 +	Δ_2 −	Δ_3 +	Δ_4 −	log. $\frac{2}{\sqrt{\pi}}\epsilon^{-t^2}+10$.
1·500	0·966 105 146 475 311	237 148 106 014	1 427 164 898	6 651 519	17 126	9·075 292 475 034 5977
2	342 294 581 325	5 727 592 635	20 513 379	34 337	182	2 684 970 965 2505
4	578 022 173 960	4 313 713 593	13 879 042	17 098	238	0 073 992 540 0482
6	812 335 887 552	2 906 451 649	07 261 944	6 599 805	293	9·067 459 539 758 9906
8	0·967 045 242 339 201	231 505 789 510	00 662 139	6 582 458	347	4 841 612 622 0778
1·510	0·967 276 748 128 712	0 111 709 830	1 394 079 680	65 058	17 400	9·062 220 211 129 3098
12	506 859 838 541	228 724 195 207	87 514 622	47 606	452	9·059 595 335 280 6865
14	735 584 033 749	7 343 228 191	80 967 017	30 102	504	6 966 985 076 2080
16	962 927 261 939	5 968 791 276	74 436 915	12 548	554	4 335 160 515 8473
18	0·968 188 896 053 215	224 600 866 908	67 924 367	6 494 944	604	1 699 861 599 6854
1·520	0·968 413 496 920 123	3 239 437 484	1 361 429 424	77 291	17 653	9·049 061 088 327 6412
22	636 736 357 607	1 884 485 351	54 952 133	59 590	701	6 418 840 699 7419
24	858 620 842 959	0 535 992 809	48 492 542	41 843	748	3 773 118 715 9873
26	0·969 079 156 835 768	219 193 942 109	42 050 700	24 049	794	1 123 922 376 3774
28	298 350 777 877	217 858 315 458	35 626 651	6 406 210	839	9·038 471 251 680 9124
1·530	0·969 516 209 093 336	6 529 095 017	1 329 220 441	6 388 326	17 884	9·035 815 106 629 5921
32	732 738 188 353	5 206 262 902	22 832 115	70 399	927	3 155 487 222 4166
34	947 944 451 255	3 889 801 186	16 461 717	52 428	970	0 492 393 459 3858
36	0·970 161 834 252 441	2 579 691 897	10 109 288	34 416	18 012	9·027 825 825 340 4998
38	374 413 944 338	211 275 917 026	03 774 872	6 316 363	053	5 155 782 865 7587
1·540	0·970 585 689 861 364	209 978 458 517	1 297 458 509	6 298 269	18 094	9·022 482 266 035 1622
42	795 668 319 880	8 687 298 277	91 160 240	80 136	133	9·019 805 274 848 7106
44	0·971 004 355 618 157	7 402 418 173	84 880 104	61 964	172	7 124 809 306 4037
46	211 758 036 330	6 123 800 034	78 618 140	43 754	210	4 440 869 408 2416
48	417 881 836 364	204 851 425 648	72 374 385	6 225 508	247	1 753 455 154 2243
1·550	0·971 622 733 262 013	3 585 276 771	1 266 148 878	07 225	18 283	9·009 062 566 544 3518
52	826 318 538 784	2 325 335 118	59 941 653	6 188 906	318	6 368 203 578 6240
54	0·972 028 643 873 902	1 071 582 371	53 752 747	70 553	353	3 670 366 257 0410
56	229 715 456 273	199 824 000 178	47 582 194	52 166	387	0 969 054 579 6028
58	429 539 456 450	198 582 570 150	41 430 028	6 133 746	420	8·998 264 268 546 3093
1·560	0·972 628 122 026 600	7 347 273 868	1 235 296 282	15 294	18 452	8·995 556 008 157 1606
62	825 469 300 469	6 118 092 881	29 180 988	6 096 810	484	2 844 273 412 1567
64	0·973 021 582 393 349	4 895 008 703	23 084 177	78 296	514	0 129 064 311 2976
66	216 482 402 053	3 678 002 822	17 005 881	59 752	544	8·987 410 380 854 5832
68	410 160 404 875	192 467 056 692	10 946 130	6 041 178	573	4 688 223 042 0136
1·570	0·973 602 627 461 567	1 262 151 741	1 204 904 951	22 577	18 602	8·981 962 590 873 5888
72	793 889 613 308	0 063 269 367	1 198 882 375	03 948	629	8·979 233 484 349 3088
74	983 952 882 675	188 870 390 940	92 878 427	5 985 292	656	6 500 903 469 1735
76	0·974 172 823 273 614	7 683 497 804	86 893 135	66 609	682	3 764 848 233 1831
78	360 506 771 419	186 502 571 278	80 926 526	5 947 902	707	1 025 318 641 3373
1·580	0·974 547 009 342 697	5 327 592 655	1 174 978 624	29 170	18 732	8·968 282 314 693 6364
82	732 336 935 351	4 158 543 201	1 169 049 453	10 415	756	5 535 836 390 0802
84	916 495 478 553	2 995 404 162	63 139 039	5 891 636	779	2 785 883 730 6689
86	0·975 099 490 882 715	1 838 156 760	57 247 403	72 835	801	0 032 456 715 4022
88	281 329 039 475	180 686 782 193	51 374 567	5 854 013	822	8·957 275 555 344 2804
1·590	0·975 462 015 821 668	179 541 261 638	1 145 520 554	35 170	18 843	8·954 515 179 617 3033
92	641 557 083 306	8 401 576 254	39 685 385	16 306	863	1 751 329 534 4710
94	819 958 659 560	7 267 707 175	33 869 078	5 797 424	883	8·948 984 005 095 7835
96	997 226 366 735	6 139 635 521	28 071 655	78 523	901	6 213 206 301 2408
98	0·976 173 366 002 255	175 017 342 389	22 293 132	5 759 603	919	3 438 933 150 8428

TABLE OF THE VALUES OF $H = \frac{2}{\sqrt{\pi}}\int_0^t \epsilon^{-t^2} dt$. (2) FROM $t = 1{\cdot}000$ TO $t = 3{\cdot}000$.

t	H	Δ_1 +	Δ_2 −	Δ_3 +	Δ_4 −	$\log \frac{2}{\sqrt{\pi}}\epsilon^{-t^2} + 10$.
1·600	0·976 348 383 344 644	173 900 808 860	1 116 533 529	5 740 667	18 936	8·940 661 185 644 5896
2	522 284 153 504	2 790 015 999	10 792 861	21 714	953	8·937 879 963 782 4812
4	695 074 169 503	1 684 944 852	05 071 147	02 746	968	5 095 267 564 5175
6	866 759 114 355	0 585 576 451	1 099 368 401	5 683 763	984	2 307 096 990 6986
8	0·977 037 344 690 806	169 491 891 813	93 684 638	5 664 765	998	8·929 515 452 061 0245
1·610	0·977 206 836 582 619	8 403 871 939	1 088 019 874	45 753	19 011	8·926 720 332 775 4952
12	375 240 454 558	7 321 497 819	82 374 120	26 729	024	3 921 739 134 1107
14	542 561 952 377	6 244 750 427	76 747 392	07 692	037	1 119 671 136 8709
16	708 806 702 804	5 173 610 728	71 139 700	5 588 644	048	8·918 314 128 783 7759
18	873 980 313 531	164 108 059 672	65 551 056	5 569 585	059	5 505 112 074 8256
1·620	0·978 038 088 373 203	3 048 078 201	1 059 981 471	50 515	19 069	8·912 692 621 010 0202
22	201 136 451 404	1 993 647 245	54 430 956	31 436	079	8·909 876 655 589 3595
24	363 130 098 650	0 944 747 726	48 899 519	12 349	088	7 057 215 812 8436
26	524 074 846 376	159 901 360 556	43 387 170	5 493 253	096	4 234 301 680 4724
28	683 976 206 932	158 863 466 638	37 893 918	5 474 149	103	1 407 913 192 2461
1·630	0·978 842 839 673 570	7 831 046 870	1 032 419 768	55 039	19 110	8·898 578 050 348 1645
32	0·979 000 670 720 440	6 804 082 140	26 964 729	35 922	117	5 744 713 148 2277
34	157 474 802 580	5 782 553 333	21 528 807	16 800	122	2 907 901 592 4356
36	313 257 355 913	4 766 441 326	16 112 007	5 397 673	127	0 067 615 680 7884
38	468 023 797 240	153 755 726 992	10 714 334	5 378 542	131	8·887 223 855 413 2859
1·640	0·979 621 779 524 232	2 750 391 200	1 005 335 792	59 407	19 135	8·884 376 620 789 9282
42	774 529 915 432	1 750 414 815	999 976 386	40 269	138	1 525 911 810 7152
44	926 280 330 247	0 755 778 698	994 636 117	21 128	140	8·878 671 728 475 6470
46	0·980 077 036 108 945	149 766 463 709	989 314 989	01 986	142	5 814 070 784 7236
48	226 802 572 654	148 782 450 707	984 013 003	5 282 843	143	2 952 938 737 9450
1·650	0·980 375 585 023 360	7 803 720 547	978 730 160	63 699	19 144	8·870 088 332 335 3112
52	523 388 743 907	6 830 254 086	973 466 461	44 555	144	8·867 220 251 576 8221
54	670 218 997 993	5 862 032 180	968 221 906	25 412	143	4 348 696 462 4778
56	816 081 030 173	4 899 035 686	962 996 494	06 270	142	1 473 666 992 2783
58	960 980 065 859	143 941 245 463	957 790 223	5 187 131	140	8·858 595 163 166 2235
1·660	0·981 104 921 311 322	2 988 642 371	952 603 093	67 993	19 137	8·855 713 184 984 3135
62	247 909 953 693	2 041 207 271	947 435 099	48 859	134	2 827 732 446 5483
64	389 951 160 964	1 098 921 031	942 286 240	29 729	130	8·849 938 805 552 9279
66	531 050 081 995	0 161 764 520	937 156 511	10 602	126	7 046 404 303 4522
68	671 211 846 515	139 229 718 611	932 045 909	5 091 481	121	4 150 528 698 1214
1·670	0·981 810 441 565 127	8 302 764 184	926 954 428	72 365	19 116	8·841 251 178 736 9352
72	948 744 329 310	7 380 882 121	921 882 062	53 255	110	8·838 348 354 419 8939
74	0·982 086 125 211 431	6 464 053 314	916 828 807	34 152	103	5 442 055 746 9973
76	222 589 264 745	5 552 258 659	911 794 655	15 056	096	2 532 282 718 2456
78	358 141 523 405	134 645 479 060	906 779 599	4 995 967	088	8·829 619 035 333 6385
1·680	0·982 492 787 002 465	3 743 695 429	901 783 632	76 837	19 080	8·826 702 313 593 1763
82	626 530 697 894	2 846 888 685	896 806 744	57 816	071	3 782 117 496 8588
84	759 377 586 578	1 955 039 756	891 848 928	38 754	062	0 858 447 044 6862
86	891 332 626 334	1 068 129 582	886 910 174	19 702	052	8·817 931 302 236 6582
88	0·983 022 400 755 916	130 186 139 110	881 990 472	4 900 660	042	5 000 683 072 7751
1·690	0·983 152 586 895 026	129 309 049 298	877 089 812	4 881 630	19 031	8·812 066 589 553 0367
92	281 895 944 324	8 436 841 115	872 208 182	62 611	019	8·809 129 021 677 4431
94	410 332 785 439	7 569 495 544	867 345 572	43 604	007	6 187 979 445 9943
96	537 902 280 983	6 706 993 575	862 501 968	24 609	18 994	3 243 462 858 6903
98	664 609 274 558	125 849 316 216	857 677 359	4 805 628	981	0 295 471 915 5310

TABLE OF THE VALUES OF $H=\frac{2}{\sqrt{\pi}}\int_0^t \epsilon^{-t^2}dt$. (2) FROM $t=1{\cdot}000$ TO $t=3{\cdot}000$.

1·700]

[1·798

t	H	Δ_1 +	Δ_2 −	Δ_3 +	Δ_4 −	$\log.\frac{2}{\sqrt{\pi}}\epsilon^{-t^2}+10.$
1·700	0·983 790 458 590 775	124 996 444 485	852 871 731	4 786 660	18 968	8·797 344 006 616 5165
2	915 455 035 259	4 148 359 413	48 085 071	67 706	954	·794 389 066 961 6468
4	0·984 039 603 394 673	3 305 042 048	43 317 365	48 767	939	·791 430 652 950 9218
6	162 908 436 721	2 466 473 451	38 568 598	29 844	924	·788 468 764 584 3416
8	285 374 910 172	121 632 634 696	33 838 754	4 710 935	908	·785 503 401 861 9062
1·710	0·984 407 007 544 868	0 803 506 878	829 127 819	4 692 043	18 892	8·782 534 564 783 6156
12	527 811 051 746	119 979 071 102	24 435 776	73 168	875	·779 562 253 349 4697
14	647 790 122 848	9 159 308 494	19 762 608	54 310	858	·776 586 467 559 4687
16	766 949 431 343	8 344 200 196	15 108 298	35 469	841	·773 607 207 413 6124
18	885 293 631 539	117 533 727 367	10 472 829	4 616 646	823	·770 624 472 911 9008
1·720	0·985 002 827 358 906	6 727 871 184	805 856 183	4 597 842	18 804	8·767 638 264 054 3341
22	119 555 230 090	5 926 612 843	01 258 341	79 057	785	·764 648 580 840 9121
24	235 481 842 933	5 129 933 559	796 679 284	60 292	766	·761 655 423 271 6349
26	350 611 776 492	4 337 814 567	92 118 992	41 546	746	·758 658 791 346 5024
28	464 949 591 059	113 550 237 121	87 577 446	4 522 821	725	·755 658 685 065 5148
1·730	0·985 578 499 828 180	2 767 182 496	783 054 625	04 117	18 704	8·752 655 104 428 6719
32	691 267 010 677	1 988 631 988	78 550 508	4 485 434	683	·749 648 049 435 9738
34	803 255 642 665	1 214 566 913	74 065 075	66 773	661	·746 637 520 087 4204
36	914 470 209 578	0 444 968 612	69 598 302	48 134	639	·743 623 516 383 0118
38	0·986 024 915 178 189	109 679 818 443	65 150 168	4 429 518	616	·740 606 038 322 7481
1·740	0·986 134 594 996 633	8 919 097 793	760 720 650	10 924	18 593	8·737 585 085 906 6290
42	243 514 094 426	8 162 788 067	56 309 726	4 392 355	570	·734 560 659 134 6548
44	351 676 882 493	7 410 870 696	51 917 371	73 809	546	·731 532 758 006 8253
46	459 087 753 189	6 663 327 134	47 543 562	55 288	521	·728 501 382 523 1406
48	565 751 080 323	105 920 138 860	43 188 274	4 336 792	496	·725 466 532 683 6007
1·750	0·986 671 671 219 182	5 181 287 378	738 851 482	318 320	18 471	8·722 428 208 488 2055
52	776 852 506 560	4 446 754 216	34 533 162	299 875	446	·719 386 409 936 9552
54	881 299 260 776	3 716 520 929	30 233 287	81 455	419	·716 341 137 029 8496
56	985 015 781 705	2 990 569 097	25 951 832	63 062	393	·713 292 389 766 8887
58	0·987 088 006 350 802	102 268 880 328	21 688 769	4 244 696	366	·710 240 168 148 0727
1·760	0·987 190 275 231 130	1 551 436 255	717 444 073	26 357	18 339	8·707 184 472 173 4014
62	291 826 667 385	0 838 218 540	13 217 716	08 046	311	·704 125 301 842 8749
64	392 664 885 925	0 129 208 870	09 009 670	4 189 763	283	·701 062 657 156 4932
66	492 794 094 795	99 424 388 963	04 819 907	71 508	255	·697 996 538 114 2562
68	592 218 483 758	98 723 740 564	00 648 399	4 153 282	226	·694 926 944 716 1640
1·770	0·987 690 942 224 322	98 027 245 448	696 495 117	35 085	18 197	8·691 853 876 962 2166
72	788 969 469 770	97 334 885 416	92 360 032	16 918	167	·688 777 334 852 4140
74	886 304 355 186	96 646 642 302	88 243 114	4 098 781	137	·685 697 318 386 7561
76	982 950 997 489	95 962 497 970	84 144 333	80 674	107	·682 613 827 565 2430
78	0·988 078 913 495 458	95 282 434 310	80 063 659	4 062 597	076	·679 526 862 387 8747
1·780	0·988 174 195 929 768	94 606 433 248	676 001 062	44 552	18 045	8·676 436 422 854 6512
82	268 802 363 016	93 934 476 738	71 956 510	26 538	014	·673 342 508 965 5724
84	362 736 839 755	93 266 546 766	67 929 972	08 556	17 982	·670 245 120 720 6384
86	456 003 386 520	92 602 625 349	63 921 416	3 990 606	950	·667 144 258 119 8492
88	548 606 011 870	91 942 694 538	59 930 811	3 972 688	918	·664 039 921 163 2048
1·790	0·988 640 548 706 408	91 286 736 415	655 958 123	54 803	17 885	8·660 932 109 850 7051
92	731 835 442 823	90 634 733 095	52 003 320	36 951	852	·657 820 824 182 3502
94	822 470 175 918	89 986 666 725	48 066 370	19 132	818	·654 706 064 158 1401
96	912 456 842 644	89 342 519 488	44 147 237	01 348	785	·651 587 829 778 0747
98	0·989 001 899 362 132	88 702 273 599	40 245 889	3 883 597	751	·648 466 121 042 1541

TABLE OF THE VALUES OF $H = \frac{2}{\sqrt{\pi}}\int_0^t \epsilon^{-t^2} dt$. (2) FROM $t = 1{\cdot}000$ TO $t = 3{\cdot}000$.

t	H	Δ_1 +	Δ_2 −	Δ_3 +	Δ_4 −	log. $\frac{2}{\sqrt{\pi}}\epsilon^{-t^2}$ + 10.
1·800	0·989 090 501 635 731	88 065 911 307	636 362 292	3 865 881	17 716	8·645 340 937 950 3783
2	178 567 547 037	87 433 414 895	2 496 412	48 199	682	·642 212 280 502 7473
4	266 000 961 932	86 804 766 683	628 648 212	30 553	646	·639 080 148 699 2611
6	352 805 728 615	86 179 949 023	4 817 659	12 942	611	·635 944 542 539 9196
8	438 985 677 639	85 558 944 306	1 004 718	3 795 366	575	·632 805 462 024 7229
1·810	0·989 524 544 621 944	84 941 734 954	617 209 351	77 827	17 540	8·629 662 907 153 6710
12	609 486 356 899	84 328 303 430	3 431 525	60 323	503	·626 516 877 926 7638
14	693 814 660 329	83 718 632 229	609 671 201	42 857	467	·623 367 374 344 0014
16	777 533 292 557	83 112 703 884	5 928 344	25 427	430	·620 214 396 405 3838
18	860 645 996 441	82 510 500 967	2 202 918	3 708 034	393	·617 057 944 110 9110
1·820	0·989 943 156 497 408	81 912 006 083	598 494 883	3 690 679	17 355	8·613 898 017 460 5829
22	0·990 025 068 503 491	81 317 201 879	4 804 205	73 361	318	·610 734 616 454 3996
24	106 385 705 369	80 726 071 035	1 130 843	56 082	280	·607 567 741 092 3611
26	187 111 776 405	80 138 596 274	587 474 762	38 840	241	·604 397 391 374 4674
28	267 250 372 678	79 554 760 352	3 835 921	3 621 637	203	·601 223 567 300 7184
1·830	0·990 346 805 133 031	78 974 546 068	580 214 284	04 473	17 164	8·598 046 268 871 1142
32	425 779 679 099	78 397 936 258	576 609 811	3 587 348	125	·594 865 496 085 6548
34	504 177 615 357	77 824 913 796	3 022 462	70 263	086	·591 681 248 944 3402
36	582 002 529 153	77 255 461 596	569 452 199	53 217	046	·588 493 527 447 1703
38	659 257 990 749	76 689 562 614	5 898 983	3 536 210	006	·585 302 331 594 1452
1·840	0·990 735 947 553 363	76 127 199 841	562 362 772	19 244	16 966	8·582 107 661 385 2649
42	812 074 753 204	75 568 356 313	558 843 528	02 318	926	·578 909 516 820 5293
44	887 643 109 517	75 013 015 103	5 341 210	3 485 433	885	·575 707 897 899 9386
46	962 656 124 620	74 461 159 327	1 855 777	68 589	844	·572 502 804 623 4926
48	0·991 037 117 283 947	73 912 772 139	548 387 188	51 785	803	·569 294 236 991 1913
1·850	0·991 111 030 056 086	73 367 836 736	544 935 403	3 435 023	16 762	8·566 082 195 003 0349
52	184 397 892 822	72 826 336 356	1 500 380	18 302	721	·562 866 678 659 0232
54	257 224 229 178	72 288 254 279	538 082 077	01 624	679	·559 647 687 959 1563
56	329 512 483 456	71 753 573 825	4 680 454	3 384 987	637	·556 425 222 903 4342
58	401 266 057 282	71 222 278 358	1 295 467	3 368 392	595	·553 199 283 491 8568
1·860	0·991 472 488 335 640	70 694 351 283	527 927 075	51 840	16 552	8·549 969 869 724 4242
62	543 182 686 923	70 169 776 047	4 575 235	35 330	510	·546 736 981 601 1364
64	613 352 462 970	69 648 536 142	1 239 906	18 863	467	·543 500 619 121 9934
66	683 000 999 112	69 130 615 099	517 921 043	02 439	424	·540 260 782 286 9951
68	752 131 614 211	68 615 996 496	4 618 604	3 286 058	381	·537 017 471 096 1417
1·870	0·991 820 747 610 707	68 104 663 951	511 332 545	69 721	16 337	8·533 770 685 549 4330
72	888 852 274 657	67 596 601 127	508 062 824	53 428	294	·530 520 425 646 8690
74	956 448 875 784	67 091 791 731	4 809 396	37 178	250	·527 266 691 388 4499
76	0·992 023 540 667 515	66 590 219 512	1 572 218	20 972	206	·524 009 482 774 1755
78	090 130 887 027	66 091 868 266	498 351 246	3 204 811	162	·520 748 799 804 0458
1·880	0·992 156 222 755 294	65 596 721 831	495 146 435	3 188 694	16 117	8·517 484 642 478 0610
82	221 819 477 125	65 104 764 089	1 957 742	72 621	073	·514 217 010 796 2209
84	286 924 241 214	64 615 978 969	488 785 121	56 593	028	·510 945 904 758 5256
86	351 540 220 183	64 130 350 441	5 628 528	40 610	15 983	·507 671 324 364 9751
88	415 670 570 624	63 647 862 524	2 487 917	3 124 672	938	·504 393 269 615 5694
1·890	0·992 479 318 433 148	63 168 499 279	479 363 245	08 780	15 893	8·501 111 740 510 3084
92	542 486 932 427	62 692 244 814	6 254 465	3 092 933	847	·497 826 737 049 1922
94	605 179 177 241	62 219 083 282	3 161 532	77 131	802	·494 538 259 232 2208
96	667 398 260 522	61 748 998 880	0 084 401	61 375	756	·491 246 307 059 3941
98	729 147 259 403	61 281 975 855	467 023 026	3 045 666	710	·487 950 880 530 7123

TABLE OF THE VALUES OF $H=\frac{2}{\sqrt{\pi}}\int_0^t \epsilon^{-t^2}dt$. (2) FROM $t=1{\cdot}000$ TO $t=3{\cdot}000$.

t	H	Δ_1 +	Δ_2 −	Δ_3 +	Δ_4 −	$\log\frac{2}{\sqrt{\pi}}\epsilon^{-t^2}+10.$
1·900	0·992 790 429 235 257	60 817 998 495	463 977 360	3 030 002	15 664	8·484 651 979 646 1752
2	851 247 233 752	60 357 051 137	0 947 358	14 384	617	·481 349 604 405 7828
4	911 604 284 889	59 899 118 163	457 932 974	2 998 813	571	·478 043 754 809 5353
6	971 503 403 052	59 444 184 002	4 934 161	83 288	525	·474 734 430 857 4325
8	0·993 030 947 587 054	58 992 233 129	1 950 873	2 967 810	478	·471 421 632 549 4745
1·910	0·993 089 939 820 183	58 543 250 067	448 983 063	52 379	15 431	8·468 105 359 885 6613
12	148 483 070 250	58 097 219 383	6 030 684	36 994	384	·464 785 612 865 9928
14	206 580 289 633	57 654 125 694	3 093 689	21 657	337	·461 462 391 490 4691
16	264 234 415 327	57 213 953 662	0 172 032	06 367	290	·458 135 695 759 0902
18	321 448 368 988	56 776 687 996	437 265 665	2 891 124	243	·454 805 525 671 8561
1·920	0·993 378 225 056 985	56 342 313 455	434 374 541	75 929	15 195	8·451 471 881 228 7667
22	434 567 370 440	55 910 814 843	1 498 612	60 781	148	·448 134 762 429 8221
24	490 478 185 284	55 482 177 012	428 637 831	45 681	100	·444 794 169 275 0223
26	545 960 362 296	55 056 384 861	5 792 151	30 628	052	·441 450 101 764 3673
28	601 016 747 157	54 633 423 339	2 961 522	2 815 624	004	·438 102 559 897 8570
1·930	0·993 655 650 170 496	54 213 277 441	420 145 898	00 667	14 956	8·434 751 543 675 4915
32	709 863 447 937	53 795 932 210	417 345 231	2 785 759	908	·431 397 053 097 2708
34	763 659 380 148	53 381 372 739	4 559 472	70 899	860	·428 039 088 163 1949
36	817 040 752 886	52 969 584 166	1 788 573	56 087	812	·424 677 648 873 2637
38	870 010 337 052	52 560 551 680	409 032 486	2 741 324	763	·421 312 735 227 4773
1·940	0·993 922 570 888 733	52 154 260 519	406 291 162	26 609	14 715	8·417 944 347 225 8357
42	974 725 149 251	51 750 695 966	3 564 553	11 943	666	·414 572 484 868 3388
44	0·994 026 475 845 217	51 349 843 356	0 852 610	2 697 325	618	·411 197 148 154 9868
46	077 825 688 574	50 951 688 072	398 155 285	82 757	569	·407 818 337 085 7795
48	128 777 376 645	50 556 215 544	5 472 528	2 668 237	520	·404 436 051 660 7169
1·950	0·994 179 333 592 189	50 163 411 253	392 804 291	2 653 766	14 471	8·401 050 291 879 7992
52	229 497 003 442	49 773 260 728	0 150 525	39 344	422	·397 661 057 743 0262
54	279 270 264 169	49 385 749 546	387 511 181	24 971	373	·394 268 349 250 3980
56	328 656 013 716	49 000 863 337	4 886 210	10 648	323	·390 872 166 401 9146
58	377 656 877 053	48 618 587 775	2 275 562	2 596 374	274	·387 472 509 197 5759
1·960	0·994 426 275 464 828	48 238 908 587	379 679 188	82 149	14 225	8·384 069 377 637 3820
62	474 514 373 415	47 861 811 548	7 097 039	67 973	175	·380 662 771 721 3329
64	522 376 184 964	47 487 282 483	4 529 065	53 848	126	·377 252 691 449 4286
66	569 863 467 457	47 115 307 265	1 975 218	39 771	076	·373 839 136 821 6690
68	616 978 774 712	46 745 871 818	369 435 447	2 525 744	027	·370 422 107 838 0543
1·970	0·994 663 724 646 530	46 378 962 116	366 909 703	11 767	13 977	8·367 001 604 498 5843
72	710 103 608 646	46 014 564 180	4 397 935	2 497 840	927	·363 577 626 803 2590
74	756 118 172 826	45 652 664 085	1 900 096	83 962	878	·360 150 174 752 0785
76	801 770 836 911	45 293 247 951	359 416 134	70 134	828	·356 719 248 345 0429
78	847 064 084 862	44 936 301 952	6 945 999	2 456 356	778	·353 284 847 582 1519
1·980	0·994 892 000 386 814	44 581 812 309	354 489 643	42 628	13 728	8·349 846 972 463 4058
82	936 582 199 122	44 229 765 294	2 047 015	28 950	678	·346 405 622 988 8044
84	980 811 964 416	43 880 147 229	349 618 065	15 322	628	·342 960 799 158 3478
86	0·995 024 692 111 644	43 532 944 486	7 202 743	01 744	578	·339 512 500 972 0360
88	068 225 056 130	43 188 143 487	4 800 999	2 388 216	528	·336 060 728 429 8690
1·990	0·995 111 413 199 617	42 845 730 704	342 412 783	74 738	13 478	8·332 605 481 531 8467
92	154 258 930 321	42 505 692 659	0 038 045	61 310	428	·329 146 760 277 9692
94	196 764 622 980	42 168 015 924	337 676 735	47 933	378	·325 684 564 668 2365
96	238 932 638 904	41 832 687 123	5 328 802	34 605	327	·322 218 894 702 6485
98	280 765 226 026	41 499 692 926	2 994 196	2 321 328	277	·318 749 750 381 2054

2·000] [2·098

t	H	Δ_1 +	Δ_2 −	Δ_3 +	Δ_4 −	log. $\frac{2}{\sqrt{\pi}}\epsilon^{-t^2}$ + 10.
2·000	0·995 322 265 018 953	41 169 020 058	330 672 868	2 308 101	13 227	8·315 277 131 703 9070
2	363 434 039 011	40 840 655 292	328 364 766	294 925	177	·311 801 038 670 7533
4	404 274 694 303	514 585 450	6 069 842	281 799	126	·308 321 471 281 7445
6	444 789 279 754	190 797 408	3 788 043	268 723	076	·304 838 429 536 8804
8	484 980 077 161	39 869 278 087	1 519 320	2 255 697	026	·301 351 913 436 1611
2·010	0·995 524 849 355 248	550 014 464	319 263 623	242 722	12 975	8·297 861 922 979 5866
12	564 399 369 712	232 993 562	7 020 902	229 797	925	·294 368 458 167 1568
14	603 632 363 274	38 918 202 457	4 791 105	216 922	875	·290 871 518 998 8718
16	642 550 565 731	605 628 273	2 574 183	204 098	824	·287 371 105 474 7316
18	681 156 194 004	38 295 258 188	0 370 086	2 191 324	774	·283 867 217 594 7362
2·020	0·995 719 451 452 192	37 987 079 426	308 178 762	178 600	12 724	8·280 359 855 358 8855
22	757 438 531 618	681 079 264	6 000 162	165 927	673	·276 849 018 767 1796
24	795 119 610 882	377 245 030	3 834 235	153 304	623	·273 334 707 819 6185
26	832 496 855 912	075 564 099	1 680 930	140 732	572	·269 816 922 516 2022
28	869 572 420 011	36 776 023 901	299 540 199	2 128 210	522	·266 295 662 856 9306
2·030	0·995 906 348 443 912	478 611 912	297 411 989	115 738	12 472	8·262 770 928 841 8038
32	942 827 055 824	183 315 661	5 296 251	103 316	421	·259 242 720 470 8218
34	979 010 371 484	35 890 122 726	3 192 935	2 090 945	371	·255 711 037 743 9845
36	0·996 014 900 494 210	599 020 737	1 101 989	078 625	321	·252 175 880 661 2921
38	050 499 514 947	35 309 997 372	289 023 365	2 066 354	270	·248 637 249 222 7444
2·040	0·996 085 809 512 320	023 040 362	286 957 010	054 134	12 220	8·245 095 143 428 3415
42	120 832 552 681	34 738 137 485	4 902 876	041 964	170	·241 549 563 278 0833
44	155 570 690 167	455 276 573	2 860 912	029 844	120	·238 000 508 771 9699
46	190 025 966 740	174 445 505	0 831 068	017 775	069	·234 447 979 910 0013
48	224 200 412 245	33 895 632 212	278 813 293	2 005 756	019	·230 891 976 692 1775
2·050	0·996 258 096 044 457	618 824 674	276 807 538	1 993 786	11 969	8·227 332 499 118 4985
52	291 714 869 131	344 010 923	4 813 751	981 867	919	·223 769 547 188 9642
54	325 058 880 054	071 179 039	2 831 884	969 999	869	·220 203 120 903 5747
56	358 130 059 093	32 800 317 154	0 861 885	958 180	819	·216 633 220 262 3300
58	390 930 376 247	32 531 413 449	268 903 705	1 946 411	769	·213 059 845 265 2300
2·060	0·996 423 461 789 696	264 456 154	266 957 294	934 692	11 719	8·209 482 995 912 2748
62	455 726 245 850	31 999 433 552	5 022 602	923 024	669	·205 902 672 203 4644
64	487 725 679 403	736 333 974	3 099 579	911 405	619	·202 318 874 138 7988
66	519 462 013 377	475 145 800	1 188 174	1 899 836	569	·198 731 601 718 2779
68	550 937 159 177	31 215 857 462	259 288 338	1 888 317	519	·195 140 854 941 9018
2·070	0·996 582 153 016 638	30 958 457 440	257 400 022	876 847	11 469	8·191 546 633 809 6705
72	613 111 474 079	702 934 266	5 523 174	865 428	420	·187 948 938 321 5840
74	643 814 408 345	449 276 520	3 657 746	854 058	370	·184 347 768 477 6422
76	674 263 684 865	197 472 831	1 803 688	842 738	320	·180 743 124 277 8452
78	704 461 157 696	29 947 511 881	249 960 951	1 831 467	271	·177 135 005 722 1930
2·080	0·996 734 408 669 576	699 382 397	248 129 483	820 246	11 221	8·173 523 412 810 6856
82	764 108 051 974	453 073 160	6 309 237	809 075	171	·169 908 345 543 3229
84	793 561 125 133	208 572 998	4 500 162	1 797 053	122	·166 289 803 920 1050
86	822 769 698 131	28 965 870 788	2 702 210	786 880	073	·162 667 787 941 0319
88	851 735 568 919	28 724 955 459	0 915 329	1 775 857	023	·159 042 297 606 1035
2·090	0·996 880 460 524 378	485 815 986	239 139 472	764 883	10 974	8·155 413 332 915 3200
92	908 946 340 364	248 441 397	7 374 589	753 958	925	·151 780 893 868 6812
94	937 194 781 761	012 820 765	5 620 631	743 083	876	·148 144 980 466 1871
96	965 207 602 526	27 778 943 217	3 877 549	732 256	827	·144 505 592 707 8379
98	992 986 545 743	27 546 797 924	2 145 293	1 721 479	777	·140 862 730 593 6334

TABLE OF THE VALUES OF $H=\frac{2}{\sqrt{\pi}}\int_0^t \epsilon^{-t^2}dt$. (2) FROM $t=1{\cdot}000$ TO $t=3{\cdot}000$.

2·100] [2·198

t	H	Δ_1 +	Δ_2 −	Δ_3 +	Δ_4 −	$\log\frac{2}{\sqrt{\pi}}\epsilon^{-t^2}+10.$
2·100	0·997 020 533 343 667	27 316 374 110	230 423 814	1 710 750	10 729	8·137 216 394 123 5737
2	047 849 717 777	087 661 046	228 713 064	700 071	679	·133 566 583 297 6588
4	074 937 378 823	26 860 648 053	7 012 993	1 689 440	631	·129 913 298 115 8886
6	101 798 026 876	635 324 499	5 323 554	678 858	582	·126 256 538 578 2632
8	128 433 351 375	26 411 679 803	3 644 696	1 668 324	533	·122 596 304 684 7826
2·110	0·997 154 845 031 178	189 703 431	221 976 372	657 840	10 485	8·118 932 596 435 4468
12	181 034 734 609	25 969 384 900	0 318 532	647 403	436	·115 265 413 830 2557
14	207 004 119 509	750 713 771	218 671 128	637 016	388	·111 594 756 869 2095
16	232 754 833 280	533 679 658	7 034 113	626 676	339	·107 920 625 552 3080
18	258 288 512 938	25 318 272 222	5 407 436	1 616 385	291	·104 243 019 879 5512
2·120	0·997 283 606 785 161	104 481 172	213 791 051	606 143	10 243	8·100 561 939 850 9393
22	308 711 266 332	24 892 296 264	212 184 908	595 948	10 195	·096 877 385 466 4721
24	333 603 562 596	681 707 304	0 588 960	585 801	147	·093 189 356 726 1497
26	358 285 269 899	472 704 145	209 003 159	575 703	099	·089 497 853 629 9720
28	382 757 974 044	24 265 276 689	7 427 456	1 565 652	051	·085 802 876 177 9391
2·130	0·997 407 023 250 733	059 414 886	205 861 804	555 649	10 003	8·082 104 424 370 0511
32	431 082 665 619	23 855 108 731	4 306 154	545 694	9 955	·078 402 498 206 3077
34	454 937 774 350	652 348 271	2 760 460	535 787	908	·074 697 097 686 7092
36	478 590 122 621	451 123 598	1 224 674	525 927	860	·070 988 222 811 2554
38	502 041 246 219	23 251 424 851	199 698 747	1 516 114	813	·067 275 873 579 9464
2·140	0·997 525 292 671 070	053 242 218	198 182 633	506 349	9 765	8·063 560 049 992 7822
42	548 345 913 288	22 856 565 934	6 676 284	496 631	718	·059 840 752 049 7627
44	571 202 479 222	661 386 281	5 179 653	486 960	671	·056 117 979 750 8881
46	593 863 865 503	467 693 589	3 692 692	477 337	624	·052 391 733 096 1582
48	616 331 559 092	22 275 478 233	2 215 356	1 467 760	577	·048 662 012 085 5730
2·150	0·997 638 607 037 325	084 730 638	190 747 595	458 230	9 530	8·044 928 816 719 1327
52	660 691 767 963	21 895 441 273	189 289 365	448 748	483	·041 192 146 996 8371
54	682 587 209 237	707 600 656	7 840 617	439 311	436	·037 452 002 918 6863
56	704 294 809 893	521 199 350	6 401 306	429 922	390	·033 708 384 484 6803
58	725 816 009 242	21 336 227 965	4 971 384	1 420 578	343	·029 961 291 694 8190
2·160	0·997 747 152 237 208	152 677 159	183 550 806	411 282	9 297	8·026 210 724 549 1025
62	768 304 914 367	20 970 537 635	2 139 524	402 031	250	·022 456 683 047 5308
64	789 275 452 002	789 800 142	0 737 493	1 392 827	204	·018 699 167 190 1039
66	810 065 252 145	610 455 476	179 344 666	383 669	158	·014 938 176 976 8217
68	830 675 707 621	20 432 494 479	7 960 997	1 374 557	112	·011 173 712 407 6843
2·170	0·997 851 108 202 100	255 908 039	176 586 441	365 490	9 066	8·007 405 773 482 6917
72	871 364 110 139	080 687 089	5 220 950	356 470	021	·003 634 360 201 8439
74	891 444 797 227	19 906 822 608	3 864 481	347 495	8 975	7·999 859 472 565 1408
76	911 351 619 835	734 305 622	2 516 986	338 565	929	·996 081 110 572 5825
78	931 085 925 457	19 563 127 201	1 178 421	1 329 681	884	·992 299 274 224 1690
2·180	0·997 950 649 052 659	393 278 462	169 848 739	320 843	8 839	7·988 513 963 519 9003
82	970 042 331 121	224 750 566	8 527 896	312 049	793	·984 725 178 459 7763
84	989 267 081 687	057 534 719	7 215 847	303 301	748	·980 932 919 043 7971
86	0·998 008 324 616 405	18 891 622 172	5 912 546	1 294 598	703	·977 137 185 271 9627
88	027 216 238 578	18 727 004 224	4 617 949	1 285 939	658	·973 337 977 144 2731
2·190	0·998 045 943 242 801	563 672 214	163 332 009	277 325	8 614	7·969 535 294 660 7282
92	064 506 915 016	401 617 530	2 054 684	268 756	569	·965 729 137 821 3281
94	082 908 532 546	240 831 603	0 785 928	260 232	525	·961 919 506 626 0728
96	101 149 364 149	081 305 907	159 525 696	251 752	480	·958 106 401 074 9622
98	119 230 670 056	17 923 031 963	8 273 944	1 243 316	436	·954 289 821 167 9964

TABLE OF THE VALUES OF $H = \frac{2}{\sqrt{\pi}}\int_0^t \epsilon^{-t^2} dt$. (2) FROM $t = 1{\cdot}000$ TO $t = 3{\cdot}000$.

t	H	Δ_1 +	Δ_2 −	Δ_3 +	Δ_4 −	$\log \frac{2}{\sqrt{\pi}}\epsilon^{-t^2} + 10$.
2·200	0·998 137 153 702 018		157 030 628		8 392	7·950 469 766 905 1754
		17 766 001 334		1 234 924		
2	154 919 703 352		5 795 704		348	·946 646 238 286 4992
		610 205 630		226 576		
4	172 529 908 982		4 569 128		304	·942 819 235 311 9677
		455 636 502		218 273		
6	189 985 545 484		3 350 855		260	·938 988 757 981 5811
		302 285 647		210 013		
8	207 287 831 130		2 140 842		216	·935 154 806 295 3392
		17 150 144 804		1 201 797		
2·210	0·998 224 437 975 934		150 939 046		8 173	7·931 317 380 253 2420
		16 999 205 758		1 193 624		
12	241 437 181 693		149 745 422		129	·927 476 479 855 2897
		849 460 336		185 495		
14	258 286 642 029		8 559 927		086	·923 632 105 101 4821
		700 900 409		177 409		
16	274 987 542 437		7 382 519		043	·919 784 255 991 8193
		553 517 890		169 366		
18	291 541 060 327		6 213 153		8 000	·915 932 932 526 3012
		16 407 304 737		1 161 366		
2·220	0·998 307 948 365 065		145 051 786		7 957	7·912 078 134 704 9280
		262 252 951		153 410		
22	324 210 618 016		3 898 377		914	·908 219 862 527 6995
		118 354 575		145 496		
24	340 328 972 590		2 752 881		871	·904 358 115 994 6158
		15 975 601 694		137 625		
26	356 304 574 284		1 615 256		829	·900 492 895 105 6768
		833 986 437		129 796		
28	372 138 560 721		0 485 460		786	·896 624 199 860 8826
		15 693 500 977		1 122 010		
2·230	0·998 387 832 061 698		139 363 451		7 744	7·892 752 030 260 2333
		554 137 526		114 266		
32	403 386 199 224		8 249 185		702	·888 876 386 303 7286
		415 888 341		106 564		
34	418 802 087 565		7 142 621		660	·884 997 267 991 3688
		278 745 720		098 904		
36	434 080 833 286		6 043 717		618	·881 114 675 323 1537
		142 702 004		091 287		
38	449 223 535 289		4 952 430		576	·877 228 608 299 0834
		15 007 749 573		1 083 711		
2·240	0·998 464 231 284 863		133 868 719		7 534	7·873 339 066 919 1579
		14 873 880 854		076 176		
42	479 105 165 717		2 792 543		493	·869 446 051 183 3771
		741 088 311		068 683		
44	493 846 254 027		1 723 860		451	·865 549 561 091 7412
		609 364 451		061 232		
46	508 455 618 478		0 662 628		410	·861 649 596 644 2500
		478 701 823		053 822		
48	522 934 320 302		129 608 806		369	·857 746 157 840 9035
		14 349 093 017		1 046 453		
2·250	0·998 537 283 413 319		128 562 353		7 328	7·853 839 244 681 7019
		220 530 664		039 125		
52	551 503 943 983		7 523 228		287	·849 928 857 166 6450
		093 007 436		031 838		
54	565 596 951 419		6 491 391		246	·846 014 995 295 7329
		13 966 516 045		024 591		
56	579 563 467 464		5 466 800		206	·842 097 659 068 9656
		841 049 246		017 385		
58	593 404 516 710		4 449 414		165	·838 176 848 486 3430
		13 716 599 832		1 010 220		
2·260	0 998 607 121 116 542		123 439 194		7 125	7·834 252 563 547 8652
		593 160 637		1 003 095		
62	620 714 277 179		2 436 100		084	·830 324 804 253 5322
		470 724 538		996 010		
64	634 185 001 716		1 440 090		045	·826 393 570 603 3440
		349 284 448		988 965		
66	647 534 286 164		0 451 125		005	·822 458 862 597 3005
		228 833 323		981 960		
68	660 763 119 487		119 469 165		6 965	·818 520 680 235 4018
		13 109 364 158		974 995		
2·270	0·998 673 872 483 645		118 494 170		6 926	7·814 579 023 517 6479
		12 990 869 988		968 069		
72	686 863 353 634		7 526 101		886	·810 633 892 444 0388
		873 343 888		961 183		
74	699 736 697 521		6 564 917		847	·806 685 287 014 5744
		756 778 970		954 337		
76	712 493 476 492		5 610 581		807	·802 733 207 229 2548
		641 168 390		947 529		
78	725 134 644 881		4 663 052		768	·798 777 653 088 0800
		12 526 505 338		940 761		
2·280	0·998 737 661 150 219		113 722 291		6 729	7·794 818 624 591 0500
		412 783 047		934 031		
82	750 073 933 266		2 788 260		691	·790 856 121 738 1647
		299 994 787		927 341		
84	762 373 928 053		1 860 919		652	·786 890 144 529 4242
		188 133 868		920 689		
86	774 562 061 921		0 940 230		613	·782 920 692 964 8285
		077 193 638		914 075		
88	786 639 255 559		0 026 155		575	·778 947 767 044 3775
		11 967 167 482		907 500		
2·290	0·998 798 606 423 041		109 118 655		6 537	7·774 971 366 768 0714
		858 048 828		900 963		
92	810 464 471 869		8 217 692		499	·770 991 492 135 9100
		749 831 136		894 465		
94	822 214 303 005		7 323 227		461	·767 008 143 147 8933
		642 507 909		888 004		
96	833 856 810 914		6 435 223		423	·763 021 319 804 0215
		536 072 686		881 581		
98	845 392 883 599		5 553 642		385	·759 031 022 104 2944
		11 430 519 044		875 196		

TABLE OF THE VALUES OF $H=\frac{2}{\sqrt{\pi}}\int_0^t \epsilon^{-t^2}dt$. (2) FROM $t=1{\cdot}000$ TO $t=3{\cdot}000$.

t	H	Δ_1 +	Δ_2 −	Δ_3 +	Δ_4 −	$\log.\frac{2}{\sqrt{\pi}}\epsilon^{-t^2}+10.$
2·300	0·998 856 823 402 643	11 325 840 598	104 678 446	868 848	6 348	7·755 037 250 048 7121
2	868 149 243 241	222 031 000	3 809 598	62 538	310	·751 040 003 637 2746
4	879 371 274 241	119 083 939	2 947 060	56 265	273	·747 039 282 869 9818
6	890 490 358 180	016 993 144	2 090 796	50 029	236	·743 035 087 746 8338
8	901 507 351 323	10 915 752 377	1 240 767	843 830	199	·739 027 418 267 8306
2·310	0·998 912 423 103 700	815 355 440	100 396 937	37 668	6 162	7·735 016 274 432 9722
12	923 238 459 140	715 796 171	99 559 269	31 542	125	·731 001 656 242 2585
14	933 954 255 311	617 068 444	8 727 727	25 453	089	·726 983 563 695 6896
16	944 571 323 755	519 166 171	7 902 273	19 401	052	·722 961 996 793 2655
18	955 090 489 926	10 422 083 298	7 082 872	813 385	016	·718 936 955 534 9862
2·320	0·998 965 512 573 224	325 813 811	96 269 488	07 405	5 980	7·714 908 439 920 8516
22	975 838 387 035	230 351 728	5 462 083	01 461	944	·710 876 449 950 8618
24	986 068 738 763	135 691 106	4 660 622	795 552	908	·706 840 985 625 0168
26	996 204 429 869	041 826 036	3 865 070	89 680	873	·702 802 046 943 3166
28	0·999 006 246 255 904	9 948 750 645	3 075 390	783 843	837	·698 759 633 905 7611
2·330	0·999 016 195 006 550	856 459 097	92 291 548	78 041	5 802	7·694 713 746 512 3504
32	026 051 465 647	764 945 591	1 513 507	72 275	766	·690 664 384 763 0845
34	035 816 411 238	674 204 358	0 741 232	66 543	731	·086 611 548 657 9634
36	045 490 615 596	584 229 669	89 974 689	60 847	696	·682 555 238 196 9870
38	055 074 845 265	9 495 015 827	9 213 842	755 185	661	·678 495 453 380 1554
2·340	0·999 064 569 861 092	406 557 170	88 458 657	49 559	5 627	7·674 432 194 207 4686
42	073 976 418 262	318 848 072	7 709 098	43 966	592	·670 365 460 678 9265
44	083 295 266 334	231 882 940	6 965 132	38 408	558	·666 295 252 794 5292
46	092 527 149 274	145 656 216	6 226 723	32 885	524	·662 221 570 554 2767
48	101 672 805 490	9 060 162 378	5 493 839	727 395	490	·658 144 413 958 1690
2·350	0·999 110 732 967 868	8 975 395 934	84 766 443	21 940	5 456	7·654 063 783 006 2061
52	119 708 363 802	891 351 431	4 044 504	16 518	422	·649 979 677 698 3879
54	128 599 715 232	808 023 445	3 327 986	11 130	388	·645 892 098 034 7145
56	137 407 738 677	725 406 589	2 616 856	05 775	355	·641 801 044 015 1858
58	146 133 145 266	8 643 495 509	1 911 080	700 454	321	·637 706 515 639 8020
2·360	0·999 154 776 640 775	562 284 882	81 210 626	695 166	5 288	7·633 608 512 908 5629
62	163 338 925 658	481 769 423	80 515 460	89 911	255	·629 507 035 821 4686
64	171 820 695 080	401 943 874	79 825 548	84 690	222	·625 402 084 378 5190
66	180 222 638 954	322 803 015	9 140 859	79 501	189	·621 293 658 579 7143
68	188 545 441 969	8 244 341 657	8 461 358	674 344	156	·617 181 758 425 0543
2·370	0·999 196 789 783 626	166 554 643	77 787 014	69 220	5 124	7·613 066 383 914 5391
72	204 956 338 269	089 436 849	7 117 794	64 129	091	·608 947 535 048 1686
74	213 045 775 119	012 983 184	6 453 665	59 070	059	·604 825 211 825 9430
76	221 058 758 303	7 937 188 589	5 794 595	54 042	027	·600 699 414 247 8621
78	228 995 946 892	7 862 048 036	5 140 553	649 047	4 995	·596 570 142 313 9260
2·880	0·999 236 857 994 929	787 556 531	74 491 505	44 084	4 963	7·592 437 396 024 1346
82	244 645 551 460	713 709 109	3 847 422	39 152	932	·588 301 175 378 4880
84	252 359 260 569	640 500 840	3 208 269	34 252	900	·584 161 480 376 9862
86	259 999 761 409	567 926 822	2 574 018	29 383	869	·580 018 311 019 6292
88	267 567 688 231	7 495 982 188	1 944 634	624 545	838	·575 871 667 306 4170
2·390	0·999 275 063 670 419	424 662 099	71 320 089	19 739	4 807	7·571 721 549 237 3495
92	282 488 332 518	353 961 749	0 700 350	14 963	776	·567 567 956 812 4268
94	289 842 294 267	283 876 362	0 085 387	10 219	745	·563 410 890 031 6489
96	297 126 170 629	214 401 194	69 475 168	05 505	714	·559 250 348 895 0157
98	304 340 571 824	7 145 531 531	8 869 663	600 821	684	·555 086 333 402 5273

TABLE OF THE VALUES OF $H = \frac{2}{\sqrt{\pi}}\int_0^t \epsilon^{-t^2} dt$. (2) FROM $t = 1{\cdot}000$ TO $t = 3{\cdot}000$.

2·400] [2·498

t	H	Δ_1 +	Δ_2 −	Δ_3 +	Δ_4 −	log. $\frac{2}{\sqrt{\pi}}\epsilon^{-t^2}$ + 10.
2·400	0·999 311 486 103 355	7 077 262 689	68 268 842	596 168	4 653	7·550 918 843 554 1837
2	318 563 366 044	009 590 014	7 672 675	91 545	623	·546 747 879 349 9849
4	325 572 956 057	6 942 508 884	7 081 130	86 952	593	·542 573 440 789 9309
6	332 515 464 941	876 014 706	6 494 178	82 389	563	·538 395 527 874 0216
8	339 391 479 648	6 810 102 917	5 911 789	577 856	533	·534 214 140 602 2571
2·410	0·999 346 201 582 565	744 768 984	65 333 933	73 352	4 503	7·530 029 278 974 6373
12	352 946 351 549	680 008 403	4 760 581	68 879	474	·525 840 942 991 1624
14	359 626 359 952	615 816 701	4 191 702	64 434	445	·521 649 132 651 8322
16	366 242 176 654	552 189 433	3 627 268	60 019	415	·517 453 847 956 6468
18	372 794 366 087	6 489 122 184	3 067 249	555 633	386	·513 255 088 905 6061
2·420	0·999 379 283 488 271	426 610 567	62 511 617	51 275	4 357	7·509 052 855 498 7103
22	385 710 098 838	364 650 226	1 960 341	46 947	328	·504 847 147 735 9592
24	392 074 749 064	303 236 832	1 413 394	42 647	300	·500 637 965 617 3529
26	398 377 985 896	242 366 085	0 870 747	38 376	271	·496 425 309 142 8913
28	404 620 351 981	6 182 033 714	0 332 371	534 133	243	·492 209 178 312 5745
2·430	0·999 410 802 385 694	122 235 476	59 798 238	29 919	4 214	7·487 989 573 126 4026
32	416 924 621 170	062 967 157	9 268 319	25 733	186	·483 766 493 584 3753
34	422 987 588 328	004 224 571	8 742 586	21 574	158	·479 539 939 686 4929
36	428 991 812 899	5 946 003 559	8 221 012	17 444	131	·475 309 911 432 7552
38	434 937 816 458	5 888 299 991	7 703 568	513 341	103	·471 076 408 823 1623
2·440	0·999 440 826 116 449	831 109 763	57 190 227	09 266	4 075	7·466 839 431 857 7142
42	446 657 226 212	774 428 801	6 680 962	05 218	048	·462 598 980 536 4108
44	452 431 655 013	718 253 057	6 175 744	01 197	021	·458 355 054 859 2523
46	458 149 908 071	662 578 511	5 674 547	497 204	3 993	·454 107 654 826 2385
48	463 812 486 581	5 607 401 168	5 177 343	493 238	966	·449 856 780 437 3694
2·450	0·999 469 419 887 749	552 717 063	54 684 105	489 298	3 939	7·445 602 431 692 6452
52	474 972 604 811	498 522 255	4 194 807	85 385	913	·441 344 608 592 0657
54	480 471 127 067	444 812 834	3 709 422	81 499	886	·437 083 311 135 6310
56	485 915 939 901	391 584 912	3 227 922	477 640	860	·432 818 539 323 3411
58	491 307 524 813	5 338 834 629	2 750 282	473 807	833	·428 550 293 155 1959
2·460	0·999 496 646 359 442	286 558 154	52 276 476	469 999	3 807	7·424 278 572 631 1955
62	501 932 917 596	234 751 677	1 806 476	66 218	781	·420 003 377 751 3399
64	507 167 669 273	183 411 419	1 340 258	62 463	755	·415 724 708 515 6291
66	512 351 080 692	132 533 625	0 877 795	458 734	729	·411 442 564 924 0630
68	517 483 614 317	5 082 114 564	0 419 060	455 031	704	·407 156 946 976 6417
2·470	0·999 522 565 728 881	032 150 535	49 964 030	51 352	3 678	7·402 867 854 673 3652
72	527 597 879 416	4 982 637 857	9 512 677	447 700	653	·398 575 288 014 2335
74	532 580 517 273	933 572 880	9 064 978	44 072	627	·394 279 246 999 2465
76	537 514 090 152	884 951 974	8 620 905	40 470	602	·389 979 731 628 4043
78	542 399 042 127	4 836 771 539	8 180 435	436 893	577	·385 676 741 901 7069
2·480	0·999 547 235 813 666	789 027 997	47 743 542	33 340	3 552	7·381 370 277 819 1542
82	552 024 841 663	741 717 795	7 310 202	429 813	528	·377 060 339 380 7464
84	556 766 559 458	694 837 406	6 880 389	26 310	503	·372 746 926 586 4833
86	561 461 396 863	648 383 326	6 454 080	22 831	479	·368 430 039 436 3649
88	566 109 780 189	4 602 352 077	6 031 249	419 377	454	·364 109 677 930 3914
2·490	0·999 570 712 132 266	556 740 205	45 611 872	15 947	3 430	7·359 785 842 068 5626
92	575 268 872 471	511 544 279	5 195 926	12 541	406	·355 468 531 850 8786
94	579 780 416 751	466 760 895	4 783 385	409 158	382	·351 127 747 277 3394
96	584 247 177 645	422 386 668	4 374 226	05 800	358	·346 793 488 347 9449
98	588 669 564 313	4 378 418 242	3 968 426	402 466	335	·342 455 755 062 6953

TABLE OF THE VALUES OF $H=\frac{2}{\sqrt{\pi}}\int_0^t \epsilon^{-t^2}dt$. (2) FROM $t=1{\cdot}000$ TO $t=3{\cdot}000$.

t	H	Δ_1 +	Δ_2 −	Δ_3 +	Δ_4 −	$\log\frac{2}{\sqrt{\pi}}\epsilon^{-t^2}+10.$
2·500	0·999 593 047 982 555	4 334 852 281	43 565 961	399 155	3 311	7·338 114 547 421 5903
2	597 382 834 836	291 685 475	166 806	5 867	3 288	·333 769 865 424 6302
4	601 674 520 311	248 914 537	42 770 939	2 603	64	·329 421 709 071 8149
6	605 923 434 848	206 536 201	378 336	389 362	41	·325 070 078 363 1443
8	610 129 971 049	4 164 547 227	41 988 974	386 144	18	·320 714 973 298 6185
2·510	0·999 614 294 518 275	122 944 396	41 602 831	2 948	3 195	7·316 356 393 878 2374
12	618 417 462 671	081 724 513	219 882	379 776	72	·311 994 340 102 0012
14	622 499 187 184	040 884 407	40 840 106	6 626	50	·307 628 811 969 9097
16	626 540 071 592	000 420 927	463 480	3 499	27	·303 259 809 481 9630
18	630 540 492 519	3 960 330 946	089 981	370 394	05	·298 887 332 638 1610
2·520	0·999 634 500 823 465	920 611 360	39 719 586	367 312	3 082	7·294 511 381 438 5039
22	638 421 434 825	881 259 085	352 275	4 252	60	·290 131 955 882 9915
24	642 302 693 911	842 271 062	38 988 023	1 213	38	·285 749 055 971 6239
26	646 144 964 973	803 644 253	626 810	358 197	16	·281 362 681 704 4010
28	649 948 609 225	3 765 375 640	268 613	355 202	2 995	·276 972 833 081 3229
2·530	0·999 653 713 984 865	727 462 229	37 913 411	2 229	2 973	7·272 579 510 102 3896
32	657 441 447 094	689 901 047	561 182	349 278	51	·268 182 712 767 6011
34	661 131 348 141	652 689 144	211 904	6 348	30	·263 782 441 076 9574
36	664 784 037 284	615 823 588	36 865 556	3 439	09	·259 378 695 030 4584
38	668 399 860 872	3 579 301 471	522 117	340 552	2 888	·254 971 474 628 1042
2·540	0·999 671 979 162 343	543 119 906	36 181 565	337 685	2 867	7·250 560 779 869 8948
42	675 522 282 249	507 276 025	35 843 880	4 839	46	·246 146 610 755 8301
44	679 029 558 274	471 766 985	509 041	2 015	25	·241 728 967 285 9102
46	682 501 325 259	436 589 959	177 026	329 211	04	·237 307 849 460 1351
48	685 937 915 218	3 401 742 143	34 847 815	326 427	2 784	·232 883 257 278 5048
2·550	0·999 689 339 657 361	367 220 755	34 521 388	323 664	2 763	7·228 455 190 741 0193
52	692 706 878 115	333 023 030	197 725	0 921	43	·224 023 649 847 6785
54	696 039 901 146	299 146 227	33 876 803	318 199	23	·219 588 634 598 4825
56	699 339 047 372	265 587 622	558 605	5 496	02	·215 150 144 993 4312
58	702 604 634 995	3 232 344 513	243 109	312 814	2 683	·210 708 181 032 5248
2·560	0·999 705 836 979 508	199 414 218	32 930 295	0 151	2 663	7·206 262 742 715 7631
62	709 036 393 727	166 794 074	620 144	307 508	43	·201 813 830 043 1462
64	712 203 187 801	134 481 438	312 636	4 885	23	·197 361 443 014 6740
66	715 337 669 239	102 473 687	007 751	2 281	04	·192 905 581 630 3467
68	718 440 142 926	3 070 768 216	31 705 471	299 696	2 584	·188 446 245 890 1641
2·570	0·999 721 510 911 143	039 362 442	31 405 774	7 131	2 565	7·183 983 435 794 1263
72	724 550 273 585	008 253 800	108 643	4 585	46	·179 517 151 342 2332
74	727 558 527 385	2 977 439 742	30 814 058	2 058	27	·175 047 392 534 4850
76	730 535 967 127	946 917 743	521 999	289 550	08	·170 574 159 370 8815
78	733 482 884 869	2 916 685 293	232 449	287 061	2 489	·166 097 451 851 4228
2·580	0·999 736 399 570 163	886 739 905	29 945 388	4 590	2 471	7·161 617 269 976 1088
82	739 286 310 068	857 079 107	660 798	2 138	52	·157 133 613 744 9396
84	742 143 389 175	827 700 448	378 660	279 705	34	·152 646 483 157 9152
86	744 971 089 623	798 601 493	098 955	7 290	15	·148 155 878 215 0356
88	747 769 691 116	2 769 779 828	28 821 665	274 893	2 397	·143 661 798 916 3008
2·590	0·999 750 539 470 943	741 233 055	28 546 772	2 514	2 379	7·139 164 245 261 7107
92	753 280 703 999	712 958 797	274 258	0 153	61	·134 663 217 251 2654
94	755 993 662 795	684 954 692	004 105	267 810	43	·130 158 714 884 9649
96	758 678 617 487	657 218 397	27 736 295	5 485	25	·125 650 738 162 8091
98	761 335 835 884	2 629 747 587	470 810	263 178	07	·121 139 287 084 7981

TABLE OF THE VALUES OF $H = \frac{2}{\sqrt{\pi}}\int_0^t \epsilon^{-t^2} dt$. (2) FROM $t = 1{\cdot}000$ TO $t = 3{\cdot}000$.

t	H	Δ_1 +	Δ_2 −	Δ_3 +	Δ_4 −	$\log \frac{2}{\sqrt{\pi}}\epsilon^{-t^2} + 10.$
2·600	0·999 763 965 583 471	2 602 539 955	27 207 632	260 888	2 290	7·116 624 361 650 9319
2	66 568 123 426	575 593 211	26 946 744	258 616	72	·112 105 961 861 2105
4	69 143 716 637	548 905 083	688 128	6 361	55	·107 584 087 715 6338
6	71 692 621 720	522 473 316	431 767	4 123	38	·103 058 739 214 2019
8	74 215 095 036	2 496 295 672	177 644	251 903	21	·098 529 916 356 9148
2·610	0·999 776 711 390 708	470 369 930	25 925 742	249 699	2 204	7·093 997 619 143 7725
12	79 181 760 638	444 693 887	676 043	7 512	2 187	·089 461 847 574 7749
14	81 626 454 526	419 265 357	428 530	5 343	70	·084 922 601 649 9221
16	84 045 719 883	394 082 169	183 188	3 190	53	·080 379 881 369 2141
18	86 439 802 052	2 369 142 171	24 939 998	241 053	36	·075 833 686 732 6509
2·620	0·999 788 808 944 224	344 443 226	24 698 945	238 933	2 120	7·071 284 017 740 2324
22	91 153 387 450	319 983 215	460 012	6 830	04	·066 730 874 391 9587
24	93 473 370 665	295 760 033	223 182	4 742	2 087	·062 174 256 687 8298
26	95 769 130 697	271 771 593	23 988 440	2 671	71	·057 614 164 627 8457
28	98 040 902 290	2 248 015 825	755 768	230 617	55	·053 050 598 212 0063
2·630	0·999 800 288 918 115	224 890 673	23 525 152	228 578	2 039	7·048 483 557 440 3117
32	02 513 408 789	201 194 100	296 574	6 555	23	·043 913 042 312 7619
34	04 714 602 888	178 124 080	070 019	4 547	07	·039 339 052 829 3568
36	06 892 726 969	155 278 608	22 845 472	2 556	1 992	·034 761 588 990 0966
38	09 048 005 577	2 132 655 692	622 916	220 580	76	·030 180 650 794 9811
2·640	0·999 811 180 661 268	110 253 356	22 402 337	218 619	1 961	7·025 596 238 244 0103
42	13 290 914 625	088 069 638	183 718	6 674	45	·021 008 351 337 1844
44	15 378 984 263	066 102 595	21 967 044	4 744	30	·016 416 990 074 5032
46	17 445 086 857	044 350 295	752 299	2 830	15	·011 822 154 455 9668
48	19 489 437 153	2 022 810 826	539 470	210 930	00	·007 223 844 481 5752
2·650	0·999 821 512 247 978	001 482 286	21 328 540	209 045	1 885	7·002 622 060 151 3283
52	23 513 730 264	1 980 362 792	119 494	7 176	70	6·998 016 801 465 2262
54	25 494 093 056	959 450 473	20 912 319	5 321	55	·993 408 068 423 2689
56	27 453 543 529	938 743 475	706 998	3 481	40	·988 795 861 025 4564
58	29 392 287 004	1 918 239 958	503 517	201 655	26	·984 180 179 271 7886
2·660	0·999 831 310 526 963	1 897 938 096	20 301 862	199 844	1 811	6·979 561 023 162 2656
62	33 208 465 059	877 836 078	102 018	8 047	1 797	·974 938 392 696 8874
64	35 086 301 136	857 932 107	19 903 971	6 265	82	·970 312 287 875 6540
66	36 944 233 243	838 224 400	707 707	4 496	68	·965 682 708 698 5653
68	38 782 457 643	1 818 711 190	513 210	192 742	54	·961 049 655 165 6214
2·670	0·999 840 601 168 833	1 799 390 722	19 320 468	1 002	1 740	6·956 413 127 276 8223
72	42 400 559 555	780 261 257	129 465	189 276	26	·951 773 125 032 1680
74	44 180 820 812	761 321 068	18 940 189	7 564	12	·947 129 648 431 6584
76	45 942 141 880	742 568 443	752 625	5 865	1 699	·942 482 697 475 2936
78	47 684 710 322	1 724 001 683	566 760	184 181	85	·937 832 272 163 0736
2·680	0·999 849 408 712 006	705 619 104	18 382 579	2 509	1 671	6·933 178 372 494 9983
82	51 114 331 110	687 419 034	200 070	0 851	58	·928 520 998 471 0679
84	52 801 750 144	669 399 816	019 218	179 207	44	·923 860 150 091 2822
86	54 471 149 959	651 559 804	17 840 012	7 576	31	·919 195 827 355 6412
88	56 122 709 763	1 633 897 368	662 436	175 958	18	·914 528 030 264 1451
2·690	0·999 857 756 607 132	616 410 890	17 486 478	4 353	1 605	6·909 856 758 816 7937
92	59 373 018 021	599 098 765	312 125	2 761	1 592	·905 182 013 013 5871
94	60 972 116 787	581 959 401	139 364	1 182	79	·900 503 792 854 5253
96	62 554 076 188	564 991 219	16 968 182	169 616	66	·895 822 098 339 6082
98	64 119 067 407	1 548 192 653	798 566	168 063	53	·891 136 929 468 8360

TABLE OF THE VALUES OF $\dot{H}=\frac{2}{\sqrt{\pi}}\int_0^t \epsilon^{-t^2}dt$. (2) FROM $t=1{\cdot}000$ TO $t=3{\cdot}000$.

2·700] [2·798

t	H	Δ_1 +	Δ_2 −	Δ_3 +	Δ_4 −	$\log \frac{2}{\sqrt{\pi}}\epsilon^{-t^2}+10.$
2·700	0·999 865 667 260 059	1 531 562 150	16 630 503	166 522	1 541	6·886 448 286 242 2084
2	7 198 822 209	15 098 169	463 981	4 994	28	·881 756 168 659 7257
4	8 713 920 378	1 498 799 182	298 987	3 478	16	·877 060 576 721 3878
6	0·999 870 212 719 559	182 663 673	135 509	1 975	03	·872 361 510 427 1946
8	1 695 383 232	1 466 690 139	15 973 533	160 484	1 491	·867 658 969 777 1462
2·710	0·999 873 162 073 372	50 877 090	15 813 049	159 006	1 479	6·862 952 954 771 2425
12	4 612 950 462	35 223 047	654 043	7 539	66	·858 243 465 409 4837
14	6 048 173 509	19 726 543	496 504	6 085	54	·853 530 501 691 8696
16	7 467 900 053	04 386 124	340 419	4 643	42	·848 814 063 618 4003
18	8 872 286 177	1 389 200 348	185 776	153 212	31	·844 094 151 189 0757
2·720	0·999 880 261 486 525	74 167 784	15 032 564	1 793	1 419	6·839 370 764 403 8959
22	1 635 654 309	59 287 013	14 880 771	0 386	07	·834 643 903 262 8610
24	2 994 941 322	44 556 628	730 385	148 991	1 395	·829 913 567 765 9707
26	4 339 497 950	29 975 235	581 394	7 607	84	·825 179 757 913 2253
28	5 669 473 185	1 315 541 449	433 786	146 235	72	·820 442 473 704 6246
2·730	0·999 886 985 014 633	01 253 897	14 287 551	4 874	1 361	6·815 701 715 140 1687
32	8 286 268 531	1 287 111 221	142 677	3 525	50	·810 957 482 219 8576
34	9 573 379 751	73 112 068	13 999 152	2 186	38	·806 209 774 943 6913
36	0·999 890 846 491 820	59 255 103	856 966	0 859	27	·801 458 593 311 6697
38	2 105 746 923	1 245 538 997	716 106	139 543	16	·796 703 937 323 7929
2·740	0·999 893 351 285 919	31 962 434	13 576 563	8 238	1 305	6·791 945 806 980 0608
42	4 583 248 353	18 524 109	438 325	6 944	1 294	·787 184 202 280 4736
44	5 801 772 462	05 222 729	301 380	5 661	83	·782 419 123 225 0311
46	7 006 995 191	1 192 057 010	165 719	4 389	72	·777 650 569 813 7334
48	8 199 052 201	1 179 025 679	031 331	133 127	62	·772 878 542 046 5805
2·750	0·999 899 378 077 880	66 127 476	12 898 204	1 876	1 251	6·768 103 039 923 5723
52	0·999 900 544 205 356	53 361 148	766 328	0 635	41	·763 324 063 444 7089
54	1 697 566 503	40 725 455	635 693	129 405	30	·758 541 612 609 9903
56	2 838 291 958	28 219 167	506 288	8 185	20	·753 755 687 419 4165
58	3 966 511 125	1 115 841 065	378 102	126 976	09	·748 966 287 872 9874
2·760	0·999 905 082 352 190	03 589 938	12 251 127	5 777	1 199	6·744 173 413 970 7031
62	6 185 942 128	1 091 464 588	125 350	4 588	89	·739 377 065 712 5636
64	7 277 406 716	79 463 826	000 762	3 409	79	·734 577 243 098 5689
66	8 356 870 542	67 586 473	11 877 353	2 240	69	·729 773 946 128 7189
68	9 424 457 015	1 055 831 360	755 113	121 081	59	·724 967 174 803 0137
2·770	0·999 910 480 288 375	44 197 328	11 634 032	119 932	1 149	6·720 156 929 121 4533
72	1 524 485 703	32 683 228	514 100	8 793	39	·715 343 209 084 0377
74	2 557 168 930	21 287 920	395 307	7 663	30	·710 526 014 690 7668
76	3 578 456 851	10 010 276	277 644	6 543	20	·705 705 345 941 6407
78	4 588 467 127	998 849 175	161 101	115 433	10	·700 881 202 836 6594
2·780	0·999 915 587 316 302	987 803 507	11 045 668	4 332	1 101	6·696 053 585 375 8228
82	6 575 119 809	976 872 172	10 931 335	3 241	91	·691 222 493 559 1311
84	7 551 991 981	966 054 078	818 094	2 159	82	·686 387 927 386 5841
86	8 518 046 059	955 348 142	705 935	1 086	73	·681 549 886 858 1818
88	9 473 394 201	944 753 294	594 849	110 023	63	·676 708 371 973 9244
2·790	0·999 920 418 147 495	934 268 468	10 484 826	108 969	1 054	6·671 863 382 733 8117
92	1 352 415 963	923 892 611	375 857	7 924	45	·667 014 919 137 8438
94	2 276 308 574	913 624 678	267 933	6 888	36	·662 162 981 186 0207
96	3 189 933 252	903 463 633	161 045	5 861	27	·657 307 568 878 3423
98	4 093 396 886	893 408 449	055 185	104 842	18	·652 448 682 214 8087

TABLE OF THE VALUES OF $H = \frac{2}{\sqrt{\pi}}\int_0^t \epsilon^{-t^2} dt$. (2) FROM $t = 1{\cdot}000$ TO $t = 3{\cdot}000$.

2·800] [2·898

t	H	Δ_1 +	Δ_2 −	Δ_3 +	Δ_4 −	$\log \frac{2}{\sqrt{\pi}}\epsilon^{-t^2} + 10.$
2·800	0·999 924 986 805 335	883 458 107	9 950 342	103 833	1 009	6·647 586 321 195 4199
2	5 870 263 441	873 611 598	846 509	102 832	1 001	·642 720 485 820 1759
4	6 743 875 039	863 867 921	743 677	101 841	992	·637 851 176 089 0766
6	7 607 742 960	854 226 085	641 836	100 857	983	·632 978 392 002 1222
8	8 461 969 046	844 685 107	540 979	99 883	975	·628 102 133 559 3124
2·810	0·999 929 306 654 152	835 244 011	9 441 096	98 916	966	6·623 222 400 760 6475
12	0·999 930 141 898 163	825 901 831	342 179	97 959	958	·618 339 193 606 1273
14	0 967 799 994	816 657 611	244 221	97 009	949	·613 452 512 095 7520
16	1 784 457 605	807 510 399	147 211	96 068	941	·608 562 356 229 5213
18	2 591 968 004	798 459 256	051 143	95 136	933	·603 668 726 007 4355
2·820	0·999 933 390 427 260	789 503 248	8 956 008	94 211	925	6·598 771 621 429 4944
22	4 179 930 508	780 641 452	861 797	93 294	916	·593 871 042 495 6981
24	4 960 571 960	771 872 949	768 502	92 386	908	·588 966 989 206 0466
26	5 732 444 909	763 196 833	676 116	91 486	900	·584 059 461 560 5399
28	6 495 641 743	754 612 203	584 631	90 593	892	·579 148 459 559 1779
2·830	0·999 937 250 253 945	746 118 165	8 494 038	89 709	885	6·574 233 983 201 9607
32	7 996 372 110	737 713 836	404 329	88 832	877	·569 316 032 488 8883
34	8 734 085 946	729 398 339	315 497	87 963	869	·564 394 607 419 9606
36	9 463 484 285	721 170 804	227 534	87 101	861	·559 469 707 995 1778
38	0·999 940 184 655 090	713 030 371	140 433	86 248	854	·554 541 334 214 5397
2·840	0·999 940 897 685 461	704 976 186	8 054 185	85 402	846	6·549 609 486 078 0463
42	1 602 661 647	697 007 403	7 968 784	84 563	839	·544 674 163 585 6978
44	2 299 669 050	689 123 182	884 220	83 732	831	·539 735 366 737 4940
46	2 988 792 232	681 322 694	800 488	82 908	824	·534 793 095 533 4350
48	3 670 114 925	673 605 113	717 580	82 092	816	·529 847 349 973 5208
2·850	0·999 944 343 720 039	665 969 625	7 635 488	81 283	809	6·524 898 130 057 7513
52	5 009 689 664	658 415 420	554 205	80 481	802	·519 945 435 786 1266
54	5 668 105 084	650 941 696	473 724	79 686	795	·514 989 267 158 6467
56	6 319 046 779	643 547 658	394 038	78 899	788	·510 029 624 175 3116
58	6 962 594 437	636 232 519	315 139	78 118	780	·505 066 506 836 1212
2·860	0·999 947 598 826 956	628 995 498	7 237 021	77 345	773	6·500 099 915 141 0756
62	8 227 822 454	621 835 823	159 676	76 578	766	·495 129 849 090 1748
64	8 849 658 277	614 752 726	083 097	75 819	760	·490 156 308 683 4188
66	9 464 411 002	607 745 448	007 278	75 066	753	·485 179 293 920 8075
68	0·999 950 072 156 450	600 813 236	6 932 212	74 320	746	·480 198 804 802 3410
2·870	0·999 950 672 969 686	593 955 344	6 857 892	73 581	739	6·475 214 841 328 0193
72	1 266 925 030	587 171 033	784 311	72 848	733	·470 227 403 497 8423
74	1 854 096 063	580 459 571	711 462	72 123	726	·465 236 491 311 8102
76	2 434 555 633	573 820 231	639 340	71 403	719	·460 242 104 769 9228
78	3 008 375 864	567 252 294	567 937	70 690	713	·455 244 243 872 1802
2·880	0·999 953 575 628 159	560 755 048	6 497 246	69 984	706	6·450 242 908 618 5823
82	4 136 383 207	554 327 786	427 262	69 284	700	·445 238 099 009 1292
84	4 690 710 994	547 969 809	357 978	68 591	694	·440 229 815 043 8209
86	5 238 680 802	541 680 422	289 387	67 903	687	·435 218 056 722 6574
88	5 780 361 224	535 458 938	221 484	67 223	681	·430 202 824 045 6387
2·890	0·999 956 315 820 162	529 304 677	6 154 261	66 548	675	6·425 184 117 012 7647
92	6 845 124 839	523 216 964	087 713	65 879	669	·420 161 935 624 0355
94	7 368 341 802	517 195 130	021 834	65 217	662	·415 136 279 879 4510
96	7 885 536 932	511 238 513	5 956 617	64 561	656	·410 107 149 779 0114
98	8 396 775 445	505 346 456	892 057	63 910	650	·405 074 545 322 7165

TABLE OF THE VALUES OF $H=\frac{2}{\sqrt{\pi}}\int_0^t \epsilon^{-t^2}dt$. (2) FROM $t=1{\cdot}000$ TO $t=3{\cdot}000$.

2·900] [3·000

t	H	Δ_1 +	Δ_2 −	Δ_3 +	Δ_4 −	$\log\frac{2}{\sqrt{\pi}}\epsilon^{-t^2}+10.$
2·900	0·999 958 902 121 901					6·400 038 466 510 5664
		499 518 310	5 828 146		644	
2	9 401 640 210			63 266		·394 998 913 342 5611
		493 753 429	764 881		638	
4	9 895 393 639			62 627		·389 955 885 818 7005
		488 051 176	702 253		633	
6	0·999 960 383 444 815			61 995		·384 909 383 938 9847
		482 410 917	640 258		627	
8	0 865 855 732			61 368		·379 859 407 703 4137
		476 832 027	578 890		621	
2·910	0·999 961 342 687 760			60 747		
		471 313 884	5 518 143		615	6·374 805 957 111 9875
12	1 814 001 643			60 132		
		465 855 873	458 011		610	·369 749 032 164 7060
14	2 279 857 516			59 522		
		460 457 384	398 489		604	·364 688 632 861 5693
16	2 740 314 899			58 918		
		455 117 813	339 570		598	·359 624 759 202 5774
18	3 195 432 713			58 320		
		449 836 563	281 250		593	·354 557 411 187 7303
2·920	0·999 963 645 269 275			57 727		
		444 613 040	5 223 523		587	6·349 486 588 817 0279
22	4 089 882 316			57 140		
		439 446 657	166 383		582	·344 412 292 090 4703
24	4 529 328 973			56 558		
		434 336 833	109 824		576	·339 334 521 008 0575
26	4 963 665 806			55 982		
		429 282 991	053 842		571	·334 253 275 569 7894
28	5 392 948 797			55 411		
		424 284 560	4 998 431		566	·329 168 555 775 6662
2·930	0·999 965 817 233 357			54 845		
		419 340 974	4 943 586		560	6·324 080 361 625 6877
32	6 236 574 332			54 285		
		414 451 673	889 301		555	·318 988 693 119 8539
34	6 651 026 005			53 730		
		409 616 102	835 571		550	·313 893 550 258 1650
36	7 060 642 108			53 180		
		404 833 711	782 391		545	·308 794 933 040 6208
38	7 465 475 819			52 635		
		400 103 955	729 756		540	·303 692 841 467 2214
2·940	0·999 967 865 579 774			52 095		
		395 426 294	4 677 661		535	6·298 587 275 537 9668
42	8 261 006 068			51 561		
		390 800 194	626 100		530	·293 478 235 252 8569
44	8 651 806 263			51 031		
		386 225 125	575 069		525	·288 365 720 611 8918
46	9 038 031 388			50 506		
		381 700 562	524 563		520	·283 249 731 615 0715
48	9 419 731 950			49 987		
		377 225 986	474 576		515	·278 130 268 262 3960
2·950	0·999 969 796 957 936			49 472		
		372 800 882	4 425 104		510	6·273 007 330 553 8652
52	0·999 970 169 758 818			48 962		
		368 424 740	376 142		505	·267 880 918 489 4793
54	0 538 183 557			48 457		
		364 097 054	327 686		500	·262 751 032 069 2380
56	0 902 280 611			47 956		
		359 817 325	279 729		496	·257 617 671 293 1416
58	1 262 097 936			47 461		
		355 585 057	232 268		491	·252 480 836 161 1899
2·960	0·999 971 617 682 993			46 970		
		351 399 758	4 185 298		486	6·247 340 526 673 3831
62	1 969 082 751			46 484		
		347 260 943	138 815		482	·242 196 742 829 7209
64	2 316 343 695			46 002		
		343 168 131	092 813		477	·237 049 484 630 2036
66	2 659 511 825			45 525		
		339 120 843	047 288		473	·231 898 752 074 8310
68	2 998 632 668			45 052		
		335 118 607	002 236		468	·226 744 545 163 6032
2·970	0·999 973 333 751 275			44 584		
		331 160 955	3 957 651		464	6·221 586 863 896 5202
72	3 664 912 230			44 121		
		327 247 425	913 531		459	·216 425 708 273 5820
74	3 992 159 655			43 662		
		323 377 556	869 869		455	·211 261 078 294 7885
76	4 315 537 210			43 207		
		319 550 893	826 662		450	·206 092 973 960 1398
78	4 635 088 104			42 756		
		315 766 987	783 906		446	·200 921 395 269 6359
2·980	0 999 974 950 855 091			42 310		
		312 025 391	3 741 596		442	6·195 746 342 223 2767
82	5 262 880 482			41 868		
		308 325 664	699 727		438	·190 567 814 821 0624
84	5 571 206 146			41 431		
		304 667 367	658 297		433	·185 385 813 062 9928
86	5 875 873 514			40 997		
		301 050 068	617 299		429	·180 200 336 949 0679
88	6 176 923 582			40 568		
		297 473 337	576 731		425	·175 011 386 479 2879
2·990	0·999 976 474 396 919			40 143		
		293 936 750	3 536 588		421	6·169 818 961 653 6526
92	6 768 333 669			39 722		
		290 439 884	496 866		417	·164 623 062 472 1621
94	7 058 773 553			39 305		
		286 982 323	457 561		413	·159 423 688 934 8164
96	7 345 755 876			38 892		
		283 563 655	418 668		409	·154 220 841 041 6154
98	7 629 319 531			38 483		
		280 183 470	380 185		405	·149 014 518 792 5592
3·000	0·999 977 909 503 001			38 078		
			3 342 107		401	6·143 804 722 187 6478

(3) TABLE OF VALUES OF $H=\frac{2}{\sqrt{\pi}}\int_0^t \epsilon^{-t^2}dt$, AND $G=\int_t^\infty \epsilon^{-t^2}dt$, FROM $t=3{\cdot}0$ TO $t=6{\cdot}0$.

L denotes the value of Laplace's continued fraction (§9).

t	H	L	ϵ^{-t^2}	G
3·0	0·999 977 909 503 001	·951 813 839 183 927	·000 123 409 804 087	·000 019 577 193 237
3·1	988 351 342 633	·954 514 373 156 224	·000 067 054 824 303	·000 010 323 353 804
3·2	993 974 238 848	·957 000 847 840 583	·000 035 712 849 642	·000 005 340 191 779
3·3	996 942 290 204	·959 294 708 327 178	·000 018 643 742 332	·000 002 709 824 752
3·4	998 478 006 638	·961 414 842 914 146	*·(5)9 540 162 873 079	·000 001 348 831 499
3·5	0·999 999 258 901 628	·963 377 932 668 129	·(5)4 785 117 392 129	·000 000 658 553 786
3·6	644 137 007	·965 198 747 506 567	·(5)2 352 575 200 010	315 375 366
3·7	832 848 942	·966 890 397 828 628	·(5)1 133 727 138 748	148 133 768
3·8	922 996 073	·968 464 548 822 273	·(5)0 535 534 780 279	068 242 954
3·9	965 207 751	·969 931 604 907 714	·(5)0 247 959 601 805	030 833 828
4·0	0·999 999 984 582 742	·971 300 864 958 029	·(5)0 112 535 174 719	·000 000 013 663 189
4·1	932 999 724	·972 580 654 473 280	·(5)0 050 062 180 208	005 937 745
4·2	997 144 506	·973 778 466 897 719	·(5)0 021 829 577 951	002 530 616
4·3	998 806 528	·974 901 013 211 320	·(8)9 330 287 574 505	001 057 687
4·4	999 510 829	·975 954 360 776 017	·(8)3 908 938 434 265	000 433 517
4·5	0·999 999 999 803 384	·976 943 983 556 604	·(8)1 605 228 055 186	·000 000 000 174 246
4·6	922 504	·977 874 833 415 583	·(8)0 646 143 177 311	068 679
4·7	980 048	·978 719 571 814 619	·(8)0 254 938 188 039	026 544
4·8	988 648	·979 577 750 614 522	·(8)0 098 595 055 760	010 061
4·9	995 781	·980 357 595 871 849	·(8)0 037 375 713 279	003 739
5·0	0·999 999 999 998 463	·981 094 307 287 316	·(8)0 013 887 943 865	·000 000 000 001 363
5·5	999 993	·984 229 800 386 619	·(12)072 877 240 958	·(8)0 000 006 520 723
6·0	1—·(15)021 516 075	·986 653 109 231 165	·(15)231 952 283 024	·(8)0 000 000 019 069
∞	1·0	1·0	·0	0·0

* The figures in parentheses indicate the number of ciphers between the decimal point and the figures that follow. The value of H for $t=6$ is 0·999 999 999 999 999 978 483 925.

ERRATA.

Page 257, last line, *for* $\int_0$ *read* $\int_0^t$

„ 258, line 4, *for* $\int_t^\infty \epsilon - {}^{t^2}dt$, *read* $\int_t^\infty \epsilon^{-t^2}dt$

„ 261, note †, *for* 1·283791 670, etc., *read* 1·128 397 167 0, etc., twice.

„ 263, note *, *for* Probabilities, *read* Probabilities,"

„ 266, note †, *for* Mr W. T. B., *read* Mr W. S. B.

„ 271, line 3, *for* Δ_8), *read* Δ^8)

„ 273, line 14, *for* + etc. }. *read* + etc. }.*

„ 276, line 22, *for* ϵ^{-t^2}, *read* ϵ^{-t^2}

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MUSEUM

X.—*The Relations between the Coaxial Minors of a Determinant of the Fourth Order.* By Thomas Muir, LL.D.

(Read January 31, 1898.)

1. The existence of relations between the coaxial minors of a determinant was first discovered by MacMahon in 1893. The whole literature of the subject is comprised in three papers, viz.:—

MacMahon, *Phil. Trans.*, clxxxv. pp. 111–160.
Muir, *Phil. Mag.*, 5th series, xli. pp. 537–541.
Nanson, *Phil. Mag.*, 5th series, xliv. pp. 362–367.

My present object is to continue the investigation of the relations in question, and more particularly to draw attention to an *explicit* expression for a determinant of the 4th order in terms of its own coaxial minors. At the outset some fresh considerations regarding determinants in general will be found useful.

2. As is well known, the coaxial minors of a determinant of the nth order are $2^n - 1$ in number, the determinant itself and each of the elements of its primary diagonal being counted. For example, the coaxial minors of $|a_1b_2c_3d_4|$ are

$$
\begin{array}{l}
|a_1b_2c_3d_4|, \\
|a_1b_2c_3|, \quad |a_1b_2d_4|, \quad |a_1c_3d_4|, \quad |b_2c_3d_4|, \\
|a_1b_2|, \quad |a_1c_3|, \quad |a_1d_4|, \quad |b_2c_3|, \quad |b_2d_4|, \quad |c_3d_4|, \\
a_1, \quad b_2, \quad c_3, \quad d_4.
\end{array}
$$

Of these the first $2^n - 1 - n$ may be devertebrated, if we may say so, by substituting zeros for the elements of their primary diagonals; and the determinants thus resulting are found to be of considerable interest. They appear in Cayley's well-known expansion-theorem, which for a determinant of the 3rd order is

$$
\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = \begin{vmatrix} . & a_2 & a_3 \\ b_1 & . & b_3 \\ c_1 & c_2 & . \end{vmatrix} + a_1 \begin{vmatrix} . & b_3 \\ c_2 & . \end{vmatrix} + b_2 \begin{vmatrix} . & a_3 \\ c_1 & . \end{vmatrix} + c_3 \begin{vmatrix} . & a_2 \\ b_1 & . \end{vmatrix} + a_1b_2c_3.
$$

Indeed this theorem may be described as giving an expression for a determinant in terms of its own devertebrated coaxial minors and its primary diagonal elements.

Now, if we use Cayley's expansion in connection with each of the first $2^n - 1 - n$ coaxial minors, we obtain $2^n - 1 - n$ equations, linear in respect to the devertebrated minors. So that, on solving for the latter, there must result an expression for each

devertebrated coaxial minor in terms of the vertebrate coaxial minors and the primary diagonal elements. The general theorem thus obtained is

$$\begin{vmatrix} . & a_2 & a_3 & a_4 & \ldots\ldots \\ b_1 & . & b_3 & b_4 & \ldots\ldots \\ c_1 & c_2 & . & c_4 & \ldots\ldots \\ d_1 & d_2 & d_3 & . & \ldots\ldots \\ \ldots & \ldots & \ldots & \ldots & \ldots\ldots \end{vmatrix} = |a_1b_2c_3d_4\ldots.| - \Sigma a_1|b_2c_3d_4\ldots.| + \Sigma a_1b_2|c_3d_4\ldots.| + \ldots. \qquad (A)$$

It may be viewed as a sort of converse of CAYLEY's, which in outward form it very closely resembles.

3. The truth of it may be established by proceeding in the manner just indicated; but there is another available process which has the advantage of presenting it merely as the ultimate case of a more general theorem, viz., a theorem for similarly expanding a determinant which is only *partially* devertebrated.

Taking determinants of the 3rd order, we have in succession and without any difficulty of verification,

$$\begin{vmatrix} . & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = |a_1b_2c_3| - a_1|b_2c_3|,$$

$$\begin{vmatrix} . & a_2 & a_3 \\ b_1 & . & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = |a_1b_2c_3| - a_1|b_2c_3| - b_2|a_1c_3| + a_1b_2c_3,$$

$$\begin{vmatrix} . & a_2 & a_3 \\ b_1 & . & b_3 \\ c_1 & c_2 & . \end{vmatrix} = |a_1b_2c_3| - a_1|b_2c_3| - b_2|a_1c_3| - c_3|a_1b_2| + 2a_1b_2c_3.$$

Proceeding to the 4th order, we have with equal simplicity in the first case

$$\begin{vmatrix} . & a_2 & a_3 & a_4 \\ b_1 & b_2 & b_3 & b_4 \\ c_1 & c_2 & c_3 & c_4 \\ d_1 & d_2 & d_3 & d_4 \end{vmatrix} = |a_1b_2c_3d_4| - a_1|b_2c_3d_4|. \qquad (A_1)$$

For the next case we have similarly

$$\begin{vmatrix} . & a_2 & a_3 & a_4 \\ b_1 & . & b_3 & b_4 \\ c_1 & c_2 & c_3 & c_4 \\ d_1 & d_2 & d_3 & d_4 \end{vmatrix} = \begin{vmatrix} . & a_2 & a_3 & a_4 \\ b_1 & b_2 & b_3 & b_4 \\ c_1 & c_2 & c_3 & c_4 \\ d_1 & d_2 & d_3 & d_4 \end{vmatrix} - b_2 \begin{vmatrix} . & a_3 & a_4 \\ c_1 & c_3 & c_4 \\ d_1 & d_3 & d_4 \end{vmatrix},$$

and as each of the determinants on the right has already been expanded in the new form, there is at once obtained by substitution

$$|a_1b_2c_3d_4| - a_1|b_2c_3d_4| - b_2|a_1c_3d_4| + a_1b_2|c_3d_4|. \qquad (A_2$$

Again, we have

$$\begin{vmatrix} . & a_2 & a_3 & a_4 \\ b_1 & . & b_3 & b_4 \\ c_1 & c_2 & . & c_4 \\ d_1 & d_2 & d_3 & d_4 \end{vmatrix} = \begin{vmatrix} . & a_2 & a_3 & a_4 \\ b_1 & . & b_3 & b_4 \\ c_1 & c_2 & c_3 & c_4 \\ d_1 & d_2 & d_3 & d_4 \end{vmatrix} - c_3 \begin{vmatrix} . & a_2 & a_4 \\ b_1 & . & b_4 \\ d_1 & d_2 & d_4 \end{vmatrix},$$

$$= |a_1b_2c_3d_4| - a_1|b_2c_3d_4| - b_2|a_1c_3d_4| + a_1b_2|c_3d_4| - c_3\{|a_1b_2d_4| - a_1|b_2d_4| - b_2|a_1d_4| + a_1b_2d_4\},$$

$$= |a_1b_2c_3d_4| - a_1|b_2c_3d_4| - b_2|a_1c_3d_4| - c_3|a_1b_2d_4| + a_1b_2|c_3d_4| + a_1c_3|b_2d_4| + b_2c_3|a_1d_4| - a_1b_2c_3d_4. \qquad (A_3)$$

And lastly, by proceeding in exactly the same way, we have the theorem of the preceding section, viz.:—

$$\begin{vmatrix} . & a_2 & a_3 & a_4 \\ b_1 & . & b_3 & b_4 \\ c_1 & c_2 & . & c_4 \\ d_1 & d_2 & d_3 & . \end{vmatrix} = |a_1b_2c_3d_4| - \Sigma a_1|b_2c_3d_4| + \Sigma a_1b_2|c_3d_4| - 3a_1b_2c_3d_4,$$

where the Σ refers to combinations of the four elements, a_1, b_2, c_3, d_4.

4. MacMahon's problem of expressing the determinant of the 4th order in terms of its coaxial minors may thus be transformed into something apparently simpler, viz., expressing the determinant in terms of its *devertebrated* coaxial minors and the primary diagonal elements.

In the case of the determinant $|a_1b_2c_3d_4|$ the eleven (*i.e.*, 2^4-1-4) devertebrated coaxial minors are

$$\begin{vmatrix} . & a_2 & a_3 & a_4 \\ b_1 & . & b_3 & b_4 \\ c_1 & c_2 & . & c_4 \\ d_1 & d_2 & d_3 & . \end{vmatrix} \quad i.e., \quad a_2b_1c_4d_3 + a_3b_4c_1d_2 + a_4b_3c_2d_1 - \left\{ \begin{matrix} a_2b_4c_1d_3 + a_3b_1c_4d_2 \\ + a_2b_3c_4d_1 + a_4b_1c_2d_3 \\ + a_4b_3c_1d_2 + a_3b_4c_2d_1 \end{matrix} \right\} = \text{D say},$$

$$\begin{vmatrix} . & a_2 & a_3 \\ b_1 & . & b_3 \\ c_1 & c_2 & . \end{vmatrix} \quad i.e., \quad a_2b_3c_1 + a_3b_1c_2 = \text{C}_1 \text{ say},$$

$$\begin{vmatrix} . & a_2 & a_4 \\ b_1 & . & b_4 \\ d_1 & d_2 & . \end{vmatrix} \quad i.e., \quad a_2b_4d_1 + a_4b_1d_2 = \text{C}_2 \text{ say},$$

$$\begin{vmatrix} . & a_3 & a_4 \\ c_1 & . & c_4 \\ d_1 & d_3 & . \end{vmatrix} \quad i.e., \quad a_3c_4d_1 + a_4c_1d_3 = \text{C}_3 \text{ say},$$

$$\begin{vmatrix} . & b_3 & b_4 \\ c_2 & . & c_4 \\ d_2 & d_3 & . \end{vmatrix} \quad i.e., \quad b_3c_4d_2 + b_4c_2d_3 = \text{C}_4 \text{ say},$$

$$\begin{vmatrix} . & a_2 \\ b_1 & . \end{vmatrix} \quad i.e., \; -a_2b_1 = \mathrm{B}_1 \text{ say},$$

$$\begin{vmatrix} . & a_3 \\ c_1 & . \end{vmatrix} \quad i.e., \; -a_3c_1 = \mathrm{B}_2 \text{ say},$$

$$\begin{vmatrix} . & a_4 \\ d_1 & . \end{vmatrix} \quad i.e., \; -a_4d_1 = \mathrm{B}_3 \text{ say},$$

$$\begin{vmatrix} . & b_3 \\ c_2 & . \end{vmatrix} \quad i.e., \; -b_3c_2 = \mathrm{B}_4 \text{ say},$$

$$\begin{vmatrix} . & b_4 \\ d_2 & . \end{vmatrix} \quad i.e., \; -b_4d_2 = \mathrm{B}_5 \text{ say},$$

$$\begin{vmatrix} . & c_4 \\ d_3 & . \end{vmatrix} \quad i.e., \; -c_4d_3 = \mathrm{B}_6 \text{ say}.$$

Using the last six equations to eliminate b_1, c_1, d_1, c_2, d_2, d_3—these being the elements on one side of the primary diagonal of $|a_1b_2c_3d_4|$—from the preceding five equations, we have

$$\left.\begin{aligned} \mathrm{B}_1\mathrm{B}_6 + \mathrm{B}_2\mathrm{B}_5 + \mathrm{B}_3\mathrm{B}_4 - \begin{vmatrix} & \mathrm{B}_2\mathrm{B}_6\dfrac{a_2b_4}{a_3c_4} + \mathrm{B}_1\mathrm{B}_5\dfrac{a_3c_4}{a_2b_4} & \\ -\mathrm{B}_3\dfrac{a_2b_3c_4}{a_4} & - \mathrm{B}_1\mathrm{B}_4\mathrm{B}_6\dfrac{a_4}{a_2b_3c_4} & \\ +\mathrm{B}_2\mathrm{B}_5\dfrac{a_4b_3}{a_3b_4} & + \mathrm{B}_3\mathrm{B}_4\dfrac{a_3b_4}{a_4b_3} & \end{vmatrix} &= \mathrm{D}, \\ -\mathrm{B}_2\dfrac{a_2b_3}{a_3} + \mathrm{B}_1\mathrm{B}_4\dfrac{a_3}{a_2b_3} &= \mathrm{C}_1, \\ -\mathrm{B}_3\dfrac{a_2b_4}{a_4} + \mathrm{B}_1\mathrm{B}_5\dfrac{a_4}{a_2b_4} &= \mathrm{C}_2, \\ -\mathrm{B}_3\dfrac{a_3c_4}{a_4} + \mathrm{B}_2\mathrm{B}_6\dfrac{a_4}{a_3c_4} &= \mathrm{C}_3, \\ -\mathrm{B}_5\dfrac{b_3c_4}{b_4} + \mathrm{B}_4\mathrm{B}_6\dfrac{b_4}{b_3c_4} &= \mathrm{C}_4. \end{aligned}\right\}$$

But the four fractional quantities $\dfrac{a_2b_3}{a_3}, \dfrac{a_2b_4}{a_4}, \dfrac{a_3c_4}{a_4}, \dfrac{b_3c_4}{b_4}$ — or say $\gamma_1, \gamma_2, \gamma_3, \gamma_4$ — in the last four equations are connected by the relation

$$\gamma_1\gamma_3 = \gamma_2\gamma_4,$$

and the three similar quantities in the remaining equation of the set are expressible in terms of these four, viz.:—

$$\frac{a_2b_4}{a_3c_4} = \frac{\gamma_2}{\gamma_3} \text{ or } \frac{\gamma_1}{\gamma_4},$$

$$\frac{a_2b_3c_4}{a_4} = \gamma_1\gamma_3 \text{ or } \gamma_2\gamma_4,$$

$$\frac{a_4b_3}{a_3b_4} = \frac{\gamma_1}{\gamma_2} \text{ or } \frac{\gamma_4}{\gamma_3}.$$

It is thus possible by the elimination of $\gamma_1, \gamma_2, \gamma_3, \gamma_4$ to deduce five equations, not more than two of which, however, can be independent.

5. Taking the first four equations of the set of five, and using γ_1, γ_2, γ_3 as just indicated, we have

$$\left.\begin{aligned} B_1B_6+B_2B_5+B_3B_4-D &= \left(B_2B_6\frac{\gamma_2}{\gamma_3}+B_1B_5\frac{\gamma_3}{\gamma_2}\right)-\left(B_3\gamma_1\gamma_3+B_1B_4B_6\frac{1}{\gamma_1\gamma_3}\right)+\left(B_2B_5\frac{\gamma_1}{\gamma_2}+B_3B_4\frac{\gamma_2}{\gamma_1}\right)\\ C_1 &= -B_2\gamma_1+B_1B_4\frac{1}{\gamma_1},\\ C_2 &= -B_3\gamma_2+B_1B_5\frac{1}{\gamma_2},\\ C_3 &= -B_3\gamma_3+B_2B_6\frac{1}{\gamma_3}. \end{aligned}\right\}$$

Now, by means of each pair of the last three of these equations, the γ's may be eliminated from a corresponding one of the bracketed expressions in the first equation, the results of this action in fact being

$$B_2B_6\frac{\gamma_2}{\gamma_3}+B_1B_5\frac{\gamma_3}{\gamma_2} = \frac{-C_2C_3+\sqrt{C_2^2+4B_1B_3B_5}\sqrt{C_3^2+4B_2B_3B_6}}{2B_3},$$

$$B_3\gamma_1\gamma_3+B_1B_4B_6\frac{1}{\gamma_1\gamma_3} = \frac{C_1C_3+\sqrt{C_1^2+4B_1B_2B_4}\sqrt{C_3^2+4B_2B_3B_6}}{2B_2},$$

$$B_2B_5\frac{\gamma_1}{\gamma_2}+B_3B_4\frac{\gamma_2}{\gamma_1} = \frac{-C_1C_2+\sqrt{C_1^2+4B_1B_2B_4}\sqrt{C_2^2+4B_1B_3B_5}}{2B_1}.$$

We thus have

$$\begin{aligned} D=B_1B_6+B_2B_5+B_3B_4+\frac{C_2C_3}{2B_3}+\frac{C_3C_1}{2B_2}+\frac{C_1C_2}{2B_1}\\ -\frac{1}{2B_3}\sqrt{C_2^2+4B_1B_3B_5}\sqrt{C_3^2+4B_2B_3B_6}+\frac{1}{2B_2}\sqrt{C_3^2+4B_2B_3B_6}\sqrt{C_1^2+4B_1B_2B}\\ -\frac{1}{2B_1}\sqrt{C_1^2+4B_1B_2B_4}\sqrt{C_2^2+4B_1B_3B_5}, \end{aligned}$$

—a relation among ten of the eleven devertebrated coaxial minors of $|a_1b_2c_3d_4|$. Then as for each of the ten there is an expression in terms of the vertebrate coaxial minors, and, in the case of one of them, viz., D, this expression involves the original determinant $|a_1b_2c_3d_4|$, it is clear that we may deduce from this the result foreshadowed by MacMahon, viz., an expression for $|a_1b_2c_3d_4|$ in terms of its coaxial minors.

Making the actual substitutions in places where subsequent simplification is readily possible,* we find

$$\begin{aligned} |a_1b_2c_3d_4| = \Sigma a_1|b_2c_3d_4|+\Sigma|a_1b_2||c_3d_4|-2\Sigma a_1b_2|c_3d_4|+6a_1b_2c_3d_4+\frac{C_1C_2}{2B_1}+\frac{C_1C_3}{2B_2}+\frac{C_2C_3}{2B_3}\\ -\frac{1}{2B_1}\sqrt{C_1^2+4B_1B_2B_4}\sqrt{C_2^2+4B_1B_3B_5}+\frac{1}{2B_2}\sqrt{C_1^2+4B_1B_2B_4}\sqrt{C_3^2+4B_2B_3B_6}\\ -\frac{1}{2B_3}\sqrt{C_2^2+4B_1B_3B_5}\sqrt{C_3^2+4B_2B_3B_6}, \end{aligned}$$

* In the case of each expression under a root-sign a certain amount of simplification is also possible, *e.g.*, we find

$$\begin{aligned} C_2^2+4B_1B_3B_5 = |a_1b_2d_4|^2-\Sigma 2|a_1b_2d_4||b_2d_4|a_1+4|a_1b_2d_4|a_1b_2d_4+4|a_1b_2||a_1d_4||b_2d_4|\\ +\Sigma a_1^2|b_2d_4|^2-\Sigma 2a_1b_2|a_1d_4||b_2d_4|. \end{aligned}$$

where $B_1, B_2, \ldots B_6, C_1, C_2, C_3$ have the significations given to them in section 4, but are to be replaced by using the theorem of sections 2, 3.

6. Again, taking the last four of the set of five equations in section 4, and bearing in mind that $\gamma_1\gamma_3 = \gamma_2\gamma_4$, all that is necessary for elimination is to put

$$\gamma_1 = \frac{-C_1 + \sqrt{C_1^2 + 4B_1B_2B_4}}{2B_2} \quad \text{or} \quad \frac{2B_1B_4}{C_1 + \sqrt{C_1^2 + 4B_1B_2B_4}},$$

$$\gamma_2 = \frac{-C_2 + \sqrt{C_2^2 + 4B_1B_3B_5}}{2B_3} \quad \text{or} \quad \frac{2B_1B_5}{C_2 + \sqrt{C_2^2 + 4B_1B_3B_5}},$$

$$\gamma_3 = \frac{-C_3 + \sqrt{C_3^2 + 4B_2B_3B_6}}{2B_1} \quad \text{or} \quad \frac{2B_2B_6}{C_3 + \sqrt{C_3^2 + 4B_2B_3B_6}},$$

in the equation

$$C_4 = -B_5\frac{\gamma_1\gamma_3}{\gamma_2} + B_4B_6\frac{\gamma_2}{\gamma_1\gamma_3}.$$

The result of this action is

$$\begin{aligned} 4B_1B_2B_3C_4 + C_1C_2C_3 &- C_1\sqrt{C_2^2 + 4B_1B_3B_5}\sqrt{C_3^2 + 4B_2B_3B_6} \\ &+ C_2\sqrt{C_3^2 + 4B_2B_3B_6}\sqrt{C_1^2 + 4B_1B_2B_4} \\ &- C_3\sqrt{C_1^2 + 4B_1B_2B_4}\sqrt{C_2^2 + 4B_1B_3B_5} = 0; \end{aligned}$$

and similar equations can be got for C_3 in terms of C_1, C_2, C_4; for C_2 in terms of C_1, C_3, C_4; and for C_1 in terms of C_2, C_3, C_4.

7. On comparison of these results with those of Professor NANSON it will be found that instead of an explicit expression for $|a_1b_2c_3d_4|$ in terms of its coaxial minors, and an explicit expression for one of the coaxial minors of the 3rd order in terms of the three others and those of lower order, he obtains in each case an unsolved biquadratic equation. The presumption therefore is that each of his biquadratics must be resolvable into linear factors. This will now be shown to be the case. The series of necessary transformations is among the most interesting of the kind, and therefore well worthy of attention apart altogether from the problem with which they are here connected.

8. The latter of the two biquadratics is

$$\begin{vmatrix} DL & CQ & BR & AQRL+2BCD \\ CP & DM & AR & BRPM+2CAD \\ BP & AQ & DN & CPQN+2ABD \\ AL & BM & CN & DLMN+2ABC \end{vmatrix} = 0,$$

where

$$D, C, B, A; R, Q, L, P, M, N$$

correspond to but are not identical with the

$$C_1, C_2, C_3, C_4; B_1, B_2, B_3, B_4, B_5, B_6$$

of the present paper.

Now this determinant is easily seen to be the same as

$$\frac{1}{L^2M^2N^2}\begin{vmatrix} DLMN & CQMN & BRMN & AQRLMN+2BCDMN \\ CPNL & DMNL & ARNL & BRPMNL+2CADNL \\ BPLM & AQLM & DNLM & CPQNLM+2ABDLM \\ AL & BM & CN & DLMN+2ABC \end{vmatrix}.$$

Taking BC/L times each element of the last row from the corresponding element of the 1st row, CA/M times each element of the last row from the corresponding element of the 2nd row, and AB/N times each element of the last row from the corresponding element of the 3rd row, we transform this new determinant into

$$\begin{vmatrix} DLMN-ABC & CQMN-B^2C\frac{M}{L} & BRMN-BC^2\frac{N}{L} & AQRLMN+BCDMN-\frac{2AB^2C^2}{L} \\ CPNL-A^2C\frac{L}{M} & DLMN-ABC & ARNL-AC^2\frac{N}{M} & BRPLMN+CADLN-\frac{2A^2BC^2}{M} \\ BPLM-A^2B\frac{L}{M} & AQLM-AB^2\frac{M}{N} & DLMN-ABC & CPQLMN+ABDLM-\frac{2A^2B^2C}{N} \\ AL & BM & CN & DLMN+2ABC \end{vmatrix}.$$

Diminishing now each element of the last column by BC/L times the corresponding element of the 1st column, by CA/M times the corresponding element of the 2nd column, and by AB/N times the corresponding element of the 3rd column, we change the last column into

$$\left.\begin{matrix} AQRLMN-AC^2QN-AB^2RM+\frac{AB^2C^2}{L} \\ BRPLMN-BA^2RL-BC^2PN+\frac{A^2BC^2}{M} \\ CPQLMN-CB^2PM-CA^2QL+\frac{A^2B^2C}{N} \\ DLMN-ABC \end{matrix}\right\} \text{ or } \left\{\begin{matrix} \frac{A}{L}(NLQ-B^2)(LMR-C^2) \\ \frac{B}{M}(LMR-C^2)(MNP-A^2) \\ \frac{C}{N}(MNP-A^2)(NLQ-B^2) \\ DLMN-ABC \end{matrix}\right. ;$$

and if, merely for shortness' sake, we put

$$PL\left(1-\frac{A^2}{MNP}\right)=\lambda^2,$$
$$QM\left(1-\frac{B^2}{NLQ}\right)=\mu^2,$$
$$RN\left(1-\frac{C^2}{LMR}\right)=\nu^2,$$

the determinant becomes

$$\begin{vmatrix} DLMN-ABC & CN\mu^2 & BM\nu^2 & AL\mu^2\nu^2 \\ CN\lambda^2 & DLMN-ABC & AL\nu^2 & BM\nu^2\lambda^2 \\ BM\lambda^2 & AL\mu^2 & DLMN-ABC & CN\lambda^2\mu^2 \\ AL & BM & CN & DLMN-ABC \end{vmatrix}.$$

Dividing the columns by λ, μ, ν, $\lambda\mu\nu$ respectively, and multiplying the rows in order by the same, we obtain

$$\begin{vmatrix} DLMN-ABC & CN\lambda\mu & BM\nu\lambda & AL\mu\nu \\ CN\lambda\mu & DLMN-ABC & AL\mu\nu & BM\nu\lambda \\ BM\nu\lambda & AL\mu\nu & DLMN-ABC & CN\lambda\mu \\ AL\mu\nu & BM\nu\lambda & CN\lambda\mu & DLMN-ABC \end{vmatrix},$$

—a determinant which is seen to have all the elements of the primary diagonal alike, all the elements of the secondary diagonal alike, and to be symmetric with respect to both diagonals. Such a determinant, when of the 4th order, must clearly be a function of the four elements which necessarily recur in every line; and, as a matter of fact, it is known to be expressible as the product of four factors, the first of which is the sum of the said four elements, and differs from each of the others in the sign of two of its last three terms. The biquadratic we began with is thus the same as

$$\begin{aligned}&(DLMN-ABC+CN\lambda\mu+BM\nu\lambda+AL\mu\nu)\\ .&(DLMN-ABC+CN\lambda\mu-BM\nu\lambda-AL\mu\nu)\\ .&(DLMN-ABC-CN\lambda\mu+BM\nu\lambda-AL\mu\nu)\\ .&(DLMN-ABC-CN\lambda\mu-BM\nu\lambda+AL\mu\nu) = 0;\end{aligned}$$

so that if we put back the values of λ, μ, ν and solve, we have

$$D=\frac{1}{LMN}\{ABC \pm C\sqrt{A^2-MNP}\sqrt{B^2-NLQ} \pm B\sqrt{C^2-LMR}\sqrt{A^2-MNP} \pm C\sqrt{A^2-MNP}\sqrt{B^2-NLQ}\},$$

and this, on the required changes being made, will be found to be identical with the result of section 6.

9. The other biquadratic referred to is

$$\begin{vmatrix} \theta & (1-C)\sqrt{1-B^2} & (1-B)\sqrt{1-C^2} & (1-A)\sqrt{1-B^2}\sqrt{1-C^2} \\ (1-C)\sqrt{1-A^2} & \theta & (1-A)\sqrt{1-C^2} & (1-B)\sqrt{1-C^2}\sqrt{1-A^2} \\ (1-B)\sqrt{1-A^2} & (1-A)\sqrt{1-B^2} & \theta & (1-C)\sqrt{1-A^2}\sqrt{1-B^2} \\ 1-A & 1-B & 1-C & \theta \end{vmatrix} = 0,$$

where θ stands for $(A-1)(B-1)(C-1)-\frac{1}{2}\Delta$. It is the biquadratic not for the general determinant $|a_1b_2c_3d_4|$ but for the very special instance

$$\begin{vmatrix} 1 & h & g^{-1} & 1 \\ h^{-1} & 1 & f & 1 \\ g & f^{-1} & 1 & 1 \\ 1 & 1 & 1 & 1 \end{vmatrix} \quad \text{or} \quad \Delta.$$

In this case the required transformation is very easy. All that is necessary is to divide the first three rows by $\sqrt{1-B^2}\sqrt{1-C^2}$, $\sqrt{1-C^2}\sqrt{1-A^2}$, $\sqrt{1-A^2}\sqrt{1-B^2}$ respectively, and then multiply in order the first three columns by the same. The result is

$$\begin{vmatrix} \theta & (1-C)\sqrt{1-A^2}\sqrt{1-B^2} & (1-B)\sqrt{1-C^2}\sqrt{1-A^2} & (1-A)\sqrt{1-B^2}\sqrt{1-C^2} \\ (1-C)\sqrt{1-A^2}\sqrt{1-B^2} & \theta & (1-A)\sqrt{1-B^2}\sqrt{1-C^2} & (1-B)\sqrt{1-C^2}\sqrt{1-A^2} \\ (1-B)\sqrt{1-C^2}\sqrt{1-A^2} & (1-A)\sqrt{1-B^2}\sqrt{1-C^2} & \theta & (1-C)\sqrt{1-A^2}\sqrt{1-B^2} \\ (1-A)\sqrt{1-B^2}\sqrt{1-C^2} & (1-B)\sqrt{1-C^2}\sqrt{1-A^2} & (1-C)\sqrt{1-A^2}\sqrt{1-B^2} & \theta \end{vmatrix} = 0,$$

where again the determinant has the elements of the primary diagonal all alike, the elements of the secondary diagonal all alike, and is symmetric with respect to both diagonals. As before, therefore, it resolves into four factors, and we have on substituting the value of θ

$$(A-1)(B-1)(C-1) - \tfrac{1}{2}\Delta \pm (1-C)\sqrt{1-A^2}\sqrt{1-B^2} \pm (1-B)\sqrt{1-C^2}\sqrt{1-A^2} \pm (1-A)\sqrt{1-A^2}\sqrt{1-B^2} = 0,$$

or

$$\Delta = 2ABC - 2\Sigma AB + 2\Sigma A - 2 \pm 2(1-C)\sqrt{1-A^2}\sqrt{1-B^2} \pm 2(1-B)\sqrt{1-B^2}\sqrt{1-A^2} \pm 2(1-A)\sqrt{1-A^2}\sqrt{1-B^2},$$

which is readily shown to be in agreement with the more general result in section 5.*

10. Not only does the determinant

$$\begin{vmatrix} DL & CQ & BR & AQRL+2BCD \\ CP & DM & AR & BRPM+2CAD \\ BP & AQ & DN & CPQN+2ABD \\ AL & BM & CN & DLMN+2ABC \end{vmatrix}$$

resolve into factors, but each of the two determinants into which it may be partitioned is also so resolvable. For, multiplying the columns in order by $\sqrt{MNQR}$, $\sqrt{NLRP}$, $\sqrt{LMPQ}$, 1, and then dividing the rows in order by $\sqrt{LQR}$, $\sqrt{MRP}$, $\sqrt{NPQ}$, $\sqrt{LMN}$, we obtain the new form

$$\begin{vmatrix} D\sqrt{LMN} & C\sqrt{NPQ} & B\sqrt{MRP} & A\sqrt{LQR} + \dfrac{2BCD}{\sqrt{LQR}} \\ C\sqrt{NPQ} & D\sqrt{LMN} & A\sqrt{LQR} & B\sqrt{MRP} + \dfrac{2CDA}{\sqrt{MRP}} \\ B\sqrt{MRP} & A\sqrt{LQR} & D\sqrt{LMN} & C\sqrt{NPQ} + \dfrac{2DAB}{\sqrt{NPQ}} \\ A\sqrt{LQR} & B\sqrt{MRP} & C\sqrt{NPQ} & D\sqrt{LMN} + \dfrac{2ABC}{\sqrt{LMN}} \end{vmatrix};$$

* Instead of the biquadratic of this section another might readily have been obtained from the single equation numbered (9) in Professor NANSON's paper, viz.,

$$\mu + \sqrt{1-B^2}\sqrt{1-C^2} + \sqrt{1-C^2}\sqrt{1-A^2} + \sqrt{1-A^2}\sqrt{1-B^2} = 0,$$

where

$$\mu = A+B+C+D-\tfrac{1}{2}\Delta - 1 - BC - CA - AB.$$

Observe also that this equation gives a much simpler expression for Δ, viz.:—

$$\Delta = -2 + 2\Sigma A - 2\Sigma AB + 2\Sigma\sqrt{1-B^2}\sqrt{1-C^2}.$$

and on partitioning this into two the first is seen to be

$$\begin{aligned}&= (D\sqrt{LMN}+C\sqrt{NPQ}+B\sqrt{MRP}+A\sqrt{LQR})\\&\quad .(D\sqrt{LMN}+C\sqrt{NPQ}-B\sqrt{MRP}-A\sqrt{LQR})\\&\quad .(D\sqrt{LMN}-C\sqrt{NPQ}+B\sqrt{MRP}-A\sqrt{LQR})\\&\quad .(D\sqrt{LMN}-C\sqrt{NPQ}-B\sqrt{MRP}+A\sqrt{LQR}),\end{aligned}$$

and the second to be

$$\begin{vmatrix} D\sqrt{LMN} & C\sqrt{NPQ} & B\sqrt{MRP} & D\sqrt{LMN}\,.C\sqrt{NPQ}\,.B\sqrt{MRP} \\ C\sqrt{NPQ} & D\sqrt{LMN} & A\sqrt{LQR} & C\sqrt{NPQ}\,.D\sqrt{LMN}.A\sqrt{LQR} \\ B\sqrt{MRP} & A\sqrt{LQR} & D\sqrt{LMN} & B\sqrt{MRP}.A\sqrt{LQR}\,.D\sqrt{LMN} \\ A\sqrt{LQR} & B\sqrt{MRP} & C\sqrt{NPQ} & A\sqrt{LQR}\,.B\sqrt{MRP}\,.C\sqrt{NPQ} \end{vmatrix} \times \frac{2}{LMNPQR},$$

and therefore

$$\begin{aligned}&=(C\sqrt{NPQ}\,.A\sqrt{LQR}-D\sqrt{LMN}\,.B\sqrt{MRP})\\&\quad (A\sqrt{LQR}\,.B\sqrt{MRP}-C\sqrt{NPQ}\,.D\sqrt{LMN})\\&\quad (B\sqrt{MRP}.C\sqrt{NPQ}-A\sqrt{LQR}\,.D\sqrt{LMN})\div \tfrac{1}{2}LMNPQR,\\&=2(CAQ-DBM)(ABR-CDN)(BCP-ADL).\end{aligned}$$

11. Were it not for the divisor LMNPQR attached to the second determinant in the preceding section, the full determinant would be a function of only four variables, viz. :—

$$\begin{aligned}&A\sqrt{LQR},\\&B\sqrt{MRP},\\&C\sqrt{NPQ},\\&D\sqrt{LMN};\end{aligned}$$

and as a matter of fact the final expansion of it may be written

$$\begin{aligned}&\Sigma(A\sqrt{LQR})^4-2\Sigma(A\sqrt{LQR})^2(B\sqrt{MRP})^2\\&\quad +8(A\sqrt{LQR}\,.B\sqrt{MRP}\,.C\sqrt{NPQ}\,.D\sqrt{LMN})\\&\quad +\frac{4\Sigma(A\sqrt{LQR})^2(B\sqrt{MRP})^2(C\sqrt{NPQ})^2-4\Sigma(A\sqrt{LQR})^3.B\sqrt{MRP}.C\sqrt{NPQ}.D\sqrt{LMN}}{LMNPQR}.\end{aligned}$$

12. Standing in close connection with the subject-matter of the preceding sections—the connection of general with particular—is the problem of clearing the equation

$$x + h\sqrt{bc} + k\sqrt{ca} + l\sqrt{ab} = 0$$

of root-signs, or of transforming a fraction of which $x+h\sqrt{bc}+k\sqrt{ca}+l\sqrt{ab}$ is the denominator into one having its denominator rational. Viewing the matter in either way we reach the result

$$(x+h\sqrt{bc}+k\sqrt{ca}+l\sqrt{ab})(x+h\sqrt{bc}-k\sqrt{ca}-l\sqrt{ab})(x-h\sqrt{bc}+k\sqrt{ca}-l\sqrt{ab})(x-h\sqrt{bc}-k\sqrt{ca}+l\sqrt{ab})$$

or

$$\begin{aligned}&x^4+h^4b^2c^2+k^4c^2a^2+l^4a^2b^2\\&\quad -2x^2(h^2bc+k^2ca+l^2ab)\\&\quad -2abc(h^2k^2c+k^2l^2a+l^2h^2b)\\&\quad -8xhklabc.\end{aligned}$$

This, however, is well known to be equal to the determinant

$$\begin{vmatrix} x & h\sqrt{bc} & k\sqrt{ca} & l\sqrt{ab} \\ h\sqrt{bc} & x & l\sqrt{ab} & k\sqrt{ca} \\ k\sqrt{ca} & l\sqrt{ab} & x & h\sqrt{bc} \\ l\sqrt{ab} & k\sqrt{ca} & h\sqrt{bc} & x \end{vmatrix};$$

and we may consequently say that *the rationalizant of the expression* $x+h\sqrt{bc}+k\sqrt{ca}+l\sqrt{ab}$ *is the biaxisymmetric determinant of the 4th order which has the terms of the expression for the elements of its first row, all the elements of its primary diagonal alike, and all the elements of its secondary diagonal alike.*

Another determinant form of the result is obtained by using the dialytic method of elimination. Taking the original equation and multiplying in succession by $\sqrt{bc}$, $\sqrt{ca}$, $\sqrt{ab}$, we have

$$\left.\begin{aligned} x + h\sqrt{bc} + k\sqrt{ca} + l\sqrt{ab} &= 0 \\ hbc + x\sqrt{bc} + lb\sqrt{ca} + kc\sqrt{ab} &= 0 \\ kca + la\sqrt{bc} + x\sqrt{ca} + hc\sqrt{ab} &= 0 \\ lab + ka\sqrt{bc} + hb\sqrt{ca} + x\sqrt{ab} &= 0 \end{aligned}\right\},$$

and therefore on eliminating $\sqrt{bc}$, $\sqrt{ca}$, $\sqrt{ab}$ there results the rationalizant

$$\begin{vmatrix} x & h & k & l \\ hbc & x & lb & kc \\ kca & la & x & hc \\ lab & ka & hb & x \end{vmatrix}.$$

It is easy to change the one form into the other; indeed, this change is what has been effected in sections 8, 9, Professor NANSON having obtained his results in the latter of the two forms.

13. Another closely related problem, as Professor NANSON has made clear, is that of expressing $\cos(\alpha+\beta+\gamma)$ in terms of $\cos\alpha$, $\cos\beta$, $\cos\gamma$, or say, for shortness' sake, S in terms of A, B, C.

Since

$$\cos(\alpha+\beta+\gamma) - \cos\alpha\cos\beta\cos\gamma + \cos\alpha\sin\beta\sin\gamma + \cos\beta\sin\gamma\sin\alpha + \cos\gamma\sin\alpha\sin\beta = 0,$$

we have

$$S - ABC + A\sqrt{1-B^2}\sqrt{1-C^2} + B\sqrt{1-C^2}\sqrt{1-A^2} + C\sqrt{1-A^2}\sqrt{1-B^2} = 0,$$

and the problem is seen to be a case of the preceding, the result being either

$$\begin{vmatrix} S-ABC & A\sqrt{1-B^2}\sqrt{1-C^2} & B\sqrt{1-C^2}\sqrt{1-A^2} & C\sqrt{1-A^2}\sqrt{1-B^2} \\ A\sqrt{1-B^2}\sqrt{1-C^2} & S-ABC & C\sqrt{1-A^2}\sqrt{1-B^2} & B\sqrt{1-C^2}\sqrt{1-A^2} \\ B\sqrt{1-C^2}\sqrt{1-A^2} & C\sqrt{1-A^2}\sqrt{1-B^2} & S-ABC & A\sqrt{1-B^2}\sqrt{1-C^2} \\ C\sqrt{1-A^2}\sqrt{1-B^2} & B\sqrt{1-C^2}\sqrt{1-A^2} & A\sqrt{1-B^2}\sqrt{1-C^2} & S-ABC \end{vmatrix}.$$

or

$$\begin{vmatrix} S-ABC & A & B & C \\ A(1-B^2)(1-C^2) & S-ABC & C(1-B^2) & B(1-C^2) \\ B(1-C^2)(1-A^2) & C(1-A^2) & S-ABC & A(1-C^2) \\ C(1-A^2)(1-B^2) & B(1-A^2) & A(1-B^2) & S-ABC \end{vmatrix},$$

which latter can be simplified, as Professor NANSON shows, into

$$\begin{vmatrix} S+2ABC & A & B & C \\ A+2BCS & S & C & B \\ B+2ACS & C & S & A \\ C+2ABS & B & A & S \end{vmatrix}.$$

This, however, can be obtained much more directly from the use of another expression for $\cos(\alpha+\beta+\gamma)$, viz. :—

$$\cos(\alpha+\beta+\gamma) = \cos\alpha\cos(\beta+\gamma) + \cos\beta\cos(\gamma+\alpha) + \cos\gamma\cos(\alpha+\beta) - 2\cos\alpha\cos\beta\cos\gamma,$$

where nothing but cosines appears, the angles being

$$\alpha,\ \beta,\ \gamma;\ \ \beta+\gamma,\ \gamma+\alpha,\ \alpha+\beta;\ \ \alpha+\beta+\gamma.$$

Making in this equation the substitutions

$$\begin{cases} \alpha = \alpha+\beta+\gamma, \\ \beta = -\gamma, \\ \gamma = -\beta, \end{cases} \quad \begin{cases} \alpha = -\gamma, \\ \beta = \alpha+\beta+\gamma, \\ \gamma = -\alpha, \end{cases} \quad \begin{cases} \alpha = -\beta, \\ \beta = -\alpha, \\ \gamma = \alpha+\beta+\gamma, \end{cases}$$

we obtain three other perfectly similar identities * connecting the same seven cosines, the complete set of four identities being in the notation above employed

$$\left.\begin{aligned} S + 2ABC - A\cos(\beta+\gamma) - B\cos(\gamma+\alpha) - C\cos(\alpha+\beta) &= 0 \\ A + 2SCB - S\cos(\beta+\gamma) - C\cos(\gamma+\alpha) - B\cos(\alpha+\beta) &= 0 \\ B + 2CSA - C\cos(\beta+\gamma) - S\cos(\gamma+\alpha) - A\cos(\alpha+\beta) &= 0 \\ C + 2BAS - B\cos(\beta+\gamma) - A\cos(\gamma+\alpha) - S\cos(\alpha+\beta) &= 0 \end{aligned}\right\}.$$

From these $\cos(\beta+\gamma)$, $\cos(\gamma+\alpha)$, $\cos(\alpha+\beta)$ can be eliminated, and the desired result at once obtained.

14. It may be noticed in passing that the substitution of $90^\circ-\alpha$, $90^\circ-\beta$, $90^\circ-\gamma$ for α, β, γ gives the similar relation between $\sin(\alpha+\beta+\gamma)$, $\sin\alpha$, $\sin\beta$, $\sin\gamma$.

It should also be noted that the corresponding expression for $\cos(\alpha+\beta)$ in terms of $\cos\alpha$ and $\cos\beta$ is obtained from an identity of a different type, viz., $\sin(\alpha+\beta) = \sin\alpha\cos\beta+\cos\alpha\sin\beta$, the set of equations being

$$\left.\begin{aligned} \sin\beta + \cos(\alpha+\beta).\sin\alpha - \cos\alpha\sin(\alpha+\beta) &= 0 \\ \cos(\alpha+\beta).\sin\beta + \sin\alpha - \cos\beta\sin(\alpha+\beta) &= 0 \\ \cos\alpha.\sin\beta + \cos\beta.\sin\alpha - \sin(\alpha+\beta) &= 0 \end{aligned}\right\}$$

* In effect the substitutions are the same as the circular substitution $\begin{pmatrix} S & A & B & C \\ A & B & C & S \end{pmatrix}$ if we consider $\cos(\beta+\gamma)$ $\cos(\gamma+\alpha)$, $\cos(\alpha+\beta)$ as invariant.

and the resulting equation *

$$\begin{vmatrix} 1 . & \cos(\alpha+\beta) & \cos\alpha \\ \cos(\alpha+\beta) & 1 & \cos\beta \\ \cos\alpha & \cos\beta & 1 \end{vmatrix} = 0 .$$

The same identity almost suffices to give the corresponding relation between $\sin(\alpha+\beta)$, $\sin\alpha$, $\sin\beta$, the set of equations now being

$$\left.\begin{array}{rrrrrrrrr} & - & \sin\beta.\cos\alpha & - & \sin\alpha.\cos\beta & + & \sin(\alpha+\beta) & = 0 \\ \sin\beta.\cos(\alpha+\beta) & & & - & \sin(\alpha+\beta).\cos\beta & + & \sin\alpha & = 0 \\ \sin\alpha.\cos(\alpha+\beta) & - & \sin(\alpha+\beta).\cos\alpha & & & + & \sin\beta & = 0 \\ \sin(\alpha+\beta).\cos(\alpha+\beta) & - & \sin\alpha.\cos\alpha & - & \sin\beta.\cos\beta & + & 2\sin\alpha\sin\beta\sin(\alpha+\beta) & = 0 \end{array}\right\},$$

whence on the elimination of $\cos\alpha$, $\cos\beta$, $\cos(\alpha+\beta)$ we have

$$\begin{vmatrix} . & \sin\beta & \sin\alpha & \sin(\alpha+\beta) \\ \sin\beta & . & \sin(\alpha+\beta) & \sin\alpha \\ \sin\alpha & \sin(\alpha+\beta) & . & \sin\beta \\ \sin(\alpha+\beta) & \sin\alpha & \sin\beta & 2\sin\alpha\sin\beta\sin(\alpha+\beta) \end{vmatrix} = 0 .$$

15. The consideration of the relation between $\cos(\alpha+\beta+\gamma+\delta)$ and $\cos\alpha$, $\cos\beta$, $\cos\gamma$, $\cos\delta$ leads at once to the question of the rationalization of the equation

$$a + b\sqrt{xy} + c\sqrt{xz} + d\sqrt{xw} + e\sqrt{yz} + f\sqrt{yw} + g\sqrt{zw} + h\sqrt{xyzw} = 0,$$

because

$$\cos(\alpha+\beta+\gamma+\delta) = \cos\alpha\cos\beta\cos\gamma\cos\delta - \Sigma\cos\gamma\cos\delta\sin\alpha\sin\beta + \sin\alpha\sin\beta\sin\gamma\sin\delta .$$

By proceeding in exactly the same manner as in section 12 the result of the rationalization is obtained in three forms, viz., (1) the product

$$\begin{aligned} &(a + b\sqrt{xy} + c\sqrt{xz} + d\sqrt{xw} + e\sqrt{yz} + f\sqrt{yw} + g\sqrt{zw} + h\sqrt{xyzw}) \\ .&(a + b\sqrt{xy} + c\sqrt{xz} + d\sqrt{xw} - e\sqrt{yz} - f\sqrt{yw} - g\sqrt{zw} - h\sqrt{xyzw}) \\ .&(a + b\sqrt{xy} - c\sqrt{xz} - d\sqrt{xw} + e\sqrt{yz} + f\sqrt{yw} - g\sqrt{zw} - h\sqrt{xyzw}) \\ .&(a + b\sqrt{xy} - c\sqrt{xz} - d\sqrt{xw} - e\sqrt{yz} - f\sqrt{yw} + g\sqrt{zw} + h\sqrt{xyzw}) \\ .&(a - b\sqrt{xy} + c\sqrt{xz} - d\sqrt{xw} + e\sqrt{yz} - f\sqrt{yw} + g\sqrt{zw} - h\sqrt{xyzw}) \\ .&(a - b\sqrt{xy} + c\sqrt{xz} - d\sqrt{xw} - e\sqrt{yz} + f\sqrt{yw} - g\sqrt{zw} + h\sqrt{xyzw}) \\ .&(a - b\sqrt{xy} - c\sqrt{xz} + d\sqrt{xw} + e\sqrt{yz} - f\sqrt{yw} - g\sqrt{zw} + h\sqrt{xyzw}) \\ .&(a - b\sqrt{xy} - c\sqrt{xz} + d\sqrt{xw} - e\sqrt{yz} + f\sqrt{yw} + g\sqrt{zw} - h\sqrt{xyzw}) , \end{aligned}$$

* It is interesting to note the mode in which the more general relation connecting $\cos(\alpha+\beta+\gamma)$, $\cos\alpha$, $\cos\beta$, $\cos\gamma$, passes over into this on putting $\gamma=0$ in the former. The result of the substitution is

$$\begin{vmatrix} \cos\alpha & 1 & \cos(\alpha+\beta) & \cos\beta+2\cos\alpha\cos(\alpha+\beta) \\ \cos\beta & \cos(\alpha+\beta) & 1 & \cos\alpha+2\cos\beta\cos(\alpha+\beta) \\ 1 & \cos\alpha & \cos\beta & \cos(\alpha+\beta)+2\cos\alpha\cos\beta \\ \cos(\alpha+\beta) & \cos\beta & \cos\alpha & 1+2\cos\alpha\cos\beta\cos(\alpha+\beta) \end{vmatrix},$$

where the elements of the 4th column are easily transformed into zeros with the exception of the last element which becomes

$$1 + 2\cos\alpha\cos\beta\cos(\alpha+\beta) - \cos^2(\alpha+\beta) - \cos^2\beta - \cos^2\alpha,$$

so that the value of the determinant is seen to be

$$\begin{vmatrix} \cos\alpha & 1 & \cos(\alpha+\beta) \\ \cos\beta & \cos(\alpha+\beta) & 1 \\ 1 & \cos\alpha & \cos\beta \end{vmatrix}^2 .$$

With this mode of degeneration may be compared that seen on p. 377 of *Proc. Roy. Soc. Edin.*, xx

(2) the biaxisymmetric determinant

$$\begin{vmatrix}
a & b\sqrt{xy} & c\sqrt{xz} & d\sqrt{xw} & e\sqrt{yz} & f\sqrt{yw} & g\sqrt{zw} & h\sqrt{xyzw} \\
b\sqrt{xy} & a & e\sqrt{yz} & f\sqrt{yw} & c\sqrt{xz} & d\sqrt{xw} & h\sqrt{xyzw} & g\sqrt{zw} \\
c\sqrt{xz} & e\sqrt{yz} & a & g\sqrt{zw} & b\sqrt{xy} & h\sqrt{xyzw} & d\sqrt{xw} & f\sqrt{yw} \\
d\sqrt{xw} & f\sqrt{yw} & g\sqrt{zw} & a & h\sqrt{xyzw} & b\sqrt{xy} & c\sqrt{xz} & e\sqrt{yz} \\
e\sqrt{yz} & c\sqrt{xz} & b\sqrt{xy} & h\sqrt{xyzw} & a & g\sqrt{zw} & f\sqrt{yw} & d\sqrt{xw} \\
f\sqrt{yw} & d\sqrt{xw} & h\sqrt{xyzw} & b\sqrt{xy} & g\sqrt{zw} & a & e\sqrt{yz} & c\sqrt{xz} \\
g\sqrt{zw} & h\sqrt{xyzw} & d\sqrt{xw} & c\sqrt{xz} & f\sqrt{yw} & e\sqrt{yz} & a & b\sqrt{xy} \\
h\sqrt{xyzw} & g\sqrt{zw} & f\sqrt{yw} & e\sqrt{yz} & d\sqrt{xw} & c\sqrt{xz} & b\sqrt{xy} & a
\end{vmatrix},$$

and (3) the axisymmetric determinant

$$\begin{vmatrix}
a & b & c & d & e & f & g & h \\
bxy & a & ey & fy & cx & dx & hxy & g \\
cxz & ez & a & gz & bx & hxz & dx & f \\
dxw & fw & gw & a & hxw & bx & cx & e \\
eyz & cz & by & hyz & a & gz & fy & d \\
fyw & dw & hyw & by & gw & a & ey & c \\
gzw & hzw & dw & cz & fw & ez & a & b \\
hxyzw & gzw & fyw & eyz & dxw & cxz & bxy & a
\end{vmatrix}.$$

The third form is easily changed into the second by multiplying the columns in order by

$$1,\ \sqrt{xy},\ \sqrt{xz},\ \sqrt{xw},\ \sqrt{yz},\ \ldots\ldots,\ \sqrt{xyzw},$$

and then dividing the rows in order by the same. The mode of resolution of the second form into factors is well known.*

16. There is still another variant of the problem of sections 6, 8, viz., to express the relation

$$\cos^{-1}x + \cos^{-1}y + \cos^{-1}z + \cos^{-1}w = 0$$

in purely algebraical form. In essence it is the same as the variant dealt with in section 13.

Subtracting $\cos^{-1}w$ from both sides, and then taking the cosines of the two equals, we have

$$xyz - x\sqrt{1-y^2}\sqrt{1-z^2} - y\sqrt{1-z^2}\sqrt{1-x^2} - z\sqrt{1-x^2}\sqrt{1-y^2} = w,$$

which is at once seen to be an equation of the form dealt with in section 12. The result of the rationalization is

$$\begin{vmatrix}
x & y & z & w+2xyz \\
y & x & w & z+2yxw \\
z & w & x & y+2zwx \\
w & z & y & x+2wzy
\end{vmatrix} = 0,$$

or

$$\Sigma x^4 - 2\Sigma x^2y^2 + 8xyzw + 4\Sigma x^2y^2z^2 - 4\,\Sigma\, x^3yzw = 0.$$

* See *Quart. Journ. of Math*, xviii. pp. 170, 171.

Similarly we have for the equation

$$\cos^{-1}x + \cos^{-1}y + \cos^{-1}z = 0$$

the purely algebraical equivalent

$$\begin{vmatrix} 1 & x & z \\ x & 1 & y \\ z & y & 1 \end{vmatrix} = 0;$$

and for the equation

$$\cos^{-1}x + \cos^{-1}y + \cos^{-1}z + \cos^{-1}w + \cos^{-1}v = 0$$

a purely algebraical equivalent essentially the same as that referred to in section 15 as giving the relation between $\cos(\alpha+\beta+\gamma+\delta)$ and $\cos\alpha$, $\cos\beta$, $\cos\gamma$, $\cos\delta$.

17. This suggests a very simple and perfectly symmetrical mode of expressing the relation of section 6 between the coaxial minors of an order lower than the fourth, viz.:—

$$\cos^{-1}\frac{C_1}{2\sqrt{-B_1B_2B_4}} + \cos^{-1}\frac{C_2}{2\sqrt{-B_1B_3B_5}} + \cos^{-1}\frac{C_3}{2\sqrt{-B_2B_3B_6}} + \frac{C_4}{2\sqrt{-B_4B_5B_6}} = 0.$$

The law of formation of the denominators is perhaps not clear, but this is due merely to a defect in the notation. If we substitute for B's and C's their values as given in terms of the coaxial minors of $|a_1b_2c_3d_4|$ we have

$$\sum \cos^{-1}\frac{|a_1b_2c_3| - a_1|b_2c_3| - b_2|a_1c_3| - c_3|a_1b_2| + 2a_1b_2c_3}{2(-1)^{\frac{1}{2}}(|a_1b_2| - a_1b_2)^{\frac{1}{2}}(|a_1c_3| - a_1c_3)^{\frac{1}{2}}(|b_2c_3| - b_2c_3)^{\frac{1}{2}}} = 0;$$

and, further, if we denote by

$$\begin{vmatrix} a_1b_2c_3 \\ 0\,0\,0 \end{vmatrix}$$

the determinant got from $|a_1b_2c_3|$ by changing the elements of the primary diagonal into zeros, the relation may be written

$$\sum \cos^{-1}\frac{\begin{vmatrix} a_1b_2c_3 \\ 0\,0\,0 \end{vmatrix}}{2\left\{-\begin{vmatrix} a_1b_2 \\ 0\,0 \end{vmatrix}\begin{vmatrix} a_1c_3 \\ 0\,0 \end{vmatrix}\begin{vmatrix} b_2c_3 \\ 0\,0 \end{vmatrix}\right\}^{\frac{1}{2}}} = 0.$$

18. Another matter which has light thrown upon it by certain of the preceding paragraphs is SYLVESTER's original illustration of the dialytic method of elimination as applied to ternary quadrics. It will be remembered that from the equations

$$\left.\begin{aligned} Bx^2 - 2C'xy + Ay^2 &= 0 \\ Cy^2 - 2A'yz + Bz^2 &= 0 \\ Az^2 - 2B'zx + Cx^2 &= 0 \end{aligned}\right\}$$

he deduced three others

$$\left.\begin{aligned} C'z^2 + Cxy - A'zx - B'yz &= 0 \\ A'x^2 + Ayz - B'xy - C'zx &= 0 \\ B'y^2 + Bzx - C'yz - A'xy &= 0 \end{aligned}\right\},$$

and thus obtained the eliminant in the form

$$\begin{vmatrix} . & C & B & -2A' & . & . \\ C & . & A & . & -2B' & . \\ B & A & . & . & . & -2C' \\ A' & . & . & A & -C' & -B' \\ . & B' & . & -C' & B & -A' \\ . & . & C' & -B' & -A' & C \end{vmatrix},$$

which, it was afterwards shown,

$$= 2 \begin{vmatrix} A & C' & B' \\ C' & B & A' \\ B' & A' & C \end{vmatrix}^2.$$

Now the given equations may be written

$$\left.\begin{aligned} \frac{y\sqrt{A}}{x\sqrt{B}} + \frac{x\sqrt{B}}{y\sqrt{A}} &= \frac{2C'}{\sqrt{AB}} \\ \frac{z\sqrt{B}}{y\sqrt{C}} + \frac{y\sqrt{C}}{z\sqrt{B}} &= \frac{2A'}{\sqrt{BC}} \\ \frac{x\sqrt{C}}{z\sqrt{A}} + \frac{z\sqrt{A}}{x\sqrt{C}} &= \frac{2B'}{\sqrt{CA}} \end{aligned}\right\};$$

consequently it is seen that there exists the relation

$$\cos^{-1}\frac{A'}{\sqrt{BC}} + \cos^{-1}\frac{B'}{\sqrt{CA}} + \cos^{-1}\frac{C'}{\sqrt{AB}} = 0,$$

and therefore

$$\begin{vmatrix} 1 & \frac{A'}{\sqrt{BC}} & \frac{C'}{\sqrt{AB}} \\ \frac{A'}{\sqrt{BC}} & 1 & \frac{B'}{\sqrt{CA}} \\ \frac{C'}{\sqrt{AB}} & \frac{B'}{\sqrt{CA}} & 1 \end{vmatrix} = 0.$$

Similarly the resultant of

$$\left.\begin{aligned} Bx^2 - Dxy + Ay^2 &= 0 \\ Cy^2 - Eyz + Bz^2 &= 0 \\ Lz^2 - Kzw + Cw^2 &= 0 \\ Aw^2 - Gwx + Lx^2 &= 0 \end{aligned}\right\}$$

is

$$\cos^{-1}\frac{D}{2\sqrt{AB}} + \cos^{-1}\frac{E}{2\sqrt{BC}} + \cos^{-1}\frac{K}{2\sqrt{CA}} + \cos^{-1}\frac{G}{2\sqrt{LA}} = 0,$$

and therefore from section 16 is

$$\begin{vmatrix} \frac{D}{2\sqrt{AB}} & \frac{E}{2\sqrt{BC}} & \frac{K}{2\sqrt{CL}} & \frac{G}{2\sqrt{LA}} + \frac{DEK}{4BC\sqrt{LA}} \\ \frac{E}{2\sqrt{BC}} & \frac{D}{2\sqrt{AB}} & \frac{G}{2\sqrt{LA}} & \frac{K}{2\sqrt{CL}} + \frac{EDG}{4AB\sqrt{CL}} \\ \frac{K}{2\sqrt{CL}} & \frac{G}{2\sqrt{LA}} & \frac{D}{2\sqrt{AB}} & \frac{E}{2\sqrt{BC}} + \frac{KGD}{4LA\sqrt{BC}} \\ \frac{G}{2\sqrt{LA}} & \frac{K}{2\sqrt{CL}} & \frac{E}{2\sqrt{BC}} & \frac{D}{2\sqrt{AB}} + \frac{GKE}{4CL\sqrt{AB}} \end{vmatrix} = 0.$$

Multiplying the rows in order by $4BC\sqrt{LA}$, $4AB\sqrt{CL}$, $4LA\sqrt{BC}$, $4CL\sqrt{AB}$ we change this determinant into

$$\begin{vmatrix} 2DC\sqrt{BL} & 2E\sqrt{ABCL} & 2KB\sqrt{AC} & 2GBC+DEK \\ 2EA\sqrt{BL} & 2D\sqrt{ABCL} & 2GB\sqrt{AC} & 2KAB+EDG \\ 2KA\sqrt{BL} & 2G\sqrt{ABCL} & 2DL\sqrt{AC} & 2ELA+KGD \\ 2GC\sqrt{BL} & 2K\sqrt{ABCL} & 2EL\sqrt{AC} & 2DCL+GKE \end{vmatrix},$$

and now dividing the columns in order by $2\sqrt{BL}$, $2\sqrt{ABCL}$, $2\sqrt{AC}$, 1 we have finally

$$\begin{vmatrix} DC & E & KB & DEK+2GBC \\ EA & D & GB & EDG+2KAB \\ KA & G & DL & KGD+2ELA \\ GC & K & EL & GKE+2DCL \end{vmatrix} = 0,$$

which agrees with what has been obtained otherwise.*

* *Proc. Roy. Soc. Edin.*, xxi. p. 333.

XI.— *Chapters on the Mineralogy of Scotland. Chapter VIII.*—Silicates.* By M. Forster Heddle, M.D., Past President of the Mineralogical Society of Great Britain, Emeritus Professor of Chemistry in the University of St Andrews.

(Read December 6th, 1897.)

The earlier mineralogists laboured under two great disadvantages. They could not readily, on account of the small number of students of chemistry, call in the aid of that science: and at the time when mineralogy was becoming a distinct branch of science chemistry was in itself crude as well as cumbrous. They were thus forced to rely chiefly upon external properties; and, where crystalline form was absent, they were confined to what may be called physical properties alone.

Their knowledge of the composition of bodies being thus limited and uncertain, the old nomenclature was to a considerable extent founded upon external features alone.

It is the habit of many of the silicates to run out into lengthened crystals, the greatest amount of their concreting material being deposited in the direction of the main axis of the crystal, and when a multiplicity of crystals are concreted, these are thrown out from a common centre of crystallising growth, to radiate through the matrix, very much after the manner of such crystals as have grown in what we term empty or free space, where no matrix is present to interfere with a tendency to divergence. This fact, the evident displacement of that which is not now displaceable, gives us, in the first place, some information as to the condition of the matrix of divergent crystal groups at the time of their formation; and leads us, in the second, to consider whether that matrix was in a very different condition, or held in degree any very different relationship (as a *body foreign* to the substance crystallising in it) from the liquid or the vapour present in those cavities in which we usually find divergent crystalline groups.

The Swedish mineralogist Wallerius, who wrote in 1747, was one of the earliest authors who instituted group-arrangements. After considering the gems, and rock-

* Chapter I. The Rhombohedral Carbonates. Part I., . . . *Trans. R.S.E.*, vol. xxvii. p. 493.
„ II. The Felspars. Part I., ... „ „ xxviii. p. 197.
„ III. The Garnets, ... „ „ xxviii. p. 299.
„ IV. Augite, Hornblende, and Serpentinous Change, . . „ „ xxviii. p. 453.
„ V. The Micas; with description of Haughtonite, a new Mineral Species, ... „ „ xxix. p. 1.
VI. "Chloritic Minerals," . . . . . „ „ xxix. p. 55.
VII. Ores of Manganese, Iron, Chromium, and Titanium, . „ „ xxx. p. 427

species, such as felspar, mica, talc, and asbestos, he instituted a family which he termed *Hornbärg* (the *Roche de Corne* of the French); this embraced, as a sub-family, Skiörl (Schörl of the Germans).

CRONSTEDT (1758) adopted the same family, and threw into it, "as a convenient pocket," many of the recently discovered species.

It is not altogether easy to make out what species were entitled to get into this pocket, or which side of its medial partition it was intended that they should lodge themselves in, beyond this that *corneus* was to hold "cheap or worthless stones," "mostly of colours from black to dull green."

CRONSTEDT introduced a little more method as regards the "*Skorl*" side of the pocket, making it nearly synonymous with the *corneus crystallisatus* of WALLERIUS, and destined to receive *prismatic minerals* of black, brown, green, and reddish colours, but still having some resemblance to horn in lustre.

He, however, again introduces confusion—a confusion which was continued by WALLERIUS in an edition of 1778—through adopting the term "Basaltes" instead of Skorl; and HILL, in his work on *Fossils*, 1771, fortifies the error, when, in speaking of the *Shirls*, he says, "as to size we see them from that of barleycorn up to the Giant's Causeway," the columns of which he calls "Basaltes Hibernicus" or "Irish Shirl."

ROMES DE LISLE, however, in 1783, bringing crystallography to bear, at once got rid of such excrescences as basaltic pillars, helleflintas, and rocks; but on account of chemistry being still a lagging science, he was forced to throw in many new and indeed old species, and to increase the number of adjective distinctions; and though we do find these to be all Silicates, yet he departs from the prismatic elongation by introducing such species as axinite, staurolite, and harmotome.

When chemistry came to lend a hand to the structural erection of the science, the disintegration of the great "schorl group" commenced. BERGMANN, by his researches, published in 1780, went far to disband it; the five which lingered last were kyanite, "blue schorl"; staurolite, "cruciform schorl"; andalusite, "a red schorl"; rutile, red schorl; tourmaline, black schorl; and these were extruded from the family in the above order. There is this much indication of these forming a natural group that, with the exception of rutile, they are all silicates of alumina.

It is somewhat singular that the substance—namely, tourmaline—which last retained the name schorl seems to have been that to which the term was first applied. In MATTHESIUS' *Sarepta*, 1562, we find that the name "schurl" was used for the "sterile black little stones" accompanying tin ore and gold, and which were thus probably *tourmaline*; and as they were metallurgically worthless, it has been suggested that the word originally was derived from the old German word *Schor*, meaning *refuse*.

"Schorl" is a name still applied to an inferior fibrous, opaque, black tourmaline; it is the sole representative of a great family; but we still use the adjective *schorlous*—as "schorlous beryl"—to imply crystals, thinner and more elongated than usual, which are *imbedded in a matrix with more or less of a radiating arrangement.*

It is more especially the minerals retained longest in this old family of the schorls which fall to be considered in the present chapter, and in chemical simplicity the first of these is Andalusite, ~~Ä~~ Si, right prismatic.

1. *Red Andalusite, from Auchendoir, Aberdeen.*

"Red Schorl" from Aberdeenshire has been noticed in several old works, but Mr JAMES SOWERBY, the author of *British Mineralogy* (1804), has the credit of first *describing* the mineral. He does this with precision, but, though he shows what it is not, he draws a false conclusion as to what it is.

His description is as follows:—

"Argilla durissima, *Scotch Corundum*, spec. char. Nearly pure argil; hardest of all minerals, next to the diamond.

"This curious substance was sent me from a dealer in Aberdeen, under the name of Red Schorle from Achen-door. I figure it here because it is a substance which appears to be new to British writers. Upon inquiry I found it was very little known, nor was it to be found in any mineralogical collection in London, nor scarcely in Scotland. Even Mr JAMESON had not previously obtained it. From him I hope for a good account of it."

Then follows his description, which concludes: "Among a tolerable quantity I found very few with crystallised terminations; the faces, however, are very distinct. We find this fossil has been taken for a rubellite, and KIRWAN's description in a great measure accords with that idea. But in many respects it has been confounded with the titanite of KIRWAN. May the radiating variety be the substance of which MACQUART says the garnets are formed? He describes it as consisting of straight fibres diverging from a common centre. KIRWAN mentions red schorl, and says rubellites are so called. Another substance resembling this was found by MORVEAU in Poitou, which he presumed to be adamantine spar."

After showing how it differs from certain of the above, and giving its properties, SOWERBY writes: "This seems undoubtedly the 'Spath adamantin d'un rouge violet' of BOURNON, which he now considers a variety of corundum."

SOWERBY finally points out JAMESON's mistake as to corundum occurring at Tiree,* and concludes, "therefore, ours is the only thing known at present as corundum in Scotland."

Though Auchendoir has been given as the locality for this red andalusite, I rather think that both the localities in which I myself found it are in the parish of Kildrummy, and that only the grey variety is found in Auchendoir. These localities lie a few miles to the south-west of the village of Lumsden. The first—the south side of the Peat Hill—affords but few specimens, and these are poor. The second is the southern slopes of the hill of Clashnaree, in Clova.

* The Tiree mineral is greyish-white malacolite.—M. F. H

The specimens all lie loose, being the most enduring portions of veins which have themselves endured after the disintegration of a very micaceous gneiss. To all appearance, indeed, the specimens seem to represent the "branches" or knots upon very thin veins of quartz, which veins can, after considerable search, be seen formed in the rock, and such as I have found were barren of minerals. The loose lying fragments of veins consist of a melange of quartz, andalusite, labradorite, fibrolite, and an ill-defined black mica.* These minerals interlace in a confused manner, there being no approach to a uniformity in growth from the two sides of the vein. The andalusite crystals, indeed, sometimes pass from side to side, lacing the other ingredients together. The mineral is always rudely crystalline, but regular crystals are very rare. The most perfect I have delineated.

Though I have figured them as "complete" in the terminal planes, yet all the crystals I have seen had these planes hemihedrally disposed. The colour is a uniform dull purplish red; but there is this most important fact to be noted, that all the crystals which can be sectioned and examined, though uniform in structure and transparent in thin slices, have a central core which is deep purple, with purple spots at the four corners of the transverse section, after the manner of chiastolite. Well-crystallised andalusite thus seems to have a complex internal structure which is independent of any portion of the matrix being caught up during its concretion into a geometric solid. This fact, not, so far as I know, before noticed, comes to have an important bearing in all speculations as to the question of the mode of formation of chiastolite crystals in clay-slate rocks.

The crystals are sometimes 3 or 4 inches in length, and occasionally an inch in thickness. Rarely, as noticed by SOWERBY, they form a tube-like sheath to a central core of the felspar; there is not here the slightest appearance of any passage into felspar, as assumed by BOURNON; but there is an almost insensible passage into, or intermixture between it and colourless and brilliant lustred fibrolite.

The specific gravity of this red andalusite is 3·121.

The analysis on 1·302 grammes yielded—

Silica, ...	·442	
from Alumina, ...	·036	
	·478	= 36·712
Alumina, ...		59·678
Ferric Oxide, ...		2·302
Manganous Oxide, ...		·230
Lime, ...		·860
Magnesia, ...		trace
Water, ...		·465
		100·247

Insoluble silica, 4·184 per cent.

* See Chapter V. The Micas (*Trans. R.S.E.*, xxix. (1879) 33).

2. *Andalusite from Marnoch, Banffshire.*

The precise locality is the banks of the stream near the Mill of Achintoul, Kinnordy Castle.

The nature of the ground at Clashnaree—for it is covered with sward and peat—and the consequent impossibility of tracing the specimens into connection with the rock, as well as the confused crystalline arrangement of the constituents of the veins, prevents our arriving at any information which can have a geologic bearing on its formation, simple though it be in composition. This should not be the case, however, as regards its occurrence at Marnoch and elsewhere in Banffshire; though the light thrown therefrom is still obscure. And yet it is not so much that the amount of evidence supplied by the mode of occurrence and internal structure of such substances as the andalusite of Marnoch, the staurolite of Aldernie, the chiastolite of Portsoy, the apophyllite of Kilsyth, and the stilbite of the Long Craig is in itself small, as that we have collected so few observations on the paragenetic formations, and know so little of the physical laws which govern the formation of such crystals as are built up, not according to ordinary polar molecular concretion, but apparently by the sequential interlocking of tesselated structures, each one of which structures seems to have been constructed in defiance of all the recognised laws of crystalline accretion.

As regards internal structural arrangement—the mode of fitting of the molecular or crystallo-molecular bricks of the fabric—the imbedded andalusite crystals of Marnoch yield almost no insight. Because in the formation of the crystals—however that was effected—so much of the matrix has been caught up by the concreting andalusite substance as must be regarded as capable of interfering with the free formation of any definite structure, seeing that it has chemically interfered with the purity of the material attempting to crystallise apart. Possibly its potency to interfere may be all the greater that the intruding substance is present not in the condition of a *magma mica*, but as a perfected mineral formation—biotite mica.

The crystals of andalusite at Marnoch lie all imbedded in a fine-grained schist, which has, when fresh, a pale yellow-brown colour, due to a crypto-crystalline magma of silicate of alumina. This magma is sprinkled throughout with minute crystals of rich brown biotite, granules of quartz, specks of magnetite, and twin crystallisations of staurolite, of less than pin-head bulk. The crystals of biotite lie in all directions, pervading the whole mass; those of staurolite have some disposition to be arranged in special layers; and this is very much more marked as regards the crystals of andalusite, though there are other localities in which it is hardly observable.

The crystals of andalusite are from half to nearly one inch in length, by about one-third of that thickness, and it is to be remarked that though for the most part here disposed in layers, they are very far from invariably disposed upon their sides, as regards the rock bedding, though that position dominates. They are ash-grey in colour, and in section and even to the eye a central lozenge-shaped tessela of darker and clearer shade of colour is seen; while the whole substance of the crystals is also seen to spangle with crystals of biotite. These are equal in size to those generally occurring in the rock, are disposed like these in all directions, and are not very markedly fewer in number.

The analysis of these was made on crystals freed absolutely from the inclosing rock, and with even some portion of their outer surfaces removed to ensure as great purity as possible. They yielded—

On 1·3 grammes—

Silica,	52·538
Alumina,	39·314
Ferric Oxide,	1·094
Ferrous Oxide,	3·267
Manganous Oxide,	·461
Lime,	·861
Magnesia,	·846
Alkalies,	trace
Water,	1·11
	99·491

Loss, ·238 per cent. of water in the bath.

This result shows a very considerable intermixture with all of the ingredients of the rock, notwithstanding which the crystals are hardly affected by the knife, and have a vitreous lustre.

Three theories have been advanced to account for the presence of the crystalline constituents of clay-slates, for they occasionally bulk so largely as to entitle them to the name. According to the first of these theories, the crystals in question are regarded as the product of chemical action in the ocean in which the original material was deposited. The second theory attributes and confines the formation of the crystalline minerals to processes of metamorphism which have taken place subsequent to the solidification of the rocks. The third theory refers them to an aggregative action going on in the still plastic clay-slate mud prior to its solidification.

The first of these theories has been maintained by CREDNER; but against it numerous arguments have been adduced, and especially the difficulty of supposing an ocean capable of depositing from its waters at successive periods minerals of such different chemical composition as actinolite, andalusite, chlorite, etc.

The second theory has received the support of DELESSE, but in opposition to it the existence in the rocks in question of broken crystals which have been re-cemented by the surrounding clay-slate substance has been pointed to.

Striking facts, drawn from the microscopical structure of the rocks, have been adduced by ZIRKEL in favour of the third theory.

Later metamorphic action must not, however, be excluded in seeking to account for the origin of the crystalline constituents of clay-slates.

A review even of the theories themselves suffices to show that four distinct stages at least may be considered in the series of changes by which the rocks in question may have acquired their present character :—

1*st*, the deposition of the mud ;

2*nd*, the formation of minerals during the plastic state ;

3*rd*, the separation or segregation of other materials after solidification ; and

4*th*, the action of metamorphic processes.

If such processes have operated locally, it will have to be considered whether they most favour the second or the third of these theories, for they may be *local* in their operation either geographically or geologically. They may have operated in close proximity to igneous outbursts, or to limestone formations where there has been much crushing of the beds, or even when there has been disturbance alone. And, geologically, the change may be apparent throughout the whole sweep of a formation, but only up to a certain thickness of its deeper-seated beds.

TOURMALINE.

This substance, common in granitic veins as it is, does not often occur in Scotland either in well-developed forms or of marked purity. The finest crystal I know of, the terminal portion of which I examined, was found in the coarse granite vein of Rubislaw quarry. It occurred along with microcline, muscovite, beryl, and garnet. It was $8\frac{1}{2}$ inches in length by $1\frac{1}{4}$ in width. It was curved like the figure 6, but was perfectly terminated and formed throughout. Fine crystals are rarely found in granite veins in andalusite schist in North Glen Clova in Aberdeenshire.

Material sufficiently pure for analysis was prepared from several localities, but our want of any satisfactory method of determinating boracic acid induced the writer to postpone the analyses, except in the case of crystals which were found in the granitic belt of rock which cuts gneiss near Struay Inn, Ross-shire.

It here occurs in jet-black crystals of some inches in length along with muscovite, orthoclase granular pink, and microcline of a dove blue, garnet and beryl. Its specific gravity is * . In powder it is brown.

* The blank was in the MS. Professor GEIKIE informs me that the specific gravity lies between 3·1 and 3·24 ; Scottish examples being nearer to the latter than to the former value.—P. G. T.

On 1·3 grammes—

		P.C.
Silica,	·457	
from alumina,	·005	
	·462	= 35·538
Alumina,		35·55
Ferric Oxide,		·18
Ferrous Oxide,		7·12
Manganous Oxide,		·307
Lime,		1·108
Magnesia,		3·538
Potash,		1·072
Soda,		·429
Boracic Acid (loss),		10·768
Fluorine,		1·705
Phosphoric Acid,		trace
Water,		2·955
		100·000

Fibrolite, ~~Al~~ Si. Anorthic.

This species was first recognised as British by the writer, but there is reason to believe that it was noticed by SOWERBY, although he was ignorant of its true nature.

In speaking of the andalusite of Auchendoir, while stating that it does not merge into felspar, he remarks: "The nearest approach to mixing insensibly is by fibres, which in ours are, however, sufficiently distinct." He also remarks: "The gangue is chiefly composed of a coarse granite intermixed with indurated asbestos."

In the first, if not in the second of these observations, he must refer to fibrolite, and had he laid due weight upon the fact that the fibres were "sufficiently distinct," he would have seen that they must have been a material different from the andalusite which he was describing.

The fibrolite of Clashnaree occurs in three different modes of arrangement. First, as a corded or stalactitic-like coating to the other minerals, somewhat after the manner in which galmei coats galena. Here it forms a kind of sheath which envelopes labradorite, quartz, and andalusite alike. Second, it radiates in bundles of fibres through the labradorite, and these fibres often unite into a mass which resembles okenite. This variety is very tough. Third, it frequently is disposed with its fibres in parallel arrangement to the crystals of the red andalusite; and long slender crystals of the red andalusite are often imbedded amongst the fibres of the fibrolite.

As the fibrolite is white or colourless, and of adamantine lustre, it is easily distinguished, and there is nothing of the nature of a transition; it is a case of the main

axis of dimorphous substances lying parallel to one another, as known to occur with grenatite and kyanite, and with other di-morphs.

In this third form it is somewhat more brittle than in the others, but it is still reduced to powder with extreme difficulty. I with difficulty separated a sufficiency of the fibrolite in its third form for analysis; but when separated it was exquisitely pure and brilliant. It had a hardness fully 7· in the scale.

22·1 grains yielded—

Silica,	38·410
Alumina,	61·426
Ferric Oxide,	·215
Manganous Oxide,	·114
Water,	·23
	100·395

3. *Fibrolite from Pressendye Hill, Tarland, Aberdeenshire.*

The specimens examined I found in small quantity coating gneiss, in thin veins on the north-west side of the hill, at about 300 yards from its summit.

Its colour was dull white; it was not very lustrous; it was in fibrous and slightly matted tufts, which were very tough. No piece was got large enough for the determination of the specific gravity.

It yielded—

Silica,	39·680
Alumina,	58·822
Ferrous Oxide,	·038
Manganous Oxide,	1·100
Potash,	·860
Soda,	trace
Water,	·320
	100·820

Dr Thomas Aitken of Inverness showed me fragments of granite boulders which he had collected at Auchendown, near Cawdor. These contain a substance of an appearance very similar to the last. There is, however, some suspicion in my mind that this may be merely somewhat plicated plates of a hydrous mica, which show the edges of the plates only. The specimens, having been exposed, are not altogether fresh.

There is one fact which so far increases the probability of this being fibrolite, namely, that a black mica, which has much the appearance of that associated with the mineral at Clashnaree, is present in the Auchendown boulders.

Kyanite, ~~Ä~~ Si. Anorthic.

There is good reason to believe that this species was first found in Scotland.

SAUSSURE, *fils*, describes it under the name *Sappare*, in *Journ. de Phys.*, xxxiv. 213, 1789. His name *sappare* arose from a mistake in reading a label of the mineral, on which it was called *sapphire*; a copy of this label is given in the *Journ. de Phys.* The specimen thus labelled was from Botriphnie in Scotland, and was sent by the Duke of GORDON to SAUSSURE the father.

In the *Descr. Cat. de l'École des Mines*, p. 154, published by SAGE in 1784, it is called *Talc bleu*; but as the present writer found no "Talc bleu" in the collection of l'École des Mines, and as he among the specimens of kyanite found a Botriphnie specimen of the mineral, it is probable *that* had been the specimen termed the "Talc bleu" by SAGE, and the specimen presented by SAUSSURE, having come from the same original source.

The name *sappare* was used for the mineral by some writers up to 1823, when we find it employed by JAMES SMITHSON who, in virtue of its infusibility, used it as a support in blowpipe experiments.*

Kyanite is no longer got at Botriphnie, and the precise spot where it occurred I have not been able to find. Specimens from this locality are in the collection at Jermyn Street Museum, and in those of Edinburgh, Banff, and, I think, Montrose. They were larger and finer than any now obtained in Scotland. The only associated mineral is margarodite.

The second locality at which this mineral was found in Scotland was in the vicinity of the Burn of Boharm, about a mile above the house of Auchlankart—that at least is the spot where the writer has found it in *North* Boharm.

Dr MACCULLOCH, in writing of it at this spot, gives the following accurate description of it, one which should be pondered in considering the metamorphism of the rock matrix.

"Boharm. This sappare-disthene is said to have been originally discovered in this place. The crystals occur in a quartz vein which traverses a talcose clay-slate. They pass through both without any change of their direction or appearance; seeming to mark a common condition in the schist and the quartz at the period of their formation. Although these crystals in general penetrate and impress the quartz, they are sometimes bent and waved, as if they had accommodated themselves to its irregularities. This is not the case, however, with those imbedded in the talcose slate, which radiate in brushes of rectilinear crystals through its mass. This rock consists of a talcy clay-slate, so penetrated with hornblende as to render its character for an instant doubtful. On an accurate examination it will be seen that the body of the rock is a clay-slate, and that it is interspersed throughout with lamellar and thin crystals of hornblende. These lamellæ are generally disposed at right angles to the lamella of the schist, and are sometimes short and straight, and variously placed, interfering with each other often in every direction. More commonly they diverge from a sort of central axis in curved planes, so

* SMITHSON remarks: "Chemical analysis carries destruction along with it, and bestows knowledge of a substance only at the cost of its existence. One remedy which can be offered for this defect is to reduce the scale of operating, and thus as far as possible reduce the amount of the sacrifice."

that their section, according with that of the lamella of the schist, exhibits an appearance of curved pencilliform groups of acicular crystals, frequently an inch in length, assuming an appearance of great singularity. In this direction the schist is visible, and appears to form the largest part of the stone, while in the cross fracture, the lamellæ of hornblende alone being seen, the whole rock seems to consist of this mineral. Occasionally the hornblende displays crystals disposed in so many different ways that the schist is discernible even in the cross fracture."

To this description I have only to add that the specimens I have obtained from near Auchlankart were all of the *rhætizile* or grey variety, much impregnated with the substance of the schist, in which indeed I alone here found them, but that I found at the same spot—which is at the upper fork of the burn—crystallised staurolite in simple crystals, the mode of the occurrence of which—as regards the quartz and the rock matrix which alike hold them—was precisely as described by MACCULLOCH for disthene. These crystals of staurolite were amber coloured and transparent, but had a central structure, which will be noticed below.

Specimens nearly as fine as those from Botriphnie were formerly found by Colonel IMRIE loose lying in the neighbourhood of Millden and the Burn of Turret, North Glen Esk, Forfarshire. One of these has been figured by SOWERBY, vol. iii. p. 49. Here also margarodite is the sole associate.*

"Near Banchory, in Aberdeenshire," "near Mortlach, Banffshire," and "in quartz near the summit of Ben y Gloe," in blue radiating crystals, in quartz nodules, in clay-slate, in limestone at Ardonald, by Cunningham, are old localities at which this mineral is no longer found.

It has long been known, and is still found at Vanleep, Hillswick, Shetland. At this gash, a chasm in the cliffs of the western shore of Hillswick, kyanite occurs of three markedly dissimilar appearances.

The ordinary blue crystals generally isolated and imbedded in massive quartz are here very rare. Large plumose groupings of a reddish-grey colour, also occurring isolated in massive quartz, are less rare; but the common appearance is that of veins or large isolated nodules of smaller intermatted crystals of an anchovy-red passing into

* I analysed a specimen from Colonel IMRIE'S collection, and obtained on 1·3 grammes :—

Silica,	36·384
Alumina,	58·296
Ferric Oxide,	1·609
Ferrous Oxide,	1·123
Lime,	·861
Potash,	·252
Soda,	·423
Water,	1·445
	100·393
The loss in bath was	·282 per cent.
The insoluble silica,	1·691 „
The specific gravity,	3·538 „

white, and apparently dark green, from an intimate intermixture of chlorite plates. Occasionally a plate or two of talc occurs, and very rarely large and fine crystals of chloritoid. These veins cut the huge beds of quartz which intercept the micaceous strata of the promontory. The locality faces the picturesque sea-stacks of red porphyry termed the Drongs. The crystals analysed were picked white, somewhat tinted with pink.

On 1·2 grammes—

Silica, ...	·474	
from Alumina, . . .	·022	
	·496	= 38·153
Alumina, ...		56·979
Ferric Oxide, ...		1·867
Manganous Oxide, ...		·153
Lime, ...		·301
Water, ...		2·646
		100·099

Loses in the water-bath, ·701 per cent.
Insoluble silica, 3·024 per cent.

From near Millden in Tarffside, Forfarshire. This occurs in large flat crystals of a fine blue colour.

I have found it at the following new localities in Shetland. Cliffhill, near Woodwick, and north-west of Norwick Bay in Unst. *Magnetite* and *garnets* are its associates at the first of these localities; it is in quartzose belts at the last; the rock in both cases being gneiss, and the colour of the mineral pale blue. At the south end of the Wark of Skewsburgh, in the Mainland, associated with *ilmenite* in quartz veins in gneiss. It is here greyish-white to blue. To the east of the same hill near its north end.

Kyanite has more recently been found at the following localities:—

In minute crystals of perfect transparency and deep blue colour along with green hornblende and red garnet, forming the rock eklogite. This was found by Mr Dudgeon, to the north-east of Obb, in Harris.

Finely crystallised in the form of the figure and of a fine blue colour, at a height of about 1100 feet, on the north-west slopes of Garlat Hill, Cowie Hill, Tarffside, by Mr Robert Murray. The matrix here was gneiss and the associate finely crystallised chlorite.

In interlacing grey crystals in gneiss far up in bed of the burn which comes from the east into Glen Derry, Loch Callater, Aberdeenshire, by the Rev. Mr Peyton.

In blue crystals in gneiss in Allt Beg, Glen Rinnies, by Mr James Wilson.

In mica schist at a bridge over the Little Drumlach, in the parish of Enzie, Banffshire,* by Mr Wallace of Inverness.

* *Min. Mag.*, vol. vi. No. 28. But no description given.

By the writer it has been found—

In small blue crystals in Hebridean gneiss on the hill to the west of Ben Chaipaval in Harris.

In quartzite near summit of Carn Lia, Ben y Gloe, Perthshire.

In greyish-blue crystals, along with garnet, sphene, ilmenite, and chlorite, in mica schist, one mile north of Loch Bulg in Aberdeenshire.

In large blue-grey crystals, along with ilmenite and chlorite, in gneiss on the slopes on the east side of the corry of Meall Buidh, on the south side of the Moor of Rannoch.

In gneiss in the railway cutting west of Mulben, Banffshire.

In bright blue crystals in gneiss near limestone about one mile west of the limestone quarries at Dulnan, Inverness-shire.

Loose lying in grey and blue interlacing crystals on the south slopes of Cruach Ardran, in Perthshire.

In tufts of grey crystals impregnated with the substance of the rock in clay-slate, at the lime quarries of the Burn of Aldernie, Banff.

In large single imbedded blue crystals and fasciculitic tufts, a peculiar yellow margarodite slate, south-east of the lime quarries in Glen Urquhart.*

In quartzose veins in a clay slate which contains rosette groupings of actinolitic crystals in the rocks, about three-quarters of a mile north-west of Sandend in Banffshire. The crystals of the mineral here, an inch or two in length, pass through portions of both matrix and vein, after the manner of rivets, just as described by MACCULLOCH as occurring at Boharm. They appear to have issued from the matrix into the vein, as if formed nearly contemporaneously with the filling of the latter with the quartz; but as the terminations are not distinct, this conclusion is drawn from the greater breadth of the crystals, where they lie in the quartz, than in the schist.

These crystals, like those at many other of the above localities, are in parts colourless or pale yellow.

In lenticular quartz nodules, often morion, and sometimes prase, with pyrite, in chiastolite slate, west of the clay-slate quarry near Portsoy, sometimes colourless.

* This yielded on 1·3 grammes :—

Silica,	37·53
Alumina,	58·105
Ferric Oxide,	2·089
Lime,	·129
Magnesia,	·076
Potash,	·252
Soda,	·741
Water,	1·198
	100·12

Its specific gravity was 3·016.

EPIDOTE.

1. From Balta Island, Shetland. Occurs in a geo which cuts the island near the centre of its eastern cliff-lined shore. The epidote occurs in crystals of half an inch in size, imbedded without any associate in a quartz vein which cuts gabbro. The epidote is pale pea-green, the quartz somewhat granular.

On 1·396 grammes—

Silica,	·531	
from Alumina,	·01	
	·541	= 38·753
Alumina,		26·986
Ferric Oxide,		7·898
Ferrous Oxide,		1·806
Manganous Oxide,		·501
Lime,		20·378
Magnesia,		·786
Potash,		·25
Soda,		·21
Water,		2·376
		99·944

Insoluble silica, 7·578 per cent.

2. The above occurrence of epidote certainly in no way bears out the theory of its resulting as a product of the alteration of hornblende. This, however, may have been the case at the next locality which I quote. This is Nudista in Hillswick.

It here occurs in dull, soft-looking crystals of an olive-green colour, of about an inch in length. These crystals radiate through large foliated crystals of dark green hornblende, cutting the foliations transversely. The specific gravity of this epidote is 3·396.

On 1·304 grammes—

Silica,	·484	
from Alumina,	·009	
	·493	= 37·866
Alumina,		24·722
Ferric Oxide,		9·961
Ferrous Oxide,		·361
Manganous Oxide,		·536
Lime,		23·104
Magnesia,		·766
Water,		2·822
		100·138

3. From North Quin Geo, Hillswick. Epidote occurs here filling a small vein. It forms stellate groups of rich green crystals over an inch in length.

The analysis on 1·503 grammes yielded—

Silica,	36·127
Alumina,	20·574
Ferric Oxide,	14·921
Manganous Oxide,	·306
Lime,	23·025
Magnesia,	·306
Water,	4·568
	99·827

Insoluble silica, 7·734 per cent.
The ferric oxide is to the alumina as 1 to 2.

4. From Delnabo, Glen Gairn, Aberdeenshire. Out of the old limestone quarry. It occurs imbedded in green prehnite, in radiated crystals of a pale green colour.

On 1·501 grammes—

Silica,	38·374
Alumina,	26·087
Ferric Oxide,	10·388
Manganous Oxide,	·738
Lime,	21·647
Magnesia,	·239
Water,	2·441
	99·914

Insoluble silica, 7·812 per cent.
The ferric oxide is to the alumina as 1 to 4.

WITHAMITE.

This red variety has been found only at one spot. This is a projecting spur of amygdaloidal felspathic porphyry, which touches the road through Glencoe upon its north side, about three miles above the turn of the glen. The epidote occurs in the little druses, very rarely in bright green crystals. It is then associated with byssolite and chlorite. Much more frequently it occurs in the red modification. It forms very minute acicular crystals of a brilliant blood-red colour. These crystals radiate from the sides of the druses, a narrow layer of a milky saussurite-like substance sometimes intervening. The crystals are red and yellow respectively in two directions, at right angles to one another (MacKnight and Brewster). Minute specks of quartz sometimes occur.

On account of the extremely minute quantity in which this substance is found, the purifying of the sample analysed was executed with extreme care.

On 1·3 grammes—

Silica,	·543	
from Alumina,	·019	
	·562	= 43·23
Alumina,		23·09
Ferric Oxide,		6·675
Ferrous Oxide,		1·131
Manganous Oxide,		·138
Lime,		20·003
Magnesia,		·884
Potash,		·962
Soda,		·935
Lithia,		·253
Water,		2·4
		99·701

Insoluble silica, 1·957 per cent.

This result is by no means a satisfactory agreement with the composition of epidote.

ZOISITE.

1. This mineral was first found by me in Britain in Glen Urquhart, Inverness-shire. It occurred in the most southerly of the limestone quarries, about a mile north-east of Milltown. It was found only in one large nodule of calcite of about a hundredweight, crystals of a grey to a pale bluish-white colour, about one inch in length, interlaced in the calcite. There was a very little quartz; a few specks of chalcopyrite and brushes of light green actinolite in association with the mineral.

The crystals, and indeed the mineral, has the general appearance of tremolite, but the cleavage leaves no room for doubt. The form is well seen in these crystals, but there are no terminations. The cleavage face seems a twin face, as there are repeated re-entering angles which produce a coarse striation. The crystals are brittle. The cleavage face is somewhat pearly; fractures are vitreous. The specific gravity taken on three pieces gave 3·004, 3·111, and 3·014.

On 1·303 grammes—

Silica,	·484	
in filter,	·019	
from Alumina,	·013	
	·516	= 39·60
Alumina,		31·083
Carry forward,		70·683

Brought forward,	70·683
Ferrous Oxide,	2·071
Manganous Oxide,	·078
Lime,	23·336
Magnesia,	trace
Potash,	·566
Soda,	1·056
Water,	2·412
	100·202

Insoluble silica, 1·55 per cent.

2. From an adjacent quarry; the mineral being very similar, but the matrix was quartz. The specific gravity is 3·014.

On 25 grains—

Silica,	41·56
Alumina,	29·901
Ferrous Oxide,	3·205
Lime,	22·142
Magnesia,	·332
Potash,	·345
Soda,	·684
Water,	2·19
	100·359

Insoluble silica, 2·31 per cent.

3. From the same quarries, but in large crystals of about 2 inches by $1\frac{1}{4}$. These large crystals were lying loose in the quarry, the calcite having apparently been dissolved away by rain. They were slightly browned on the outside, but lustrous when broken. Three determinations of the gravity gave 3·312, 3·322, 3·318.

On 1·301 grammes—

Silica,	·48	
from Alumina,	·034	
	·514	= 39·508
Alumina,		30·827
Ferrous Oxide,		2·52
Manganous Oxide,		·077
Lime,		22·813
Potash,		·681
Soda,		·9
Water,		2·505
		99·831

Insoluble silica, 2·334 per cent.

4. From Laggan, Dulnan Bridge, Inverness-shire.

This was given me by Sir ARCHIBALD GEIKIE. It was got in quartz veins in the limestone quarry. It occurs in pale brown crystals entirely imbedded in the quartz. The crystals are lustrous and well defined; the associates are chlorite and sahlite. In the near neighbourhood there is much kyanite. The specific gravity is 3·438.

On 1·2 grammes—

Silica,	·463	
from Alumina,	·002	
	·465	= 38·75
Alumina,		28·144
Ferric Oxide,		6·547
Manganous Oxide,		·916
Lime,		22·026
Magnesia,		·416
Water,		3·333
		100·132

Loses in the bath, ·155 per cent.

5. From Loch Garve, Ross-shire. This locality was found by the late W. H. BELL. It is that in which the mineral occurs in much largest quantity in Scotland, and also in much the largest crystals. It occurs in a quartzose vein, which, starting from near the centre of the south shore of the lake, strikes right up the hill for two or three hundred feet. Sometimes it is almost massive, occasionally in crystals of a stouter habit than those of Urquhart. The colour is ash-grey to white, passing to pale yellowish-brown. It is translucent. The specific gravity is 3·268.

On 1·34 grammes—

Silica,	40·066
Alumina,	30·834
Ferric Oxide,	1·58
Ferrous Oxide,	·48
Manganous Oxide,	·22
Lime,	23·66
Magnesia,	·476
Potash,	·504
Soda,	·428
Water,	2·100
	100·348

Insoluble silica, 2·2 per cent.

IDOCRASE.

From Delnabo, Glen Gairn, Aberdeenshire. Idocrase occurs abundantly, passing when massive almost insensibly into garnet in the old limestone quarry. The portion taken for the analysis was a portion of a magnificent dark brown crystal of about $7\frac{1}{2}$ inches in length by nearly 1 inch in width, which was fractured by the blow which disclosed it. Its specific gravity was 3·43.

On 1·302 grammes—

Silica,	·45	
from Alumina,	·022	
	·472	= 36·251
Alumina,		18·626
Peroxide of Iron,		·932
Ferrous Oxide,		5·036
Manganous Oxide,		·844
Lime,		33·935
Magnesia,		1·574
Potash,		·568
Soda,		·329
Water,		1·78
		99·875

Insoluble silica, 1·483 per cent.

XII.—*The Absolute Thermal Conductivity of Nickel.*

By T. C. Baillie, M.A., B.Sc., Assistant Lecturer and Demonstrator in Physics, University College of North Wales, Bangor. (With a Plate.)

(Read 16th May 1898.)

§ 1. Introduction.—The experiments described in this paper were commenced with the view, not only of determining the absolute thermal conductivity of nickel, but also of comparing the results found by Forbes's and Ängstrom's methods for the same specimen. Although some readings were taken for Ängstrom's method, that part of the investigation was not completed, because it was found that the experimental errors — unavoidable, on account of the necessity of measuring rapidly changing temperatures—would be too great for the results to be of any value. The thermal conductivity of a portion of the bar of nickel used for Forbes's method was determined by a direct method involving the determination only of steady temperatures, and the results so obtained are given in the latter portion of this paper.

Forbes's Method.

§ 2. The Statical Experiment.—The nickel bar used was kindly lent by Dr Knott, being a piece about four feet long, which he had no immediate occasion to use for his own experiments on "The Strains produced in Iron, Steel, and Nickel Tubes in the Magnetic Field" (see *Trans.*, vol. xxxviii. part iii. No. 13). This bar was turned down so as to be of uniform circular section, and holes for thermometers were drilled into it by Messrs Aitken & Allan, Edinburgh. Four thermometer holes were drilled in each of the end portions of the bar, so as to leave a length of 19 inches intact for a tube required by Dr Knott at a later period.

The bar was set up in Professor Tait's private laboratory, with the same fittings, altered to suit the size of the nickel bar, as were used by Professor Tait, and afterwards by Dr Mitchell, in their experiments on thermal conductivities (see *Trans.*, 1878, xxx., and *Trans.*, 1887, xxxiii. part ii. p. 535). One end of the bar was fitted with white lead into a round hole in the side of a cast-iron pot, which was afterwards nearly filled with solder. This end of the bar was heated by a bunsen flame placed under the pot of solder. A constant temperature was maintained at this end of the bar by keeping the gas supplied to the bunsen burner at constant pressure by means of Professors Tait and Crum Brown's gas regulator. This regulator is like a small gasometer, one of the balancing weights of which, on descending, bends a piece of soft, flexible rubber tubing conducting the gas supply to it, so as to diminish the internal cross-section of the piece of soft tubing;

on the weight ascending, the cross-section of the tubing is increased. The nickel bar was protected by a double metal screen from thermal disturbances due to the heater. A constant stream of cold water was kept playing on the unheated end of the bar. The thermometers used were some of the Kew standard thermometers used by Professor TAIT and Dr MITCHELL. Temperatures above 200° C. were not used, and as the readings on the majority of the thermometers varied in the course of an experiment through a range of about 1° C., the thermometers were simply read by the naked eye to the nearest quarter of a degree. One could be quite certain of avoiding making any error due to parallax of more than that amount. A little mercury was put in each of the thermometer holes to give good thermal contact between the bulbs of the thermometers and the bar. No amalgamation of the nickel has ever been observed.

A steady state as regards the distribution of temperature along the bar was not reached in five hours from the time the gas at the heater was lighted, and the readings on the thermometers usually increased at the rate of about a quarter of a degree per hour for a few hours more. The following table shows the readings (uncorrected) obtained over an interval of nearly twenty-four hours on 18th July 1894 and the following morning. The gas was lighted at 8.30 A.M.

Time.	1	2	3	4	5	6	7	8	Temperature of Air.
12.30	175	130·5	100	78	26	23	20·5	18·2	19·0
1.30	177	133	102·5	80·5	28	25	22	19·1	19·1
2.0	177	133·3	103	81	28·9	25·4	22·5	19·5	19·1
2.15	176·5	133	103	81·5	29	25·8	22·5	19·7	19·1
2.45	176·5	133	103	81·5	29·4	26	23	20	19·1
6.0	177·8	134	104	82·2	30	26·5	23·2	20	19·1
6.30	178	134·3	104·5	82·5	30	26·5	23	20	19·2
7.0	178	134·3	104	82·5	30	26·5	23·1	20·1	19·2
7.30	177·5	134	104	82·3	30	26·5	23·2	20·1	19·4
8.0	178·5	134·5	104	82·5	30·1	26·7	23·4	20·2	19·3
8.30	178	134	104	82·2	30	26·6	23·1	20·0	19·3
9.0	178	134·3	104	82·3	30	26·5	23	20	19·4
9.30	178	134·3	104	82·5	30	26·5	23	20	19·4
10.0	178	134	104	82·5	30	26·5	23	20	19·3
11.20	179	135	105	83	30·1	26·6	23·1	20	19·4
1.30 a.m.	178	135	105	83	30·1	26·5	23·1	19·9	19·3
2.15	178	135	104·7	83	30·1	26·5	23·1	19·9	19·3
3.30	179·7	135·5	105	83·4	30·2	26·6	23	19·9	19·3
4.25	179	135·1	105	83·4	30·2	26·6	23	19·9	19·2
5.0	179	135	105	83·2	30·2	26·6	23	19·8	19·2
6.0	178·5	135	104·8	83	30	26·5	23	19·7	19·0

After several sets of readings had been taken, the bar was reversed, and heated at the other end. The thermometers were never shifted from their positions, except when the bar was reversed. They were read on each morning before the burner was lighted, and their readings on those occasions never differed by more than one-third of a degree. The thermometers were corrected for stem exposure by adding to the reading V. the

product ·000113 V^2. This correction seems to me to be probably too high for some of the thermometers, but it is less than that which Dr MITCHELL applied to the same thermometers, viz., ·00016 V^2. The value of the stem correction which I have chosen is based on the results of experiments to be described later in connection with the other method of determining the conductivity.

The dimensions, etc., of the bar were as follows :—

Length of bar, ...	128·5 cm.
Mass of bar, ...	19·28 kilo.
Diameter of bar, ...	4·67 cm.
Density of nickel, *i.e.*, mass ÷ volume, . . .	8·724 gms. per c.cm.
Specific gravity of a small piece cut off, determined by weighing in air and in water, . . .	8·75

Distances of thermometer holes from one end of the bar :—

Number of Hole.	Distance to Centre of Hole.
1	14·75 cm.
2	23·02 „
3	31·34 „
4	39·73 „
5	88·72 „
6	97·02 „
7	105·26 „
8	113·59 „

The statical experiment was frequently repeated at the same and different temperatures, and the following table contains the readings chosen for calculation corrected for stem exposure. On 6th August and the following days the bar was heated at the opposite end to that at which it was heated on previous occasions. The table shows that any effect due to tarnishing of the surface during the few weeks occupied by these experiments is not noticeable.

Date of Experiment.	Numbers of Thermometer Holes.								Temperature of Air.
	1	2	3	4	5	6	7	8	
July 12	150·5	114·8	90·2	72·3	28·9	26·0	23·2	20·6	20·2
„ 13	180·6	135·0	103·5	81·7	28·9	25·4	22·5	19·7	19·4
„ 16	181·7	135·6	104·4	82·1	29·3	26·0	22·8	20·0	18·8
„ 17	181·0	136·0	104·9	82·8	29·8	26·1	23·0	20·0	19·1
„ 18	181·6	136·0	105·2	83·2	30·1	26·5	23·0	20·1	19·3
„ 27	69·8	57·0	47·7	40·8	22·0	20·7	19·2	18·0	18·5
„ 30	116·7	100·1	72·8	59·9	26·5	24·1	22·0	20·0	18·9
„ 31	116·6	99·9	72·5	59·6	26·5	24·4	22·4	20·5	19·1
August 1	72·5	59·4	49·4	42·3	23·0	21·8	20·4	19·0	19·0
„ 2	67·5	55·4	46·8	40·3	22·7	21·5	20·4	19·1	18·6
„ 3	67·5	55·3	46·7	40·3	23·9	23·0	22·4	21·9	18·7
„ 6	150·3	114·2	89·7	71·3	28·5	25·7	23·1	20·9	18·9
„ 7	198·8	147·9	113·8	89·2	31·6	27·9	24·5	21·4	19·2
„ 8	109·3	85·9	69·5	57·0	25·9	23·7	21·6	19·5	18·9
„ 9	70·0	57·4	48·3	41·2	22·8	21·4	20·0	18·8	18·3
„ 10	165·3	124·9	97·6	77·6	29·8	26·6	23·8	21·1	18·9
„ 13	163·5	123·2	96·0	75·6	28·3	25·3	22·7	20·0	18·4

§ 3. Reduction of the Readings.—The equation for the conduction of heat in a bar, each part of which is at a steady temperature, is :—

$$KA\frac{d^2\theta}{dx^2}=Ep\theta,$$

where K is the thermal conductivity, A the cross-section, E the emissivity, and p the perimeter of a part of the bar, at temperature θ above the surrounding air, and at a distance x from some fixed point in the axis of the bar. Since K, A, E, and p are either constants or functions of θ only, it follows that $\frac{d^2\theta}{dx^2}$ is a function of θ only, and therefore the value of $d^2\theta/dx^2$ for any given value of θ should be the same, no matter which of the sets of readings it is derived from. This affords a means of testing the concordance of the various sets of readings. The determination of $d^2\theta/dx^2$ directly—by drawing a curve representing θ as a function of x, taking the tangents at various points, and thus getting another curve showing $d\theta/dx$ as a function of x; and from this, by a similar process, another showing $d^2\theta/dx^2$ as a function of x, and using the first and last curves to get $d^2\theta/dx^2$ as a function of θ—does not give $d^2\theta/dx^2$ with sufficient accuracy. A common proceeding is to find suitable values of the constants in some empirical equation representing θ as a function of x, and to differentiate the equation to obtain $d\theta/dx$. The following method of reducing the readings obtained in the statical experiment was adopted after trying others. Curves were made from the sets of readings on the first four thermometers only, in which log. θ was shown as a function of x. The gradient of these curves increased, but not very rapidly, with log. θ, and therefore $d(\log.\ \theta)/dx$ increased as θ increased. The curves were drawn by a lath, to the ends of which couples were applied so as to give it the slight curvature necessary to make the curve produced by its means pass in close proximity to each of the four points given by the corrected readings of the thermometers. It was noticed that the value of $d(\log.\ \theta)/dx$ was practically the same, for the same value of θ, for all curves. A new curve was then constructed, in which $d(\log.\ \theta)/dx$ was shown as a function of θ. The different points found on this curve lay very approximately in a straight line—that is to say, $\frac{d}{d\theta}\left(\frac{d(\log.\ \theta)}{dx}\right)$ was practically constant. The equation to the statical curves must then be of the form $\frac{1}{bc}\log.\frac{\theta}{\theta+b}=x+B$, where b and c have the same value for each curve. The simplicity of this method of finding the average values of $d^2\theta/dx^2$ for all sets of readings was what led to its adoption. In any case, $d(\log.\ \theta)/dx$ does not vary so rapidly as $d\theta/dx$, and it is therefore easier to get $d\theta/dx$ with accuracy, when using graphical methods, by multiplying $d(\log.\ \theta)/dx$ by θ, than it is to get $d\theta/dx$ directly. The values found for the constants in the above equation were $c=\cdot0000505$, and $b=670$. The value of $d^2\theta/dx^2$ is $c^2(2\theta+b)(\theta+b)\theta$. The following table contains the values of $d^2\theta/dx^2$ calculated, not from the expression just given, but from the

numbers found in the curve, using the formula

$$\frac{d^2\theta}{dx^2} = \theta \frac{d\ (\log.\ \theta)}{dx} \left\{ \frac{d\ (\log.\ \theta)}{dx} + \theta \frac{d}{d\theta} \left(\frac{d\ (\log.\ \theta)}{dx} \right) \right\}.$$

The temperatures given in the tables are actual temperatures, not temperature excesses. They have been got by adding 19—about the average temperature of the air during the experiments—to the numbers used in the curves which were differences of temperature between the bar and the air.

Temperatures.	$\frac{d\ (\log.\ \theta)}{dx}$	$\frac{d^2\theta}{dx^2}$	Temperatures.	$\frac{d\ (\log.\ \theta)}{dx}$	$\frac{d^2\theta}{dx^2}$
40	·0335	·0239	110	·0370	·138
50	·0340	·0369	120	·0375	·159
60	·0345	·0509	130	·0380	·181
70	·0350	·0661	140	·0385	·205
80	·0355	·0823	150	·0390	·230
90	·0360	·0997	160	·0395	·256
100	·0365	·1183	170	·0400	·284
			180	·0405	·313
			190	·0410	·344
			200	·0415	·376

§ 4. The Cooling Experiment.—For this experiment a short bar turned down from a left-over portion of Dr Knott's nickel bars was used. It was a piece of the same rod as the bar used in the statical experiment: it was turned down in the same way, at the same time, and to the same diameter, as was found by careful measurement. The length of the cooling bar was 21·55 cm., and as its diameter was 4·67 cm., the surface exposed at the ends was $9\frac{3}{4}$ per cent. of the whole surface. This involves an increase in the rates of cooling of about ten per cent. This is a serious drawback in these experiments. It has been allowed for by diminishing the observed rates of cooling in the ratio of the whole area of the cooling bar to the area of the curved portion only. It is possible that in air the emissivity of a vertical surface is, *ceteris paribus*, greater than the average emissivity of a curved cylindrical surface of the same diameter. As there is heat lost from the ends of the cooling bar, there must be some fall of temperature between the centre and the ends. I have given up all attempts at making allowance for this. The best way of meeting difficulties of that kind is to make the end correction negligible altogether. The bar was heated over a row of bunsen burners, without the previous warming necessary to avoid "sweating." The bar was heated pretty rapidly, and turned round rapidly while being heated, and very little moisture condensed upon it. A Kew mercury thermometer was used to measure the rate of cooling of the short bar which was heated to about 250° C. Readings were not taken until the bar had cooled for some time with the thermometer in position, since the distribution of temperature in the thermometer itself is at first irregular. This is discussed very fully by Professor Tait in his paper already referred

to. The thermometer was observed through the telescope of a cathetometer, and the time at which the top of the mercury column passed each degree division mark was noted by looking at that instant at the dial of a watch. After some practice it was found easy to note the time of such transits to within a couple of seconds from the position of the seconds' hand, without paying much attention to the divisions round the dial. The time of transit set down for each degree division was late by the time taken to look from the thermometer to the watch, but as this is small and affects each reading, it is of no consequence. When the cooling became comparatively slow, as it did below 100° C., it was possible to see the top of the mercury column disappear behind a degree division mark, note the time, and have the eye in position at the telescope again in time to see the top of the column reappear on the under edge of the division mark.

§ 5. Reduction of the Cooling Readings.—The method employed for reducing these readings was as follows:—On paper ruled in squares temperatures were plotted as *abscissae*, and the times (in seconds) taken by the bar to cool through one degree were plotted as ordinates corresponding to the mean temperatures for those degrees: thus, for example, the ordinate corresponding to the temperature 102·5° C. was the observed time taken by the bar to cool from 103° C. to 102° C. The advantage of this method of reduction is the simplicity of correcting for errors of observation, &c. Suppose, for example, that the time set down for the transit across the 175 degree division mark is too late, the time noted for cooling from 176° to 175° is too great, and the amount by which it is unduly increased is deducted from the time of cooling from 175° to 174°; but the average time of cooling for a range including 176° to 174° is not affected by the supposed error. An error in graduation, by which one of the division marks is displaced, produces a similar effect. The ordinates would, if there were no errors of any kind, increase in length continuously as the temperature diminishes. If the curve formed by the ends of the ordinates is not continuous, all that is necessary is to make a continuous curve by reducing the lengths of those ordinates which are obviously too long and increasing the lengths of adjacent ordinates, and *vice versa*, so as to keep the sum total of the lengths of all the ordinates constant. This treatment will get rid of the effects of errors such as those considered above. The way in which this was carried out was to form a new curve in which for each reading was substituted the average of the five nearest readings. This gave a curve which was smooth but with small "waves" along it. A mean curve was then drawn by means of a lath planed thinner towards one end so as to produce the necessary variation of curvature along it. The ordinate at any temperature of the curve so constructed is the reciprocal of the rate of cooling at that temperature.

A cooling experiment was done alongside of the statical experiment on several days, until the surface of the cooling bar was thought to be just perceptibly dimmer than that of the long bar. It was confidently expected that the repeated heating of the short bar, especially as "sweating" was not entirely avoided, would affect the surface and increase its emissivity. The readings taken show that each time the bar was heated its emis-

sivity was increased before the tarnish on the surface became even perceptible. The following table will show this, and it provides the means of allowing for it.

Date of Experiment.	Time of Cooling from 200° to 150° in Seconds.	Temperature of Air.	Time of Cooling from 150° to 100° in Seconds.	Temperature of Air.	Time of Cooling from 100° to 70° in Seconds.	Temperature of Air.	Time of Cooling from 70° to 40° in Seconds.	Temperature of Air.
1894								
July 12	1375	20·4°	2210	20·3°	2295	20·3°	5125	20·2°
„ 13	1355	19·5°	2175	19·3°	2280	19·3°		
„ 17			2087	19·1°	2193	19·1°	4795	19·1°
„ 18			2105	19·1°				
August 2	1348	18·8°	2150	18·7°	2201	18·6°	4660	18·55°
„ 3					2191	18·7°	4503	18·7°
„ 7			2178	18·3°	2214	18·5°		
„ 8	1341	19·2°	2149	19·2°	2233	19·1°	4740	19·1°

The table also shows irregularities in the cooling, due possibly to changes in the state of the atmosphere, or to variations in the unavoidable draughts of the second order of magnitude. The readings obtained on 8th August gave the best curves, and they have been used to determine the rates of cooling given in a later table. A reduction of 2 per cent. was made in the rates of cooling found, in order to allow for the increase in the emissivity which had taken place by 8th August. A correction is necessary in the cooling experiment for the exposed stem of the thermometer. In the statical experiment the stem correction was made by adding to the observed reading V. the product ·000113 V^2. Using the same form of correction in the cooling experiment, let V. be the observed, and θ the true temperature, then since $\theta = V. + ·000113\ V^2.$, the true rate of cooling $d\theta/dt$ is equal to $(1 + ·000226\ V.)$ times the apparent rate of cooling dv/dt got from the curves in which stem exposure is not allowed for. Since the thermometer parts with its heat to the cooling bar, its temperature must always during the cooling be higher than that of the bar; in other words, the thermometer lags behind the bar by an amount depending on the rate of cooling. No attempt has been made to allow for that in these experiments. Some mercury was put into the hole for the thermometer to give good thermal contact between the bar and the thermometer. The following table gives the rates of cooling of the short bar found after applying the corrections referred to above, except that due to lag, and that due to loss of heat at the ends of the bar.

Temperature.	Rate of Cooling.	Temperature.	Rate of Cooling.	Temperature.	Rate of Cooling.
40	·00362	90	·0149	150	·0308
50	·00556	100	·0174	160	·0336
60	·00770	110	·0200	170	·0363
70	·01005	120	·0225	180	·0392
80	·01250	130	·0251	190	·0423
		140	·0279	200	·0455

§ 6. Specific Heat of Nickel.—The determination of the specific heat of the nickel has been found by far the most troublesome part of these experiments. A portion of the cooling bar about 2·5 inches long had a hole drilled into it to receive the thermometer. A little mercury was put into the hole along with the thermometer. It was then heated and allowed to cool. At some instant the temperature was noted just as it was let fall into a large calorimeter, and the heat given out by the nickel was measured in the ordinary way by the method of mixtures. This was repeated at the same and different temperatures, and the results were not quite concordant, but indicated that the specific heat increased with temperature. As it was not quite certain that the temperature in all parts of the interior of the piece of nickel was that of the mixture when the readings were taken nickel turnings were tried. Several pieces of the turnings made in turning down the nickel bars were tied together with a short piece of thread, whose mass was negligible, and heated in the inner chamber of a double cylinder of copper containing glycerine between the cylinders. This was heated to over 200°C. and packed up with cotton wool in a wooden case provided with a contrivance for opening a slide at the bottom and allowing the nickel to fall into the calorimeter at the moment of opening. The calorimeter used was a small glass beaker of suitable dimensions. With it the cooling correction was smaller than with a copper calorimeter of the same size. It was hoped that as the heater cooled very slowly it would be safe to assume that the temperature of the turnings after being in the heater some time would be that of the thermometer whose bulb was inserted amongst them. As the heater cooled, determinations of the specific heat could be done on the same day at lower and lower temperatures. It was found that the sets of determinations obtained on separate days did not agree no matter how long the nickel was kept in the heater, and as all the quantities involved could be measured within 1 per cent., and the correction for cooling was only about 1 per cent., the heater was regarded as the cause of the irregularities.

In some subsequent experiments the nickel turnings were heated in a steam jacket of the usual laboratory pattern, and results agreeing within 2 per cent. were obtained when the nickel was in the heater for not less than two hours. As the heat required to raise the temperature of 1 gramme of water is not constant, but varies in a manner depending on the thermometer used in the calorimeter, closer agreement than this is not to be expected. The specific heat thus obtained was higher than that got for the same temperature from the large mass cut off the cooling bar. At the same time there is *no* reason for supposing that the specific heat would *not* be affected by the nickel being cut up and distorted as it is in the form of turnings.

The values of the specific heat given below were found from a solid piece of the nickel weighing nearly 100 grammes, and as a glass calorimeter could not be used with so large a mass, a copper calorimeter was made of thin sheet copper, the depth of it being 5 inches, and the diameter $2\frac{1}{4}$ inches. There was a slight recess along one side to accommodate the thermometer and a flange round the lip by which it was suspended

in the interior of a large copper vessel which protected it from draughts in the room. The thermometer used in the calorimeter was one of DUCRETET'S précision thermometers divided into tenths of degrees centigrade, and it was read by means of a telescope fixed a little distance off, hundredths of a degree being estimated by eye. The steam heater was used in the ordinary way, but in order to get determinations at different temperatures the same heater was used with methylated spirits instead of water. A tube was arranged to conduct all the spirits which condensed in the apparatus back to the boiler by a pipe leading in at the bottom of it. Only a small portion of the spirits was distilled off, as the flame of the burner heating the boiler was so arranged that very little vapour was formed over and above that required to produce heat enough by its condensation to maintain the heater at a uniform temperature. The boiling point rose by less than one degree in the course of a day on account of loss by distillation of the more volatile constituents of the spirits. The top of the chamber in the heater, in which the nickel was suspended while being heated, was closed by a large cork in which were two holes, one letting in the thermometer which indicated the temperature of the nickel, and another through which passed a fine wire supporting the nickel. The bottom of the chamber was closed by a slide padded with cotton wool. This slide was drawn aside when the nickel was dropped into the calorimeter, an arrangement being made for doing all this with great rapidity. The wire which had supported the nickel in the heater remained attached to it, and the end of it, which was usually found projecting out of the mouth of the calorimeter, was at once seized and the contents of the calorimeter stirred by moving the nickel up and down and to and fro in the water. While this was being done the thermometer was being watched through the telescope. Two persons were thus necessary. No correction has been made for the thermal equivalent of the work done in stirring, as it has been assumed to be negligible. It was impossible to observe if stirring the water produced an appreciable quantity of heat as under all circumstances its effect was quite obscured by the disturbances produced by other causes.

The correction for cooling was applied in the following way:—The observer at the telescope had in his hands a stop watch, with two hands so arranged that, by pressing one stop, they would start together; pressing another stop made one of them remain where it was at the instant of pressing; a second press made it overtake and go on as before with the other hand. In this way, the time at which the thermometer indicated any reading could be noted to a fraction of a second, the reading being written down subsequently; and a fresh reading could be then taken in the same way. A curve was then plotted from the readings thus obtained, with temperatures as ordinates, and times as *abscissae*. The part of the curve corresponding to times after the maximum temperature had been reached was produced backwards to the axis of zero time, and in this way the temperature which the calorimeter must have cooled from, had its rise of temperature been instantaneous, was found. This is very nearly the temperature which would have been reached if the calorimeter had not lost any heat at all. This correction is probably

too much by something less than one-half per cent. An example of such a curve of correction for cooling is given in fig. 1.

At temperatures over 100° C. another form of heater was used. An iron tube was surrounded by a conical-shaped iron chamber riveted on to it, and mercury was put in the space between. This was heated by a circular gas burner, the flame of which was regulated by the volume of the mercury, which, on expanding above a certain limit, cut off all the gas, except what found its way through a small by-pass. The by-pass was arranged to allow just sufficient gas through it to keep the burner lighted, and thus to save the trouble of lighting the gas when the mercury had contracted sufficiently. The arrangement was similar to that shown at E in fig. 2. The flame rose and fell about ten times in a minute. The temperature at any one place in the heater was very steady after it had been in action for an hour or two, but the temperature near the top of the inner tube was 2 or 3 degrees lower than that of the hottest part. The thermometer used for reading the temperature of the nickel in this heater was put in so that the bulb touched the nickel. The nickel dropped into the calorimeter through the centre of the ring burner. The inner tube of the heater was prolonged below the burner. Corrections for stem exposure have been applied to the readings of the thermometers. It was possible to obtain only an approximation to the stem corrections on account of the manner in which the thermometers were placed, with some part of their stem at unknown temperatures; but as the correction is only 2 per cent. in the greatest instance, they are probably accurate enough.

The following tables give the data obtained in the last sets of experiments done :—

SET I.

Mass of nickel, ... 99·3 grammes.
Mass of water in calorimeter, . . . 156·7 „
Water equivalent of calorimeter, etc., . . 3·5

Date of Experiment.	Initial Temp. of Water.	Corrected Temp. of Mixture.	Rise of Temp. of Water.	Initial Temp. of Nickel.	Fall of Temp. of Nickel.	Log. Ratio Rise of Temp. of Water to Fall of Temp. of Nickel.
1897						
July 7	16·94	21·87	4·93	99·1	77·2	$\bar{2}$·8052
„ „	18·80	23·65	4·85	99·2	75·2	$\bar{2}$·8072
„ „	19·98	24·83	4·85	99·3	74·5	$\bar{2}$·8135
„ „	20·88	25·72	4·84	99·4	73·7	$\bar{2}$·8173
„ 8	18·65	23·54	4·89	99·4	75·9	$\bar{2}$·8091
„ „	19·60	24·46	4·86	99·4	74·9	$\bar{2}$·8121
„ „	19·12	23·99	4·87	99·4	75·4	$\bar{2}$·8101
Arithmetic mean of observed values	19·14	24·01	4·87	99·3	75·3	$\bar{2}$·8107

Average value of specific heat, ·104.

SET II.

Mass of nickel, ... 99·3 grammes.
Mass of water in calorimeter, . . . 126·7 „
Water equivalent of calorimeter, etc., . . 3·3

Date of Experiment.	Initial Temp. of Water.	Corrected Temp. of Mixture.	Rise of Temp. of Water.	Initial Temp. of Nickel.	Fall of Temp. of Nickel.	Log. Ratio Rise of Temp. of Water to Fall of Temp. of Nickel.
1897						
July 9	17·10	23·13	6·03	99·4	76·3	$\bar{2}$·8978
„ „	18·10	24·08	5·98	99·4	75·3	$\bar{2}$·8999
„ „	19·40	25·34	5·94	99·4	74·1	$\bar{2}$·9040
„ „	19·88	25·72	5·84	99·5	73·8	$\bar{2}$·8983
„ 10	18·19	24·28	6·09	99·6	75·3	$\bar{2}$·9078
„ „	20·25	26·12	5·87	98·6	72·5	$\bar{2}$·9083
Arithmetic mean of observed values	18·82	24·78	5·96	99·3	74·55	$\bar{2}$·9027

Average value of specific heat of nickel, ·105.

SET III.

Mass of nickel, ... 99·3 grammes.
Mass of water in calorimeter, . . . 126·7 „
Water equivalent of calorimeter, etc., . . 3·3

Date of Experiment.	Initial Temp. of Water.	Corrected Temp. of Mixture.	Rise of Temp. of Water.	Initial Temp. of Nickel.	Fall of Temp. of Nickel.	Log. Ratio Rise of Temp. of Water to Fall of Temp. of Nickel.
1897						
July 13	21·05	25·27	4·22	77·3	52·0	$\bar{2}$·9093
„ „	22·27	26·31	4·04	77·5	51·2	$\bar{2}$·8971
„ „	23·80	27·82	4·02	78·1	50·3	$\bar{2}$·9026
„ 14	21·01	25·23	4·22	78·2	53·0	$\bar{2}$·9010
„ „	22·13	26·20	4·07	77·5	51·3	$\bar{2}$·8995
„ „	23·55	27·51	3·96	77·5	50·0	$\bar{2}$·8987
Arithmetic mean	22·30	26·39	4·09	77·7	51·3	$\bar{2}$·9015

Average value of specific heat of nickel, ·105.

SET IV.

Mass of nickel, ... 99·3 grammes.
Mass of water in calorimeter, . . . 156·7 „
Water equivalent of calorimeter, etc., . . 3·5

Date of Experiment.	Initial Temp. of Water.	Corrected Temp. of Mixture.	Rise of Temp. of Water.	Initial Temp. of Nickel.	Fall of Temp. of Nickel.	Log. Ratio Rise of Temp. of Water to Fall of Temp. of Nickel.
1897						
July 20	21·60	29·75	8·15	145·7	116·0	$\bar{2}$·8467
,, ,,	22·80	30·93	8·13	146·4	115·5	$\bar{2}$·8475
,, 21	20·31	28·73	8·42	147·3	118·6	$\bar{2}$·8511
,, ,,	21·12	29·32	8·20	147·3	118·0	$\bar{2}$·8419
,, ,,	21·90	30·08	8·18	147·3	117·2	$\bar{2}$·8438
,, 22	18·78	27·35	8·57	148·1	120·8	$\bar{2}$·8510
,, ,,	18·99	27·50	8·51	148·8	121·3	$\bar{2}$·8461
,, ,,	19·63	28·05	8·42	149·1	121·0	$\bar{2}$·8425
,, 23	21·06	29·43	8·37	149·0	119·6	$\bar{2}$·8449
Arithmetic mean	20·69	29·02	8·33	147·67	118·65	$\bar{2}$·8462

Average value of specific heat of nickel, ·113.

The average value of the specific heat of the nickel turnings for a range varying from just under 100° C. to about 20° C. was about ·11. This shows that either the thermal capacity is altered in the process of disintegration, or that there is some error in the determination depending upon the size of the pieces employed. The latter I believe to be the case. During the month of June, I did several determinations at the same time with a bundle of copper washers, and with the piece of nickel referred to. After a few trials I found the mass of copper (114·8 grammes) which had the same thermal capacity as the 99·3 grammes of nickel. I tried to discover a difference between the rate of rise of temperature in the calorimeter when the copper was employed from that when the nickel was used. The difference in the times taken to reach the maximum reading was only about six seconds, the whole time being about one minute to one and a quarter. Probably the lag of the thermometer behind the calorimeter obscured the greater part of the actual difference.

The effect of not receiving all the heat from the nickel would be to make the apparent specific heat less than the true specific heat. This error would obviously be greater at low temperatures than at high temperatures, and thus would make the apparent specific heat increase more rapidly with temperature than the true specific heat actually does. Probably this is the real reason why the specific heats of carbon and silicon—so-called bad conductors of heat—have been found to be much lower at ordinary temperatures than that expected from DULONG and PETIT's law of constant atomic heats, whereas at very high temperatures their specific heats are much greater and nearly great enough to fulfil the law. Errors of this kind are reduced to a minimum by using WATERMAN'S calorimeter. A description of this apparatus, and a short discussion of the determinations of specific heats is given in a paper by WATERMAN in the *Physical Review* (vol. iv. No. 3) for December 1896.

§ 7. It seems to me a disadvantage of FORBES's method that its accuracy has to

depend on that of the determination of specific heat. While I have no confidence in the values found for the specific heat of the nickel, I give the values of the conductivity found by using them. I hope to be able at a future time to supplant these figures by others which can be relied on.

The following table gives the values of the ratio of the conductivity to the specific heat after applying the end correction to the rates of cooling given in a previous table, and the values of the conductivity using the values of the specific heat in the adjacent column. No corrections have been applied for changes of the dimensions of the nickel with temperature as these are really negligible.

Temperature.	Ratio of Conductivity to Specific Heat.	Specific Heat.	Conductivity.
40	1·19	·098	·118
50	1·19	·102	·121
60	1·19	·105	·125
70	1·20	·108	·130
80	1·20	·111	·133
90	1·18	·114	·135
100	1·16	·118	·137
110	1·14	·121	·138
120	1·12	·124	·139
130	1·09	·127	·139
140	1·07	·130	·140
150	1·06	·134	·142
160	1·04		
170	1·01		
180	·99		
190	·97		
200	·96		

§ 8. Experimental and other Errors in Forbes's Method.—The sources of error may be classified as follows:—

Statical Experiment.

(1) Thermometric errors.

(2) Errors in reduction of results, for example in differentiating the temperature curve.

(3) Want of uniformity or regularity in the substance or surface of the Forbes bar.

Cooling Experiment.

(4) Radiation from ends of bar.

(5) Lag of thermometer behind bar due to gradient of temperature necessary to cause flow of heat from thermometer to bar.

(6) Thermometric errors.

(7) Errors of observation in taking cooling readings.

(8) Errors in reduction of rate of cooling from these readings.

(9) Difference in the emissive powers of the surfaces of the cooling bar and statical bar. Some of the causes of such differences may be—

(a) tarnish.

(b) differences in the amount of polish.

(c) difference in the surroundings or in the state of the atmosphere during the cooling and statical experiments.

(d) differences in the radiation due to the temperature of the cooling bar always falling while that of the statical bar is steady.

(10) Errors in the determination of the specific heat.

Of these the chief are—

(a) the specimen used for this may not be a fair average specimen.

(b) want of uniformity in its temperature when put into the calorimeter.

(c) the calorimeter not receiving the whole of the heat supposed to be given out from the specimen.

(d) changes in the thermal capacity of water with temperature as measured by the thermometer used (or, in the case of ice calorimeters, errors in the value of the latent heat or other constants used).

Of these there is little difficulty in arranging the errors from (1), (2), (4), (6), (7), (8) to be small by using proper care and suitably arranging the apparatus or the bars used. Serious error from (3) could be detected by taking a sufficient number of properly varied sets of readings. Small errors due to (9) (b) and (9) (c) are difficult to avoid and it is impossible to discover their existence. (10) (c) is the most serious cause of error in the ordinary method of mixtures. There is probably some error from this cause even in BUNSEN'S calorimeter, as it usually gives lower values than other methods. (10) (d) is unavoidable in all thermometric thermal measurements. (5) and (9) (d) are inherent to the method and are not to be avoided by the use of thermo-electric junctions instead of thermometers. It is also impossible to estimate the errors arising therefrom. In ÄNGSTROM'S method errors from (10) affect the result in the same way, and as all the temperatures measured are varying temperatures, errors of the same sort as (5) and (9) (d) may occur. ANGSTROM'S method is unreliable on other grounds. It is essentially based on the assumption that the ratio of the conductivity to the emissivity is constant.

The values of the conductivity of copper found by Professor TAIT were (reduced to C.G.S. units) for good conducting copper $1{\cdot}08\,(1+{\cdot}0013t)$; for bad conducting copper $\cdot71\,(1+{\cdot}0014t)$. The ratio of these values is independent of nearly every source of error mentioned, and yet Dr STEWART (vide *Trans. Roy. Soc.*, 1893, p. 569) found $1{\cdot}12\,(1-{\cdot}001t)$; while KIRCHHOFF and HANSEMANN (*vide* WIEDEMANN'S *Annalen*, 9, p. 1; 13, p. 406) found $\cdot51\,(1+{\cdot}0057t)$ both for pure copper. One has doubts about believing that the wide range of variations of these values is due only to differences in the specimens of metal used. I, therefore, determined to find the conductivity of the nickel I had used by a method with fewer sources of error.

Direct Method.

§ 9. THE APPARATUS.—The method of determining thermal conductivity by direct measurement of the rate of flow of heat and gradient of temperature is that adopted in the following experiments. This method was used long ago by CLEMENT and by PÉCLET (vide *Ann. de Chimie et de Physique*, 3ᵉ tom. ii. p. 107, 1841), and in their hands did not yield satisfactory results as they did not measure the temperatures of the metal itself, but it has been used with success by E. H. HALL, who utilised the metal experimented upon as one of a thermo-electric couple to measure its own gradient of temperature (vide *Pro. American Academy*, vol. xxxi. p. 271).

In the present investigation one end of the nickel bar used for FORBES's method was cut off, and an extra thermometer hole was drilled into it. Its surface was repolished. The dimensions were as follows:—

Diameter, from 4·660 to 4·667 cm.
Length, 42·55 cm.
Density, 8·75 grammes per c.cm.

No. of Thermometer Hole.	Distance in Centimetres from end at which the Rate of Flow of Heat was measured.
1	2·84
2	11·20
3	19·54
4	27·88
5	36·17

A shorter length would have sufficed, and it would have been an advantage to have made more thermometer holes. The bar was fitted up so that one end could be kept at any constant high temperature, while a flow of water could be kept cooling the other, the rise of temperature of the water and the mass of water passing per unit of time being measured. These data were sufficient to measure the rate at which heat left the end of the bar. The gradient of temperature at any point is given by the tangent to the curve drawn from the readings given by the thermometers.

A slide bench was erected in front of the table carrying the apparatus, and was arranged to carry a telescope which could be raised or lowered in a vertical line, and at the same time moved to and fro along the bench which was placed parallel to the axis of the nickel bar. The thermometers used in the bar were some of Professor TAIT's Kew thermometers from the same stock as those used in the FORBES bar, and they were placed so as to hang vertically, this being tested by a plumb line. As the telescope could only move so as to be always horizontal, parallax was avoided. When the telescope was adjusted so that one of the thermometers was in focus, all were in focus for that same adjustment, which was never altered. The readings were estimated by eye to the nearest tenth of a degree.

A diagram representing a vertical section through the axis of the bar is given in fig. 2. The heater was the cast-iron pot, J, which was used with the FORBES bar. The bar was fixed into the circular hole (at K) in the side of it with red lead. A Jena glass flask,

F, with a fairly long neck was filled to the bottom of the neck with mercury and put into the pot; and the space, H, between was filled nearly full of mercury. To prevent mercury leaking through the cast-iron pot it was previously lined with pipe-clay, a paste of pipe-clay and water being painted in with a brush and allowed to dry. Into the neck of the flask was fitted a glass piece made as shown in fig. 2. The arrows show the path taken by the gas to reach the burner, and the temperature was kept constant by the mercury cutting off the gas supply at E on reaching a certain temperature. The by-pass, B, was opened to allow a full supply of gas while the heater was being warmed up to the proper temperature, the mercury in the flask being allowed to run over at D, which was at all other times closed. When the desired temperature was reached the by-pass, B, was nearly closed, enough gas being allowed to pass through it to keep the Argand burner, G, from going out. This gas regulator worked so well that a thermometer hung in the same place in the pot of mercury showed no variation exceeding one-tenth of a degree centigrade during a whole day.

At first a large steel cap was fitted on the end of the bar, with mercury inside it, the idea being to make it at once the heater and the regulator. It showed a steady, slow rise of temperature, and, although there was no visible leakage, in a few days fine drops of mercury were seen on the iron tray placed under the burner to catch the mercury in case of accident. No leakage of mercury could be noticed from it even under greater pressure from the inside while standing cold, and therefore the mercury must have leaked through pores too small to be noticed while the flame played upon them. The variation of the temperature of cut-off is a very delicate test of such leakage.

The rate at which heat was given out at the other end of the bar was obtained by measuring the rise of temperature and the rate of flow of the stream of water which played on the end of the bar. A brass cap, M, was fitted on the end of the bar, the water entering the space between it and the bar having its temperature measured at O, and the temperature of the water leaving the bar was measured at P. The thermometers at O and P were Anschutz thermometers graduated in fifths of a degree centigrade. The rate of flow of the water was found by observing the time taken to fill the flask, Q, of known capacity to the fiducial mark. The water was supplied at constant level from a chamber, S, containing the well-known inverted bottle device, R. Distilled water was used, but great difficulty was found in keeping the rate of flow regular until the plan was tried of making the outlet of a piece of glass tubing drawn out fine and broken off at the capillary portion. With this improvement the flow was very uniform, and the temperature of the water (at O) reaching the bar was also very steady, but the temperature of the water leaving the bar (at P) varied. When the water had been once used it was cooled by being put in the inner chamber of a double copper tank, while cold tap water was circulating in the outer chamber surrounding it. The same water was thus used over and over again.

The order of taking readings was as follows:—1°, the thermometers in the bar; 2°,

the temperature of the cold water (at O) going to the bar; 3°, the time at which the empty flask, Q, was put to catch the overflowing water; 4°, the temperature of the water leaving the bar (at P) was read every half-minute while the flask was filling; 5°, the time the flask was exactly filled to the fiducial mark; 6°, the temperature of the water entering the cap at O; 7°, the thermometers in the bar. All these readings varied little in the course of one evening, and the rate at which heat was given out at the end of the bar varied within 2 per cent. The following table gives the readings (uncorrected) taken on 6th January 1898.

Average Temperature of Water.	Average Rate of Flow of Heat in Calories per Second.	Temperature of Air.	Temperatures of Holes in Bar.				
			No. 1.	No. 2.	No. 3.	No. 4.	No. 5.
21·7	4·85	13·2	39·6	61·65	88·35	122·2	168·2
		13·2	39·6	61·7	88·4	122·4	168·1
21·8	4·86	13·2	39·6	61·7	88·4	122·4	168·1
		13·3	39·8	61·8	88·5	122·45	168·2
21·85	4·88	13·3	39·65	61·9	88·55	122·5	168·25
		13·3	39·65	61·8	88·5	122·55	168·45
21·86	4·83	13·3	39·65	61·8	88·5	122·55	168·45
		13·3	39·7	61·85	88·6	122·6	168·35
Mean 21·8	4·86	13·3	39·65	61·8	88·5	122·45	168·25

§ 10. Correction of the Thermometers.—The corrections of the thermometers were not found in the ordinary way, as there is always more or less doubt attached to any allowance that may be made for stem exposure on account of the impossibility of knowing the exact distribution of temperature along the stem of the thermometer. The method adopted was simple and allowed the testing to be carried out with the thermometers in as nearly as possible the same circumstances as they are in during the experiments.

The thermometers were tested at three different temperatures, at 0° C., about 100° C., and about 218° C. At 0° C., the correction was found by hanging the thermometers in a vertical position with their bulbs, and as much of their stem as was under the surface of the bar, embedded in powdered ice washed with distilled water. At the other two temperatures the apparatus, a vertical section through the centre of which is shown in fig. 3, was used. A piece of brass tubing of nearly the same diameter as the bar of nickel was cut into three lengths, E, F, and G, and these were brazed together as shown. The end, H, was closed, and three small tubes, A, B, and C, were brazed in the middle piece F. These tubes were about half full of Wood's alloy. The piece E was half-filled with water which was heated by the burner D, which was adjusted until just a small quantity of water vapour escaped at the open end K. The tubes, A, B, and C, were of about the same depth as the thermometer holes in the bar, and during the test the thermometers were suspended in a vertical position with their bulbs near the bottom of the tubes. Asbestos screens were fitted up at L and M to shield the thermometers

from the disturbing effects of the burner on the one side and the escaping vapour on the other. The arrangement is just that of a modified reflux condenser.

The thermometers were suspended with their bulbs in the mercury of the heater (at H in fig. 2), and the temperature of the heater was gradually raised to about 100° C., when the regulator was adjusted to act. The thermometers were left there under these conditions from morning till evening. As readings were always taken in the evenings, while the heater was set working in the mornings, the thermometers were never read until they had been at the same temperature for several hours. It was therefore thought necessary to keep the thermometers the same length of time at 100° C. before testing them at that temperature, so as to allow the glass to take on the same set that it had in the bar at the same temperature. It is possible that if an ordinary mercury thermometer is kept for hours at some temperature before it is read, its reading at the same temperature on some other occasion will only be the same after it has remained at that temperature for some hours.

The burner D was lit, and after the testing apparatus had been at 100° C. for some time, one of the thermometers was taken out of the mercury heater and quickly put into tube A. After the first two or three occasions, it was found easy to do this so dexterously that the reading on the thermometer did not fall more than 2° in the interval. After it had been in A for some time, during which the reading was constant, it was rapidly transferred to B, and by and by to C. The thermometers were hung up vertically by means of a plumb line, and the readings taken with the telescope. It was found that when E was too full of water, even when it was just over half-full, the readings in A, B, and C were not alike. When that was the case, the thermometer was left in one of the tubes until enough of the water had evaporated. The barometer was read sometime during the test and the true temperature of the bulb of the thermometer found from REGNAULT's tables. The difference between the observed reading on the thermometer and the temperature of the water vapour gave the whole correction at that temperature, the graduation correction and the stem exposure correction being thus lumped together. The same thing was gone through for each of the thermometers.

The same sort of process was repeated with the same apparatus, after the water had been dried out and naphthalene put in its place. Pure naphthalene was used, and as the boiling point of pure naphthalene has been determined on the air thermometer scale by CRAFTS, and has been found to be very constant, it is as satisfactory a "fixed" point on the scale of temperatures as one can wish for. The total correction of each of the thermometers was thus found at the temperature of the boiling point of naphthalene. The graduation corrections on the Kew thermometers used were known to be small, and hence it was only to be expected that an expression of the form $a+bt^2$ would represent the correction. This expression suited the values of the corrections found for all the thermometers except one to within a fifth of a degree, but the value of b was not the same in all cases, as it varied from ·00008 to ·000115. Curiously enough, b was smaller for those thermometers graduated up to 300° C. than for those which could not read

above 220° C. The corrections at 0° C. were zero for most of the thermometers. One read ·55° too low, but that was due to a small particle of the mercury having been shaken up into the top of the stem—probably during transit—from which it could not be again dislodged. It was on the strength of these results that $\cdot000113t^2$ was used to give the stem correction in the FORBES bar experiments.

The following table gives the corrected mean readings obtained from the last three experiments, together with the values of the conductivity calculated from them.

Date of Experiment.	Temp. of Air.	Corrected Mean Temperatures of Holes in Bar.					Temp. at End of Bar.	Gradient at End of Bar.	Mean Temp. of Water.	Flow of Heat in Calories per Second.	Conductivity at End of Bar.
		No. 1.	No. 2.	No. 3.	No. 4.	No. 5.					
31/12/97	14·8	39·3	74·1	115·9	168·1	242·7	28·5	3·66	16·8	8·12	·130
4/1/98	13·2	47·0	79·0	118·9	170·0	243·2	37·7	3·23	22·1	7·225	·131
6/1/98	13·3	40·5	62·3	89·4	123·9	170·3	34·3	2·08	21·8	4·86	·136

§ 11. THEORY OF THE METHOD.—Let K be the conductivity, θ the temperature, X the distance from some fixed point on the axis of the bar, of a section of the bar of area A, across which H units of heat pass in unit of time, then

$$KA\frac{d\theta}{dx}=H.$$

Corresponding values of θ, H, and $\frac{d\theta}{dx}$ are given in the above table for the end section of the bar whose cross-section is 17·1 square centimetres (diameter is 4·663 cms.). The values of H given are subject to two corrections: (1) a correction for heat lost by radiation from the brass cap; (2) correction for the changes in the thermal capacity of unit mass of water with temperature. An estimate of the former error shows that it never exceeded 1 per cent., so that it is probable that these corrections combined do not exceed 2 per cent. They are rather smaller than the corresponding corrections in a specific heat determination.

The values of $\frac{d\theta}{dx}$ are liable to error from two sources: (1) thermometric errors in the temperature of the nearest thermometer hole; (2) arithmetical or geometrical errors in differentiating the temperature curve. Errors from both of these causes would have been reduced by having more thermometer holes, and what discordance there is between the values of the conductivity found from the three sets of readings given above is probably mostly due to errors in estimating $d\theta/dx$. Differences amounting to 2 or 3 per cent. are only to be expected. All these sources of error effect FORBES's method,—and, of course, also ÄNGSTROM's—but to these are added in FORBES's method all those arising from the cooling experiment.

The measurements referred to only determine the conductivity at temperatures somewhat above that of the air, but the conductivity could be found in a similar manner at other temperatures (such as slightly over 100° C., by allowing the water in the cap to be

evaporated into steam). Also, by using an electrical heater, the heat supplied at the hot end (subject to corrections for radiation) could be measured and the gradient of temperature at that end. Such experiments, however, were not carried out in this case, because it was seen, in the manner described below, that the conductivity varied little with temperature.

§ 12. Change of Conductivity with Temperature.—Before the brass cap was fitted on the end of the bar for the experiments just described readings were taken with the bar losing heat only by radiation. After the distribution of temperature became steady, the heat which passed any cross-section of the bar was lost by radiation from the rest of the bar beyond. The following table gives the temperatures obtained, thermometric corrections being applied.

Temperature of Air.	Corrected Mean Temperatures of Holes in Bar.				
	No. 1.	No. 2.	No. 3.	No. 4.	No. 5.
13·6	63·85	68·9	78·7	95·35	119·5
8·4	83·55	91·0	107·35	134·15	174·4
14·3	102·7	112·75	132·8	167·35	219·1
9·2	110·15	121·8	146·2	185·4	248·3

Curves were drawn from these readings, and differentiated. By supposing the bar prolonged by an amount slightly over the length of the radius, and producing the temperature curve to that point, one obtained the curve which would suit the bar if no heat had been lost from the end, at which place $d\theta/dx$ would then be zero. From the first set of readings the value of $d\theta/dx$ at the section which had the temperature 120·65° C. was found to be 3·55, and its distance from the point at which $d\theta/dx$ vanished was 38 centimetres. The average excess of the temperature of those 38 centimetres of the bar over the temperature of the surrounding air was 67·45°. This gives the following relation :—

$$KA \times 3{\cdot}55 = Ep \times 67{\cdot}45 \times 38,$$

where K is the conductivity at 120·65° C. and E the average emissivity under the conditions referred to. From the set of readings obtained on 31st December 1897, and given on page 19, the gradient at 120·65° C. was found to be 5·66, the gradient at 63·2° C. to be 4·33 ; and the distance between the points at these two temperatures was 11·72 centimetres. The average excess of the temperature of those 11·72 centimetres of the bar over the temperature of the surrounding air was 76·0°. If we *assume* the average emissivity to be the *same* in these two cases, we find that 1·23 is the value of that part of the gradient which is required to account for the heat lost by cooling over the 11·72 centimetres in the latter instance. For if

$$KA \times 3{\cdot}55 = Ep \times 67{\cdot}45 \times 38,$$

then

$$KA \times 1{\cdot}23 = Ep \times 76{\cdot}0 \times 11{\cdot}72.$$

If we deduct 1·23 from the gradient, 5·66, at 120·65° C. in the latter experiment (date 31st December 1897), we find the gradient (the remaining 4·43) which would cause the same heat to pass the cross-section at 120·65° C. as passes the cross-section at 63·2° C. with its gradient at 4·33. In other words 4·43 and 4·33 would be corresponding values of the gradients at 120·65° and 63·2° respectively if no heat were lost by radiation from the bar. The conductivities at these two temperatures are inversely as these numbers. This shows a diminution of conductivity of 2½ per cent., with a rise in temperature of about 60°. This is within the limits of experimental error. The assumption that the average emissivities for temperature excesses of 67·45° and 76° are the same is not likely to be correct. The emissivity in the latter case will be greater, probably by something of the order of 2 per cent. The effect of the increase of emissivity with temperature will be to reduce the apparent diminution of conductivity with rise of temperature, and might even change it into an increase, but in any case it would be very small and within the limits of experimental error.

The following tables give two sets of data obtained from the curves drawn from the corrected readings already given in tabular form.

Corresponding Values of			Average Temperature Excess.	Corresponding Values at another Section of Bar of		
x	θ	$d\theta/dx$		x	θ	$d\theta/dx$
0	63·2	0	67·45	38·0	120·65	3·55
8·73	63·2	4·33	76·0	20·45	120·65	5·66
0	108·5	0	125·0	30·0	188·7	6·00
18·23	108·5	5·335	130·8	30·55	188·7	8·13

From these are deduced the following :—

	θ	$d\theta/dx$
Corresponding values of θ and $d\theta/dx$ which would be found if no heat were lost from surface of bar,	63·2	4·33
	120·6	4·43
Ditto,	108·5	5·335
	188·7	5·55

These figures indicate a diminution of conductivity of the amount ·000066 per rise of temperature of 1° C. The conductivity cannot fall so much as this, and in any case the change of conductivity with temperature is within the limits of error of such experiments up to a temperature of 200° C.

§ 13. Conclusion.—The conductivity of nickel found by the direct method is ·132. There is some doubt about the third figure after the decimal point, and that figure is the only one affected by changes of temperature up to 200°C. It is interesting to note that

the value of the specific heat of nickel found by using nickel turnings, viz., ·11, would, if multiplied by the ratio of the conductivity to the specific heat at the mean temperature 60°, give a result in exact agreement with the above. It should, however, be stated that the specimen of nickel showed slight fissures. These were not serious enough to affect the readings sufficiently to make it noticeable in the appearance of the temperature curves, and the readings obtained from the FORBES bar do not show irregularities from such a cause. The nickel used was also very pure. I am much indebted to my colleague, Mr F. V. DUTTON, for analysing it for me with the following result :—

Analysis of Nickel.

Manganese,	1·63	per cent.
Magnesium,	0·28	„
Iron,	0·75	„
Nickel,	97·22	„
Total,	99·88	

The FORBES's method experiments were carried out in Edinburgh University Physical Laboratory; the other method was done in the Physical Laboratory of the University College of North Wales, and from time to time the work was carried on partly in Edinburgh, partly in Bangor. I have to thank Professor TAIT and Professor GRAY for affording me every facility in carrying out these determinations.

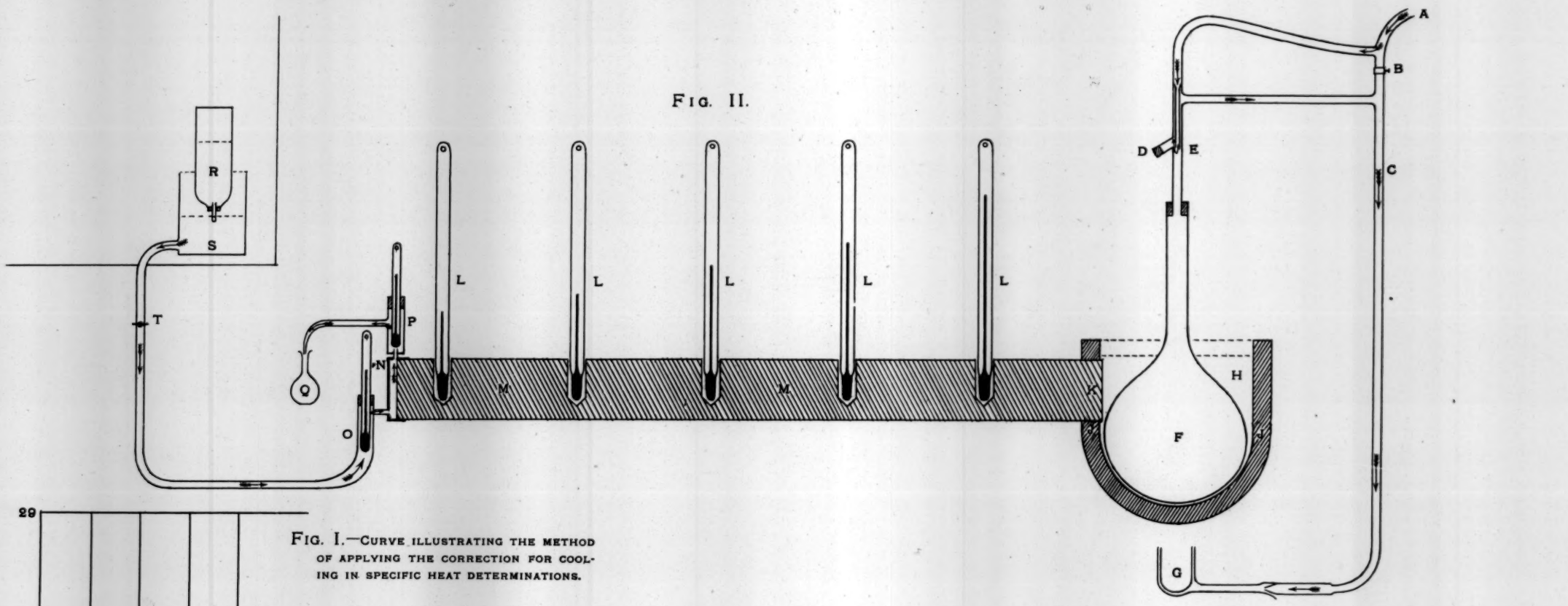
EXPLANATION OF FIGURES.

Fig. 2. A. Gas supply.
B. By-pass.
C. Tube leading gas to burner.
D. Opening for letting out mercury to regulate temperature of cut-off.
E. Place at which mercury acts on gas supply.
F. Glass flask containing mercury.
G. Argand gas burner.
H. Mercury.
J. Cast-iron pot.
K. End of bar heated.
L. Thermometers.
M. Bar of Nickel.
N. Brass cap.
O. Water inlet with thermometer.
P. Water outlet with thermometer.
Q. Flask for measuring rate of flow of water.
R. Inverted bottle
S. Water tank } on reduced scale.

Fig. 3. A, B, C. Tubulures for thermometers containing Wood's alloy.
D. Gas burner.
E. Chamber of water or naphthalene.
K. Open end of apparatus.
L, M. Asbestos screens.

Mr. Baillie on Absolute Thermal Conductivity of Nickel.

Fig. II.



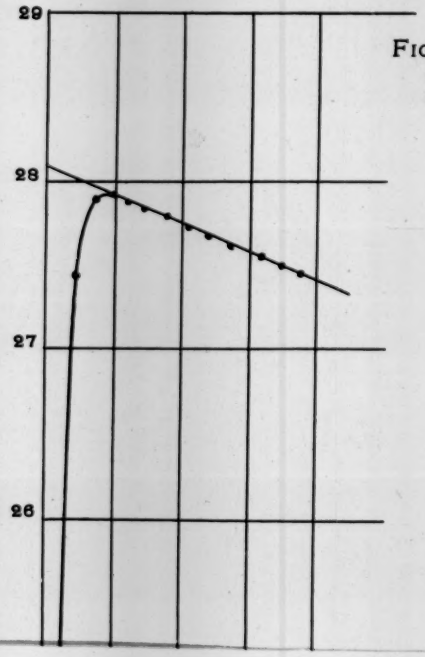
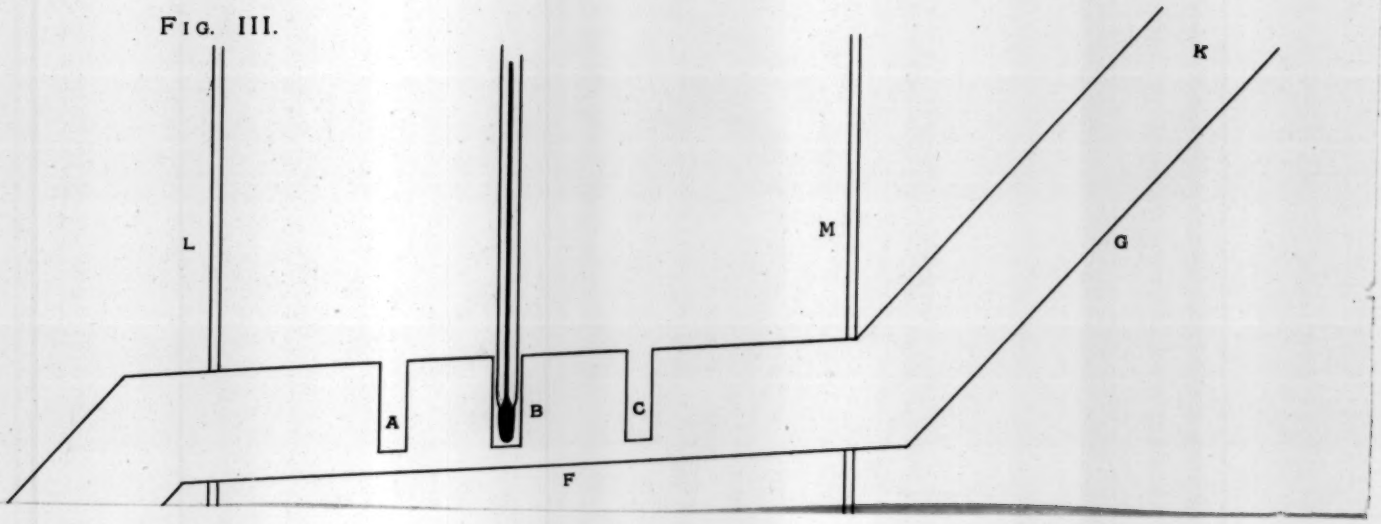
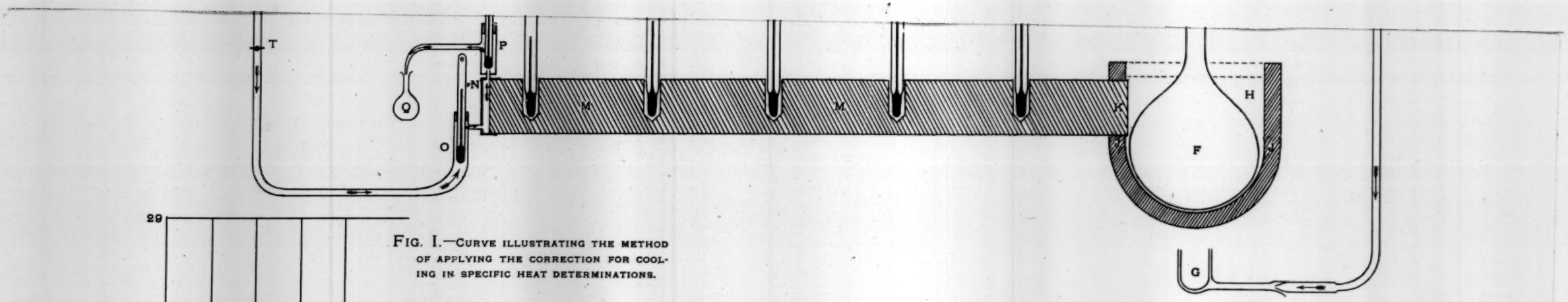


Fig. I.—Curve illustrating the method of applying the correction for cooling in specific heat determinations.

Fig. III.





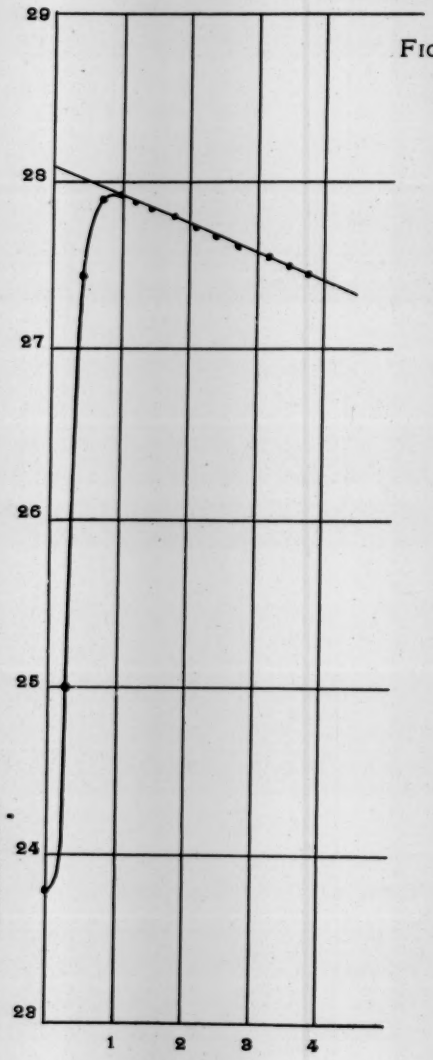
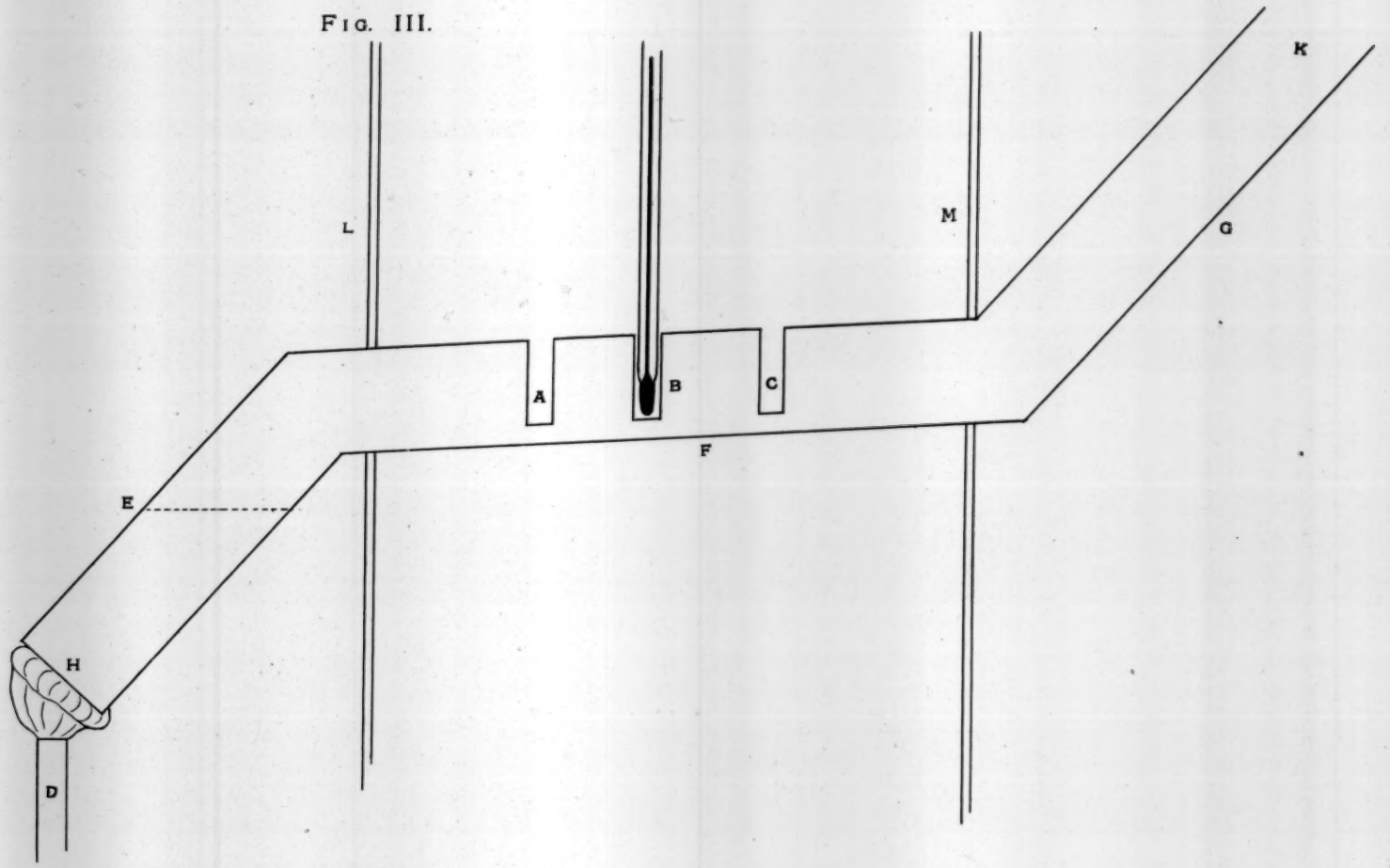


FIG. I.—CURVE ILLUSTRATING THE METHOD OF APPLYING THE CORRECTION FOR COOLING IN SPECIFIC HEAT DETERMINATIONS.

INITIAL TEMPERATURE OF WATER 23·82°C.
CORRECTED TEMPERATURE OF MIXTURE 28·08°C.

FIG. III.



XIII.—*The Old Red Sandstone of the Orkneys.* By John S. Flett, M.B., B.Sc. (With a Map.)

(Read 17th January 1898.)

Literature.

The first geologist to examine critically the Old Red Sandstone of the Orkneys seems to have been Professor Jameson, who in 1800 spent six weeks in a mineralogical tour through the county, and so barren did he find the islands, from his point of view, that he counted his journey one of the most uninteresting he had ever made. As yet, the rich store of organic remains which the dark grey flagstones contained had not been brought to light, but the stimulus given to this branch of investigation by the work of Hugh Miller and Agassiz awakened interest in the subject, and we find that a number of collections was formed, especially from the quarries in the neighbourhood of the town of Stromness. Hence when, at a later period (1848), Hugh Miller paid a visit to this district, as narrated in his *Footprints of the Creator, or the Asterolepis of Stromness*, many of the fossils of these rocks were already well known to local collectors, among whom he mentions particularly the late Mr W. Watt of Skaill and Dr Garson of Stromness. Professor Traill of Edinburgh University had for many years been forming a collection, and specimens had been forwarded by him to Agassiz, who makes mention of the fact. Hugh Miller, in the work above cited, and in his *Cruise of the Betsy* (1858), gives a description of his visit to Kirkwall, Stromness, and various parts of the West Mainland, which contains many interesting facts relating to the occurrence and distribution of the fossils in these districts. Further reference to his work will be found in a subsequent part of this paper. The general similarity of the rocks around Stromness to the sandstones of Cromarty and the flagstones of Caithness, as regards the fossils they contained, may be regarded as well established at this date, and the subsequent descriptions of Orcadian specimens contained in Professor M'Coy's *Synopsis of Classification of British Palæozoic Rocks* (1858) served in some measure to confirm this opinion. So far, there had been no attempt to ascertain the structure of the county, but in 1858 Sir R. Murchison* made a brief survey of the islands. He ascertained that there were at least two main types of sedimentary deposits in the Old Red Sandstone of Orkney,—a lower series of flagstones and, overlying them, conformably, as he believed, a series of yellow sandstones, well seen in the island of Hoy. The lower series at Stromness rested, by means of a basement conglomerate, upon an axis of crystalline rock. A great advance was made in 1878 by the appearance of the first part of Sir Archibald Geikie's mono-

* Sir R. Murchison, *Quart. Jour. Geol. Soc.*, vol. xv.

graph on the Old Red Sandstone of Western Europe.* As the result of two visits to Orkney, in which he was accompanied by Mr B. N. Peach, he pointed out that the yellow sandstones of Hoy did not pass down conformably into the flagstones which form the basis of that island, but were separated from them by a marked unconformity. At the base of the upper sandstones lay a series of contemporaneous lavas and ash beds, which were in all probability erupted from certain 'necks' in the low-lying district at the foot of the Hoy Hills. These rocks he regarded as belonging to the upper Old Red Sandstone. The lower Old Red Sandstone consisted principally of a great thickness of flagstones, with which were interstratified beds of yellow and red sandstone, and occasionally of conglomerate. The fossils belonged exclusively to this lower series; and a table is given, compiled by Mr C. W. Peach, showing the distribution of fossil fishes in the lower Old Red Sandstone of Lake Orcadie, including those of Orkney so far as known at that time. As Sir Archibald Geikie anticipated, subsequent revision has necessitated "considerable pruning of the fossil lists." The conglomerates around the granite axis of Stromness formed merely a local base, "due to the uprise of an old ridge of rock from the surface of the sheet of water in which these strata were accumulated," and were presumably not on the same horizon as the thick conglomerates on which, in Caithness, the lowest flagstones rest. The sandstones interbedded with the flagstones in South Ronaldshay were regarded as in all probability the northward continuation of the similar rocks at Gill's Bay, Huna, and John o' Groats, on the south side of the Pentland Firth. From a geological point of view, the brief notice of the Old Red Sandstone of the Orkneys contained in this paper forms by far the most important contribution to the knowledge of the subject published up to that time.

In two papers on the Geognosy of Orkney,† published in December 1879, Professor Foster Heddle showed the existence of a well-marked syncline beginning in the North Isles in the island of Eday, and continuing thence through Shapinshay and Inganess Bay to Scapa and the north-west corner of South Ronaldshay. The beds which occupy the centre of this trough are coarse arenaceous freestones, which rest perfectly conformably on the ordinary blue flags of the islands, and at Heclabir, in Sanday, contain thin beds of conglomerate. These sandstones cannot, in consequence, be the same as the yellow sandstones of Hoy, which unconformably overlie the flags. In these papers many interesting details are given of the minerals occurring in the islands, and of the structural peculiarities of the flagstones, especially as seen in the magnificent coast sections.

In 1880 Messrs Peach and Horne made a much more detailed examination of the islands than had previously been attempted, and the result was an important paper on the Old Red Sandstone of Orkney.‡ They showed that in all probability the upper Old Red Sandstone of the district was confined to the island of Hoy, while the rest of the

* Sir Arch. Geikie, "The Old Red Sandstone of Western Europe," pt. i., *Trans. Roy. Soc. Edin.*, vol. xxviii. pp. 409 and 410.

† *Mineralogical Magazine*, "The Geognosy and Mineralogy of Scotland," part v.—Orkney, M. Foster Heddle, M.D., 1880, p. 102.

‡ *Proc. Roy. Phys. Soc. Edin.* 1880

county consisted of the flagstones and sandstones of the lower series. The distribution of these two members was described, and sections given to show their relation to one another. In their paper on the Glaciation of the Orkney Islands* a map was published, which reappears in the chapter contributed by them to Tudor's *The Orkneys and Shetland*,† and leaves little to be desired so far as regards a knowledge of the distribution of the different lithological types which constitute the Old Red Sandstone of the Orkneys. The structure of the county, they regarded, with Professor HEDDLE, as, in the main, a syncline which runs from Eday to South Ronaldshay, broken in the Mainland by two great faults which cross it and follow the shores of Scapa Bay. In the centre of this syncline lie the sandstones which form the uppermost member of the lower series, while the flagstones form the rest of the district, with the exception of the area occupied by the upper Old Red Sandstone in the island of Hoy. They showed also that in Shapinshay, among the yellow sandstones of the lower Old Red, occurred a belt of contemporaneous volcanic rocks, consisting of a single outflow of a diabasic lava.‡

I.—The Palæontological Subdivisions of the Orcadian Old Red Sandstone.

The Eday Sandstones.—So far, those geologists who had endeavoured to make out the structure and succession of the Orcadian Old Red Sandstone had relied mostly on the different types of sedimentary rocks to establish their conclusions, without reference to the fossils the rocks contained. But in 1896, in a paper read to the Royal Physical Society of Edinburgh,§ the present writer showed that among the yellow sandstones of the lower Old Red Sandstone of Deerness, Orkney, occurred three fossils not previously recorded from Orkney, and known only to occur in the John o' Groats sandstones of Caithness, viz., *Dipterus macropterus* (Traq.), *Tristichopterus alatus* (Egert.), and *Microbrachius Dicki* (Traquair). In this way the opinion, already expressed by previous authors,‖ that the sandstones which conformably overlie the flagstones in Orkney were the northern representatives of the similar beds at John o' Groats, Caithness, was confirmed by palæontological evidence. During the following summer investigation was made whether the sandstones in other districts of Orkney, to which had been assigned the same position, contained the same suite of fossils, with the result that in several of the localities examined (in Shapinshay, Inganess Bay, and Eday) one or other of them was proved to occur, and it was established that they constituted the type fossils of a palæontological zone of the Orcadian Old Red Sandstone, which was at the same time distinguished by the lithological characters of its rocks. This may, in consequence, be designated the zone of *Tristichopterus alatus* (Egert.), or, from the locality in Orkney in which they have been principally studied, *the Eday sandstones.*

* *Quart. Jour. Geol. Soc. Lond.*, vol. 36.
† London, 1883.
‡ The occurrence of this basalt was noted by JAMESON, *Mineralogy of the Scottish Isles*, ii. 235.
§ *Proc. Roy. Phys. Soc. Edin.*, vol. xiii.
‖ PEACH and HORNE, *op. cit.* Sir A. GEIKIE *Old Red Sandstone*, p. 409.

As will be shown in a subsequent part of this paper, they fall naturally into two subdivisions, a yellow series beneath and a red series above; and it is the thin layers of flag intercalated in the yellow sandstones which have furnished the fossils described. A no less striking characteristic of these beds is the occurrence in them of that zone of volcanic rocks of which the first mention was made by Professor JAMESON.*

The Rousay Beds.—The inquiry was next advanced into the beds which underlie this zone, and were known to consist of a series of flagstones, presumably of great thickness, and of wide distribution throughout the county. All efforts to break up this series into recognisable subdivisions by means of belts of rock, with sufficiently well-developed peculiarities to ensure their recognition in different districts, had hitherto failed;† and, from an extensive knowledge of these rocks, the present writer felt that success was hardly to be hoped for in such an attempt. But should the distribution of their fossils show that certain forms occurred only on particular horizons, this great series could be broken up into zones, which could be identified wherever they occurred, if only they contained a sufficient number of organic remains in a satisfactory state of preservation. The base of the Eday sandstones was chosen as forming a well defined horizon, from which it would be possible to work downwards into the flagstone series in search of type fossils. These underlying beds were then followed from Eday, Westray, and Sanday in the north to South Ronaldshay in the south; the geological structure being carefully mapped, and a record of the fossils observed in each district compiled at the same time. The flagstones of these districts proved to be barren and unfossiliferous compared with the well known localities, chiefly in the West Mainland of Orkney, from which for many years fossils had been obtained in great numbers. Yet in every district decipherable fragments were to be found; and in some localities the fossils were quite as satisfactory as in the better known beds of the West Mainland. By far the most common were the sculptured bones and scales of *Glyptolepis paucidens* (Agassiz), which occurred in every district examined, often in great profusion, and with them *Dipterus valencienesii* (Sedgwick and Murchison), in every locality, and almost equally abundant. In fact, both these fossils occur right up to the base of the Eday sandstones, though as yet in Orkney not known with certainty to pass up into these overlying rocks. In Deerness, Holm, and Eday the beds immediately below the sandstones are crowded with *Dipterus valencienesii* (Sedgwick and Murchison), often in fine preservation, and covering the surface of whole slabs of rock. After these in frequency comes *Homosteus Milleri* (Traquair), of which the large and usually broken plates are often to be seen. Other fossils were relatively few. In Crook Bay, Shapinshay, I found a Cheiracanthus, which when submitted to Dr TRAQUAIR was determined to be *Cheiracanthus Murchisoni* (Agassiz). At Dingieshowie, Deerness, at Kirkwall, and elsewhere, *Osteolepis macrolepedotus* (Ag.) is found. *Diplopterus Agassizi* (Traill) occurs in the East Mainland, *Estheria membranacea* at Kirkwall, Rendall, and Westray. *Coccosteus decipiens* (Ag.) at Kirkwall, Dingieshowie, and even in the sandstones at Deerness, as I learned from Mr

* *Op. cit.*

† ARCHIBALD GEIKIE, *op. cit.*, p. 410.

MAGNUS SPENCE of Deerness, who forwarded a specimen he found in Newark Bay to Dr TRAQUAIR. To these we must add a new and undescribed species of Asterolepis, of which scattered plates were found by Mr SPENCE of Deerness and myself in Deerness, Holm, and South Ronaldshay. These have been presented by us to the Edinburgh Museum of Science and Art, and Dr TRAQUAIR has kindly consented to draw up and publish a description of them. This interesting fossil is, so far as we know at present, confined to a narrow belt of the flagstones immediately underlying the Eday sandstones, where it occurs with *Dipterus valencienesii* (Sedgwick and Murchison), and *Glyptolepis paucidens* (Ag.); and should further investigations confirm this restricted distribution, it may eventually be taken to mark the existence of a palæontological sub-zone immediately beneath that of *Tristichopterus alatus* (Egert.). That already it should be known from three localities widely separated, and in each case from precisely the same horizon, shows that it can hardly be called a rare fossil in Orkney, and in the future further specimens may be confidently expected to turn up should these beds be submitted to careful and extended investigation. With this exception, this list of fossils contains none which is not of very general distribution throughout the whole thickness of the Orcadian flags.

But when, in the progress of the mapping, a layer of rocks occupying a somewhat lower position was reached, fossils were obtained which were new to Orkney, or among the very rarest of those recorded from it. In the island of Rousay I found along the west side a belt of rocks containing *Coccosteus minor* (Miller), the best specimens being obtained in a quarry of thin slaty flagstones near Sacquoy Head. With it occurred the large enamelled scales of a ganoid fish, of which the fragmentary remains were not sufficient for satisfactory determination. Application was made to the proprietor of the island, General Burroughs, for liberty to quarry, and permission was at once granted. Better material was thus procured, and all doubt removed by the discovery of well preserved remains of *Thursius pholidotus* (Traquair), an addition to the list of the fossil fishes of Orkney. Both occurred on the same bed of rock, and are here recorded from Orkney, one for the first time, the other after a lapse of almost forty years, during which the knowledge of its occurrence seems practically to have disappeared. Curiously enough, when, at a subsequent time, at my request, Dr TRAQUAIR examined for me certain plates of *Coccosteus minor* (Miller) preserved in the British Museum,* which, I presumed, had come from another locality mentioned by HUGH MILLER, he informed me that these specimens, which belonged to the Egerton Collection, were derived from the same locality, but when or by whom they were collected is not known. A very careful search, a year or more previously, among all the local collections of fossil fishes, had failed to bring under my notice any remains of this fish, and none seem to have passed through Dr TRAQUAIR'S hands, as he comments on its apparent absence from the north side of the Pentland Firth.†

* A. SMITH WOODWARD, B.M. Cat.—Fossil Fishes, pt. ii. p. 291.

† "Achanarras Revisited," *Proc. Roy. Phys. Soc. Edin.*, xii. 285.

Hugh Miller, in his *Cruise of the Betsy* (1858), p. 358, narrates how, during his stay in Kirkwall, he paid a visit to a quarry a few hundred yards to the east of the town, where he observed numerous specimens of a species of Coccosteus, which he regarded as the same as those he had received from the neighbourhood of Thurso (collected by Robert Dick), and as certainly distinct from, and not merely young forms of, the common *Coccosteus decipiens* (Agassiz). For these he extemporises the name of *Coccosteus minor*. As no specimens of this fossil from Orkney were contained in his collection, and no further material had been obtained from this locality for many years, the accuracy of this observation remained open to some doubt, in spite of his careful identification. Unfortunately, these quarries are now practically worked out and deserted, but I can remember, years ago, seeing in the stones of some old houses in Kirkwall, which had evidently come from this quarry, great numbers of very minute specimens of a Coccosteus. With the rediscovery of this species, however, these doubts in great measure are removed; and as I shall subsequently show, the horizon of these rocks in the vicinity of Kirkwall is identical with that of the beds which in Rousay contain the same fossils. Hence, there is every presumption that this is another locality in Orkney for this species.

In the extreme south end of South Ronaldshay, I found at Banks Geo further examples of the same species, and as here they occur at no great distance from the Eday sandstone series of this island, it would seem that the horizon is a somewhat higher one than that in which it occurs in Rousay and in Kirkwall; but as the island is traversed by a number of faults, no very great reliance can be placed on any estimates of the thickness of the intervening rocks.

Here, then, we have from three localities—one in the north, one in the centre, and one in the south of the county, the extreme stations being over thirty miles apart—the occurrence of a distinct and characteristic fossil in the flagstones. With it occurs another *Thursius pholidotus* (Traquair), which is nowhere known except accompanying it. From the many quarries in the West Mainland, from which for seventy years innumerable specimens have been obtained, not one case is known in which these have been found, and it may safely be presumed that there they do not occur. Their absence, at any rate, cannot be accounted for by imperfect preservation or insufficient search. They may be assumed, in consequence, to constitute the type fossils of a zone of the Orcadian Old Red Sandstone beneath that already defined for the Eday sandstones, and the beds in which they occur I shall designate, from the locality in which the fossils are best preserved, *the Rousay beds.*

List of the fossils contained in the Rousay beds of Orkney :—

Thursius pholidotus (Traq.), Rousay.
Coccosteus minor (Miller), Rousay, Kirkwall, S. Ronaldshay.
Glyptolepis paucidens (Ag.), Kirkwall, Rousay, Eday, Tankerness, Westray, Sanday, Evie, etc.
Dipterus valencienesii (S. and M.), Kirkwall, Tankerness, Rousay, Eday, Evie, Firth, Westray, Sanday, etc.

Homosteus Milleri (Traq.), Kirkwall, Firth, Rousay, Westray, Sanday, Tankerness.
Cheiracanthus Murchisoni (Ag.), Shapinshay.
Coccosteus decipiens (Ag.), Deerness, Tankerness, Kirkwall, S. Ronaldshay.
Osteolepis macrolepidotus (Ag.), Kirkwall, Deerness.
Diplopterus Agassizi (Traill), Toab.
Estheria membranacea, Kirkwall, Rendall, Westray.
Asterolepis, sp. nov., Holm, Deerness, S. Ronaldshay.

The Stromness Beds.—A careful examination of the list above given will show that not only does it include certain fossils new or rare to Orkney, but that certain others well known to occur there are wanting. It may be said that practically all the fossils in the museums of the world or in private collections which have been furnished by the Orkney flagstones come from a restricted district in the West Mainland, and in the vicinity of the town of Stromness. Here the richness in fossil remains, and their fine preservation, is in striking contrast to the Rousay beds which occupy the remainder of the county. And not only are the fossils more numerous, but species occur which have never been obtained from other districts. Of these, there are two species of Pterychthys—*P. Milleri* (Ag.) and *P. productus* (Ag.)—*Cheirolepis Trailli* (Ag.), *Diplacanthus striatus* (Ag.), and *Gyroptychius angustus* (M'Coy). These, then, in turn constitute the type fossils of still another zone of the Old Red of Orkney, which from the locality of their typical development we will call *the Stromness beds.* With them others occur which are present also in the Rousay beds, viz.—

Coccosteus decipiens (Ag.).
Homosteus Milleri (Traquair).
Dipterus valenciencsii (Sedgw. and Murch.).
Osteolepis macrolepidotus (Ag.).
Diplopterus Agassizi (Traill).
Cheiracanthus Murchisoni (Ag.).

No value attaches to these latter as zone fossils, while there can be no doubt that the former, or some of them at any rate, are entitled to this rank. Much remains to be done before the knowledge of the distribution of the various fossil fishes in the Orcadian Old Red Sandstone can be said to be complete, but, from the Stromness beds at any rate, we have the result of seventy years of the activity of collectors, and the main facts must be regarded as already sufficiently established. That in no case have the type fossils of the Rousay beds been obtained in this locality is perfectly certain, and is a striking fact when we remember that the present writer has obtained these species from two localities in other parts of the county (South Ronaldshay and Rousay) in the course of a short space of time; while in no place have the type fossils of the Stromness beds been obtained along with those of the Rousay beds, or, for that matter, in any locality in which, according to the geological structure of the county, these latter are present; and further, as will be subsequently shown, these results, obtained from a study of the distribution of the fossil fishes of Orkney alone, are in substantial

accordance with the facts already known regarding their distribution in the other districts in which they occur. A mutually exclusive occurrence of this nature can only be regarded as due to the disappearance of one series of forms before the arrival or evolution of the other, and clearly establishes that the successive stages of the deposition of the Old Red Sandstone of the Orkneys were accompanied by changes in the fauna which inhabited the waters in which the rocks were being formed.

II.—The Structure of the Orkneys.

I. *Stromness Beds.*

To the geologist who endeavours to unravel the structure of the Orkneys, a magnificent opportunity is afforded by the excellent and numerous coast sections. So completely is the country cut up by sounds and bays, that at no place can there be any doubt as to the general structure; and even in the larger areas of land, as in the West Mainland, wherever cultivation is to be found, dwelling-houses and stone dykes have been built, and one is, as a rule, at no difficulty in finding stone quarries within a comparatively short distance of one another. If we add to these the many opportunities provided by the inland lochs and streams for an examination of the underlying rocks, it will readily be understood how it is possible, in a comparatively short time, to map with satisfactory detail very considerable areas of country. Only in a very few places do superficial accumulations of boulder clay or peat moss conceal the relations of the rocks beneath, through any extensive tract of land. Wherever the flagstones are present, the structure may almost be said to be writ large on the face of the country. As has been frequently observed by writers on the scenery and geology of Orkney, the hills have then markedly terraced contours, the harder beds of flag resisting erosion and forming a terrace, while the softer beds between, by their more rapid decay, form miniature escarpments. These terraces are everywhere present in flagstone districts of Orkney, and to the experienced eye at once reveal the secret of the underlying structure. In some places, as in Rousay and in Westray, they form so noticeable a feature of the landscape, as to remind one at once of the terraced volcanic districts of many parts, both of Eastern and of Western Scotland. That they are preglacial in origin is proved by the glacial striations with which they are often covered,* and no doubt they have suffered during that epoch a considerable amount of rounding and obliteration; their fine development on the west side of Rousay and of Westray is thus a relic of the old preglacial Orcadian landscapes, which owes its preservation to the fact that the ice movement being from east to west, the west side of these hills was spared the intense erosion to which the rest of the country was being subjected.

The Stromness beds of Orkney, although, as a matter of fact, probably the least extensively developed of any of the subdivisions of the lower Old Red Sandstone, have,

* Peach and Horne, *Proc. Roy. Phys. Soc., Edin.*, 1880, p. 3.

curiously enough, received hitherto by far the greatest share of attention. This is due, without doubt, to the number and excellent preservation of their fossils, of which Hugh Miller was led to make the somewhat hyperbolical statement, that were the trade once fairly opened, they could supply with ichthyolites, by the ton and by the shipload, all the museums of the world.* The list of collectors who have searched these beds is a long one, and includes many eminent names,—Hugh Miller, Professor Traill, Mr C. W. Peach, Mr W. Watt of Breckness, the Rev. J. H. Pollexfen, Dr Clouston, to mention only a few of those who, in a previous generation, were the first to develop their palæontological resources. The district to which they are confined is compact and of no great area, lying mostly in the West Mainland, in the parishes of Stromness, Sandwick, Birsay, and Harray. If to this we add the flagstones which unconformably underlie the sandstones of the west end of Hoy, and those also around the granite area in Graemsay, we include the entire district from which have been obtained the many Orkney fossils which are deposited in the museums of the world. The rest of Orkney is a district relatively barren and uninteresting to the collector, with the exception of certain areas of the Eday sandstones, such as Deerness—where, indeed, the abundance of the fossils hardly compensates for the paucity of specific forms.

The granite of Stromness.—Professor Jameson seems to have been the first to recognise the relation between the ancient crystalline rocks of the granite axis of Stromness and the flagstones of Old Red Age which rest on them by means of a thin basal conglomerate. As it has already been more than once described, a brief notice here will suffice. The area occupied is elliptical in shape, and stretches from the Ness of Stromness to the Point of Inganess on the west coast, a distance north-west of about five miles, with a breadth of about a mile. In the hand specimen it is mostly a pink, sometimes a grey granite, of medium grain, and with only a black mica. In many places it is markedly schistose, as at the Ness of Stromness and behind the town, sometimes passing even into a flaggy garnetiferous† mica schist. Numerous veins traverse it, fine-grained elvans and quartz porphyries, with stony matrix and large quartz phenocrysts, and very coarse pegmatites, usually without mica, and showing traces of graphic structure. The microscope shows the rock to be a pretty normal granitite, with orthoclase, plagioclase, and microcline (in small quantities), quartz, biotite, and, especially in the segregation veins, occasional micropegmatite. Sections cut from the gneiss show it to be of similar constitution, but the pressure twinning of the polysynthetic felspars and the strain shadows in the quartz show that in these bands the rock has been subjected to a deforming force.

The basal conglomerates.—Wherever the actual contact between the granite and the flags is exposed, it proves to be an unconformable junction, the rock immediately resting on the granite being always a conglomerate composed of fragments of the crystalline rock. Admirable sections are to be obtained at the Ness of Stromness and

* Hugh Miller, *Footprints of the Creator*, p. 2.
† Heddle, *Geognosy of Scotland*—'Orkney,' p. 135.

at the Point of Inganess. Both have been frequently described, and of the latter locality Professor HEDDLE has given a map. The granite conglomerate is also seen at the Point of Ness, and in the flag quarry at Garson Burn on the Kirkwall road. In no case is it of any considerable thickness, 30 feet being probably the greatest depth anywhere exposed. With it are mixed sandy flags and coarse arkoses, but it is not a little remarkable how soon it gives place to a normal fine-grained dark grey flag, exactly similar to those which cover such wide districts of the county. In fact, such flags are in many places interbedded with layers of a coarse conglomerate. At Yeskenaby, near Inganess, occurs a series of beds of a coarse sandy millstone grit, in which there is a well known quarry for millstones; and though its junction with the granite and conglomerate of Inganess is by means of a small fault, it is easy to see that it is really the rock just overlying the conglomerate let down by this fault against the granite. In fact, on the north-west corner of Inganess, similar beds occur in the cliff where they rest on the granite and granite conglomerate, which form the low shore below. This is in Orkney the only representative of the thick sandstones which elsewhere rest on the basal conglomerate, a fact which strongly supports Sir A. GEIKIE's opinion that the granite axis of Stromness is a mere local base. Yet the shores on which these fine flags were laid down must have been tranquil and tideless, as deposits so fine could not possibly rest on an exposed or tide-swept shore. The innumerable sun-cracked and ripple-marked surfaces everywhere present in the Orkney flags show that they are the accumulations of a shallow sea, yet they can hardly be regarded as littoral deposits; they were rather the finer sediment of landlocked areas of fresh water, in which the coarser material rapidly sank to the bottom, and was deposited immediately around the river mouths.

The Stromness flags.—The flagstones of the Stromness series encircle this granite and conglomerate, and are beautifully exposed in the magnificent sections of the west coast of the Mainland of Orkney, from the Ness of Stromness to the Brough of Birsay. This most interesting coast has been described by almost every writer on the geology of Orkney. Many of the well known localities for Orcadian fossils occur along this shore (*e.g.*, Rocket House, Breckness, Belyacroo, Ramnageo, Quoyloo). Starting from Stromness we find the rocks have a westerly dip along the shore to Breckness, W.S.W., then along the Black Craig, W.S.W., at Yeskenaby, W.N.W., at Skaill, W. and N.W., and, north of Skaill Bay along Outshore Point to Marwick Head and the Brough of Birsay, about N.W. for almost the whole way. The dips roll somewhat, being S.W., W., and N.W., as is best seen between Inganess and Skaill Bay, but everywhere there is a persistent westerly component. Here we are, in fact, on the west side of a great anticline, which forms the chief structural feature of the West Mainland of Orkney. For about four miles back from the cliff, in all the quarries and burns of Stromness, Sandwick, and Birsay, there is the same universal westward dip. The anticlinal axis runs approximately from Waulkmill Bay in the south to Crustan Point, a mile west of the Brough of Birsay, for to the east of this line, in Firth, Harray, and Evie, easterly dips are consistently

present. The long axis of the Loch of Harray corresponds very closely with the crest of the anticline, as on the different sides of the loch the dips are opposite, and at Brodgar Bridge, at Ness in Harray, and at Dounby we have the flat or gently rolling beds which occupy the summit of the arch. A transverse section of the anticline is exposed on the north coast of the Mainland, from Marwick Head in Birsay to Costa Head in Evie. At Marwick Head the dip is N.W. about 10°, and this continues, with occasional variation and a few small faults, seen in the Bay of Birsay, to Skip Geo, just east of the Brough. Thereafter, along the coast by Crustan to the mouth of Swannay Burn, the rocks lie very flat, with gentle and frequently changing dips, in which, on the whole, those to the east and north-east preponderate. In Costa Head the east dip is persistent, and, gentle at first, constantly increases along the shore line to Burgar, and thence to Aikerness Point in Evie. In this entire and perfect section no disturbance of the flags is anywhere seen sufficient to indicate the existence of a fault of any importance.

If we traverse the Mainland along an east and west line through its centre, the result is the same. Starting at Skaill Bay, we find that the rocks are rolling, but the dips are always westward. Between this and Dounby the low lands are in many places covered with boulder clay, but in all the quarries the dips are west till we arrive within a few yards of the village, where it rolls to north-east. More exposures can be examined by following a line past the Loch of Clumly to the Bridge of Brodgar, which separates the Lochs of Harray and Stenness, as along this line there is an abundance of stone quarries, and the loch shore yields valuable natural sections. At Aith, W., at Sandwick Manse, W. 10° N., at Clumly Loch, W., at Lyking, W. 10° N., finally at Bookan, in one of the most prolific in fossils of all the quarries in Orkney, we have an unbroken chain of west dips, which ends only in the isthmus on which are placed the Standing Stones of Stenness. Along the shore of the Harray Loch, from Voy to Brodgar, the section is very complete, and not quite so simple as the inland exposures would have led us to expect. The rocks which form the Ness of Tenston have indeed a prevalent west dip, but sometimes roll to the east, while reefs of vertical beds run out into the loch in a direction N. 10° W., and everywhere there is much contortion and slickensiding, the organic matter of the dark flags having been deposited as a brightly polished layer on the bedding planes. These are the symptoms which everywhere in Orkney indicate the presence of a considerable fault; and as these broken rocks of Tenston Ness occupy a belt of the breadth of about half a mile, the dislocation can hardly be supposed to be a trivial one. Traced southwards, the same phenomena are to be seen in the rocks around the Bridge of Waithe. From Garson farm, near Stromness, by Bu Point, to the Bridge of Waithe, the rocks are folded into many sharp little anticlines and synclines, with mostly a north and south strike. At the bridge and down the Ireland shore by Cumaness, reefs of vertical slickensided and crushed rock are seen in several places running N. 10° W., and from here along the shore to Houton we have again a continual and rapidly changing succession of little folds (as was remarked by Messrs

PEACH and HORNE *). Along the shores of the Stenness Loch from Onston to below Deepdale, the same phenomena are repeated. Yet in this district the amount of actual crushing and fracture is much less than on Tenston Ness, and there can be no doubt that the throw of the fault is rapidly diminishing as it passes south. Similarly, to the north, on the shore of the Harray Loch at Kirkness, these appearances are repeated, and no doubt the fault runs northward to the west of Dounby village, though here not easily traceable, owing to the thick boulder clay sheet which covers these low grounds. Here, too, it is dying out, and no trace of it is to be found on the north shore of the Mainland.

Continuing our traverse across this fault, we find that the persistent west dips practically cease at the Standing Stones, where, for a time, the beds are gently rolling, and they are last seen in the quarries to the north-west of Maeshowe. In all Harray the dips are gently eastwards, except on the shore of the loch at the Point of Ness, and these east dips continue through the whole of the range of hills which, starting at Finstown, runs northwards to Costa Hill in Evie, and separates the parishes of Birsay and Harray from Evie, Rendall, and Firth. Similarly, in Greenay Hill, Birsay, in Hunland, and in the hills to the east of the village of Dounby, the easterly dips prevail. It is only in the extreme east of the Mainland, in Woodwick, Evie, in Rendall, and in several places along the shores of Firth Bay, that this direction is reversed, the rocks of this district having in many places a very gentle inclination to the west, and forming thus a little marked syncline.

Such being in its main features the structure of the West Mainland of Orkney, we would naturally expect to find the Sandwick and Stromness beds repeated on the eastern limb of the anticline in Harray and Stenness. This, however, is not the case, as the richly fossiliferous beds on the west side of the Stenness lochs do not reappear on the east, where the rocks in many points resemble the Rousay beds of the North Isles and the East Mainland. They are comparatively poor in fossil remains, and have never yielded, to my knowledge, the type fossils of the Stromness zone. This is, there can be little doubt, the effect of the north and south fault, which has let down these comparatively barren beds against the Stromness series which encircles the granite axis. It is only in the northern part of this area, at Dounby, Greenay Hill, and other localities in Birsay, that the fossils of the Stromness beds are to be found in quarries with an easterly dip, and here the evidence points to the theory that the fault is rapidly dying out, as it passes northwards to the west of Dounby. The Firth and Harray beds may be, in consequence, relegated to the passage beds between the Stromness and the Rousay series of the Old Red of the Orkneys, and seem to be on the same horizon as those which occupy the wide area which stretches from Stenness, through Orphir, into Kirkwall. As we shall see later, when we continue the section through Rousay and Egilshay into the Eday sandstones, we have a constantly ascending succession; and as nowhere do the Stromness fossils recur, the inference is obvious—as might have been anticipated from

* PEACH and HORNE, *Old Red Sandstone of Orkney*, p. 10

the fact that at Stromness they rest upon the granite axis—that *the Stromness beds form the lowest zone* of the Old Red Sandstone of the Orkneys.

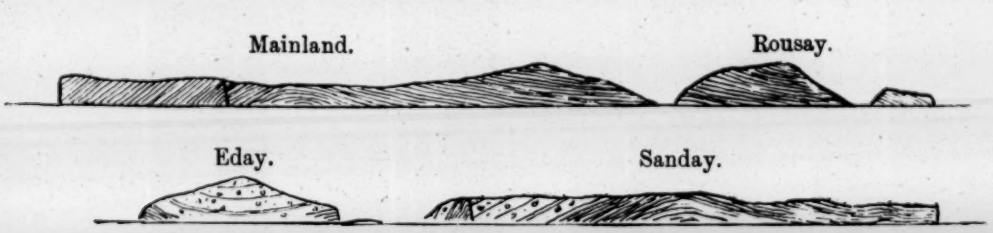
It is a matter of great difficulty to form a reliable estimate of the thickness of this series in Orkney, as will be evident when we consider that its true base is nowhere seen, and that its upper boundary must, in our ignorance of any but the general facts regarding the distribution of fossils throughout the county, be of necessity an arbitrary one. By far the best continuous section of these beds is that exposed along the shore from the Ness of Stromness to Breckness, nearly three miles to the westward. The section runs in a W. or W.N.W. direction, and during its whole course there is a continuous exposure of the rocks at low water. They dip along the shore about W. 10° S., and during the first half of the distance the average amount of dip is 15°. In the little sandy bay beyond the churchyard the dip swings southwards, and is more gentle for a little, but on the west side resumes its previous direction and amount. If we draw a line perpendicular to the strike and measure the distance, it is almost exactly two miles, and the thickness, allowing for an average dip of 12°, is about 2000 feet, which is exactly the thickness estimated by Sir A. Geikie for a section parallel to this and a mile further south, from the centre of Graemsay to the base of the Hoy Hills.* As a matter of fact, as the flagstones at Ness rest on the granite conglomerate, and the rocks at Breckness, if prolonged northwards along their strike, are seen to be on a level not greatly differing from those which at Inganess rest on the west end of the same granite axis, we are led to the conclusion that the western conglomerates must be on a much higher level than those at the east end of the granite outcrop. But the lowest rocks in this district must be those which have been uplifted by the Tenston fault along the axis of the West Mainland anticline. This fault is prolonged southwards through the Bay of Ireland; and if we carry the section backwards from Stromness to Bu Point, we find that along this shore the rocks are so rolling that no great thickness is required to be added to our estimate, the same beds being probably again and again repeated by means of gentle folds.

Results in substantial accordance with this are obtained by taking a section some six miles to the northward, from the fault on Tenston Ness on the Loch of Harray, to Skaill Bay on the west shore of the Mainland. The length of a section from Tenston due west to the Atlantic is nearly four miles, and in the intervening country the dips never vary greatly from a true W. In amount they differ, being 12° or more at Lyking, at Voy nearly flat, at Sandwick manse 5°, at Rango 5°, at Skaill 3 to 7°. If we accept 5° as an average, the thickness is 1760 feet. In this case the conditions are not so satisfactory as in the preceding, the exposure of rock not being a continuous one.

To this must now be added the rocks which lie between those of Breckness and Skaill and the base of the Rousay series. That at both these places we are well within the Stromness zone is evident from the fact that they are among the best known localities for its type fossils. The district in the N.E. corner of the West Mainland

* Sir A. Geikie, *Old Red Sandstone*, pt. i. p. 410.

(Birsay and Evie) will, in my opinion, be found the most suitable for this purpose. If we take a section from Crustan Point in Birsay, the centre of the West Mainland anticline, to Burgar in Evie, where we cannot be far from the level of those beds which in the west of Rousay contain *Thursius pholidotus* (Traq.) and *Coccosteus minor* (Miller), and strike southwards across the narrow Eynhallow Sound, the total distance is five miles, measured across the strike of the beds. The dips throughout are eastwards, and their average amount is about 3°. There is no evidence of any important fault. The thickness must in consequence be about 1300 feet. The exact position of the Crustan beds in the Stromness series is difficult to fix, but, as along the western shore from Skaill Bay by Outshore Point to the Brough of Birsay, the dips are mostly N.W., as we travel northwards the section is a constantly ascending one, and the beds which occupy the centre of the anticline at the northern shore must be far higher in the series than those which occupy a similar position in the neighbourhood of the Harray Loch. The Crustan beds in consequence are, in all probability, on a similar level to those in the vicinity of Skaill Bay; and if we add the lower half of the thickness between Crustan and Burgar to that from Tenston to Skaill, we obtain a total thickness of about 2500 feet for the Stromness beds of Orkney. The beds of Evie may, on the other hand, be relegated to the basal part of the Rousay series, and as yet there is no palæontological evidence to prevent such a step. These passage beds, in fact, between the Stromness and Birsay series below, and the Rousay beds above, are comparatively unfossiliferous, and have yielded little of value to the most careful search.



SECT. 1.—From Skaill Head (Sandwick) to Start Point (Sanday).

II. *The Rousay Beds.*

The Rousay beds of Orkney lie mostly to the north and east of the county, where they cover a much more extensive area than the better known Stromness series. As yet, however, little attention has been paid to them and their fossil contents, and the scarcity and imperfect state of their fossils is indeed disappointing to one who has been accustomed to investigate the West Mainland beds. One may travel for days along the shores or among the quarries on this group of rocks without bringing home more than one or two imperfect specimens. Yet they are never entirely barren, and careful search is always rewarded with recognisable organic remains, usually scattered bones and scales, while in a few places we may find even entire fishes, as perfect in every detail as those which abound in certain of the quarries of Sandwick and Stromness. Very characteristic of these rocks are the scattered bones, the teeth, and sculptured scales of

Glyptolepis paucidens (Ag.), and with it *Homosteus Milleri* (Traq.) is the most abundant fossil,—if we except only the head plates and scattered fragments of *Dipterus valencienesii* (Sedgw. and Murch.). But the last is quite as common, and probably commoner, in the Orcadian beds, while the two former have certainly their principal seat in the beds now to be described. With these a not unfrequent fossil is the little crustacean *Estheria membranacea*, which, as at Thurso, sometimes covers the whole surface of slabs of rocks, and is, so far as I know, confined to this zone. Other fishes occur—*Cheiracanthus*, sp., *Osteolepis macrolepidotus* (Ag.), *Diplopterus Agassizi* (Traill), *Coccosteus decipiens* (Ag.); but their principal development seems to have been in a previous time, as they are much more numerous in the lower series. Of the fishes peculiar to this zone, *Coccosteus minor* (Miller) can hardly be said to be rare, seeing that already we know it in three separate and widely distant localities. It is a very suitable fossil for zonal work, as even its scattered bones are so characteristic as to establish its identity readily. Of the different species of *Thursius*, only one is as yet known to occur; and indeed, until a means is discovered for diagnosing these fishes from scattered head plates, bones, or scales, it is unlikely that they will ever be recognised as common fishes in this region of Orkney. The state of preservation requires, in their case, to be much more perfect than holds good as a rule of the fossils of these rocks.

The North Isles District.

If we now continue eastwards our section through Orkney from Evie, through Rousay and Egilshay (sect. 1), we find that in Eynhallow the east dips which prevail in Evie are repeated, and these beds strike evidently across the narrow Eynhallow Sound into the west side of Rousay. In the latter island the east dips which mark this side of the great West Mainland anticline may be said to prevail throughout, but are everywhere very gentle, and are occasionally subjected to a temporary reversal. The terraced faces of the hills, most marked on the west side, show at a glance the simple structure and the almost horizontal disposition of the beds. Along the western coast, the dips are gentle and frequently changing, being mostly north and north-east in the northern half and south and south-west near Westness, but from Hullion along the south coast to Avalshay the dips are very persistently east, except for a brief space below Trumland House, where a very insignificant anticline occurs. East dips are constant on the shore of Rousay Sound. On the north shore the magnificent range of cliffs from Sacquoy Head to the Knee of Rousay around the whole shore of Saviskail Bay exposes an ideal section, which shows a structure slightly more complicated than that seen on the south side of the island. On Sacquoy Head the dips are east, but on Saviskail Head a small anticline, on the south shore of Saviskail Bay another, and in Scockness a third, throw the rocks into gently undulating folds, whose axis is nearly north and south, without anywhere a dislocation of any importance. The island is thus a geological plateau, out of which the agents of denudation have carved the valleys and modelled the surface features. Its heather-clad

hills are the highest in the North Isles of Orkney, rising to heights of over 800 feet; and if we allow 1000 feet for the total thickness of rock exposed, we have an estimate which cannot be far from the truth. Few fossils are yet known from it: *Dipterus valencienesii* (Sedgw. and Murch.), *Homosteus Milleri* (Traq.), *Glyptolepis paucidens* (Ag.), with the characteristic fossils *Thursius pholidotus* (Traq.) and *Coccosteus minor* (Miller). These latter occur in what are about the lowest beds of the island, a belt of thin blue calcareous flags seen best at Sacquoy Head on the north-west corner, and striking southwards through the island, to outcrop again at the Taing of Tratland and the adjoining shore. At Sacquoy Head they overlie a bed of conglomeratic sandstone, with pebbles up to the size of a walnut, of gneiss and quartzite mostly, and resembling thus the rocks of Heclabir, to be subsequently described. In Egilshay the easterly dip continues, but here much steeper, with evident crushing and fracture of the rocks;* and I think it likely that through this island passes a line of dislocation, evidence of which is to be found in the Galt of Shapinshay to the south, and in the district of Rackwick in Westray to the north, in both of which places the appearances point to a similar disturbance. This would, in fact, be a north and south fault, skirting the Eday syncline, like that already described in the West Mainland anticline, and those also described in several places by Peach and Horne (Sanday, Berstane, Holm).

If the section be now continued across the Westray Firth to Eday, we find, as described by Peach and Horne,† a strip of flagstones, with a very steep easterly dip, ranging from Ferstness to Sealskerry, and bounding on the west the area of the Eday sandstones. These lie in the trough already described by these authors; and, as they showed, the only other flagstone area in the island is one which stretches from Warness to the Graand on the south shore, and thence N.N.E. to the Kirk of Skaill and the inner corner of Backaland Bay on the east side. As the centre of the syncline runs from Zoar in Sealskerry to Calf Sound in the north, these flagstones have a W.N.W. dip of about 15°, and they have been brought up by a small fault against the red sandstones which occupy the south-east corner of the island.

In Sanday the yellow and red sandstones occupy the south-east end, as shown by Prof. Heddle,‡ broken by a fault which, running north and south through Spurness Promontory, brings up again for a brief space the underlying dark grey flags.§ Beyond them, to the north and east, the whole island consists of flags which form a well-marked anticline, their westerly members dipping to the west like the Eday beds, under which they pass, but arching over on the south shore of Otterswick Bay and near Geramont House, so that at Taftsness, Newark, and the Start the prevalent dips are to the east. These Sanday flags yielded little of value to my search, *Glyptolepis paucidens* (Ag.), *Dipterus valencienesii* (Sedgw. and Murch.), with a few well preserved fragments of an Osteolepid fish being all I noticed. There can be no doubt that they are a repetition of

* Noted by Jameson, *Scottish Isles*, ii. 239.
† Peach and Horne, *Old Red Sandstone of Orkney*, pp. 8 and 9.
‡ Heddle, *Geognosy of Scotland*, part v. p. 101.
§ Peach and Horne, *Old Red Sandstone of Orkney*, p. 7.

the Rousay beds on the east side of the Eday syncline, though as yet they have not yielded the characteristic fossils. In the same group must also be placed the North Ronaldshay flags, which time did not permit me to visit and examine in detail. Professor HEDDLE tells us that here the east and west dips are about equally common.* The island of Stronsay, which lies to the south of Sanday, has on the whole a similar structure. It consists for the most part of flags, with one or two areas of John o' Groats sandstones in the south and south-east. The dips along Linga Sound and the north-west side generally are to the north-west, while on the south side, near Housebay, they roll over to the south-east (sect. 2). The structure is thus an anticline like that of the more northern island. I was not able to obtain any data as to the fossils they contain.

In Westray, as PEACH and HORNE† remarked, the structure again is an anticline, though a careful examination showed it was not a simple one (sect. 2). The axis runs from Garth in Tuquoy Bay, to the Sneuk on the north shore. To the west of this, the flags have a persistent though gentle dip to the westward, only reversed for a short space at Noup and Noup Head. To the east of this line the flags form a rolling series, as is well seen along the south shore, where two or three small anticlines and synclines succeed one



SECTION 2.—N.E. and S.W. from Noup Head (Westray) to Lamb Head (Stronsay). 3 miles = 1 inch.

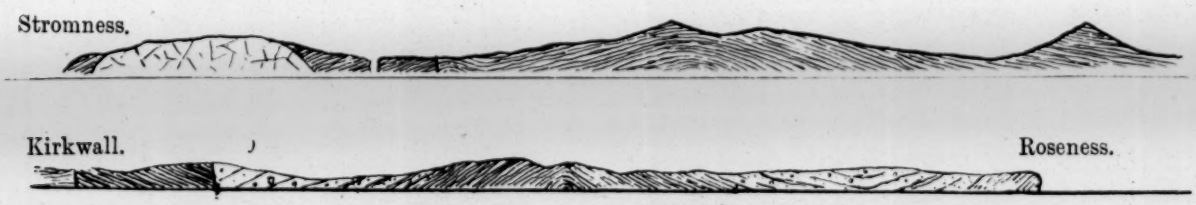
another. On the north shore, the dips are similarly rolling. From the Point of Tafts along the west shore of Rackwick runs a line of dislocation already mentioned as probably a continuation of that seen in Egilshay, and in Rapness the dips are mostly east, though in the extreme south end the flags on the western shore have a west dip. If we neglect the fault, the same strata are thus constantly repeated. There is no doubt they are the same as those of Rousay and of Sanday, the structure being only a continuation northward of that already seen in the northern shore of Rousay. The fossils I found there were *Glyptolepis paucidens* (Ag.), *Homosteus Milleri* (Traq.), *Dipterus valencienesii* (Sedgw. and Murch.), *Osteolepis*? *Estheria membranacea.*

In Shapinshay we have the two series of rocks—the Rousay beds in the north and west, and an area of Eday sandstones in the south and east. On the east side the beds have a strong south-east dip, but on the north-west corner, around the Galt and in Veantrow Bay, the dips roll greatly, and this is probably the effect of a series of faults which disturbs them: one seems to run from the Galt in the north to the Telegraph hut near Elswick on the south, while the fault which starts at Howquoy Head and runs under the town of Kirkwall must pass just to the west of the shore of the island. As has been pointed out by PEACH and HORNE,‡ the area of sandstones on the south-east

* HEDDLE, *op. cit.*, p. 122. † PEACH and HORNE, *op. cit.*, p. 2. ‡ PEACH and HORNE, *op. cit.*, p. 9.

of the island is probably a continuation southward of the rocks which occupy the centre of the Eday syncline, and the eastward dipping flags of Shapinshay will then correspond to those of Egilshay, Ferstness, and Westray, as the west dipping flags in Stronsay correspond with those of Sanday.

As will be evident from this brief summary, the North Isles of Orkney are composed of two members of the Old Red Sandstone—the Rousay beds and the Eday sandstones. The chief structural feature is the Eday syncline. The Rousay beds of Rousay and Westray, dipping eastwards, pass under the sandstones, and emerge again with a westward dip in Sanday and Stronsay, only to roll over again before they finally disappear beneath the waters of the North Sea. The beds which in Rousay contain the type fossils are, in all probability, the lowest of the flags of this area; and although the sections are frequently interrupted by the sounds which separate the islands, and by not a few important faults, it is quite evident that the entire thickness of rock required to explain the geological facts is by no means great. We have already stated 1000 feet as the maximum required for the Rousay flags, and in no other island is so great a thickness exposed. If we add to these the upper half of our estimate for the east side of the Birsay and Evie series, we have a total thickness along this section of not more than 1500 feet. No more than an approximate estimate can possibly be formed in this district, as the sections are so broken up by water, and no recognisable subdivisions can be established, either lithologically or palæontologically, with which we might ascertain the throw of the respective faults.



SECTION 3.—From Stromness, through Kirkwall, to Roseness (Holm).

The West Mainland District.

Owing to the prevalent north and south strike, the rocks of Rousay may be expected to cross into Evie and Rendall, where they lie in very nearly horizontal but slightly rolling folds, and from here to pass southwards into the district between Finstown and Kirkwall. A similar conclusion is arrived at by the examination of the rocks which stretch eastward from the Bridge of Waithe in Stenness (sect. 3). Here we are among the rolling beds which mark the termination to the south of the fault which runs along the side of the western anticline. These beds are undoubtedly to be placed, with their more northern representatives in Harray, among the upper beds of the Stromness series. Further east in Stenness we find the effects of the western anticline, though here little marked, and evidently dying out. Through most of Stenness and throughout the Ward Hill of Orphir the dips are south-west. The anti-

clinal axis passes almost through Maeshowe down Summersdale into the Kirbuster district of Orphir. In the Heddle Hills of Firth, to the east of this line, the dips are mostly east and north-east, very gentle in the flag quarries, now disused, which crown the hills on both sides of Finstown. From the latter village to Kirkwall we have a rolling succession of gentle anticlines and synclines with axes striking north and south; seen well in the shores of Firth and Kirkwall Bays, where the same beds crop out again and again. There are no steep dips and no traces of any important dislocation, but from Summersdale to Kirkwall, on the whole, the dip is eastward, and we are ascending very gradually in the series. In the quarries to the west of Kirkwall there is a very slight north-west dip, while along the shore to the east of Kirkwall Bay the dips are strongly east. The change is marked by a line of crushed rock which runs under Kirkwall in a N.N.E. direction, and emerges on the shore at Cromwell's Fort. This seems to be the northward continuation of the fault described by PEACH and HORNE as running from Howquoy Head in Holm, northwards along the shore, and forming the eastern boundary of the sandstones of Scapa Flow.* This may be possibly a continuation of that already described as passing through Egilshay into Westray. At any rate it is an important feature in the structure of this part of the Mainland of Orkney, for to the west of it lie the gently rolling beds described, while to the east the dips are steep as a rule, and the rocks thrown into very pronounced folds. In other words, it forms a natural geological boundary to the East Mainland of Orkney.

The East Mainland District.

The second area in which it has been proved that the Rousay group of fossils occurs in Orkney is that around the town of Kirkwall, in which HUGH MILLER remarked their presence more than forty years ago. The structure of the East Mainland has not that simplicity which characterises the West Mainland. To the south-west it is bounded by the fault described by PEACH and HORNE, which brings down the sandstones of Scapa against the flags. The flags along this fault are probably the lowest rocks exposed, for through the whole area there is a constant tendency to a northerly dip, varied, of course, by the subsidiary folds, and the highest rocks occur only in the northern half of the district. Two series of rocks occur—the Eday sandstones in two areas, Berstane Bay and Deerness, the Rousay beds elsewhere.

The structure is clearly defined, an anticlinal axis occupied by the flags passing up the centre of the district in a north-east direction, and forming the Ness of Tankerness, while on each side a syncline brings in the overlying rocks, the sandstones (sect. 3). The section along the public road from Kirkwall to Dingieshowie, Deerness, affords a very good index to the general structure. For a mile or more after we leave Kirkwall, the rocks are steeply inclined to the east and north-east, disturbed, no doubt, by the great fault whose

* PEACH and HORNE, *op. cit.*, p. 11.

outcrop we are crossing, and through the promontory between Kirkwall and Inganess Bay the general dip is to the north-east. At the south-west corner of the latter bay the fault already described by PEACH and HORNE, forming the western boundary of this area of John o' Groats beds, is well seen in the shore, letting down the red sandstones sharply against the blue grey flags. These are the flags which in the old quarries at the East Hill, Kirkwall, rather over a mile away, contain the Thurso fossils, according to the observations of HUGH MILLER.* They form a triangular area between two considerable faults; and though in the land north-east dips prevail, as also along the east shore of Kirkwall Bay, along the northern coast from Carness to Meil Bay, a succession of folds repeats them.

Continuing our section eastwards, we find that the sandstones of Inganess dip north-west to the fault, and at their eastern edges are bounded by grey flags with a similar dip. About five miles from Kirkwall, at Quoyburray, in a quarry near the road, the beds lie nearly quite horizontal, and from that point onwards the dips are south-east and generally steep. The axis of the anticline runs approximately from Sebay Mill to the Ness of Tankerness in an E.N.E. direction, as along this north-west shore of Deersound the dips are slight and rolling; and while, to the east of this, at Yinistay Head and through Tankerness we have the north-west dips, in Deerness these have rolled over to the south-east. At Dingieshowie the yellow sandstones are let down by a fault, but maintain the general south-east dip; and from here, along the shore to the Castle, they lie in a little trough, the dips swinging first to east, then to north-east, when they are succeeded by grey flags, which up to Roseness Point have a north dip. Along the shore of Holm Sound the north and north-east dips show that here, too, we are on the south side of a syncline which runs approximately north-east and south-west, but as we pass westwards beyond Graemshall the rocks are much disturbed, and the dips are inconstant and frequently changing.

In spite, then, of their generally steep dips, the flagstones of the East Mainland are so repeated by these folds that they cannot be regarded as of very considerable thickness, and the disturbance to which they have been subjected renders any estimate exceedingly conjectural. Their fossils are few, yet I found in different places *Glyptolepis paucidens* (Ag.), *Dipterus valencienesii* (Sedgw. and Murch.), *Osteolepis macrolepidotus* (Ag.), *Coccosteus decipiens* (Ag.), and *Diplopterus Agassizi* (Traill).

It is interesting to observe how the section drawn east and west from the Bridge of Waithe to Roseness, through Kirkwall, repeats the main features of that drawn from Skaill Bay to the Start Point of Sanday (sects. 1 and 3). The Tenston fault passes south through Waithe, and the West Mainland anticline is distinctly to be traced in Summersdale, the rolling beds between Finstown and Kirkwall are those of Rendall and Firth, the Rousay beds recur at Kirkwall, and the broken dislocated flagstones to the east of Kirkwall repeat the structure of the west of Shapinshay and Egilshay. The Eday syncline passes south through Shapinshay to Inganess Bay. The anticline of Tanker-

* *Cruise of the Betsy*, p. 394.

ness is that of Sanday and Stronsay, while the sandstones of Deerness and Holm belong to a syncline unrepresented in the northern section, except it be by the limited areas of yellow and red sandstones in the island of Stronsay.

The South Isles District.

South of the Scapa faults not one of these features reappears, and the South Isles of Orkney form a distinct district, with a well-developed structure of its own. It consists of a geological basin, in the centre of which lie the higher beds, the sandstones.* They form the shores of Scapa Flow, from the Old Kirk of Orphir to near Howquoy Head. They reappear in Hunda, the west of Burray, and the north-west of S. Ronaldshay, here dipping west and north-west, and constitute also the north end of Flotta. Around them pass the underlying flags of Orphir, Holm, the east of Burray, the south-east of S. Ronaldshay, Swona, and the south of Flotta. In the north, the junction is a fault; and through South Ronaldshay and Burray it is evident that several faults run north-east and south-west parallel to the strike of the rocks. Yet in some places the succession is an interrupted one, as, for example, to the west of Grimness Head and in the island of Flotta. In Burray the flags dip west, in S. Ronaldshay north-west, in Flotta north, the strike thus sweeping gradually round. Much broken up as the district is by the sea, it is yet sufficiently clear what the general structure of the whole area must be. The Eday syncline is rapidly dying out in Inganess Bay, and I could find no proof that the yellow sandstones pass across the East Mainland near Kirkwall, to unite with those of Scapa Flow. Even should they ultimately prove to be continuous, it is clear that the broad basin of the South Isles cannot fairly be regarded as a continuation of the Eday syncline, which already at the south end of Inganess Bay has narrowed to less than a mile in breadth, and has, furthermore, to cross the powerful dislocation of the east side of Scapa Bay. In all the features of its structure, the South Isles area shows no point of comparison with that around Kirkwall, still less with that of the North Isles of Orkney.

The largest continuous area of these rocks is that of South Ronaldshay, which alone I had time to examine in detail. It consists of two series, the grey flags of the south-eastern district, and the yellow and red sandstones of the north-west. The general dip throughout is N. to N.W., but the structure is by no means simple, as it is evident from the coast sections that powerful dislocations cross the island from N.E. to S.W. On the west side the flags extend from Brough Head to Barswick, much disturbed in many places; and from thence to St Margaret's Hope, and for a mile further east along Water Sound, the shore consists of yellow and red sandstones (faulted apparently in two places at Barswick, where they are brought down against the flags, and at Sandwick). The Hoxa promontory consists of an anticline of blue flags, and is bounded by a fault which runs across the narrow isthmus. On the east shore, again (sect. 4), the dip is continu-

* Peach and Horne, *op. cit.*, p. 12.

ously north, the flagstones stretching from the Old Head to Halcrow Head, whence a small area of sandstones extends to Windwick. Here a fault brings up the flags with a steep north dip, and at Stews Head these again are overlaid by yellow sandstones which stretch along the shore to St Peter's Church, where again the blue flags are faulted up to form the promontory of Grimness and the north-eastern corner of the isle, and to pass conformably into the yellow sandstones along the shores of Water Sound.

Of these rocks the lowest are evidently the flags of Brough Head and Old Head in the southern shore, and here, at Banks Geo, with remains of *Coccosteus decipiens* (Ag.) and of an undetermined osteolepid, I found numerous plates of *Coccosteus minor* (Miller), which have been determined by Dr Traquair. The chief importance of this lies in the fact that it establishes the zonal identity of the flags which encircle the sandstones of Scapa Flow with those which accompany the Eday beds of the North Isles. Here, however, the horizon is, to all appearance, a higher one, as the distance between the *Coccosteus minor* beds and the sandstones of Halcrow Head is not much over a mile; and though there is evidence of faulting in the intervening section, it would seem, as stated by PEACH and HORNE,* that these faults are not of any great magnitude.

A further interest is lent to the rocks of South Ronaldshay by the occurrence in them of the new species of Asterolepis previously mentioned. Of this I found a plate in a flag quarry on Hest Head. The horizon is that which is, so far as at present known, characteristic of this fish, being in the grey flags about forty feet beneath the base of the Eday sandstones. Another plate of this species was found by Mr Spence of Deerness at the Castle of Claisdie, near Stembuster, in St Andrews, and still another, a year before, by him and myself, a short distance north of Sandside in Deerness. In both these places the geological position is precisely the same; and it seems, in consequence, to be a fish of very restricted vertical range, and may ultimately prove to be the type fossil of a subzone of the Old Red Sandstone of the Orkneys at the top of the Rousay series. That it is to be united with these rather than with the overlying beds is shown by the accompanying fossils, of which the commonest by far is *Dipterus valencienesii* (Sedgw. and Murch.), which occurs often in very great numbers in this particular belt of rock. Remains of osteolepid fishes also occur, but there is no trace of the distinctive fauna of the Eday sandstones.

Lithology of the Flagstones.

When we pass from an examination of their fossil contents to the study of the rocks themselves, at first glance we are apt to be greatly impressed by their monotony, and the endless repetition of beds in no way differing greatly from one another. The effect on Professor JAMESON we have already mentioned: his six weeks' journey in Orkney proved the most uninteresting he had ever made. The geologist who is bent on the search for easily recognisable lithological zones which can assist him in the completion of his map is sure to suffer a like disappointment. Immense as is the variety in these beds, no

* PEACH and HORNE, *op. cit.*, p. 11.

two being in every respect similar, there are yet no recognisable and definite alternations which could with certainty be used in dividing up the whole into an established succession. This is true of the Orkney flags as a whole, as was pointed out by Messrs PEACH and HORNE. They vary greatly, the principal types being a sandy flag, a clay flag or mudstone, and a brittle calcareous or even bituminous flag. The sandy flags never amount to pure sandstones, there being always a certain amount of clay and of silky weathered and bleached mica, with very usually a calcareous cement between the grains of sand. The clay flag is the purest and most abundant type. They are relatively soft, fine-grained, and light grey in colour, except when darkened by organic material. On their bedding planes the pale lustrous mica is often to be seen as a shimmering film, while the microscope shows that in worn, tattered, and crumpled flakes it is an important constituent of their mass. Sand in fine rounded grains and calcite in greater or less abundance are constant constituents. Where these softer beds occur mixed with harder beds on a cliff face, they weather out rapidly into pale grey hollows, and this is the origin of a frequently remarked feature of the Orcadian cliff scenery. The calcareous and bituminous flags are the chief receptacles of the fossil remains inclosed in these rocks. The fossil collector very soon learns that the best specimens are obtained in a brittle, hard, usually slaty and thin-bedded rock, which rings to the hammer like a piece of metal. This is in some measure due to the compactness and impermeability which is conferred on these rocks by their abundant calcareous matter. But there can be no doubt that, in turn, the presence of the organic remains facilitates in some way the accumulation of carbonate of lime in the rock, as frequently around the fossil is a well marked nodule, compact and hard, and evidently calcareous in nature from the rapidity with which it weathers out, leaving the surrounding rock comparatively unaffected. These are especially common in the dark flags among the sandstones of the Eday series. The prevalent colour of these calcareous flags is dark blue-grey, and they are fine-grained, and mostly free from the concretions so abundant in the more argillaceous rocks. In these latter they are so common that hardly a stone could be found without some trace of them. Of all sizes, from that of a melon to less than a pea, and of a remarkable and often grotesque variety of shapes, they show most clearly in the weathered face of an old dry-stone dyke, or on the bare surface at the edge of the high cliffs of the coast. From the manner in which they resist the weather, they are in most cases probably siliceous—they are certainly harder and more difficult to break than the rock surrounding them. Of these concretions the best known example is the horse-tooth rock of Yeskenaby, to which Professor HEDDLE* and other authors have devoted some attention. The rock itself occurs *in situ* at Borwick, near the great trap dyke there. But this is merely an interesting variety of a phenomenon of universal distribution throughout these flags. Their surfaces are often mottled and pitted with innumerable little concretions, which it would be easy to mistake for coprolites or for rain pittings. Not uncommonly these consist of pyrites and of marcasite, which on

* HEDDLE, *op. cit.*, pl. xiv.

weathering give a rusty colour to the surrounding rock. When the flagstones weather, the siliceous concretions, owing to their greater durability, stand out in high relief upon the bedding planes, and give the rock often a curiously fretted and ornamented appearance, and so numerous are they that frequently they resemble a solid mass of fretwork or of repousée ornament upon the surface of the stone. On weathering the flags lose also their prevalent pale or dark grey colours. Many of the dark calcareous flags around Stromness weather with a creamy yellow crust, which resembles that of certain impure carboniferous limestones. Yellow and different shades of brown are the prevalent tints of the weathered stone. The changes are principally the removal of the lime in solution and the oxidation and hydration of the iron. It is the latter which stains the rock, as is seen when we consider the source of the white colour which marks the weathered flags in a peat bed, and which is due to the organic acids of the peat having removed the iron from the rock. The decomposition gradually proceeding inward from the surfaces and cracks, produces sometimes a curious effect on a seashore where a bed of calcareous flag is divided up by many joints into polygonal areas, around the outside of which is a soft, rusty, decomposed film, an inch or more in depth, while the centre area is hard, grey, and comparatively fresh. The innumerable sun-cracked and rippled surfaces were well described by Sir A. Geikie * in the flags around Thurso.

In thickness the beds vary from an inch up to perhaps 18 inches. In every district of Orkney, flags of 2 or 3 inches thick and in large flags can be obtained for paving purposes. A favourite kind at present is a coarse sandy flag in thick beds (6 inches), obtained from Orphir. Thinner slabs, used formerly for roofing slates, are also of very wide distribution. The thick beds are valued for building purposes, especially if the bedding planes are smooth and the joints well marked. In the latter case they need no dressing, as the builder places the smooth joint face, often covered with a fine layer of glancing calcite, to the outside of the wall. In some places a variety of flag occurs, very dark in colour and seemingly much crumpled, the minute laminæ of which it consists being contorted in every conceivable fashion. Such beds are of restricted distribution, and usually markedly lenticular, as they thin out abruptly in no great distance. They bear a superficial resemblance to certain curly oil shales in the Edinburgh district, but when broken open they consist of an ordinary grey flag, the contorted layers being often covered with a dark film. They are not due to earth movement and crushing, as they occur in perfectly undisturbed rocks, and they probably result from peculiar conditions of deposit, perhaps the escape of gases or the decomposition of organic matter having produced their irregular internal structure. Where the flags are crossed by a fault the disturbance is often very great, and quite out of proportion to the magnitude of the dislocation. The rocks are bent and twisted, their surfaces slickensided and blackened, or a dark breccia produced, in which the flagstone particles glance with organic matter till they resemble broken bits of coal. In some cases the fault is marked by a layer of crushed rock powder, intensely black in colour, and mixed with calcite and iron pyrites.

* Sir A. Geikie, "Old Red Sandstone," *Trans. Roy. Soc. Edin.*, vol. xxviii. p. 393.

The peculiar nature of this flagstone deposit is so strikingly new to the geologist accustomed to the study of other districts that it cannot fail to suggest a consideration of the question of its origin. Sir ARCHIBALD GEIKIE * has insisted strongly on the marked difference between these and the sandstones which in other parts of Scotland are so characteristic of the Old Red. This striking contrast in the nature of the strata points to markedly dissimilar conditions of deposit. As we trace upwards the Old Red Sandstone of the Orkneys, we shall see that in process of time this type of sediment was replaced by the more familiar one of yellow and red sandstones and red marls. There can be no doubt that this was the result of marked changes in the physical geography of the region; and when we remember that at Cromarty beds of yellow sandstone contain precisely the fossils of the flagstone beds around Stromness, and, beyond reasonable doubt, were being formed at the same time, we see clearly the truth of Sir A. GEIKIE's conclusion that the flagstones of Orkney are merely the result of certain peculiar conditions of deposit. From their rippled and sun-cracked surfaces, they were certainly originally laid down in shallow water; and from the extensive area they now occupy, they must in many cases have been laid down far from land. That this area was tranquil I have shown to be probable, from the way the fine flags lie among the conglomerates of Stromness right against the old granitic shore. A similar mixture of deposits is to be found at the present day only in the land-locked areas of our river mouths and inland lochs. The other striking feature of these flags is the way in which they combine materials in other formations confined to different rocks. All contain sand, clay, and carbonate of lime in varied proportions, yet sandstones, limestones, or true shales are never typically developed in this peculiar formation.

III. *The Eday Sandstones, or John o' Groats Series.*

The Rousay beds of Orkney, as described by many previous writers, pass upwards conformably into an overlying series of yellow and red sandstones and marls, which contain in many places the fossils which characterise the John o' Groats beds of Caithness, and are to be regarded as on the same horizon with them. This is a very different series, and much more varied than the Rousay beds of Orkney. An entire change in the nature of the sedimentary deposits indicates a complete and comparatively rapid change in the physical conditions of the area. The yellow sandstones, with their flag beds grading upwards into red sandstones and marls, must have been the formation of shallow areas of water, with currents sufficiently strong to introduce now and then even layers of coarse gravel. The unvarying and monotonous Rousay beds, the deposit of still, though comparatively shallow water, come suddenly to an end. It is interesting to observe that these changes were accompanied by the outburst of volcanic action in a district which had for ages been the seat of uninterrupted quiet sedimentation. In the whole thickness of the Stromness and Rousay beds of Orkney there is no trace of

* Sir A. GEIKIE, "Old Red Sandstone," p. 363.

contemporaneous volcanic activity. The same conditions prevailed in the Thurso area, as was shown by Sir A. GEIKIE, the first trace of volcanic rocks being the necks on the shore at Huna, which pierce the red beds of the John o' Groats sandstones.* These physical changes heralded also the appearance of a completely new fauna in the district. It is long since it was shown by the late C. W. PEACH that at John o' Groats occurred certain fossils nowhere else to be found, viz., *Tristichopterus alatus* (Egert.) and *Microbracheus Dicki* (Traq.),† and to these *Dipterus macropterus* (Traq.) was subsequently added‡ by Dr TRAQUAIR. The same species occur in Orkney, as I have elsewhere shown, and here they form practically the only known fossils of these beds. With the single exception of a specimen of *Coccosteus decipiens* (Ag.) collected in Newark Bay, Deerness, by Mr MAGNUS SPENCE, and forwarded by him to Dr TRAQUAIR, I know of no other fossils which have been found in them. How sudden and complete the change must have been is shown by the following facts. In Eday, *Glyptolepis paucidens* (Ag.) and *Dipterus valencienesii* (Sedgw. and Murch.) occur within a few feet of the base of the yellow sandstones. In the Deerness district *Asterolepis*, sp. nov., *Osteolepis macrolepidotus* (Ag.), *Dipterus valencienesii* (Sedgw. and Murch.), *Glyptolepis paucidens* (Ag.), and *Coccosteus decipiens* (Ag.) occur in the rocks immediately underlying these beds, *Dipterus valencienesii* (Sedgw. and Murch.) in some places in vast numbers and curiously small in size. With the single exception already mentioned, not one recurs in the richly fossiliferous flags among the yellow sandstones. It would seem as if these species had been unsuited to the new environment in some manner or other, and their extinction had been rapid and complete. The flags so crowded with remains of *Dipterus valencienesii*, only a few of which have attained their full size, irresistibly impress on the mind the idea of a sudden extermination. At a higher level we find the same confused aggregation of fishes in the flagstone belts among the yellow sandstones, but this is on the horizon of the volcanic rocks, and we shall probably be right in regarding it as a consequence of the volcanic activity. The rocks of this series, unlike those they overlie, fall perfectly naturally into two main subdivisions, a yellow below and a red above, the latter possibly an index to the change which ensued on a contraction of the area of the old lake, and rendered it the seat of chemical operations resulting in a new type of deposit.

In their paper on the Old Red Sandstone of Orkney, Messrs PEACH and HORNE described with great accuracy the boundaries of these rocks, which they named the 'upper sandstone series' of the lower Old Red. It will be sufficient if I here give merely a brief account of their distribution. They occur in the centre of the Eday syncline, forming most of the island of Eday and the Red Holm between it and Westray, and lying in a gentle syncline, which is broken by a fault bringing up a strip of flags which stretches from Warness to the Kirk of Skaill. As described by these authors, the

* Sir A. GEIKIE, *Old Red Sandstone*, pt. i. p. 405.

† British Association Meeting at Aberdeen, 1858.

‡ EGERTON, Geological Survey Decade. TRAQUAIR, *Geological Magazine*, Nov. 1888. *Proc. Roy. Phys. Soc. Edin.*, 1896.

succession between the lower and the upper series is a perfectly conformable one. An extension of this syncline occupies Spurness, the S.W. corner of Sanday, and the Calf of Eday. It stretches southwards into Shapinshay, where it forms the south-east corner of the island. These beds have mostly a south-east dip, and belong to the west edge of the syncline. Thence it extends into the opposite shores of the Mainland, and occupies an area which stretches from Holland Head around the shores of Inganess Bay and in a narrow strip to the Skerry of Yinistay in Tankerness. The west boundary of this is a considerable fault already described as seen in the south-west corner of the bay, on the old Kirkwall road, and running thence along the shore and by Berstane House to the centre of the Bay of Meil. On the eastern boundary the sandstones pass perfectly conformably downwards into the flags.

The second area of these rocks is that of Deerness, first described by the present writer in a previous paper. It is separated by the Tankerness anticline from the Inganess Bay area, and the Rousay flags appear on the west corner of Deerness, near Mirkady, and pass up conformably into the John o' Groats beds. The whole area forms a well marked syncline, which includes almost the whole of Deerness, and stretches thence into Holm, where a narrow area of these rocks surround the farm of Stembuster. The dips throughout the south-east half of the sandstones of Deerness are south and south-east. At Stembuster the south-east dips gradually swing round to E.N.E., and finally to nearly north, near the Castle of Claisdie. Several faults occur in the area, one at the Mull head letting down the red and yellow sandstones against the grey flags, which at Sandside contain *Asterolepis*, sp. nov., and *Dipterus valencienesii* (Sedgw. and Murch.), but none of the John o' Groats fossils. These flags in turn, as we pass southwards, graduate upwards into the yellow sandstones. Another fault must run into Newark Bay (though not seen, the area being occupied by blown sand), for to the east of it the dips are south, while to the west the dips are mostly E.S.E., and the yellow sandstones of one side strike at the red beds on the other. Much of this syncline must lie out to sea, and possibly, as already suggested, the red rocks of Stronsay are really part of it, though it is worth mentioning that the rocks of Copinshay are grey flags, undoubtedly belonging to a lower horizon.

In the south isles of Orkney the sandstones occupy the centre of the basin.* A narrow strip of sandstones bound Scapa Bay from Orphir Kirk to near Scapa Distillery and thence along the eastern shore to Howquoy Head, in Holm. They form the west end of Rousay and the island of Hunda, here dipping west, the north-west corner of South Ronaldshay, with a general north-west dip; and on the east side, at Windwick and St Peter's Church, small areas of sandstones are faulted down among the flags of the south and east side of the island. In Flotta they occupy principally the northern half of the island and the adjacent Calf of Flotta, having here a north dip, and passing down conformably into the grey flags of the southern shore.† Lastly, in the island of Hoy they are found in that part of Walls to the north of Longhope, around the Burn of Ore,

* Peach and Horne, *op. cit.*, pp. 11 and 12.

† Peach and Horne, *op. cit.*, p. 12.

and are separated by a fault by the upper Old Red Sandstone, which extends over the most of the remainder of the island.

The Yellow Sandstones and Flags of the John o' Groats Series.

Starting at the northern extremity of their area in Orkney, we find that in Eday these beds occupy a comparatively small area and are of very limited development. At the Kirk of Skaill, on the eastern shore of Eday, a belt of yellow sandstones immediately overlies the top flags of the Rousay series. These are followed by a thin zone of red marls, which in turn are overlaid by thin-bedded calcareous flags, rich in fossil remains, of which *Dipterus macropterus* was the only one I found in satisfactory preservation. Above these we find a series of yellow and red beds (with thin layers of conglomerate), which form a gradual transition to the red and brown sandstones and marls so largely developed in the centre and north end of the island. The whole thickness of this series is not over 100 feet, and it is, in fact, their most insignificant development in any part of Orkney. Were it not for the very convincing sections elsewhere obtained, it would be impossible to regard these beds as other than a merely local facies of the basal series of the red beds. Messrs PEACH and HORNE* give the following estimated thickness :—

Red and yellow sandstones—
Flagstones, 40 feet.
Reddest shales, 15 feet.
Hard white sandstone, 20 feet.
Gray calcareous flagstones.

—the last being the underlying Rousay series, as I regard them, as they contain no trace of John o' Groats fossils of the group of flagstones interbedded with the sandstones, while *Dipterus valencienesii* and *Glyptolepis paucidens* are not infrequent in them. These yellow beds and flags stretch across London Bay, and emerge again at Millbounds, where the section is very similar to that described.

On the west side of the syncline the same beds crop out again just to the east of Fersness, where they furnish the chief supply of yellow freestone used for building purposes in Kirkwall and throughout the islands. A hundred yards to the east of the pier the yellow beds come in gradually below the red, which here dip about E.S.E. Among them occur again a belt of thin flags and an insignificant red series. The section, in fact, repeats in every respect that to the east, and *D. macropterus* is found in the flags to the west of the pier, but here the thickness must be somewhat greater, as the average dip is about 20°, and the area of shore occupied is about 400 yards. At Warness, again, to the south-west corner of the island, the underlying flags, with here and there a yellow bed, pass up into a yellow sandstone series, 70 to 80 feet thick, over-

* PEACH and HORNE, *op. cit.*, p. 5.

laid by a few feet of red beds, and these by 20 feet of coarse flags (in which I found no fossils). Over these flags, which no doubt are the same as those of London Bay, come a few yellow beds, which rapidly give place to the red sandstones of Sealskerry Bay.

In the south end of Sanday these beds recur, and form the western edge of the promontory of Spurness, disturbed and set on end by a north and south fault, which brings up with them the underlying beds of flagstones in a narrow strip. A thick conglomerate occurs among them at Heclabir, but in other respects they differ little from the Eday sandstones, though, from their limited distribution, no very complete idea of their features can be formed. After we cross the fault above mentioned, we find the red sandstones in great strength, forming the shore to near the Noust of Boloquoy on the north coast. Here yellow and red beds, mixed, strike along the shore, and, slightly faulted at Grunnavi Head, continue with a dip W.N.W. to Blue Geo, where the flags again come in. The thickness here is not great; but owing to the presence of several small faults, an exact estimate is not possible. These beds, traced along the strike, emerge at Quoyness on the south shore, where, however, they are covered by the blown sand of the beach. The yellow sandstones of Sanday show the same features as those of Eday, and, like them, are of comparatively small thickness.

The conglomerates which occur in these rocks of Eday and Sanday have already been the subject of discussion by several writers.* Professor HEDDLE noted that at Heclabir, in Sanday, occurred a bed of conglomerate about 14 feet in thickness, and that the pebbles it contained consisted of "granites, more than one variety, gneisses, often chloritic, porphyrys, and seemingly of quartzite,—rocks which are entirely different from the primitive rocks near Stromness, and therefore rocks not occurring in the islands."† He states also that both the pebbles and the cementing paste have a highly vitrified aspect, and that he had a strong impression this was a volcanic conglomerate. Messrs PEACH and HORNE state with regard to the beds of Eday, which form very insignificant belts at the base of the red series—nowhere over a few inches in thickness—that "the included pebbles consist of fragments of mica schist, quartzite, gneiss, granite, and other metamorphic rocks, all stained of a reddish colour."‡ According to my own observations, all those mentioned occur with one exception; the commonest by far at Heclabir being a creamy or white lustrous quartzite, in much rounded and waterworn pebbles, up to 6 inches in diameter. At the latter locality I was unable to find any volcanic rocks, but there were very numerous pebbles of grey limestone, which microscopic sections showed to be entirely holo-crystalline and true marbles, without any trace of organic structure. With these were others which at first puzzled me; but on referring to Mr PEACH, he at once recognised them as cherts and cherty limestones from the Eillean Dhu series of Durness (Cambrian); and the microscope showed that, like these, they were of oolitic structure, though, so far as my examination went, by no means so perfect as in the

* JAMESON, *Mineralogy of the Scottish Isles*, vol. ii. p. 257.
† HEDDLE, *Geognosy of Scotland*, v. p. 103.
‡ PEACH and HORNE, *op. cit.*, p. 5.

sections shown me by Mr PEACH. By his advice I searched carefully, on a subsequent visit to the spot, for traces of the piped quartzites and other Cambrian rocks, but failed to observe any. The presence of these pebble beds shows very clearly how great must have been the physical changes which the area had undergone, before sediment so coarse reached districts which had long been the seat of a deposit of the finest grain and the most uniform nature. They are very local in distribution, no trace of the thick beds at Heclabir being found among the yellow sandstones in other areas of Sanday, or indeed anywhere in the district, except on the opposite shore of Eday, where their thickness is quite trivial in comparison.

The yellow beds of Eday and Sanday stretch southward into Shapinshay, where they attain a much greater importance, forming, in fact, the whole thickness of the John o' Groats series in that island. Here the outcrop forms the south-east corner, and is bounded by a line running N.E. from the angle of the bay below the Established Church on the south shore to the Bay of Crook on the east. The underlying flags seem to pass up quite conformably and without any important break into a series of yellow current-bedded sandstones, mixed with numerous thin beds of dark-coloured flags. Along the east side the structure is simplest, the prevalent dips being S.E. and E.S.E., but elsewhere the dips roll greatly, and the beds are evidently being constantly repeated. The yellow sandstones overlying these mixed beds are very pure and massive, and cannot, with any probability, be estimated at less than 400 to 500 feet. Only very rarely is a red-coloured bed of clay to be seen; but at more than one place there occur belts of flags intercalated between yellow sandstones, and in some places 30 feet in thickness. These flags may be the counterparts in this area of the flagstones which in Eday occupy a similar position, and, like them, they contain the characteristic John o' Groats fossils, one specimen of *Tristichopterus alatus* (Egert.) having been found by me at Store Point in a coarse grey flag. It is among them also that the volcanic rocks * occur which PEACH and HORNE described as the only evidence of contemporaneous volcanic action in the lower Old Red of Orkney. They consist of a single lava flow, which, though much weathered, is recognisable as an olivine diabase, and is distinctly vesicular at the top surface, while it rests quite conformably on the underlying flag, which is considerably baked and altered.† To their observations I have only a few to add. The interbedded character of the volcanic rock is shown also by the occurrence at its south-western corner of a bed of ash several inches thick immediately overlying it, while in several places thin layers of sprinkled ash can be traced in the overlying flags a few inches apart, and to a distance of 10 feet above the surface of the lava. This shows that though the volcanic activity resulted apparently in only one outflow of lava, it continued for a time to produce occasional showers of ashes, which were spread out over the sea-bottom, and mixed with the sediment accumulating there. At its base the lava contains here and there a bit of an angular baked flag, but its upper surface is vesicular

* JAMESON, *Mineralogy of the Scottish Islands*, ii. 235.
† PEACH and HORNE, *op. cit.*, pp. 9 and 13.

and very irregular, the sandstone filling up all these irregularities quite unaltered and undisturbed in bedding. In several places the lava is 30 feet thick, but in one little creek its top and bottom surfaces were seen in section, and here it was not over 12 feet in thickness. Its greatest development is to the south and east, from which direction it seems to have flowed from a source now, no doubt, concealed by the sea; and this conclusion is strengthened by the occurrence on the same horizon of similar volcanic rocks in the sandstones of Deerness.

The southern termination of this area of John o' Groats beds corresponds very closely with the shores of Inganess Bay. At more than one place in this district the flagstones have yielded *Dipterus macropterus* (Traq.) and *Tristichopterus alatus* (Egert.), and in it occur both types of sediment characteristic of these rocks; but so completely is it occupied by the sea that little certainty can be attained as to its exact geological structure. Along the eastern shores the rocks are yellow sandstones, with many thin beds of dark brittle flag, dipping mostly N.W. at gentle angles. On its western side, again, the red sandstones and marls of Holland Head are underlain by a fine pure yellow sandstone below Berstane House, which at its lower part contains belts of flagstone, and even an occasional red bed. The proximity to the great fault which runs out to sea in Meil Bay disturbs these rocks somewhat, but there can be no doubt that this is the order of the succession, and that, on the whole, these are higher in the series than the yellow sandstones, which on the other side of the bay rest on the flags of the East Mainland anticline. The area must be somewhat disturbed by faults, for on the shore to the southward, at the west corner of Inganess Bay, we find a patch of red marls which belong undoubtedly to the overlying red series. The yellow sandstones of this area bear a close resemblance to those of Shapinshay, from which they differ chiefly in the absence of any interbedded volcanic rocks. They show also that the Shapinshay rocks are merely the basal part of the series, and that overlying the yellow beds in this area, as in Eday, there is a series of red sandstones and marls of considerable thickness.

In Deerness occurs an area of John o' Groats beds which in some respects is the most varied and interesting of any in Orkney. Separated from the previous series by the anticline which brings up the lower flags through the parish of St Andrews, it forms in turn a syncline or basin, of which only the northern half is accessible to observation. The axis of this syncline runs probably E.N.E. from Stembuster on the shore south of Dingieshowie, and on the south side of this axis we have only a very short stretch of sandstones along the shore to just south of the Castle of Claisdie, where they pass down into the grey flags of the parish of Holm. Northwards along the shore the dips sweep round, till at Dingieshowie they are E.S.E.; and E.S.E. and S.E. dips, as already remarked, are far the most prevalent throughout the parish of Deerness. One of the most complete and trustworthy sections is that described by me in a previous paper * as stretching from Dingieshowie to Newark Bay along the south shore, but this is in so far incom-

* *Trans. Roy. Phys. Soc. Edin.*, vol. xiii.

plete that the fault which crosses the isthmus at Dingieshowie cuts out the passage beds underlying the yellow sandstones. These are seen in the shores farther west, at Stembuster, where they consist of thin courses of yellow sandstone with slaty flags between, forming a very gradual transition between the two types of sediment. Above these 'passage beds' lies a series of red marls, with thin yellow and brown sandstones (40 feet) between, recalling in some ways the beds described as occupying a similar position in Eday. The exact point at which the base of this series should be drawn is a matter of some doubt, as among the lowest of them occur beds crowded with *Dipterus valenciennesii* (Sedgw. and Murch.), and containing also *Asterolepis*, sp. nov., but containing no other fishes, an observation due to Mr Magnus Spence. Traced upwards, the red beds pass gradually into a pure massive yellow sandstone, which forms the high cliff below Tornpike, and is, no doubt, the same as that of Delday's Banks. These lowest beds are exposed also in other parts of the parish, as at Braebuster and the shores to the south of it, where thin grey flags pass gradually up into yellow sandstones. In the north shore of Deerness a very similar series occurs at Halle, and extend thence to near the Covenanters Monument, lying very flat, and forming the extreme N. edge of the syncline; and these rocks must again outcrop in Sandside Bay, between the flags which form its northern side and the yellow sandstones to the south, though here the rock is concealed by the blown sand which occupies the centre of the bay. Just north of the Brough of Deerness the presence of a few red beds beneath the yellow sandstones is well seen in a lofty cliff, and again the same feature is to be observed in the shore below Horraquoy. The yellow sandstones recall, in very many respects, those of Shapinshay. They are of much the same thickness, 400 to 500 feet, and through them lie here and there belts of thin grey calcareous flags, which are the chief source of the John o' Groats fossils of Orkney. They contain *Dipterus macropterus* (Traq.), *Tristichopterus alatus* (Egert.), and *Microbrachius Dicki* (Traq.), the first especially in great abundance, and often in fine preservation; and it is probable that through the low lying centre of the parish these largely replace the yellow sandstone series.

A further point of similarity to the yellow sandstones of Shapinshay is furnished by the presence in these beds of a zone of contemporaneous volcanic rocks of basic composition. These occur rather above the middle of the yellow series, and, as in the district previously described, they are immediately associated with a belt of grey flags intercalated among the sandstones. They consist of both ashes and lavas, and in addition there are several intrusive sheets which, from their composition and general character, are undoubtedly to be ascribed to the same volcanic source.

At the extreme south-east corner of the parish, at the Point of Ayre, a series of volcanic rocks form a narrow belt running W.N.W. in the land, and outcropping on the seashore. The general dip in this quarter is S. and S.S.E., and from Horraquoy southward along the east shore we pass over a gradually ascending section of the lower members of the yellow sandstones. This dip continues to the Point of Ayre, which consists of beds of flagstone, and these, though somewhat faulted, evidently are to be

assigned to the upper part of the yellow beds. From this point westward they strike along the shore, which they form up to the Bay of Newark, where they are covered by blown sand. On the west side of the bay, beds occur striking N.N.E., and evidently let down by means of a dislocation covered by the superficial accumulations in the centre of the bay. The principal mass of volcanic rock at the Point of Ayre forms a narrow area which runs E.S.E. out to sea, and is in breadth about 40 yards. Its base is not seen, and its lower member is a thick bed of dark green volcanic ash, with large spherical bombs up to 2 feet in diameter, vesicular, especially in the centre, and much decomposed. A few bits of baked flag occur in the ash, and it weathers in a markedly spheroidal manner, resembling, in fact, very closely many of the basaltic ash beds around the shores of the Firth of Forth, as at Kinghorn and Elie. In general it shows no trace of bedding, but here and there a few thin irregular lenticles of sand are to be seen, which prove that though rapidly accumulated, it is not the product of a single outburst. A curious feature is the existence in it of flagstone veins. These are very tortuous and irregular, an inch or two in thickness, and filled with a normal, somewhat calcareous flagstone, in which little or no trace of any metamorphism is to be found. They are vertical, and show no sign of bedding or contortion, and are to be regarded as due to the formation of cracks in the thick accumulation of volcanic ash, into which the ordinary sediment of the sea-bottom was washed. At first glance, this bed of agglomerate suggests at once that it is a volcanic neck, and the elongated form of its outcrop would support this explanation. But its junction with the flags to the south is a small fault, and these show none of that alteration which is to be expected in the walls of a volcanic neck. And, moreover, the bed itself is seen in the low cliff to be overlaid by a thin lava, and that again by well-bedded flags. Still, it is in every way probable that an accumulation of this sort was formed in the immediate proximity of a volcanic orifice. The overlying lava is some three feet in greatest thickness, vesicular at its upper surface, the vesicles being large, not markedly elongated, and filled with calcite and other secondary minerals. It is greatly decomposed, but shows little of the spheroidal weathering of the agglomerate, being rather divided by well-marked joints into polygonal vertical columns. Under the microscope it turns out to be an olivine basalt, so greatly decomposed that few of the original minerals remain. At the western corner of the outcrop this lava is seen to be, in turn, overlaid by ordinary flags, which are in nowise altered by the heat of the underlying rock, and contain little or no fragmental volcanic matter. These rocks are bounded to the south, and probably also to the north, by small faults. A few yards to the west of them, what seems to be a quite distinct outflow is exposed in the shore. This is the edge of a small lava flow, three feet in thickness, and thinning out in a few yards to the south, while the flags close over it. It is dark in colour, with large steam cavities in its upper surface, and bears a striking resemblance to the volcanic rock at Haco's Ness, Shapinshay. The sea has removed the overlying rocks, except at the thin edge, where a layer of dark green ashes mixed with sand is seen to immediately overlie the lava, succeeded in turn by a normal unbaked ordinary flag. The lava rests upon a similar

flagstone, and hence cannot be the same as that already described to overlie the thick agglomerate bed, a few yards further to the east.

Among the yellow sandstones, about two miles further to the west along the shore, and about a hundred feet below where they pass into the red sandstones, occurs another belt of contemporaneous volcanic rock. It is associated here, also, with a series of flagstones, and no doubt is on the same level as the rocks just described. In a little bay to the east of the Castle, a bed of dark green ashy sandstones, mostly fine-grained, but with here and there lapilli of a couple of inches in diameter, is to be seen, interbedded with yellow sandstones and flags. It is very similar in character to the ash beds in Shapinshay which overlie the lava; but while these are mostly of very inconsiderable thickness, it is in some places three or four feet thick. No lava is associated with it, and in the sandstones above and below I found no trace of any recurrence of the volcanic activity. In all probability it is the representative, in this section, of the coarse agglomerate already described, which must have greatly thinned out in the intervening distance. The striking feature of this volcanic zone is its very diminutive thickness. Still, the occurrence in Orkney of such a zone is a remarkable confirmation of the opinion expressed by Sir A. Geikie, that the "ancient volcano of John o' Groats might be one of a series which might hopefully be sought for among the Orkney Islands."*

Rocks of an intrusive origin occur also in this district, the principal mass being exposed in the locality last mentioned, about 50 yards west of the ashy sandstone. It forms a mass of about 25 feet in thickness, though its base is not exposed, a dark green rock, which is first seen in the shore, and runs out to sea in a series of picturesque stacks and reefs. Its intrusive character is shown by the absence of any amygdaloidal upper surface, and the evidently unconformable junction with the overlying sandstones. Yet these were, so far as I could make out, not markedly altered, though they are so decomposed that this would not be easy to determine. The rock is about 30 feet beneath the ashy sandstone, and in structure is a much weathered diabase, with crystals of plagioclase felspar, augite, and probably olivine, almost entirely decomposed into green chloritic products, which show traces of ophitic structure. Throughout Deerness, in several places, occur masses of volcanic rock so decomposed and so obscured in their geological relations by the surface accumulations that it is not easy to form an opinion as to their true character. They all occur among the yellow sandstones and the flags associated with them. One is seen to the south of the Free Church, and several outcrops are known in the vicinity of the Public School. I am greatly indebted to Mr Magnus Spence for specimens and observations on these outcrops. From their microscopic structure and the absence of any accompanying tuffs, they are in all probability intrusive sheets. The freshest specimen I obtained was a dark green diabase, with well-marked ophitic structure and pseudomorphs of serpentine after olivine. It came from a deep pit, at one time sunk in a field behind the Public School.†

* Sir A. Geikie, "Old Red Sandstone," *Trans. Roy. Soc. Edin.*, vol. xxviii. p. 406.

† The Black Holm of Copinshay consists of an intrusive sheet of olivine diabase about 30 feet thick, enclosing a large mass of baked flag penetrated by numerous veins. This is probably that referred to by Jameson, *Scottish Islands*, ii. p. 235.

An outcrop of special interest occurs in a field 400 yards west of Smiddybanks. Here, in an old gravel-pit, a face some ten feet high is exposed, now much broken down by weathering. The rock is a coarse red sandy ash, with green spots. In it occur very numerous sandstone and flagstone fragments, some as large as a man's head,—the sandstones baked into quartzites; the flags fused and slaggy on their surfaces, and with their edges rounded. Materials such as these form a considerable proportion of the whole mass. It seems unbedded, or rather the few traces of bedding planes showed a dip discordant with that of the surrounding sandstones. No similar bed crops out along the shore, and the outcrop seems to be limited in area and rudely circular in outline, though, as it occurs in the midst of cultivated land, its exact margins cannot be traced. It is difficult to understand what this is, unless it be regarded as a small volcanic neck, the mixed nature of its fragments being so different from that of the other ash beds, while its position in the centre of the intrusive sheets and lavas and ashes already described renders such a hypothesis, to say the least, highly probable.

There can be no doubt that all these volcanic rocks owe their origin to the same period of volcanic activity. Their situation, almost in the direct line between the Neck of Huna and the lava of Haco's Ness, points to the existence of a north and south fracture or line of weakness, which may be ascribed to the earth movements, which, at the close of the deposition of the Rousay rocks of Orkney, introduced new types of sediment and new forms of life. To the westwards, at any rate, no trace of similar structures has been found. At two subsequent periods volcanic rocks rose to the surface in this district: one series forms the lavas and ash beds of Hoy, described by Sir A. GEIKIE. These, too, are of basaltic character, but they are separated from those we are at present considering by a great conformity. The others form the trap dykes, which traverse the flagstones mostly in an E.N.E. and W.S.W. direction. But these latter are in no place connected with surface outflows, and differ so widely in structure and composition from the rocks of Deerness and Shapinshay, as undoubtedly to have proceeded from quite distinct sources. They are, in fact, chiefly developed in the West Mainland, and are comparatively few in regions occupied by John o' Groats rocks.



SECTION 4.—N. and S., through South Ronaldshay. 2 miles = 1 inch.

The only remaining district of the yellow sandstones is the basin of the South Isles. A complete examination of this area I was unable to overtake, but was compelled to confine myself to the islands of South Ronaldshay and Burray, in which they occupy the largest area of any of the South Isles, and very clear sections are to be obtained. Here, also, the underlying yellow series is well developed, and passes down by means of a series of flaggy passage beds into the grey flags, which at the south end of South Ronaldshay contain the Rousay fossils. These passage beds are well seen on the south

shore of Watersound, just east of St Margaret's Hope. At Stews Head they contain a few reddish bands. In South Ronaldshay the yellow series is largely developed, and, with the exception of the district from Widewall to St Margaret's Hope, and thence to Hoxa, they occupy all the areas marked on the map as belonging to John o' Groats beds. A fine section of massive yellow sandstones, with a few flag-beds, is seen extending from Barswick on the west side, north to Herston Head. It is broken by several faults, but there can be no doubt that in thickness it is greater than any other section of the same rocks elsewhere exposed in Orkney. Among these beds no trace of a volcanic zone has yet been discovered, and as yet no John o' Groats fossils have been obtained from any of the South Isles. Their relationships are such, however, as to leave no doubt whatever of their position in the series.

In the district around Melsetter in the island of Hoy, according to PEACH and HORNE, bands of yellow sandstone occur, overlying conformably the flags which form the south end of the island. These resemble greatly the upper Old Red Sandstones of the west end of Hoy, which unconformably overlie the flags. Now, at the west side of Hoy, opposite Graemsay, the upper sandstones rest on flags which are to be correlated with the Orcadian beds of the opposite shores of Stromness. This is clear proof of the great erosion which must have preceded the deposition of the upper Old Red series in Orkney, as time sufficient for the removal of all the Rousay rocks and all the John o' Groats rocks of Orkney must have elapsed before the upper beds were laid down on the up-turned edges of the Stromness flags which form the base of the Old Man of Hoy.

The Red Sandstones of the John o' Groats Beds.

The red sandstones of the John o' Groats beds of Orkney have their greatest development in South Ronaldshay, in the extreme south, and in Eday, at the extreme north of the country, while in the intervening districts their thickness is small. In Eday, they form the entire north end of the island, and thence pass down the centre to Sealskerry Bay. Some of the highest elevations along this line have a height of 350 feet, and the least possible estimate of the thickness of the whole series cannot be less than 600 feet. The yellow sandstones of this island are, however, of only slight thickness, and it is possible that the red beds, in fact, replace the yellow, which further south have a much greater development. Red sandstones form also the south-east corner of the island around the point of Veness. To the geologist these beds are somewhat uninteresting. No fossils have been found in them, and they contain no contemporaneous volcanic rocks. The absence of fossils is perhaps due to the fact that there are no beds of close-grained flag suitable for the preservation of organic remains. The beds themselves consist of coarse red sandstones, often in thick beds, alternating with red shales and marls, with sometimes a greenish or greyish shale. In Eday the sandstones greatly preponderate, and in some places are so coarse as to deserve the title of 'grits.' No traces of any chemical deposit, such as rock salt or gypsum, occur any-

where, and the red matter is uniformly disposed through the rock, except where leached out by percolating water, or where aggregated into irregular layers of iron pan.

In Sanday, along the west shore, the beds have a very similar character, but are more friable, owing to the admixture of dark red clay. In Shapinshay red beds practically do not occur, the only representatives of the John o' Groats beds being the yellow sandstones and flags; but on Holland Head red beds again appear, with every peculiarity to be found in those of Eday. Here, again, the beds are mostly massive sandstones, the red shales being of only secondary importance. The total thickness in this section is about 200 feet. In Deerness, red beds form the western shore of Newark Bay, and stretch westwards nearly to the Castle. Here thick coarse sandstones are mixed with green and red marls. The extreme north point of the parish consists of similar rocks, which are let down by a fault running east and west just south of the Mull Head. They have little of the massive uniformity which characterises the beds of Eday, the alternations in the nature of the sediment being comparatively frequent. Red beds form also the cliff above the Scapa Pier, but in the South Isles area, their best exposure is that from Widewall in South Ronaldshay, by Roeberry, to Hoxa, and thence to St Margaret's Hope along the shore. Here the dip is gentle to north and north-west, and the underlying beds of yellow sandstone pass up very gradually into the deep red marls beneath Roeberry House. The thickness of these marls—which contain thin beds of red sandstone—is considerable, and they resemble closely the beds seen in Calf Sound, in Eday, in every respect, except their greater thickness. Similar beds are to be seen below Smiddybanks in St Margaret's Hope. Overlying these there come in massive coarse red sandstones, which occupy the rest of the area up to Hoxa, where they are faulted against the flags of Hoxa Head. The thickness exposed in this section is about 500 feet, and not greatly less than that of Eday, where the yellow sandstones are so insignificant. The whole thickness of the John o' Groats beds of Orkney may thus be put down at about 1000 feet in its greatest development. Red beds occur also in Burray and Hunda, but these present no features of special interest to merit a separate description.

With these red sandstones the long history of the Orcadian Old Red of Orkney comes to a close. A complete change in the nature of the sediment accompanied what must have been considerable changes in the physical conditions of the area. Yet it is, after all, only a reversion to that type of deposit which elsewhere had been the main one for vast periods of time. In the nature of its rocks and in the limited development of volcanic activity, this area had long been a great contrast to the Old Red of Southern Scotland; only at its close do we find a partial resemblance to make its appearance. The red sandstones are the least important part of the Orkney Old Red. Neither in Caithness nor Orkney do we find them conformably overlaid by any other rock. The new conditions which supervened were marked by the precursors of a new fauna, of which the first example is the Asterolepis, a fish so characteristic of the upper Old Red of the southern shores of the Moray Firth. But before that fauna was to attain its greatest development great changes in the physical geography of Scotland had to take

place, and vast periods of time to elapse. Before the deposit of the upper Old Red of Hoy, much of the Orcadian Old Red had been stripped from the surface of the Orkneys, and very considerable dislocations had modified entirely the old physiography and structure of the country.

III.—*Comparison with the Old Red of other Districts.*

Such being in its main features the structure of the Orkneys, and the subdivisions which can be established by the distribution of the fossils, it remains to be considered how far these conclusions can be applied to other districts in which rocks of like age and similar fossils occur.

The John o' Groats Beds and the Eday Sandstones.

As regards the uppermost beds, the inquiry is a simple one. Rocks containing the same fossils occur in only one locality—the north-eastern angle of Caithness; and here their lithological characters so strikingly resemble those of the Orkney beds that no difficulty whatever can be felt in accepting their zonal identity. The John o' Groats beds of Caithness are, then, to be correlated with the Eday, Deerness, and South Ronaldshay sandstones of Orkney. Sir A. Geikie gives a list of the fossils which have been found in this series in Caithness.* He enumerates, in addition to the three type fossils, *Acanthodes Peachi* (Eg.) and *Glyptolepis leptopterus* (Ag.), neither of which is known to be present in the similar beds of Orkney. It is remarkable how in both counties the fishes characteristic of the lower rocks have been superseded by new types so completely that almost no trace of their persistence is to be obtained. The uppermost zone of the Orcadian Old Red is thus a well characterised one, and may be designated, from the locality in which alone it was known to occur for many years, The John o' Groats Sandstones (zone of *Tristichopterus alatus*, Egert.).

The Thurso and Rousay Beds.

For the representatives elsewhere in Scotland of the lower zones, we must look to two localities, to Cromarty, from which Hugh Miller and Agassiz early in the century furnished a list of fossils, and to Caithness, where, since the time of Hugh Miller and Robert Dick, much work has been done in the palæontology of the Old Red. The earlier work has subsequently been subjected to thorough revision, and a wealth of new material been brought to light by Dr Traquair, to whose papers I am greatly indebted, and on whose published statements I shall rely in comparing the lists of fossils from each locality. In his paper, "Achanarras Revisited" (1894),† he has briefly stated the

* Sir A. Geikie, *Old Red Sandstone*, p. 404.
† Traquair, *Proc. Roy. Phys. Soc. Edin.*, vol. xii., 1894.

results of a comparison of lists of fossils from Caithness, Orkney, and Cromarty, and the result is a division of the known fossils into three groups. One is that we have already considered—the John o' Groats group. The second contains a series of fossils which occur together only in the neighbourhood of Thurso. The list is as follows :—

Homacanthus borealis (Traq.).
Rhadinacanthus longispinus (Ag.).
Mesacanthus Peachi (Egert.).
Cheiracanthus, sp. (perhaps 2 sp.).
Coccosteus decipiens (Ag.).
Coccosteus minor (H. Miller).
Homosteus Milleri (Traq.).
Dipterus valencienesii (Sedgw. and Murch.).
Glyptolepis paucidens (Ag.).
Thursius macrolepidotus (Sedgw. and Murch.).
Thursius pholidotus (Traq.).
Osteolepis microlepidotus (Pander.).
(*Scales, doubtfully resembling those of* Gyroptychius).

It will be observed that this list contains the type fossils of the Rousay series of Orkney, *Coccosteus minor* (H. Miller) and *Thursius pholidotus* (Traq.); and when we compare it with the list of the fossils I have found in those rocks, we find that the following species occur in both :—

Coccosteus minor (H. Miller).
Thursius pholidotus (Traq.).
Dipterus valencienesii (Sedgw. and Murch.).
Glyptolepis paucidens (Ag.).
Cheiracanthus, sp.
Coccosteus decipiens (Ag.).
Homosteus Milleri (Traq.).

With the exception of the first two, these are all contained in the list of fossils which occur throughout the whole thickness of the Orkney flagstones. In Orkney occurs one species not yet found in Caithness, *Asterolepis*, sp. nov., which, considering that it is a fossil of limited range, and confined to a few beds of rock, is an exception of no great importance; and two others present in Caithness, but not known from the vicinity of Thurso, *Osteolepis macrolepidotus* (Ag.) and *Diplopterus Agassizi*. Of these, the latter is one of the rarest of Caithness species, while in Orkney it is quite common, especially in the quarries of Sandwick and Stromness. From the Rousay beds of Orkney I have seen only one satisfactory specimen. It is probable that we have here a case of local distribution, and that the absence of this fossil from the rocks around Thurso is due, not to adverse conditions of preservation, but that rather it was from the first a species characteristic of the more northern area, and hence more likely to persist there, and occur on a higher horizon. On the other hand, we have a number of forms known

to occur near Thurso, but not found as yet in the Rousay beds of Orkney. These are :—

Homacanthus borealis (Traq.).
Rhadinacanthus longispinus, (Ag.).
Mesacanthus Peachi (Egert.).
Thursius macrolepidotus (Sedgw. and Murch.).
Osteolepis microlepidotus (Pander.).

Of these, the first is a rare fossil, and only described for the first time in 1892.* The second cannot be regarded as very abundant, seeing that the British Museum Catalogue (1891) does not enumerate it as a Caithness species. The third has not, so far, been mentioned in the literature of Orcadian geology, though Dr TRAQUAIR, I believe, has obtained a species of Mesacanthus from Orkney this last summer. That these three rarities should be known from the carefully examined rocks around Thurso, and not as yet from the Rousay beds of Orkney, to which attention has only lately been directed, cannot be regarded as a strong argument against the theory that the one series is the northern representative of the other. The two remaining fossils are of more importance, seeing that they are regarded by Dr TRAQUAIR as typical of the Thurso rocks, and confined to them. One of these, *Osteolepis microlepidotus* (Pander.), is very characteristic of them, and abundant in some of the beds ; but I have, at many different times, examined collections of Orcadian fossils, and carefully searched the rocks for this species, without ever obtaining a specimen which Dr TRAQUAIR would admit belonged to it. No doubt it has figured more than once in lists of fossils from Orkney, but the identification is at present more than doubtful. It is possible that we have here a case of local distribution the converse of that of Diplopterus, but at any rate the discrepancy is one which cannot be overlooked, and it is to be hoped that further search in Orkney will bring this fish to light. *Thursius macrolepidotus* (Ag.), it may also be anticipated, will turn up in the Rousay beds, or at any rate its absence is not very remarkable when we remember that only one satisfactory specimen of the other species of the same genus has yet been discovered. Yet that, in that case, in the same quarry, two species which, according to Dr TRAQUAIR, are typical of the Thurso rocks, should have been found together for the first time in Orkney, is a surprising confirmation of the views he enunciated in 1894, that they are type species of a special subdivision of the Orcadian Old Red ; and that their distribution in Orkney, so far as yet known, is in complete accordance with this supposition, has already been proved to be the case. They occur always on practically the same horizon, and in the lowest beds they have never yet been found.

No other locality for these two fossils is at present known, and from the district in which they have been longest and most thoroughly investigated they may be named the Thurso Beds, or the Zone of *Coccosteus minor* (Hugh Miller) and *Thursius pholidotus* (Traq.).

* *Trans. Geol. Soc. Edin.*, 1892.

The Cromarty, Achanarras, and Stromness Beds.

The third group of fossils recognised by Dr TRAQUAIR is that which HUGH MILLER first described from Cromarty, and he himself, on several occasions, from Achanarras (Caithness), and which was long believed to be the only one present in the Orkneys.

The following is a list of the fossils of Cromarty, Achanarras, and the Stromness beds of Orkney .—

Palæo spondylus Gunni,		A.	
Diplacanthus striatus (Ag.),	C.	A.	O.
„ *tenuistriatus* (Traq.),	C.		
Rhadinacanthus longispinus (Traq.),	C.	A.	O.
Mesacanthus pusillus (Traq.),	C.	A.?	O.?
Cheiracanthus Murchisoni (Ag.),	C.	A.	O.
„ *latus* (Egert.),	C.		
„ *grandispinus* (M'Coy),			O.
Pterichthys Milleri (Ag.),	C.	A.	O.
„ *productus* (Ag.),	C.	A.	O.
„ *oblongus* (Ag.),	C.	A.	
Dipterus valencienisii, (Sedgw. and Murch.),	C.	A.	O.
Coccosteus decipiens (Ag.),	C.	A.	O.
Homosteus Milleri (Traq.),	C.	A.	O.
Glyptolepis paucidens (Ag.),		A.	O.
„ *leptopterus* (Ag.),	C.		O.
Gyroptychius microlepidotus (Ag.),	C.	?	O.
Diplopterus Agassizi (Traill),	C.	A.	O.
Osteolepis macrolepidota (Ag.),	C.	A.	O.
Cheirolepis Trailli (Ag.),	C.	A.	O. *

A glance will show the very complete accordance of these lists. All the more frequently occurring fishes are common to all the localities, except possibly *Glyptolepis paucidens* (Ag.), which in the Cromarty district is replaced by the closely allied *Glyptolepis leptopterus* (Ag.). *Gyroptychius microlepidotus* (Ag.) seems to be absent from the Caithness area. The other fishes found in one area only are all rare fossils.

If, now, we examine the list to ascertain which fossils are confined to these areas, we find that—

Palæospondylus Gunni (Traq.)
Diplacanthus, 2 sp.
Pterychthys, 3 sp.
Cheirolepis Trailli (Ag.)

and possibly

Gyroptychius microlepidotus (Ag.)

are not known to occur elsewhere.

* This list has been compiled from—TRAQUAIR, "Fossil Vertebrates of the Moray Firth"; TRAQUAIR, "Achanarras Revisited"; A. S. WOODWARD, British Museum Catalogue of Fossil Fishes.

To these Dr TRAQUAIR adds two—*Osteolepis macrolepidotus* (Ag.), which certainly occurs in the East Mainland of Orkney, and *Diplopterus Agassizi* (Traill), which, he says, he has not been able to establish with certainty as a member of the Thurso group. If we except the rare *Palæospondylus Gunni* (Traq.), which is known only from Achanarras, we have three genera and six species which, so far as our present knowledge of the distribution of the fossil fishes of the Scottish Old Red Sandstone goes, may serve as type fossils for this group of rocks; and these, it will be remembered, are the genera which I found in Orkney to characterise the Stromness beds; and we may regard it as established that this is a distinct zone of the Orcadian Old Red Sandstone, of which the representatives are the sandstones of Cromarty, Lethen, Gamrie, Clunie, and Tynet, the flagstones of Achanarras in Caithness, and the Stromness beds of the Orkneys.

In conclusion, I wish to acknowledge my indebtedness to those who have assisted me in this work—to Professor JAMES GEIKIE, D.C.L., LL.D., F.R.S., without whose encouragement and advice it would never have been undertaken; to Dr R. H. TRAQUAIR, LL.D., F.R.S., who has determined for me all the more important specimens collected, and has kindly undertaken the description of the new material which turned up in the course of the investigation; to Messrs BENJAMIN PEACH, F.R.S., and JOHN HORNE, F.G.S., of the Geological Survey of Scotland, who have at all times placed at my service their great knowledge of field work, and their intimate acquaintance with the geology of the district. Mr JAMES W. CURSITER, F.S.A. Scot., of Kirkwall, kindly placed at my disposal his fine library of books relating to the county, and his collection of Orkney fossils; Mr THOMAS M'CRIE, of Kirkwall, allowed me also to examine his collection; and Mr MAGNUS SPENCE, of Deerness, gave me most valuable assistance in the field work in that district and elsewhere.

MAP OF THE GEOLOGY OF THE ORKNEYS

by John S. Flett.

Scale
0 1 2 3 4 5 6 miles
1 inch = 6 miles

3°

59

Mull Head
Bow Hd.
Noup Hd.
Rackwick
Head of Moclett
Russatarig
Bisgeos
Pierowall
Pt. of Cott
Sponess
Skelwick
Westray
Ingoness
Tuquoy Bay
Bakie Skerry
Stanger Hd.
Red Hd.
Twinness
Weathernes
Pt. of Huro
Fara
Cuthes Bay
Calf of Eday
Bologuoy
Lambaness
Millbay
London Bay
Skaill
Eday
Fers Ness
Fersness Bay
Spurness
Hackness
Sacquoy Hd.
Sauskaill Hd.
Bring Hd.
Sauskaill Bay
Quoynalonga Ness
Stockness
Seulskerry
ROUSAY
Egilsay
Costa Hd.
Scabra Hd.
Eynhallow
Crustan
Brough
Birsay B.
Barony
Swannay L.
Hundland
Loch
Marwick Hd.
EVIE
Woodwick
Hillside
Greenay Hill
BIRSAY
Outshare Pt.
Vestra Field
WEST
Doushy
Comgal
RENDALL
Bay of Skaill
Row Hd.
MAINLAND
SANDWICK
HARRAY
Borwick
Yesnaby
Inganess
Nebar
Seater
Black Craig
STENNESS
Breckness
Ireland
Ward Hill
The Ness
Hoy Sound
Stromness
Clestrom
Swanbister Bay
St. John's Hd.
Kame
Quoys Bay
Houton Hd.
Scapa Flow
Ward Hill
The Brunt
Old Man
Cava
Roray Hd.
Lyrawa B.
HOY
Rackwick
Lit. Rackwick
Hope
Long Hope
Hackness
Cantick Hd.
Torness
Ayth Hope
Wide Firth
Lobster Bay
Burness
Bay of Firth
Carness
Rainberry
Kirkwall
Bay
Holland Hd.
Yesnaby Hd.
Inganess Bay
Berwick Hd.
Mull Hd.
Brough
Sandside
Deer Sound
Deerness
Pool
Pt. of Ayre
Newark B.
Stembuster
Claisdie
Black Holm
Copinsay
Gaitnip
Deepdale
St. Mary's
Howequoy Hd.
Banks
Haas
Roseness
Burray
Ness
Echnaloch Bay
Calf of Flotta
Fara
Roan Hd.
Panhope
Hoxa
Water Sound
St. Mary's Hd.
Grimness
St. Peters Ch.
South Ronaldsay
Stews Hd.
Hesta Hd.
Windwick
Halcro Hd.
Old Hd.
Banksgea
Burwick
Brough Ness
Barth Hd.
Swora
Sandwick
Barswick
Roberm
Herston
Winewall
Shapinsay
Ness of Ork
The Galt
Veantrow Bay
Crook Bay
The Foot
Haco's Ness
Hd. of Work
Auskerry
North Ronaldsay
Torness
Dennis Hd.
Linklet Bay
Twiness
Brides Ness
Burrian Pt.
Holm of Ire
Whitemill Head
Tafts Ness
Sandoun Bay
Southvie Bay
Otters Wick
North B.
Brough N.
Lamness Bay
Start Pt.
Newark
Scuthi Hd.
Sanday
Tresness
Quoyness
Elsness
Huip
Linksness
Griceness
Lunga Holm
Mill Bay
St. Catherine's Bay
Odin Ness
Stronsay
Burgh Hd.
Holland Bay
Lamb Hd.
House Bay
Rothiesholm Hd.
Torness
Pentland Skerries
PENTLAND FIRTH

— Trap dykes
volcanic rocks
Old Red Sandstone (upper) — Hoy Beds
Old Red Sandstone (Orcadian) — John o'Groats Beds; Rousay Beds; Stromness Beds
granite gneiss etc.
Faults
Faults not seen
Dips

3°

A. RITCHIE & SON, EDIN.ᴿ

XIV.—*On Torsional Oscillations of Wires.* By Dr W. Peddie. (With Two Plates.)

(Read 20th June 1898.)

This paper is in continuation of two others, on the same subject, previously communicated to the Society. In the First Paper (*Philosophical Magazine*, July 1894) it was shown that the formula

$$y^n(x+a)=b,$$

where n, a, and b are constants in any one experiment, represents with accuracy the relation between y, the range of oscillation, and x, the number of oscillations which have taken place since torsion was first applied and the wire was left to itself, so that the oscillations gradually diminished. The apparatus employed, and the method of observation used, were identical with those described in the Second Paper above referred to. The wire which was experimented upon was the same as that used on the previous occasions. Its length, as given in the First and Second Papers, was 89·1 cm. A measurement made on the date 19.10.1897, in the course of the last series of experiments described in the present paper, showed that the length had become 89·3 cm. This increase was doubtless due to the fact that the heavy lead oscillator had been left attached to the wire during the whole of the intervening period. On the date given, it was also found that, with the same oscillator as was used in the experiments first described, ten oscillations were performed in 81 seconds, when the range was large, while 79 seconds were occupied when the range was small. This observation verified the result stated in the First Paper, that the period slightly increases as the range increases. It also showed that the wire was practically in the same condition as it was at first, in so far as elastic qualities are concerned; for the corresponding periods were only slightly less in earlier experiments, the difference being largely accounted for by the slight increase of length of the wire.

In the First Paper, the above equation was also deduced as an approximation, from the assumption that the defect of the potential energy of the system, at any given distortion, from the value which it would have had in accordance with Hooke's Law, was proportional to a power of the distortion. It was pointed out that the value of n seemed to approximate to zero when the range of oscillation was very small; and that, when n becomes zero, the equation changes form and becomes the well-known exponential equation, which was first proved by Lord Kelvin to hold when the oscillations are small.

An improved method of calculating the values of the quantities n, a, and b was

described in the Second Paper. That method was employed in the calculations to be given subsequently. Since

$$n \log y + \log (x+a) = \log b,$$

if $\log (x+\alpha)$ be plotted against $\log y$, the corresponding points lie on a straight line which intersects the axis along which $\log y$ is measured at an angle whose tangent is n—provided that the proper value of a is used. The value of b can then be obtained. If a wrong value of a be used, the points will not lie on a straight line. If too large a value of a is taken, the curve on which they lie is convex towards the origin; if too small a value is taken, the curve is concave towards the origin. In this way the true values of the constants are obtained in any experiment. Fig. 1 illustrates the method.

First Series of Experiments.

Previous attempts to separate the effects of the magnitude of the initial oscillation and of fatigue upon the values of the quantities n and b had not been successful. An attempt was therefore made to eliminate entirely the effect of magnitude of range by inducing very great fatigue in the wire. Before this was done a single experiment was made on the date 8.6.96, the wire having practically not been oscillated since the conclusion, on the date 24.12.95, of the third series of experiments described in the Second Paper. After the date 8.6.96, the wire was oscillated three or four times per week, by from 20 to 40 complete oscillations of large magnitude, until the date 10.7.96, when 150 large oscillations were given. Then, on the dates 14.7.96 and 15.7.96, respectively, 40 and 5 large oscillations were given. No readings of the decrease of range with increase of number of oscillations, when the wire was left to itself so that the oscillations died away, were taken on any of these occasions—the object being merely to induce excessive fatigue as a permanent condition in the wire. Such readings were taken on ten succeeding occasions. On each occasion the wire received 25 complete large oscillations, and was then brought to rest before being started anew in oscillation, when the readings were commenced.

Table I. gives the results obtained, the quantities a, n, and b being calculated in the manner already referred to. The magnitude of the initial range y_0 varied greatly in different experiments. The table also includes the results of the experiment made on the date 8.6.96. These show that the wire was practically in the same condition that it had been left in at the conclusion of the previous experiments. On the other hand, the results of the experiments made under conditions of great fatigue of the wire show a marked change in the state of the wire. The value of the product nb has attained a practically constant value, about equal to one-half of its previous value. The values of n and b are practically constant also, though the initial range varies greatly. The double sets of results given under two dates correspond to slightly different inclinations of the line in the diagram used to determine n and b.

Fig. 2 shows the result of taking $n = 1{\cdot}02$, $b = 98$, and choosing a for each experiment, so as to make the points taken from observation in each experiment lie, as far as possible, on a single curve. Ordinates (y) represent range of oscillation, and abscissæ represent number of oscillations (x) plus a. The diagram shows that an improvement might be made by taking n larger, the product nb being still kept equal to 100. The result is given in fig. 3, the value of n being 1·03, while that of b is 97. It appears from that figure that an increase of b would introduce further improvement. The result of making $n = 1{\cdot}03$ and $b = 100$ is shown in fig. 4. The closeness with which the points lie on the curve is quite sufficient to justify the adoption of the general equation

$$y^{1{\cdot}03}(x + a) = 100$$

to represent the results of the whole series of experiments. As a rule, the points which correspond to the first readings taken after the oscillations were started in each experiment are those which lie furthest off the curve. If the first readings were as accurate as the others we should have

$$a = 100\, y_0^{-1{\cdot}03}$$

where y_0 is the first reading. It is desirable to determine whether or not a slight modification of this expression for a will apply when the actually observed values of y_0 are used. The data below show that this is the case. The first row gives the observed values of y_0. The second gives the values of a, which were employed in order to make the points agree well with the curve shown in fig. 4. The third row gives the values of a, calculated by the above expression; the fourth gives the values of the differences between the observed and the calculated values of a; and the fifth gives the values of a, if we assume 1·4 to be the true value of that difference, and calculate a from the expression

$$a = 1{\cdot}4 + 100\, y_0^{-1{\cdot}03}.$$

The initial reading, 8·05, taken on the date 22.7.96, totally disagrees with the second, third, and subsequent readings, and seems to have been a mistake. A value 7·5 is much more in accordance with the others.

7·5	16·5	20·3	26·2	29	30	31·5	32·5	35·1	45·2
14·2	7	6	5	4·5	4·4	4	4	4	3·8
12·6	5·6	4·5	3·7	3·1	3·0	2·86	2·77	2·56	2·36
1·6	1·4	1·5	1·3	1·4	1·4	1·1	1·2	1·4	1·4
14·0	7	5·9	5·1	4·5	4·4	4·3	4·2	4·0	3·8

The numbers in the last row agree sufficiently well with those in the second to justify the adoption of the general formula

$$y^{1\cdot03}(x+1\cdot4+y_0{}^{-1\cdot03})=100$$

for the representation of the results of the whole series of experiments made under the condition of equal large fatigue.

Table II. contains a comparison, in the case of each experiment, of the results of observation with those of calculation. The middle column in each case contains the observed values of y, when x has successively the values 1, 2, 3, 5, 7, 10, 15, 20, 25, 30, 35, 40, 45, and 50. The numbers in the left hand column are those calculated for the same values of x, with the values of a, n, and b, given in Table I.; those in the right hand column are the corresponding values obtained by means of the general formula just given. The latter have been kindly calculated for me by Mr W. Thomson, formerly Donald Fraser bursar in the Physical Laboratory. In practically all cases, excepting the one in which the initial range had its largest value, the numbers in the third column agree at least as well with those in the second as do those in the first.

Discussion of the Initial Ranges in Previous Experiments.

If we take the data for the experiments detailed in Tables IV. and V. of the Second Paper (*Trans. R.S.E.*, 1896), and calculate from them, for these experiments, the values of p in the expression

$$y^n(x+p+b\,y_0{}^{-n})=b,$$

we get interesting evidence of the effect of magnitude of initial range and of fatigue upon the value of p. The results are given in Table III. In the first set, the initial range, y_0, is fairly constant. The numbers in the column headed N give the number of large oscillations to which the wire was subjected before readings were taken. These numbers, therefore, to some extent, indicate the amount of fatigue. They do not do so entirely, since the effect of previous fatigue persists to some extent from day to day. This is indicated by the smaller values of p on succeeding dates, when N had a given value. When fatigue is small, p bears a large ratio to a; when fatigue is great, p bears a small ratio to a.

In the second set, fatigue was practically constant while the initial range varied between wide limits. As was to be expected, p practially vanishes in comparison with a when the initial range is very small, so that the curve $y^n(x+\alpha)=b$ is very flat.

Re-calculation of Data in Table I. of the First Paper.

The values of n, a, and b, given in Table I. of the First Paper (*Philosophical Mag-*

azine, July 1894), were obtained by superposing the experimental curves upon sets of curves of the required form, and choosing the one which gave best correspondence. A re-calculation of the values, by the method now employed, was made, in order to get a strict comparison of the earlier results with those more recently obtained. Table IV. contains the values so found. The columns headed n', a', b' contain the values of the quantities n, a, and b given in the First Paper. The column headed b'' contains the values of b, calculated by the present method, with the old unit for y (0·364 times the new unit used in the Second Paper and the present paper). The columns headed n, a, and b give the values found by the present method in the new unit. The values of n and a are independent of the y–unit. Table VI. is, in part, a reproduction of Table II. of the First Paper. Values of y are given in the top row, and corresponding values of $x+a'$ are given in sets of three rows, each set corresponding to one experiment. The middle row of each set gives the experimentally observed values of $x+a'$; the upper row of each gives the values of $x+a'$ calculated by means of the values of n', a', and b', given in Table IV.; and the lower row gives the values of $x+a'$ calculated by means of the values of n, a', and b'', given in that table. The new values are, on the whole, just as suitable as the old values, and are accordingly used in the subsequent discussion.

Relations between n *and* b.

It was pointed out, in the Second Paper, that, throughout the three series of experiments therein described, the value of the product nb was, within possible experimental errors, constant. The basis for this statement is exhibited graphically in figs. 5, 6, 7. In these figures the values of log nb are plotted as ordinates against the values of n as abscissæ. The average values of log nb was in each case taken to be 2·3. By means of the re-calculated values of n and b for the series described in the First Paper, a similar diagram (fig. 8) was obtained for that series. With the single exception of experiment P, all the points group very well about a straight line having a positive slope. This implies the existence of a *Critical Angle* (see Second Paper) throughout the series of experiments described in the First Paper; so that, by a proper choice of the y–unit, the value of nb might have been made constant in that series also. For the equation

$$ny^n(x+a)=nb$$

may be written in the form

$$ny'^n(x+a)=nb\left(\frac{1}{k}\right)^n$$

by making $ky'=y$, *i.e.*, by taking as the unit a quantity k times greater than the

unit in terms of which y was measured. And, if we denote the quantity on the right hand side of the equation by B, we get

$$\log (nb) = \log \mathrm{B} + n \log k,$$

which, when k is constant, is the linear relation above referred to.

But the value of n is such, throughout each series of experiments, that it is impossible to determine whether that relation, or a linear relation between log b and n, is the more accurate. If one were strictly accurate in a given series, the other cannot be so simultaneously. Yet the possible variations in the determined values of n and b, for any experiment in a given series, are such that either relation may be regarded as practically correct. The results for the latter are exhibited graphically in figs. 9, 10, 11, and 12.

Just as the maintenance of a linear relation between log nb and n, in a given series, implies the existence, throughout that series, of a Critical Angle at which the loss of energy per oscillation is independent of n; so the maintenance of a linear relation between log b and n, in a given series, implies the existence, throughout that series, of an angle at which the loss of energy per oscillation varies inversely as n. For the equation

$$y^n(x+a) = b$$

may be put into the form

$$y'^n(x+a) = b\left(\frac{1}{k'}\right)^n$$

by taking as the y-unit a quantity k' times greater than the unit in terms of which y was measured. And k' can always be chosen so that the right hand side of the equation has a given constant value, β say. We then have

$$\log b = \log \beta + n \log k',$$

which, when k' is constant, is the second linear relation. Also

$$\frac{dy'}{dx} = -\frac{1}{n\beta} y'^{n+1}.$$

Hence, when y' is unity, *i.e.*, when $y = k'$, dy'/dx and $y'dy'/dx$ vary inversely as n, the latter quantity is practically proportional to the loss of energy per oscillation. For convenience of reference we may call k' the *Inverse Angle.*

Existence of an Oscillation Constant.

As we have just seen, we can always choose a unit k'', which will make the relation between y and x take the form

$$y^n(x+a) = \mathrm{A},$$

where A is an absolute constant. We may call this quantity, k'', the *Unifying Angle,*

since it gives the value of a y–unit, which, in each case, makes b take the absolutely constant value A. Its magnitude is given by the relation

$$k'' = \left(\frac{b}{\mathrm{A}}\right)^{\frac{1}{n}}.$$

If a simple expression such as this, connecting the Unifying Angle with the observed quantities n and b in each experiment, did not exist, we could not regard that angle as a quantity possessing any physical importance whatsoever. Indeed, we could not regard it as such unless the quantity A is found by experiment to correspond to some physical constant.

A glance at figs. 5–12 makes it apparent that, in each series of experiments, the lines representing the linear relations already discussed, pass with great accuracy through the point corresponding to $n = 1$, $\log b = 2{\cdot}3$. The value $b = 200$ is therefore of distinct physical importance in all the series. By giving A this value, and eliminating B and β from the linear equations, we get

$$k' = \left(\frac{b}{\mathrm{A}}\right)^{\frac{1}{n-1}},$$

and

$$k = \left(\frac{nb}{\mathrm{A}}\right)^{\frac{1}{n-1}}.$$

Thus the Inverse and Critical Angles have also simple expressions in terms of b and n.

The quantity A is an *Oscillation Constant* which depends essentially upon the material of which the wire is made. Further evidence regarding its constancy will be given immediately.

Second Series of Experiments.

In order to obtain further evidence on points already referred to, a second series of experiments, commencing on the date 14.10.97, was made. Between that date and the date 30.7.96, on which the first series was concluded, the wire had not been oscillated except on a few occasions in November 1896, and again in March 1897. The results are given in Table V.

At the end of the first experiment it was found that $36\frac{1}{2}$ full oscillations took place in 5 minutes when the oscillations were large, while 37 took place in the same time when the oscillations were small. At the end of the experiment dated 15.11.97 (1), 38 half oscillations took place in $2\frac{1}{2}$ minutes when the oscillations were small.

The values of a, n, and b, which are obtained when y_0 is very small, are extremely uncertain; yet there is no doubt that the value of n is considerably less than unity under that condition, and that the value of b is large.

In the earlier experiments of this series there is evidence that the wire had recovered to a slight extent from the state of fatigue induced in the first series. But

the subjection of the wire to a comparatively small number of full oscillations (given in brackets in Table V.) before an experiment was made, reduced n and b to values like those which were obtained in the first series. This was the case even when y_0 was comparatively small—see experiment 12.11.97 (1).

The most important object of the present series of experiments was to determine whether or not, under different initial conditions, points representing simultaneous values of log b and n still practically lay upon straight lines passing through the point (2·3, 1). This was found to be the case. At first the slope of the line was found to be positive, as it was in the experiments described in the First Paper. The slope of the line increased, under increased fatigue, until it became practically vertical. The wire was very sensitive to variations of fatigue, whether due to magnitude of initial range or to repeated oscillations. Increased fatigue causes an increase of n and a diminution of b: see, for example, experiments 11.11.97 (1) and (2); experiments 16.11.97 (1) and (2); and experiments 17.11.97 (1), (2) and (3).

Fig. 13 represents a number of the results graphically. The group of three points marked thus ⊙ corresponds to the first three experiments. The group marked × corresponds to the next nine experiments; those marked ⊡ correspond to the next ten; those marked v correspond to succeeding experiments in which fatigue was large; and those marked by single points correspond to some of the experiments in which fatigue was small. It is evident that the various groups throughout each of which fatigue was fairly constant are collected in the neighbourhood of straight lines passing through the point (2·3, 1). Variations may be due to slight differences of condition as to fatigue or to the fact that a is always chosen as a whole number, while the most suitable value may lie between two consecutive whole numbers. If, in any case in which a is small, an error of unity were made in the value of a, the corresponding value of n would change by 0·06 or 0·07, while the value of log b would only change by about 0·015 or 0·02. As an error of unity, when a is small, is impossible, it is evident that the grouping of the points round the lines cannot be regarded as accidental.

It therefore appears that the *Oscillation Constant*, A, *is truly a constant throughout all the treatment to which the wire has been subjected.*

Recovery from Fatigue.

The data given, Table V., show that the wire recovers partially from the effect of fatigue with considerable rapidity. Compare, for example, the data for the experiments 16.11.97 (2) and 25.11.97. This is most marked in the case of small oscillations—see 12.11.97 (1) and 17.11.97 (1), the former experiment being made immediately after heavy fatigue, while the latter was made one day after heavy fatigue.

There is another fact which may possibly bear on the question. In some of the

curves obtained by plotting log $(x+a)$ against log y, when the initial oscillation is small, though a straight line passes with considerable accuracy in the neighbourhood of the points, leaving as many points on one side as on the other on the average, yet almost absolute accuracy would be obtained by drawing *two* lines meeting at a very slight inclination—the smaller value of n corresponding to the smaller oscillations. The crossing point of these lines may possibly indicate an angle of torsion, such that molecular groups which break at a less angle have recovered from fatigue, while those which break at a greater angle have not yet recovered from fatigue. I first observed this in the experiment 17.11.97 (1), but it was found subsequently in other experiments, and had also occurred in previous experiments, as detailed below.

It first appeared in the experiment 3.11.97 (2) with $y_0 = 12{\cdot}8$, and it appears slightly also in the succeeding experiment 4.11.97 (1) with $y_0 = 20{\cdot}7$. It occurred also in the experiment 9.11.97. In the case of the three experiments of date 10.11.97, it appeared markedly in the first, very slightly, if at all, in the second, and not at all in the third—each experiment apparently aiding in its obliteration. The initial angles in these cases were 13·1, 11·0, and 11·2 respectively. It could not be said to be evident in the experiment 11.11.97 (2), $y_0 = 9{\cdot}3$, which followed immediately after the experiment 11.11.97 (1), $y_0 = 35{\cdot}6$; and it did not appear in the experiment 12.11.97 (1), $y_0 = 9{\cdot}4$, which was immediately preceded by 40 large oscillations. In the experiment 15.11.97 (1), $y_0 = 8{\cdot}6$, made after the wire had remained at rest for three days, it again appeared markedly, the point of junction of the two lines corresponding to an angle about one and a half times as large as that indicated in the experiment 10.11.97 (1). It could not be observed in the experiment 16.11.97 (1), which followed a large oscillation on the preceding day, though it would appear if a smaller value of a were chosen. But a smaller value of a would increase the value of n, and it is to be noticed that the values of n and b, found for that experiment and the preceding one, are abnormally large (see 18.11.97 (1)). As already mentioned, the peculiarity appears in the experiment 17.11.97 (1), $y_0 = 14{\cdot}3$, the wire having been considerably fatigued on the preceding day. It did not appear in the subsequent experiments on that date. It was evident in the experiment 18.11.97 (1), $y_0 = 9{\cdot}8$. In the succeeding experiment on the same date, $y_0 = 10$, it was also apparent, but the joining point of the lines occurred at a smaller angle. It could not be said to appear in any of the succeeding experiments. In these the initial range was very small, or very large; or, the initial range being of intermediate size, the experiments were made when the wire had been only slightly oscillated for some days, in which case the joining point might be expected to occur at smaller angles than those which were observed.

The phenomenon, although not very readily observed, occurs with such persistency that I scarcely think that it can be due to accidental causes. The facts that the joining point occurs at a larger angle when fatigue is small than when it is large, and that repetition of an experiment with small initial range makes the joining point pass to smaller angles, seem to indicate that there is a fairly sharply-marked limiting angle,

below which recovery from fatigue has proceeded to a greater extent than it has for larger angles of distortion.

Zero Effect of Period of Oscillation.

In order to determine whether or not the period of oscillation had any influence on the values of n and b, on the date 27.10.97, the large oscillator was replaced by the oscillator of smaller moment of inertia, which was used in the experiments described in the first paper. The results are given in fig. 14. A comparison of the results given in Table V., for the experiment 27.10.97 (2), with the results for previous experiments with the large oscillator, *e.g.*, with the results for the experiment 20.10.97, shows that no change by halving the period. With such speeds of oscillation we must therefore regard the results as independent of "after-action."

Law of Oscillation.

We have already found that the period of a complete oscillation is very nearly constant, being slightly greater for large oscillations than for small oscillations. Some additions were made to the apparatus in order to make possible determinations of the times of outward and inward motions over a given range. Fig. 17 shows the details. The torsion head, to which the upper end of the vertical wire is attached, is seen at the top of the diagram. The horizontal lead ring is seen attached to the lower end of the wire. A Wimshurst machine is seen on the left side of the wire. A vertical glass tube is seen at one extremity of a diameter of the lead ring. Its lower end is drawn to a fine point, and it is filled with a coloured liquid. A similar tube is placed at the other end of the diameter of the ring to secure symmetry in the oscillator. The liquid in the tube is placed, by means of a copper wire, visible in the diagram, in electric connection with the lead ring; and a copper wire also connects the torsion head (which is insulated by means of blocks of paraffin from the support to which it is clamped) to one pole of the Wimshurst machine. When the machine is worked, the liquid is driven out of the tube in a fine jet. On the right hand side of the diagram, at a lower level than the lead ring, are seen massive iron blocks, between which is clamped a horizontal steel wire, which is weighted at its outer end in order to give it a sufficiently long period of vibration. This wire supports a horizontal sheet of paper, which vibrates with the wire. If this paper be at rest while a torsional oscillation is given to the vertical wire under test, the jet of liquid will trace a circle on the paper. But if the paper now oscillates on the whole transversely to the motion of the jet, a waved curve will be traced, which crosses the circle at each semi-vibration. The interval of time between two successive crossings is constant (equal to the period of semi-vibration of the steel wire), and we can thus obtain a comparison of the times of outward and inward motions over a given range.

Two of these curves are shown in fig. 16. The part of a curve which corresponds to the outward motion can easily be distinguished from that which corresponds to the inward motion by its greater amplitude. In the first curve, 20 semi-vibrations take place in the range AB in the outward motion, while 20 take place in the range CA in the inward motion. The difference BC corresponds (allowing for the slight difference at the end A) to about one-third of a semi-vibration. Thus *the outward motion over the range* AC *occupies less time than the inward motion over the same range*, the difference being about 1 in 60.

Result of Heating the Wire to Redness.

[*Added* 18*th July* 1898.—It is to be expected that the molecular freedom which is introduced by heating the wire to redness will undo, to a great extent at least, the effect of fatigue. Before testing this point the wire was subjected to greater fatigue than on any previous occasion, and an experiment was then made on the date 1.7.98. The results were

$$a=4, \quad n=1{\cdot}015, \quad b=89{\cdot}6, \quad nb=91, \quad y_0=36{\cdot}7.$$

Thus by excessive fatigue the value of b was made smaller than it had ever been, while n, as formerly under such conditions, approximated to unity.

On the date 14.7.98 the wire was heated to redness by a Bunsen flame, the lead ring being removed to prevent stretching. An experiment was then made, and the results were

$$a=7, \quad n=1{\cdot}253, \quad b=680, \quad nb=852, \quad y_0=43{\cdot}4.$$

A comparison with the results given in the last column of Table IV. shows that b has become much more than twice as large as the greatest previous value.

It is interesting to compare this result with the results of two experiments made on the date 19.7.93, but not published in the first paper. In these experiments the wire hung inside a long solenoid composed of two similar coils of stout copper wire. In the first experiment a heavy current was run, in opposite directions, through the coils. The effect was to maintain the wire at a temperature of about 80° C. The results were

$$a=2, \quad n=1{\cdot}747, \quad b=536, \quad nb=936.$$

The difference between the conditions now considered and those above described is that now the wire is *maintained* at a comparatively high temperature during the experiment, while formerly it was heated to redness and was then experimented upon *when cold.* Though b is not quite so large in the latter case as in the former, n is considerably greater than formerly—so much so that nb is greater in the case now under discussion than in the other. Hence, when the temperature is maintained high, the loss of energy

per oscillation is much greater at large angles, much less at small angles, than it is when the temperature is normal, even after heating to redness.

In the second of the two experiments, performed immediately after the first, the only change made was that the current was sent in the *same* direction round the two coils. Thus, in addition to the maintenance of the wire at a temperature of about 80° C., *a steady state of magnetisation was maintained.* The results were

$$a=2,\quad n=2{\cdot}312,\quad b=2210,\quad nb=5110.$$

The effects just described are, therefore, in all respects greatly intensified. The molecular theory of magnetisation would lead one to expect decreased loss of energy at small angles, and increased loss at high angles, when the magnetisation is great.]

Theory of the Oscillations of an Imperfectly-Elastic Solid.

The first attempt at a theoretical investigation of the properties of a ductile solid was made by JAMES THOMSON (*Camb. and Dub. Math. Journ.*, 1848) in a paper "On the Strength of Materials, as influenced by the existence or non-existence of certain Mutual Strains among the Particles composing them." In applying his investigation to the case of torsion of a wire, he assumed that a certain definite tangential stress per unit area could be sustained without the production of permanent distortion, while an infinitesimal increase of the stress over this value caused continuous sliding until the stress diminished to the given definite value. In this way he explained the existence of elastic limits, and the greater strength of a wire as regards torsion in one direction or the opposite.

A mathematical development of MAXWELL'S views of the molecular constitution of a material substance is given by J. G. BUTCHER (*Proc. Lond. Math. Soc.*, vol. viii.) in a paper "On Viscous Fluids in Motion." In it, molecular groups are considered as consisting of two classes—those in which finite strain can be sustained without rupture, and those in which no strain can be sustained; and the properties of substances are regarded as depending upon the relative proportions in which those groups are present. The investigation deals only with those cases in which fluidity is manifest. The question of "elastic after-action" is included.

In the present investigation, the question of an imperfectly-elastic solid is alone considered, and elastic after-action is neglected. The case of torsion of a wire is explicitly developed. The fact that the period of oscillation had no effect on the experimental results obtained in the preceding part of the paper justifies the omission of the consideration of after-action in the application of the theory to these cases.

The time which elapses between the breaking down of a group and its formation into a new configuration is regarded as being zero in comparison with the time of motion of the wire through any finite range.

Consider unit length of the wire. Let ξ be the relative linear displacement per unit length at which a particular group breaks down, and let $\nu d\xi$ be the number of such groups which break in the increment of displacement $d\xi$. Then, in the element of volume $2\pi r dr$, the number $2\pi\nu r dr d\xi$ break down in the increment $d\xi$. Let θ be the angular distortion per unit length of the wire. Then $r\theta$ is the shear in the element of volume under consideration. Let

$$\xi = \frac{1}{m} r\theta, \quad \xi' = \frac{1}{m-1} r\theta,$$

where m is a whole number. If we assume that a group which breaks at the shear ξ is, on the average, formed again into a group which also breaks at the shear ξ, those groups which break at ξ and ξ' will also break at $r\theta$. Now take

$$\xi'' = \xi + p(\xi' - \xi) = \xi\left(1 + \frac{p}{m-1}\right),$$

where p is a proper fraction.

A group which breaks at ξ'', has had, when the total shear is $r\theta$, $m-1$ breaks, its last being at $(m-1)\xi'' = (m-1+p)\xi$. The shear to which it is subjected, when the total shear is $r\theta$, is therefore

$$(m-1)(\xi' - \xi'') = (1-p)\xi.$$

Hence, if we divide the shear $\xi' - \xi$ into an infinite number of equal parts $d\xi$, the average value of p is $\frac{1}{2}$, so that the average value of the stretch to which the group which breaks at ξ'' is subjected, when the total shear is $r\theta$, is $r\theta/2m$.

Now the number $2\pi\nu r dr d\xi$, when summed over the range corresponding to two consecutive values of m, becomes

$$\frac{2\pi\nu r dr}{m(m-1)} \cdot r\theta.$$

So, if the stress to which a group is subjected when it sustains a shear x is, on the average, kx, the total stress for the above number of groups is

$$\frac{\pi k\nu r^3\theta^2 dr}{m^2(m-1)}.$$

And the total stress due to groups which break at shears lying between 0 and $r\theta$ is

$$\theta^2\pi k\nu \sum_{2}^{\infty} \cdot \frac{1}{m^2(m-1)} \int_0^a r^3 dr = \frac{1}{4}\pi k\nu\theta^2 a^4 \sum_{2}^{\infty} \frac{1}{m^2(m-1)}, \quad . \quad . \quad . \quad . \quad (1)$$

where a is the radius of the wire, and ν and k are assumed to be constants.

If N be the total number of groups per unit volume, the number of unbroken groups is, in the volume $2\pi r dr$,

$$\left(\mathrm{N} - \int_0^{r\theta} \nu d\xi\right) 2\pi r dr;$$

and the total stress due to such groups is

$$\int_0^a (N - \nu r\theta) \cdot kr\theta \cdot 2\pi r dr = 2\pi k a^2 \left(\frac{N}{3} a\theta - \frac{\nu}{4} a^2\theta^2 \right). \quad . \quad . \quad . \quad . \quad . \quad (2)$$

The total force tending to diminish the torsion is therefore

$$\frac{2}{3}\pi k N a^2 (a\theta) - \frac{1}{4}\pi k \nu a^2 \left[2 - \sum_2^\infty \frac{1}{m^2(m-1)} \right] (a\theta)^2 .$$

The *single* force which, acting at the distance a from the axis, would equilibrate this is

$$\frac{1}{2}\pi k N a^2 (a\theta) - \frac{1}{5}\pi k \nu a^2 \left[2 - \sum_2^\infty \frac{1}{m^2(m-1)} \right] (a\theta)^2 .$$

$$= \frac{1}{2}\pi k N a^2 (a\theta) - \frac{1}{5}\pi k \nu a^2 \sum_1^\infty \frac{1}{m^2} \cdot (a\theta)^2 . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Hence *the deviation from Hooke's Law is represented by a negative term involving the square of the distortion*, provided that the quantity ν is constant.

But ν is the rate at which groups break down per unit change of distortion. Thus (3) gives the theoretical deviation from Hooke's Law *when the range of distortion at which a group breaks down is, on the average for all groups, uniformly distributed over all possible ranges.*

If ν were zero there would be no internal loss of energy in the wire; and, if the wire were once set in oscillation, the oscillations would, so far as this cause is concerned, continue for ever without any loss of amplitude. If ν is very small, the difference between the quantities of energy stored up in the wire in two successive maximum twists is practically proportional to ydy/dx, where y is the scale-reading and x represents number of oscillations, since Hooke's Law is nearly obeyed; and we can easily prove (see below) that the loss of energy in an outward oscillation is proportional to the cube of the distortion. Also, since, by our fundamental assumptions, every group which broke down at a certain stage in the outward motion breaks down again at the same point in the inward motion, the total loss of energy, in the form of heat, in the inward motion to the zero is equal to that in the outward motion from zero. Hence we get $-bdy = y^2 dx$, which gives

$$y(x + a) = b .$$

This is, as we have seen, precisely the equation which was found experimentally to connect range of oscillation with number of oscillations when the wire is greatly fatigued. If, therefore, our theoretical assumptions correctly represent the physical conditions, *the effect of great fatigue is to produce averagely uniform distribution of breaking range over all possible values.*

The apparatus which was used in the experimental investigations was not suitable for the purpose of testing the expression (3) directly in its application to the torsion of wires. Table VII. has been drawn up for me by Mr P. S. HARDIE, formerly Neil Arnott scholar in the Physical Laboratory, to test the applicability to the bending of bars of the equation

$$y = ax - bx^2,$$

where y represents distorting force and x represents distortion. The data used in the calculation are some of those given by HODGKINSON and FAIRBAIRN in the *B. A. Reports*, 1837. The columns headed x and y give observed values of these quantities; the columns headed y' give calculated values of y. The correspondence is extremely close, in some cases remarkably so, when it is considered that any flaw in the homogeneity of the material tends to introduce irregularities in the action under stress. Fig. 15 exhibits graphically the results in one case. The full curve represents a curve $y = ax - bx^2$, and the points on or near it are obtained from the experiments. The straight full line in the diagram represents the Hooke's Law line $y = ax$. The coordinate, $y = a^2/4b$, of the vertex of the parabola corresponds theoretically to the breaking stress. The material always, as is to be expected, breaks at a smaller stress.

We have now to investigate the inward motion. At any stage, all groups which give rise to an inward force in the outward motion give rise to the same inward force in the inward motion, provided that their last breaking-point has not been repassed. On the other hand, those groups whose last breaking-point has been repassed do not exert an inward force, but in general exert an outward force. Hence the inward force at any stage on the inward motion to zero is less than the inward force at the same stage on the outward motion. Thus we deduce at once from the theory the observed result that *the time of outward motion over a given range is less than the time of inward motion over the same range.*

Let us suppose now that the angular distortion ϕ, in the inward motion, has become less than half the maximum angular distortion θ. *Every* group which broke down in the outward motion is now exerting an outward force. In the volume $2\pi r dr$, since we are assuming that the breaking range of distortion for different groups is, on the average, uniformly distributed over all possible values, all groups which broke *first* between ϕ and θ are now exerting on the average an outward force $\frac{1}{2}kr(\theta - \phi)$. All those which broke at a range less than ϕ are now exerting an outward force which is proportional to the distance between $r\phi$ and their *last* breaking-point on the inward motion. To find the total value of this force, consider $m\xi = r\phi, (m-1)\xi' = r\phi$. A group which broke at

$$\xi'' = \xi + p(\xi' - \xi) = \frac{r\phi}{m}\left(1 + \frac{p}{m-1}\right)$$

had its nearest breaking-point outside $r\phi$ at $m\xi''$. Its distortion is therefore $m\xi'' - r\phi = pr\phi/(m-1)$. Now, at the fixed point $r\phi$, when ξ'' ranges over $\xi - \xi'$, p takes all values from 0 to 1 uniformly, so that its average value is $\frac{1}{2}$. Hence we find that the outward

pull exerted by all groups which broke first in the range $\xi' - \xi$ is

$$\int_0^a k\nu(\xi' - \xi)\frac{1}{2}\frac{r\phi}{m-1} \cdot 2\pi r dr = \frac{1}{4}\pi k\nu a^2(a^2\phi^2)\frac{1}{m(m-1)^2}.$$

Thus the total outward force due to those groups whose breaking-range ξ is less than $r\phi$ is

$$\frac{1}{4}\pi k\nu a^2(a^2\phi^2)\sum_2^\infty \frac{1}{m(m-1)^2} = \frac{1}{4}\pi k\nu a^2(a^2\phi^2)\sum_2^\infty \cdot \frac{1}{m^2}.$$

The single force, equivalent to this, acting at a distance a from the axis, is

$$\frac{1}{5}\pi k\nu a^2(a^2\phi^2)\sum_2^\infty \cdot \frac{1}{m^2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

The outward force due to groups which broke first between θ and ϕ is

$$\int_0^a \frac{1}{2}kr(\theta - \phi) \cdot 2\pi r dr \cdot \nu r(\theta - \phi) = \frac{1}{4}\pi k\nu a^2[a^2(\theta - \phi)^2].$$

Referred to a this becomes

$$\frac{1}{5}\pi k\nu a^2[a^2(\theta - \phi)^2]. \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

The whole inward force due to unbroken groups is

$$\int_0^a (\mathrm{N} - \nu r\theta) \cdot 2\pi r dr \cdot kr\phi = 2\pi k a^2(a\phi)\left[\frac{1}{3}\mathrm{N} - \frac{1}{4}\nu(a\theta)\right].$$

When referred to distance a this becomes

$$2\pi k a^2(a\phi)\left[\frac{1}{4}\mathrm{N} - \frac{1}{5}\nu(a\theta)\right]. \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

The total inward force is therefore

$$\frac{1}{2}\pi k\mathrm{N}a^2(a\phi) - \frac{1}{5}\pi k\nu a^2(a^2\phi^2) \cdot \sum_1^\infty \frac{1}{m^2} - \frac{1}{5}\pi k\nu a^2(a^2\theta^2). \quad . \quad . \quad . \quad . \quad (7)$$

By comparison of the expressions (3) and (7) we see that *when, in the inward motion, the range is less than half its maximum value, the inward force is less than the inward force at the same stage on the outward motion by an amount which depends only on the square of the maximum range.*

When, in the inward motion, the zero is reached, every group which has broken breaks and re-forms into its initial condition, so that the oscillation proceeds, as formerly, on the other side of the zero, but with less initial energy,—so giving rise to the lessening of amplitude.

Now, as a given increase of maximum range decreases the inward force at any stage of the inward motion more and more as that range is greater, the time of inward motion increases when the range increases. But the form of (3) shows that the time of outward motion is less when the range of oscillation is small than when it is large. Therefore *the period of complete oscillation is greater for large oscillations than for small.* This was shown in the first paper. KUPFFER pointed it out first in 1853.

The result that the zero of oscillation is a point at which groups re-form into their original condition explains the fact of the constancy of that zero which was found to obtain as oscillations proceed (see Second Paper).

The expression (7) vanishes when

$$\left(a\phi - \frac{5N}{4\nu\sum_1^\infty \frac{1}{m^2}}\right)^2 = \left(\frac{\cdot 5N}{4\nu\sum_1^\infty \frac{1}{m^2}}\right)^2 - \frac{a^2\theta^2}{\sum_1^\infty \frac{1}{m^2}} \quad . \quad . \quad . \quad . \quad . \quad (8)$$

This is, according to the theory, the relation which connects the angle of set with the angle of maximum twist, provided that the former does not exceed half the latter, and provided also that ν is constant—a condition which seems to hold, as we have seen, when the wire is greatly fatigued. This equation represents an ellipse whose semi-axes have a ratio of about 13 to 10, and would imply that the wire would flow round under the action of continued stress when the set equalled about ten-thirteenths of the distortion, if we could apply the equation to sets beyond half distortion (see Note).

If the inward motion were stopped just short of the zero, and the wire were then given an outward motion, the conditions differ from those in the first outward motion. When the angle reaches a value ψ, equation (6) gives the inward force due to unbroken groups if ϕ be replaced by ψ. With the same substitution, (5) represents the outward pull due to groups which broke first between ψ and θ. So also, ψ being substituted for θ, (1) gives the inward pull due to groups which broke between 0 and ψ. Hence, the expression in (1) being referred also to distance a from the axis, the total inward force in this case is

$$\tfrac{1}{2}\pi k N a^2(a\psi) - \tfrac{1}{3}\pi k\nu a^2\,(a^2\psi^2)\sum_2^\infty \frac{1}{m^2} - \tfrac{1}{3}\pi k\nu a^2\,(a^2\theta^2) \quad . \quad . \quad . \quad . \quad . \quad (9)$$

This differs from the expression (7) in the multiplier of the middle term. The value of $\sum_1^\infty \frac{1}{m^2}$ is very closely 5/3 and that of $\sum_2^\infty \frac{1}{m^2}$ is closely 2/3.

The expressions (3) and (9) have identical values when $\psi = \theta$, after which, the angle θ not being exceeded, the inward motion again obeys the law of force given

by (7); the next outward motion, the in motion being stopped just short of the zero, again obeys the law of force given by (9); and so on. By taking $\sum_{2}^{\infty}\frac{1}{m^2}$ instead of $\sum_{1}^{\infty}\frac{1}{m^2}$ in equation (8) we get an expression for the angle of set in the first part of the outward motion under these circumstances.

We can easily get a simple graphical construction for the two extreme positions of set. Plot forces as abscissæ and angles as ordinates. Draw the Hooke's Law line as indicated by the first term of (3). Draw also the parabolic curve given by (3), and the parabolic curve indicated by the first two terms of (9). Take three-fifths of the difference of abscissæ of the Hooke's Law line and the former parabola at the ordinate corresponding to the maximum angle θ, and plot it along the line of abscissæ. The ordinate drawn through the point so found intersects the two parabolæ at points whose ordinates are the extreme angles of set. The method is shown in fig. 15.

The dotted curve in fig. 15 is the second parabola above referred to, the full curve being the first. The position of set being taken as origin, the dotted curve does not greatly differ from a straight line, the deviations at the larger forces being in the direction of too great distortion. This result explains WIEDEMANN'S observation (*Philosophical Magazine*, vol. ix., 1880) that, *after a wire has been twisted a few times in opposite directions alternately by a given couple, and is then twisted by increasing couples in the direction of the last twist, Hooke's Law is nearly obeyed, provided the original couple is not exceeded, the slight deviations being in the direction of too great twist.*

In order to deduce the expression

$$y^n(x+a)=b$$

as the more general relation connecting range of oscillation with number of oscillations, we have only to assume that the quantity ν, employed in the preceding investigation, varies as a power of the strain. Take $\xi=r\theta/(m+p)$ where m is a whole number and p is a proper fraction; and, instead of ν, let us write

$$\nu\left(\frac{r\theta}{m+p}\right)^{\mu}$$

where ν and μ are regarded as constants. Each group which breaks at ξ has, when it breaks, potential energy $\frac{1}{2}k\xi^2$, which is transformed into heat. Also each such group, p varying from 0 to 1, breaks m times. Hence the heat developed in the range 0 to θ, is, in the volume $2\pi rdr$,

$$\tfrac{1}{2}km\int_{p=1}^{p=0}\left(\frac{r\theta}{m+p}\right)^{2}.\nu\left(\frac{r\theta}{m+p}\right)^{\mu}d\left(\frac{r\theta}{m+p}\right).2\pi rdr=\pi k\nu\frac{(r\theta)^{3+\mu}}{3+\mu}\frac{rdr}{m^{2+\mu}(m+1)^{3+\mu}}.$$

The total loss of energy is therefore

$$\frac{\pi k \nu a^2 (a\theta)^{3+\mu}}{(3+\mu)\,(5+\mu)} \sum_1^\infty \frac{m}{[m(m+1)]^{3+\mu}}.$$

If this loss is a small fraction of the whole energy we may write it proportional to $\theta d\theta/dx$, and, by integration, obtain, in the former notation, the result

$$y^{1+\mu}(x+a)=b\,.$$

The theory therefore indicates that *n is greater or less than unity, according as groups breaking at large distortions are more or less numerous than groups breaking at small distortions.*

We can easily, as above, determine the more general relation which connects set with torsion, but it is sufficient to note that the preceding considerations justify, from the point of view of theory, the adoption of the approximate expression used in the first paper on this subject, and that they are therefore justified, in turn, by the experimental confirmation therein given.

It is not to be supposed that the agreement of the results of the above theory with the results of observation necessarily proves the truth of the particular assumptions therein made. The object of the investigation is rather to show how well a theory based upon simple and reasonable assumptions concerning molecular statistics can account for general phenomena exhibited by imperfectly elastic solid media.

NOTE. *Added 6th October* 1898.

It is of interest to determine the general law of motion at all stages of the inward motion. Let θ and ϕ have the same meanings as formerly, and take

$$r\theta=(1+p)r\phi$$

with the condition

$$\frac{1}{p}=\mu+\lambda,$$

where μ is a whole number and λ is a proper fraction. Consider the various stages $r\phi/(m+1)$ to $r\phi/m$, where m is a whole number.

A group which breaks at

$$\frac{r\phi}{m+1}+x\frac{r\phi}{m(m+1)}$$

has its $(m+1)^{\text{th}}$ break at

$$r\phi+x\frac{r\phi}{m}.$$

For all values of x from 0 to 1 this point lies between $r\phi$ and $r\theta$, provided that we have

$$m > \frac{1}{p}.$$

When the stage ϕ on the inward motion is reached, all such groups exert outward force, and their average stretch is

$$\frac{1}{2}\left[\frac{m+1}{m} r\phi - r\phi\right] = \frac{1}{2}\frac{r\phi}{m}.$$

The total outward pull due to them is therefore

$$\sum_{\mu+1}^{\infty} \cdot \int_0^a 2\pi r dr \cdot \frac{1}{2} k \frac{r\phi}{m} \cdot \nu \frac{r\phi}{m(m+1)}, \qquad (10)$$

the summation being with respect to m.

When we have

$$m < \frac{1}{p},$$

we must take the fraction x so that its largest value is given by $r\phi + xr\phi/m = r\theta$, *i.e.*,

$$x = mp.$$

Then the number of groups

$$\nu mp \frac{r\phi}{m(m+1)} = \nu \frac{r\theta - r\phi}{m+1}$$

break in the range $r\phi$ to $r\theta$ with an average stretch $\frac{1}{2}(r\theta - r\phi)$. Hence their outward pull is

$$\sum_{1}^{\mu} \frac{1}{m+1} \cdot \int_0^a 2\pi r dr \cdot \frac{\nu}{2} k (r\theta - r\phi)^2. \qquad (11)$$

In the case of the remaining number

$$(1 - mp) \frac{\nu r\phi}{m(m+1)} = \left[\frac{r\phi}{m} - \frac{r\theta}{m+1}\right]\nu,$$

we have to consider the mth break. Now the mth break of a group which broke at $r\phi/(m+1) + mpr\phi/m(m+1)$ occurs at $mr\theta/(m+1)$, so that the average stretch for this number is

$$\frac{1}{2} m \left[\frac{r\phi}{m} - \frac{r\theta}{m+1}\right].$$

Hence the total inward pull of these groups is

$$\sum_{1}^{\mu} \cdot \int_0^a 2\pi r dr \cdot \nu \frac{1}{2} k m \left[\frac{r\phi}{m} - \frac{r\theta}{m+1}\right]^2 \qquad (12)$$

To these expressions we have to add the outward pull of groups which break only between $r\phi$ and $r\theta$. This is

$$\int_0^a 2\pi r dr \,.\, \tfrac{1}{2} k\nu (r\theta - r\phi)^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

By integration of the expressions (10), (11), (12), and (13), and by supposing, as formerly, that the forces act at a distance a from the axis, we find that the total inward force is

$$\tfrac{1}{5}\pi k\nu a^4 \left\{ \sum_1^\mu m\left[\frac{\phi}{m} - \frac{\theta}{m+1}\right]^2 - (\theta-\phi)^2 \sum_1^\mu \frac{1}{m+1} - \phi^2 \sum_{\mu+1}^\infty \frac{1}{m^2(m+1)} - (\theta-\phi)^2 \right\} + 2\pi k a^3 \phi \left[\frac{N}{4} - \frac{1}{5}\nu a\theta\right]$$

if we take account of the pull (6) due to unbroken groups. This can be put in the form

$$\tfrac{1}{2}\pi k N a^2 (a\phi) - \tfrac{1}{5}\pi k\nu a^2 (a^2\phi^2) \sum_{\mu+1}^\infty \frac{1}{m^2} - \tfrac{1}{5}\pi k\nu a^2 (a^2\theta^2) \sum_1^{\mu+1} \frac{1}{m^2} \quad . \quad . \quad . \quad (14)$$

which reduces, when we put $\mu = 0$, to the expression (7) applying to the second half of the inward motion.

The points $\left(1 - \frac{1}{m}\right)r\theta$ are points such that, in the intermediate ranges, the multipliers of the second and third terms in (14) remain constant. The sudden changes in the magnitudes of these terms are equal and opposite. For, when ϕ reaches the value $\mu\theta/(\mu+1)$, λ having become zero in the expression $1+\mu+\lambda$, μ is to be suddenly changed to $\mu-1$ in the affixes of the summations, so that the second term is suddenly increased by the amount

$$\tfrac{1}{5}\pi k\nu a^2 \left(a^2 \frac{\mu^2}{(\mu+1)^2}\theta^2\right)\frac{1}{\mu^2} = \tfrac{1}{5}\pi k\nu a^2 (a^2\theta^2) \frac{1}{(\mu+1)^2}$$

which is also the decrease of the third term. Thus the force varies continuously.

The amount by which (14) differs from (3) at any definite value of the angle is

$$-\tfrac{1}{5}\pi k\nu a^2 \left[\sum_1^{\mu+1} \frac{1}{m^2} \,.\, a\theta^2 - \sum_1^\mu \frac{1}{m^2} \,.\, a^2\phi^2\right].$$

This is therefore the continuously varying expression for the defect of the inward force at a given stage in the inward motion from the inward force at the same stage in the outward motion.

The limiting boundary of the space included by the series of ellipses represented by equating (14) to zero indicates the general relation between torsion and set when ν is constant. These ellipses intersect consecutively at points where $2\phi = \theta$, $3\phi = 2\theta$, $4\phi = 3\theta$, etc. At these points the rate of variation of set with torsion changes suddenly.

TABLE I.—*Results of the First Series of Experiments.*

Date.	a	n	b	nb	y_0
8.6.96	5	1·148	178	204	44·5
16.7.96	4	1·037	100	104	35·1
18.7.96	4	1·030	99·7	103	32·5
20.7.96	4	1·040	99	103	31·5
21.7.96	4	1·045	100	104	30·1
22.7.96	13	1·033	93·5	96·6	8·05?
23.7.96	7	1·040	101	105	16·5
23.7.96	7	1·020	98	100	16·5
24.7.96	3	1·067	99	106	45·2
27.7.96	5	1·000	97	97	29·0
27.7.96	4	1·060	100	106	29·0
28.7.96	5	1·017	100	102	26·2
30.7.96	6	1·033	100	103	20·3

TABLE II.—*Test of the Results of the First Series.*

16.7.96			18.7.96			20.7.96		
17·9	19·8	18·3	18·2	18·5	17·6	17·6	18·4	17·3
15·1	15·5	15·3	15·3	14·9	14·9	14·8	14·9	15·0
13·0	13·0	13·2	13·2	12·9	12·9	12·8	12·8	12·7
10·2	10·2	10·3	10·3	10·2	10·1	10·0	10·1	10·0
8·4	8·4	8·5	8·5	8·4	8·4	8·3	8·4	8·3
6·7	6·6	6·7	6·7	6·6	6·7	6·6	6·6	6·6
5·0	4·9	5·0	5·0	4·9	5·0	4·9	4·9	4·9
4·0	4·0	4·0	4·0	4·0	4·0	3·9	3·9	3·9
3·3	3·3	3·3	3·3	3·3	3·3	3·2	3·2	3·3
2·8	2·9	2·9	2·8	2·8	2·8	2·8	2·9	2·8
2·5	2·5	2·5	2·5	2·5	2·5	2·5	2·6	2·5

TABLE II.—*Continued.*

21.7.96			22.7.96			23.7.96		
17·5	18·4	17	6·3	6·4	6·2	11·4	11·8	11·6
14·7	14·8	14·4	5·9	5·9	5·9	10·2	10·4	10·4
12·7	12·8	12·5	5·5	5·6	5·5	9·2	9·4	9·4
10·0	10·0	9·9	5·0	4·9	5·0	7·7	7·7	7·8
8·3	8·3	8·2	4·6	4·4	4·5	6·7	6·6	6·7
6·6	6·4	6·6	3·9	3·9	4·0	5·5	5·4	5·6
4·9	4·9	4·9	3·2	3·2	3·3	4·3	4·3	4·3
3·9	3·9	3·9	2·7	2·7	2·8	3·6	3·6	3·6
3·3	3·3	3·3	2·4	2·4	2·5	3·0	3·0	3·0
2·8	2·9	2·8	2·1	2·2	2·2	2·6	2·7	2·6
2·5	2·6	2·5	1·9	2·0	2·0	2·3	2·4	2·3
2·2	2·3	2·2	1·7	1·9	1·8	2·1	2·1	2·1
2·0	2·0	2·0	1·6	1·8	1·7	1·9	1·9	1·9
1·8	1·8	1·9	...	...	...	...	...	...

23.7.96			24.7.96			27.7.96		
11·7	11·8	11·6	17·7	20·3	19·1	16·7	16·8	16·7
10·4	10·4	10·4	14·8	15·5	15·9	14·3	14·1	14·2
9·4	9·4	9·4	13·1	13·3	13·6	12·5	12·3	12·4
7·8	7·7	7·8	10·3	10·3	10·6	10·0	9·7	9·8
6·7	6·6	6·7	8·4	8·5	8·7	8·3	8·1	8·2
5·6	5·4	5·6	6·6	6·5	6·8	6·7	6·4	6·5
4·2	4·3	4·3	4·9	4·7	5·1	5·0	4·8	4·9
3·6	3·6	3·6	3·9	3·9	4·0	4·0	3 8	3·9
3·0	3·0	3·0	3·2	3·2	3·3	3·3	3·2	3·3
2·6	2·7	2·6	2·8	2·8	2·9	2·9	2·8	2·8
2·3	2·4	2·3	2·5	2·5	2·5	2·5	2·5	2·5
2·0	2·1	2·1	2·2	2·2	2·2	2·2	2·2	2·2
1·9	1·9	1·9	1·9	1·9	2·0	2·0	2·0	2·0
...	...	...	1·8	1·8	1·8	1·8	1·8	1·8

TABLE II.—*Continued.*

27.7.96			28.7.96			30.7.96		
16·7	16·8	16·7	15·9	16·7	15·1	13·1	13·4	13·4
14·1	14·1	14·2	13·1	13·7	13·0	11·5	11·7	11·8
12·2	12·3	12·4	12·0	11·9	11·5	10·3	10·4	10·5
9·6	9·7	9·8	9·8	9·5	9·3	8·5	8·5	8·6
8·0	8·1	8·2	7·5	8·2	7·8	7·2	7·1	7·3
6·4	6·4	6·5	6·4	6·3	6·3	5·9	5·8	6·0
4·8	4·8	4·9	4·9	4·7	4·7	4·5	4·4	4·6
3·8	3·8	3·9	4·0	3·8	3·8	3·7	3·7	3·7
3·2	3·2	3·3	3·2	3·1	3·2	3·1	3·1	3·1
2·8	2·8	2·8	2·8	2·8	2·8	2·7	2·7	2·7
2·4	2·5	2·5	2·5	2·5	2·4	2·4	2·4	2·4
2·2	2·2	2·2	2·2	2·2	2·2	2·1	2·0	2·2
2·0	2·0	2·0	2·0	2·0	2·0	1·9	1·9	2·0
1·8	1·8	1·8	...	...	...	1·7	1·7	1·8

TABLE III.—*Tests of Initial Deviations from Formulæ.*

Date.	y_0	N	p	a	Date.	y_0	p	a
16.7.95	37·1	1	2·13	6	9.12.95	37·2	0·35	3
17.7.95	51·3	10	2·06	4	12.12.95	36·8	1·25	3
18.7.95	44·4	20	2·00	4	17.12.95	14·2	0·00	9
19.7.95	41·2	30	2·49	5	18.12.95	14·3	−0·70	9
20.7.95	36·6	1	2·09	5	19.12.95	9·6	0·20	22
20.7.95	48·7	50	1·58	3	19.12.95	7·0	−0·20	25
20.7.95	39·7	1	1·47	4	20.12.95	5·3	0·80	80
22.7.95	40·0	80	1·29	3	20.12.95	3·0	0·00	120
23.7.95	42·0	120	0·73	2	24.12.95	1·6	−2·00	219
25.7.95	30·2	160	0·23	2	24.12.95	8·5	−0·30	47
26.7.95	38·7	1	1·98	4	...	...	...	...
26.7.95	43·9	200	0·93	2	...	...	...	...
27.7.95	41·5	50	0·95	2	...	...	...	...

TABLE IV.—*Re-calculated Data for Table I. in the First Paper.*

Date.	n'	a'	b'	b''	n	a	b
5.7.93	1·05	6·5	574	543	1·02	7·5	196
7.7.93	1·18	7·5	802	723	1·13	8·5	231
10.7.93	1·18	6·6	802	770	1·16	6·6	238
10.7.93	1·18	6·4	802	847	1·20	6·4	246
10.7.93	1·18	6·4	802	820	1·19	6·4	247
14.7.93	1·18	7·3	842	822	1·17	7·3	252
14.7.93	1·18	6·7	802	781	1·167	6·7	240
17.7.93	1·32	4·4	1074	1080	1·326	4·4	283
18.7.93	1·18	6·6	802	820	1·19	6·6	247
18.7.93	1·18	7·0	802	824	1·19	7·0	248
18.7.93	1·40	2·6	761	738	1·38	3·0	183

TABLE V.—*Data for Second Series of Experiments.*

Date.	a	n	b	nb	y_0
14.10.97	7	0·9	129	116	37·5
15.10.97	7	0·89	117	104	39·0
18.10.97	7	0·87	107	93	40·3
19.10.97	6	0·95	119	113	33·9
20.10.97	6	0·92	112	103	43·6
21.10.97	6	0·935	117	109	39·2
22.10.97	6	0·917	110	101	41·2
25.10.97 (1)	6	0·95	122	116	39·0
25.10.97 (2)	6	0·91	107	97	41·0
26.10.97	6	0·92	111	102	39·3
27.10.97 (1)	6	0·912	107	98	40·1
27.10.97 (2)	6	0·92	111	102	43·7
28.10.97	5	0·96	105	101	37·1 (N = 5)

TABLE V.—*Continued.*

Date.	a	n	b	nb	y_0
29.10.97 (1)	5	0·957	104	100	38·7 (N = 5)
29.10.97 (2)	5	0·957	101	96	38·3 (N = 10)
1.11.97 (1)	5	0·985	113	111	39·6
1.11.97 (2)	6	0·990	119	118	22·9
2.11.97 (1)	7	0·970	127	123	28·4
2.11.97 (2)	5	0·975	105	102	37·1
3.11.97 (1)	5	1·000	114	114	40·5
3.11.97 (2)	10	0·990	124	123	12·8
4.11.97 (1)	7	0·965	119	115	20·7
4.11.97 (2)	5	0·968	100	97	35·7 (N = 20)
5.11.97	4	1·022	100	102	37·1 (N = 40)
8.11.97 (1)	5	1·025	116	119	40·7
8.11.97 (2)	5	0·985	99	98	36·1 (N = 20)
9.11.97	12	0·992	148	147	12·7
10.11.97 (1)	12	1·010	165	167	13·1
10.11.97 (2)	17	0·913	152	139	11·0
10.11.97 (3)	16	0·900	147	132	11·2
11.11.97 (1)	5	1·012	116	117	35·6
11.11.97 (2)	15	0·950	125	119	9·3
12.11.97 (1)	11	1·008	101	102	9·4 (N = 40)
12.11.97 (2)	4	1·020	99	101	39·0
15.11.97 (1)	50	0·680	213	145	8·6
15.11.97 (2)	4	1·042	127	132	39·8
16.11.97 (1)	17	1·017	166	175	9·8
16.11.97 (2)	4	1·030	101	104	36·5 (N = 60)
17.11.97 (1)	11	0·953	134	128	14·3
17.11.97 (2)	10	0·982	136	134	12·2
17.11.97 (3)	4	1·030	106	109	32·2
18.11.97 (1)	30	0·857	151	129	9·8
18.11.97 (2)	18	0·950	159	152	10·0
19.11.97	220	0·270	562	152	4·3
22.11.97 (1)	60	0·523	313	164	9·6
22.11.97 (2)	25	0·695	164	114	15·2
23.11.97	20	0·740	160	118	17·0
24.11.97	8	0·925	135	125	29·9
25.11.97	6	0·968	123	119	38·0
14.12.97	300	0·590	655	386	3·5
15.12.97	6	1·010	144	145	34·3
9.2.98	220	0·363	600	218	4·5

TABLE VI.—*Former and Re-calculated Data for Table II. in the First Paper.*

Date.	65	60	55	50	45	40	35	30	25	20	17	15	12	10	9	8	7	6	5	4
		7·7	8·5	9·3	10·5	11·7	13·7	16·0	19·5	24·7	28·5	33·2	42·1	51·0	57·1	64·5	…	…	…	…
5.7.93		8·0	8·5	9·2	10·2	11·5	13·3	16·0	19·6	24·6	28·9	33·3	42·0	51·6	57·6	65·6	…	…	…	…
		7·3	9·0	9·4	10·2	11·6	13·4	15·9	19·4	24·6	29·2	33·3	42·1	50·9	56·7	64·1	…	…	…	…
				7·9	8·9	10·3	12·1	14·5	18·0	23·4	27·6	32·9	42·7	53·0	60·0	69·0	…	…	…	…
7.7.93				8·0	8·9	10·3	12·1	14·9	18·2	23·4	28·1	32·4	42·5	53·0	60·0	69·5	…	…	…	…
				7·7	8·8	10·2	12·0	14·5	18·0	23·5	28·4	32·9	42·6	52·6	59·4	68·0	…	…	…	…
			7·1	7·9	8·9	10·3	12·1	14·5	18·0	23·4	27·6	32·9	42·7	53·0	60·0	69·0	…	…	…	…
10.7.93			7·4	8·0	8·9	10·3	12·1	14·8	18·3	23·3	28·0	32·1	41·6	52·6	60·4	69·0	…	…	…	…
			7·4	8·2	9·3	10·7	12·5	14·9	18·4	23·9	28·8	33·3	43·1	53·3	60·7	69·9	…	…	…	…
			7·1	7·9	8·9	10·3	12·1	14·5	18·0	23·4	27·6	32·9	42·7	53·0	60·0	69·0	…	…	…	…
10.7.93			7·0	7·8	8·7	10·0	11·9	14·4	18·1	23·4	28·0	32·7	42·5	53·4	60·4	70·4	…	…	…	…
			6·9	7·7	8·8	10·1	11·9	14·3	17·8	23·3	28·3	32·9	42·9	53·4	60·7	69·9	…	…	…	…
			7·1	7·9	8·9	10·3	12·1	14·5	18·0	23·4	27·6	32·9	42·7	53·0	60·0	69·0	…	…	…	…
10.7.93			7·0	7·8	8·7	10·0	11·8	14·5	18·0	23·3	27·8	32·1	42·2	52·4	60·4	70·4	…	…	…	…
			7·0	7·8	8·8	10·2	11·9	14·3	17·8	23·2	28·2	32·7	42·6	52·9	61·4	70·6	…	…	…	…
			7·4	8·3	9·3	10·8	12·7	15·2	18·9	24·5	29·0	34·5	44·8	55·6	63·0	72·4	…	…	…	…
14.7.93			8·0	8·6	9·5	10·8	12·7	15·4	19·1	24·6	29·4	34·3	44·7	55·8	62·8	73·3	…	…	…	…
			7·6	8·5	9·6	11·0	12·8	15·4	19·0	24·7	29·9	34·5	44·9	55·5	62·9	72·2	…	…	…	…
			7·1	7·9	8·9	10·3	12·1	14·5	18·0	23·4	27·6	32·9	42·7	53·0	60·0	69·0	…	…	…	…
14.7.93			7·4	8·0	8·8	10·2	12·1	14·7	18·4	23·6	28·7	33·2	43·1	53·3	60·0	69·7	…	…	…	…
			7·3	8·1	9·2	10·5	12·3	14·8	18·2	23·7	28·6	33·1	43·0	53·2	60·1	69·0	…	…	…	…
	4·35	4·8	5·3	6·1	7·1	8·2	9·85	12·0	15·3	20·5	24·8	30·1	40·4	51·4	59·0	69·0	82·3	…	…	…
17.7.93	4·60	4·8	5·2	5·9	6·9	8·1	9·80	12·0	15·1	20·5	25·0	29·6	39·7	51·0	58·4	69·4	81·4	…	…	…
	4·3	4·7	5·3	6·0	6·9	8·1	9·7	11·9	15·1	20·3	25·2	29·8	40·0	51·0	58·7	68·5	81·8	…	…	…
			7·1	7·9	8·9	10·3	12·1	14·5	18·0	23·4	27·6	32·9	42·7	53·0	60·0	69·0	…	…	…	…
18.7.93			7·3	7·9	8·8	10·0	11·9	14·5	18·0	23·6	27·2	32·3	42·6	52·9	60·1	69·1	…	…	…	…
			7·0	7·8	8·8	10·2	11·9	14·3	17·8	23·2	28·2	32·7	42·6	52·9	61·4	70·6	…	…	…	…
			7·1	7·9	8·9	10·3	12·1	14·5	18·0	23·4	27·6	32·9	42·7	53·0	60·0	69·0	…	…	…	…
18.7.93			7·4	7·9	8·7	10·2	11·7	14·4	18·0	23·4	28·3	32·5	42·5	53·0	60·0	70·0	…	…	…	…
			7·0	7·8	8·9	10·2	12·0	14·4	17·9	23·3	28·3	32·8	42·8	53·2	61·7	71·0	…	…	…	…
			2·8	3·2	3·7	4·3	5·2	6·4	8·4	11·7	14·0	17·2	23·4	30·3	35·1	41·4	49·9	61·9	79·9	109·3
18.7.93			2·7	3·1	3·5	4·1	5·2	6·6	8·8	11·6	14·2	17·1	23·6	30·0	33·6	40·6	49·6	61·6	80·6	109·0
			2·5	2·9	3·5	4·1	5·1	6·4	8·3	11·4	14·4	17·2	23·5	30·4	35·2	41·5	49·9	61·9	79·9	108·6

TABLE VII.—*Results for Hodgkinson's and Fairbairn's Experiments.*

H. Exp. I, 1.			H. Exp. I, 2.			H. Exp. I, 3.		
x	y	y'	x	y	y'	x	y	y'
3·7	3·2	3·17	5·1	4·6	4·34	3·8	3·2	3·32
5·2	4·6	4·34	6·7	6·0	5.68	5·2	4·6	4·52
7·0	6·0	5·83	12·9	11·2	10·91	7·0	6·0	6·06
13·2	11·2	10·90	26·1	22·4	21·63	13·3	11·2	11·45
27·1	22·4	21·48	56·1	44·8	44·80	27·6	22·4	22·76
58·8	44·8	45·80	90·0	67·2	68·76	59·8	44·8	45·47
94·0	67·2	70·02	129·7	89·6	93·99	95·8	67·2	65·95
136·0	89·6	95·60	...	...	...	138·8	89·6	83·60
$a=0·839$ $b=0·001$			$a=0·855$ $b=0·001$			$a=0·880$ $b=0·002$		

H. Exp. I, 4.			H. Exp. I, 5.			H. Exp. I, 8.		
x	y	y'	x	y	y'	x	y	y'
1·5	2	1·83	2·5	5	3·78	7	8	7·74
3·2	4	3·87	4·5	7·5	6·79	11	12	11·93
4·6	6	5·55	6·5	10	9·79	15	16	15·98
13·0	16	15·53	13·4	20	20·12	24	24	24·48
27·3	32	31·07	27·0	40	40·59	33	32	32·18
44·4	48	48·25	58·0	80	81·43	44	40	40·45
61·8	64	63·94	89·5	120	119·98	50	44	44·50
81·3	80	79·36	122·4	160	156·04	53	45	46·37
103·0	96	93·9	158·5	200	189·76	...	...	...
$a=1·22$ $b=0·003$			$a=1·52$ $b=0·002$			$a=1·14$ $b=0·005$		

H. Exp. I, 9.			H. Exp. I, 13.			H. Exp. II, 1.		
x	y	y'	x	y	y'	x	y	y'
7	8	7·88	8·5	10·82	10·75	3·3	2·2	2·2
10·2	12	11·35	10·6	13·43	13·19	6·2	4·2	4·2
14	16	15·38	13·0	16·05	15·85	12·0	8·0	8·0
22	24	23·43	15·6	18·66	18·62	24·0	16·0	15·8
31	32	31·90	12·5	21·26	23·15	37·0	24·0	23·8
40	40	39·82	21·2	23·88	24·13	51·0	32·0	32·3
51	48	47·55	24·3	26·49	26·72	64·9	40·0	40·3
62	56	56·11	27·2	29·10	28·33	79·8	48·0	48·4
...	...	...	30·7	31·72	32·02	95·3	56·0	56·5
...	...	...	34·0	34·33	34·44	112·0	64·0	64·8
...	...	...	37·8	36·94	36·83	131·0	72·0	73·5
$a=1·153$ $b=0·004$			$a=1·35$ $b=0·01$			$a=0·679$ $b=0·0009$		

TABLE VII.—*Continued.*

H. Exp. II, 7.

x	y	y'
2·1	4	4·47
3·0	6	6·29
4·0	8	8·25
5·0	10	10·13
6·0	12	11·95
7·1	14	13·86
8·2	16	15·62
10·9	20	19·81
13·9	24	23·89
17·6	28	27·81
23·0	32	31·94
29·5	36	34·14
31·5	37	34·23
...	...	...
...	...	...
...	...	...
...	...	...
...	...	...
...	...	...

$a = 2·205$ $b = 0·0355$

H. Exp. II, 8.

x	y	y'
7	8	8·04
10·5	12	11·96
12	14	13·56
14·5	16	16·22
18	20	19·84
22	24	23·81
26	28	27·65
31	32	31·86
41	40	40·65
45	44	43·75
51	48	48·10
56	52	51·47
62	56	55·20
	...	...
...	...	...
...	...	...
...	...	...
...	...	...
...	...	...

$a = 1·188$ $b = 0·0048$

H. Exp. III, 3.

x	y	y'
4	8	7·8
8	16	15·4
12	24	22·9
17	32	31·9
22	40	40·7
26	48	47·5
31	56	56·0
36	64	63·8
42	72	73·0
47	80	80·4
52	88	87·5
58	96	95·6
64	104	103·4
71	112	111·9
79	120	121·0
85	128	127·4
96	136	137·9
105	146	145·6
116	154	153·6

$a = 1·974$ $b = 0·0056$

H. Exp. III, 4.

x	y	y'
6·8	8·96	9·87
7·3	10·08	11·56
7·7	10·82	11·12
9·4	13·06	13·62
10·8	15·30	15·26
12·5	17·54	16·59
14·7	19·78	20·23
16·2	22·02	22·06
18·0	24·26	24·19
20·0	26·50	26·50
21·8	28·74	28·50
24·0	30·98	30·89
26·1	33·22	33·06
28·6	35·46	35·53
31·0	37·70	37·80
33·3	39·94	39·87
36·0	42·18	42·17
38·8	44·42	44·41
42·0	46·66	46·78
45·0	48·90	48·82
49·5	51·14	52·57
53·0	53·38	53·44

$a = 1·517$ $b = 0·0096$

H. Exp. V, 1.

x	y	y'
31	22·1	22·50
68	45·0	45·37
114	67·0	67·67
141	78·0	77·70
171	84·6	84·90

$a = 0·776$ $b = 0·0016$

H. Exp. VI, 1.

x	y	y'
28·0	32	31·18
61·2	64	64·11
100·0	96	97·00
122·0	112	113·00
146·0	128	120·50

$a = 1·17$ $b = 0·002$

H. Exp. VI, 3.

x	y	y'
27·5	32	30·7
61·0	64	63·9
100·0	96·2	97·0
123·0	112	113·7
149·0	128	129·8

$a = 1·17$ $b = 0·002$

F. Exp. I, 3.

x	y	y'
7·2	8	8·5
12·5	16	14·0
26·9	32	31·0
42·0	48	47·3
58·4	64	64·2
74·8	80	80·4
92·4	96	96·7
110·5	112	112·4
131·5	128	129·4

$a = 1·196$ $b = 0·0016$

TABLE VII.—*Continued.*

F. Exp. I, 4.			F. Exp. II, 3.			F. Exp. II, 4.		
x	y	y'	x	y	y'	x	y	y'
2·8	4	3·98	3·1	4	3·66	3	4	3·70
6·0	8	8·11	7·0	8	8·00	6·6	8	7·90
9·2	12	12·10	10·9	12	11·98	10·3	12	11·95
12·5	16	15·99	15·2	16	16·04	14·4	16	16·11
16·2	20	20·46	20·0	20	20·20	18·8	20	20·21
20·3	24	24·24	25·1	24	24·05	23·8	24	24·39
24·2	28	27·94	30·7	28	27·66	29·0	28	28·22
29·0	32	31·84	34·3	30	29·52	35·5	32	32·18
31·6	34	33·78	...	...	...	39·0	34	33·78
$a=1·417$ $b=0·011$			$a=1·214$ $b=0·0102$			$a=1·263$ $b=0·01$		

F. Exp. III, 3.			F. Exp. III, 4.			F. Exp. IV, 3.		
x	y	y'	x	y	y'	x	y	y'
3·0	4	3·62	3·1	4	4·11	3·7	4	4·53
6·8	8	8·06	6·0	8	7·86	7·3	8	8·03
10·2	12	11·80	9·2	12	11·95	10·9	12	12·01
14·0	16	15·96	12·2	16	15·75	14·7	16	16·25
17·8	20	19·88	15·6	20	20·70	18·2	20	20·76
21·7	24	23·74	18·9	24	24·01	22·1	24	23·84
30·0	33	31·32	22·1	28	27·86	26·0	28	27·84
34·9	37	35·41	30·0	36	37·11	30·2	32	32·09
37·7	39	37·62	34·0	40	41·64	34·9	36	36·76
40·8	41	40·07	...	...	...	37·8	38	39·60
43·9	43	42·15	...	...	...	...	...	...
$a=1·224$ $b=0·006$			$a=1·327$ $b=0·003$			$a=1·123$ $b=0·002$		

F. Exp. IV, 4.			F. Exp. V, 4.			F. Exp. VI, 4.		
x	y	y'	x	y	y'	x	y	y'
3·5	4	3·81	3·2	4	3·97	3·3	4	3·86
7·0	8	7·73	6·6	8	7·79	7·0	8	7·91
10·8	12	11·81	10·1	12	11·60	11·0	12	11·99
14·6	16	15·79	14·1	16	16·10	15·3	16	16·02
18·3	20	19·58	18·1	20	20·04	20·0	20	20·00
22·0	24	23·41	22·9	24	24·37	25·0	24	23·75
26·1	28	27·25	27·6	28	29·20	30·6	28	27·36
30·4	32	31·53	33·0	32	32·11	37·2	32	31·27
32·8	34	33·69	35·5	33	33·75	...	...	...
35·2	36	35·89	...	...	...	...	...	...
38·0	38	38·42	...	...	...	...	...	...
$a=1·125$ $b=0·003$			$a=1·27$ $b=0·009$			$a=1·2$ $b=0·01$		

TABLE VII.—*Continued.*

F. Exp. VII, 3.			F. Exp. VIII, 4.			F. Exp. IX, 2.			F. Exp. X, 1.		
x	y	y'	x	y	y'	x	y	y'	x	y	y'
7·0	8	8·43	3·2	4	3·66	6·7	8	7·98	7·9	8	8·19
13·8	16	16·23	6·1	8	7·92	13·3	16	15·99	16·0	16	16·33
27·0	32	31·52	9·7	12	11·97	26·7	32	30·75	32·6	32	32·31
42·2	48	47·82	12·8	16	16·22	42·1	48	48·19	50·7	48	48·24
58·7	64	64·60	16·1	20	19·95	58·9	64	64·04	70·0	64	64·78
74·9	80	80·02	19·8	24	24·11	76·7	80	80·68	90·9	80	79·19
92·8	96	96·78	23·2	28	27·88	96·1	96	97·33	114·1	96	94·08
112·2	112	111·40	27·3	32	32·00	117·7	112	113·90	142·8	112	109·60
...	...	...	29·3	34	33·99	142·0	128	130·78	157·1	118	116·00
...	...	...	31·8	36	35·43	155·4	136	136·94	...	...	...
...	...	...	34·0	38	38·49	...	...	...	...	...	...
$a=1·218$ $b=0·002$			$a=1·336$ $b=0·006$			$a=1·205$ $b=0·002$			$a=1·053$ $b=0·002$		

PLATE I.

FIG. 1.

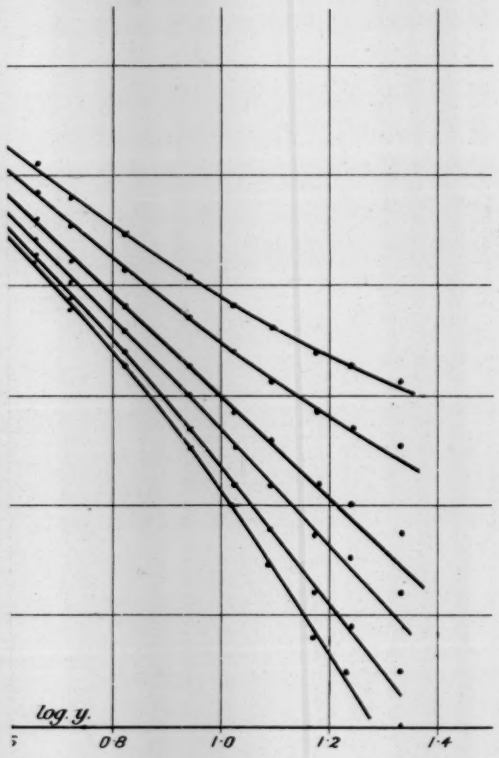


FIG. 2.

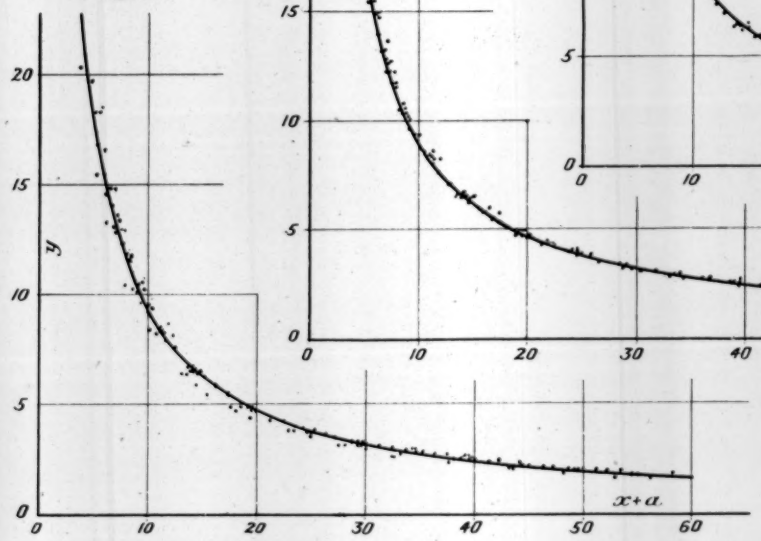


FIG. 3.

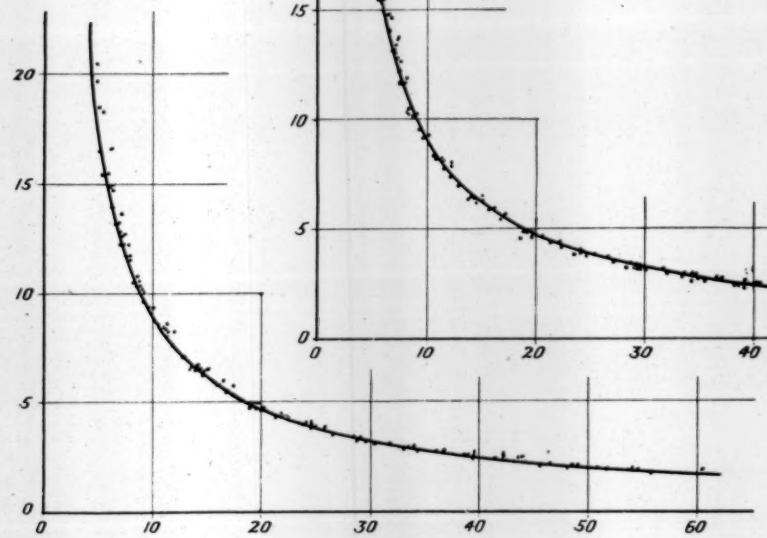
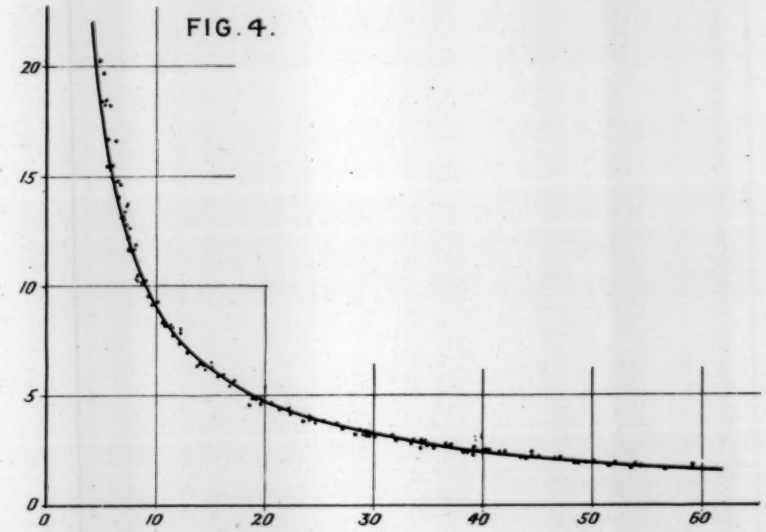


FIG. 4.



5.

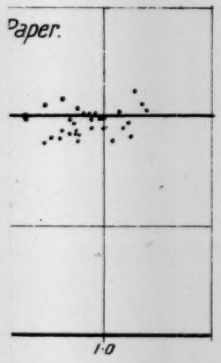


FIG. 7.

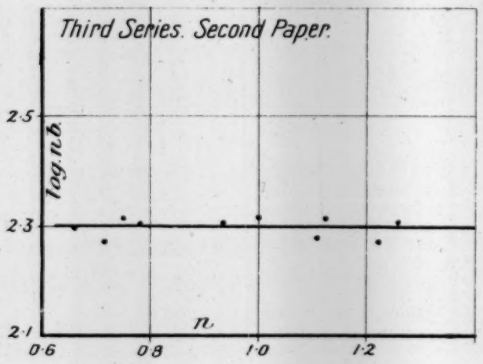


FIG. 9.

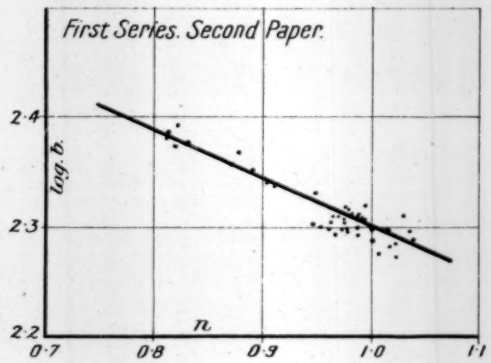
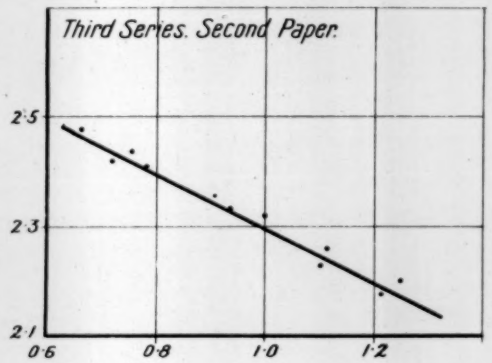


FIG. 11.



6.

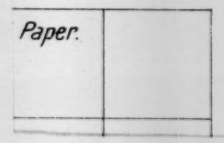


FIG. 8.

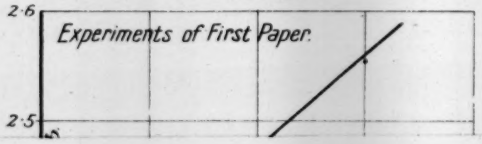


FIG. 10.

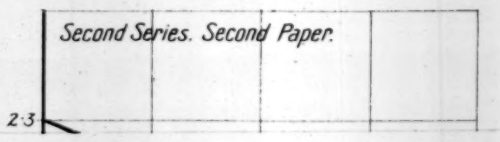
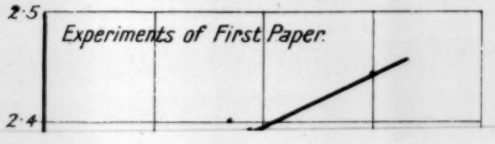


FIG. 12.



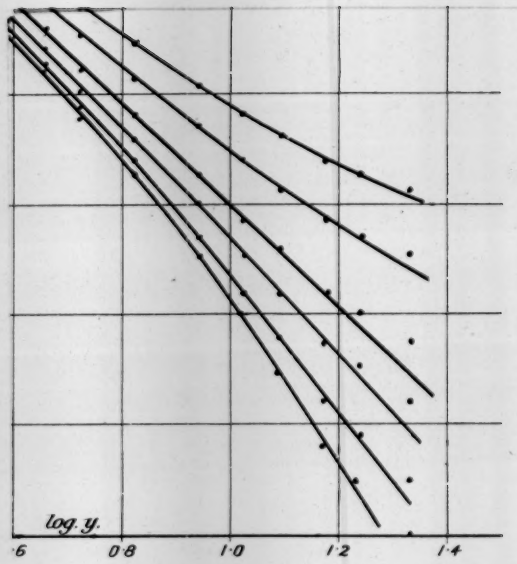
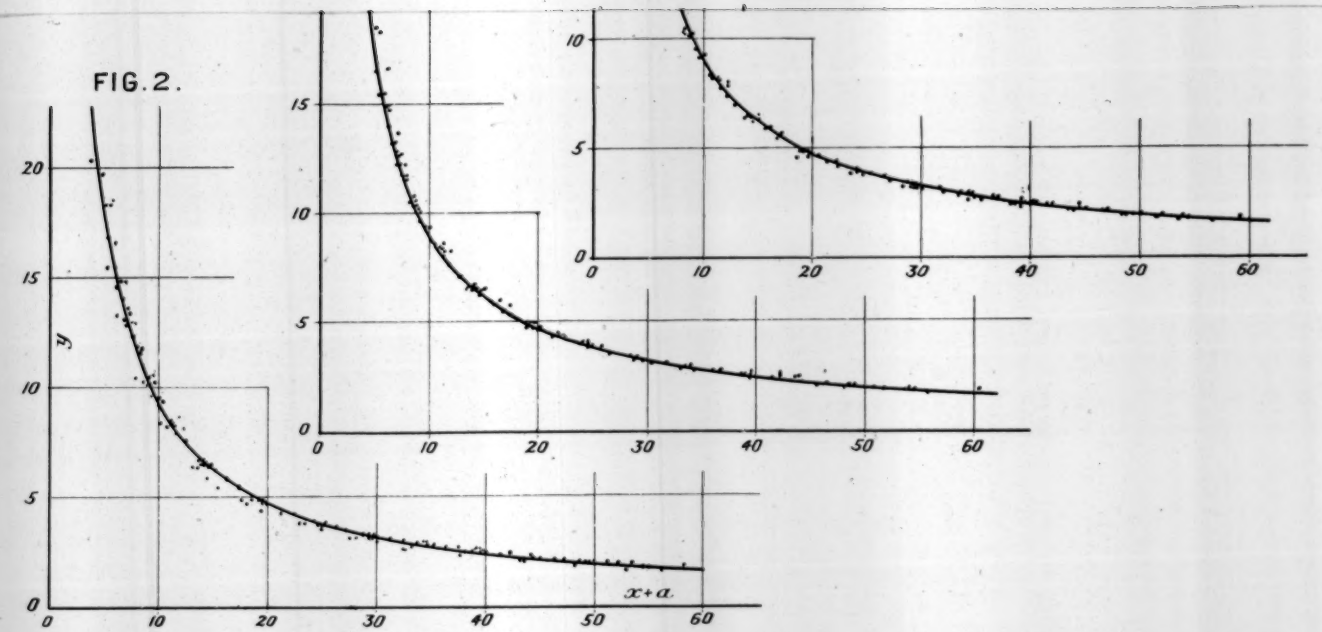


FIG. 2.



. 5.

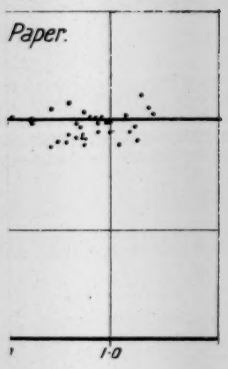


FIG. 7.

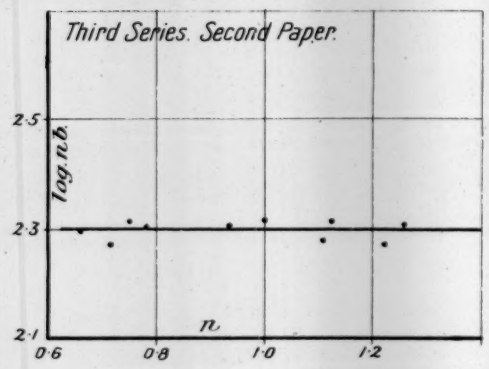


FIG. 9.

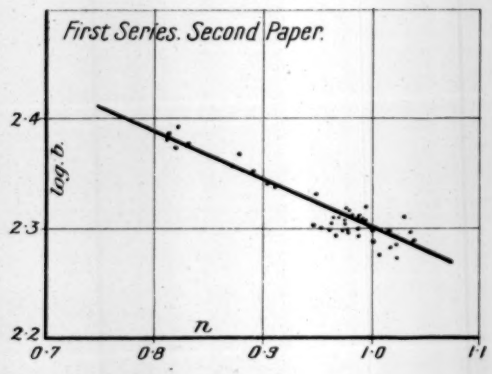
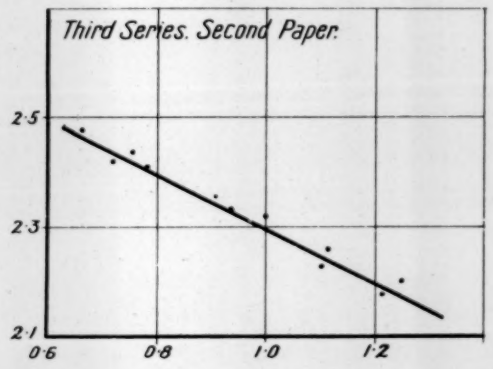


FIG. 11.



6.

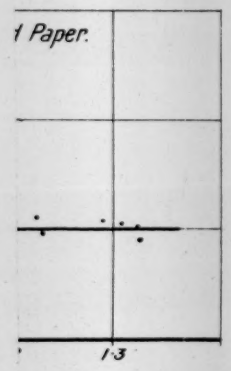


FIG. 8.

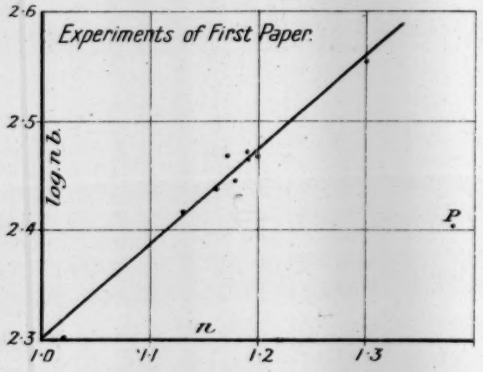


FIG. 10.

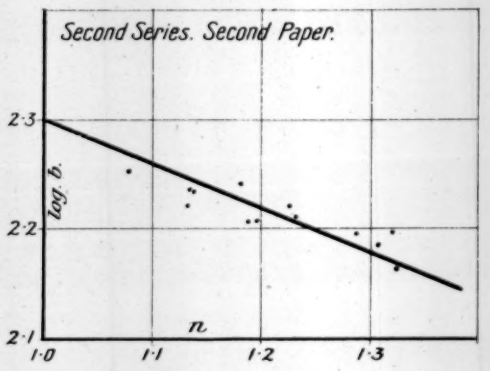


FIG. 12.

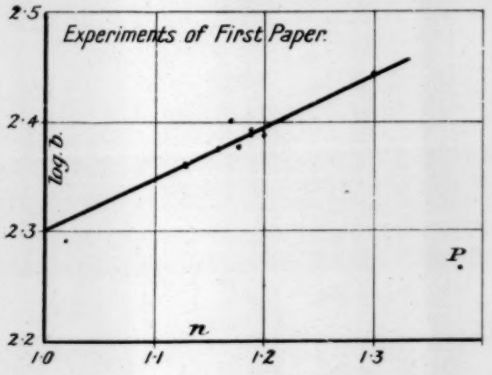


PLATE I.

FIG. 1.

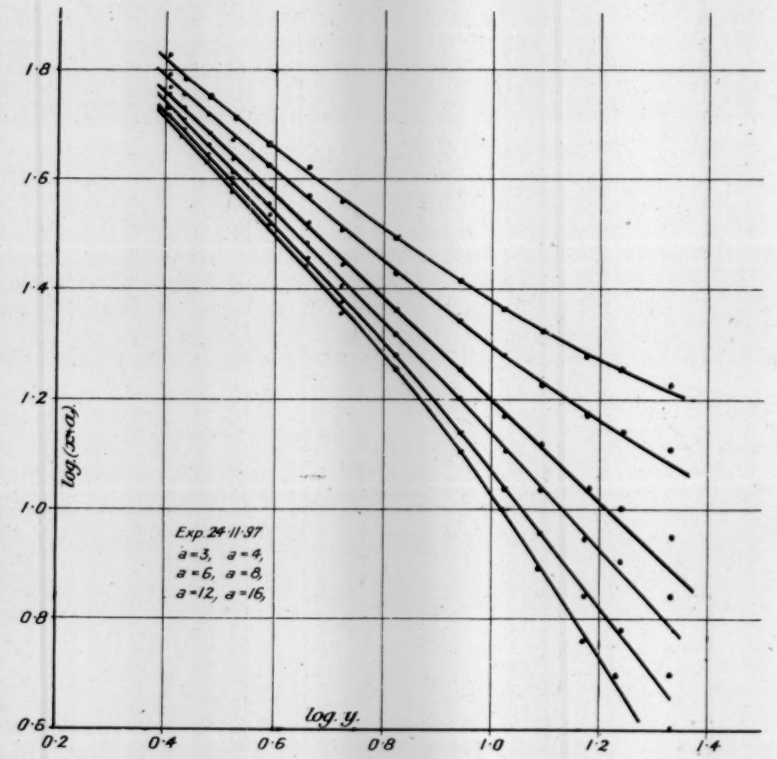


FIG. 2.

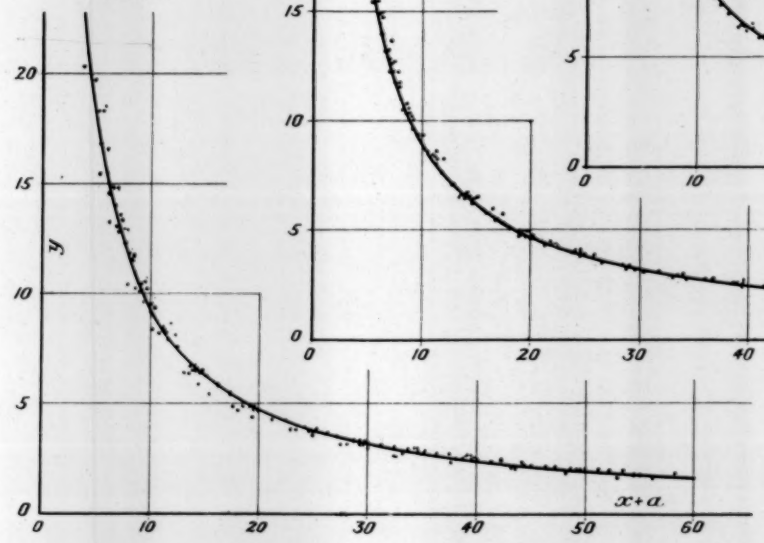


FIG. 3.

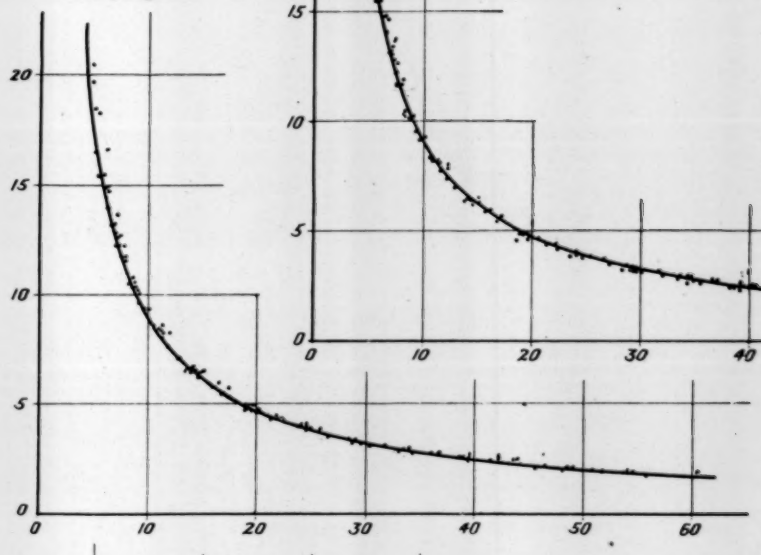


FIG. 4.

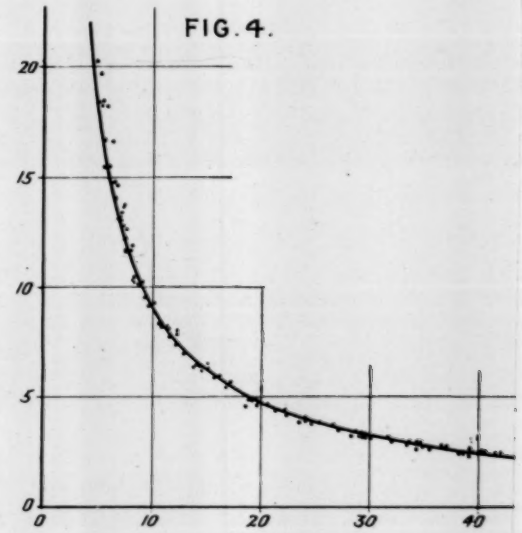


FIG. 5.

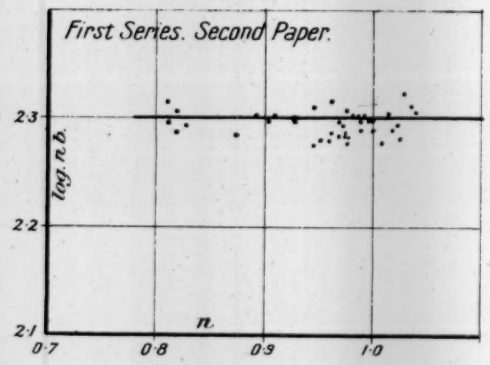


FIG. 7.

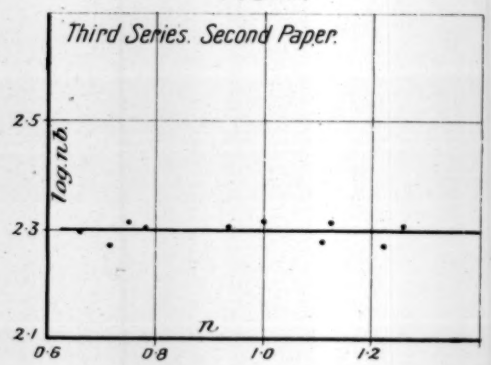
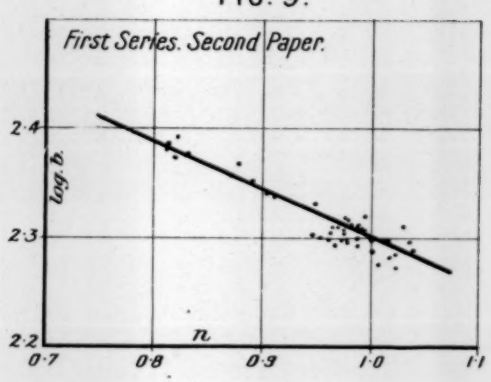


FIG. 9.



FIG

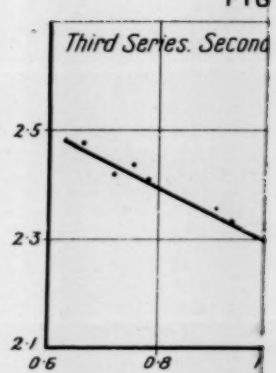


FIG. 6.

FIG. 8.

FIG. 10.

FIG

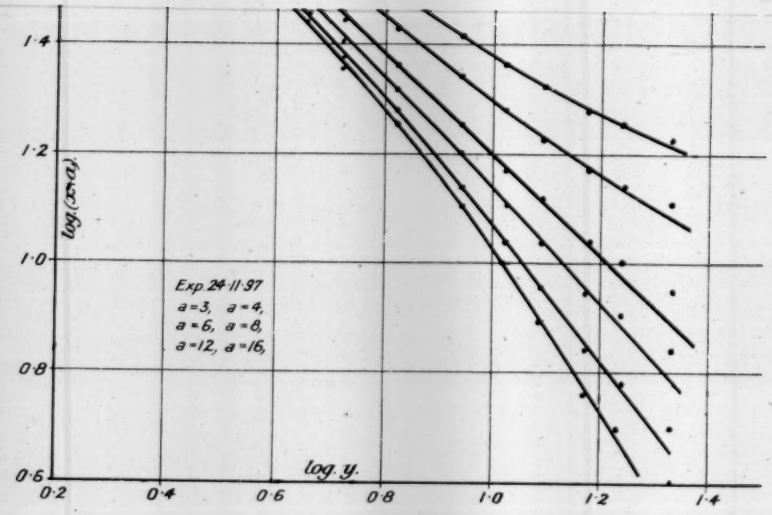


FIG. 2.

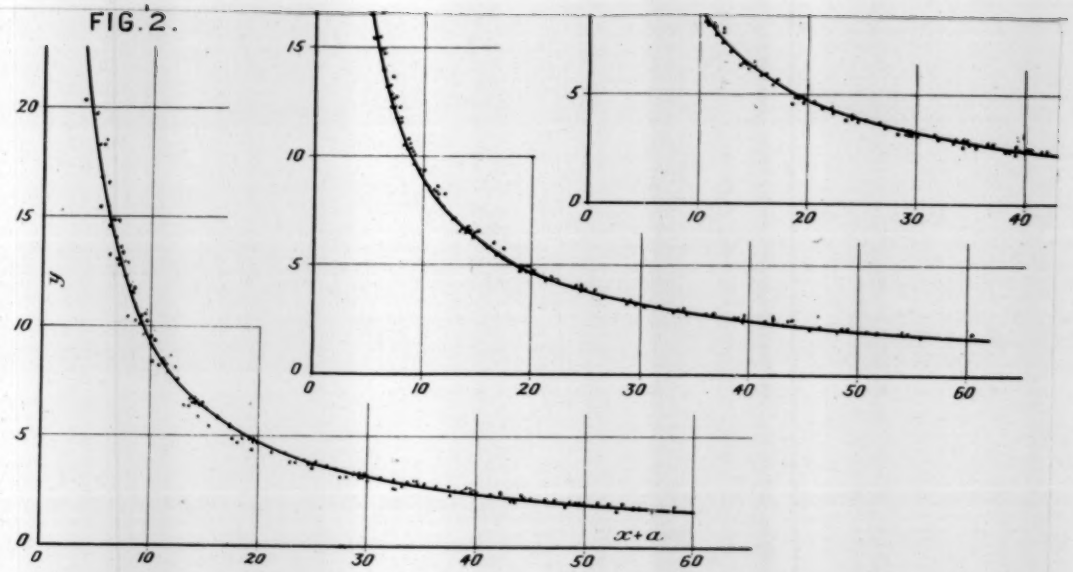


FIG. 5.

FIG. 7.

FIG. 9.

FIG

FIG. 6.

FIG. 8.

FIG. 10.

FIG

PLATE II.

FIG. 14.

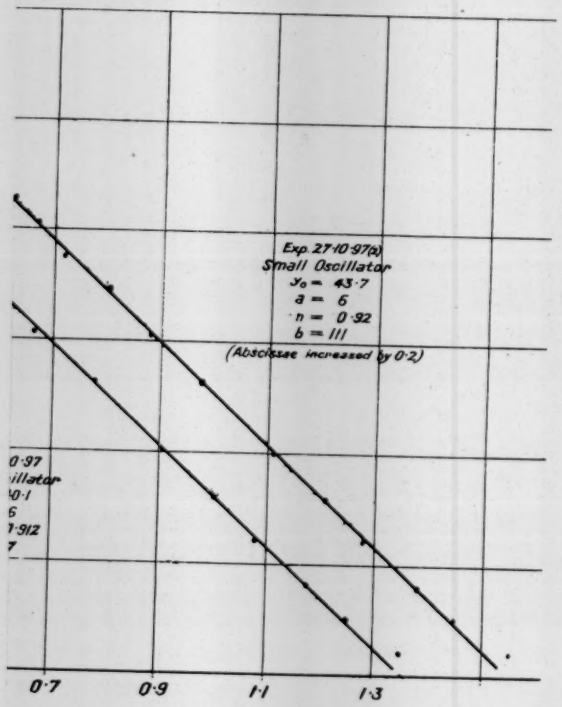


FIG. 13.

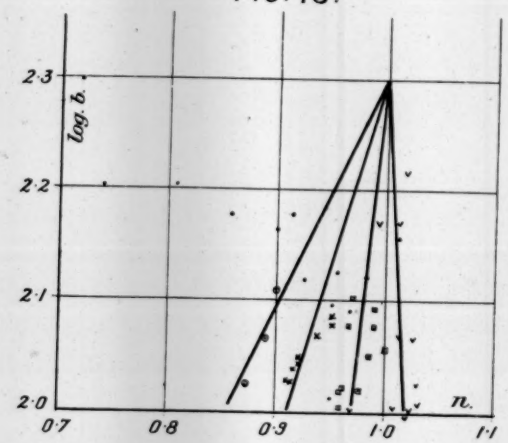


FIG. 15.

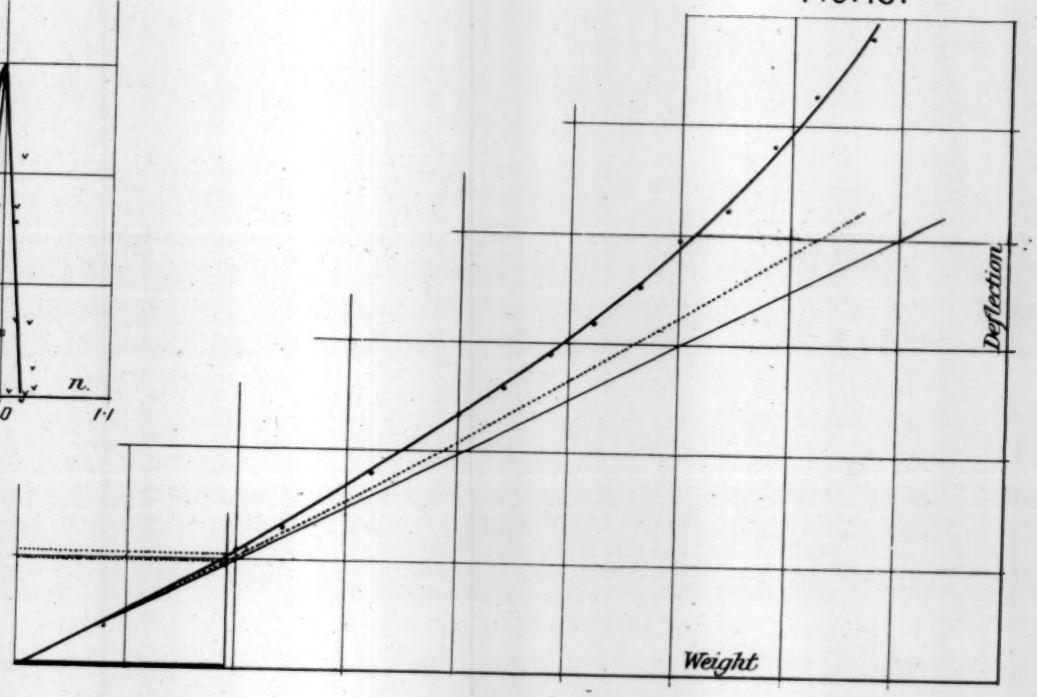


FIG. 16.

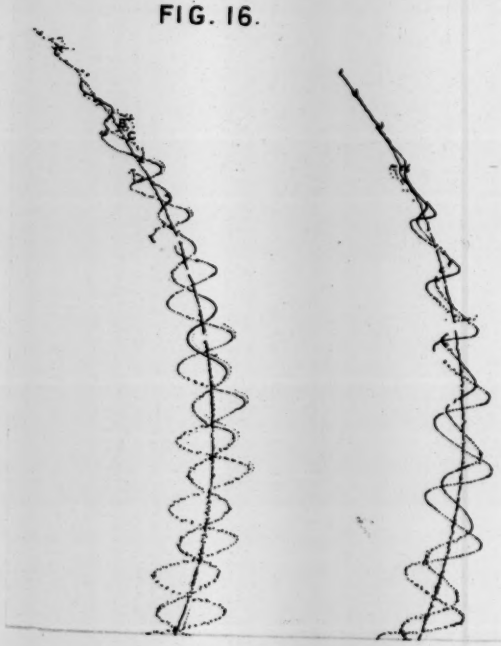
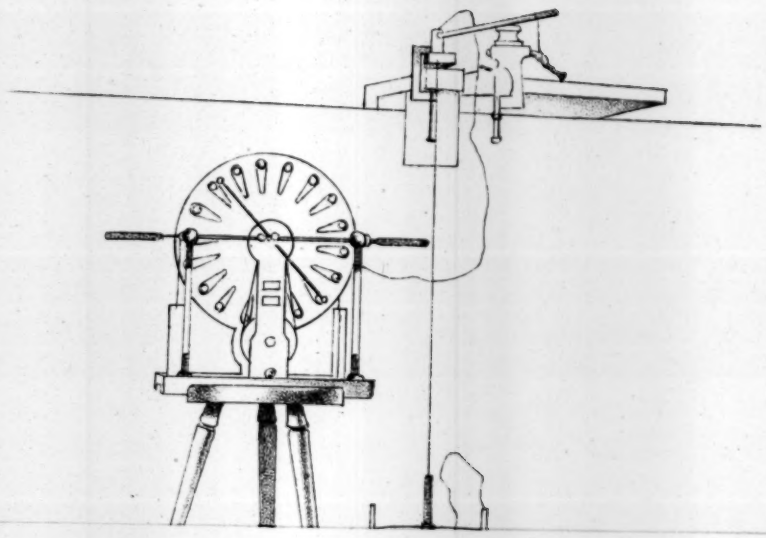
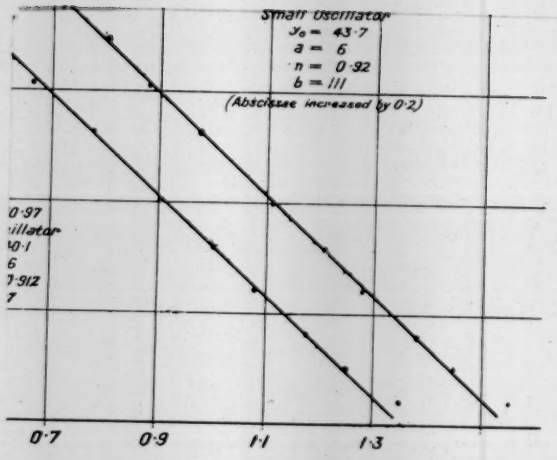
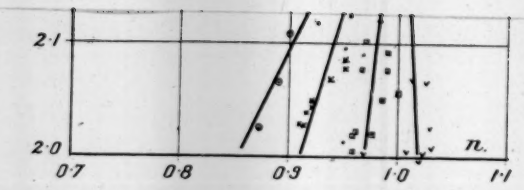


FIG. 17.




Small Oscillator
$y_0 = 43{\cdot}7$
$a = 6$
$n = 0{\cdot}92$
$b = 111$
(Abscissae increased by 0·2)
0·97
illator
0·1
6
·912
7
0·7
0·9
1·1
1·3


2·1
2·0
0·7
0·8
0·9
1·0
1·1
n.

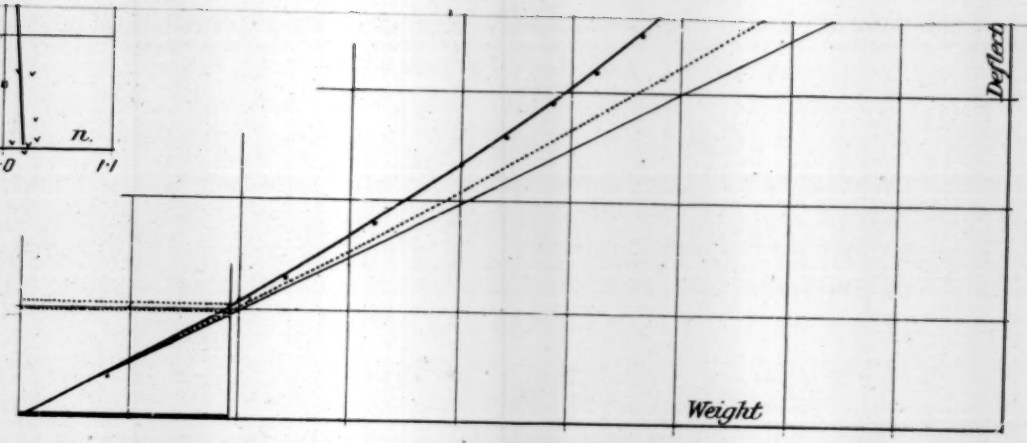

Deflect
Weight

FIG. 16.

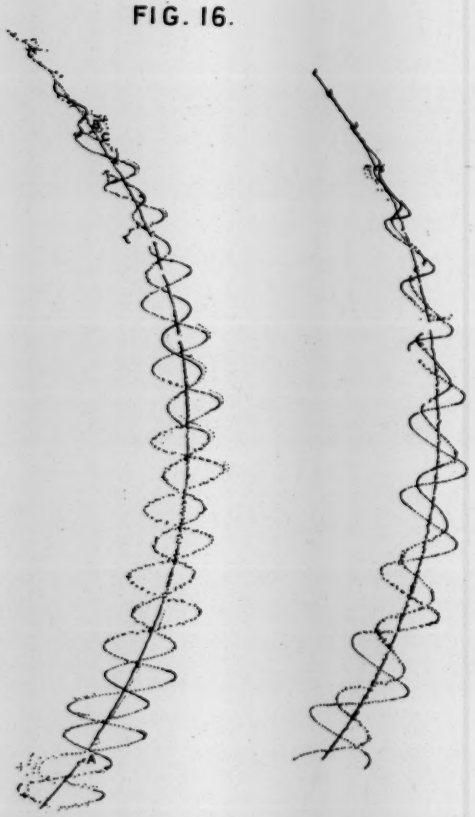


FIG. 17.

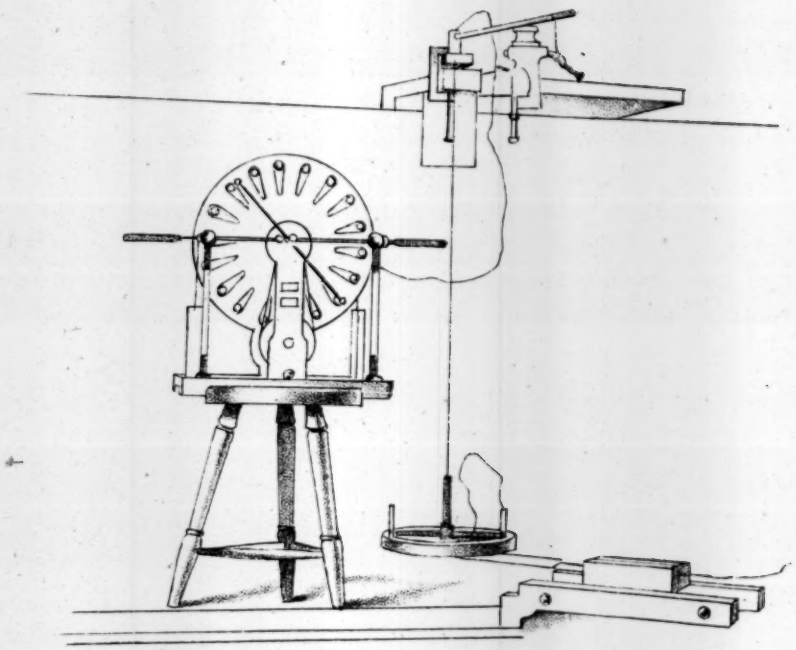


PLATE II.

FIG. 14.

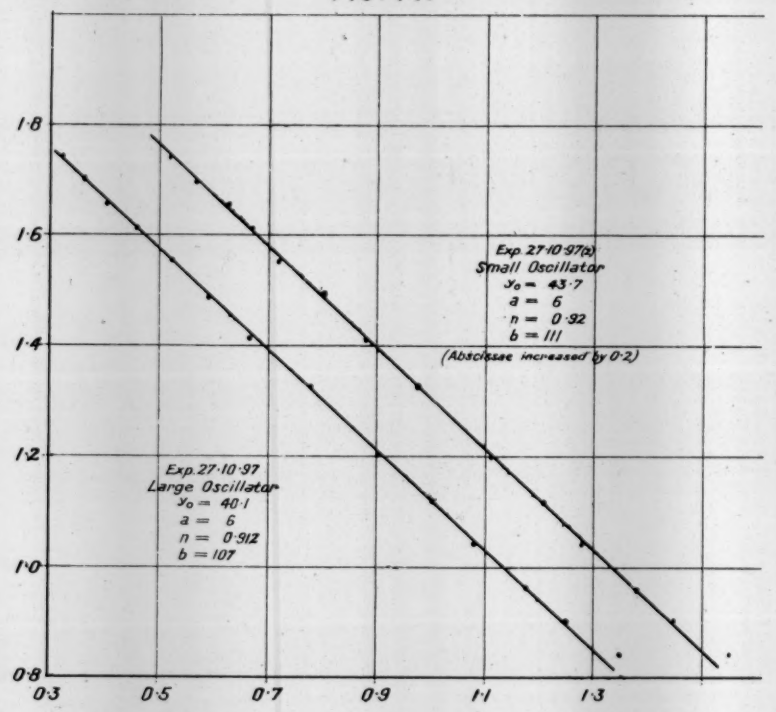


FIG. 13.

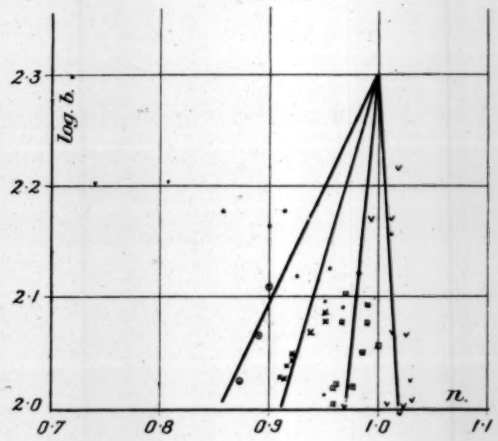


FIG. 15.

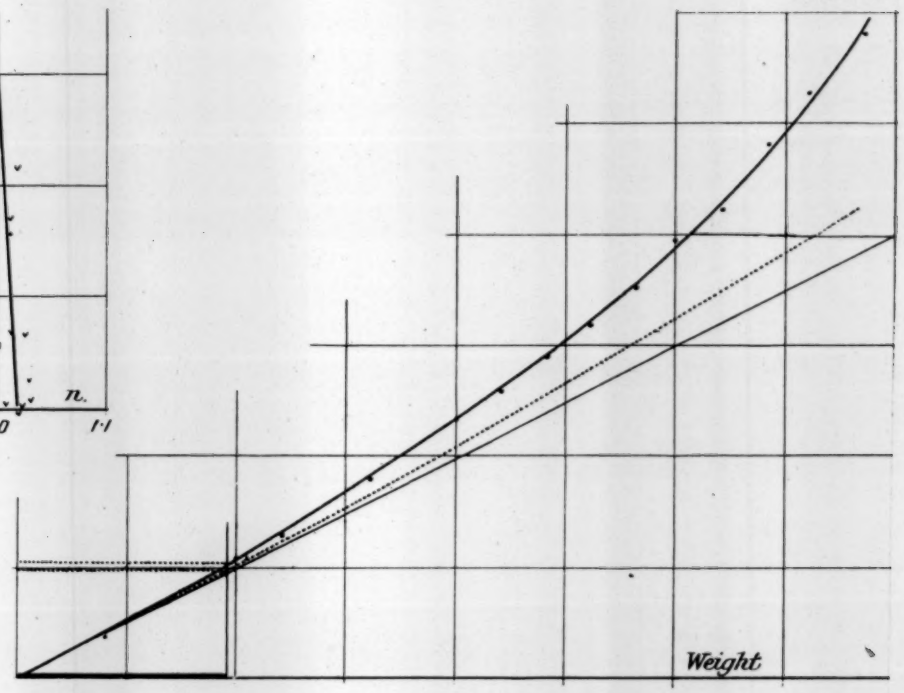


FIG. 16.

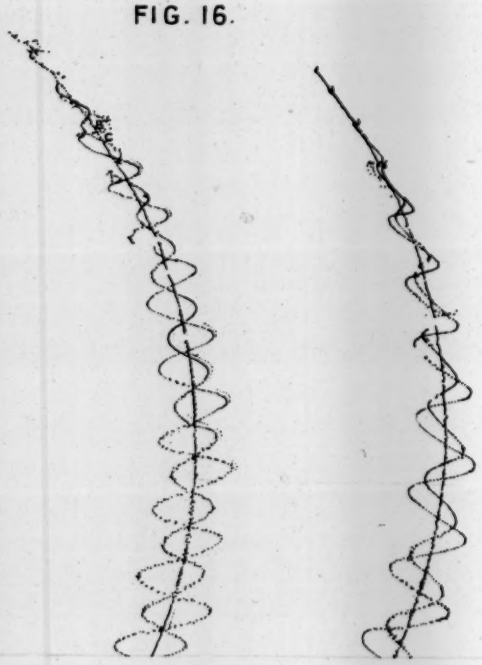
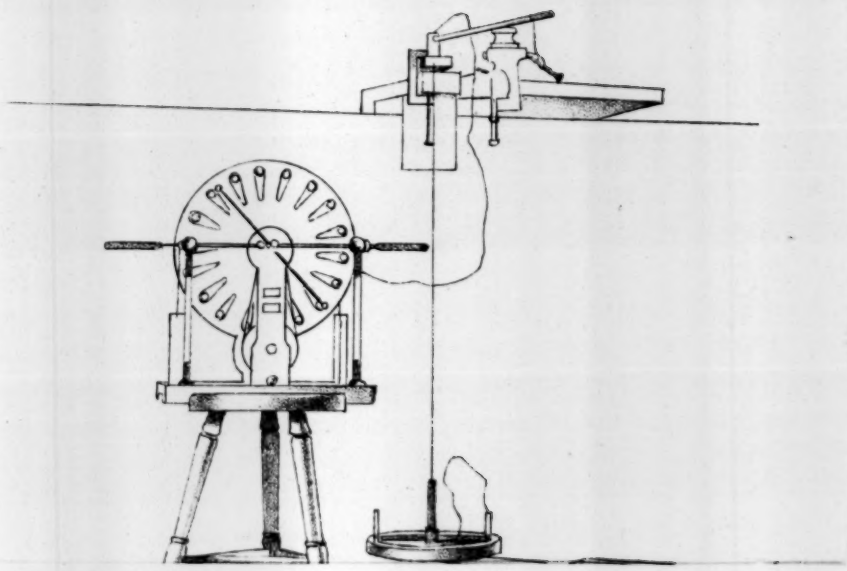


FIG. 17.



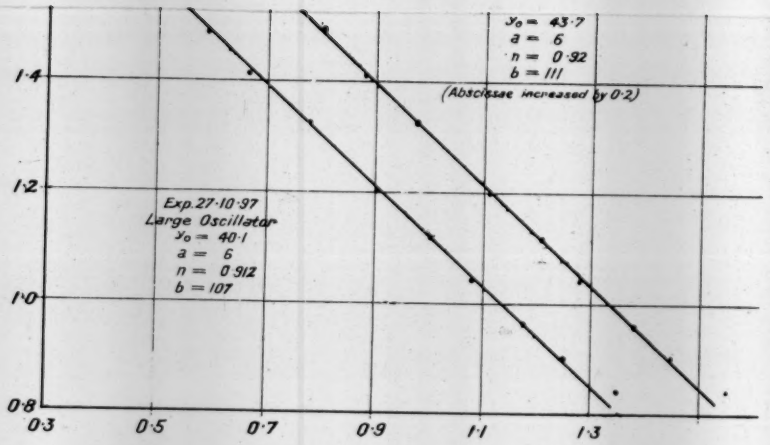

$y_0 = 43{\cdot}7$
$a = 6$
$n = 0{\cdot}92$
$b = 111$
(Abscissae increased by 0·2)
Exp. 27·10·97
Large Oscillator
$y_0 = 40{\cdot}1$
$a = 6$
$n = 0{\cdot}912$
$b = 107$
1·4
1·2
1·0
0·8
0·3
0·5
0·7
0·9
1·1
1·3

FIG. 16.

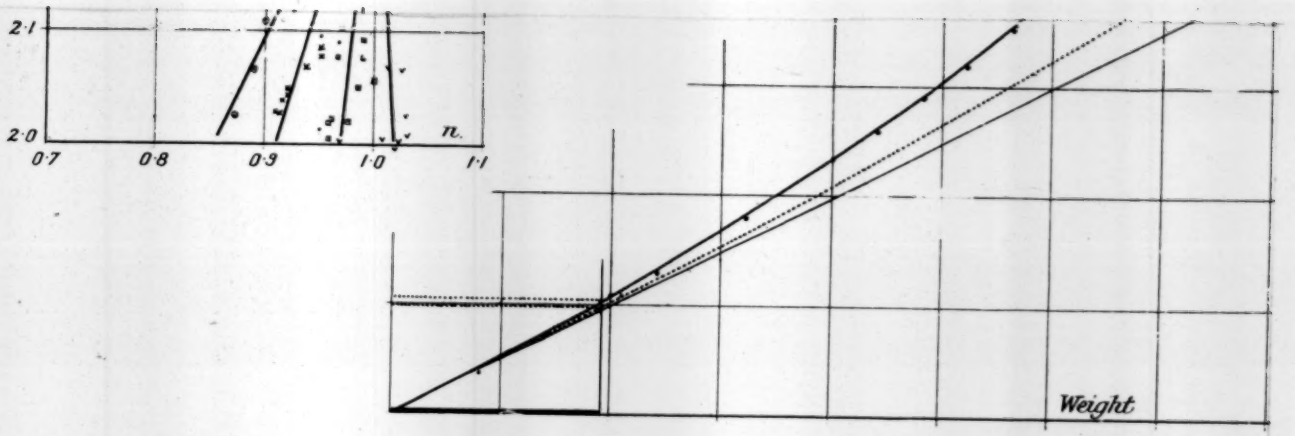
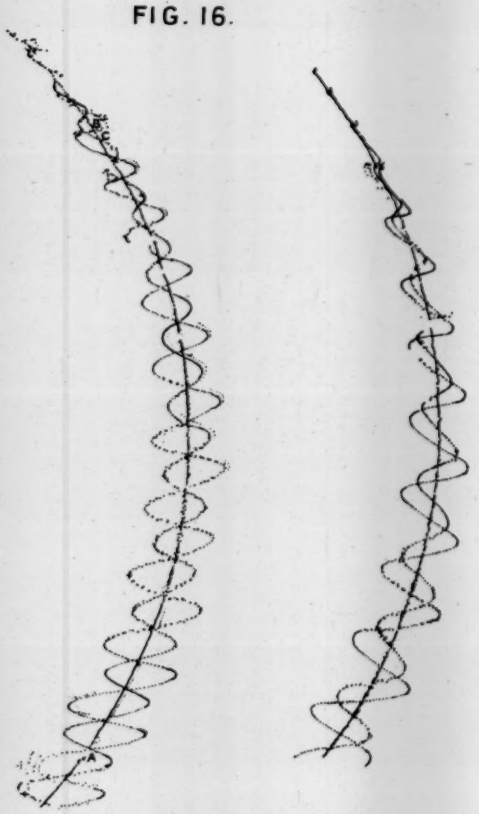
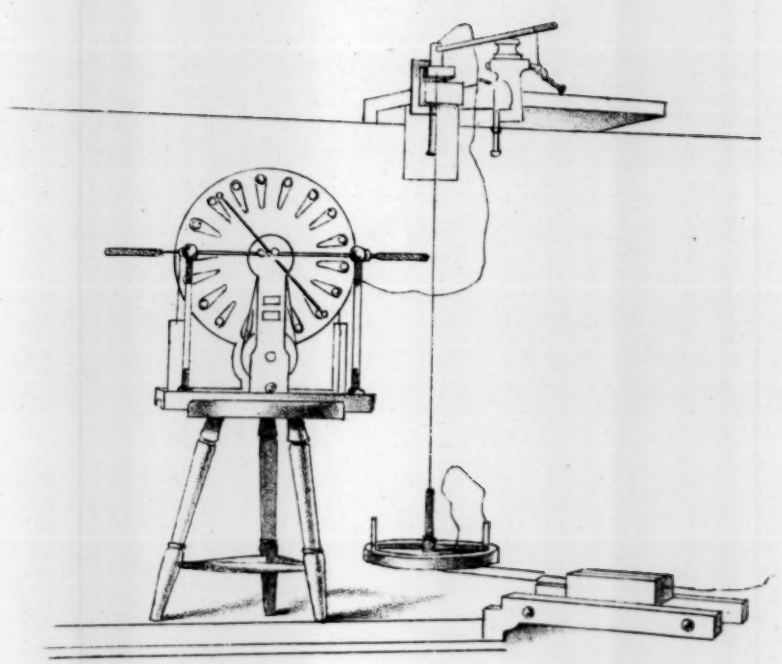

2·1
2·0
0·7
0·8
0·9
1·0
1·1
n.
Weight

FIG. 17.





XV.—*The Strains produced in Iron, Steel, Nickel, and Cobalt Tubes in the Magnetic Field.* PART II.

By Professor C. G. KNOTT, D.Sc., F.R.S.E. (Plates I. and II.)

(Read 6th June 1898.)

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§ 1. INTRODUCTION.—The remarkable changes produced by magnetization in the internal volumes of hollow cylinders of iron, steel, and nickel have been described in Part I. (see *Trans. R.S.E.*, vol. xxxviii. pp. 531–555). As pointed out in the closing paragraph, a complete discussion of these changes had to be "deferred until direct measurements of elongation had been obtained with the various tubes under the same magnetic influences." It was not possible, of course, to measure the elongations of *all* the tubes that had been experimented with; for of these, eighteen (Nos. I. to VI. of each inclusive) were no longer in existence, having been the successive stages through which No. VII. was brought from the condition of small bore and thick walls to that of wide bore and thin walls.

As the investigation with the existing tubes proceeded, it became more and more matter for regret that the idea of measuring the elongations as well as the volume changes of the successive tubes had not occurred to me at an earlier stage. I therefore resolved to carry out a complete series of experiments with a new set of iron tubes, all successive stages in the life-history of one and the same bar. These are distinguished below as Nos. I′. to VIII′. inclusive. The changes of length and the changes of volume of bore of each of these tubes were measured, and from these measurements certain interesting results were obtained.

But it now became evident that a much clearer insight into the character of the strain accompanying magnetization in iron and nickel tubes would be obtained if the cubical dilatation of the *material* of these tubes could be measured directly. This, unfortunately, could not be effected with the tubes in use, which nearly filled the core of the magnetizing coil (Part I., § 6).

Led by these considerations, I proceeded to study the volume and length changes of sets of smaller tubes, each of which could be enclosed in a strong brass tube inserted in the core of the magnetizing coil. The new nickel tubes, distinguished as the B tubes, were formed by successive borings from a nickel bar, 20·2 cm. long and 2·72 cm.

in diameter. Two new sets of iron tubes, distinguished as the A tubes and the B tubes, were studied. The B tubes correspond in dimensions with the nickel B tubes; the A tubes are twice as long. Each original iron bar gave in succession seven tubes, the internal diameters of which increased from two-eighths of an inch (No. I.) to fully an inch (No. VII.). The internal diameters of the nickel tubes range from three-eighths (No. II.) to six-eighths (No. V.) of an inch. A further boring was in this case out of the question, since the material had begun to crack, and the tube consequently to leak. The dimensions of these and the other new tubes are given in numerical detail in Table I. at the end of the paper.

Having thus briefly sketched the history of the research, I propose in what follows (1) to discuss in full the results for the A and B tubes, and (2) to compare with these, and elucidate by their means, the comparatively incomplete results obtained with the large tubes, both old and new.

The existing old tubes are No. VII., 3, 5, 7 of iron and of steel; and No. VII., 1, 4, 7 of nickel.

The iron and steel tubes, No. 9, are the thinnest walled of all the large tubes, and differ from the others in having no internal ledge on which a washer could be screwed down under the cap. The measurement of the volume changes in these thin tubes required a different method of fitting the cap, and, indeed, a different cap altogether. The results originally obtained with them were not regarded as altogether satisfactory, and were accordingly omitted in Part I. They are given below, for the sake of completeness, along with the elongations.

Excluding the eighteen temporary tubes which formed the successive stages of the iron, steel, and nickel tubes, No. VII., we have in all forty tubes, whose changes of form and volume in various magnetic fields are now to be discussed.

§ 2. Methods of Experiment.—With each of the A and B tubes four distinct experiments were made. These were:—

(1) Measurements in various magnetic fields of the corresponding changes of volume of bore.

(2) Measurements in the same fields of changes of length.

(3) Measurements in the same fields of changes of volume of the *material* of the tube.

(4) Measurements in the same fields of *apparent* external changes of volume, the tube being plugged and treated as a bar.

The first form of experiment was conducted after the manner described in Part I., § 7. The metal tube and the connected capillary glass tube were filled with water, and the changes of volume measured by the displacements of the end of the water column in the capillary.

In the second form of experiment the change of length was measured by means of a lever and mirror arrangement, similar in essence to the arrangements used by other experimenters (Joule, Barrett, Bidwell, etc.). The tube under investigation rested

inside the brass tube already mentioned, which was adjusted within the magnetizing coil so that the iron or nickel tube was centrally placed in the coil. The mirror rested by two colinear knife-edges on supports firmly attached to a brass cap screwed to the top of the brass tube. A glass rod of suitable length rested by its lower end on the top of the iron or nickel tube, and supported on its upper end a third knife-edge also fixed to the mirror. This knife-edge was parallel to, but lay 1·1 mm. behind, the common line of the other two knife-edges. Any change of length in the inner tube would produce a rise or fall of the support of the single knife-edge, while the two colinear knife-edges would be unaffected. The consequent tilt given to the mirror, which was set approximately vertical, was measured by means of a telescope and reflected scale in the usual manner. A simple calculation gave the corresponding change of length of the tube, and from that the longitudinal dilatation could at once be found.

Experiments (1) and (2) were made with the large tubes also. In the measurement of the change of length, however, the method was slightly modified. The brass cap supporting the colinear knife-edges was screwed on to the top of the iron, steel, or nickel tube, while the glass rod supporting the single knife-edge passed up through the hollow core from the base of the tube.

In the third form of experiment the iron or nickel tube was placed within the brass tube, which was otherwise filled with water, and to which the capillary was attached. Any change of volume of the material of the immersed tube produced its effect on the position of the end of the water column in the capillary.

In the fourth form of experiment the iron or nickel tube was plugged up (air only being inside), and in this condition was dropped into the brass tube, while everything else was exactly as in experiment (3).

Unless the differences in the mechanical constraints to which any tube was subjected in these various experiments produce really important disturbances, we should expect to find the *apparent* volume change of experiment (4) to be equal to the algebraic sum of the volume changes of experiments (1) and (3).

A glance at the numbers given in Tables II. and VI. below will show how satisfactorily the experiments establish this relation.

§ 3. The Strain Coefficients.—In these experiments the quantities directly measurable are :—

V, the volume of the material;

v, the volume of the bore;

v', the volume of the space enclosed by the outer surface and ends—in other words, the volume of the original bar from which the tubes were formed;

δV, δv, $\delta v'$, the changes of these volumes in given fields;

λ, the longitudinal dilatation of the tube; and

δ, $= \delta V/V$, the average cubical dilatation of the material of the tube in these same fields.

Now we may write

$$\delta v/v = \lambda + 2\mu \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where μ represents the transverse dilatation of the core. It may be regarded as measuring the elongation at each point of the inner surface of the tube in the direction perpendicular to the axial plane passing through the point. If we suppose δ to be the cubical dilatation at this point, we have the equation

$$\delta = \lambda + \mu + \nu \quad . \quad . \quad . \quad . \quad . \quad (2)$$

where ν is the elongation in the direction of the radius.

Similarly, $\frac{1}{2}(\delta v'/v' - \lambda)$ gives μ', the "tangential" elongation at the outer surface of the tube; and then the radial elongation is given by

$$\nu' = \delta - \lambda - \mu' \quad . \quad . \quad . \quad . \quad . \quad (2)$$

In calculating these strain coefficients I make the two assumptions:—First, that δ, which really measures the average cubical dilatation throughout the metal, also measures the cubical dilatations of the elements at the surfaces; and second, that λ has the same value at every point of the tube. There seems no way of testing the truth of the first assumption; but the second was tested by direct experiment, and no indication was found of λ having different values at the outer and inner walls.

The precise significance of the ratios λ, μ, ν, μ', ν' may be thus indicated. Imagine a small spherical element of diameter $2e$ at the inner surface of the tube. After the application of the magnetic stress this sphere becomes an ellipsoid, whose principal axes are $2e(1+\lambda)$ in a direction parallel to the axis of the tube, $2e(1+\nu)$ in a radial direction, and $2e(1+\mu)$ in a direction at right angles to these—that is, tangential. The ratios $1+\lambda$, $1+\nu'$, $1+\mu'$ have similar meanings for an originally spherical element at the outer surface.

Again, if r, R, are the inner and outer radii of the tube, $r\mu$ and $R\mu'$ represent the outward displacements of the corresponding surfaces.

Although it is not possible to calculate accurately these strain coefficients in the case of the large tubes, we may obtain an approximate estimate of their values on the further assumption that the cubical dilatations are the same for all tubes of the same metal. Thus, since $V + v = v'$, we have $\delta V + \delta v = \delta v'$ in any given field. Hence

$$\frac{\delta V}{V} + \frac{\delta v}{v}\frac{v}{V} = \frac{\delta v'}{v'}\frac{v'}{V}$$

or,

$$\frac{V}{v'}\left\{\delta + (\lambda + 2\mu)\frac{v}{V}\right\} = \lambda + 2\mu' \quad . \quad . \quad . \quad (3)$$

Now, in the case of the large tubes, λ and $\lambda + 2\mu$ are measured by direct experiments, and the volumes v, v', V are known. Consequently, if δ be assumed, the value of $\lambda + 2\mu'$ at once follows. Hence μ and μ' may be calculated. The values of ν and ν' are then found from the equations (2) above.

This process has been applied to the old iron tubes 1, 3, 5, VII., to the new iron tubes I′. to VIII′., and to tube A VII.

As will be seen immediately, the cubical dilatation in the case of nickel is negligibly small compared with the elongations, so that μ' is to be found from the simplified form of equation

$$\lambda + 2\mu' = \frac{v}{v'}(\lambda + 2\mu) \quad . \quad . \quad . \quad . \quad . \quad (3)'$$

The ratios ν and ν' follow as above.

The values for the Nickel Tubes 1, 4, 7, and I. to VII. have been obtained in this way, the additional assumption being made that the elongation λ is the same for the non-existent Tubes I. to VI. as for the final existent form VII. A consideration of all the measured values of λ for the various tubes, large and small, will show that this assumption, though not strictly true, does not involve an error of magnitude sufficient to modify seriously the final conclusions.

§ 4. The Bored Nickel Tubes.—These are best considered first, because of the comparative simplicity of the results obtained. The volume changes and dilatations of the B tubes in various magnetic fields are given in Table II., and are shown graphically in the first two rows of Plate I.

Especially noteworthy is the smallness of the material volume change in comparison with the other measured volume changes. So minute is δV, that in calculating the strain coefficients we may, without any risk of serious error, put

$$\lambda + \mu + \nu = 0.$$

The δv and $\delta v'$ curves lie very close together. It is, in fact, hardly possible, on the chosen scale, to draw them distinct in the case of B II. For B V. one curve only is given, that, namely, which shows how the apparent external volume change increases with the field. A tiny crack in the wall of the tube prevented any good observation of the bore change being made.

On the whole, there is a tendency for the volume changes δv and $\delta v'$ to differ more as the bore increases, that is, as the walls get thinner. This may be referred to the different conditions of constraint in the two forms of experiment. When δv was being measured, the brass cap, by means of which the capillary was attached to the upper end of the tube, was screwed on to the outside surface. On the other hand, in order to permit the tube to slide easily within the brass tube when $\delta v'$ was to be measured, the nickel tube was in this case closed by a brass plug which screwed on to the inside surface. Thus in any lateral expansion of the tube, there would be more constraint with the outside fitting cap than with the inside fitting plug.

Again, the manner in which the cubical dilatation diminishes with the thickness of the wall suggests the possibility that part of the measured change of volume δV may be due to empty spaces within the metal—in other words, to its vesicular structure.

Passing to the consideration of the coefficients of strain, we notice a steady, though

small numerical increase in the values of the longitudinal elongations (λ) as the bore of the tube is increased—that is, as the thickness of the wall is diminished. This is quite in accordance with what might be expected if the elongation depends on the induction rather than on the field. For, as I found by direct experiment in the case of the large tubes, the induction in a given field is smaller in the tube of narrower bore or wider wall.

The "tangential" elongations (μ) at the inner surface are all positive and much smaller numerically than the longitudinal elongations. They show a tendency, however, to increase with the bore. Because of the comparatively small value (practically zero) of the cubical dilatation, the corresponding "radial" elongations (ν) are distinctly larger than the tangential elongations, but show no marked tendency either to diminish or increase as the bore varies.

On the other hand, the tangential and radial elongations (μ', ν') at the outer surface behave somewhat in contrary fashion, μ' increasing, and ν' correspondingly decreasing, as the bore increases. For any given field and tube the four ratios μ, μ', ν', ν are in order of magnitude, μ' and ν' approximating to equality when the bore is narrow, and gradually diverging in value as the bore increases. These relations are well shown in the curves.

Had it been possible to obtain wider bores without hopelessly damaging the tube, which already showed signs of cracking, it is highly probable that μ' and ν' would again approximate to equality, just as was found to be the case with the like quantities for the iron tube (see below).

By application of equation (3)′ of last paragraph, the ratio μ' was calculated for the large Nickel Tubes 1, 4, 7, and VII. Similar calculations were also made for Tubes I. to VI., the elongation λ being assumed to be the same for these as for their final form VII. Although this assumption is not strictly accurate, it is sufficiently near the truth not to lead to any serious error in the calculations of the other ratios.

The results for the four existing tubes are given in Table III., and are represented graphically in the third row of Plate I.

An epitome of the results for Tubes I. to VII. forms Table IV., and some of the features are shown graphically in the fourth row of Plate I.

The volume changes measured are, of course, much larger for these tubes than for the B tubes. Nevertheless, the linear dilatations come out with values practically identical in the two sets. This will appear the more remarkable when the different conditions under which the large and small tubes are magnetized are borne in mind. Each large tube when inserted in the magnetizing coil extended to within a few inches of each end, and could not therefore be magnetized so uniformly as one of the short tubes which lay much more completely within the magnetizing coil.

One interesting point brought out by the large tubes is the negative value of μ in the tubes of narrowest bore under high magnetizing forces. See, for example, the μ-graphs of No. 1, II. and III. (Plate I.). Also there is an interesting gradation in the effect as the bore increases. Thus the values of μ for Nos. 1 and I. are positive in fields lower than 140 and 180 respectively, and negative in higher. In Tube No. II.

the same feature is shown, but the change of sign occurs about Field 270. In Tube III. the *form* of the graph is the same, but no negative value is reached. In Tube IV., however, this characteristic has disappeared, and the behaviour is, broadly speaking, the same as in Tube VII. Notwithstanding this peculiarity in the sign of μ in the narrow-bored tubes, the calculated values of μ' ν' come out nearly the same for all. It will be readily seen, both from the curves and from Table IV., that the comparative values of μ' ν' follow the same law of change as in the B tubes, approximating in value in the tube of narrowest bore, and gradually drawing apart as the bore increases.

§ 5. The Coiled Nickel and Cobalt Tubes.—These are formed from sheets of metal, each tube being about 10 inches long and 1 inch diameter. After the long edges had been soldered together to form a hollow tube, the changes of length and the changes of volume of the material in various fields were measured, as already described. The tube was then plugged up with brass discs at both ends, and measurements were made of its changes of bulk.

From these measurements the dilatations λ, μ', ν' follow at once. It seems hardly necessary to trouble calculating μ and ν in these cases of very thin-walled tubes. A glance at formula (3), § 3, shows that the comparative smallness of V makes the quantities μ, ν differ very slightly from μ', ν'.

The three Nickel Tubes C I., C II., C III. were formed from three sheets of different thicknesses. The dimensions are given in Table I. The nickel was obtained as pure as possible, and in this respect is better than the nickel of the bored tubes, which contains $2\frac{1}{2}$ to 3 per cent. of impurities. This may account for the fact that the longitudinal contraction is distinctly greater in the C tubes than in any of the others.

It is, however, in the external volume changes that the greatest difference is shown between the coiled tubes of very thin wall and the bored tubes of comparatively thick wall. With all of the C tubes there is *increase* of external volume except in the lowest fields; in every other instance *decrease* was the characteristic feature. Compare, for example, the Curves C I., C II., C III. in the lowest left-hand corner of Plate I. with the curves in the first row and with similar curves in the former paper.

It is interesting to note that the volume increase is greatest in the tube of thinnest wall, and falls off as the wall is made thicker. With a still thicker wall the volume change might change sign and become negative. It would not be safe, however, to institute any strict comparison between tubes formed by coiling sheets and tubes formed from solid bars by boring.

Excepting that the μ' curve lies higher than the ν' curve, there is not any great diversity shown in the nature of the linear dilatations in the various types of tube. The C I. and C III. curves, occupying the middle of the last row in Plate I., are very similar to the λ, μ', ν' curves in the third and fourth rows of the same plate. It may be mentioned that C II. differs so very little from C III. as hardly to require a separate set of curves. The greater divergence between the values of μ' and ν' in C I. than in either of the others is noteworthy as being somewhat unexpected.

But the differences that exist seem of comparatively small importance beside the general agreement, even to numerical details, among the dilatations of the different types of nickel tube.

The results for the cobalt tube call for little remark. The linear dilatations are much smaller than for nickel, as a glance at the curves on Plate I. shows. The cubical dilatation of the metal is, however, greater, being appreciable enough to be measured.

The broad difference between the two metals is that the molecular groups yield more readily to the magnetizing force in nickel than in cobalt. The nickel curves all show an approximately saturated condition in the substance; but there is no evidence of such a condition being approached in the case of cobalt.

§ 6. The Bored Iron and Steel Tubes.—The results for iron are, as compared with those for nickel, of a very complex character.

Consider, first of all, the volume changes of bore in the various sets of tubes as given in the column headed δv in Tables VI., VIII., IX. A gradual change in the behaviour of the tube as the bore is made larger is very apparent in the case of Tubes A and B (Table VI. and Plate II., first and third rows of graphs). Also there is an evident parallelism in the two series A and B. Thus, in I. and II. of both sets, the volume of bore diminishes steadily as the field increases. In III., however, the change of volume passes through a curious minimum distinctly shown in the graphs. In IV. and V. this minimum becomes more evident, the negative change of volume diminishing markedly in the higher fields. Finally, in VI. and VII. this diminishing negative change becomes an increasing positive change. In the A set a particular peculiarity is associated with a field which is lower than that associated with the corresponding peculiarity in the B set. For example, the fields associated with the minimum volume change or greatest *diminution* of volume are :—

In	A	III.	105	and	260 in	B III.;
,,	A	IV.	90	,,	200 ,,	B IV.;
,,	A	V.	70	,,	165 ,,	B V.;
,,	A	VI.	40	,,	90 ,,	B VI.;
,,	A	VII.	20	,,	40 ,,	B VII.;

while the change of sign from negative to positive volume change occurs in fields

70, 35, 140, and 75

in A VI., A VII., B VI., and B VII. respectively.

A similar parallelism is shown in the two sets of results for the changes in external volume of the plugged tubes. Compare, for example, the $\delta v'$ curves in the first and third rows of Plate II.

The same feature is reproduced in the columns headed λ, the measurements, namely, of the linear dilatation in the direction of the magnetizing force. A glance at the curves figured in the second and fourth rows of Plate II. shows

that the maximum value of λ in each A tube occurs in a lower field than it does in the corresponding B tube. Thus the field of maximum λ is :—

In	A	I.	165	and	370	in	B	I.
,,	A	II.	150	,,	310	,,	B	II.
,,	A	III.	140	,,	270	,,	B	III.
,,	A	IV.	125	,,	225	,,	B	IV.
,,	A	V.	110	,,	190	,,	B	V.
,,	A	VI.	95	,,	150	,,	B	VI.
,,	A	VII.	80	,,	100	,,	B	VII.

Thus the B tubes reproduce, but in higher fields, the peculiarities shown by the A tubes. The reason is not far to seek. It depends on the fact that the shorter B tubes have a larger *demagnetizing factor* than the larger A tubes. Not only so, but, in accordance with well-known results, the demagnetizing factor diminishes with the area of section of the material, the length being constant. This consideration explains at once the gradual shifting of the critical points (maximum λ, minimum δv and $\delta v'$) into lower fields as the bore of a tube of given length is gradually increased.

The shifting of the crest of the longitudinal elongation curve as we pass through the series A I. to A VII. and B I. to B VII. is also well shown with tubes I′. to VIII′. An inspection of Table IX. will bring this out clearly enough; but the feature is most distinctly shown in the following table constructed from the curves corresponding to Table IX., which curves, however, being broadly similar to those in Plate II., I have not thought it necessary to publish.

FIELDS FOR MAXIMUM ELONGATION IN IRON TUBES I′. TO VIII′.

Tube.	I′.	II′.	III′.	IV′.	V′.	VI′.	VII′.	VIII′.
Field	230	215	200	180	170	155	140	120

According to the commonly accepted theory, the demagnetizing factor in a cylindrical bar is proportional to the square of the ratio of the diameter to the length. In the case of a cylindrical *tube* this law requires modification. Perhaps the most probable *simple* hypothesis is to compare the tube, as regards its demagnetizing factor, to a bar of the same length and the same cross-section of material. How far this applies to the present case is tested immediately by a comparison of Tubes A II. and B VI., which have their maximum elongations in about the same field. Presumably their demagnetizing factors are nearly the same. Now the cross-sections of the material of A II. and B VI. are as 5·96 to 1·94. Dividing these respectively by 4 and 1, which are as the squares of the lengths of the A and B tubes, we get for the ratio of the demagnetizing factors 194 : 144 or

27 : 20. The hypothesis suggested above requires this ratio to be unity. In making this comparison, however, we should bear in mind that the B tube is, because of its shorter length, more uniformly magnetized throughout, and has therefore less leakage of lines of force from the sides than the A tube. This would tend to accelerate the shifting of the maximum to lower fields, an effect which in less uniform fields would be produced by widening the bore of the tube.

The cross-sections of A V. and B VII. are as 3·24 to ·82; hence their demagnetizing factors are as 81 to 82. These tubes should, according to the theory, have their maximum elongations in the same field. As a matter of fact, the field corresponding to the maximum elongation of A V. is somewhat higher than that of B VII.

A similar comparison of Tubes VII′. and A III., in both of which the field of maximum elongation is 140, brings out 39 : 32 as the ratio of the demagnetizing factors. The deviation of this ratio from unity may also be partly accounted for by the greater uniformity of magnetization in the somewhat shorter A tube.

The steadiness with which in all cases the field of maximum elongation diminishes as the bore is widened is particularly noteworthy.

Contrary to the usual experience, I have had no difficulty in measuring the changes of volume of the magnetized material. A glance at the δV columns of Table VI. shows that these changes in the case of iron are by no means insignificant. The corresponding dilatations (δ in Table VII.) have fairly similar values in the A tubes until A VII. is reached, but vary considerably from tube to tube in the B series.

The tendency is for the cubical dilatation to diminish as the walls of the tube become very thin. Also, both in A VII. and B VII. there is a maximum dilatation in a moderate field. The existence of this maximum need in no way surprise us, for a like peculiarity appears in the longitudinal dilatation. It is rather matter for surprise that there should have been no hint at a maximum cubical dilatation with the other tubes. I have already suggested that the cubical dilatation may be appreciably affected by a vesicular condition in the material, and it is well known that such molecular changes are influenced by the form of the body. Now the thinner the wall, the less chance is there of the presence of vesicular cavities; and it is also conceivable that in very thin walled tubes there is increased uniformity of magnetization with possibly less leakage of the lines of induction from the sides. Either or both of these considerations seem to give a sufficient explanation of the phenomenon just described.

§ 7. Curious Behaviour of Iron Tubes under Certain Conditions.—In all experiments in which the iron tubes were enclosed in the brass tube—that is, in experiments of type (3) and (4) of § 2—a curious effect was observed, of which, so far, I have been unable to find a satisfactory explanation. I can best indicate its nature by transcribing from the experimental note-book the unreduced numbers which measured the volume changes. These represent the number of divisions on the micrometer scale through which the water meniscus in the capillary tube appeared

to move at the instant the magnetic field was established or removed. The positive sign means that the meniscus moved outwards along the capillary, showing an increase of volume of the iron tube contained in the brass tube. The negative sign means, of course, a movement in the opposite direction, indicating a decrease in the bulk of the contained iron tube. When two numbers are entered side by side in the third column, the first (in brackets) gives the greatest excursion in the indicated direction made by the meniscus before it comes to its final position of rest after removal of the magnetic field.

IRON TUBE A III., November 12th, 1897.

APPARENT CHANGE OF EXTERNAL VOLUME; TUBE PLUGGED.

Field.	Field on.	Field off.
531	−25	(−23) +11
403	−29	(−13) +19
302	−30·5	(−11) +22
...	...	... ...
202	−33	(− 8) +27
...	...	... ...
100	−42	(− 1) +38
...	...	... ...

A III., November 15th, 1897.

CHANGE OF VOLUME OF METAL.

536	+18	−34
408	+15	−28
308	+14	−24
...	...	...
100	+ 5·5	− 8·5
...	...	...

Thus, taking the very first result given above, we see that, when a field of 531 was established, the water meniscus in the capillary moved back 25 divisions of the micrometer scale, and there came to rest. On removal of the field, the meniscus darted back 23 other divisions, then turned and moved forward along the capillary, stopping finally +11 divisions from the position it occupied just before the field was removed. Its final position was therefore −14(= −25+11) from that originally occupied just before the field was established.

Again, taking the first result of the experiment of November 15th, we see that the establishment of Field 536 produced a forward motion of the meniscus through 18 divisions of the scale, but that, on the removal of the field, the meniscus moved back through 34 divisions, occupying a final position −16 from that occupied before the field was established.

The results are the same for both directions of field. In every case the negative reading is greater than the positive. In the experiment of November 12th, the

phenomenon has a strong resemblance to hysteresis; but, as proved by the other experiment, in which the return value is the greater, it is merely a resemblance and nothing more. That the phenomenon does not depend only on the iron, but has something to do with the brass tube, is proved by the fact that there is not the slightest evidence of its existence in the experiment for measuring the change of volume of bore. Thus, compare with the foregoing tables the following :—

A III., November 12th, 1897.

Change of Volume of Bore.

Field.	Field on.	Field off.
531	+47	−47·5
403	+47	−47·8
302	+46·5	−47
...	...	...
100	+47	−47
...	...	...
48	+14	−13

In this case the capillary was in connection with the interior of the tube, and a positive reading means a contraction of the volume.

The complete absence of the peculiar effect in this last case quite disposes of any attempt at explanation in terms of hysteresis or possible change of temperature. The effect is as if at every make and break of the current in the magnetizing coil the brass tube containing the iron were permanently increased in internal capacity, or as if a certain quantity of water in the tube were removed from it. It is conceivable that a sudden increase of volume of the inclosed iron might push out a small quantity of water so that a positive reading might be followed by a greater negative reading, as in the experiment of November 15th; but it is altogether inconceivable that a contraction of the iron should be accompanied by a like pushing out of water, as would have to be the case were the explanation to hold good for the first experiment of November 12th.

One early suggestion was that the slight difference of diameter between the iron tubes and the bore of the brass tube might produce a certain constraint on the film of water between. But it was found on trial that the peculiar effect persisted in the case of an iron bar whose diameter was made distinctly smaller than the diameter of bore of the brass tube.

Thus, by a process of exclusion, we seem to be driven to the view that the brass tube must experience, when the field is established, an abrupt change of volume from which it does not immediately recover, or from which it recovers slowly when the field is removed. Care was taken to have the iron tube as nearly as possible centrally placed in the magnetizing coil. But no doubt there was some lack of perfect symmetry, so that,

when the iron tube became magnetized, extra pressures between it and the bottom or walls of the brass tube might very easily come into existence; and these might reasonably enough be supposed to produce minute but appreciable changes of volume. In one experiment the tube was set 2 inches above the usual position in the magnetizing coil; but this displacement from the central position did not to any decisive extent affect the readings—a point which rather tells against the explanation just given. But the most serious difficulty is the magnitude of the strain, which, though small, represents stresses of considerable magnitude. The micrometer-reading 14 means a volume change of 34×10^{-6} cub. cm. The bore of the brass tube was 46 cms. long and 3 cms. in diameter, giving a cubical dilatation of fully 10^{-7} within the tube. We may get an approximate idea of the pressure required to produce this dilatation by solving the equation

$$10^{-7} = \frac{\mathrm{P}a^2 - \mathrm{P}'b^2}{b^2 - a^2} \quad \frac{1}{k}$$

where P P′ are the internal and external pressures, a and b the inner and outer radii of the tube, and k the reciprocal of the compressibility. In the present case b is 2·1 cms., k may be put equal to 10^{12} C.G.S. units, and P′ is the atmospheric pressure, say 10^6. Thus—

$$\mathrm{P}a^2 = 10^6(4{\cdot}41 + {\cdot}22)$$

and

$$\mathrm{P} = 10^6 \times 2{\cdot}06\,.$$

Hence it would need an *additional* pressure within of fully an atmosphere to produce the dilatation observed. This seems hardly possible under the conditions, for the interior communicates with the outside through the capillary.

The extraordinary manner in which the plugged Tube A III. passed through its changes when the field was removed was common to the tubes from A I. to A IV., and from B I. to B III. But there was no evidence of it with A V., B IV., and the tubes of wider bore than these. In all cases in which it occurred, the effect was the same—an increased negative change at the instant the magnetizing current was broken, followed immediately by a return to a final positive or diminished negative change. The phenomenon may be explained as an effect of the current of self-induction, producing a short-lived but intense magnetization in the *outer* layers of the iron tube, for the phenomenon is not observed in the experiments for measuring the change of volume of bore—that is, in experiments on the behaviour of the tube at the inner wall. If this, however, were the sole cause, it is hardly likely that it would so entirely disappear in A V. and B IV., which still have fairly thick walls. It is conceivable that there may also be a tendency for the transverse dilatation μ' to disappear more quickly than the longitudinal dilatation λ. A slight tendency in λ to persist when the field was removed—in other words, an appreciable time-lag—would, in the higher fields, give rise to a momentary increase in the diminution of volume.

The quantities tabulated under δV and $\delta v'$ are the means of the readings obtained on application and withdrawal of the magnetizing force. In the majority of cases, the sum $\delta V + \delta v$ is not very different from $\delta v'$, a result which gives a check on the accuracy of the experiment.

§ 8. The Dilatations in Iron and Steel Tubes (Bored).—From the volume and length changes, in the manner described in § 3, the dilatations δ, λ, μ, ν, μ', ν' were calculated, and are entered in the appropriate columns in Table VII. A selection of the results is given graphically in the second and fourth rows of Plate II.

There is a tendency for the dilatations μ and ν to be of opposite sign from λ—that is to say, they are, as a rule, negative when λ is positive, and positive when λ is negative. The curves obtained by plotting the dilatations in terms of the magnetic fields are, with one exception, remarkably smooth, and in no way reflect the peculiar features of the δv and $\delta v'$ curves, from which they have been derived by a purely arithmetical process. Compare, for example, the groups of curves for A V., A VI., B V., and B VI. The grouping of the λ, μ, ν curves is broadly the same in all; and yet, at first sight, nothing is more striking than the differences between the δv and $\delta v'$ curves for V. and VI. The reason for this is not far to seek. The ratios $\delta v/v$ and $\delta v'/v'$ become for the thinner walled tubes distinctly smaller than λ, so that the calculated values of μ and μ' are more affected by the peculiarities of λ than by those of $(\lambda + 2\mu)$ and $(\lambda + 2\mu')$.

One interesting feature in regard to the relative magnitudes of μ and ν is that they, so to speak, change places in the transition from Tube A V. to A VI., and from B VI. to B VII. In other words, we find, on plotting the curves, that the μ curves lie below the corresponding ν curves up to A V. and B VI., and that thereafter the μ curves lie above the ν curves. In this respect the coiled tubes discussed in next paragraph are comparable to A VII. and B VII. This result seems to be one that could hardly have been expected. A gradual convergence to equality might more reasonably have been looked for, as the tube was taken thinner and thinner. The interchange of μ and ν in regard to magnitude is, however, characteristic of both the nickel and the iron, and must have some fundamental significance.

The results for the other iron tubes given in Tables VIII. and IX. are very similar to those for the A and B series. In these tables an average value for δ is assumed, so that the values of ν, μ', and ν' are, at the best, tentative. It is, however, noteworthy that the large Tubes 3, 5, 7, VII., and 9, and I′. to VIII′. should give results, broadly speaking, identical with those obtained from the shorter and narrower tubes of the A and B series. It is instructive to note how comparatively little the values of the transverse and radial dilatations are affected by the sign of the change of volume of bore. As already pointed out, the longitudinal dilatation λ has a preponderating influence upon the character of the other dilatations. To express it otherwise, the ratio $\lambda + 2\mu$, which was the particular object of study in the first paper, is subject to relatively great changes of values, simply because it is the difference of two ratios, λ and -2μ, of which sometimes one and sometimes the other is the greater. Thus, the

measured volume changes of bore of the Iron Tubes I. to VII. (see Plates I. and II. of the previous paper) differ markedly from those of the Iron Tubes I′. to VIII′., and 3, 5, and 7. Nevertheless, the dilatations of Tube VII., as given in Table VIII. below, do not appreciably differ from those of other tubes.

It is this consideration which fully explains the apparently extraordinary behaviour of the steel tubes as described in the first paper (p. 537 and Plates III. and IV., *l.c.*). I have no measurement of the volume change of steel, so that it is not possible to calculate even the *tentative* values of ν, μ', and ν'. Probably δ is small compared with λ, and some idea of the character of the strain in steel might be got by assuming δ to be negligible. The results would not, however, differ essentially from those for iron. I have given in Table X. simply the undoubted results for the Steel Tubes, 3, 5, 7, VII.$_1$, VII.$_2$, VII.$_3$, and 9. There are three sets of columns of numbers. The first contains the ratios $\lambda + 2\mu$ calculated mainly from the data given in Part I.; the second contains the measured values of λ; and the third contains the calculated values of μ. Broadly speaking, they are very similar to the corresponding values for iron.

The history of Steel No. VII., as given in the Appendix to Part I., is very extraordinary. In its earliest condition distinguished as VII.$_1$ after the final boring which changed VI. into VII., it behaved, as regards bore dilatation, in a manner altogether peculiar. There was great positive dilatation up to Field 280, and negative dilatation in higher fields. Two months later, the law of the dilatation was found to be just reversed. This condition is distinguished as VII.$_2$. The tube was then annealed by slow cooling; and the bore dilatations were found to be greatly diminished in value, but otherwise to resemble roughly the corresponding quantities in the second condition. The contrasts between these three successive states of the same tube are shown in the numbers in Table X. The transverse dilatations μ for Steel VII.$_1$ and Steel VII.$_2$ are calculated on the assumption that the longitudinal dilatation of each is the same as that for VII.$_3$. A consideration of the three cases shows very plainly the effect of comparatively small changes in the values of the tangential dilatations. In none of them does 2μ differ greatly from $-\lambda$. Thus the ratio $\lambda + 2\mu$ is comparatively small in all. In VII.$_1$, λ has the preponderating influence; in VII.$_2$, 2μ preponderates. In VII.$_3$, 2μ still preponderates, but very slightly; so that in this case the bore dilatations are very insignificant compared to the linear dilatations.

§ 9. The Coiled Iron Tubes C I., C II. (Table V.)—These were formed of sheet-iron ·37 mm. in thickness. They were each about 10 inches long. The diameter of C I. was shorter than 1 inch by 4 per cent., and the diameter of C II. longer by about the same fraction.

No appreciable change of volume of the metal was obtained, but this simply means that it was too small to be measured. Under the most favourable conditions of experiment half a small division of the micrometer scale was about the limit of certainty. This would mean a volume change of about 2×10^{-6} cub. cm., giving a cubical dilatation of $\cdot 25 \times 10^{-6}$. I believe half of this value might have been detected. We may safely conclude that the cubical dilatation of this kind of iron, when placed in a magnetic field

of 500, does not exceed $\cdot 2 \times 10^{-6}$. In the calculation of the radial dilatations (ν') the cubical dilatation has been assumed to be zero. If, as is highly probable, the cubical dilatation is positive, the values for ν' will be slightly greater. There is, however, no pronounced difference between the values of μ' and ν', such as exists in the case of the coiled nickel tubes.

For the sake of easy comparison the $\delta v'$ curves for C I. and C II. are shown on Plate II., drawn to the same reference point and axes as the curves giving the volume changes for A V. They are the dotted curves, and are characterised by a maximum in Field 25, followed by a minimum in Field 120 or 130. They thus bear a certain resemblance to the $\delta v'$ curve for A I., in which, however, the early maximum is not strongly marked.

The maximum elongation occurs in both cases about Field 60, a distinctly higher value than what is usually obtained with wires.

The broad distinction between the results for the bored and coiled tubes is the comparatively large distortion experienced by the latter in strong magnetic fields.

§ 10. General Conclusions.—It remains to give a general summary of the results, with special reference to the characteristics of the strains which accompany the magnetization of iron, nickel, and cobalt tubes. In a broad sense, the results for any one metal are much the same, whatever the dimensions of the tube may have happened to be. The changes in the values of the various dilatations, cubical or linear, as we pass from tube to tube of the same metal, are, for the most part, of secondary importance, and seem to belong to the same category of phenomena as the influence of form upon the susceptibility.

Of all the quantities measured, the changes of volume of bore and the changes of external volume of the plugged tube obey what, in certain cases, appear to be most capricious rules. But as soon as we calculate the strain coefficients, we see at once the reason for this apparent capriciousness. The strain coefficients themselves have, as already mentioned, very similar values in all the tubes of one metal; but as the longitudinal elongation (λ) usually differs in sign from the transverse elongation (μ), the ratio $(\lambda + 2\mu)$ is arithmetically a difference of two numbers not very different in value. Sometimes the one term predominates, sometimes the other. Thus the A and B tubes Nos. V. and VI. are very similar as regards their dilatations (Table VII.), but there is at first sight a marked dissimilarity as regards their volume changes, δv and $\delta v'$ (Table VI.).

In my former paper I found great difficulty in interpreting the apparently capricious results obtained there. This difficulty now in great measure disappears, and the interest is transferred from the measured volume changes to the dilatations to which they lead.

As a general rule, the elongation in the direction of the magnetizing force is the most important. Mr Bidwell* has made us familiar with the laws of its variation in iron, nickel, and cobalt; and the values obtained by me are in full accord with his. The steady manner in which the maximum elongation point shifts into lower fields

* *Phil. Trans.*, Series A., vol. 179, 1888.

as the bore of the tube is widened is a new experimental fact; but it is one which might almost have been predicted from our previous knowledge of the magnetic properties of iron (see above, p. 465).

In the case of the narrow bored Nickel Tubes 1 and I. the radial elongation is greater numerically than the longitudinal dilatation, at least in the higher fields. This feature is slightly shown in Tube II., and is associated with a tangential elongation of the same sign as the longitudinal elongation. In these tubes, accordingly, the internal radius shortens, and the external radius lengthens, so that we are led to the conception of a cylindrical surface within the metal which experiences no displacement outwards or inwards.

A similar feature is characteristic of the narrower bored iron tubes in high fields, as a glance down the columns λ and ν in Table VII. will show.

Now, if we compare the measured quantity $\lambda + 2\mu'$ for a nickel tube of very narrow bore with the quantity δ obtained for the bar from which the tube was formed, we find that the former quantity is numerically much the greater. This shows that the removal of a comparatively small amount of material from the core of a nickel bar alters, in a remarkable degree, the character of the strain which accompanies powerful magnetization. In the bar, therefore, the molecules must be subject to considerable mechanical constraint. The mere formation of a second free surface of small extent in the heart of the metal produces, so to speak, a relief, whose effects extend appreciably to the outer surface. There is, indeed, more resemblance between the narrowest and the widest bored tubes as regards their condition of strain in a magnetic field than between the narrowest bored tube and the original bar.

On the other hand, if we compare the quantity $\lambda + 2\mu'$ for an iron tube of very narrow bore with the quantity δ, we find that the former is the smaller, or, at least, is never the larger. A comparison of the quantities $\delta v'$ and δV for A I. and B I. in Table VI. or in Plate II. indicates this clearly enough. That is to say, the removal of a small amount of material in the heart of the bar alters, to a comparatively slight extent, the displacement of the outer surface under magnetization. The alteration is, nevertheless, quite unmistakable, the formation of a second free surface in the heart of the metal distinctly modifying its behaviour. The tendency, in both iron and nickel, is for $\delta v'$ to be *algebraically less* that δV; but numerically the difference is very much less in iron than in nickel.

Another point of interest is the *form* of ellipsoid into which a small spherical element at either surface is changed; and here again the comparative simplicity of the results for nickel is noteworthy. In every case the shortest axis of the magnetic strain ellipsoid is in the direction of magnetization, and in the great majority of cases the longest is in the direction of the radius. Occasionally in the tubes of narrowest bore the third axis, that which corresponds to μ, the tangential elongation, is one of contraction, but generally it is, like the radial axis, one of elongation. Thus, in all cases of the bored tubes, a circular element in any plane perpendicular to the axis of the cylinder

becomes an ellipse drawn out radially. In the coiled tubes, on the other hand, this ellipse is drawn out more in the tangential than in the radial direction, a feature which also belongs to the (coiled) cobalt tube. A very similar, indeed almost identical, result is obtained with the iron tubes. The central section of the strain ellipsoid in a plane perpendicular to the axis of the cylinder has its greater axis parallel to the radius of the cylinder; or, in other words, μ is either a greater negative ratio or a smaller positive ratio than ν. The only exceptions are tubes A VI. and VII., B VI. and VII. and the coiled tube C II.

In general, then, we may draw the following conclusions regarding the behaviour of tubes of the magnetic metals, iron, nickel, and cobalt:—

1. *The strain is one in which there is comparatively little change of volume, but considerable change of form; in other words, the magnetic stresses, acting on the molecular groups, are mainly shearing stresses. This is particularly true of nickel.*

2. *Except in the case of very thin walls, a circular element in any transverse plane perpendicular to the axis of the cylinder becomes, when the cylinder is magnetized parallel to its axis, an ellipse with its major axis pointing towards the axis of the tube. When the walls are thin, the ellipse into which a small circular element is changed has its minor axis pointing towards the axis of the tube. The ellipse, in both cases, increases in excentricity as the distance from the axis diminishes.*

To obtain similar results with purely mechanical stresses acting on the surfaces of a tube, we should have to take the normal stress on the external surface greater than that on the internal surface for all but the tubes of widest bore; and for the wide bored tubes we should have to take the internal normal stress the greater. For nickel and cobalt in all fields, and for iron in high fields, these surface stresses would be tensions; but for iron in low fields they would be pressures. This comparison is merely intended to illustrate the character of the strain, for there can be no fundamental resemblance between the magneto-elastic problem discussed in this paper and the purely elastic problem hinted at. In the one case we are dealing with the effect of surface tractions upon a tube of elastic material; in the other, with the effect of magnetic body forces upon a tube of elastic material of high susceptibility.

One invariable characteristic of the strain ellipsoid in nickel and cobalt is that the minimum axis is parallel to the magnetizing force. This characteristic holds for iron when the magnetizing force is distinctly higher than that which corresponds to the change of sign of the longitudinal elongation. In other words, when the iron shows marked contraction in the direction of magnetization, its behaviour, as determined by the strain ellipsoid, is very similar to the behaviour of the other metals.

Then, again, when the magnetizing force is such as to produce distinct positive elongation in a direction parallel to its line of action, in this direction also lies the maximum axis of the strain ellipsoid. It is only during the transition condition, as the longitudinal dilatation changes sign, that the corresponding axis of the strain ellipsoid loses its maximum or minimum characteristic.

These broad results are deduced from the numbers contained chiefly in Tables II. and VII. To facilitate somewhat the comparison between iron and nickel I have drawn up the following table giving for the Tubes B V. (iron and nickel) the dilatations in chosen fields at the inner and outer surfaces, thus indicating the ellipsoidal form into which an originally spherical element is changed under the action of magnetic force. The chosen fields are the fields of maximum elongation and of zero elongation in the Iron Tube B V., and the greatest field reached in the experiments. The ratios λ, μ, ν are one millionfold the elongations of three mutually perpendicular unit lines, which when added to unity give the principal semi-axes of the ellipsoid into which the unit sphere is changed.

PRINCIPAL ELONGATIONS IN IRON AND NICKEL TUBES B V., EXPRESSED IN MILLIONTHS (10^{-6}).

	Field = 190.		Field = 310.		Field = 500.	
	Iron.	Nickel.	Iron.	Nickel.	Iron.	Nickel.
λ	+2·1	−16·2	0	−21·8	−2·18	−24·8
μ	−1·5	+ 4·2	−·4	+ 6·7	+ ·8	+ 6·9
ν	− ·4	+12·0	+·6	+15·7	+1·57	+17·9
λ'	+2·1	−16·2	0	−21·8	−2·18	−24·8
μ'	−1·25	+ 6·5	−·1	+ 8·4	+1·01	+ 9·6
ν'	− ·65	+ 9·7	+·3	+13·4	+1·26	+15·2

We have no right to assume that these values hold for every point along the inner and outer surfaces of the tube. Being calculated from observed total changes of length and volume for the whole tube, they are, at best, only average values; and it is highly probable that they vary as we pass from points at the middle to points near the ends. Nevertheless, since much longer tubes give very similar results, these average values must be fair approximations to the true values at most parts of the tube.

It is also a fair presumption that elements in the heart of the metal suffer strains intermediate in character to those belonging to the surface elements.

Taking, then, the mean of the two strains in each case just given, let us calculate the stresses associated with these strains, assumed for the moment to be elastic.

Let P, Q, R be the principal stresses corresponding to the elongations λ, μ, ν. Then P is of the form $M\delta + N\lambda$; and the corresponding expressions for Q and R are obtained by cyclical permutation of λ, μ, ν.

Expressed in terms of YOUNG's modulus E and the rigidity n, the stresses become

$$P = n\left(\frac{E-2n}{3n-E}\,\delta + 2\lambda\right), \quad Q = \text{etc.}, \quad R = \text{etc.}$$

We may take, for rough estimation, $E = 2 \times 10^{12}$ and $n = \cdot 8 \times 10^{12}$ in both metals,* giving the following values in kilogrammes per square centimetre for the stresses corresponding to the mean strains in the foregoing table, if we consider these strains to be purely elastic.

PRINCIPAL MEAN STRESSES IN IRON AND NICKEL TUBES B V. IN KGMS. PER SQ. CM.

	Field = 190.		Field = 310.		Field = 500.	
	Iron.	Nickel.	Iron.	Nickel.	Iron.	Nickel.
P	+1·8	−13	+·08	−18	−1·6	−20
Q	−1·0	+ 4	−·1	+ 6	+ ·8	+ 7
R	− ·6	+ 9	+·3	+12	+1·2	+13

From these numbers we may calculate for each state the quantity

$$\tfrac{1}{2}(P\lambda + Q\mu + R\nu) = \tfrac{1}{2}n\{\delta^2 + 2(\lambda^2 + \mu^2 + \nu^2)\}$$

which measures the potential energy stored up in unit volume of an elastic substance strained to the extent and in the manner indicated by the ratios λ, μ, ν. We find, then, for the potential energy of strain in the three fields named, the quantities

5·3, 0·2, and 6·1 ergs in iron.
327 , 595 , „ 766 „ „ nickel.

These are calculated by taking into account only the *final* values. But there is a fundamental difference between the manner in which the magnetic force of, say, 310 carries the iron through its intermediate conditions of strain, and the manner in which an application of surface tractions effects the same succession of states. At every stage in the process, whether the strain coefficients be increasing or decreasing, the magnetizing force is producing a change which we must suppose to be always resisted by the elastic forces—that is to say, between the field corresponding to the maximum elongation in iron and the zero elongation, there is no real *assisting* of the forces which are producing the strain. We have, in short, no warrant in assuming a giving back of energy by the strained metal during this stage, which, in the purely elastic problem, answers to a condition of work done *by* the substance as it recovers. Since the elastic constants are not appreciably changed by magnetization, we may plausibly enough assume the work done during the second half of the positive elongation stage to be equal to the work done during the first half. Consequently, instead of 0·2 erg being the amount of work done against the elastic forces in the Field 310, we should take 10·6 + 0·2 (= 10·8) as in all probability the better value.

* Mr Mitchell, a student in the Physical Laboratory, Edinburgh University, determined for me the value of YOUNG'S modulus by flexure experiments on the nickel sheets from which the coiled Tubes C_2 C_3 had been formed. The values found were respectively $2{\cdot}1 \times 10^{12}$ and $2{\cdot}5 \times 10^{12}$ in C.G.S. units.

Assuming then that the applied magnetic force strains the substance at every stage against the molecular forces which determine the elasticity of the substance, we find that the amounts of work done per unit volume in so straining the two metals in the three fields named are respectively :—

5·3, 10·8, and 16·7 ergs in iron.
327 , 595 , „ 766 „ „ nickel.

A similar calculation applied to the Iron and Nickel Tubes B II., which are much thicker in the walls than B V., gives for the amounts of work done per unit volume in straining the substance by the magnetic forces 200, 300, and 500, the quantities

6·5, 17, and 27·2 ergs in iron.
385 , 730, „ 1114·6 „ „ nickel.

From both these examples, so very different in the details of the strains, we find, by taking the ratios of the corresponding pairs, that 60 to 40 times as much work is done in straining nickel as is done in straining iron by means of magnetic forces whose values lie between 200 and 500.

Applying a similar calculation to the coiled Tubes C II., of which the iron tube is, however, somewhat thinner in the wall than the nickel tube, we find for the amounts of work stored up in unit volume in Fields 60, 180, 300, and 500,

12·1, 24·1, 32·3, 66 ergs in iron.
99·7, 596 , 818·4, 961 „ „ nickel.

These quantities show that in a field of 250 the work done in straining nickel against the elastic stresses is about 27 times the corresponding work done in straining the iron. Because of the different thicknesses of the tubes, this comparison has not the same claim to attention as the other two.

The assumptions underlying the calculations that have just been made are, of course, open to criticism.

The magnetic force acts directly on the molecular groupings, which break up to form new configurations. The mutual action between every contiguous pair of these new configurations is in all probability the effective cause of the strains produced. We know nothing definite regarding this mutual action, but it is not an altogether unreasonable hypothesis to suppose that the strains are produced against the elastic forces which bind the molecules together. It is, at all events, a fact worthy of note, that the less susceptible material is the one on which the greater amount of molecular work is done.

I have made no attempt to bring these experimental facts into line with the theories of magneto-striction as developed by Helmholtz, Kirchhoff, J. J. Thomson, and others. Cantone's calculation* of Kirchhoff's constants from the results of experiment on iron and nickel ovoids can hardly be regarded as a test of the applicability of Kirchhoff's

* *Atti d. R. Accad. d. Lincei*, vi. 1890.

theory. Dr TAYLOR JONES has proved* that only a small part of the longitudinal contractions of magnetized nickel wire can be accounted for by means of KIRCHHOFF's or THOMSON's theory. NAGAOKA and JONES† have shown that KIRCHHOFF's theory, when applied to an anchor ring uniformly magnetized, leads to the conclusion that the cubical dilatation should be three times the linear dilatation—a result not borne out by BIDWELL's experiments.‡

Under the influence of strong magnetic forces, iron, nickel, and cobalt are no doubt brought into an æolotropic state, very different from the state before the magnetic forces were applied. The molecules or molecular groups are thrown into new configurations, which, by their mutual action, give rise to the accompanying strains. To get some idea of the nature of these strains under conditions favourable for fairly accurate measurement has been the object of this paper. The significance of many of the facts described is by no means clear, and no recognised theory of magnetic stress seems able to elucidate them. The introduction of terms representing the rotation of molecules§ adds greatly to the complexity of the equations, and it is difficult to see how these could be experimentally tested. It is not, however, so much the rotation of the individual molecule that we have to consider, as the resultant effect of new configurations of molecular groups.

APPENDIX—*September* 1898.—As this paper was passing through the press, I received from my former pupil and colleague, Professor NAGAOKA, of the Imperial University, Japan, an important and masterly discussion‖ of the applicability of KIRCHHOFF's theory of magneto-striction to the co-ordination of the inter-relations of magnetism and strain. In this paper, NAGAOKA and HONDA give measurements of the volume changes due to magnetization in iron and nickel, which are fairly concordant with the values given here. The nickel rod they used was much smaller in section than my nickel B tube, which may, perhaps, account for the greater values of the cubical dilatation obtained by them. They compare their results with my measurements of the changes of volume of *bore* as given in Part I.; but such a comparison cannot really be made. It is not merely, as they suggest, that their "measurements of the volumes were *external*," while mine "were made on the changes in the *internal* capacity of a nickel tube." As already pointed out (see above, p. 473), the boring out of a nickel bar completely changes its behaviour in a magnetic field, the outer surface becoming subject to a displacement much greater than what was found with the bar solid throughout. For the same reason their remark that a certain inconvenience inseparable from my earlier form of experiment "will disappear if the change of volume

* *Phil. Trans.*, Series A., vol. 189, pp. 189–200, 1895.

† *Phil. Mag.*, May 1896, p. 454.

‡ *Proc. Roy. Soc.*, 1890.

§ See DUHEM, *American Journal of Mathematics*, xvii. p. 117, 1895.

‖ "Researches on Magneto-striction." By Professor NAGAOKA and Mr HONDA. *Journal of the College of Science*, Imperial University, Japan, vol. ix. p. 353; also in *Phil. Mag.* for September 1898.

of the magnet itself be observed" has no real relevancy; for the change of volume of the magnet itself was not the subject of inquiry.

NAGAOKA and HONDA's experimental determination of the very slight effect of hydrostatic pressure on the magnetization is of high interest. It emphasises the view expressed above that the strain accompanying magnetization is mainly a shear. Whether we accept KIRCHHOFF's theory or not, we should expect to find small cubical dilatation under magnetization to be associated with small magnetic change under increased hydrostatic pressure. This result, consequently, can hardly be regarded as a verification of KIRCHHOFF's theory. A similar remark may be made in regard to other reciprocal relations; and before such a theory as KIRCHHOFF's can be accepted as established, there should be approximate *numerical* identity between theory and experiment. Mere *qualitative* agreement in a few particulars between theory and experiment may be largely a matter of chance, and cannot be put in the same category with one serious discrepancy. NAGAOKA and HONDA admit that "KIRCHHOFF's theory is a rough approximation," but conclude "that, excepting the theoretical deduction as to the effect of hydrostatic pressure on the magnetization of iron, there are no serious discrepancies between theory and experiment." Their high class experimental work, by bringing to light one serious discrepancy and other discrepancies of a less serious character, seems to me to demonstrate the insufficiency of KIRCHHOFF's theory. This, of itself, is important enough; but, in the present state of our knowledge, the new experimental facts discovered by NAGAOKA and HONDA are of much greater importance.

Explanation of Symbols used in the Following Tables.

V = volume of material of tube.
v = ,, bore ,,
v' = ,, bar of same length and breadth as tube.

} All expressed in cubic centimetres.

δV, δv, $\delta v'$ = the measured changes of the volumes; unit, 10^{-6} cub. cm.

$\delta = \delta V/V$ = cubical dilatation.
λ = elongation parallel to axis of tube.
μ, μ' = tangential elongations at inner and outer surfaces.
ν, ν' = radial ,, ,, ,,

} All expressed in millionths, 10^{-6}.

The value of the magnetic field in every case is the value at the centre of the magnetizing coil and is expressed in C.G.S. magnetic units.

TABLE I.—Dimensions of the Various Tubes.

The Nickel Tubes (Bored).

Tube.	Length.	Diameter.		Volume of		Tube.	Length.	Diameter.		Volume of	
		External.	Internal.	Bore.	Metal.			External.	Internal.	Bore.	Metal.
B II.	20·2	2·77	·953	14·04	103·36	I.	47	4·2	·635	17·43	633·57
" III.	"	"	1·270	25·18	92·22	II.	"	"	·953	33·57	617·43
" IV.	"	"	1·588	38·80	78·60	III.	"	"	1·270	59·77	591·23
" V.	"	"	1·905	58	59·4	IV.	"	"	1·588	95·81	555·19
1	47	4·2	·635	17·88	633·12	V.	"	"	1·905	136·35	514·65
4	"	"	1·588	87·58	563·42	VI.	"	"	2·222	173·72	477·28
7	"	"	2·540	224·47	426·53	VII.	"	"	2·540	228·6	422·4

The Coiled Tubes.

Tube.	Length.	Diameter.	Thickness.	Volume of	
				Bore.	Metal.
Nickel C I.	25	2·5	·027	117	5·85
" C II.	"	"	·050	113	9·78
" C III.	"	"	·105	102·8	20·01
Cobalt.	25·2	2·42	·038	112·4	7·6
Iron C I.	25·5	2·63	·037	127·2	8·3
" C II.	25·6	2·4	"	118·25	8·25

Iron Tubes A and B.

Tube.	Length.	Diameter.		Volume of		Tube.	Length.	Diameter.		Volume of	
		External.	Internal.	Bore.	Metal.			External.	Internal.	Bore.	Metal.
A I.	40·6	2·62	·635	13·68	205·12	B I.	20·3	2·62	·635	7·52	101·88
" II.	"	"	·953	27·7	191·1	" II.	"	"	·953	14·44	94·96
" III.	"	"	1·270	52·12	166·68	" III.	"	"	1·270	25·78	83·62
" IV.	"	"	1·588	79·18	139·62	" IV.	"	"	1·588	39·11	70·29
" V.	"	"	1·905	112·35	106·45	" V.	"	"	1·905	55·26	54·14
" VI.	"	"	2·222	152·48	66·32	" VI.	"	"	2·222	76·17	33·23
" VII.	"	"	2·420	184·63	34·17	" VII.	"	"	2·420	93·59	15·8

Iron and Steel Tubes (Original Set).

Iron.						Steel.					
Tube.	Length.	Diameter.		Volume of		Tube.	Length.	Diameter.		Volume of	
		External.	Internal.	Bore.	Metal.			External.	Internal.	Bore.	Metal.
3	46·2	3·84	1·270	67·7	470·1	3	45·7	3·84	1·270	61·93	467·17
5	"	"	1·905	127·4	410·4	5	"	"	1·905	127·95	401·15
7	"	"	2·540	223·4	314·4	7	"	"	2·540	227·45	301·65
VII.	"	"	2·696	252·08	285·72	VII.	"	"	2·696	243·51	285·59
9	"	"	3 19	343·7	194·1	9	"	"	3·19	343·3	185·8

Iron Tubes I′. to VIII′.

Tube.	Length.	Diameter.		Volume of	
		External.	Internal.	Bore.	Metal.
I′.	46·2	3·84	·635	17·4	511·7
II′.	"	"	·953	31·96	497·14
III′.	"	"	1·270	59·6	469·5
IV′.	"	"	1·588	90·38	438·72
V′.	"	"	1·905	128 8	400 3
VI′.	"	"	2·222	173·13	355·97
VII′.	"	"	2·540	220·82	308·28
VIII′.	"	"	2·858	286·71	242·39

TABLE II.

NICKEL TUBES B – II, III, IV, V.

Field.	Volume Changes × 10^6 c.c.			Dilatations × 10^6					
	δV.	δv.	$\delta v'$.	δ.	λ.	μ.	ν.	μ'.	ν'.
50	– ·5	0	+ 2·5	– ·00	– ·5	+ ·25	+ ·25	+ ·25	+ ·25
100	– 2·5	– 5	– 5·5	– ·02	– 3·5	+ 1·6	+ 1·9	+ 1·7	+ 1·8
150	– 5·4	– 55	– 57	– ·05	– 8·5	+ 2·3	+ 6·2	+ 4·0	+ 4·5
200	– 8·0	– 102	– 109·3	– ·08	– 12·4	+ 2·6	+ 9·7	+ 5·7	+ 6·6
250	– 7·8	– 140	– 150	– ·08	– 15·2	+ 2·6	+ 12·5	+ 7·0	+ 8·1
300	– 6·2	– 167	– 171·7	– ·06	– 16·9	+ 2·5	+ 14·4	+ 7·2	+ 9·7
400	– 2·8	– 192	– 197·8	– ·03	– 19·3	+ 2·8	+ 16·5	+ 8·8	+ 10·5
500	+ 3·3	– 210	– 212·7	+ ·03	– 21	+ 3	+ 18	+ 9·6	+ 11·4
50	– ·8	0	0	– ·01	– 1·2	+ ·6	+ ·6	+ ·6	+ ·6
100	– 2·7	– 51	– 59	– ·03	– 5·5	+ 1·8	+ 3·7	+ 2·5	+ 3
150	– 5·5	– 172	– 190	– ·06	– 10·2	+ 1·7	+ 8·5	+ 4·3	+ 5·9
200	– 7·8	– 278	– 294	– ·09	– 14·7	+ 1·9	+ 12·7	+ 6·1	+ 8·5
250	– 7·7	– 346	– 362	– ·09	– 17·4	+ 1·9	+ 15·4	+ 7·2	+ 10·1
300	– 6·2	– 387	– 403	– ·07	– 19·4	+ 2·0	+ 17·4	+ 8	+ 11·4
400	– 2·8	– 435	– 444	– ·03	– 21·7	+ 2·2	+ 19·5	+ 9	+ 12·7
500	+ 3·1	– 456	– 459	+ ·04	– 23·1	+ 2·5	+ 20·6	+ 9·6	+ 13·5
50	0	+ 1	– 2	– ·00	– 1·6	+ ·8	+ ·8	+ ·8	+ ·8
100	– ·5	– 80	– 80	– ·01	– 6·4	+ 2·2	+ 4·2	+ 2·9	+ 3·5
150	– 1	– 208	– 217	– ·01	– 11·4	+ 3·0	+ 8·4	+ 4·8	+ 6·6
200	– 1·3	– 303	– 321	– ·02	– 15·5	+ 3·8	+ 11·7	+ 6·4	+ 9·1
250	– 2	– 364	– 384	– ·03	– 18·2	+ 4·4	+ 13·8	+ 7·5	+ 10·7
300	– 2·7	– 401	– 416	– ·03	– 20	+ 4·7	+ 15·3	+ 8·2	+ 11·8
400	– 2·8	– 436	– 448	– ·04	– 22·2	+ 5·5	+ 16·7	+ 9·2	+ 13
500	+ 3·7	– 450	– 463	– ·05	– 23·8	+ 6·1	+ 17·7	+ 10	+ 13·8
50	0		– 80	– ·00	– 2	+ ·8	+ 1·2	+ ·9	+ 1·1
100	– ·7		– 185	– ·01	– 8·3	+ 2·6	+ 5·7	+ 3·4	+ 4·9
150	– 1		– 331	– ·02	– 13·3	+ 3·8	+ 9·5	+ 5·2	+ 8·1
200	– ·8		– 481	– ·01	– 17	+ 4·4	+ 12·6	+ 6·5	+ 10·5
250	– 1·2		– 527	– ·02	– 19·8	+ 5·4	+ 14·4	+ 7·7	+ 12·1
300	– 1·5		– 583	– ·03	– 21·7	+ 5·9	+ 15·8	+ 8·4	+ 13·3
400	– 2·5		– 615	– ·04	– 24	+ 6·8	+ 17·2	+ 9·4	+ 14·6
500	– 3·4		– 627	– ·06	– 24·8	+ 6·9	+ 17·9	+ 9·6	+ 15·2

TABLE III.

NICKEL TUBES 1, 4, 7, VII.

Field.	λ.	μ.	ν.	μ′.	ν′.		δν.
50	− 1·6	+ ·8	+ ·8	+ ·8	+ ·8		− 1
100	− 6·8	+ 1·1	+ 5·7	+ 3·3	+ 3·5		− 82
150	− 11·9	− ·9	+ 12·8	+ 5·8	+ 6·1		− 245
200	− 15·5	− 3·0	+ 18·5	+ 7·5	+ 8·0	1	− 383
300	− 19·3	− 4·6	+ 23·9	+ 9·2	+ 10·1		− 509
400	− 21·7	− 6·1	+ 27·8	+ 10·4	+ 11·3		− 605
500	− 23·1	− 6·2	+ 29·3	+ 11·1	+ 12·0		− 634
50	− 3·2	+ 1·5	+ 1·7	+ 1·6	+ 1·6		− 26
100	− 9·8	+ 2·9	+ 6·9	+ 4·6	+ 5·2		− 358
150	− 14·6	+ 3·7	+ 10·9	+ 6·8	+ 7·8		− 639
200	− 17·5	+ 4·2	+ 13·3	+ 8·1	+ 9·4	4	− 794
300	− 20·8	+ 5·1	+ 15·7	+ 9·7	+ 11·1		− 924
400	− 22·4	+ 5·7	+ 16·7	+ 10·5	+ 11·9		− 967
500	− 23·2	+ 6·0	+ 17·2	+ 10·9	+ 12·3		− 975
50	− 3·8	+ 1·4	+ 2·4	+ 1·7	+ 2·1		− 210
100	− 10·4	+ 2·5	+ 7·9	+ 4·3	+ 6·1		− 1230
150	− 15·3	+ 3·3	+ 12·0	+ 6·2	+ 9·1		− 1920
200	− 18·2	+ 3·8	+ 14·4	+ 7·3	+ 10·9	7	− 2390
300	− 21·5	+ 4·7	+ 16·8	+ 8·6	+ 12·9		− 2730
400	− 22·8	+ 5·2	+ 17·6	+ 9·2	+ 13·6		− 2800
500	− 23·6	+ 5·4	+ 18·2	+ 9·6	+ 14·0		− 2870
50	− 3·6	+ 1·7	+ 1·9	+ 1·8	+ 1·8		− 45
100	− 9·6	+ 3·3	+ 6·3	+ 4·7	+ 4·9		− 710
150	− 14·5	+ 4·3	+ 10·2	+ 6·2	+ 8·3		− 1350
200	− 17·5	+ 4·9	+ 12·6	+ 7·4	+ 10·1	VII.	− 1750
300	− 20·8	+ 5·8	+ 15·0	+ 8·8	+ 12·0		− 2130
400	− 22·4	+ 6·2	+ 16·2	+ 9·5	+ 12·9		− 2285
500	− 23·5	+ 6·7	+ 16·8	+ 10·0	+ 13·5		− 2335

TABLE IV.

NICKEL TUBES, I.–VII.

Field.	λ.	μ.	ν.	μ′.	ν′.		δν.
50	− 3·6	+2·0	+ 1·6	+ 1·9	+ 1·7	I.	+ 8
150	−14·5	+1·7	+12·8	+ 7·1	+ 7·4		− 195
300	−20·8	− 3·8	+24·6	+10·0	+10·8		− 495
500	−23·5	− 4·2	+27·2	+11·3	+12·2		− 555
50	− 3·6	+2·0	+ 1·6	+ 1·9	+ 1·7	II.	+ 15
150	−14·5	+3·2	+11·3	+ 7·0	+ 7·5		− 270
300	−20·8	+ ·9	+19·9	+ 9·9	+10·9		− 640
500	−23·5	− ·9	+24·4	+11·1	+12·4		− 850
50	− 3·6	+2·2	+ 1·4	+ 1·8	+ 1·8	III.	+ 45
150	−14·5	+3·6	+10·9	+ 6·9	+ 7·6		− 440
300	−20·8	+ ·8	+20	+ 9·5	+11·3		−1160
500	−23·5	+ ·3	+23·2	+10·6	+12·9		−1380
50	− 3·6	+2·2	+ 1·4	+ 1·9	+ 1·7	IV.	+ 75
150	−14·5	+4·9	+ 9·6	+ 6·9	+ 7·6		− 460
300	−20·8	+4·4	+16·4	+ 9·5	+11·3		−1160
500	−23·5	+4·4	+19·1	+10·7	+12·8		−1400
50	− 3·6	+2·2	+ 1·4	+ 1·9	+ 1·7	V.	+ 110
150	−14·5	+5·5	+ 9·0	+ 6·9	+ 7·6		− 480
300	−20·8	+6·4	+14·4	+ 9·6	+11·2		−1100
500	−23·5	+7·1	+16·4	+10·8	+12·7		−1280
50	− 3·6	+1·9	+ 1·7	+ 1·8	+ 1·8	VI.	+ 25
150	−14·5	+4·0	+10·5	+ 6·4	+ 8·1		−1150
300	−20·8	+4·7	+16·1	+ 8·9	+11·9		−2000
500	−23·5	+5·5	+18·0	+10·1	+13·4		−2190
50	− 3·6	+1·7	+ 1·9	+ 1·8	+ 1·8	VII.	− 45
150	−14·5	+4·3	+10·2	+ 6·2	+ 8·3		−1350
300	−20·8	+5·8	+15·0	+ 8·8	+12·0		−2130
500	−23·5	+6·7	+16·8	+10	+13·5		−2335

TABLE V.

Nickel C Tubes (Coiled).

Field.	$\delta v'$.			C I.			C II.			C III.		
	C I.	C II.	C III.	λ.	μ'.	ν'.	λ.	μ'.	ν'.	λ.	μ'.	ν'.
25	− 54	− 22	− 22	− 4	+ 1·8	+ 2·2	− 2	+ 1·0	+ 1·0	− 1·3	·6	·7
50	+ 72	+ 30	− 33	− 9·5	+ 5·0	+ 4·5	− 7·1	+ 3·7	+ 3·4	− 5·3	2·5	2·8
100	+430	+200	+ 46	−18·2	+10·9	+ 7·4	−15·3	+ 8·5	+ 6·8	−13·6	7·0	6·6
150	+618	+345	+155	−22·5	+13·8	+ 8·7	−20·1	+11·5	+ 8·6	−18·8	10·0	8·8
200	+718	+410	+233	−25·4	+15·6	+ 9·8	−22·9	+13·1	+ 9·8	−22·3	12·1	10·2
300	+737	+483	+320	−28·2	+17·1	+11·1	−26	+15·0	+11·0	−25·5	14·1	11·4
400	+735	+497	+349	−29·3	+17·7	+11·6	−27·2	+15·6	+11·6	−27·5	15·2	12·3
500	+728	+498	+361	−30	+18·0	+12·0	−28·2	+16·1	+12·1	−28·5	15·7	12·8

Cobalt Tube (Coiled).

Field.	$\delta v'$.	δV.	δ.	λ.	μ'.	ν'.
50	− 6	− ·1	− ·01	− ·2	+ ·1	+ ·1
100	− 26	− ·2	− ·03	− ·8	+ ·3	+ ·5
200	− 82	− ·9	− ·12	− 2·3	+ ·8	+ 1·4
300	−158	− 1·3	− ·17	− 4·0	+ 1·4	+ 2·4
400	−240	− 1·4	− ·18	− 5·8	+ 1·9	+ 3·7
500	−320	− 1·4	− ·18	− 7·6	+ 2·5	+ 4·9

Iron C Tubes (Coiled).

Field.	$\delta v'$.		C I.			C II.		
	C I.	C II.	λ.	μ'.	ν'.	λ.	μ'.	ν'.
25	+15·2	+51	+ 1·9	− ·94	− ·96	+ 2·1	− ·87	− 1·23
50	−15·3	+20	+ 2·21	− 1·17	− 1·04	+ 3·1	− 1·48	− 1·62
100	− 35	+ 0·8	+ 1·67	− ·98	− ·69	+ 2·17	− 1·08	− 1·09
150	−36·2	+ 1	+ ·49	− ·40	− ·09	+ ·9	− ·45	− ·45
200	−32·1	+ 8	− ·63	+ ·18	+ ·45	− ·3	+ ·18	+ ·12
300	−20·7	+24	− 2·92	+ 1·38	+ 1·54	− 2·61	+ 1·39	+ 1·22
400	− 8·7	+42	− 4·83	+ 2·38	+ 2·45	− 4·78	+ 2·54	+ 2·24
500	+ 3·2	+62	− 6·1	+ 3·07	+ 3·03	− 5·9	+ 3·18	+ 2·72

TABLE VI.—Volume Changes in Iron.

Field.	A Tubes. δV.	A Tubes. δv.	A Tubes. δv′.		B Tubes. δV.	B Tubes. δv.	B Tubes. δv′.
50	+ 3	− 2·2	+ ·3	I.	+ 1	− ·1	+ 1
100	+ 15	− 26·7	− 11·5		+ 2·4	− ·5	+ 1·7
150	+ 28·6	− 36	− 9·5		+ 4·3	− 2·2	+ 2·9
200	+ 37·2	− 37·5	− 1·8		+ 8·1	− 6·8	+ 4·4
250	+ 45·2	− 37·9	+ 4·2		+ 13	− 14·2	+ 5·8
300	+ 50·5	− 38·2	+ 11		+ 18·3	− 17·9	+ 7·0
400	+ 62·7	− 39·3	+ 25		+ 27·8	− 22·8	+ 10·0
500	+ 75·2	− 40·8	+ 41·7		+ 37·8	− 26·7	+ 14·5
50	+ 5·4	− 10	− 5	II.	+ ·7	− ·8	+ ·3
100	+ 14·8	− 53·5	− 35·5		+ 3·2	− 2·4	+ ·5
150	+ 29	− 63	− 30·5		+ 7·4	− 7·8	− ·7
200	+ 38·4	− 64·1	− 23·8		+ 11·7	− 16·3	− 5·1
250	+ 46·3	− 65·1	− 18·8		+ 18·3	− 32·3	− 14·2
300	+ 52·2	− 66·1	− 15·8		+ 23·7	− 41	− 17·3
400	+ 62·3	− 68·3	− 9·5		+ 32·6	− 43·5	− 12·3
500	+ 73·1	− 69	− ·8		+ 37·5	− 47	− 9·5
50	+ 4·3	− 35	− 30	III.	+ 1	− 1·2	− ·7
100	+ 16·5	− 115	− 95		+ 3·1	− 5	− 2·5
150	+ 29	− 112	− 81		+ 7	− 15	− 9·4
200	+ 35·2	− 113	− 74·8		+ 13·2	− 41	− 28·4
250	+ 40	− 113	− 69·8		+ 19·9	− 54	− 32·5
300	+ 43	− 113	− 68		+ 25·6	− 51·3	− 23·4
400	+ 49·2	− 114	− 64		+ 36·2	− 52·2	− 12·0
500	+ 58·2	− 115	− 57·3		+ 43·3	− 52·2	− 6·7
50	+ 4	− 52	− 50	IV.	+ ·5	− 3·5	− 3·5
100	+ 12	− 125	− 111		+ 1·5	− 12·8	− 13
150	+ 21	− 122	− 101		+ 4·3	− 34	− 34
200	+ 26·2	− 122·5	− 96		+ 8·1	− 59·2	− 56·8
250	+ 29·7	− 122·5	− 91		+ 11·3	− 55	− 48
300	+ 33·5	− 123	− 88		+ 13·8	− 51·7	− 42·7
400	+ 41·7	− 124	− 83		+ 16·2	− 48	− 38·5
500	+ 51·2	− 125	− 80		+ 17·3	− 48	− 35·2
50	+ 3·6	− 100	− 100	V.	+ ·7	− 7	− 5·5
100	+ 11·5	− 109	− 93		+ 1·8	− 25	− 22
150	+ 17·8	− 100	− 81		+ 4·1	− 51·5	− 46·8
200	+ 22·2	− 97	− 73		+ 7·5	− 49·7	− 42·3
250	+ 24·7	− 95	− 67		+ 9·1	− 43·5	− 34
300	+ 26	− 94	− 67		+ 9·7	− 40·8	− 29·4
400	+ 32	− 91	− 62		+ 9·9	− 36·3	− 23·8
500	+ 39·7	− 88	− 53		+ 10·2	− 32·2	− 18·2
50	+ 4·8	− 38	− 33	VI.	+ 1	− 15·5	− 15
100	+ 8·9	+ 32	+ 46		+ 2·7	− 29·6	− 25
150	+ 12·6	+ 55·5	+ 71		+ 5·2	+ 4	+ 16
200	+ 16·2	+ 63·5	+ 80		+ 7·2	+ 22·5	+ 37·9
250	+ 20	+ 68	+ 86		+ 7·7	+ 32·3	+ 48
300	+ 22	+ 71·5	+ 91		+ 7·8	+ 39·2	+ 53·8
400	+ 26	+ 78·1	+ 98		+ 7·2	+ 47·5	+ 59·7
500	+ 28·7	+ 84·3	+ 128		+ 7·3	+ 53·5	+ 61·8
25	+ 2	− 18		VII.	+ ·2	− 8	− 7·7
50	+ 4	+ 42			+ ·5	− 13·2	− 10·2
100	+ 5·5	+ 97			+ ·9	+ 12·2	+ 17·5
150	+ 6·2	+ 107	Not observed.		+ 1·8	+ 20·4	+ 28·3
200	+ 6·5	+ 113			+ 2·7	+ 22·5	+ 31·5
300	+ 5	+ 118			+ 3·5	+ 23	+ 33
400	+ 3·2	+ 122			+ 3·2	+ 23·5	+ 33·5
500	+ 1·5	+ 132			+ 2·3	+ 26	+ 35·7

TABLE VII.—Dilatations in Iron.

Field.	A Tubes.							B Tubes.						
	δ.	λ.	μ.	ν.	μ′.	ν′.		δ.	λ.	μ.	ν.	μ′.	ν′.	
50	+·02	+ ·6	− ·38	− ·20	− ·3	− ·28	I.	+·01	+ ·15	− ·08	− ·06	− ·07	− ·07	I.
100	+·08	+1·8	−1·88	+ ·16	− ·93	− ·8		+·02	+ ·48	− ·28	− ·18	− ·23	− ·23	
150	+·14	+2·65	−2·64	+ ·13	−1·35	−1·16		+·04	+ ·91	− ·6	− ·27	− ·44	− ·43	
200	+·19	+2·5	−2·62	+ ·31	−1·25	−1·06		+·08	+1·46	−1·18	− ·20	− ·71	− ·67	
250	+·23	+1·96	−2·37	+ ·64	− ·96	− ·77		+·13	+2·08	−1·99	+ ·04	−1·02	− ·93	
300	+·25	+1·34	−2·07	+ ·98	− ·63	− ·44		+·18	+2·5	−2·44	+ ·12	−1·22	−1·10	
400	+·31	− ·2	−1·34	+1·85	+ ·17	+ ·36		+·27	+2·79	−2·91	+ ·39	−1·35	−1·17	
500	+·38	−1·6	− ·69	+2·71	+ ·90	+1·08		+·37	+2·42	−2·99	+ ·94	−1·15	− ·9	
50	+·03	+ ·6	− ·48	− ·09	− ·31	− ·26	II.	+·01	+ ·17	− ·12	− ·04	− ·08	− ·08	II.
100	+·08	+2·2	−2·07	− ·05	−1·18	− ·94		+·03	+ ·5	− ·34	− ·13	− ·25	− ·22	
150	+·15	+2·72	−2·5	− ·07	−1·43	−1·14		+·08	+1·0	− ·77	− ·15	− ·51	− ·41	
200	+·20	+2·43	−2·37	+ ·14	−1·27	− ·96		+·12	+1·63	−1·37	− ·14	− ·84	− ·67	
250	+·24	+1·82	−2·09	+ ·51	− ·95	− ·63		+·19	+2·28	−2·26	+ ·17	−1·21	− ·88	
300	+·27	+1·1	−1·74	+ ·91	− ·58	− ·28		+·25	+2·54	−2·69	+ ·40	−1·35	− ·94	
400	+·33	− ·32	−1·08	+1·73	+ ·15	+ ·50		+·34	+2·24	−2·66	+ ·76	−1·18	− ·72	
500	+·38	−1·63	− ·43	+2·44	+ ·8	+1·21		+·39	+1·3	−2·28	+1·37	− ·70	− ·21	
50	+·03	+ ·9	− ·79	− ·08	− ·52	− ·35	III.	+·01	+ ·24	− ·14	− ·09	− ·13	− ·10	III.
100	+·10	+2·48	−2·35	− ·03	−1·46	− ·92		+·04	+ ·51	− ·35	− ·12	− ·27	− ·20	
150	+·17	+2·75	−2·45	− ·13	−1·56	−1·02		+·08	+ ·96	− ·77	− ·11	− ·53	− ·35	
200	+·21	+2·42	−2·29	+ ·08	−1·38	− ·83		+·16	+1·45	−1·52	+ ·23	− ·86	− ·43	
250	+·24	+1·8	−1·98	+ ·42	−1·05	− ·51		+·24	+2·28	−2·19	+ ·15	−1·29	− ·75	
300	+·26	+1·05	−1·61	+ ·82	− ·67	− ·12		+·30	+2·17	−2·08	+ ·21	−1·20	− ·67	
400	+·30	− ·4	− ·90	+1·60	+ ·06	+ ·64		+·43	+1·27	−1·65	+ ·81	− ·69	− ·15	
500	+·35	−1·86	− ·17	+2·38	+ ·80	+1·41		+·52	− ·1	− ·96	+1·58	+ ·02	+ ·60	
50	+·03	+ ·95	− ·81	− ·11	− ·59	− ·33	IV.	+·01	+ ·1	− ·10	+ ·1	− ·07	− ·02	IV.
100	+·09	+2·55	−2·07	− ·39	−1·53	− ·93		+·02	+ ·43	− ·38	− ·03	− ·28	− ·13	
150	+·15	+2·58	−2·06	− ·37	−1·52	− ·91		+·06	+1·0	− ·94	·00	− ·66	− 28	
200	+·19	+2·08	−1·82	− ·07	−1·26	− ·63		+·12	+2·0	−1·76	− ·12	−1·26	− ·62	
250	+·21	+1·4	−1·48	+ ·29	− ·9	− ·29		+·16	+2·0	−1·71	− ·13	−1·22	− ·62	
300	+·24	+ ·7	−1·13	+ ·67	− ·54	+ ·08		+·20	+1·5	−1·41	+ ·11	− ·95	− ·35	
400	+·30	− ·7	− ·44	+1·44	+ ·17	+ ·83		+·23	+ ·45	− ·84	+ ·62	− ·45	+ ·23	
500	+·37	−1·97	+ ·20	+2·14	+ ·80	+1·54		+·25	− ·7	− ·26	+1·21	+ ·19	+ ·76	
50	+·03	+1·15	−1·02	− ·1	− ·81	− ·31	V.	+·01	+ ·1	− ·12	+ ·03	− ·08	− ·01	V.
100	+·11	+2·16	−1·57	− ·48	−1·30	− ·75		+·03	+ ·7	− ·58	− ·09	− ·46	− ·21	
150	+·17	+1·95	−1·42	− ·36	−1·16	− ·62		+·08	+1·5	−1·22	− ·20	− ·97	− ·45	
200	+·21	+1·37	−1·12	− ·04	− ·85	− ·31		+·14	+2·1	−1·50	− ·46	−1·25	− ·71	
250	+·23	+ ·61	− ·73	+ ·35	− ·46	+ ·08		+·17	+1·04	− ·92	+ ·05	− ·68	− ·19	
300	+·24	− ·12	− ·36	+ ·72	− ·10	+ ·46		+·18	+ ·13	− ·44	+ ·47	− ·20	+ ·25	
400	+·30	−1·66	+ ·43	+1·53	+ ·69	+1·27		+·18	−1·15	+ ·25	+1·08	+ ·47	+ ·76	
500	+·37	−2·7	+ ·96	+2·11	+1·23	+1·84		+·19	−2·18	+ ·8	+1·57	+1·01	+1·26	
50	+·07	+1·8	−1·03	− ·7	− ·98	− ·75	VI.	+·03	+ ·3	− ·25	− ·02	− ·22	− ·05	VI.
100	+·13	+2·35	−1·07	−1·15	−1·07	−1·15		+·08	+1·7	−1·05	− ·57	− ·97	− ·65	
150	+·19	+1·74	− ·69	− ·86	− ·70	− ·85		+·15	+2·3	−1·12	−1·03	−1·08	−1·07	
200	+·24	+1·07	− ·33	− ·50	− ·35	− ·48		+·22	+2·0	− ·85	− ·93	− ·83	− ·95	
250	+·30	+ ·22	+ ·12	− ·04	+ ·09	− ·01		+·23	+1·3	− ·44	− ·59	− ·43	− ·64	
300	+·33	− ·61	+ ·54	+ ·40	+ ·52	+ ·42		+·24	+ ·4	+ ·06	− ·22	+ ·1	− ·26	
400	+·39	−2·05	+1·28	+1·16	+1·25	+1·19		+·22	− ·96	+ ·79	+ ·39	+ ·76	+ ·42	
500	+·43	−3·1	+1·83	+1·70	+1·84	+1·69		+·22	−2·1	+1·40	+ ·92	+1·34	+ ·98	
25	+·06	+1·2	− ·65	− ·49	− ·64	− ·50	VII.	+·01	+ ·15	− ·11	− ·03	− ·13	− ·01	VII.
50	+·12	+2·2	− ·99	−1·09	− ·99	−1·09		+·03	+1·15	− ·64	− ·48	− ·63	− ·49	
100	+·16	+2·4	− ·92	−1·32	− ·99	−1·25		+·06	+2·0	− ·94	−1·00	− ·92	−1·02	
150	+·18	+1·7	− ·56	− ·96	− ·59	− ·93		+·12	+1·4	− ·59	− ·69	− ·57	− ·71	
200	+·19	+ ·9	− ·15	− ·56	− ·18	− ·53		+·17	+ ·6	− ·16	− ·27	− ·14	− ·29	
300	+·15	− ·55	+ ·60	+ ·1	+ ·56	+ ·14		+·22	−1·42	+ ·84	+ ·80	+ ·86	+ ·78	
400	+·09	−1·63	+1·15	+ ·57	+1·11	+ ·61		+·20	−2·96	+1·61	+1·55	+1·63	+1·53	
500	+·04	−2·6	+1·66	+ ·98	+1·61	+1·03		+·15	−4·48	+2·38	+2·25	+2·40	+2·23	

TABLE VIII.

IRON TUBES 3, 5, 7, VII., 9.

Field.	δ.	λ.	μ.	ν.	μ'.	ν'.		δv.
50	+·03	+ ·40	− ·21	− ·16	− ·19	− ·18		− ·9
100	+·09	+1·48	−1·13	− ·26	− ·76	− ·63		− 52·9
150	+·17	+2·75	−2·42	− ·16	−1·44	−1·14		−141
200	+·22	+3·07	−2·70	− ·15	−1·59	−1·26	3	−158
300	+·28	+1·92	−2·29	+ ·65	−1·01	− ·63		−181
400	+·33	+ ·50	−1·90	+1·73	− ·32	+ ·11		−223
500	+·43	− ·96	−1·37	+2·76	+ ·43	+ ·96		−249
50	+·03	+ ·17	− ·21	+ ·07	− ·11	− ·03		− 31
100	+·09	+1·05	−1·27	+ ·31	− ·67	− ·29		−187
150	+·17	+1·90	−1·79	+ ·06	−1·09	− ·64		−213
200	+·22	+1·94	−1·68	− ·04	−1·06	− ·66	5	−182
300	+·28	+ ·90	−1·19	+ ·57	− ·52	− ·10		−187
400	+·33	− ·53	− ·54	+1·40	+ ·20	+ ·66		−207
500	+·43	−1·84	+ ·03	+1·94	+ ·87	+1·40		−226
50	+·03	+ ·38	− ·33	− ·02	− ·24	− ·11		− 60
100	+·09	+1·60	−1·16	− ·35	− ·92	− ·59		−159
150	+·17	+2·00	−1·23	− ·6	−1·05	− ·78		−102
200	+·22	+1·60	−1·00	− ·38	− ·82	− ·56	7	− 89
300	+·28	+ ·15	− ·24	+ ·37	− ·07	+ ·20		− 74
400	+·33	−1·33	+ ·51	+1·15	+ ·70	+ ·96		− 69
500	+·43	−2·71	+1·19	+1·95	+1·41	+1·73		− 75
50	+·03	+ ·58	− ·38	− ·17	− ·33	− ·22		− 46
100	+·09	+2·52	−1·21	−1·22	−1·21	−1·29		+ 28
150	+·17	+2·98	−1·16	−1·65	−1·30	−1·51		+166
200	+·22	+2·50	− ·83	−1·45	−1·00	−1·28	VII.	+212
300	+·28	+1·27	− ·16	− ·83	− ·34	− ·65		+242
400	+·33	− ·18	+ ·59	− ·08	+ ·41	+ ·10		+252
500	+·43	−1·61	+1·31	+ ·73	+1·16	+ ·88		+254
50	+·03	+1·1	− ·73	− ·34	− ·66	− ·41		−121
100	+·09	+2·7	−1·45	−1·16	−1·40	−1·21		− 69
150	+·17	+2·45	−1·15	−1·13	−1·14	−1·14		+ 55
200	+·22	+1·77	− ·76	− ·79	− ·76	− ·79	9	+ 90
300	+·28	− ·03	+ ·18	+ ·13	+ ·17	+ ·14		+110
400	+·33	−1·90	+1·13	+1·10	+1·12	+1·11		+121
500	+·43	−3·55	+1·94	+2·04	+1·96	+2·02		+110

TABLE IX.

IRON TUBES, I'. TO VIII'.

Field.	δ.	λ.	μ.	ν.	μ'.	ν'.		δν.
50	+·03	+ ·33	− ·01	− ·29	− ·15	− ·15	I'.	+ 5·3
100	+·09	+ ·80	+ ·09	− ·80	− ·39	− ·32		+ 16·8
150	+·17	+1·80	− ·87	− ·76	− ·82	− ·81		+ 1·0
200	+·22	+2·25	−1·43	− ·60	−1·03	−1·00		− 10·5
300	+·28	+2·00	−1·65	− ·07	− ·89	− ·83		− 22·5
500	+·43	+ ·20	−1·11	+1·33	+ ·08	+ ·15		− 35·1
50	+·03	+ 48	− ·24	− ·21	− ·23	− ·22	II'.	− ·2
100	+·09	+1·10	− ·66	− ·35	− ·51	− ·50		− 7
150	+·17	+2·00	−1·94	+ ·11	− ·98	− ·85		− 60
200	+·22	+2·48	−2·41	+ ·15	−1·22	−1·04		− 75
300	+·28	+1·85	−2·27	+ ·70	− ·88	− ·69		− 86
500	+·43	− ·50	−1·36	+2·29	− ·36	+ ·57		−103
50	+·03	+ ·42	− ·25	− ·14	− ·20	− ·19	III'.	− 4
100	+·09	+1·30	− ·91	− ·30	− ·64	− ·57		− 30·4
150	+·17	+2·48	−2·28	− ·03	−1·28	−1·03		−124
200	+·22	+2·80	−2·54	− ·04	−1·43	−1·15		−136
300	+·28	+2·00	−2·32	+ ·60	−1·02	− ·7		−157
500	+·43	− ·60	−1·30	+2·33	+ ·31	+ ·72		−190
50	+·03	+ ·56	− ·31	− ·22	− ·27	− ·26	IV'.	− 5·5
100	+·09	+1·43	− ·95	− ·39	− ·72	− ·62		− 42·2
150	+·17	+2·80	−2·08	− ·55	−1·45	−1·18		−124
200	+·22	+2·93	−2·30	− ·41	−1·52	−1·19		−151
300	+·28	+1·95	−1·99	+ ·32	−1·04	− ·63		−183
500	+·43	− ·74	− ·87	+2·04	+ ·34	+ ·83		−224
50	+·03	+ ·40	− ·28	− ·09	− ·21	− ·16	V'.	− 2
100	+·09	+1·64	−1·28	− ·27	− ·81	− ·74		−117
150	+·17	+2·70	−2·26	− ·27	−1·51	−1·02		−233
200	+·22	+2·64	−2·27	− ·18	−1·47	− ·95		−244
300	+·28	+1·47	−1·78	+ ·59	− ·89	− ·30		−270
500	+·43	−1·13	− ·56	+2·12	+ ·45	+1·11		−295
50	+·03	+ ·45	− ·28	− ·14	− ·23	− ·19	VI'.	− 17
100	+·09	+2·10	−1·41	− ·16	−1·19	− ·82		−125
150	+·17	+2·94	−1·99	− ·78	−1·59	−1·18		−180
200	+·22	+2·60	−1·86	− ·52	−1·41	− ·97		−192
300	+·28	+1·38	−1·32	+ ·22	− ·8	− ·30		−218
500	+·43	−1·20	− ·12	+1·75	+ ·51	+1·12		−247
50	+·03	+ ·55	− ·44	− ·08	− ·38	− ·14	VII'.	− 72
100	+·09	+2·07	−1·64	− ·34	−1·26	− ·72		−268
150	+·17	+2·47	−1·81	− ·49	−1·43	− ·87		−254
200	+·22	+2·11	−1·64	− ·25	−1·24	− ·65		−256
300	+·28	+ ·79	− ·98	+ ·47	− ·56	+ ·05		−255
500	+·43	−1·89	+ ·37	+1·95	+ ·83	+1·49		−254
50	+·03	+ ·55	− ·49	− ·03	− ·39	− ·13	VIII'.	−120
100	+·09	+2·25	−1·45	− ·71	−1·28	− ·88		−185
150	+·17	+2·10	−1·30	− ·63	−1·15	− ·78		−142
200	+·22	+1·48	− ·98	− ·28	− ·82	− ·44		−139
300	+·28	·00	− ·23	+ ·51	− ·06	+ ·34		−130
500	+·43	−2·80	+1·19	+2·04	+1·39	+1·84		−120

TABLE X.

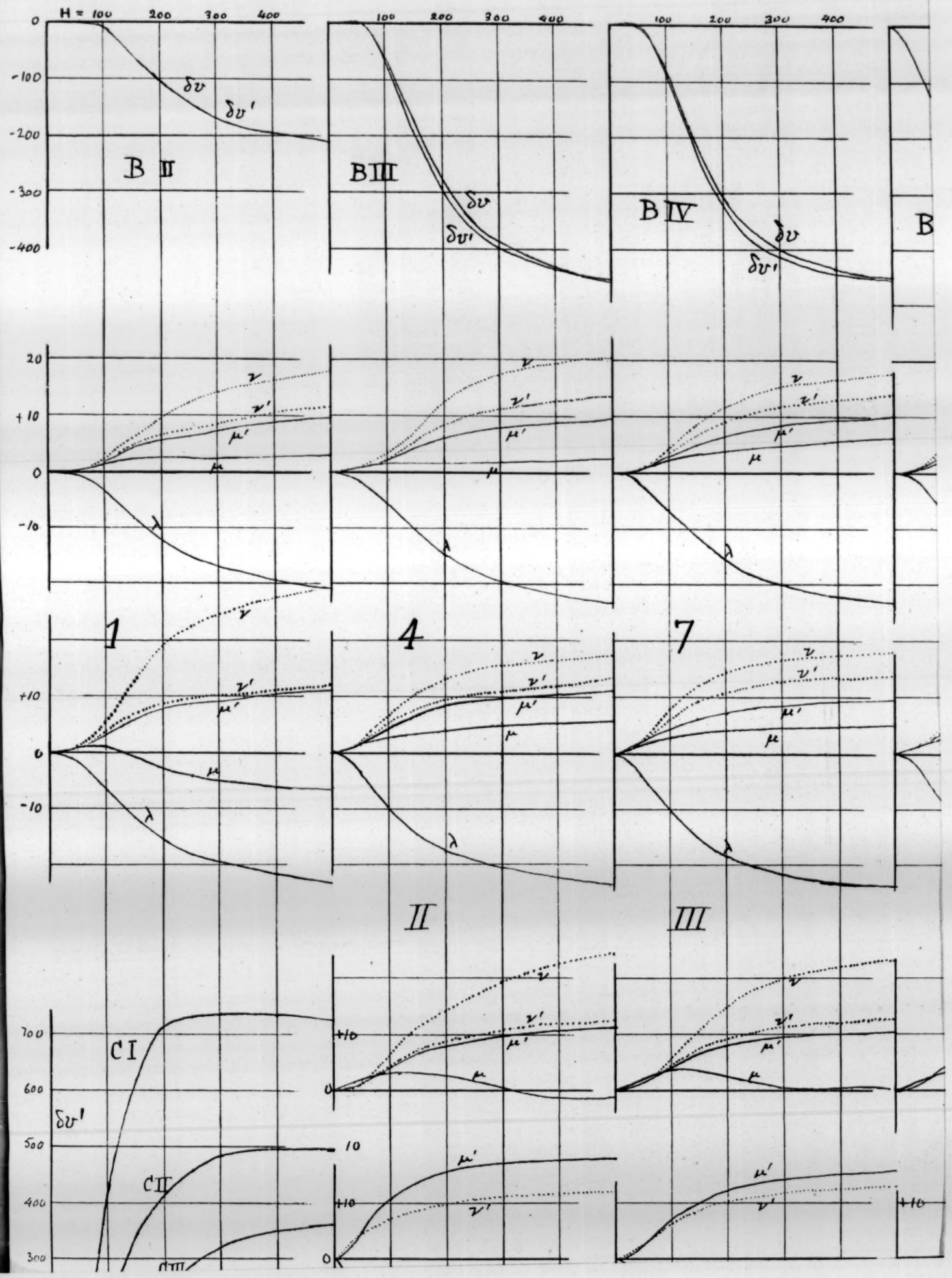
Steel Tubes.

Field.	3.	5.	7.	VII_1.	VII_2.	VII_3.	9.
			Bore Dilatations, $(\lambda+2\mu)10^6$.				
50	+ ·13	+ ·05	− ·19	+ ·05	− ·29	− ·10	− ·45
100	+ ·38	+ ·20	− ·30	+ ·25	− ·59	− ·02	+ ·08
150	+ ·16	+ ·68	+ ·21	+ ·40	− ·4	+ ·07	+ ·56
200	+ ·33	+1·25	+ ·52	+ ·31	− ·19	+ ·10	+ ·70
300	+ ·68	+2·14	+ ·76	− ·06	+ ·04	+ ·11	+ ·86
400	+ ·53	+2·64	+ ·96	− ·46	+ ·16	+ ·08	+ ·93
500	+ ·39	+2·89	+1·05	− ·82	+ ·24	+ ·04	+ ·98
			Longitudinal Dilatations, $\lambda\ 10^6$.				
50	+ ·25	+ ·15	+ ·51			+ ·68	+ ·95
100	+ ·68	+ ·85	+1·23			+2·20	+1·80
150	+1·40	+1·75	+1·47	Probably same as for VII_3.	Probably same as for VII_3.	+2·00	+1·50
200	+1·51	+1·80	+ ·98			+1·40	+ ·75
300	+ ·40	+ ·60	− ·45			− ·31	− ·90
400	−1·00	− ·90	−1·98			−2·00	−2·42
500	−2·40	−2·40	−3·30			−3·52	−3·8
			Tangential Dilatations, $\mu\ 10^6$.				
50	− ·06	− ·05	− ·30	− ·32	− ·49	− ·39	− ·70
100	− ·15	− ·33	− ·77	− ·98	−1·40	−1·11	− ·86
150	− ·62	− ·54	− ·63	− ·80	−1·20	− ·97	− ·47
200	− ·59	− ·28	− ·23	− ·55	− ·61	− ·65	− ·03
300	+ ·14	+ ·77	+ ·61	+ ·13	+ ·18	+ ·21	+ ·88
400	+ ·77	+1·77	+1·47	+ ·77	+1·08	+1·04	+1·68
500	+1·40	+2·65	+2·18	+1·35	+1·88	+1·77	+2·39

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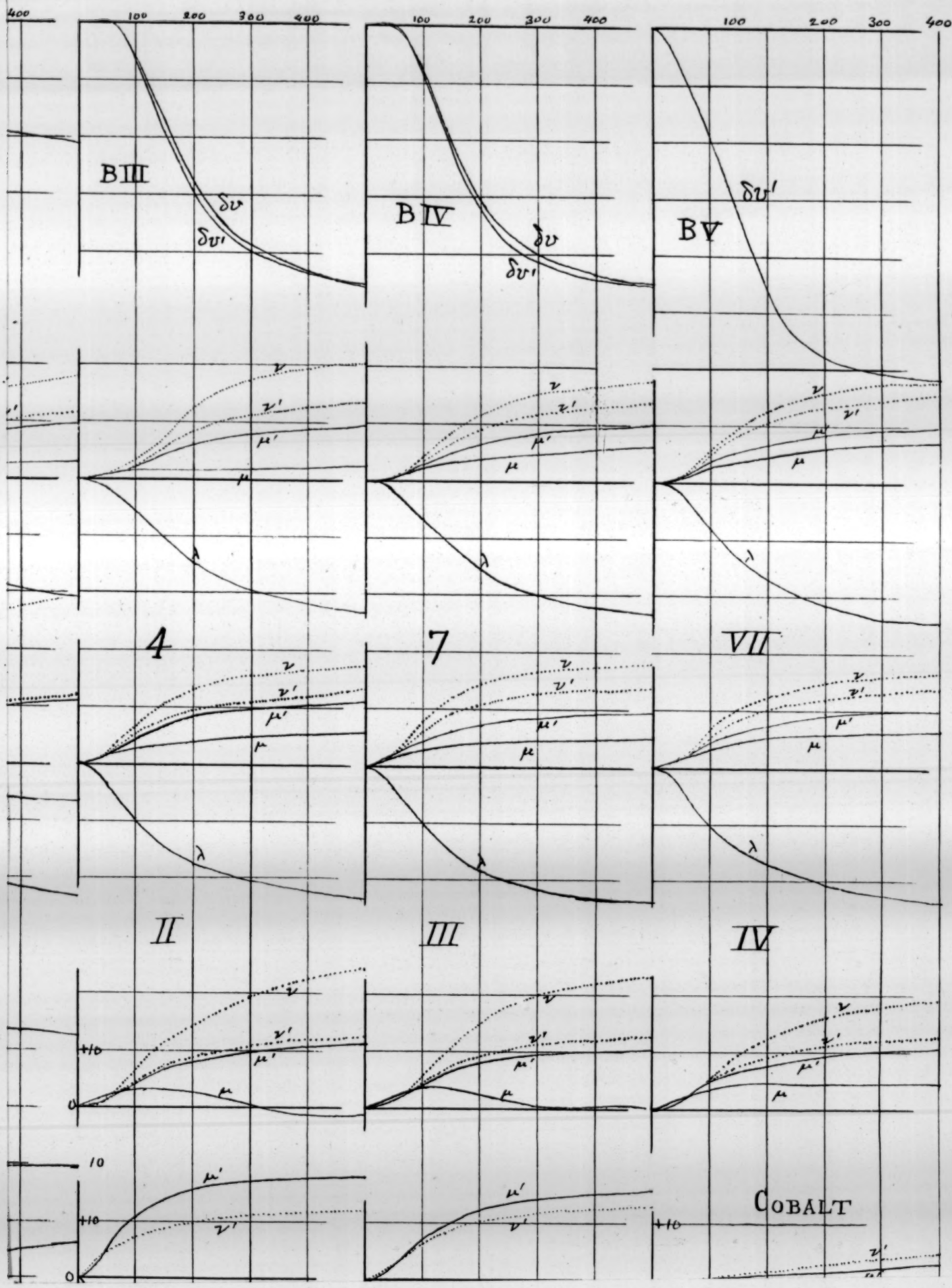
NICKEL TUBES AND COBALT TUBE.

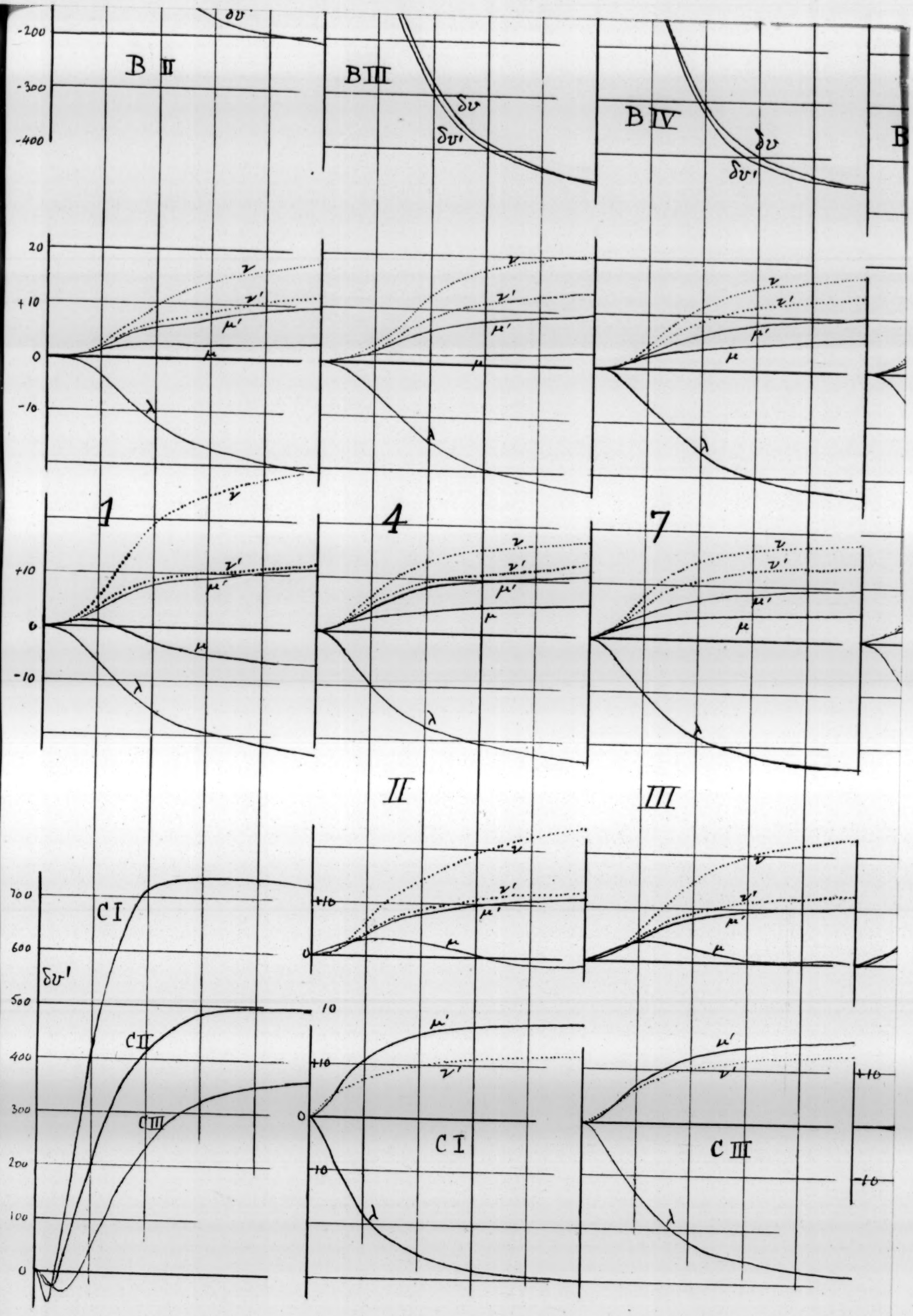
PLATE I.

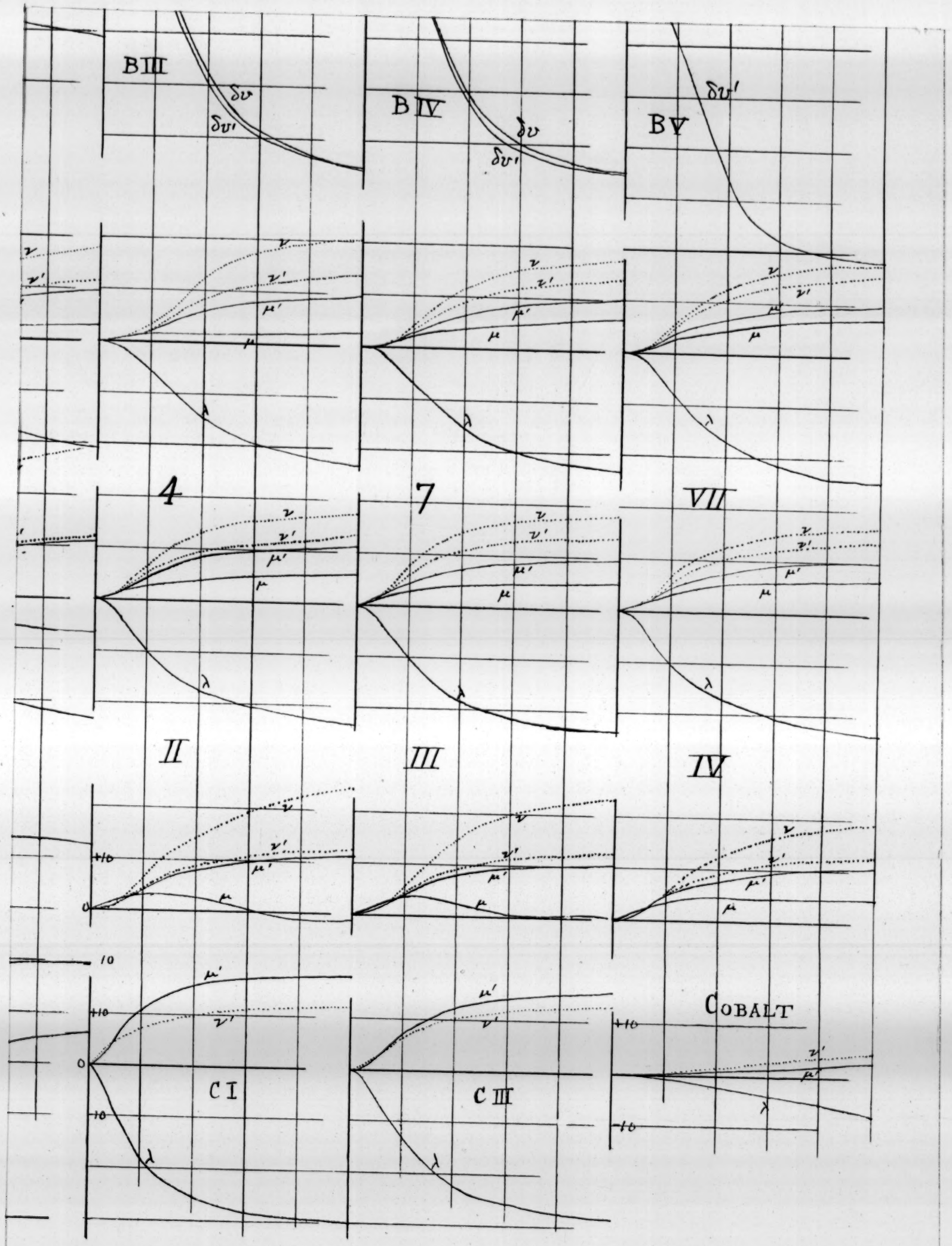


NICKEL TUBES AND COBALT TUBE.

PLATE I.



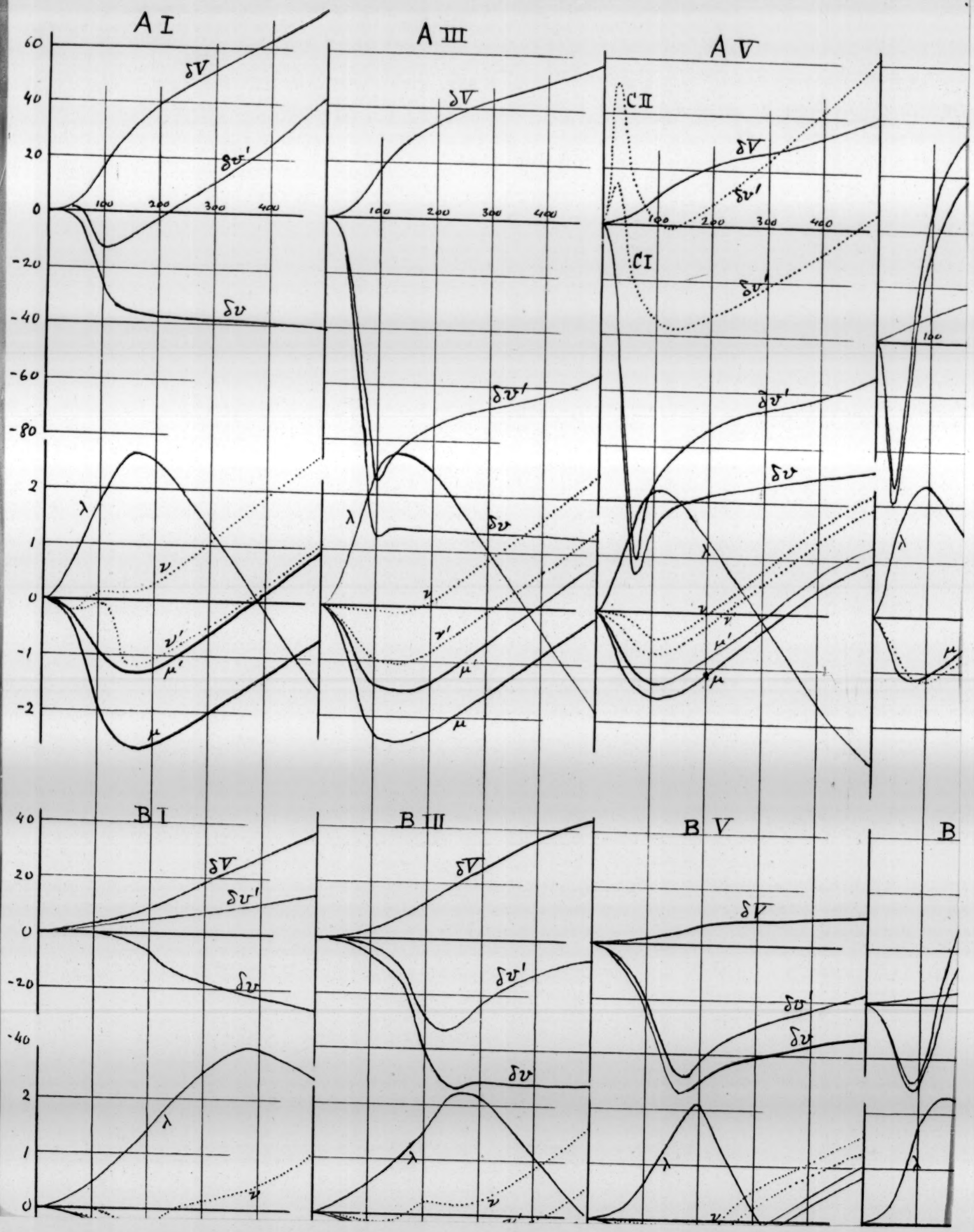

δv
-200
-300
-400
B II
B III
B IV
δv
δv'
20
+10
0
-10
ν
ν'
μ'
μ
λ
1
4
7
+10
0
-10
II
III
700
600
δv'
500
400
300
200
100
0
C I
C II
C III
10
+10
0
-10
C I
C III


BIII
δv
δv'
BIV
δv
δv'
BV
δv'
ν
ν'
μ'
μ
λ
4
7
VII
II
III
IV
+10
0
10
CI
CIII
COBALT

Trans. Roy. Soc. Edin.

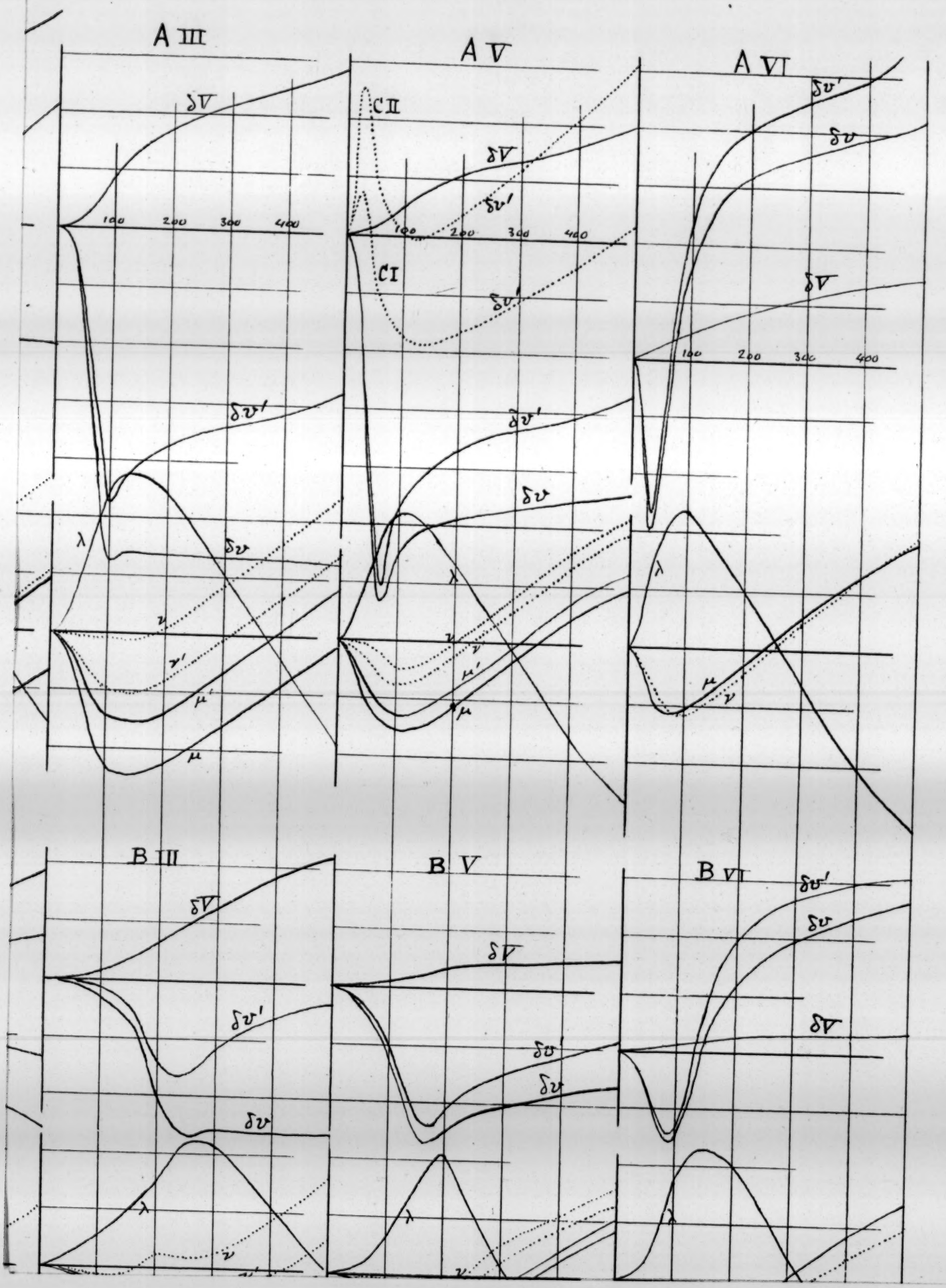
IRON TUBES. A. B. AND C.

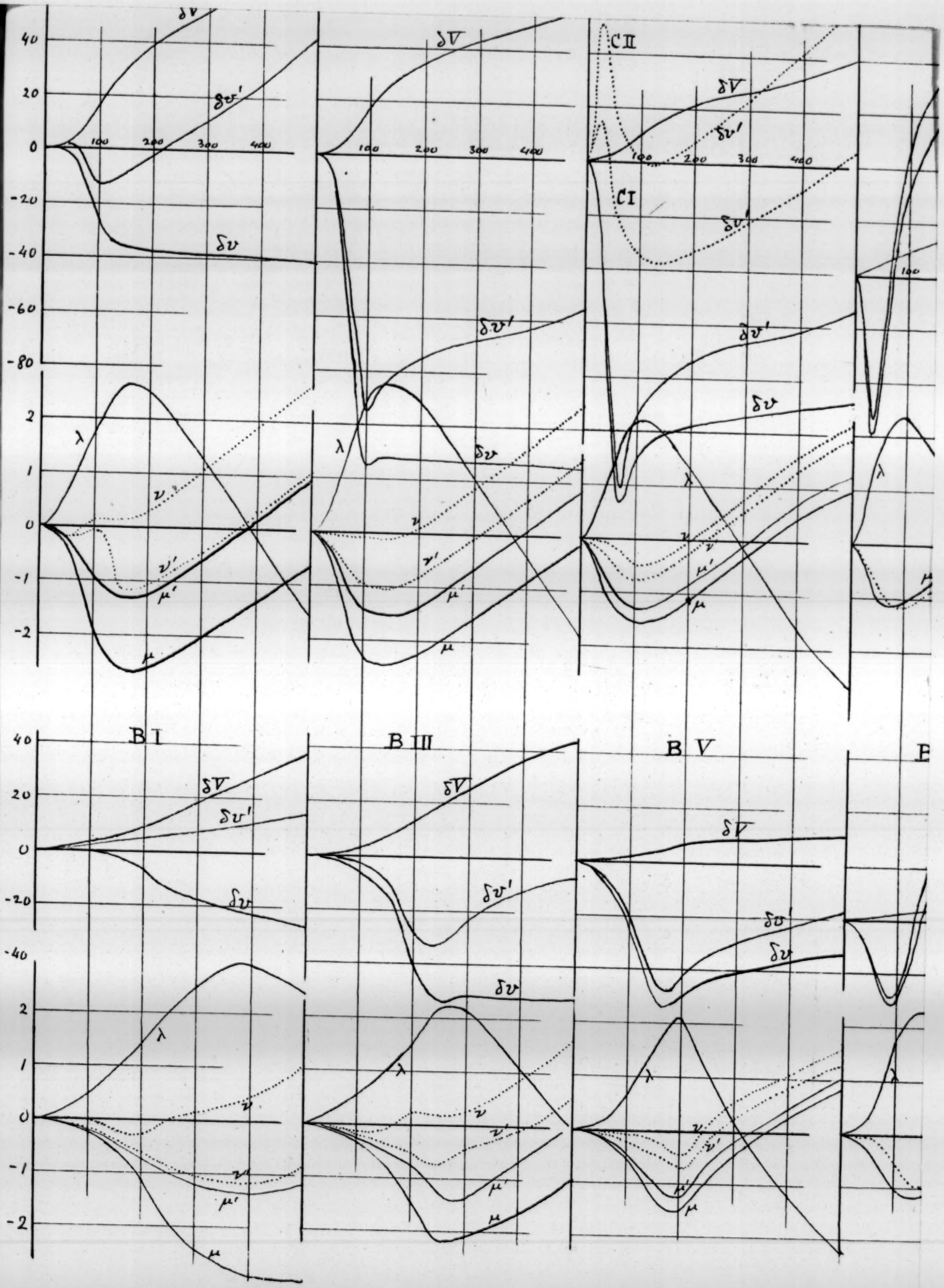
PLATE II.

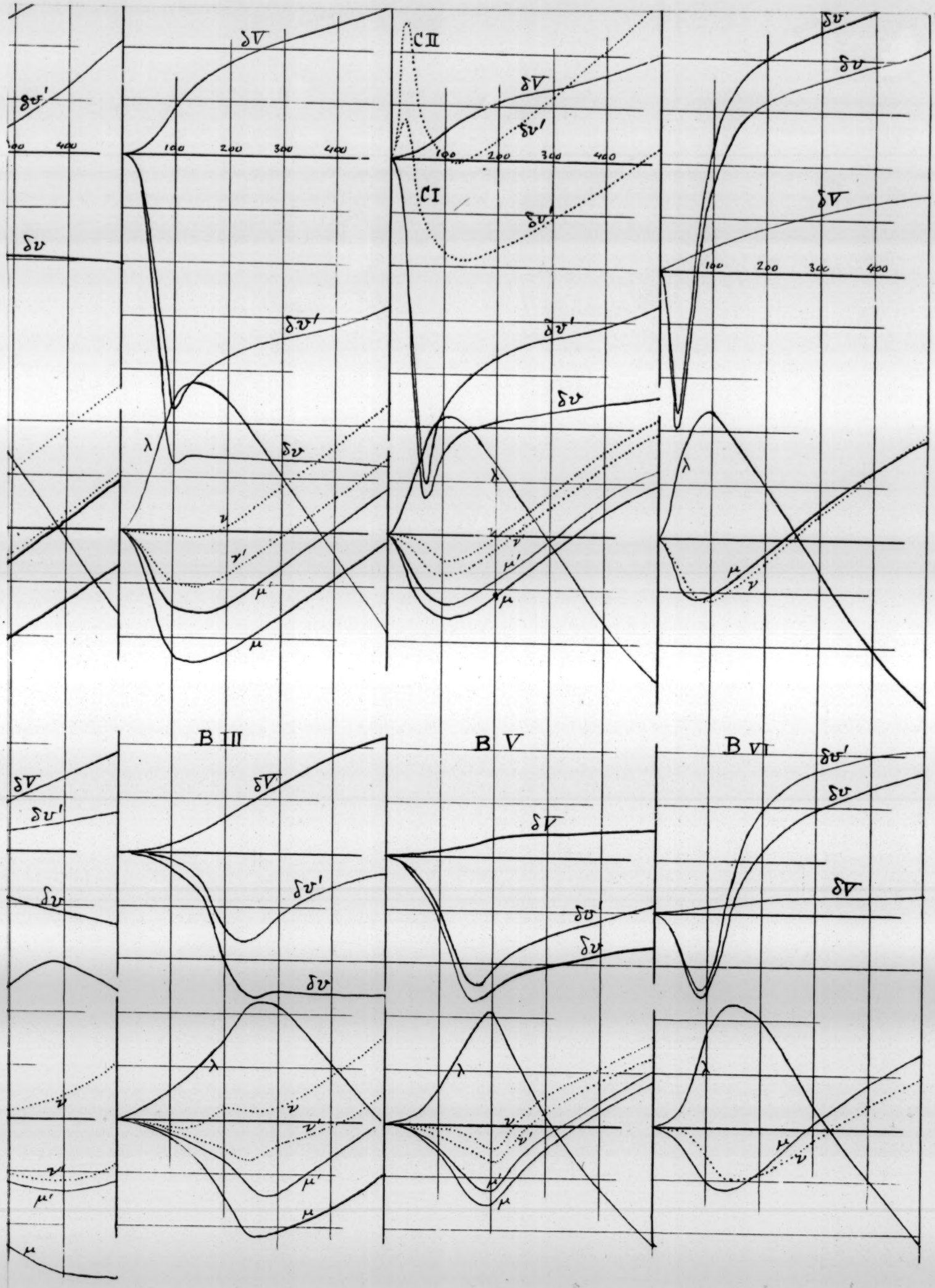


IRON TUBES, A, B, AND C.

PLATE II.




CII
CI
BI
BIII
BV
δV
δv'
δv
λ
ν
ν'
μ
μ'
40
20
0
-20
-40
-60
-80
2
1
0
-1
-2
100
200
300
400


CII
CI
B III
B V
B VI
100
200
300
400

XVI.—*On the Path of a Rotating Spherical Projectile.* II.

By Professor TAIT. (With a Plate.)

(Read 6th and 20th January 1896.)

The first instalment of this paper was devoted in great part to the general subject involved in its title, but many of the illustrations were derived from the special case of the flight of a golf-ball. Since it was read I have endeavoured, alike by observation and by experiment, to improve my numerical data for this interesting application, particularly as regards the important question of the coefficient of resistance of the air. As will be seen, I now find a value intermediate to those derived (by taking average estimates of the mass and diameter of a golf-ball) from the results of ROBINS and of BASHFORTH. This has been obtained indirectly by means of a considerable improvement in the apparatus by which I had attempted to measure the initial speed of a golf-ball. I have, still, little doubt that the speed may, *occasionally*, amount to the 300, or perhaps even the 350, foot-seconds which I assumed provisionally in my former paper:—but even the first of these is a somewhat extravagant estimate; and I am now of opinion that, even with very good driving, an initial speed of about 240 is not often an underestimate, at least in careful play. From this, and the fact that six seconds at least are required for a long carry (say 180 yards), I reckon the "terminal velocity" at about 108, giving $v^2/360$ as the resistance-acceleration.

I hope to recur to this question towards the end of the present paper:—but I should repeat that I naturally preferred the comparatively recent determination to the much older one, and that in formerly assuming a resistance even greater than that which BASHFORTH'S formula assigns, I was to some extent influenced by the consideration of the important effects of roughening or hammering a golf-ball. For I fancied that this might increase the direct resistance, as well as the effects due to rotation, by the better grip of the air which it gives to the ball. [See last sentence of § 11. Of course the assumption of increased coefficient of resistance *required* a corresponding increase of the estimate of initial speed.] The time of describing 180 yards *horizontally*, *i.e.*, when gravity is not supposed to act, if the initial speed is 240 and the "terminal velocity" 108, is about $5^s{\cdot}2$; and this has to be increased by at least 1^s, if we allow for the curvature of the path and the effect of gravity. I have employed this improved value of the coefficient of resistance in all the calculations which have been made since I obtained it. But various considerations have led me to the conclusion that the resistance, towards the end of the path, may be somewhat underrated because of the assumption that it is, throughout, proportional to the square of the speed. This point, also, will be referred to later, as I wish to make at once all the necessary comments and improvements on the part already published.

Though the present communication is thus specially devoted to some curious phenomena observed in the game of golf, it contains a great deal which has more extended application:—to which its results can easily be adapted by mere numerical alterations in the data. Therefore I venture to consider its subject as one suitable for discussion before a scientific Society.

In my short sketch of the history of the problem I failed to notice either of two comparatively recent papers whose contents are at least somewhat closely connected with it. These I will now very briefly consider.

The first is by CLERK-MAXWELL* "*On a particular Case of the Descent of a Heavy Body in a Resisting Medium.*" The body is a flat rectangular slip of paper, falling with its longer edges horizontal. It is observed to rotate about an axis parallel to these edges, and to fall in an oblique direction. The motion soon becomes approximately regular; and the deflection of the path from the vertical is to the side towards which the (temporarily) lower edge of the paper slip is being transferred by the rotation. [When the rectangle is not very exact, or the longer edges not quite horizontal, or the slip slightly curved, the appearance, especially when there is bright sunlight, is often like a spiral stair-case.] MAXWELL examines experimentally the distribution of currents, and consequently of pressure, about a non-rotating plane upon which a fluid plays obliquely; and shows that when the paper is rotating the consequent modification of this distribution of pressure tends to maintain the rotation. The reasoning throughout is somewhat difficult to follow, and the circumstances of the slip are very different from those of a ball:—but the direction of the deflection from the unresisted path is always in agreement with the statement made by NEWTON.

Much more intimately connected with our work is a paper by Lord RAYLEIGH† "*On the Irregular Flight of a Tennis Ball*," in which the "true explanation" of the curved path is attributed to Prof. MAGNUS. The author points out that, in general, the statement that the pressure is least where the speed is greatest, is true only of perfect fluids unacted on by external forces; whereas in the present case the whirlpool motion is directly due to friction. But he suggests the idea of short blades projecting from the ball, the pressure on each of which is shared by the contiguous portion of the spherical surface. Here we have practically NEWTON's explanation—*i.e.* the "pressing and beating of the contiguous air." Lord RAYLEIGH's paper contains an investigation of the form of the stream-lines when a perfect fluid circulates (without molecular rotation) round a cylinder, its motion at an infinite distance having uniform velocity in a direction perpendicular to the axis of the cylinder. And it is shown that the resultant pressure, perpendicular to the general velocity of the stream, has its magnitude proportional alike to that velocity and to the velocity of circulation. [There are some comments on this paper, by Prof. GREENHILL, in the ninth volume of the journal referred to.]

* *Cambridge and Dublin Mathematical Journal*, ix. 145 (1854).
† *Messenger of Mathematics*, vii. 14 (1878).

In the *Beiblätter zu d. Ann. d. Phys.* (1895, p. 289) there appears a somewhat sarcastic notice of my former paper. The Reviewer, evidently annoyed at my remarks on MAGNUS' treatment of ROBINS, which he is unable directly to controvert, refers to HÉLIE, *Traité de Balistique*, as containing an anticipation of my own work. I find nothing there beyond a very small part of what was perfectly well known to NEWTON and ROBINS; except a few of the more immediately obvious mathematical consequences, deduced from the hypothesis (for which no basis is assigned, save that it is the simplest possible) that the transverse deflecting force due to rotation is proportional to the first power of the translational speed.

In the present article I give first a brief account of my recent attempts to determine the initial speed of a golf-ball, and consequently to approximate to the coefficient of v^2 in the assumed expression for the resistance.

Next, instead of facing the labour of the second approximation (suggested in § 10) to the solution of the differential equations, I have attempted by mere numerical calculation to take account of the effect of gravity on the speed of the projectile, and have thus been enabled to give improved, though still rough, sketches of the form of the trajectory when it is not excessively flat. This process furnishes, incidentally, the means of finding the time of passage through any arc of the trajectory.

Third, I treat of the effects of wind, regarded as a uniform horizontal translation of the atmosphere parallel, or perpendicular, to the plane of the path.

Finally, recurring to the limitation of a very flat trajectory, I have treated briefly the effects of gradual diminution of spin during the flight. This loss is shown to be inadequate to the explanation of the unexpectedly small inclination of the calculated path when the projectile reaches the ground. Hence some other mode of accounting for its nearly vertical fall is to be sought, and it is traced to the rapid diminution of the resistance (assigned by ROBINS' law) when the speed has been greatly reduced.

Determination of Initial Speed.

16. The bob of my new ballistic pendulum was a stout metal tube, some 3 feet long, suspended horizontally, near the floor, by two parallel pieces of clock-spring about 2·5 feet apart, and 8·63 feet long. On one end of the tube was fixed transversely a circular disc, 1 foot in diameter, covered with a thick layer of moist clay into which the ball was driven from a distance of 4 feet or so. The whole bob had a mass of about 33 lbs.; and, in the most favourable circumstances, its horizontal displacement was about 3·5 to 4 inches. As the ball's mass is 0·1 lb., the average indicated speed was thus about 200 foot-seconds.* Though I had the assistance of two long drivers, whose

* If l be the length (in feet) of the supporting straps, d the (small) horizontal deflection of the bob, its vertical rise is obviously $d^2/2l$, so that its utmost potential energy is

$$(M+m)gd^2/2l,$$

where M is its mass and m that of the ball. But, if V was the horizontal speed of the ball, that of bob and ball was

habitual carry is 180 yards or upwards, the circumstances of the trials were somewhat unfavourable, for there was great difficulty in hitting the disc of clay centrally. The pendulum was suspended in an open door-way; and heavy matting was disposed all about the clay so as (in ROBINS' quaint language) "to avoid these dangers, to the braving of which in philosophical researches no honour is annexed"; so that the whole surroundings were absolutely unlike those of a golf-course. I therefore make an allowance of 20 per cent., and (as at present advised) regard 240 foot-seconds or something like it as a fair average value of the initial speed of a really well-driven ball:—while thinking it quite possible that, under exceptionally favourable circumstances, this may be increased by 20 or 30 per cent. at least. Now, it is certain that the time of flight is usually about six seconds when the range is about 180 yards:—considerably more for a very high trajectory, and somewhat less for a very flat one. As we have by § 5 the approximate formula

$$t=\frac{a}{V}(\epsilon^{s/a}-1),$$

we may take $a=360$ as a reasonable estimate. This number is possibly some 10 per cent. in error, but it is very convenient for calculation, and golf-balls differ considerably from one another in density as well as in diameter. With it the "terminal velocity" of a golf-ball is about 108 foot-seconds; intermediate to the values deduced from the formulæ of ROBINS and of BASHFORTH, which I make out to be 114 and 95 respectively. With this value of a, it is easy to see that air-resistance, alone, reduces the speed of a golf-ball to half its initial value in a path of 83 yards only. This is the utmost gain of range obtainable (other conditions remaining unchanged) by giving four-fold energy of propulsion! With the value (282) of a deduced from BASHFORTH's formula, this gain would have been 65 yards only! [So far for the higher speeds, but it is obvious from all ordinary experience of pendulums (with a golf-ball as bob) that slow moving bodies suffer greater resistance than that assigned by this law.]

In passing, I may mention that, on several occasions, I fastened firmly to the ball a long light tape, the further end being fixed (after all twist was removed) to the ground so that the whole was perpendicular to the direction of driving. After the 4 foot flight of the ball, the diameter at first parallel to the tape preserved its initial direction, while the tape was found twisted (in a sense corresponding to underspin) and often through one or two *full* turns, indicating something like 60 or 120 turns per second. This is clearly a satisfactory verification of the present theory.

$mV/(M+m)$. Equating the corresponding kinetic energy to the potential energy into which it is transformed, we find at once $(M+m)gd^2/2l=m^2V^2/2(M+m)$ leading to the very simple expression

$$V=\frac{M+m}{m}d\sqrt{g/l}.$$

With the numerical values given in the text we easily find that this is equivalent to

$$V=331\frac{D}{12}1{\cdot}93=53{\cdot}2D;$$

where V is, of course, in foot-seconds, but the deflection is now (for convenience) expressed in inches, and called D. Hence the numerical result in the text.

Numerical Approximation to Form of Path.

17. The differential equations of the trajectory were integrated approximately in § 10 by formally omitting the term in g in the first of them, that is so far as the speed is concerned. In other words:—by assuming that ϕ is always very small, or the path nearly horizontal throughout. It was pointed out that if the value of ϕ, thus obtained from the second, were substituted for $\sin\phi$ in the first, equation, we should be able to obtain a second approximation to the intrinsic equation of the path, amply sufficient for all ordinary applications. But the process, though simple enough in all its stages, is long and laborious:—and it is altogether inapplicable to the kinked path, discussed in § 15, which furnishes one of the most singular illustrations of the whole question.

The fact that one of my Laboratory students, Mr JAMES WOOD, had shown himself to be an extremely rapid and accurate calculator led me to attempt an approximate solution of the equations by means of differences:—treating the trajectory as an equilateral polygon of 6-foot sides, and calculating numerically the inclination of each to the horizon, as well as the average speed with which it is described. For we may write the differential equations in the form

$$\frac{1}{2}\frac{d(v^2)}{ds}+\frac{v^2}{a}=-g\sin\phi,$$

$$\frac{d\phi}{ds}=\frac{k}{v}-\frac{g}{v^2}\cos\phi,$$

and these involve approximately

$$v'^2-v^2+2\left(\frac{v^2}{a}+g\sin\phi\right)\delta s=0,$$

$$\phi'-\phi=\left(\frac{k}{v}-\frac{g}{v^2}\cos\phi\right)\delta s.$$

Thus we find, after a six-foot step, the new values

$$v'^2=\left(1-\frac{12}{a}\right)v^2-384\sin\phi,$$

$$\phi'=\phi+\frac{6k}{v}-\frac{192\cos\phi}{v^2}.$$

[If we take account of terms in $(\delta s)^2$, we find that we ought to write for $12/a$ the more accurate expression $12/a.(1-6/a)$. But this does not alter the *form* of the expression for v'^2. It merely increases by some 2 per cent. the denominator of the coefficient of resistance, of which our estimate is, at best, a very rough one; so that it may be disregarded. But the successive values of v^2 are all on this account too large; and thus the values of ϕ, in their turn, are sometimes increased, sometimes diminished, but only by trifling amounts. This is due to the fact that the change of ϕ depends upon terms having opposite signs; and involving different powers of v, so that their *relative* as well as their *actual* importance is continually changing. These remarks

require some modification when k is such that ϕ may have large values, as for instance in the kinked path treated below. But I do not pretend to treat the question exhaustively, so that I merely allude to this source of imperfection of the investigation.]

Let, now, $a=360$, $k=1/3$, and suppose ϕ to be expressed in degrees. We have, to a sufficient approximation,

$$v'^2=(v^2-400\sin\phi)\left(1-\frac{1}{30}\right),$$

$$\phi'=\phi+\frac{120}{v}-\frac{12000}{v^2}\left(1-\frac{1}{30}\right),$$

and successive substitutions in these equations, starting from any assigned values of v and ϕ, will give us the corresponding values for the next side of the polygon, with the more recent estimate of the coefficient of resistance. See the two last examples in § 19 below, which lead to the trajectories figured as 5 and 6 in the Plate.

Unfortunately, many of Mr Wood's calculations were finished before I had arrived at my new estimate of the value of a; but their results are all approximately representative of possible trajectories:—the balls being regarded as a little larger, or a little less dense, than an ordinary golf-ball; in proportion as the coefficient of resistance assumed is somewhat too great. And no difficulty arises from the assumption of too great an initial speed; for we may simply *omit* the early sides of the polygon, until we come to a practically producible rate of motion.

18. To discover how far this mode of approximation can be trusted, we have only to compare its consequences with those of the *exact* solution. For the intrinsic equation can easily be obtained in finite terms when there is no rotation. In fact, by elimination of g between the differential equations of § 10, assuming $k=0$, we have at once the complete differential of the equation

$$\epsilon^{s/a}v\cos\phi=V\cos\phi_0=V_0 \text{ suppose};$$

where it is to be particularly noticed that V_0 is the speed of the *horizontal component* of the velocity of projection, *not* the total speed. By means of this the second of the equations becomes

$$\frac{d\phi}{ds}=-\frac{g}{V_0^2}\epsilon^{2s/a}\cos^3\phi,$$

whence

$$\frac{ag}{V_0^2}(\epsilon^{2s/a}-1)=\sec\phi_0\tan\phi_0-\sec\phi\tan\phi+\log\frac{\sec\phi_0+\tan\phi_0}{\sec\phi+\tan\phi}.$$

The following fragments show the nature and arrangement of the results in one of the earlier of Mr Wood's calculated tables. Having assumed (for reasons stated in the introductory remarks above) that $a=240$, I supplied him with the following formulæ:—

$$v'^2=\left(1-\frac{1}{20}\right)v^2-400\sin\phi(1-0{\cdot}04),$$

$$\phi'=\phi-\frac{12000}{v^2}\cos\phi\,(1-0{\cdot}04),$$

and I took as initial data $V=300$, $\phi=15°$; [whence, of course, $V_0^2=84{,}000$ nearly. This is required for comparison with the *exact* solution.]

Working from these he obtained a mass of results from which I make a few extracts:—

$s/6$	v^2	v	$1/v$	$\Sigma(1/v)$	ϕ	$\sin\phi$	$\Sigma(\sin\phi)$	$\cos\phi$	$\Sigma(\cos\phi)$
1.	90,000	300	·003̇	·003̇	15°	·2588	·2588	·9659	·9659
2.	85,401	292·2	·00342	·00675	14·876	·2565	·5153	·9665	1·9324
3.	81,032	284·6	·00351	·01026	14·746	·2546	·7699	·9671	2·8995
	*		*		*		*		*
20.	33,045	181·8	·00550	·08666	11·028	·1914	4·6102	·9815	19·4569
21.	31,319	177·0	·00565	·09231	10·686	·1854	4·7956	·9826	20·4395
	*		*		*		*		*
40.	11,440	106·9	·00935	·23391	− 1·023	− ·0178	6·6163	·9998	39·3178
41.	10,875	104·3	·00959	·24350	− 2·030	− ·0355	6·5808	·9994	40·3172
	*		*		*		*		*
60.	5453	73·8	·01354	·46935	− 30·748	− ·5113	1·4677	·8595	58·3988
61.	5377	73·3	·01363	·48298	− 32·564	− ·5383	·9294	·8428	59·2416
	*		*		*		*		*

This table gives simultaneous values of s, v, and ϕ directly. t is obviously to be found by multiplying by 6 feet the numbers in column fifth; while by the same process we obtain rectangular cöordinates, vertical and horizontal, from the eighth, and the last, columns respectively. Thus for instance we have simultaneously

s	v	t	ϕ	y	x
120	181·8	0ˢ·52	11°·028	27·66	116·74
240	106·9	1·404	− 1 ·023	39·69	235·9

(The trajectory is given as fig. 3 in the Plate, and will be further analysed in the next section of the paper.)

From the complete table we find that, in this case, ϕ is positive up to the 38th line inclusive, and then changes sign. It vanishes for $s=233$ (approximately) after the lapse of $1^s·35$. The rectangular cöordinates of the vertex are about 230 and 40, and the speed there is reduced to 110. From the exact equation we find $s=232$ for $\phi=0°$. This single agreement is conclusive, since the earlier tabular values of s for a given value of ϕ ought to be somewhat in excess of the true values; while the later, and especially those for negative values of ϕ greater than 30° or so, should be somewhat too small:—*i.e.* the calculated trajectory has at first somewhat too little curvature, but towards the end of the range it has too much. It is easy to see that this is a necessary consequence of the mode of approximation employed:—look, for instance, at the fact that the initial speed is taken as constant through the first six feet. See also the remarks in § 17. On the whole, therefore, though the carry may possibly be a little underrated, the numerical method seems to give a very fair approximation to the truth. This admits of easy verification by the help of the value of $d\phi/ds$ last written, for it enables us to calculate the exact value of s for any assigned value of ϕ by a simple difference calculated from the result obtained from an assumed value.

19. Taking the method for what it is worth, the following are a few of the results

obtained from it by Mr Wood. I give the numerical data employed, plotting the curves from a few of the calculated values of x and y. But I insert, at the side of each trajectory, marks indicating the spaces passed over in successive seconds. This would have been a work of great difficulty if we had adopted a direct process, even in cases where the intrinsic equation can be obtained exactly :—and it *must* be carried out when we desire to find the effects of wind upon the path of the ball.

Fig. 1 represents the path when $a=240$ (properly 234), $V=300$, $\phi_0=0°$, and $k=1/3$. This will be at once recognised as having a very close resemblance to the path of a well-driven low ball. The vertex (at 0·76 of the range) and the point of contrary flexure are indicated. This trajectory does not differ very much from that given (for the same initial data) by the roughly approximate formula of § 10; which rises a little higher, and has a range of some ten yards greater. But the assumed initial speed, and consequently the coefficient of resistance, are both considerably too great.

In fig. 2 all the initial data are the same except k, which is now increased to 1/2 :—*i.e.* the spin is 50 per cent. greater than in fig. 1. We see its effect mainly in the increased height of the vertex, and in the introduction of a second point of contrary flexure. A further increase of k will bring these points of contrary flexure nearer to one another, till they finally meet in the vertex, which will then be a cusp, a point of momentary rest, and *the path throughout will be concave upwards*! This is one of the most curious results of the investigation, and I have realized it with an ordinary golf-ball :—using a cleek whose face made an angle of about 45° with the shaft and was furnished with parallel triangular grooves, *biting downwards*, so as to ensure great underspin. [The data for this case give extravagant results when employed in the formula of § 10. The vertex it assigns is 510 feet from the starting point and at nearly 172 feet of elevation: —while the range is increased by 60 or 70 yards. And that formula can never give more than one point of contrary flexure. All this was, however, to be expected; since the formula was based on the express assumption that gravity has no direct effect on the speed of the projectile.]

Fig. 3 shows the result of dispensing altogether with initial rotation, while endeavouring to compensate for its absence by giving an initial elevation of 15°. This figure, also, will be recognised as characteristic of a well-known class of drives; usually produced when too high a tee is employed, and the player stands somewhat behind his ball. Notice, particularly, how much the carry and the time of flight are reduced, though the initial speed is the same. The slight underspin makes an extraordinary difference, producing as it were an unbending of the path throughout its whole length, and thus greatly increasing the portion above the horizon. But of course the pace of the ball, when it reaches the ground, is very much greater than in the preceding cases, it usually falls more obliquely, and it has no back-spin. On all these accounts we should expect to find that the "run" will in general be very much greater. Still, in consequence partly of the greater coefficient of resistance at low speeds, presently to be discussed, over-spin (due to the disgraceful act called "topping") is indispensable for

a really long run. In such a case the carry will, of course, be still further reduced, unless the initial elevation be very considerably increased. (Some of Mr Wood's numerical results, from which fig.3 was drawn, were given in the preceding section.)

In fig.4, a and V are as in fig.1, but $k=1$ and $\phi_0=45°$. Here we have the kink, of which a provisional sketch (closely resembling the truth) was given in the former instalment of the paper. I have not yet obtained it with a golf-ball, though as already stated I have got the length of producing the cusp above spoken of. But the kink can be obtained in a striking manner when we use as projectile one of the large balloons of thin india-rubber which are now so common. We have only to "slice" the balloon sharply downwards (in a nearly vertical plane) with the flat hand. This is a most instructive experiment, and its repetition presents no difficulty whatever. It is to be specially noticed that, in the particular kink sketched, there is a point of minimum speed somewhat beyond the vertex, and a point of maximum speed, both nearly in the same vertical with the point of projection. The first (where the speed is reduced to 58·7) is reached in a little more than two seconds, the other (where it has risen to 73·8) in rather more than four.

It may be interesting to give a few details of Mr Wood's calculations for this case:— selecting specially those near the points of maximum and minimum speed, and along with them those for closely corresponding elevations on the ascending side. Also near the vertex. The equations were

$$v_1^2=v^2\left(1-\frac{1}{20}\right)-400\sin\phi\,(1-0{\cdot}04)$$

$$\phi_1=\phi+\frac{360}{v}-\frac{12000}{v^2}\cos\phi\,(1-0{\cdot}04)$$

$s/6$	v^2	v	$1/v$	$\Sigma(1/v)$	ϕ	$\sin\phi$	$\Sigma(\sin\phi)$	$\cos\phi$	$\Sigma(\cos\phi)$
1.	90000	300	·003	·003	45°	·7071	·7071	·7071	·7071
	*		*		*		*		*
23.	24582	156·8	·00638	·10693	78°·72	·9807	19·6186	·1956	11·3075
	*		*		*		*		*
41.	5583	74·7	·01359	·27640	145°·3	·5693	35·8751	−·8221	6·2814
	*		*		*		*		*
44.	4278	65·4	·01529	·32038	166°·46	·2343	36·9422	−·9722	3·4951
45.	3974	63·0	·01586	·33624	174°·58	·0944	37·0366	−·9955	2·4996
46.	3739	61·1	·01636	·35260	183°·16	−·0553	36·9813	−·9981	1·5015
	*		*		*		*		*
48.	3475	59·0	·01697	·38630	201°·3	−·3633	36·4078	−·9317	−·5921
49.	3441	58·7	·01704	·40334	210°·5	−·5075	35·9003	−·8616	−1·4537
50.	3464	58·9	·01700	·42034	219°·5	−·6363	35·2640	−·7714	−2·2251
	*		*		*		*		*
67.	5434	73·7	·01357	·67179	313°·1	−·7302	20·0274	·6833	−·3162
68.	5443	73·8	·01355	·68534	316°·5	−·6880	19·3394	·7258	+·4096
69.	5435	73·7	·01357	·69891	319°·9	−·6446	18·6948	·7646	+·1742
	*		*		*		*		*

The following data belong to the last elements for which the calculations were made:—

80.	4374	66·1	·01512	·85485	352°·9	−·1224	14·6898	·9925	11·2602
81.	4202	64·8	·01542	·87027	355°·8	−·0732	14·6166	·9973	12·2575

As the last five values of ϕ have been increasing steadily by nearly 3° for each element, it is clear that the direction of motion again rises above the horizontal; but whether the path has next a point of contrary flexure, or another kink, can only be found by carrying the calculation several steps further. [The second kink is very unlikely, as the speed is so much reduced at the point where the calculations were arrested. Mr Wood has gone to Australia, and I had unfortunately told him to stop the numerical work in this particular example as soon as he found that $\Sigma(\cos\phi)$, after becoming negative, had recovered its former maximum (positive) value.]

The trajectories represented in figs. 5 and 6 may be taken as fairly representative of ordinary good play by the two classes of drivers. For we have in both $a=360$, $V=200$. These are the new data, representing (as above explained) the best information I have yet acquired. In fig. 5 $k=1/3$, $\phi_0=10°$; but in fig. 6 $k=0$, $\phi_0=15°$. In spite of its 50 per cent. greater angle of initial elevation, the carry of the non-rotating projectile is little more than half that of the other:—and it takes only one-third of the time spent by the other in the air. But the contrast shows how much more important (so far as carry is concerned) is a moderate amount of underspin than large initial elevation. And we can easily see that initial elevation, which is always undesirable (unless there is a hazard close to the tee) as it exposes the ball too soon to the action of the wind where it is strongest, may be entirely dispensed with. This point is discussed in next section.

On account of their intimate connection with actual practice, I give a few of the numerical results for these two closely allied yet strongly contrasted cases, belonging to two different classes of driving:—choosing sides of each polygon passed at intervals of about 1^s, as well as those near the vertices and the point of contrary flexure. The formulæ for these cases are those given at the end of § 17 above:—the second term in the expression for ϕ' being omitted for the latter of the two trajectories.

For Fig. 5.

$s/6$	v^2	v	$1/v$	$\Sigma(1/v)$	ϕ	$\sin\phi$	$\Sigma(\sin\phi)$	$\cos\phi$	$\Sigma(\cos\phi)$
1.	40,000	200	·00500	·00500	10°	·1736	·1736	·9848	·9848
	*		*		*		*		*
25.	15,497	124·5	·00803	·16549	17·552	·3015	6·2345	·9534	25·2200
	*		*		*		*		*
39.	8,216	90·6	·01103	·29869	19·789	·3388	10·7983	·9410	38·4544
	*		*		*		*		*
42.	7,042	83·9	·01192	·33353	19·665	·3366	11·8116	·9417	41·2783
	*		*		*		*		*
54.	3,511	59·3	·01687	·50626	13·611	·2354	15·3925	·9719	52·7246
	*		*		*		*		*
61.	2,387	48·9	·02046	·63904	1·727	·0303	16·3078	·9996	59·6508
62.	2,296	47·9	·02088	·65992	− 0·675	− ·0120	16·2958	·9999	60·6507
	*		*		*		*		*
70.	2,249	47·4	·02109	·83155	− 21·807	− ·3714	14·5533	·9285	68·4117
	*		*		*		*		*
79.	3,157	56·2	·01780	1·00513	− 35·890	− ·5862	9·9647	·8103	76·1309
	*		*		*		*		*
89.	4,338	65·9	·01519	1·16748	− 40·840	− ·6538	3·6521	·7566	83·8830
	*		*		*		*		*
94.	4,853	69·7	·01436	1·24081	− 41·548	− ·6633	0·3507	·7484	87·6381

For Fig. 6.

1.	40,000	200	·00500	·00500	15°	·2588	·2588	·9659	·9659
	*		*		*		*		*
26.	16,035	126·6	·00790	·16507	3·523	·0613	4·5617	·9981	25·5497
	*		*		*		*		*
30.	13,940	118·1	·00847	·19809	0·472	·0082	4·6769	·9999	29·5476
31.	13,472	116·1	·00861	·20670	− 0·360	− ·0064	4·6705	·9999	30·5475
	*		*		*		*		*
44.	9,147	95·6	·01046	·33189	− 13·854	− ·2393	3·0442	·9709	43·4147
	*		*		*		*		*
52.	7,850	88·6	·01129	·41952	− 24·208	− ·4099	·3650	·9121	50·9412

I regret that Mr Wood was obliged to give up his calculations before he had worked out more than about a third of the requisite rows of figures for a trajectory differing initially from fig.5 in the *sole* particular $\phi = 5°$ instead of 10°. This would have been still more illustrative than fig.5 as a contrast with fig.6. But a fairly approximate idea of its form is obtained by taking the earlier part of fig.5, regarded as having the dotted line for its base. See a remark in § 22 below, which *nearly* coincides with this.

Effect of Wind.

20. So far, we have supposed that there is no wind. But with wind the conditions are usually very complex, especially as the speed of the wind is generally much greater at a little elevation than *close* to the ground. Hence I must restrict myself to the case of uniform motion of the air in a horizontal direction. We have in such a case merely to trace, by the processes already illustrated, *the path of the ball relatively to the air*; and thence we easily obtain the path relatively to the earth. Here, of course, it is absolutely necessary to calculate the time of passing through each part of the trajectory relative to the air. If the wind be in the plane of projection, and its speed U, the *relative* speed with which the ball starts has horizontal and vertical components $V\cos\alpha - U$, and $V\sin\alpha$, respectively. Thus, relatively to the moving air, the angle of elevation is given by

$$\tan\alpha' = \frac{V\sin\alpha}{V\cos\alpha - U},$$

and the speed is

$$V' = \sqrt{V^2 - 2UV\cos\alpha + U^2}.$$

The relative trajectory, traced from these data, must now have each of its points displaced forwards by the distance, Ut, through which the air has advanced during the time, t, required to reach that point in the relative path. Of course, for a head-wind, U is negative; and the points of the relative trajectory must be displaced backwards.

Figs. 7, 8, 9 illustrate in a completely satisfactory manner, though with somewhat exaggerated speeds and coefficient of resistance, the results of this process. Mr Wood had calculated for me the path in still air, with $\alpha = 288$ (or, rather, 282), $V = 300$, $\phi = 6°$, $k = 1/3$. Since the time of reaching each point in this path had been incidentally calculated, it had only to be multiplied by 25, and subtracted from the corre-

sponding abscissa, in order to give the actual path when the speed of the head-wind is about 17 miles an hour, and the initial speed about 275. (The exact values of this and of the actual angle of projection must be calculated by means of the preceding formulæ:—but they are of little consequence in so rough an illustration as the present, especially as ϕ_0 and U/V are both small.) The corresponding trajectory is shown in fig.7. If we use the same relative path for wind of 25·5 miles per hour, the actual initial speed must be about 262·5, and the true path is fig.8. Finally, fig.9 gives the result with actual initial speed 250, and head-wind blowing at 34 miles an hour. Here, again, a kink is produced in the actual path, but it is due to a completely different cause from that of fig.4. And it is specially to be noted how much the vertex is displaced towards (and even beyond) the end of the range.

21. It is not necessary to figure the result of a following wind, for such a cause merely lengthens the abscissæ in a steadily increasing ratio, and makes the carry considerably longer, while placing the vertex more nearly midway along the path. But it is well to call attention to a singularly erroneous notion, very prevalent among golfers, viz., that a following wind *carries the ball onwards*! Such an idea is, of course, altogether absurd, except in the extremely improbable case of wind moving faster than the actual initial speed of the ball. The true way of regarding matters of this kind is to remember that there is always resistance while there is relative motion of the ball and the air, and that it is less as that relative motion is smaller; so that it is reduced throughout the path when there is a following wind.

Another erroneous idea, somewhat akin to this, is that a ball rises considerably higher when driven against the wind, and lower if with the wind, than it would if there were no wind. The difference (whether it is in excess or in defect will depend on the circumstances of projection, notably on the spin) is in general very small; the often large apparent rise or fall being due mainly to perspective, as the vertex of the path is brought considerably nearer to, or further from, the player.

These approximations to the effect of wind are, as a rule, very rough; because in the open field the speed of the wind usually increases in a notable manner up to a considerable height above the ground, so that the part of the path which is most affected is that near the vertex. But the general character of the effect can easily be judged from the examples just given.

When the wind blows directly across the path, the same process is to be applied. It is easy to see that the trajectory is no longer a plane curve; and also that, in every case, the carry is increased. But, in general, "allowance is made for the wind," *i.e.* the ball is struck in such a direction as to make an obtuse angle with that of the wind, more obtuse as the wind is stronger. In this case the carry must invariably be shortened. But without calculation we can go little beyond general statements like these.

Effect of Gradual Diminution of Spin.

22. In my former paper I assumed, throughout, that the spin of the ball remains practically unchanged during the whole carry. That this is not far from the truth, is pretty obvious from the latter part of the career of a sliced or a heeled ball. If, however, in accordance with § 4, we assume *it* also to fall off in a geometric ratio with the space traversed :—an assumption which is probable rather than merely plausible ; so long, at least, as we neglect the part of the loss which would occur even if the ball had no translatory speed :—the equations of § 10 require but slight modification. For we must now write, instead of k,

$$k\epsilon^{-s/b}.$$

The time rate at which this falls off is proportional to itself and to v, directly, and to b inversely.

If we confine ourselves to the very low trajectories which are now characteristic of much of the best driving, we may neglect (as was provisionally done in § 10) the effect of gravity on the speed of the ball, and write simply

$$v = V\epsilon^{-s/a}.$$

Thus the approximate equation of the path becomes

$$\frac{dy}{dx} = \alpha + \frac{ka'}{V}(\epsilon^{x/a'} - 1) - \frac{ga}{2V^2}(\epsilon^{2x/a} - 1).$$

Here

$$\frac{1}{a'} = \frac{1}{a} - \frac{1}{b}; \text{ and finally}$$

$$y = \alpha x + \frac{ka'^2}{V}(\epsilon^{x/a'} - 1 - x/a') - \frac{ga^2}{4V^2}(\epsilon^{2x/a} - 1 - 2x/a),$$

where α is always very small, perhaps even negative ; and may, at least for our present purpose, be neglected. Its main effect is to elevate, or depress, each point of the path by an amount proportional to the distance from the origin ; and thus (when positive) it enables us to obtain a given range with less underspin than would otherwise be required.

23. For calculation it is very convenient to begin by forming tables of values of the functions

$$f(p) = \frac{\epsilon^p - 1}{p}, \text{ and } F(p) = \frac{\epsilon^p - 1 - p}{p^2} = \frac{f(p) - 1}{p};$$

for values of p at short intervals from 0 to 3 or so. (Note that the same tables are adaptable to negative values of p, since we have, obviously,

$$f(-p) = \epsilon^{-p} f(p), \text{ and } F(-p) = \epsilon^{-p}(f(p) - Fp)).$$

These we will take for granted. We may now write

$$y=\frac{x^2}{V^2}(kVF(x/a')-gF(2x/a))$$

$$\frac{dy}{dx}=\frac{x}{V^2}(kVf(x/a')-gf(2x/a)),$$

$$\frac{d^2y}{dx^2}=\frac{1}{V^2}(kV\epsilon^{x/a'}-g\epsilon^{2x/a}).$$

The range, and the horizontal distances of the vertex and of the point of contrary flexure, respectively, are given by the values of x which make the second factors vanish :—and it is curious to remark that (to the present rough approximation, of course, and for given values of a and a') these depend only upon the value of kV/g, *i.e.* the initial ratio of the upward to the downward acceleration. Thus so far as the *range* is concerned, the separate values of k and V are of no consequence, all depends on their product. But it is quite otherwise as regards the flatness of the trajectory, for the maximum height is inversely as the square of V. Of course we must remember that one indispensable condition of the approximation with which we are dealing is that the trajectory shall be very flat; and thus, if the range is to be considerable, V cannot be small, and (also of course) k cannot be very large. We have already seen how to obtain a fairly approximate value of a (say 360), but b presents much greater difficulty. We may, therefore, assume for it two moderate, and two extreme values, and compare the characteristics of the resulting paths. If b be infinite, we have the case already treated, in which the spin does not alter during the ball's flight; while, if b be less than a, the spin dies out faster than does the speed and we approximate (at least in the later part of the path) to the case of no spin. Hence we may take for the values of b the following :— ∞, 900, 360, and 180 :—so that a' has the respective values 360, 600, ∞, and -360. Let the carry ($\bar{x}$) be, once for all, taken as 180 yards. Then, for $y=0$, we must have $2\bar{x}/a=3$; and the respective values of $\bar{x}/a'$ are 1·5, 0·9, 0, and $-1·5$. With these arguments the values of F are, in order,

1·7873; 0·8807, 0·6908, 0·5, and 0·3258;

so that we have the following approximate values of the ratio kV/g

2·03, 2·59, 3·57, 5·49.

The first two require a moderate amount of spin, only, if we take 240 as the initial speed.

The approximate position of the vertex (x_0) of the first of these paths is given by

$$f(2x_0/a)=2·03\,f(x_0/a),\text{ or }\epsilon^{x_0/a}=3·06,\ (x_0/a=1·1184)$$

whence $x_0=402·6$, or about three-fourths of the carry.

The corresponding value of y is about 27 feet.

The point of contrary flexure is at $\epsilon^{x/a} = 2{\cdot}03$, so that $x_1 = 255$, and the value of $\frac{dy}{dx}$ there has its maximum, about 0·07 only.

In the other three paths above, the maximum ordinate and the maximum inclination both increase with the necessarily increased value of k, while the vertex and the point of inflexion both occur earlier in the path. The approximate time of flight, in all, is a little over five seconds. The paths themselves are shown, much foreshortened, in figs. 10, 11, 12, 13, where the unit of the horizontal scale is 3·6 times that of the vertical. This is given with the view of comparing and contrasting them. Fig. 14 shows the first, and flattest, of these paths in its proper form. It is clearly a fair approximation to the actual facts; and when we compare it with the others, as in the foreshortened figures, we see that the assumption of constant spin (§ 4) is probably not far from the truth. For, in the great majority of cases of drives of this character, there is observed to be very little run :—and this can be accounted for only on the assumption that there is considerable underspin left at the pitch. But it is also clear that the falling off of the spin produces comparatively little increase of the obliquity of impact on the ground, even in the exaggerated form in which these paths are drawn. Their actual inclinations to the ground have tangents about 0·49, 0·66, 0·78, and 1·08 respectively. The last, and greatest, of these angles is just over 45°.

24. It is interesting to compare this set of data, and their consequences, with those of §§ 11, 14, 15. The latter were in fair agreement with many of the more easily observed features of a good drive, but they gave too high a trajectory. The new measure of initial speed, and the consequent reduction of the estimated value of the coefficient of resistance, have led to results more closely resembling the truth.

But in all, as we have seen, there is one notable defect. The ball comes down too obliquely, and this is the case more especially when the carry is a long one, and the ball's speed therefore much reduced. I was at first inclined to attribute this to my having assumed the spin to remain constant during the whole flight. This was my main reason for carrying out the investigations described in §§ 22 *sq.* But these give little help, as we have just seen, and I feel now convinced that the defect is due chiefly to the assumption that the resistance is *throughout* proportional to the square of the speed. I intend to construct an apparatus on the principle described in § 16 above, but of a much lighter type, to measure the resistance for speed of 30 feet-seconds or so, downwards. But I shall probably content myself with verifying, if I can, the idea just suggested; leaving to some one who has sufficient time at his disposal the working out of the details when the resistance is proportional (towards the end of the path) to the speed directly, or to a combination of this with the second power. The former is considerably more troublesome than ROBINS' law; and a combination of the two may probably be so laborious as to damp the ardour of any but a genuine enthusiast. The possibility that the law of resistance may change its form for low speeds (*i.e.*, towards and beyond the vertex of the path) throws some doubt upon the accuracy of the determination of the

coefficient of resistance from the range, the time of flight, and the initial speed. But, at present, I have no means of obtaining a more accurate approximation.

25. The whole of this inquiry has been of a somewhat vague character, but its value is probably enhanced, rather than lessened, in consequence. For the circumstances can never be the same in any two drives, even if they are essentially good ones, and made by the same player. To give only an instance or two of reasons for this:—Two balls of equal mass may have considerably different coefficients of resistance in consequence of an apparently trifling difference of diameters, or of the amount or character of the hammering:—or they may have very different amounts of resilience, due to comparatively slight differences of temperature or pressure during their treatment in the mould. The pace which the player can give the club-head at the moment of impact depends to a very considerable extent on the *relative* motion of his two hands (to which is due the "nip") during the immediately preceding two-hundredth of a second, while the amount of beneficial spin is seriously diminished by even a trifling upward concavity of the path of the head during the ten-thousandth of a second occupied by the blow. It is mainly in apparently trivial matters like these, which are placidly spoken of by the mass of golfers under the general title of "knack," that lie the very great differences in drives effected, under precisely similar external conditions, by players equal in strength, agility, and (except to an extremely well-trained and critical eye) even in style.

[*Oct.* 5, 1898.—The printing of this paper has been postponed for nearly three years in the hope, not as yet realised, that I might be able to determine accurately by experiment the terminal speed of an average golf-ball, as well as the average value of k, when (as in § 5) $k\omega v$ represents the transverse acceleration, in terms of the rates of spin and translation. Another object has been to measure the effect of rapid rotation upon the coefficient of resistance to translatory motion. These experiments, in various forms, are still being carried out by means of various modes of propulsion, from a cross-bow to a harpoon-gun. I hope also to procure data, for speed and resistance, applicable to various other projectiles such as cricket-balls, arrows, bird-bolts, etc.]

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PROF. TAIT ON THE PATH OF A ROTATING SPHERICAL PROJEC

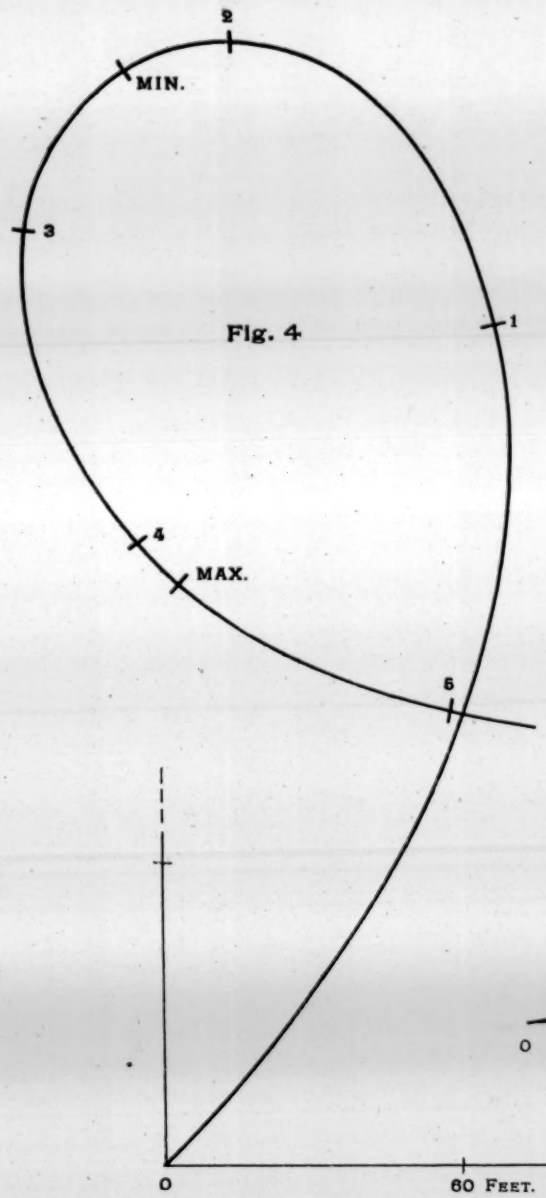
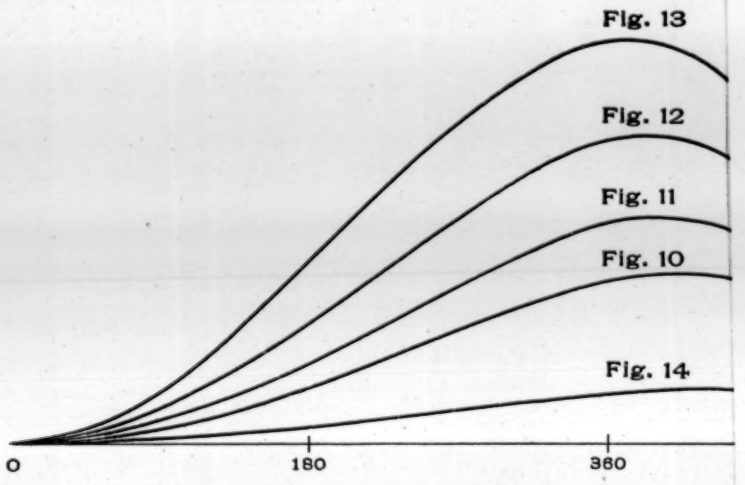
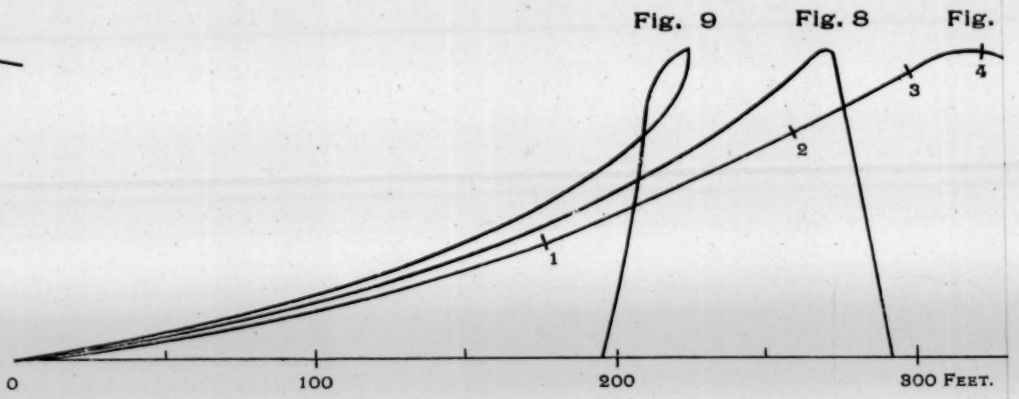
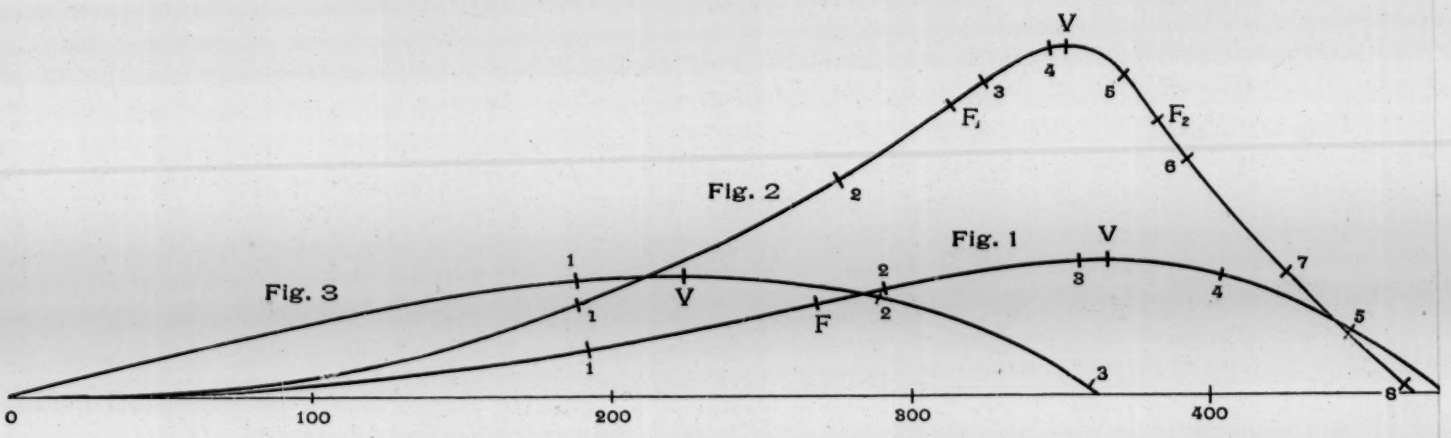
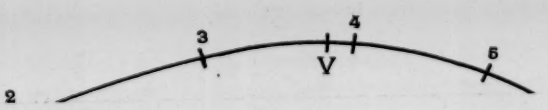


Fig. 4

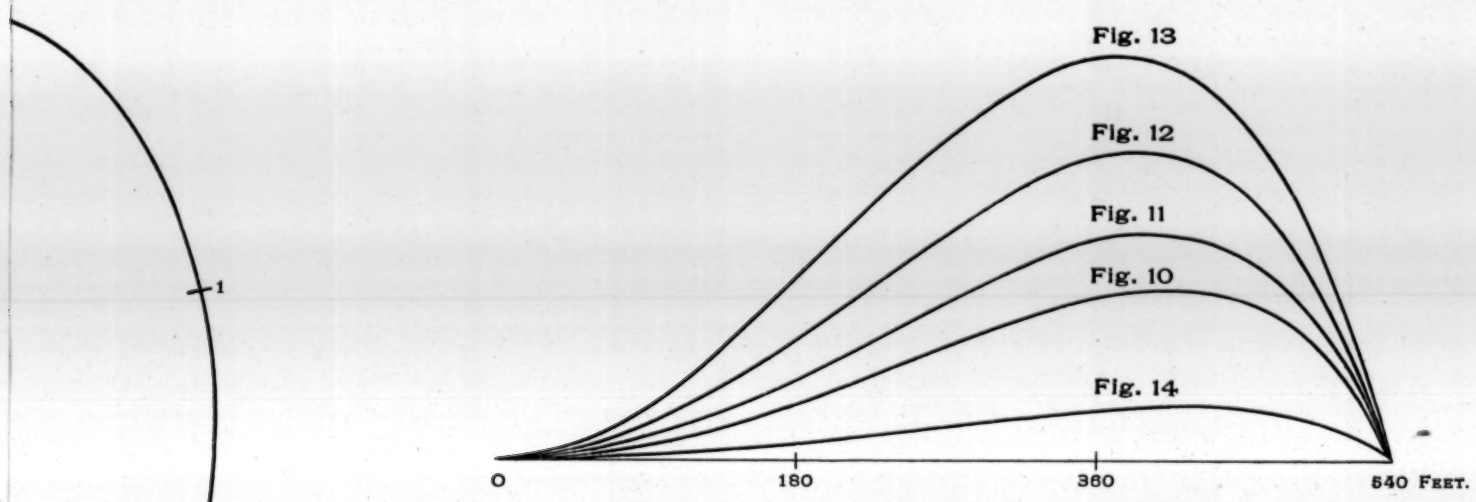


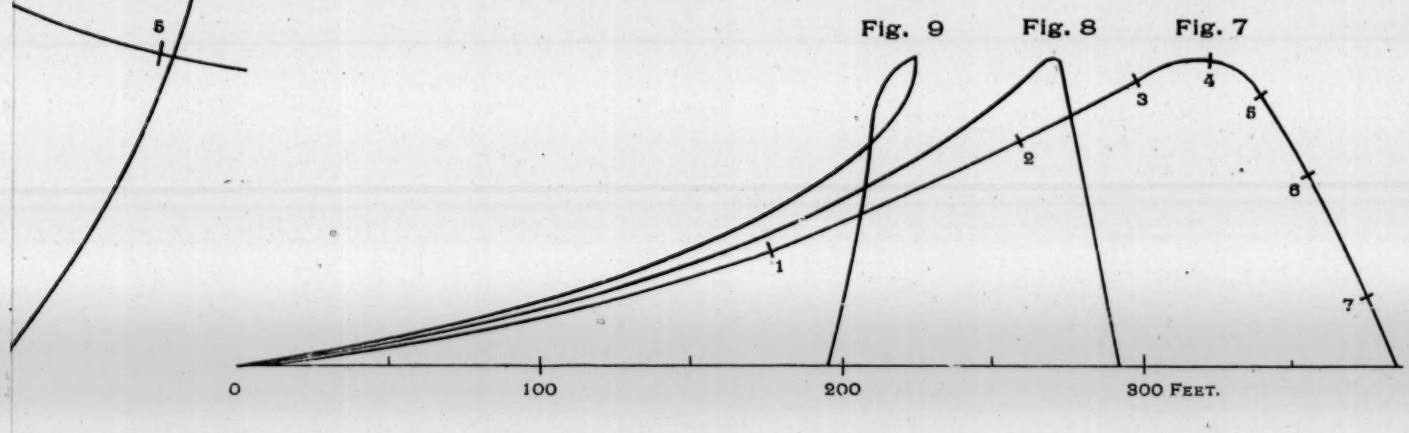


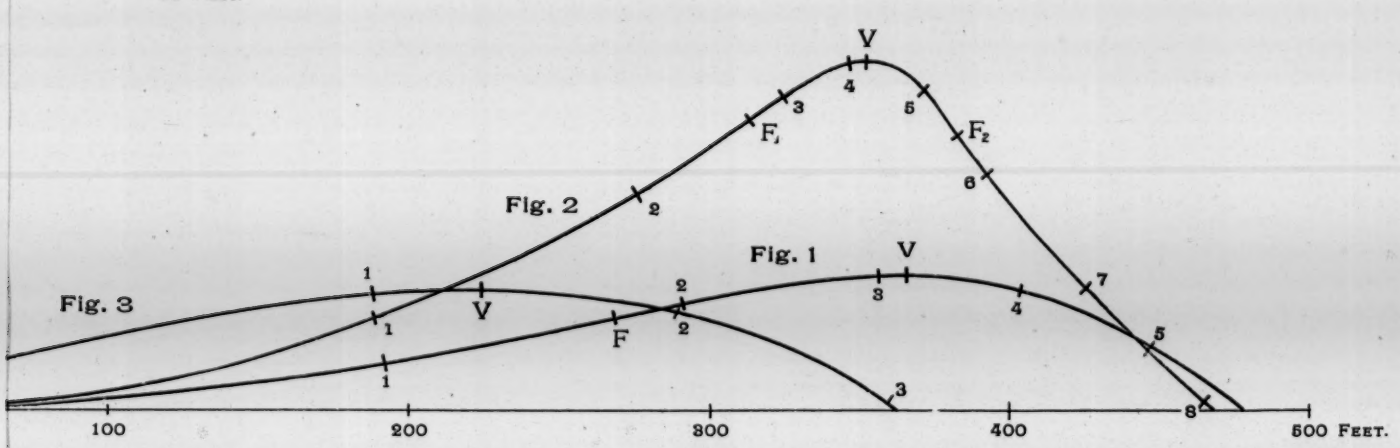


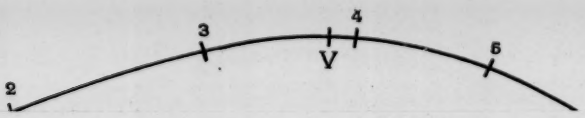


ON THE PATH OF A ROTATING SPHERICAL PROJECTILE.









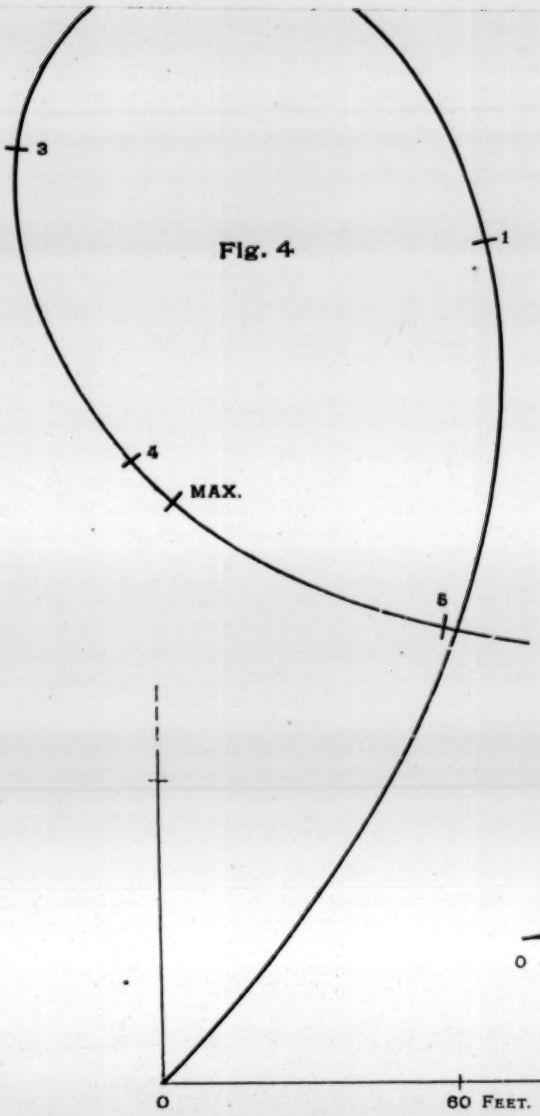
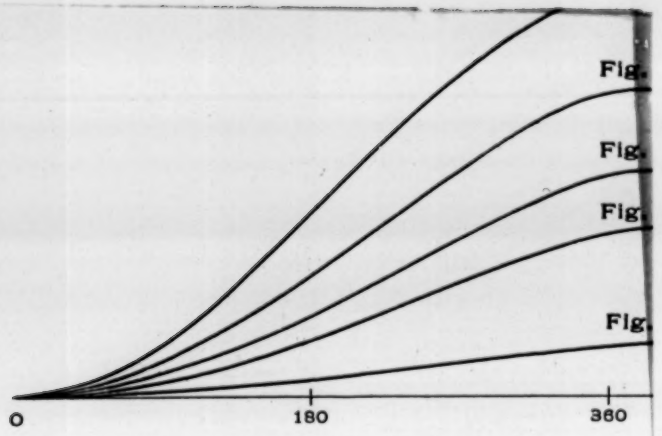
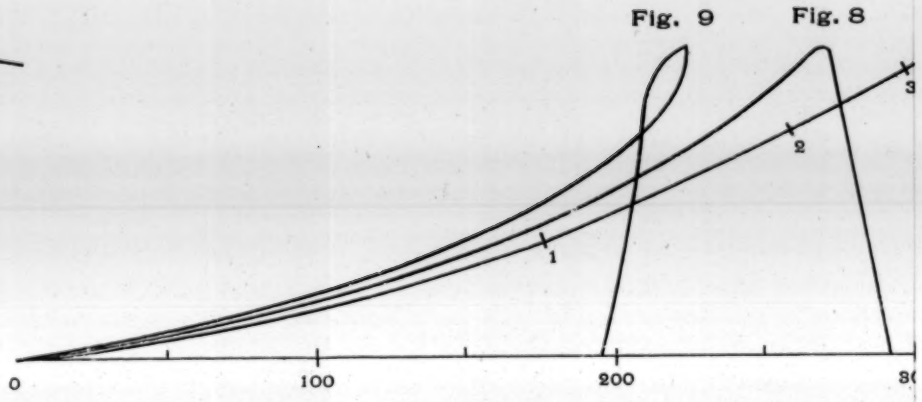
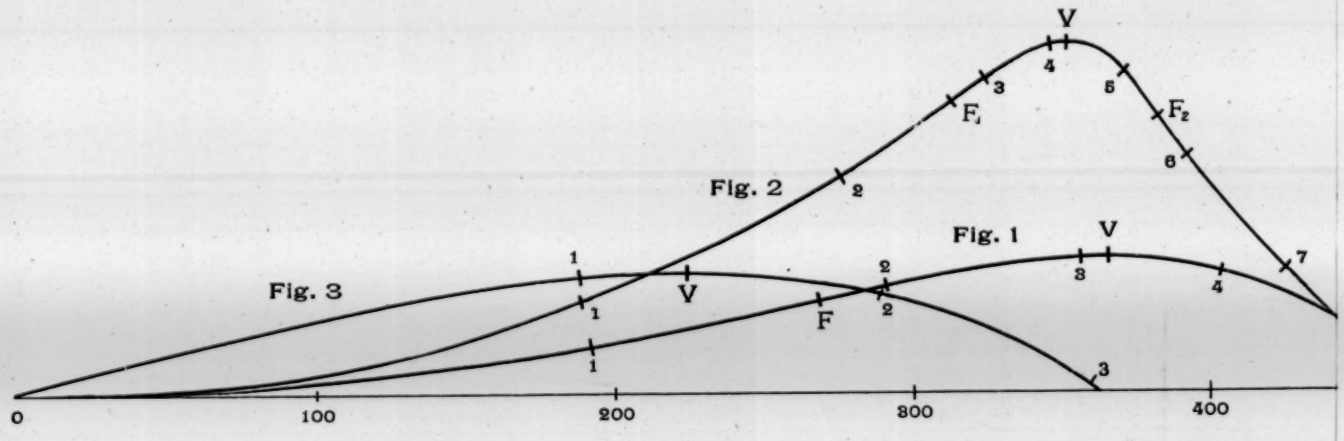

3
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4
MAX.
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60 Feet.

Fig. 4


Fig.
Fig.
Fig.
Fig.
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180
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Fig. 9
Fig. 8
1
2
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100
200


V
4
3
5
F1
F2
6
Fig. 2
2
Fig. 1
V
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2
7
Fig. 3
V
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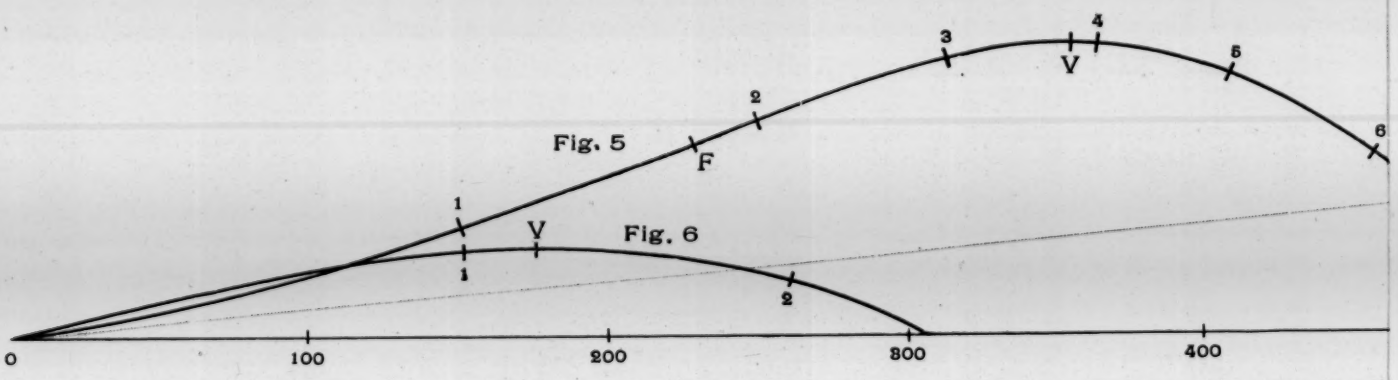
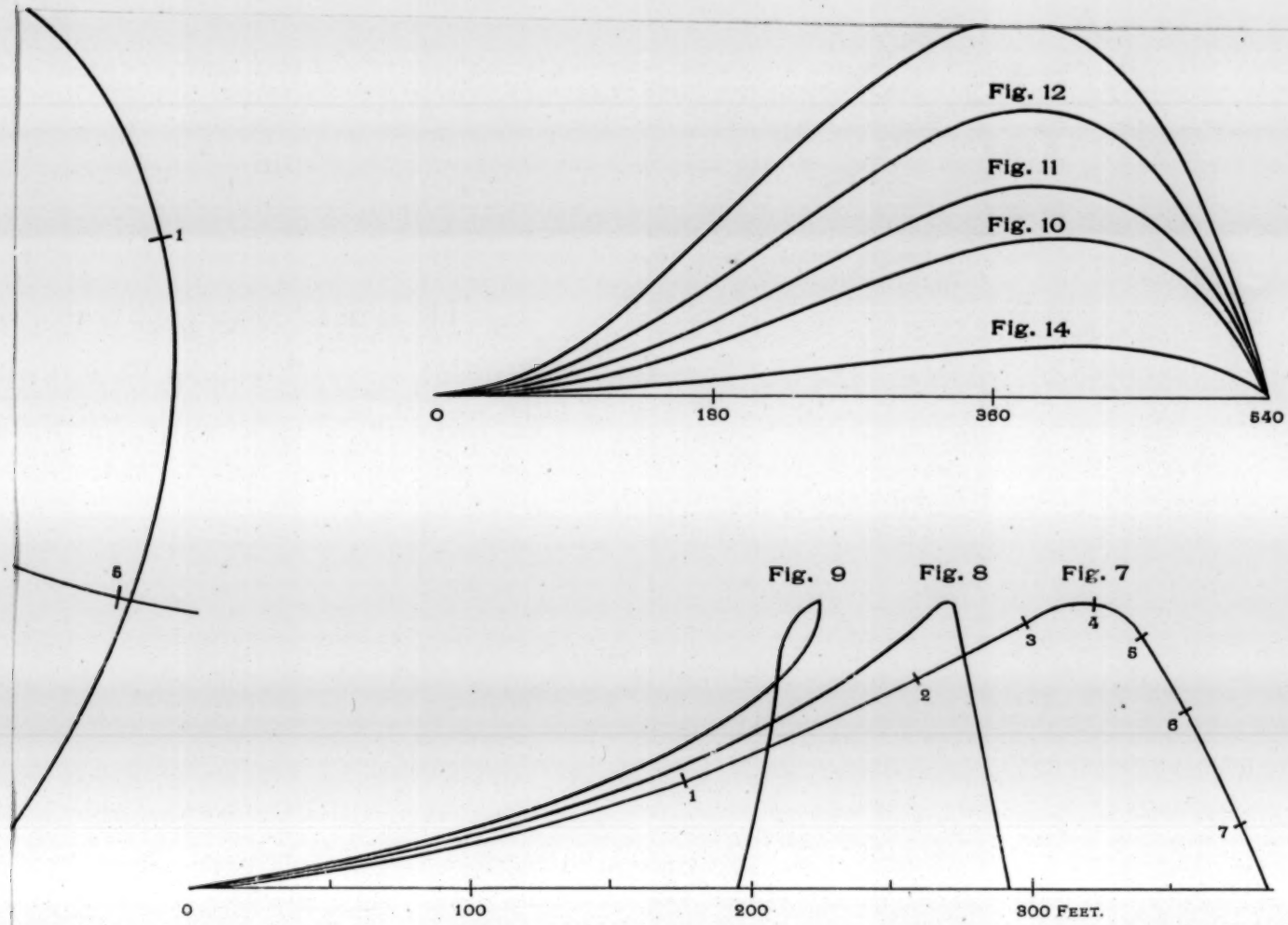
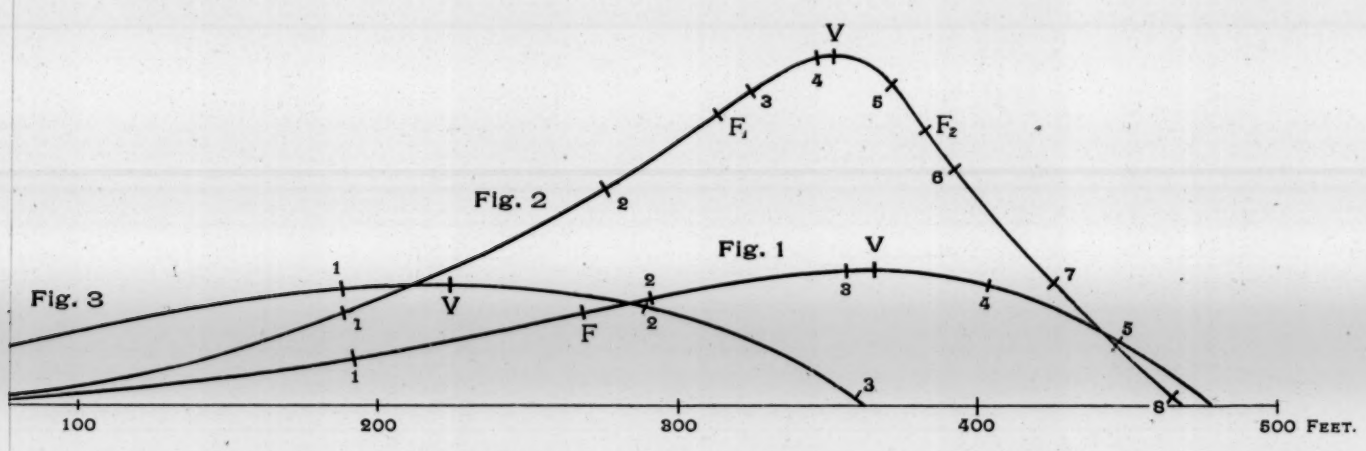
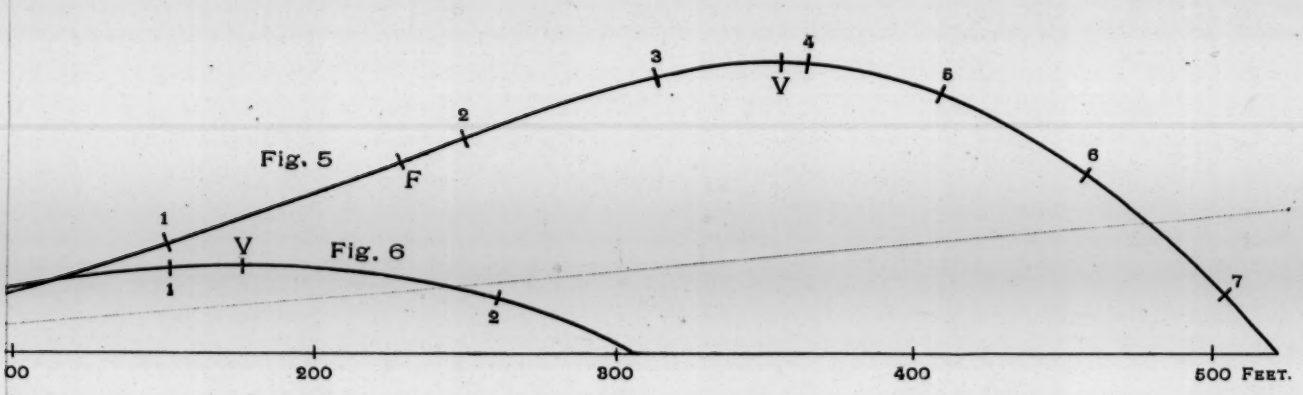

3
4
V
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2
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Fig. 5
F
1
V
Fig. 6
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200
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Fig. 12
Fig. 11
Fig. 10
Fig. 14
0
180
360
540 Feet.


1
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Fig. 9
Fig. 8
Fig. 7
1
2
3
4
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7
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300 Feet.
60 Feet.


V
F1
F2
Fig. 2
Fig. 1
Fig. 3
F
100
200
300
400
500 Feet.


V
Fig. 5
F
Fig. 6
100
200
300
400
500 Feet.

TRANSACTIONS

OF THE

ROYAL SOCIETY OF EDINBURGH.

VOL. XXXIX. PART III.—FOR THE SESSION 1898–99.

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XVII.—*On the Further Anatomy and the Budding Processes of Cephalodiscus dodecalophus* (*M'Intosh*).

By Arthur T. Masterman, B.A., D.Sc., F.R.S.E.; Lecturer and Research Fellow in the University of St Andrews. (With Five Plates.)

(Read 6th June 1898.)

The former paper (5) on the Anatomy of *Cephalodiscus* was incomplete in certain particulars, which I am now in a position to add. These are mainly in regard to the alimentary canal, the nervous system, and the reproductive organs. To the present description of these will be appended an account of the budding processes, which are a very conspicuous feature of the species.

It has been pointed out that the twelve plumes, which are definitely arranged and spread out dorsally in front of the head region, have longitudinal grooves running down their ventral side into the mouth region. "One cannot doubt that in *Cephalodiscus* the cilia of the ventral surface cause currents down the ventral grooves of the plumes, and thence into the mouth, ... carrying food-particles, entangled in slime, into the alimentary canal, a mode of alimentation analogous to that of *Phoronis*, and indeed of the *Urochorda*" (5). We might add to this, the *Lamellibranchiata*, *Brachiopoda*, and *Amphioxus*. This characteristic method of ingestion by means of ciliary currents must be regarded as a primitive one, and it is interesting to notice the several methods by which the superfluous water current is got rid of, and the food retained. The primary origin of gill-slits has been suggested by both Harmer and Brooks as connected with this function. The single pair of pharyngeal clefts in *Cephalodiscus* certainly appear to agree with such a theory.

A series of horizontal longitudinal sections through an adult *Cephalodiscus* from before backwards cut the front portion of the pharynx transversely and the hind part longitudinally. Such a series may be seen in figs. 1 to 9.

In fig. 1 it may be noticed that three oral grooves (1, 2, 3) converge from each side of the mouth, passing downwards from the branchial plumes. They are directly continuous with the grooves running down the ventral side of each of the plumes, which unite in pairs to form the oral grooves here seen.

The dorsal wall of the buccal cavity is quite simple, but on the ventral wall appears a slight semi-circular depression. In fig. 2 this depression is observed to have divided into two grooves in the ventral surface of the pharynx, which deepen rapidly. In fig. 3 they present chordoid walls, and as far back as fig. 4, they can be readily

recognised as the two pharyngeal clefts (*p.c.*). In fig. 3 the first pair of oral grooves (3) have bent round so as to lie completely within the pharynx, and in fig. 4 the other two (2, 1) show similar relations. Between the two pharyngeal pouches, in the mid-ventral line, a slight depression can be seen in this figure, and it can be followed into the ventral alimentary portion of the pharynx, which is a conspicuous feature in the remaining figures. In figs. 5 and 6 the oral grooves may be seen to become merged dorsally into a large dorsal hyper-pharyngeal groove, which is at first partially divided into two by a median ridge. Further back (fig. 7) this groove extends across the middle line, and forms the median dorsal portion of the pharynx, which receives the duct of the sub-neural gland, *sn.gl.* (*cf.* plate 24, fig. 13, *loc. cit.*). This gland is seen in figs. 5 and 6. In fig. 7 it has merged into the large dorsal chamber.

In this figure the pharynx presents a symmetrical appearance. Dorsally lies the large dorsal chamber, into which opens, from in front backwards, the duct of the sub-neural gland, *sn.gl.* (in the median line), and (laterally) the two grooves which are formed by the union of the three oral grooves on each side.

The two pharyngeal pouches (*p.c.*) lie laterally and pass towards the ventral surface, whilst mid-ventrally is the large alimentary portion of the pharynx.

A little dorsal to each pharyngeal pouch has appeared a small groove with chordoid walls, which rapidly increases in size (figs. 8 and 9, *n.c.*), and proves to be the commencing pleurochord. In fig. 9 this pleurochord is seen to occupy the dorso-lateral part of the pharynx, the pharyngeal pouch being ventro-lateral. After fig. 9 the latter rapidly decreases in size, as seen in sections, and the pharynx then presents the appearance indicated in plate 25, fig. 17, *loc. cit.*

Other important points may be noticed in a series of transverse sections, such as those shown in figs. 90–99.

Fig. 100 is a semi-diagrammatic reconstruction of the pharynx of *Cephalodiscus*, viewed from the left side. The lines cutting it transversely indicate by their numbers the approximate level at which each of the sections in figs. 90–99 are cut.

Fig. 90 cuts the head (or pre-oral part) of the pharynx, into which falls the sub-neural gland (*sn.gl.*). The lumen of the latter is directly continuous with the four grooves here shown, two dorsal grooves (*d.g.*), and two ventral (*v.g.*).

Figs. 91 and 92 show the two pairs of grooves to become more pronounced. A condition very similar to these sections, especially to fig. 92, is shown in plate 24, fig. 12, of my former paper, though the significance of these grooves had not then been followed out.

Fig. 93 cuts through the mouth-opening and through the front part of the main bulk of the pharynx. Between the dorsal and the ventral grooves, the pleurochordal diverticulum (*n.c.*) and the commencing pharyngeal cleft (*p.c.*) have now become conspicuous features on either side of the section, so that the grooves are widely separated. The dorsal grooves unite more or less in one, and the ventral grooves run down laterally on either side of the mouth (*v.g.*). They were distinctly figured in this



position in a section (plate 25, fig. 16) of the whole animal (5), and they form a very marked peri-pharyngeal band around the buccal chamber. Their position is readily determined, in cross sections, by the fact that they are situated exactly internal to the line of junction of the mesentery between collar- and trunk-cavities, with the gut wall (see fig. 93).

In fig. 94 the pleurochords form the main lateral walls of the pharynx, but the two pairs of grooves are still conspicuous.

Fig. 95 is posterior to the mouth-opening and behind the pharyngeal cleft. Here the two ventral grooves, forming a peri-pharyngeal band, have united behind the mouth in a medium ventral groove, with the middle line slightly raised in a ridge. The two dorsal grooves are still separated by a very characteristic median ridge of thickened cells comparable to a dorsal lamina. In fig. 96 somewhat the same condition holds, though the notochords are becoming reduced in size. This process of reduction goes on very rapidly, till (in fig. 97) the notochordal grooves are of no more importance than the dorsal grooves, and at the same time the ventral groove has become single and median.

In fig. 98 the two dorsal grooves, and the ventral groove alone remain, and in fig. 99 they are reduced to very small calibre and posterior to this they fuse and open into the œsophagus.

We thus see that the pharynx of *Cephalodiscus* has a pair of dorsal grooves separated by a dorsal lamina, and a ventral hypo-pharyngeal groove, joined at the front end by a peri-pharyngeal groove. At the point where the dorsal grooves join the peri-pharyngeal groove, the sub-neural gland opens. The relation of the peri-pharyngeal grooves to the ventral alimentary portion of the gut in *Cephalodiscus* seems to corroborate the suggestion that the endostyle and thyroid are homologous with this part of the pharynx in *Balanoglossus*. These grooves are all too small for water-currents and their arrangement prompts an irresistible comparison with the mucus-grooves in the pharynx of *Urochorda*, and although one can only cautiously apply close homologies yet the already tentative comparison of the sub-neural gland (5 and 6) to that of the *Urochorda*, is considerably strengthened by its similar relationship to the dorsal and peri-pharyngeal grooves.

In fig. 100 is seen a lateral view of the whole pharynx. The funnel-shaped mouth (*m*) shows the three oral grooves (*o.g.*) on each side converging into the opening of the pharynx. Round this opening is the peri-pharyngeal groove, running down from the sub-neural gland (*sn.gl.*), from which also proceeds the dorsal groove.

Down the dorso-lateral region runs the large paired plęurochordal ridge, which, rising gradually from behind, bends over in front, and opens by the pharyngeal cleft (*p.c.*) downwards and outwards to the exterior.

From this description of the pharynx and its connected structures, it is not difficult to understand to a great extent the functions performed by each part.

The twelve branchial plumes with their pinnæ are spread out from behind the buccal shield, and currents of water and food-particles pass down the ciliated ventral

grooves of these which join in pairs to form the three oral grooves on each side. The current of water and food-particles then passes through the peri-pharyngeal ring, into the main body of the pharynx. It is difficult to conjecture whether the mucoid stream passes forwards ventrally up this ring, and backwards dorsally, or whether the mucus, emerging from the sub-neural gland, passes backwards both dorsally and ventrally, but in either case, the food-particles become entangled in the mucus and are eventually carried backwards through the narrow œsophageal opening into the stomach whilst the water, flowing up to this opening, in the centre of the pharynx, will return as a back-flow along the notochordal grooves.

In *Phoronis*, as has been indicated in a preliminary note (7), the water-current is got rid of by means of the epistome, but in *Cephalodiscus* a large quantity of water must be carried into the pharynx, and the pharyngeal slits are admirably situated for its removal. Their chordoid walls must serve to keep the clefts permanently open. In addition to this, it appears probable that the dorso-lateral grooves which I have termed pleurochords will also serve to carry off the superfluous water outwards and forwards. The pharynx contracts to a small œsophageal opening at the hind end, and the whole structure suggests that, whilst the mucoid strands, laden with food, pass on to the stomach, a back current of water passes forwards along the cavities of the pleurochords and out by the pharyngeal clefts (fig. 100). If this be so, then the chordoid structure, both of the pharyngeal clefts and of the pleurochords may be due to the necessity for rigid channels to allow of the escape of cloacal water. Whether as paired pleurochords or as a median notochord, this function would persist in the early *Chordata* till pharyngeal clefts were formed at the posterior end of the pharynx, or as in *Phoronis* an extra-stomial means of separating the water from food were evolved. The notochord would then persist only in those forms in which it had secondarily acquired a supporting function (*Euchorda*).

Thus the loss of pleurochords in *Phoronis* may be traced to the peculiar adaptation of the epistome and the preclusion of a saving secondary function owing to the sedentary habitat, whilst the loss of the notochord in the *Urochorda* is primarily due to the removal of the necessity for the secondary supporting function, involved in a like manner in the adoption of a sedentary habit.

Thus one is led to suggest that the notochord of the Chordata *primarily owed its origin to the necessity for a rigid channel in the endodermal walls of the pharynx, intimately connected with the primitive process of ciliary ingestion of food. The function of the notochord as an elastic primary axis to the body would thus be a secondary modification.*

This assumption appears to be justified by the fact that the pleurochord of *Cephalodiscus*, permanently in the condition found in the embryonic *Euchorda* alone, *i.e.*, as an endodermic longitudinal groove, most probably fulfils this function.

All the *Chordata* except the *Vertebrata* (or *Holochorda*) carry on their ingestive processes by ciliary action, with the consequent necessity for a separation of the water

and food-currents, hence it is certain that the chordate pharynx and allied organs have arisen, and have reached a complexity as high as that of *Amphioxus*, under these conditions. In this category may be included the notochord (or pleurochords), pharyngeal clefts, endostyle, sub-neural gland and the hyper-pharyngeal groove of *Amphioxus*, which are intimately connected with the vertebral column, gill-slits, and visceral arches, thyroid gland, hypophysis and sub-notochordal rod respectively, in the true *Vertebrata*.

Assuming that the pharynx of the earliest archi-chordate was homogeneous, and that the return water-current from the œsophagus took a dorsal course along grooves formed by the continuity of the current, then the epithelial cells lining these grooves would lose their secretory power, and might easily undergo a chordoid degeneration, which, serving as an efficient means for keeping open the cloacal grooves, would become permanent, and the pleurochords would become established. The paired condition may be primitive or secondarily acquired in the *Diplochorda*.

The Gonads.

Both Professor M'Intosh (4) and Dr Harmer (1) have figured and alluded to the ovaries and oviducts of *Cephalodiscus* and to their descriptions. I have little to add. I have been equally unsuccessful with them in the search for male elements. In one preparation of part of the cœnœcium there appears a dense cloud of minute, darkly-staining oval bodies, which might possibly be interpreted as free spermatozoa; but it is fairly certain that the colony obtained by the *Challenger* is "female" throughout. In a series of sections in the horizontal longitudinal plane, the oviducts and ovaries are cut nearly transversely, though at an angle.

In fig. 10, the left side shows the opening of the oviduct to the exterior, as a slight cup-shaped invagination of the ectodermal cells. On the right side, the oviduct (*g.d.*) is cut rather further back, and the cells are here seen to be longer, with their nuclei more irregularly arranged than is the case with the ectoderm. The oviduct is covered on the inner side by the wall of the cœlome, which is here rather thicker than elsewhere. In fig. 11 the dorsal mesentery is observed to give out a lateral mesentery on each side to the base of the oviduct. On the right hand of this figure the oviduct is free from the ectoderm, and is seen to be supported by the lateral mesentery. Here the oviduct has a fine lumen, with indications of cilia. The epithelial cells show masses of black pigment, chiefly at their inner ends.

Fig. 12 gives two views of the junction of the oviduct (*g.d.*) with the ovary (*ov.*). On the left side the cells of the oviduct, with their abundant pigment, can be observed almost encircling the lumen of the ovary, whilst, especially on the inner side, the wall of the ovary can be recognised. On the right the oviduct is represented by a mere segment, whilst the greater part of the section is occupied by the reticular tissue of the ovary.

In this section the dorsal blood-vessel gives off an ovarian vessel running in the lateral mesentery to the ovary. It appears to terminate in contact with the ovary (*l.b.s.*), but in reality it surrounds the whole ovary, as can be easily determined in other specimens.

In fig. 13, both ovaries are seen in section at their upper ends. The ovary appears to consist of a mass of typically mesodermic tissue, consisting of a fine protoplasmic meshwork with nuclei scattered throughout it. Here and there are seen large egg nuclei, with a deeply-staining nucleolus. With progression backwards these nuclei become fewer and more prominent, whilst masses of protoplasm become aggregated around them. They then (fig. 14) tend to arrange themselves in three rows, and are of so large a size that they constrict the central space into a triangular lumen, which in fig. 15 is reduced to a mere slit.

In specimens with nearly ripe ova, the ovaries press outwards upon the rectum and other surrounding parts, and as Professor M'Intosh (4) has remarked, they appear as large rounded protuberances in the body of the individual.

In the mesodermic covering of the ovary is a thin chondroid skeletal layer, which becomes greatly thickened at the posterior end.

The peculiar structure and appearance of the oviducts led them to be at first mistaken, in an external view, for eyes; and later, when their true structure was known, Dr Marcus Gunn and Professor M'Intosh (4) suggested that they were of the nature of phosphorescent organs. Considering the habitat of the animal, this appears to be by no means improbable, the light forming a possible attraction to the male element. It seems also likely that these so-called "oviducts" function solely for the introduction of the spermatozoa into the lumen of the ovary, in which fertilisation may be effected, the egg being afterwards set free, possibly by the death of the parent. This is rendered probable by the following considerations :—

Firstly, the eggs ripen from the mouth of the ovary backwards, so that the most advanced are found at the posterior end, furthest from the oviduct.*

Secondly, the ripe eggs formed in the cœnœcium are enormous in comparison with the size of the parent, and it is difficult to conceive of their extrusion through the oviducts or elsewhere without extensive rupture of the tissues.

Thirdly, the very active asexual reproduction would justify the low sexual fecundity implied in this suggestion.

In comparing these organs with those of *Phoronis*, we may note that the oviducts lie in a pair of lateral mesenteries, occupying a similar position to those of *Phoronis*, in which are situated the nephridia. The ovaries are evidently proliferations of the cœlomic wall in each case, and the main distinction appears to consist in the "closed" condition of the oviducts in *Cephalodiscus* as compared with the "open" nephridia of *Phoronis*, a difference very closely paralleled by that existing between the urogenital systems of the *Teleostei* and the *Elasmobranchii* (compare figures 16 and 17).

* *Cf.* Remarks upon ovaries of *Cribrella oculata*, by Professor M. Sars. *Fauna litt. Norvegiæ*, Christiania, 1846.

On the other hand, the similarity of the gonads of *Cephalodiscus* to those of *Balanoglossus* is patent. They differ mainly in the fact that, as in the case of the pharyngeal clefts, there is only a single pair.

Nervous System and Pedicle. (Ventral Sucker.)

In a former paper already referred to, the nervous system was described as consisting of a dorsal ganglion over the sub-neural sinus, a pre-oral ring, a post-oral ring, and a pair of lateral nerves down the trunk. In addition, there are nerves to each of the plumes.

Fig. 85 is a diagram giving the distribution of the main nervous tracts, though the more or less general nervous plexus is not indicated. In the mid-ventral line of the trunk, from the trunk backwards, there passes a broad nervous tract which can be traced directly into the pedicle. The lateral nerves can be followed into the same organ.

Fig. 18 shows a transverse section of the pedicle. In the mid-ventral line may be seen the ventral nerve (*v.n.*) cut in section and forming a long ridge, with the chondroid layer between it and the cœlome. On each side is a lateral nerve (*v.l.n.*) similar in appearance but not so prominent. In all these nerves there can be discerned a fine lumen which is more or less continuous throughout the pedicle. The three nerves gradually approach towards each other till at the tips they become indistinguishable.

Immediately inside the ectoderm is a fairly thick layer of chondroid tissue, and internal to it, is a well-defined layer of longitudinal muscles (*l.m.*). The cœlomic space in the middle is directly continuous with the trunk-cavities, and the dorsal and ventral blood-vessels are direct continuations of the similar vessels in the trunk.

The main difference between the structure of the pedicle and that of the trunk is the absence of endodermic elements (intestine) in the former. The acknowledged secondary character of a reduplicated gut leads one to conclude that the pedicle is morphologically the hind end of the body, and that the actual posterior sacculated part of the animal is a secondary dorsal protuberance. A basal fixative portion of the body in *Phoronis* can be shown on similar grounds to be the true hind end of the body, and therefore homologous with the pedicle of *Cephalodiscus*. It will be seen later that this interpretation of the pedicle agrees with its relationships as seen in the buds.

Sexual Reproduction.

In going over a series of specimens mounted by the late Professor Busk, and now in the possession of Professor M'Intosh, I was struck by the appearance of an adult, which I have figured here (fig. 86).

It will be noticed that in the cloacal region there is a group of cyst-like structures closely pressed together and having a resemblance to a number of larvæ enveloped in their egg-capsules.

After drawing the specimen, it was removed from its mounting and sectioned, but its state of preservation has prevented even the determination of the exact position of these structures, whether in the rectum or the ovary, though they are more probably in the latter.

The examination of another specimen in the same collection furnished a series of eggs free in the cœnœcium, which were at different developmental phases, and from these I have been enabled to figure at least three stages.

Fig. 87 shows the egg contained in its capsule with a stalk of attachment, very much as described by Professor M'Intosh (4), but of a more elongated outline. Fig. 88 shows a later stage in which the elongation is more pronounced and the egg has assumed a pear-shaped outline.

In fig. 89 a larval stage is seen, which is of some interest. A constriction has appeared slightly forward of the equator and divides the body into two segments. The resemblance of this larva to the early larval stage of *Balanoglossus*, as figured by *Bateson*, prior to the external differentiation of the collar-segment, is very striking.* The presence of these larvæ indicates that at any rate a certain amount of the development takes place in the cœnœcium, and that it may be possible to obtain a series of eggs and larvæ in which the early stages may be successfully followed out.

Asexual Reproduction.

Professor M'Intosh (4) has already given a general description of the budding processes in *Cephalodiscus*, but a re-investigation of the subject, with plenty of good material, has enabled me to work out further details.

The buds are borne upon the pedicle, at the extreme distal extremity and upon the ventral surface. The distal extremity of the pedicle is modified into a sucker, and it is on the ventral border of this sucker that a pair of buds are produced. Occasionally three may occur, but, at least in some cases, the presence of a third is accounted for by the fact that the pedicle of a bud has commenced to bud in its turn, so that no less than three generations may be in organic continuity with each other.

Apart from this phenomenon we may say that two buds, on either side of the mid-ventral line, are the rule. This arrangement is comparable to that found in the creeping stolon (gymnocaulus) of *Rhabdopleura*, as already pointed out by Professor Lankester (3).

The pair of buds are rarely at the same stage of development; one is often at the stage with three pairs of plumes, whilst the other is a mere knob.

The bud first appears as a small rounded protuberance of the ventral body-wall, to one side of the mid-ventral line. It consists of a single layer of ectoderm and of mesoderm surrounding a space which is part of the pedicular cœlome.

* *Cf.* also the larval *Phoronis*, as figured by Caldwell. *Quart. Journal Micros. Science*, pl. ii. fig. 3.

This protuberance becomes constricted at the base and then appears as a small knob (fig. 19) which presents no external differentiation.

The internal structure of this bud can be understood from the sections shown in figs. 29–32, selected from a series cut transversely to the long axis.

At the distal extremity (fig. 29) a thick layer of ectoderm, containing a mass of nuclei and ill-defined cell-walls, is lined on the inside by a thin layer of mesoderm, in which are scattered a few nuclei. Protoplasmic threads of mesoderm interlace across the central cœlomic lumen.

In fig. 30 the cœlome is shown to be divided in the middle by a mesentery, running as is seen later, dorso-ventrally. Within this mesentery (fig. 31) is formed a blood-sinus (*m.s.*), which may be followed through the pedicle (fig. 32) into the ventral blood-vessel and mesentery of the pedicle (see fig. 18). At the level of fig. 30, the median blood-sinus has moved dorsalwards and merged into a large dorsal sinus (*sn.s.*) underlying the ectoderm which proves to be the rudiment of the sub-neural sinus.

A sagittal longitudinal section through a similar bud is seen in fig. 36. If it had been exactly median, it would have shown the median sinus (*m.s.*) continuous with the sub-neural sinus.

Further development proceeds by a rapid growth of the apical portion of the bud resulting in the formation of the pre-oral lobe (protomere or buccal shield), one portion of which protrudes dorsally immediately in front of the sub-neural sinus, and grows out ventrally to such an extent that the bud assumes the appearance of being bent upon itself (fig. 19, left-hand bud).

A transverse section through the pre-oral lobe (fig. 33) shows a thickened ventral wall and a cœlomic cavity in continuity with that of the rest of the bud.

A section at the base of the pre-oral lobe (fig. 34) shows the sub-neural sinus and the median mesentery as in fig. 30, but, in addition, the ectoderm is invaginating, forming a diverticulum (*g.*) pushing into the median sinus and gradually downwards between the two walls of the mesentery (fig. 35). Below the level of fig. 35 a cross-section presents the same character as fig. 32.

The commencing invagination of ectoderm shown in figure 34 gives rise, as may be easily demonstrated in later stages, to the whole of the alimentary canal of the adult, the orifice of invagination persisting as the mouth, and the anus being a new formation. The endodermal layer of the parent plays no part whatever in the formation of the bud, the whole future endoderm being derived from the ectoderm of the bud.

It is important to notice that the first invagination takes place opposite the sub-neural sinus and immediately behind the buccal shield, which is therefore pre-oral in position from the outset.

In the next stage the buccal shield has grown out laterally as well as in the dorsal and ventral line (fig. 20), so that the body now appears to consist of three parts, the buccal shield, the body in which is contained the future trunk and collar and the long cylindrical pedicle, which has commenced to elongate.

A median sagittal section through this bud (fig. 37) shows but little advance in differentiation upon the last stage. The endodermic pouch has become larger and reaches further backwards than in earlier stages. At the dorsal lip of the mouth is seen a slight depression, which is the first trace of the sub-neural gland (*sn.gl.*, HARMER'S notochord). In such a case as this, where the whole endoderm is derived from the ectoderm, it is impossible to locate the origin of the sub-neural gland in any special embryonic layer, but it may be noted that its first appearance is much further outside the pharynx and away from the sub-neural sinus than later.

Subsequent growth results in a clearer definition of the three areas already referred to, and in the appearance of two dorsal rounded knobs immediately behind the buccal shield (figs. 21 and 22). Sections indicate some important changes in the internal structure. In a horizontal longitudinal section (fig. 38) it is seen that the ectoderm has become drawn in behind the buccal shield, and where the pedicle meets the body. The rapidly enlarging endodermic sac touches it at these two parts forming two circular lines of contact. In this way the single cœlomic space is nipped off into a single pre-oral cavity (protocœle), a pair of collar-cavities (mesocœles) still separated by the persistent median mesentery and a pair of trunk-cavities (metacœles) which are still continuous through the pedicle with the cœlome of the parent.

A stage very like this was figured by Dr HARMER (1) and led him to compare the bud of *Cephalodiscus* at this stage with the young *Balanoglossus*.

The cavities of the two knobs, which are the first pair of plumes, are in direct continuity with the two collar-cavities. Fig. 39 is a median sagittal section through the bud shown at fig. 22. We may here note that the endodermic sac has increased in size and the sub-neural gland has moved backwards towards the sub-neural sinus. The dorsal posterior lip of the hind-gut has commenced to move forwards along the mid-dorsal line. Between this stage and that shown in fig. 37, the hind apex of the endodermic sac touches the dorsal ectodermic wall, and then from this point the intestine grows forwards as seen in fig. 39, till it eventually opens to the exterior, a little way behind the sub-neural sinus.

In this manner the dorsal reduplication of the gut is brought about, and it seems reasonable to suppose that the spot where the endodermic sac first touches the ectoderm represents the primitive posterior position of the anus, the later forward growth of the intestine indicating a phyletic forward movement of the anus (*cf.* Segmental duct of *Vertebrata*). The endodermic sac never comes in contact with the wall of the bud between the mouth and the pedicle, so that the pedicle could not be regarded as a dorsal process.

The next important external change is the separation of the collar-area from that of the trunk (fig. 23), caused by a bulging outwards of the former. At this stage the trunk is also more clearly separated from the pedicle and their line of junction is now on the under-side. The two plumes have elongated and their distal extremities have swollen out to form the compound eyes.

A series of transverse sections of this stage is shown in figs. 40 to 48.

In fig. 40 is seen the cross-section of the buccal shield. Its ventral wall is greatly thickened, and in the deeper part of the dorsal wall can be discovered a differentiation of nervous tissue. On the left of the middle line is shown a proboscis-pore (*p.p.*) in process of development as an invagination of ectoderm. In fig. 41 it opens into the pre-oral cœlome. Above the buccal shield is seen a cross-section of the plumes. At an earlier stage they are circular, but the surface nearest the buccal shield is here depressed to form a longitudinal groove. In the mid-dorsal line the mesodermal lining of the cœlome is thickened by an accumulation of nuclei with protoplasmic processes; these are situated at the anterior end of the sub-neural sinus. In section 41 the pre-oral cœlome is still cut ventrally, but dorsally the sub-neural sinus (*sn.s.*) is seen, covered by the central nervous mass, and containing the sub-neural gland (*sn.gl.*). The cross-section of this gland is here closely similar to that already described in the adult. In fig. 43, the plumes can be traced into the collar region, and on the right-hand side a second plume is indicated. The collar-cavities lie on either side dorsally, and between them is seen the dorsal blood sinus (*d.s.*), the direct continuation of the sub-neural sinus in fig. 41.

In this series the dorsal sinus can be followed as far as the mouth (fig. 44), but in many specimens is seen in the dorsal mesentery shown in figs. 45 and 46.

From the study of a number of specimens, it is quite clear that the blood system, as in fig. 36, consisting of a median mesenteric sinus and a terminal sub-neural sinus, is invaded by the endodermic sac from the ventral side in such a way that the median sinus is separated by the post-oral part of the sac into a dorsal and ventral blood-sinus, and the pre-oral part (sub-neural gland) effects a similar separation at its base, but at its distal extremity it rests in the sub-neural sinus.

These relationships are easy to follow in such a stage as shown in figures 40 to 48, and although matters are complicated in the adult by further differentiations, there can, to my mind, be no possible interpretation of the parts other than that given here.

I emphasise this because my interpretation of the sub-neural sinus and gland was called in question recently by Dr HARMER (2). He would consider the sub-neural sinus as a structure closed on all sides by mesoderm. If such were the case, his suggested homology with the "proboscis-vesicle" of *Balanoglossus* would have some facts to recommend it, but the structure of the sinus, as depicted as accurately as may be in figures 40 to 44, and later in figures 78 to 84, seems to me to indicate that at these stages (*a*) there is no trace of a closer mesodermic vesicle; (*b*) the sub-neural gland, arising far out, eventually rests freely in the dorsal sinus and its direct continuation, the sub-neural sinus, the dorsal wall of both these sinuses being formed by ectoderm alone. I have not been able in any later stage to detect any structure developed in this region at all comparable to the "proboscis-vesicle."

So far, therefore, as the study of the buds is concerned, their structure appears to corroborate my former interpretation of the parts as re-stated in answer to Dr HARMER's criticism (6).

Returning to figure 43, we see here only the collar-cavities (*c.c.*) and the dorsal sinus, whilst the base of the sub-neural gland is shown to open to the exterior in the mouth region. On the right-hand side the oral groove is recognisable. In fig. 44 both oral grooves are seen, and in fig. 45 the mouth is freely open ventrally. The collar-cavities are divided by a pair of trunk-cavities which lie on either side of the median dorsal mesentery. On the right hand appears an oviduct (*g.d.*) arising as an invagination of the ectoderm. The cross-section of the pharynx here shows a pair of lateral grooves which are the first trace of the pleurochords.

In fig. 46 we can note the paired ectodermal oviducts (*g.d.*) and below them the trunk-cavities. Laterally are the collar-cavities, and ventrally are the two collar-pores (*c.p.*) arising in a precisely similar manner to the proboscis-pores and the "oviducts" as pair ectodermal tubes.

In fig. 47 the stomach (*st.*) presses against the dorsal wall, and the trunk-cavities and ventral mesentery complete the section. In the mid-ventral line can be discerned a commencing nervous differentiation.

Fig. 48 is cut at the commencement of the pedicle, and shows the intestine in contact with the dorsal wall.

In this feature the bud is at an earlier stage than that which is shown in fig. 39, where the intestine has commenced its forward migration.

The mesentery is now incomplete, and the ventral sinus runs down to meet the ventral sinus of the parental pedicle. The mesoderm is partially differentiated in the ventral area into muscular tissue, which will later become the longitudinal muscles. In the ectoderm outside this the ventral nerve-tracts are clearly distinguishable. They are continuous with those of the adult.

On either side of the pedicle is a lateral groove (*l.g.*) which is of constant occurrence, and gives the pedicle at this stage a characteristic cross-section. The basal-fixing extremity of *Phoronis* presents a similar pair of lateral grooves most clearly defined immediately after metamorphosis. In the buds of *Cephalodiscus* they disappear early (see fig. 59), and their meaning is doubtful.

From this series and the above description can be obtained an idea of the structure of a typical bud of *Cephalodiscus*, with one pair of plumes.

The simple archimeric segmentation into five archicœles, the simple and non-reduplicated gut, and the primitive blood-system of sinuses should be emphasised, whilst the identity in structure and origin at this stage of the proboscis-pores, the collar-pores and the "oviducts," is very striking. The proboscis-pores alone appear to open into the cœlome.

Fig. 24 is a dorsal view of a bud intermediate in external character between the bud with one pair of plumes, just described, and that with two pairs, shown in figs. 25 and 26.

The second pair of plumes are seen to arise near the base of the first, as simple horn-like protuberances, whilst the first pair have developed a number of pinnæ from

the apex downwards. The further development of the tentacles will be referred to later, though it may be here stated that the stage to which development of the plumes has reached is only a rough criterion of the amount of internal differentiation. Thus the sections of fig. 24 to be described are of as advanced a stage as those cut through figs. 25 and 26, and seem to represent, as far as is possible, the stage indicated externally by the presence of two pairs of plumes.

Figs. 49 to 59 represent this stage in sections.

In fig. 49 there is shown a considerable advance over fig. 40. The nervous layer is well developed, both dorsally and ventrally, in the buccal shield and in the dorsal ectoderm of the plumes. The sub-neural sinus has thickened mesodermic walls, probably contractile.

Fig. 50 shows the sub-neural gland in cross-section, now of much less proportionate size than in fig. 41.

In fig. 51 the post-oral lamella and collar-cavities are evident, and a cross-section of the pharynx has a very characteristic outline. The median dorsal and median ventral grooves are continuous with the aperture of the sub-neural gland. The paired dorso-lateral grooves are the commencing pleurochords, and the ventro-lateral grooves are the paired oral grooves. These latter run in fig. 52 to the exterior.

The rectum, bounded by the trunk-cavities laterally and the oviducts (*g.d.*) running in lateral mesenteries, are also seen in this figure.

In figs. 53 and 54 the ovaries (*ov.*) developing from the cœlomic wall, and the post-oral lamella should be noted, whilst the latter figure shows, on the left side of the pharynx a pharyngeal pouch, not yet opening to the exterior (*p.c.*). Its formation from an evagination of the pharyngeal wall is in accordance with its structure in the adult.

The pharynx (*ph.*) stomach (*st.*) and rectum (*in.*) are all cut in fig. 55 behind the gut (fig. 58), the dorsal and ventral sinuses merge into one, to again separate in the pedicle, in which the mesentery is interrupted (fig. 59). The longitudinal muscles, traces of which may be found at the extreme front-end of the pharynx (fig. 53), become very prominent, and are of an advanced histological structure posteriorly (fig. 57). They extend round the whole cœlome in the pedicle (fig. 59). The single ventral nerve-tract in this region spreads out further forwards (fig. 56) and can be traced upwards as a pair of lateral cords (fig. 55) and (fig. 54) into the post-oral ring.

The external appearance of the four-plumed stage is indicated in figs. 25 and 26. The buccal shield is of enormous size, greater in proportion than in the adult, and the median curved line of pigment is distinct. In front view (fig. 26) it is saddle-shaped, and in side view (fig. 25) it shows a thickened rim.

This figure shows that the pedicle, at first terminal, and in the direct axis of the body, becomes apparently ventral by the dorsal protrusion of the trunk and its contained structures. Thus the pedicle becomes shifted till it appears, in figs. 27 and 28, and still more in the adult, to be an appendage of the ventral body-wall.

A nearly median longitudinal section through the bud (fig. 26) is illustrated in fig. 60. The buccal shield attains at this stage its greatest proportionate size, and within it we may notice some important differentiations. The sub-neural gland (*sn.gl.*) has moved backwards to reach the dorsal wall, and has now come at its distal end into close relationship with the sub-neural sinus (*sn.s.*). The postero-ventral wall of the latter has become slightly pushed in (at least in the preserved specimen) and the mesodermic wall of the invaginated part becomes thickened by a grouping of mesodermic nuclei and protoplasmic strands. Certain of these cells protrude into the cavity of the sub-neural sinus in a symmetrical radiate manner, the first traces of which may be sometimes recognised in the earlier buds with two plumes (*pr.s.*). The anus now opens as a wide aperture to the exterior, in the same dorsal position as in the adult.

The stage with three pairs of plumes develops from the preceding in quite a regular manner by the formation of a third pair, at first in the same straight line as the other two, and in the ordinary course the bud becomes detached from its parent at this stage. This is effected by a simple construction of the end of the pedicle where it is fused with the parental pedicle. The connecting tissue becomes narrower and narrower, the ectoderm presses in on all sides upon the cœlomic channel, whilst at the same time sections through a late stage of the process indicate a certain amount of histolysis especially in the longitudinal muscles. Specimens can be obtained in which the junction is so narrow that a gentle shake serves to detach the bud from its attachment, and sections of the bud show that the cœlome is closed and that the ectoderm although thin is continuous over the tip of the pedicle. In these cases one might fairly assume that the bud remains in contact with the parent only by virtue of the adhesive power of the pedicle. The free buds at this stage are roughly about $\frac{1}{4}$ the bulk of the adult. No reference up till now has been made to the size of the buds which have been described. The reason for this omission is that the size of the buds corresponding to any particular degree of differentiation varies within wide limits, so that no fast rule can be stated.

The size of the early buds in fig. 19 can be gauged by comparison with the pedicle to which they are attached, and the same applies to the four-plumed stage in fig. 25.

On the other hand the rest of the figures except fig. 28 are drawn to nearly the same scale as fig. 25. Fig. 27 illustrates the appearance of the bud soon after detachment. In this instance the buccal shield and post-oral lamella have been removed to show the arrangement of the plumes, which will be alluded to later. Fig. 28 gives the entire bud in side view, at about the same stage. The buccal shield is still of enormous size and partially over-laps the short contracted pedicle. Sections through such a stage show that except for the plumes and gonads the bud may be regarded as a small-sized adult.

Fig. 61 shows that the pharyngeal clefts are now open to the exterior, and that their walls are assuming a chordoid character, and fig. 62 indicates very clearly the

three ventral nerve-tracts which are still very prominent. In the adult they are greatly reduced. In some respects they remind one of the large ventral nerve-mass in young *Sagitta*.

The muscles are now quite distinct, and in fig. 63, which is through the base of the pedicle, they are seen to extend round dorsally.

Development of the Plumes and Pinnæ.

The plumes arise throughout in pairs. They first make their appearance as a papilla, which elongates to a finger-shaped process (fig. 69), the distal extremity of which becomes slightly swollen (fig. 70), and then bulbous (fig. 71). The epithelium of this bulbous extremity then becomes modified to form the eyes. The cuticle of certain of the epithelial cells becomes thickened (fig. 75), and soon the thickening protrudes into the cavity of the cell as a lens-like body (figs. 76 and 77). Later it is detached from the cuticle and lies freely in the protoplasm. Here it becomes rounded off to form the lens.

Immediately below the eye a pair of small processes arise laterally (fig. 72), and grow rapidly to form the paired pinnæ. The other pinnæ arise as a double row in succession downwards (figs. 73 and 74).

Each plume is at first circular in outline, but at an early stage the ectoderm of one side is pushed in to form a groove (fig. 40), so that the outline in section is horseshoe-shaped.

The order of development of the plumes is peculiar. The first pair arise close to the middle line, with their groove facing towards the dorsal surface of the buccal shield (figs. 40 and 64). The second pair arise outside these (figs. 49 and 65), and are at first in the same relation to the buccal shield.

In the course of development, however, both pairs commence to rotate upon their own axis, and also to move in a posterior direction away from the buccal shield.

Thus, in fig. 49, the plumes have rotated about 90°, or a little more in the case of the first pair. The grooves are not evident at this level, but the nerve tracts are always at the opposite side to the grooves. The third pair of plumes arise outside, and at the base of the second pair, and they behave subsequently in a similar manner to these. Thus the three pairs of plumes come to lie a little way back from the buccal shield, and with their grooves and pinnæ facing away from this organ. Fig. 27 shows this arrangement, and also the rudiment on the left side, of one of the fourth pair.

The three last pairs of plumes are later in development, and differ slightly in their growth from the three first pairs. The fourth arise almost parallel to and nearer the buccal shield than the third, whilst the fifth appear first between the second and the buccal shield, and lastly, the sixth arise just in front of the first pair. These points are illustrated in figs. 66, 67, and 68. These three pairs develop grooves facing the buccal shield, as in the case of the others, but they do not rotate.

Such is the very peculiar way in which the arrangement described in the adult is obtained, and the meaning of their method of origin formed an interesting puzzle. A comparison with the mode of origin of the tentacles in *Actinotrocha* has given a solution which has satisfied me, and may, I hope, appear plausible to others.

In *Actinotrocha* the tentacles, when stretched above the head as in the adult, are arranged, viewed from above, as in fig. 68A. The thick epithelium corresponding to that of the oral grooves is on the inside of each tentacle. They arise in succession on each side on the dorsal surface of the collar region, immediately behind the pre-oral hood. A stage with only six pairs is figured for simplicity. In comparing the arrangement with that of *Cephalodiscus*, we note that whereas, in the former case, the tentacles move round ventrally to surround the mouth and epistome, in the latter this is impossible, because the buccal shield (homologous with the pre-oral hood or epistome) is used as a sucker. In this case, therefore, the plumes, after travelling part of the way towards the ventral surface (fig. 68B), turn backwards and move up again. To put the process more correctly, the actinotrochan-like ancestor of *Cephalodiscus* adopted an adhesive function for the pre-oral hood, immediately upon the assumption of a partially sedentary existence. A general morphological rule for sedentary animals with tentacles is that these organs become directed forwards in front of the head, so that in this case the three last pairs, unable to move forwards in the ventral line, moved forward and dorsally upward to take up the position found in *Cephalodiscus*.

If the ontogeny of the plumes recapitulated the whole of phylogeny, the first three pairs of plumes should migrate in procession from the point of origin of the first (which corresponds exactly with that of the tentacles of *Actinotrocha*) down to nearly the mid-ventral line, and then upwards and backwards (*ab.* fig. 68B). Such an apparently useless migration would not be likely to occur, and we do not find that these plumes migrate even as far as (*cd.* fig. 68B). The migration would be gradually shortened till the present condition is arrived at, in which each plume moves directly from its point of origin to its later position. At the same time, it is evident that the rotation through 180° of each plume results in the three plumes having their oral grooves facing away from the buccal shield, just as would occur in the phyletic migration. On the other hand, the last three pairs, occupying a position corresponding with that of the last three pairs in *Actinotrocha*, arise in an equally direct and simple way, as in this larva, and undergo no rotation.

The Development of the Sub-Neural Gland and the surrounding Organs.

In the early buds, as already stated, the sub-neural gland by a movement inwards comes to lie with its distal end in the sub-neural sinus, but as development proceeds, the cœlomic wall immediately ventral to the tip of the gland becomes thickened and tucked in, invading the sub-neural sinus as a short diverticulum of the pre-oral cœlome.

Such a condition is found in the stage with one pair of plumes, and the series of sections here (figs. 78 to 84) described, lie between figs. 40 and 43. They are drawn by camera under a Zeiss microscope with obj. F. In fig. 78 the mesodermic cells forming a thickening of the anterior wall of the sinus are cut through, whilst the next section (79) passes through the sub-neural sinus itself. In fig. 80 a few mesodermic cells are cut through, apparently loose in the blood-sinus, but in reality they are of the same nature as the cells seen in the next section, but cut obliquely. In fig. 81 is seen the pre-oral sac invaginated from the pre-oral cœlome, and it still opens freely into the latter. It appears to be filled internally with a homogeneous darkly-staining material, which, in later development, thins out to form a uniform layer around the walls of the pre-oral sac as in the adult, and then is indistinguishable from the chondroid tissue.

Attached to the mesodermic wall of the sac are a number of cells (*t.c.*) which have a characteristic form, being somewhat fusiform with the nucleus near the centre, one end free and the other attached. They radiate in all directions except posteriorly, where the head of the sub-neural gland lies. A few can be seen in fig. 82 attached to the posterior edge of the pre-oral sac, which lies slightly over the sub-neural gland. In fig. 81 the dorsal wall of the sinus is seen to consist of ectoderm alone, and the sub-neural sinus can be traced without break into the dorsal sinus in figs. 83 and 84. In fig. 82 it is seen to be limited on the left by the pre-oral cœlomic wall, and, owing to a slight obliquity, on the right by the wall of the collar-cœlome; in the following sections the collar-cœlome is alone seen. In the latter figure may be noticed the sub-neural gland, free in the blood-sinus, which surrounds it on all sides. In the sinus are a few cells, either free blood-corpuscles or cells detached from the radiate cells. Such is the peculiar morphology of these parts, and their functions are not easy to conjecture.

A stage differing little from this is found in fig. 60, in nearly median longitudinal section.

Subsequent development consists in an expansion of the pre-oral sac, which becomes of somewhat irregular outline, in a loss of regular arrangement by the radiate cells, and in a partial closure of the aperture of the pre-oral sac. The posterior wall of the sub-neural sinus grows backwards across the aperture, and comes in contact with the pre-oral cœlome immediately covering the head of the sub-neural gland. Thus, in the adult, a median longitudinal section through this region gives one the impression that the pre-oral sac is closed ventrally and has no communication with the pre-oral cœlome, thus seeming to bear out the statement of Dr HARMER (2). The aperture, however, secondarily closed on the middle line, is still open on each side. Thus, in the adult, the pre-oral sac still communicates with the pre-oral cœlome by a pair of short channels, which are easily demonstrable by longitudinal sections a little to either side of the median plane.

On the other hand, I have entirely failed to find any trace of a communication between the pre-oral sac and the dorsal blood-sinus, such as should exist if Dr HARMER'S

interpretation of this organ as the heart were accepted, and I have every reason to believe that such cannot be demonstrated.

In the discussion upon these points between this author and myself, we have been blamed for complicating the points at issue by each using a different nomenclature, so I may be excused for giving the following table illustrating the two views :—

1. "Sub-neural gland," = Harmer's "notochord."
2. "Sub-neural sinus," = ,, "proboscis-vesicle."
3. "Pre-oral sac," * = ,, "heart."

HARMER accepts the presence of my "dorsal blood-sinus," and believes that it will be proved to communicate with (3) and not with (2); and, on the other hand, he denies the communication of (3) with the pre-oral cœlome.

Whilst denying my interpretation of the "paired notochords," HARMER corroborates my description of them, and, apparently, of the other leading anatomical points in my work not mentioned above.

It appears to me that the disputed points will have to be settled by a renewed investigation of the development of *Balanoglossus*. Should the proboscis-vesicle of *Balanoglossus* be indubitably proved to arise from mesenchyme or from the proboscis-cœlome, then it is possible that *the pre-oral sac above described may represent, in* Cephalodiscus, *the more primitive condition of the homologous structure*, whereas, should the proboscis-vesicle prove to be developed from ectoderm, it would probably have, as formerly suggested (6), to be regarded as the distal extremity of the sub-neural gland.

In any case, a rhythmically pulsating organ with an internal cavity, completely closed, is so anomalous a structure that more knowledge of its structure and development is very essential. It seems possible that the proboscis-vesicle of *Balanoglossus* may yet be proved to communicate with the pre-oral cœlome by minute lateral apertures in a similar manner to the pre-oral sac in *Cephalodiscus*. The radiating cells above described would then represent the proboscis gland (*cf.* larval nephridia of *Annelida*), and the resemblance of proboscis-vesicle and pre-oral sac would be very close, both functioning as contractile vesicles for the expulsion of excretory fluids into the pre-oral cœlome.

Summary.

The following is a brief summary of the chief features dealt with in the foregoing :—

(1) The pharynx has special adaptations for the separation of food and water-currents, amongst which may be reckoned hyper- and hypo-pharyngeal

* Prior to this, the organ here referred to has been unnamed ; the above name may be suggested for adoption in that it is descriptive without in any way implying a special homology.

grooves, the peri-pharyngeal groove, sub-neural gland, the pharyngeal clefts, and the pleurochords.

(2) The notochord of the *Chordata* may be primarily derived from this source as a channel for cloacal water from the point of separation to the points of exhalation, *i.e.*, pharyngeal clefts.

(3) The gonads are suspended in lateral mesenteries, and surrounded by a blood-sinus, communicating with the dorsal sinus.

(4) The pedicle or ventral sucker has a ventral nerve cord and two ventro-lateral cords, a dorsal and a ventral blood-sinus, and complete inner layer of longitudinal muscles.

(5) The buds arise usually in pairs ventrally. The ectoderm and mesoderm of the bud are formed directly from those of the pedicle. The endoderm is formed by invagination of the ectoderm.

(6) The pharyngeal clefts arise as endodermal diverticula which break through the ectoderm to the exterior. From their structure and relationships to the pleurochords in the adult, they would seem to be simply the oral termination of the latter. The two primitive chordate characters of notochord and pharyngeal cleft would thus appear to have arisen from a common rudiment resembling the pleurochord of *Cephalodiscus*. The anal end of the intestine is at first in contact with the ectoderm posteriorly and just above the pedicle, but later moves up to the oral end and then opens to the exterior.

(7) The blood system arises as a system of sinuses between the cœlomic cavities and the ectoderm, continuous from the first with those of the adult.

(8) The sub-neural gland arises far out and moves secondarily into connection with the sub-neural sinus.

(9) The first three pairs of plumes arise near their final position, into which they move gradually with a rotation through 180°, the last three, forming the inner row of the adult, arise *in situ*.

(10) The oviducts, proboscis-pores, and collar-pores all arise in a similar manner, as paired ectodermal tubes.

(11) The gonads arise as proliferations of the mesoderm lining the trunk-cœlome.

(12) The adult characters are assumed when three to four pairs of plumes are present; and separation from the parent is then effected by constriction.

(13) The lens of the branchial eyes is formed by cuticular hypertrophy and later detachment.

(14) Sexual development commences in the egg-capsule whilst attached to the inner wall of the cœnœcium, and results in the formation of a larva segmented into two parts by an annular constriction.

(15) The pre-oral sac arises as a diverticular invagination of the posterior wall of the pre-oral cœlome, and its bounding cells become modified into spindle-shaped excretory cells free in the sub-neural sinus.

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(2) „ *Zoologischen Anzeiger*, No. 543, 1897.
(3) Lankester, E. R., *Quart. Journal Micros. Science*, 1884.
(4) M'Intosh, W. C., *Challenger* Report on "*Cephalodiscus.*"
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(6) „ *Zoologischen Anzeiger*, No. 545, 1897.
(7) „ *Proc. Royal Soc. Edin.*, March 1896.

LIST OF PLATES.

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Figs. 78–84. Series of sections through the "sub-neural" region of bud with one pair of plumes. (Zeiss. F.)

PLATE V.

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Figs. 90–99. Series of transverse sections across the pharynx of *Cephalodiscus* (camera).

Fig. 100. Diagrammatic drawing of the pharynx and connected structures of *Cephalodiscus*.

LIST OF ABBREVIATIONS.

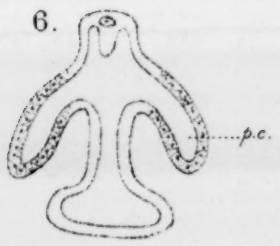
1.2.3. (in Plates I. and V.) = oral grooves.
1.2.3. (in Plate IV.) = plumes in order of development.
c.c. = collar-cavities.
c.p. = collar-pores.
d.g. = dorsal groove.
d.s. = dorsal sinus.
g. = gut diverticulum.
gd. = gonaduct.
in. = intestine.
l.b.s. = lateral blood-sinus.
l.g. = lateral grooves.
l.m. = longitudinal muscles.
m.s. = median sinus.
m. = mouth.
nc. = notochord (pleurochord).
ov. = ovary.
p.c. = pharyngeal cleft.
ph. = pharynx.
p.l.l. = post-oral lamella.
pn. = pinna.
p.p. = proboscis pore.
pr.cœ. = pre-oral cœlome.
pr.s. = pre-oral sac.
r. = rectum.
sn.gl. = sub-neural gland.
sn.s. = sub-neural sinus.
st. = stomach.
t.c. = trunk-cavity.
v.g. = ventral groove.
v.n. = ventral nerve.
v.l.n. = ventro-lateral nerve.
v.s. = ventral sinus.

Dr A. T. Masterman on Cephalodiscus dodecalophus. McI. — Plate I.

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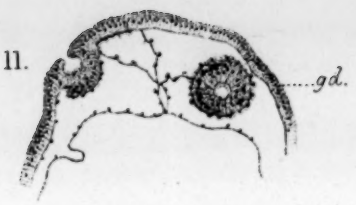
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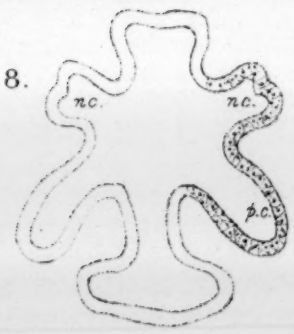
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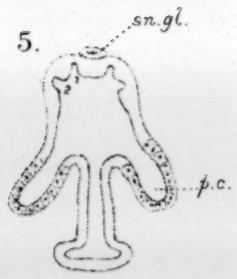
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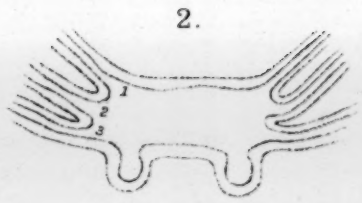
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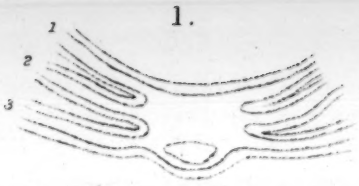
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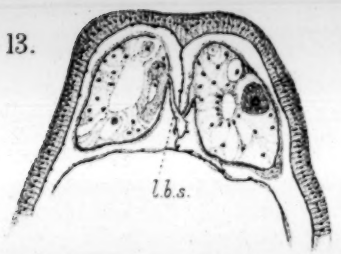
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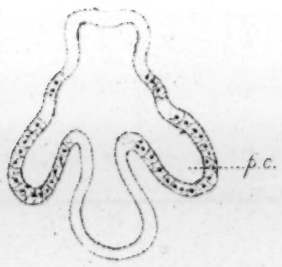
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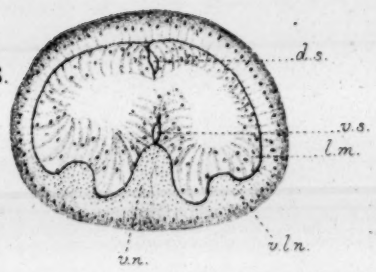
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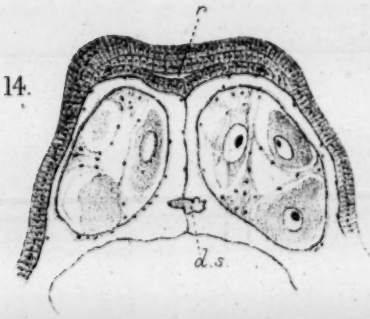
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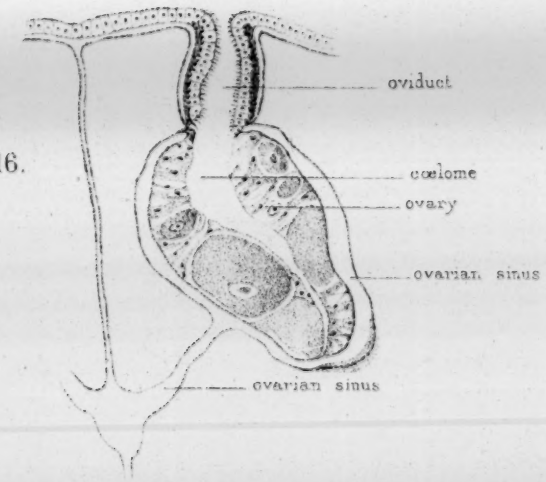
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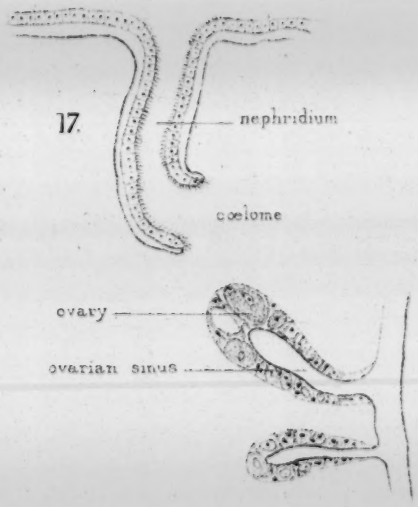
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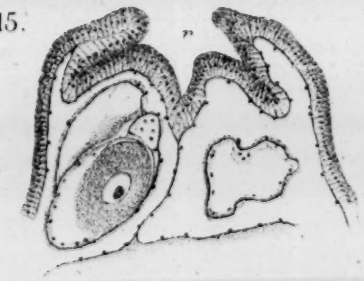
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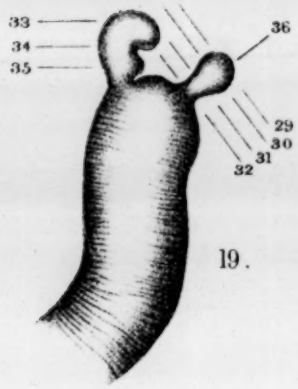


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Dr A. T. Masterman on Cephalodiscus dodecalophus. McI. — Plate II.



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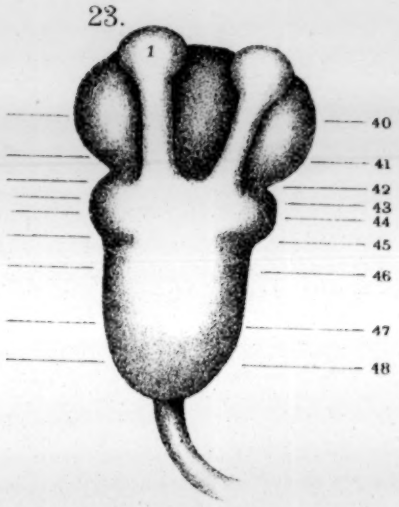


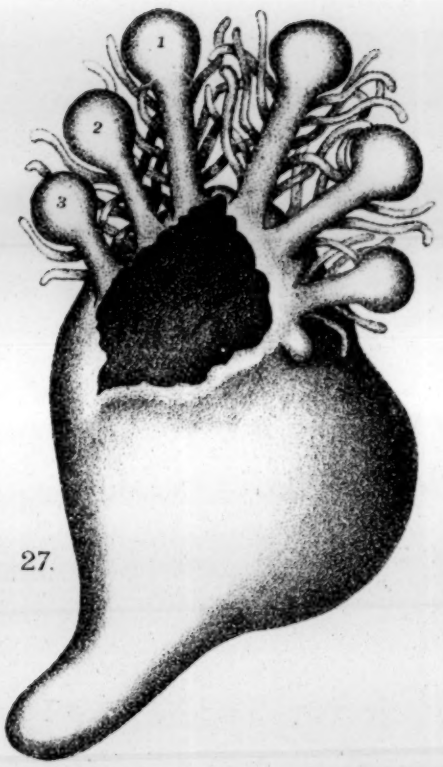
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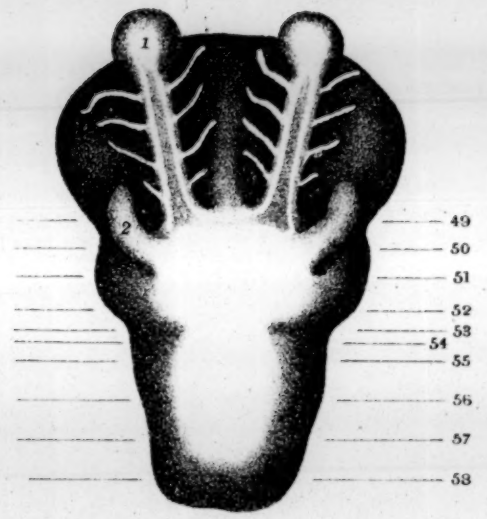
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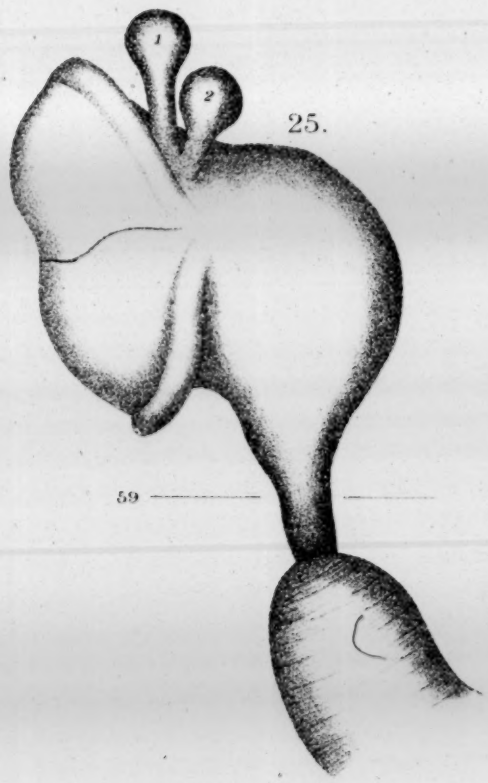




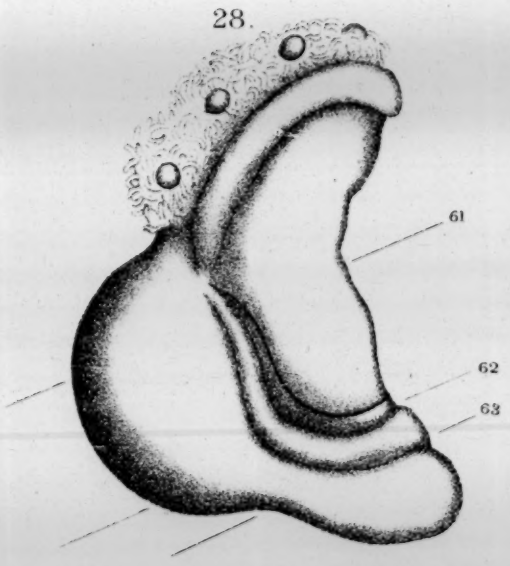
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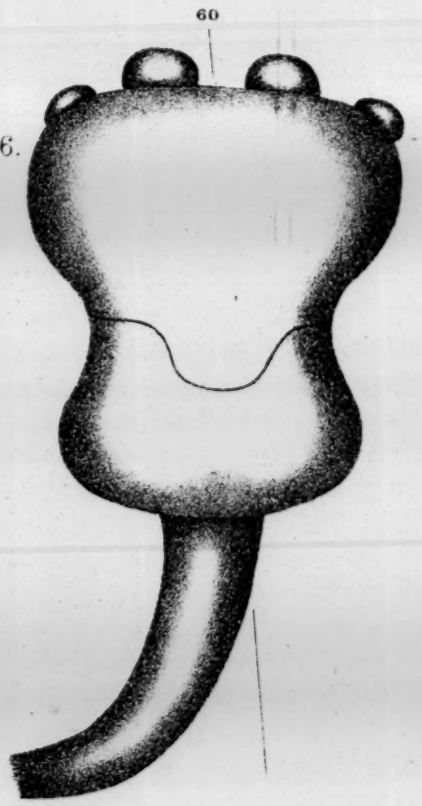




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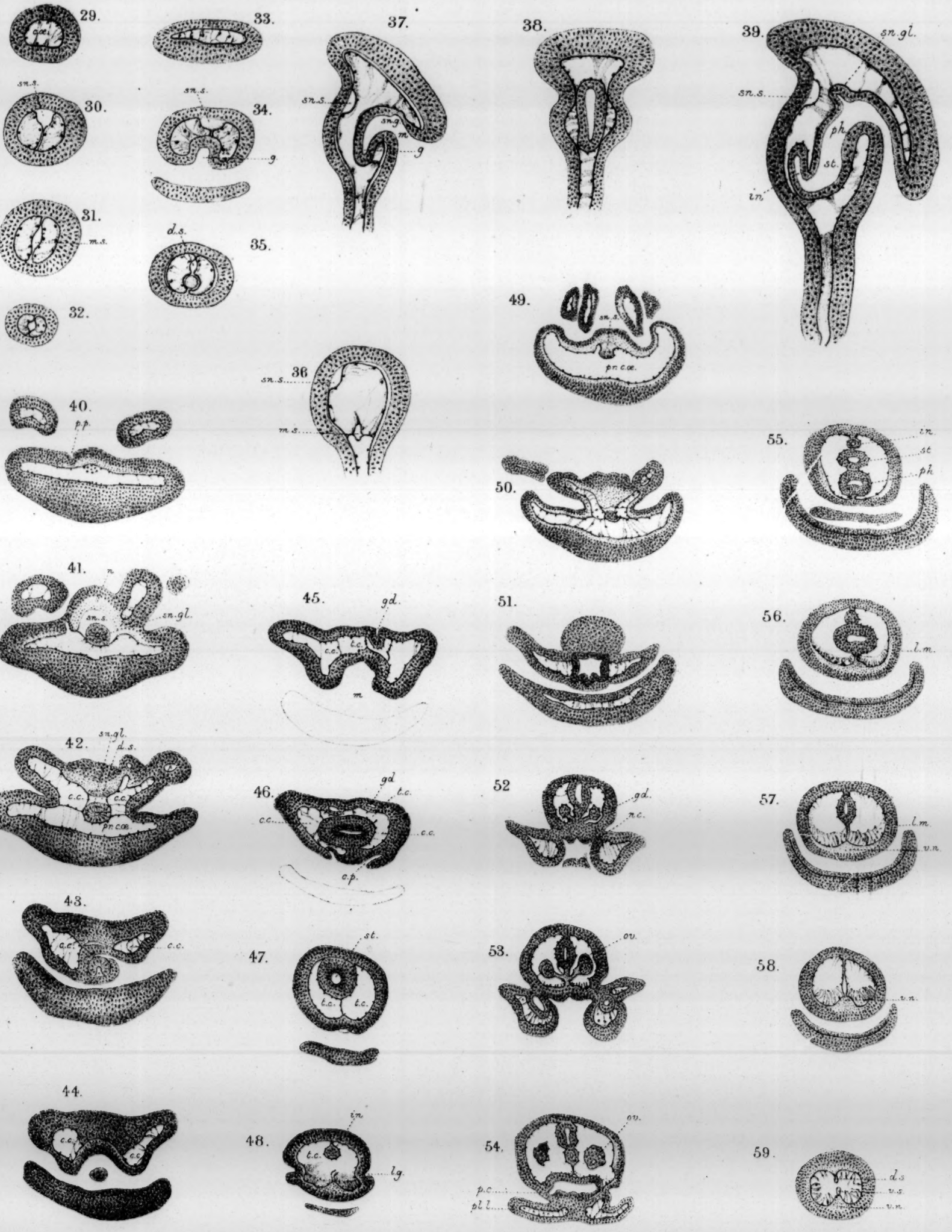


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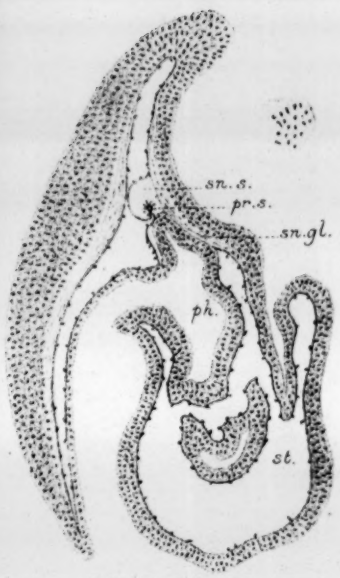


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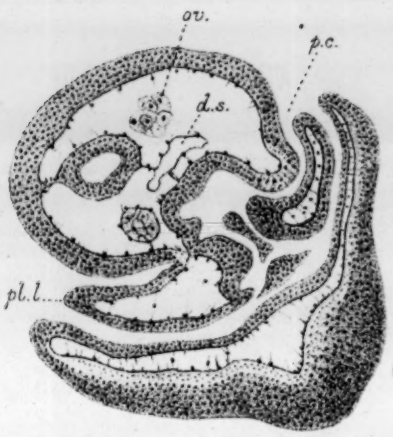
Dr A. T. Masterman on Cephalodiscus dodecalophus. McI. — Plate III.



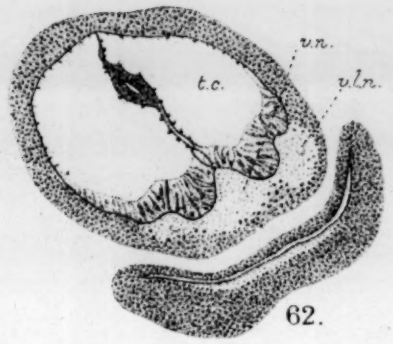
Dr A. T. Masterman on Cephalodiscus dodecalophus. McI. — Plate IV.



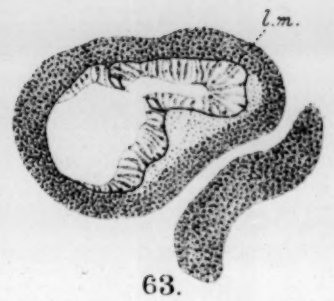
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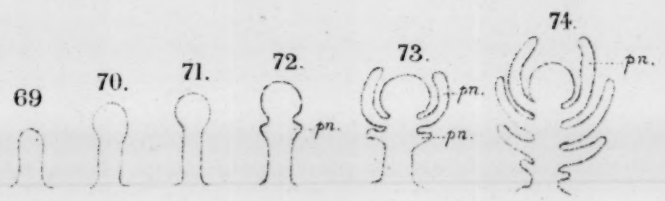
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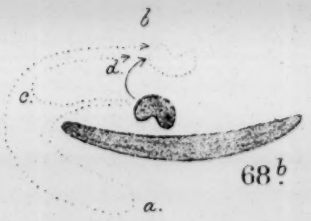
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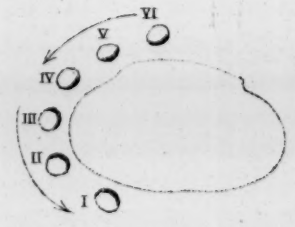
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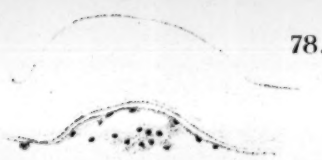
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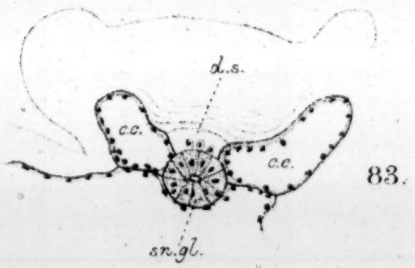


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68a.

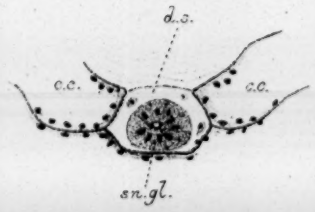

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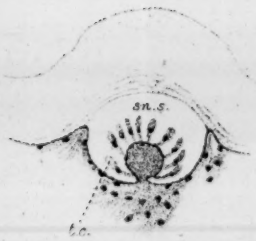
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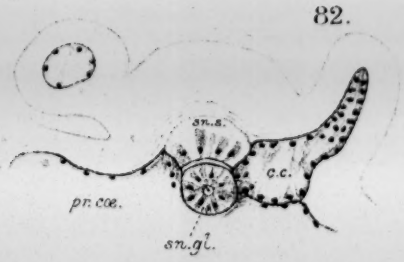
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McFarlane & Erskine, Lith. Edin^r

Dr A. T. Masterman on Cephalodiscus dodecalophus. McI. — Plate V.



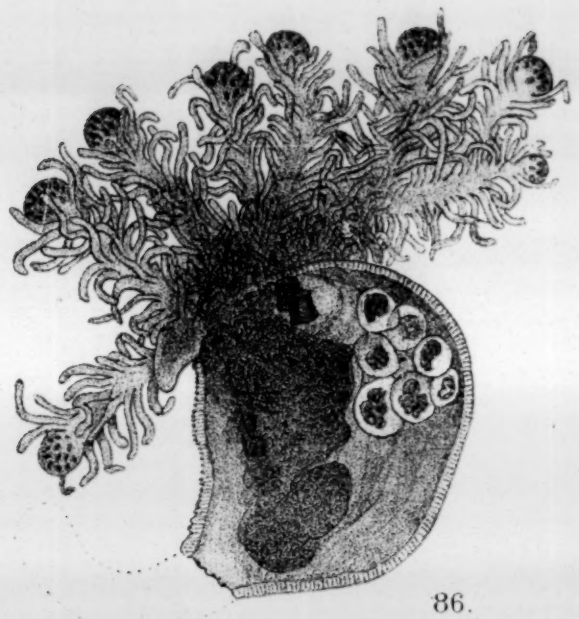
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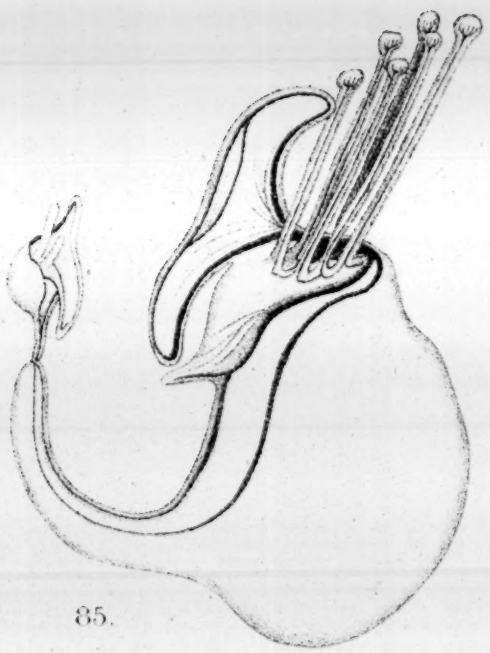
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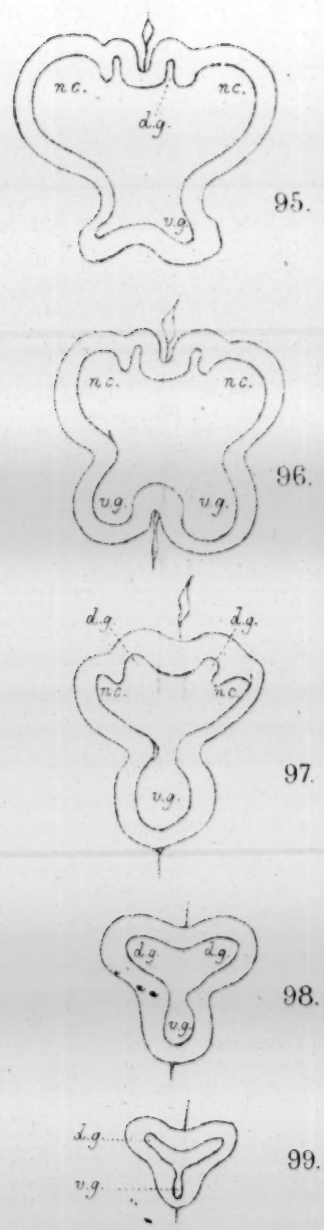


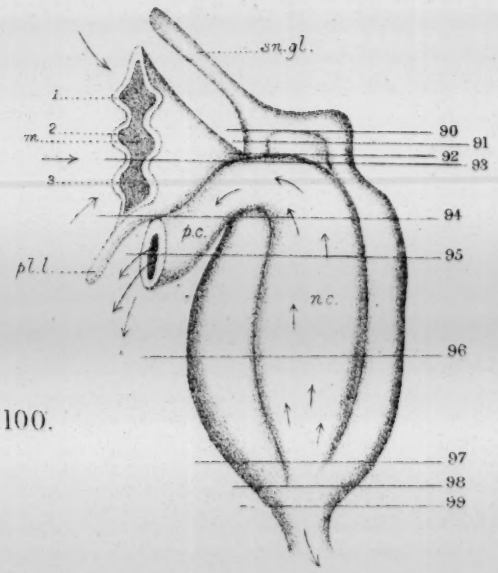
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XVIII.—*On Steam and Brines.* By J. Y. Buchanan, F.R.S.

(Read 20th June 1898.)

The immediate purpose of the present research was the investigation of the temperature at different pressures of *boiling mixtures* of steam and salts, analogous to the well-known *freezing* mixtures of ice and salt.

When steam is blown through common salt in coarse powder, it condenses to water, which dissolves some of the salt, and the resulting brine is kept boiling by the arrival of more steam. The temperature of this boiling mixture is quite constant so long as there is an abundant supply both of steam and of salt, and as the atmospheric pressure does not change, it is about 8·5° C. above the temperature of boiling water when the barometric pressure is the normal of 760 mm. When the barometric pressure is 560 mm. this excess has fallen to 8·0° C. Most other salts behave in a similar way.

The investigation was not confined to the determination of the temperature of the boiling mixture of steam, brine, and salt; the experiments were continued after all the salt was dissolved, and the passage of the steam was interrupted from time to time, and the weight of steam condensed and the temperature of the liquid observed, so as to give the concentration and the boiling temperature of solutions of the salt of different strengths. This part of the work, namely, passing steam through nonsaturated solutions, was in continuation of work begun immediately after the publication of my paper, "On Ice and Brines," in the *Proceedings of the Royal Society of Edinburgh*, 1887, vol. xiv. page 129. It is well known to all those who have tried it that it is impossible to produce pure ice by freezing a saline solution, and the purpose of the research reported in the above paper was to ascertain whether the salt, from which the ice could not be entirely freed, was an essential part of the ice, or only an inseparable adherent to it. The latter was shown to be the case, and the ice, or crystalline body which freezes out of a saline solution, to be really ice and nothing else, by showing the identity of the temperature at which snow or pure pounded ice of independent source melts in a saline solution, and that at which ice forms on freezing a saline solution of the same nature and concentration.

It was natural to pass from the freezing to the boiling temperature of solutions, and to use the temperature at which steam condenses in the solutions as a measure of their boiling temperature in the same way as the melting temperature of ice had been used for indicating the freezing temperature. It had also the advantage that no preliminary investigation was necessary. There was no doubt that the steam produced by a boiling saline solution was pure steam, there was no doubt about the temperature of the boiling solution from which the steam proceeded, and there was equally little doubt that the

steam so proceeding had the same temperature as the boiling solution, because when steam is blown into such a solution it is condensed until the solution has been heated to its boiling point. If the solution can condense outside steam, it necessarily must be able to condense its own steam, for the two substances are identical. If the bulb of a thermometer is immersed in brine boiling over a flame, it is irregular in its readings. The irregularities are due to superheating of the liquid consequent on the adhesion of the solution to the glass or metal. The ebullition is never quite regular, and the reading of the thermometer varies, showing a fall of temperature when steam is coming off freely, and a rise when there is intermission. It is very difficult in this way to obtain exact observations of the boiling temperature of a solution, or indeed of pure water.

On the other hand, it is very easy to get results of any desired degree of exactitude by using an abundant current of saturated steam to boil the water or brine. Overheating is impossible, and bumping equally so. A thermometer of any degree of delicacy may be used; there is never any room for doubt that its temperature is, to the minutest fraction of a degree, the same as that of the boiling liquid, and the value of the observation thus depends only on the trustworthiness of the instrument. In the paper on "Ice and Brines" above referred to, it was pointed out that the melting point of ice in brines of determined nature and strength could be used as fixed points on the thermometric scale, in the same way as the melting temperature of ice in pure water is habitually used. Similarly the condensing temperature of steam in saline solutions forms a ready means of fixing exactly certain points above the ordinary boiling point of water. Also the minimum temperature, or cryohydric point, of freezing mixtures is very useful, many of them being as well defined as the melting point of ice in water.

Our boiling mixtures occupy a similar position with respect to the boiling point of water that the freezing mixtures do to its freezing point. In freezing mixtures the dry salt is mixed with pounded ice or snow; if the mixture is properly made, the temperature falls at once to the true minimum, and remains quite steady for a great length of time. In boiling mixtures the dry salt is placed in a U-tube of special dimensions, to be described presently, and steam is passed into it by one leg, while the other leg carries the thermometer, and the surplus steam escapes through a side tube. The supply of steam must be abundant, while the exit tube for the steam must be sufficiently wide to make it impossible for the steam, after it has passed through the mixture of salt and brine, to have a pressure above that of the atmosphere into which it exhausts. From the time when enough steam has condensed to form a more or less liquid magma through which the steam bubbles, the mass is kept thoroughly well mixed, and the thermometer keeps the temperature with absolute steadiness until so much steam has condensed and so little solid salt remains that it cannot be assumed that every particle of steam condensed can immediately find a particle of salt to dissolve. Then the temperature begins to fall in the same way as that of a freezing mixture begins to rise. Operating in this way it is possible to obtain definite and perfectly constant temperatures with

most of the ordinary soluble salts of the laboratory. They are, however, not all equally good, and there are some that are of no use for boiling mixtures.

The effect produced when steam meets a salt depends on the properties of the salt, especially on its solubility in boiling water and on the thermal effect of its solution. Generally speaking, the greater the solubility of the salt the greater is the elevation of the condensing point of steam which it produces. At ordinary temperatures salts commonly dissolve in water with absorption of heat, which tends to increase the condensation of steam. The absorption of heat may be so great that the brine produced by the salt dissolving in the condensed steam is unable to rise to its boiling point until all the solid salt has disappeared.

Amongst anhydrous salts nitrate of ammonium is a good example of this. An experiment was made with 60 grammes of the salt. In $2\frac{1}{4}$ minutes after the steam reached the salt it was all dissolved, and the temperature barely rose to that of boiling water. Twelve grammes of steam had been condensed.

Of salts containing water of crystallisation, acetate of soda is a good example amongst those which dissolve with great absorption of heat. Sixty-two grammes of this salt treated in the same way as the nitrate of ammonium were completely dissolved in $2\frac{1}{2}$ minutes after the steam reached the salt, and the temperature had only reached 60° C. These and similar salts are not suitable for boiling mixtures.

The following are one or two examples of the solubilities observed at the boiling point of the saturated solution for one-fifth gramme in molecule of each salt :—

Salt Used.		NaCl.	KCl.	$BaCl_2$.	$(NH_4)_2SO_4$.
Temperature of Condensation of Steam on	water, .	100·44° C.	100·44°	99·95°	99·40°
	salt, . .	108·98°	108·94°	104·46°	107·03°
Weight of Steam Condensed for complete solution, ... grms.,		29·9	25·9	64·0	25

The least soluble of these salts is chloride of barium, requiring 64 grms. of condensed steam for solution, and the most soluble is sulphate of ammonium, which requires only 25 grms.

Apparatus and Method of Experimenting.—The apparatus, fig. 1, consists of the lamp, A, the steam generator or boiler, B, the U-tube or receiver, D, and the connecting tube, G. In the laboratory a gas lamp was used, except when it was wished to check the results by the fuel consumed, when a spirit lamp was used, and it was weighed before and after the experiment. In the experiments at high levels, where gas was not available, spirit was used. The lamp employed was one of a French pattern, and forming a part of the *Réchand à double flamme forcée* of smallest size, which has a large sale for domestic purposes. It is the most efficient pattern of spirit lamp with which I am acquainted, and as it is especially constructed for use in travelling, it was very suitable for my high-level work. It holds about 250 c.c. of spirit.

The steam generator or boiler, B, used at high levels, was a flask made of spun copper and of 500 c.c. capacity. A suitable charge is 300 c.c. of water. With such a charge, and heated by the French lamp, the water boiled in six minutes, with a consumption of 12 grammes of spirit. While keeping steam at the rate suitable for the experiment, the lamp consumed 21 grammes of spirit in fifteen minutes, and evaporated

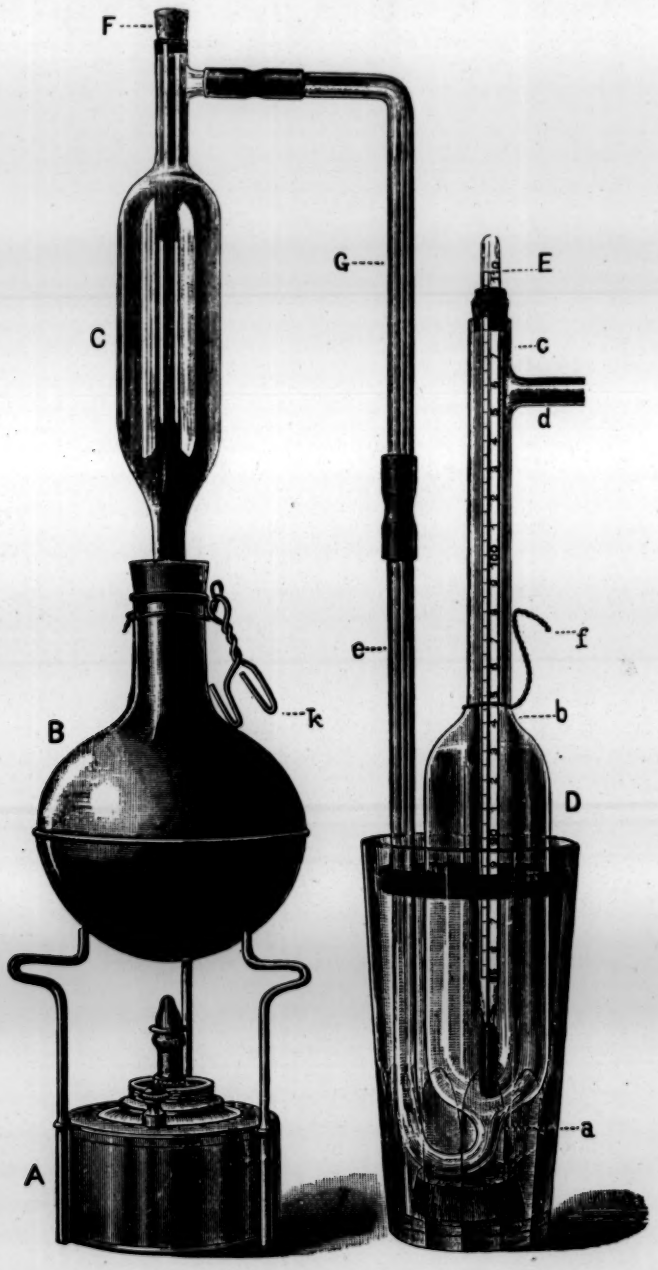


Fig. 1.

92 grammes of water. For experiments in the laboratory I use a large copper flask of 2 litres capacity. This is a very convenient laboratory vessel. They are manufactured to replace the glass flasks of Napier's coffee machines for use in restaurants, and they can be had in larger sizes. A copper flask can always be obtained at short notice

from a plumber, by getting him to fit a neck to one of the copper balls which are used as controlling floats for cistern taps. These are to be had of all sizes up to 10 inches diameter, and are very cheap.

The receiver, C, has the appearance and shape shown in the fig. The actual dimensions are variable, according to the quantity of salt on which it is proposed to operate, and according to the length of the thermometer. The working part of the thermometer must be entirely within the receiver. The uncertainty caused by the exposure of any part of the stem occupied by mercury stultifies the use of very delicate instruments. If the whole range of the thermometer is to be utilised, then the length of the receiver must be such as to take the whole thermometer. The two thermometers which I used in all my experiments differed slightly in length, and it was convenient to have a receiver made for each. Of course the longer thermometer could be used with the shorter receiver, so long as the temperature to be observed was not too high. One receiver which I have used much has the following dimensions:—Total length of the body of the tube, from the entrance, *a*, of the steam tube at the bottom to the top, *c*, where the thermometer, F, is retained by a perforated cork, is 30 cm.; the length, *a b*, of the body for the reception of the salts and brine is 16 cm., and its diameter 42 mm. The length of the neck, *b c*, is 14 cm. and diameter 13 mm. The steam exit tube, *d*, has a diameter of 7 mm., and the entry or connecting tube, *e*, has the same or a slightly less diameter. The entry tube is bent up parallel to the main body of the instrument, and is connected with the boiler by a tube, G, as shown in the fig. The steam tube, C, on the boiler is important. The straight portion which enters the boiler should have a diameter of 8 to 9 mm. Its upper part, which is fitted with a cork, should have a diameter of not more than 7 or 8 mm. Steam is kept constantly in the boiler, and connection with the receiver is made or broken instantly by inserting or removing the cork. The most convenient support for the receiver, whether it be used in or out of the laboratory, is an ordinary tumbler or drinking glass. The tube rests on a piece of cork, grooved to take the bend of the tube, and it is steadied by a ring made of a piece of india-rubber tubing. This form of support has many advantages. In the first place, the apparatus has great stability; then the glass is transparent, and it is essential to be able to see the boiling mixture during the whole course of the experiment; also, while being transparent, the glass protects the receiver from excessive loss of heat.

In the experiments to be reported, the principal thing to be observed is the difference between the temperature of pure saturated steam and the temperature produced by the condensation of this steam in the mixture of salt and brine. It is, therefore, of equal importance to observe accurately the temperature of pure saturated steam as to observe that of the boiling mixture. The same apparatus suffices for both purposes. Before charging, and being clean and dry, the receiver is connected with the boiler, and steam blown through it. Some of it condenses and collects at the bottom of the receiver, forming a pool of distilled water boiled by steam, the steam produced by which is perfect for the purpose. As the division marking 100° C. is usually some way down

the stem, the thermometer is preferably pulled up, so that the bulb is entirely in the steam. The steam so produced in an apparatus of the proportions described, must be truly saturated steam of the tension equal to the actual barometric pressure, and must, therefore, have exactly the temperature which corresponds to this pressure. This is the temperature of steam condensing on pure water.

The supply of steam is abundant, and the latent heat of steam is very great; the thermometer, therefore, must take, in a very short time, exactly the temperature of the steam with which it is completely surrounded. The thermometer may be constructed so that a millimetre on the stem corresponds to a hundredth or a thousandth of a degree. Its indication will be perfectly steady, provided that the conditions remain unchanged. The slightest change in the barometric pressure makes itself at once apparent, and at all times, especially during unsettled weather, the temperature of saturated steam must be observed at frequent intervals.

It is convenient to have a separate apparatus for this purpose. If we imagine the receiver, C, with the entry tube at the bottom straight instead of bent, so that it can take the place of the T tube in the boiler, we have a perfect apparatus for determining the temperature of saturated steam and consequently also for fixing the point corresponding to 100° C. on the scale of the thermometer. It is assumed that the steam tube is large enough to take the whole working part of the thermometer. It is unsuitable in itself or to the steam generator used if the steam makes its exit with an audible sound. Tenths of a Centigrade degree are then uncertain. Yet it is essential that there should be an abundant flow of steam through the tube; therefore, the exit tube must be sufficiently wide to allow the steam to issue in a stream of good volume, and must not be so wide as to incur any risk of regurgitation of air. Again, the entry tube for the steam must not be too narrow. It is not essential, but it is very convenient, that it should be so wide that the steam condensed on the walls and flowing back into the boiler should continue to flow down the sides of the entry tube and not collect at the bottom of the wide part of the tube. When the steam proceeds along a short straight passage from the boiler to the steam tube, it throws about any water in a violent and inconvenient way. In a tube which I use for this purpose the entrance tube has a diameter of 9 mm. and the exit tube 10 mm. The entry tube is thus wide enough to permit the condensed steam to flow back along its sides; at the same time, it is smaller than the exit tube, so that, apart from the continual condensation of a portion of the steam, there is no danger of the tube receiving more steam than it can freely get rid of.

That an apparatus such as that here described does, in fact, exclude the possibility of the steam supplied having a tension which differs at all from the pressure of the atmosphere with which it exhausts, will be evident from the following experiments. The small copper flask, the spirit lamp, and the steam tubes above described were used. The thermometer was divided into fiftieths of a degree Centigrade, the length of one degree being 35 mm. The atmospheric pressure happened to be pretty high, and

when the thermometer had taken the temperature of the steam the top of the mercury was exactly even with the centre of the line on the scale marking 100·18° C. It occupied this position when steam was being generated at its highest rate of 8·4 grammes per minute. The flame of the lamp was reduced by degrees until it reached its lowest point, when steam was being generated at the rate of 2·4 grammes per minute. It was then issuing continuously, that is, there was no regurgitation, but it partook more of the nature of an exhalation than of a stream; yet the mercury remained exactly on the centre of the line of 100·18°, and it was only when the lamp had been reduced to its very lowest that it could be said to have fallen to the lower edge of the line. The temperature then indicated by the thermometer was not lower than 180·179° C. When the reading of such a thermometer remains unaltered while the supply of steam is varied in the proportion of nearly four to one, the efficiency of the steam tube may be said to be perfect.

A further condition affecting the usefulness of these tubes is that they and the thermometer shall be perfectly clean. The thermometer is the most liable to contamination, and I generally found it convenient to wash it with soap and water before every experiment. The steam which condenses on the thermometer and on the inside of the tube should do so in a film and not as a dew, and it does so if the surfaces are perfectly clean. The inside of the tube is more difficult to deal with than the outside of the thermometer. On the other hand, when once cleaned it remains clean much longer. Soap is used here also, and the best way of using it is to smear the inside of the upper part of the tube with soap, preferably soft soap. Steam is raised, and so soon as it reaches the soap it condenses and forms a uniform film of solution which drains down back into the boiler and by continuing to boil the steam condensing washes the inside walls quite clean from everything, and for a considerable time afterwards there is no trouble about the steam condensing in the tube as dew. If the boiler has been charged for this operation with distilled water, there is the disadvantage that it immediately primes, and the steam, instead of washing down the sides of the tube, continues to blow soap bubbles at its upper end, without washing the tube. I always use ordinary tap water, which supplies equally pure steam with distilled water, but it has this advantage, that when the soap solution drains back into it, being in comparatively small quantity, it is immediately precipitated by the earthy ingredients of the water, which continues to supply pure steam without priming.

Attention to these small matters is all-important, not only in order to secure accuracy but also comfort in experimenting.

Thermometers.—Two thermometers were used for these experiments, both made especially for me by Mr Hicks, of Hatton Gardens. One of them, A, was intended for use at ordinary levels; the scale was in Fahrenheit's degrees, and ranged from 210° F. to 240° F. Each degree was divided into tenths, and had a length of 7·2 mm. The length of a Centigrade degree in this thermometer would thus have been 13 mm., and

in order that the single divisions should not be too far apart it would have had to be divided into twentieths of a degree.

The other thermometer, B, was especially made with a view to experimenting on chloride of sodium at all available heights above the sea. It was graduated into Centigrade degrees and tenths, the length of one degree being 10 mm., and the range was from 85° C. to 110° C.

By the kindness of Dr Bilwiller, director of the Central Meteorological Office of Switzerland, I was able to compare the indications of this thermometer in saturated steam at different heights with the temperatures which it ought to have shown, on the basis of the atmospheric pressure as given by the barometers of the central bureau at different stations. For higher temperatures it was verified in terms of the standard barometer of the Scottish Meteorological Society in Edinburgh, which was obligingly put at my disposal by Dr Buchan. The following are the readings in the order of height :—

Locality.	Julier.	Sils.	Zurich.	Edinburgh.
Height above sea, ... metres,	2244			
Barometer, at 0° C., . . . mm.,	581·84	616·4	720·3	764·18
Observed temperatures, ...	92·86° C.	94·38°	98·62°	100·23°
Calculated do., ...	92·69°	94·24°	98·51°	100·16°
Correction, ...	−0·17	−0·14	−0·11	−0·07

Thermometer B was compared at Kew, giving corrections amounting in the extreme to 0·2° F. All the observed temperatures have been corrected accordingly.

The General Order of the Experiment.—The temperature of saturated steam was determined in the straight steam tube. The U-shaped receiver being clean and dry, was weighed. The portion of salt, usually one-fifth of a gramme-molecule, was weighed out carefully into the receiver, which was then again weighed for the purpose of afterwards arriving at the weight of the condensed steam. The weight of the receiver, both empty and charged, includes that of the thermometer and its attachment. When steam is issuing from the boiler at the top of the T tube, the receiver is connected with it, as shown. The top of the T tube is now closed with the cork, and the passage of steam through the salt begins. The time is noted when the steam reaches the salt, and this is the beginning of the experiment, and the time is logged as a part of every entry in the note-book. It is particularly noted when the mass in the receiver forms a liquid magma through which the steam bubbles, when the thermometer attains its maximum, when it begins to be unsteady, and when steam is shut off. Before reaching the maximum the temperature is noted every half-minute, afterwards every minute, and at the end every half or quarter minute. When it is judged by the fall of temperature and the quantity of salt undissolved that these two small quantities compensate each other, the cork is

removed from the T tube and the operation is interrupted. The receiver is then quickly disconnected from the india-rubber tube, and suspended from the scale, which is within arm's reach from the working bench, and the weight ascertained to the nearest decigramme. The receiver is then immediately reconnected with the boiler, the cork inserted, and boiling recommenced. Steam is passed until the temperature has fallen to the first attainable whole number of degrees above the temperature of saturated steam, when it is interrupted, the weight observed and the receiver reconnected, to be again weighed when the next whole degree is reached, and so on until so much water has collected in the receiver that the steam can no longer be passed through it at a suitable rate without risk of throwing out some of its contents. The temperature of saturated steam is now again determined with the thermometer used in the experiments. This has been determined several times during the experiment in another apparatus and with another thermometer. This enables the effect of any change in the barometric pressure to be spread correctly over the time occupied by the experiment. This series of observations gives the concentration of solutions whose boiling points are higher than that of pure water by certain definite amounts. The small uncertainty which attaches to the determination of the concentration of the boiling saturated solution does not affect that of the less concentrated solution. When the saturated solution has been weighed and reconnected with the steam generator, it often happens that, however expeditiously the operation may be performed, some of the salt has crystallised out, and this generally requires an extra amount of heat, or steam condensed, to redissolve it. Then the boiling temperature of the solution when saturated is lowered very much by a small dilution, an effect which diminishes rapidly with increasing dilution.

As result of the series we have the temperature of the saturated boiling solution and approximately its concentration, also the boiling temperature and exact concentration of a series of more dilute solutions. When the boiling tube has been emptied and washed, steam is blown through it until the whole tube is heated up to the temperature of the steam; it is then quickly disconnected, the water ejected from it, and air blown through it from the lungs, which in a few seconds dries the inside of the receiver completely. This is the easiest way to dry the inside of all complicated glass apparatus. The glass of the apparatus is always sufficiently massive that when it has been heated to 100° C. it has more than sufficient immediately available heat to evaporate all the water that will adhere to its surface, and still not fall to such a temperature as to condense moisture from the air of the lungs.

The receiver is immediately ready for another experiment. As above described, each experiment must be expected to take from an hour and a half to two hours.

Steam Condensed in Heating the Apparatus.—When the thermometer is in its usual experimental position, that is, with its bulb in the middle of the salt and so low down that it will be immersed in the brine or water whenever enough steam has condensed to make this possible, and the whole of the working part of the stem is in the steam space,

it arrives at the maximum temperature almost simultaneously with the first exit of steam from the apparatus; but that part of the steam tube situated above the exit tube is not yet thoroughly warmed through, and a little time must be allowed during which the strength of the steam current increases until it becomes steady. In ordinary circumstances the apparatus cannot be held to be warmed through in less than 90 seconds, and, for purposes of heat calculation, we take the initial period of heating to be two minutes, during which it may be said always to be complete. Experiments made with boiling mixture tube, weighing with thermometer 239·65 grms., showed the following results:—

Steam condensed, ...	grms.,	8·3	8·3	8·5	8·35
In time, ...	seconds,	85	90	100	90

From these we find the mean amount of steam condensed in the first 90 seconds 8·25 grms. When the steam was passed through for exactly two minutes before weighing, the following weights of steam were condensed:—9·1, 8·9, and 9·0 grms., or a mean of 9·0 grms.

For vessels of the same pattern and nearly the same size the quantity of steam required to heat them or to keep them hot depends simply on the amount of glass.

Thus, our apparatus weighs 239·65 grms., and may be considered to be all glass. If the specific heat of the glass be 0·2, then the amount of water thermally equivalent to it is 48 grms. In order to raise the temperature of 48 grms. water from 15° C. to 100° C., we require 4080 g° C. (gramme-degrees-Celsius), and this can be supplied by 7·61 grms. steam saturated at 100° C., and condensing at 100° C. But the mean temperature of the apparatus during warming may be taken to be 57·5° C., and the 7·61 grms. water formed would, in cooling from 100° C. to 57·5° C., give out 323·4 g° C., a quantity which is furnished by the condensation of 0·603 grm. steam at 100° C. Deducting this from 7·61, we have 7·007, or 7 grammes as the least amount of steam required to raise the temperature of the apparatus instantaneously to 100° C. In practice, the operation takes a minute and a half, during which the apparatus is losing heat at an increasing rate. This is supplied by additional steam condensed. We have seen that the steam condensed in two minutes is 9·0 grms., and in $1\frac{1}{2}$ minutes 8·25 grms., giving 0·75 grm. of steam condensed in half a minute, or 1·5 grms. in one minute after the whole apparatus has taken the temperature of 100° C. The mean temperature during heating has been taken as 57·5° C., so we may take the rate of cooling at $\frac{6}{10}$ of the above rate, or equivalent to the condensation of 0·9 grm. steam per minute, or of 1·35 grms. for $1\frac{1}{2}$ minutes. Adding this to 7 grms., the weight of steam required for instantaneous heating, we obtain 8·35 grms. as the theoretical weight of steam required to warm the apparatus under the above conditions. The mean observed amount is 8·25 grms., which may therefore be accepted with confidence. Also, we may confidently

calculate the amount of steam required to warm another apparatus of the same type on the basis of its weight.

In order to determine more carefully the amount of steam required to keep the apparatus at a temperature of 100° C. during some time, two experiments were made, the apparatus being dry and cold to begin with. In the first, steam was passed through for 12 minutes, when 22·2 grms. were condensed; in the second, the steam was passed for 32 minutes, when 50·8 grms. were condensed. Allowing that in each case 9 grms. of steam were condensed in the first two minutes, we have 13·2 grms. condensed in 10 minutes, and 41·8 grms. in 30 minutes. The first is at the rate of 1·32 grms. per minute, and the second at the rate of 1·39 grms. per minute, or a mean of 1·35 grms. per minute.

Experiments of a similar kind were made with $\frac{1}{5}$ NaCl, or 11·7 grms. of this salt in the tube to begin with. Two minutes were sufficient for heating up to 100° C. In two experiments the amounts of steam condensed were 9·8 and 9·6 respectively, giving a mean of 9·7 grms. In a similar experiment, where the passage of steam was not stopped until the salt was all dissolved, which took $16\frac{1}{3}$ minutes, the steam condensed was 31·5 grms. Deducting 9·7 grms. we have 21·8 grms. condensed in $14\frac{1}{3}$ minutes, or 1·5 grms. per minute. The rate of condensation is naturally higher, because salt is being dissolved. With the apparatus empty at the start, 9·0 grms. steam are condensed in the first two minutes; with a charge of 11·7 grms. chloride of sodium, 9·7 grms. of steam are required; the excess, or 0·7 grm., may be taken as the steam condensed by the NaCl in the two minutes, and as constant for the same amount of NaCl in other apparatus.

The boiling tube used in all the experiments up to 26th October 1897 weighed, with thermometer, 157·3 grms. This would require 5·5 grms. of steam in order to raise it to 100° C., and the heating would be complete in one minute instead of in one and a half minutes as with the apparatus weighing 239 grms. Allowing 1·0 grm. for the amount of steam condensed in the next minute, we should have, after two minutes, 6·5 grms. steam condensed. Where $\frac{1}{5}$ NaCl was used, we should have to add 0·7, and the amount thus condensed at the end of the first two minutes would be 7·2 grms. The rate of condensation per minute, after the first two minutes, would be $\frac{2}{3}$ 1·35 or 0·9 grm., and adding 0·15 for the chloride of sodium, we have 1·05 grms. per minute, taking 30·4 grms. as the amount of steam required to be condensed for $\frac{1}{5}$ NaCl.

Localities where Experiments were Made.—The lowest station, and the one representing the sea level, was my laboratory in Edinburgh: its elevation is about 85 metres, or 279 feet, above the sea. Although a large number of experiments in this field had been made in the course of previous years, those utilised for this paper were all made after my return from Switzerland, with the same apparatus and the same thermometers that were used there, and with the experiments arranged on the same plan. Being the last series, it is also the most symmetrical.

The stations at higher levels are all in Switzerland, and they are as follows in order of elevation :—

Place.	Height above the Sea.		Place.	Height above the Sea.	
	Metres.	Feet.		Metres.	Feet.
Zurich, ...	410	1345	St Moritz, . . .	1860	6102
Fiesch, ...	1054	3458	Eggischorn, . . .	2193	7195
Andermatt, ...	1444	4738	Julier Hospiz, . . .	2244	7362
Pontresina, ...	1820	5970	Schafberg (Engadine), .	2733	8966

Of these places Pontresina, with St Moritz, was the most important. The most complete series of observations on mixtures, as well as on single salts, was made there. A corresponding series of observations on single salts was made on the Schafberg, which rises immediately behind Pontresina, and about 900 metres above it. Shelter is obtained at the top in the châlet which does duty as a restaurant, and it is approached by a well made path. Experiments on mixtures of salts could not be made at this station, because the weather became so persistently bad that the châlet was closed for the season early in September.

Of the other places on the list, St Moritz is taken as one with Pontresina, because the difference of level is less than that corresponding to ordinary fluctuations of the barometer.

Zurich and Julier Hospiz were visited for the purpose of verifying the thermometers by observing the temperatures of saturated steam, and the barometric pressure by standard barometers at the same time and place. One or two observations with salts were made at the same time, but they have only the value of isolated observations. The boiling mixture of chloride of sodium was observed at all the stations, and it was the only salt experimented with at Eggischorn, Fiesch, and Andermatt, as at that date I intended to confine my observations to chloride of sodium alone.

The salts used in this research are the chlorides of sodium, potassium, ammonium, barium ; the chlorate of potassium ; the nitrates of sodium, barium, strontium, and lead, and the sulphates of potassium and ammonium. These salts were used singly, and also in mixtures of not more than two salts each. The charge of the apparatus was usually one-fifth of a gramme-molecule, but with sparingly soluble salts, such as nitrate of barium or sulphate of potassium, one-tenth and sometimes one-twentieth of a molecule were used. A watch, giving minutes and seconds accurately, was observed during all the experiments. In the case of simple salts the time was noted when the steam reached the salt, when the salt formed a magma, with the steam condensed, when the maximum temperature was reached, when it began to fall, and when the passage of

steam was stopped previous to making the first weighing. These particular epochs were always noted, but, as a matter of fact, a complete time-log was kept of every experiment. In experiments with mixtures the temperature was noted every minute, and often every half-minute, so long as salt remained undissolved. A complete time record of this kind often furnishes valuable incidental information, and is often useful in detecting and rectifying errors of observation.

Chloride of Sodium.—The series with this salt is very complete, including sixteen independent experiments; the atmospheric pressure varied from 550·4 to 772 mm., and the temperature of saturated steam from 91·2° C. to 100·44° C. The corresponding temperatures of the boiling mixture ranged from 99·3° C. to 108·98°, so that the elevation of boiling point caused by saturation with NaCl ranges from 8·1° C. to 8·54° C., or nearly half a degree Centigrade of increase for a rise of boiling point of the salt solution of 9·68° C. Roughly, it diminishes 0·05° C. for every degree that the boiling temperature of the boiling solution falls. The results of the observations in different localities are collected in Table I., page 551.

If we consider the relation between atmospheric pressure and the vapour tension of water at the temperature of the boiling mixture, we see that it is practically constant. The mean of the sixteen values is 0·7435. The average deviation from the mean is 0·0004, and the maximum deviation 0·0009. The mean of the observations made at Edinburgh is 0·0001 below this. The five observations made at Pontresina and St Moritz give a mean of 0·0004 above, and the three observations on the Schafberg a mean of 0·0005 below 0·7435. Taking 0·7439 for the value at Pontresina, the value of $t-\mathrm{T}$ would be 0·015° C. less than when the general mean 0·7435 is used, and on the Schafberg, using the factor 0·7430, the value of $t-\mathrm{T}$ comes out 0·025° C. higher than with the general mean.

Table II. (page 552) has been constructed on the basis that $\frac{\mathrm{P}}{p_0}=0{\cdot}7435$, and therefore $\frac{p_0}{\mathrm{P}}=1{\cdot}345$. The barometric pressure, P, is given for intervals of 10 mm. from 790 mm. to 550 mm. The temperature of saturated steam at pressure, P, is given under T. Under p the vapour tension of water at the temperature of a boiling mixture of steam and NaCl at barometric pressure, P, is given, where $p=1{\cdot}345$ P. The temperature of this boiling mixture is found from Regnault's tables connecting the temperature and pressure of saturated steam, and it is given under t. The difference $(t-\mathrm{T})$ gives the elevation of the boiling point of saturated NaCl brine above that of pure water at barometric pressure P.

The figures in this table show that a boiling mixture of steam and NaCl at a known barometric pressure gives the means of obtaining an independent fixed point on a thermometer about 8 to 8·5° C. above that furnished by the boiling point of pure water at the same pressure. In most cases the normal pressure of 760 mm. would be used, but as the mean pressure in inhabited countries is less than 760 mm., it is convenient to

be able to use directly the values observed at the existing pressure, and our table affords the means of doing so, assuming that the readings of our thermometers as corrected are exact.

In Table III. are given the saturation values for the simple salts. They are arranged in order of the temperature of the boiling mixture (t_0), and this temperature indicates quite clearly the locality where the observation was made. The second column contains the values of $t_0 - T$, from which, with the first column, the values of T are at once obtained. The third column contains the relative reduction of vapour tension $\left(\frac{p_0 - P}{p_0}\right)$ produced by saturating the water with the salt at its boiling temperature.

The means of the observations at the same heights above the sea are inserted between lines.

The case of NaCl has been already discussed. The observations with KCl show a diminution of the value of $\frac{p_0 - P}{p_0}$ with a fall of barometric pressure, and consequent fall of boiling temperature. This is observed in all the other salts experimented with, and depends chiefly, if not wholly, on the diminished solubility of the salt at the lower temperature. Although, in kind, the effect is the same in all the salts, it varies much in amount. It is most pronounced in the case of $KClO_3$, for which the mean values are—

Locality :—	Edinburgh.	Pontresina.	Schafberg.
$t - T$,	3·80° C.	3·30° C.	3·06° C.
$\frac{p_0 - P}{p_0}$,	0·1250	0·1138	0·1078

It is also well marked in the case of NH_4Cl. This salt also crystallises with great promptitude so soon as the temperature falls at all. As is well known, salts differ much in this respect. A large number of cooling observations were made with NH_4Cl, and with some of the other salts, and *eutectic* points were observed, but they are not of sufficient importance for the present research to justify their being printed. The salt which appears from the value of $\frac{p_0 - P}{p_0}$ to vary least in solubility at its boiling point is $(NH_4)_2SO_4$, and in this respect it closely resembles NaCl. Nitrate of sodium was observed only at Pontresina and Schafberg. It could not be observed near sea level with either of the thermometers used in the investigation. Nitrate of potassium was excluded altogether from boiling mixtures both because of the great elevation of boiling point, and on account of the readiness with which it solidifies to a crystalline mass the moment the temperature begins to fall.

The chief part of the research is contained in Table IV. Three columns are devoted to each experiment; namely, $(t - T)$ the elevation of the boiling temperature of the mixture or brine above that of pure water at the same time and place; W the weight of steam condensed in the time from the beginning of the experiment until the value of $t - T$ has become as tabulated; and $W(t - T)$ the product of the corresponding pairs of

numbers. The experiments are numbered consecutively in the first headline (N), while under n each line in the table is numbered consecutively from 0 upwards. In this way any entry in the table can be referred to at once by its co-ordinates (N, n). The second headline gives the name and quantity of salt taken expressed in gramme-molecules. In the third headline will be found the temperature of saturated steam, or that of pure water boiling at the same time and place. The temperature of the boiling mixture or brine is obtained at once from the values of T and $(t - T)$.

The figure 0 is always used as a suffix when the boiling mixture of steam and salt or saturated brine is being dealt with. Thus t_0, p_0, W_0 always represent the temperature of the boiling saturated mixture, the steam tension of pure water of that temperature and the *dilution* of the mixture; that is, the weight of water exactly saturated by the amount of salt at temperature t_0. The temperature of the mixture when the steam was stopped for the first time, and the first weight of condensed steam, W_1, ascertained, is always t_1. It has already been pointed out that it was the custom to stop the steam while there were still some particles of solid salt present, and while the temperature of the mixture showed that there was also already unsaturated water present. The idea was that the moment might be correctly judged when the amount of free salt present would be just enough to saturate the amount of free water if time were given. As a matter of fact, this was in most cases very nearly attained, as will be seen by comparing the observed values of W_1 with the computed values of W_2 in the cases where $t_0 - t_1$ is not more than 0·1° to 0·3° C. When this difference is larger, then the passage of steam has not been interrupted until all the salt has disappeared. This is the preferable practice. When any solid salt is present, and the temperature has fallen below the maximum, we know exactly the temperature of the boiling brine and the weight of water in it, but as there is an uncertain proportion of the salt originally taken which has not passed into solution, the concentration or dilution of the brine is uncertain. For this reason it has been impossible in the majority of cases to use t_1 and W_1 in the computation of W_0. The values of W_0 have been arrived at in the following way:—The difference $W_2(t-T)_2 - W_3(t-T)_3$ is found, also the differences $t_2 - t_3$ and $t_0 - t_2$, then we assume that

$$W_0(t-T)_0 = \frac{t_0 - t_2}{t_2 - t_3}\{W_2(t-T)_2 - W_3(t-T)_3\} + W_2(t-T)_2$$

and this value divided by $(t_0 - T)$ gives W_0. Thus in experiment No. 1 on 0·2 KCl, when $T = 100·44°$ C., we have $W_2(t-T)_2 = 217·6$, and $W_3(t-T)_3 = 212·9$, their difference being 4·7. Also $t_0 - t_2 = 0·81°$ C., and $t_2 - t_3 = 1·05°$, whence we have

$$W_0(t-T)_0 = \frac{81}{105}4·7 + 217·6 = 220·6$$

whence

$$W_0 = 26·30.$$

It will be seen that the value of W_1 is 25·91, and $t_1 - T = 8·28°$, only 0·11° C. below $t_0 - T$. The passage of steam had therefore been stopped too soon; there was solid

KCl present in greater quantity than could in any length of time be dissolved in the amount of water present. Again, if we look at experiment No. 4, we find the observed value of W_1, 27·25, almost identical with the computed value of W_0, 27·21. Here the difference, $t_0 - t_1$, is 0·2° C. So that, working in the way described, with chloride of potassium, when the temperature of condensation of the steam has fallen by 0·2° C., we should expect to have the amounts of free water and free salt present in compensating amounts. No. 6 represents a case in which the steam was passed until all the salt was dissolved, and until the temperature of condensation had fallen by a whole degree. Here we use $W_1(t-T)_1$ and $W_2(t-T)_2$ for finding $W_0(t-T)_0$ and W_0. If we consider experiments Nos. 1 to 9, it will be seen that the values of W_0 in experiments 1 to 3, which were made in Edinburgh, are lower than those found in Nos. 4 to 9, which were made at high levels, and therefore at lower temperatures, in Switzerland.

It must also be noted that the weight of the salt taken was less exactly ascertained in Switzerland than in my laboratory in Edinburgh. The Swiss weighings were made with a pair of hand scales, and were exact to the nearest 0·05 grm., that is to say, generally to ±0·025 grm. In the Edinburgh experiments quoted in Table IV. the weights are exact to the nearest 0·01 grm., or to ±0·005 grm. The quantity of salt usually taken was one-fifth of a molecule in grammes. In the case of KCl, which has a medium molecular weight, this represents 14·92 grms., and we see that even the roughest of the weighings would be exact to within less than one-half per cent.

The earlier experiments in Switzerland were not always made with equivalent weights of the salts; all such cases have been recalculated for this table. While the usual quantity taken is one-fifth of a gramme-molecule, on some occasions two-fifths have been taken; and in the case of sparingly soluble salts as little as one-tenth or one-twentieth has been taken. The values of W are given for the quantity of salt quoted in the headline (M). The products have been all reduced to their value for one-fifth of a molecule salt.

The physical meaning of the expression $W(t-T)$ is important. W is a weight of water expressed in grammes, and $(t-T)$ is the excess of the boiling temperature in degrees Celsius of that water, when it holds in solution a certain amount of a given salt, above its boiling temperature when in a state of purity; therefore $W(t-T)$ expresses, in gramme-degrees (g° C.), the quantity of heat required to be in the water when it is boiling with salt dissolved in it above what is required when it is pure. If the values of $W(t-T)$ were constant for each salt at all dilutions, then the law connecting the dilution of a saline solution and the elevation of its boiling point would be graphically expressed by a hyperbola, like the law connecting the volume and pressure of a gas at constant temperature. If we look over Table IV. we see that for some salts, and mixtures of salts, the values of $W(t-T)$ are very nearly quite constant, while for the others, some deviate from constancy in the one sense, and some in the other. In the case of the chlorides of potassium and of sodium, the values of $W(t-T)$ diminish very considerably as the value of W increases. The case of ammonium chloride is peculiar, because at the sea level $W(t-T)$

diminishes as W increases; at a height of 3000 metres it increases with W, and at a height of 2000 metres it is sensibly constant. In the case of barium chloride, $W(t-T)$ diminishes with dilution; the same is the case with strontium nitrate and ammonium sulphate. Potassium chlorate, barium nitrate, and lead nitrate show $W(t-T)$ increasing with W, while sodium nitrate and potassium sulphate show almost constant values of $W(t-T)$. Mixtures of salts follow the rule of their components. There are several examples in the table of pairs of salts which individually differ in the sense in which the values of $W(t-T)$ depart from constancy, and in mixture give constant values of $W(t-T)$. Examples are Nos. 63, 70, 71, 72, 73, 78, and 79.

At the beginning of an experiment, when the steam reaches the salt, it condenses very rapidly owing to abstraction of heat by the glass and by the salt, then it condenses at a very regular rate, the salt dissolving in proportion as steam is condensed. After a certain time the exact amount of steam has condensed which is necessary to form a boiling saturated solution of the salt taken; having observed (t_0-T) and W_0 we have the value of $W_0(t_0-T)$. If the elevation of the boiling point were proportional to the concentration, this factor $W_0(t-T)$ would remain constant while the solution was diluted by further condensation of steam. But if we deny thermal importance to the salt and consider only the water, then $W_0(t_0-T)$ is the heat in the saturated water, counting from the temperature of pure boiling water. If we prevent it from losing heat externally, and provide for dilution by furnishing water of exactly the temperature of pure steam condensing on pure water at the time and place, the saturated water will mix with the free water, having a resultant temperature depending on the relative quantities of the saturated and the free water. The case, then, of sodium nitrate, for instance, in which the value of $W(t-T)$ is nearly constant, could be represented by imagining the steam to condense at the temperature of the boiling saturated solution of the salt so long as solid salt is present, and the condensation temperature of the steam remains constantly at the maximum. When the solid salt has all disappeared, then the steam condenses at the temperature at which it condenses in pure water. The two portions of water, the saturated and the free, then mix, giving the resultant temperature, depending on the relative quantities and on the assumption that the heat of the saturated solution is that which the water present in it would have if it had the same temperature.

In the cases where $W(t-T)$ is constant, we have Blagden's law of the lowering of freezing point applied to the raising of the boiling point of saline solutions; both vary directly with the concentration or inversely with the dilution. But in the case of a saline solution following Blagden's law, when ice is melted in the saturated solution already cooled to its freezing point, the solution is diluted and its temperature rises. The rise of temperature is thus the same as would have been produced if the quantity of ice which has melted had been added as pure water of 0° C. to the saturated solution at the initial temperature of its freezing point, and the two had been mixed.

The same is the case, *mutatis mutandis*, in the condensation of steam by a saline

solution. If the boiling temperature of the stronger solution be t_1, and steam be passed through it until this temperature has fallen to t_2, the temperature of steam condensing on pure water being T, and if the quantity of water in the stronger solution be W_1, and in the weaker W_2, then we should have

$$W_1t_1+(W_2-W_1)T=W_2t_2,$$

whence

$$W_1(t_1-T)=W_2(t_2-T),$$

the values of $W(t-T)$ are constant, which is the characteristic of Blagden's law, whether applied to freezing or boiling. The deviations from Blagden's law in freezing and in boiling solutions may be attributable to some deviation from the law that the capacity for heat of a saline solution is the capacity for heat of the water which it contains. It is easy to obtain from the values of $W(t-T)$ in Table IV. what must be the specific heat of the saturated water which would make $W(t-T)$ constant.

Table V. gives for dilute solutions what Table IV. gives for strong solutions. As the values of $(t-T)$ are not exactly the same for each salt, the relations of $t-T$ and $W(t-T)$ were expressed by curves and the values of $W(t-T)$ taken from them for the same values of $t-T$ for every salt. These values are given in Table VI. In all the experiments of Table V., exactly one-twentieth of a gramme-molecule of salts was taken, and the weight was exact to the nearest milligramme, and the experiments were made in every way alike. Table V. contains no mixtures; on the other hand, some simple salts are included which are not found in Table IV. They are LiCl, RbCl, CsCl, KI, KBr, KNO_3, and $AgNO_3$. Both the RbCl and the CsCl were "spectroscopically pure," and the caesium salt was quite pure. The rubidium salt, however, contained sulphate, and it is struck out in Table VI. The detailed discussion of these results, as well as of those in Table IV., must stand over to the second part of this paper, but one or two facts may be pointed out. In the case of salts for which the values of $W(t-T)$ diminish as dilution increases, a minimum is reached when $t-T$ is something between $0{\cdot}7^\circ$ and $1{\cdot}0^\circ$ C., after which it increases in all so that for values of $t-T$ between $0{\cdot}5^\circ$ and $1{\cdot}2^\circ$ the values of $W(t-T)$ for all these salts is practically constant, and they follow Blagden's law. In the case of nitrates $W(t-T)$ generally increases with dilution, and a minimum, if it exist, would have to be sought in solutions saturated under high pressure. Strontium nitrate is an exception; it has a minimum at $1{\cdot}0^\circ$ C.

In Table VII. will be found the values of p, the vapour tension of pure water at the temperature (t) of the boiling brine, the relative reduction of this tension $\left(\frac{p-P}{p}\right)$ produced by the salt in solution, and the product of this ratio into the weight of steam condensed (W). The discussion of the contents of this table is deferred.

The discussion of the observations on mixtures is also deferred. It involves a very large amount of arithmetical work, which will take some time. The behaviour of mixtures is extremely interesting. In Table IV. we have examples of two distinct types. Of the

one type, mixtures of the chlorides of potassium and sodium may be taken as an example. For a mixture of equal molecules $0{\cdot}2\frac{K+Na}{2}Cl$ $t-T$ is $11{\cdot}87^\circ$ C. at sea level, and $W_0=20{\cdot}7$ grms. The concentration of its boiling mixture is, therefore, 50 per cent. greater than that of chloride of sodium alone. In other words, a boiling saturated solution of either KCl or NaCl has still plenty of room for the other.

Mixtures of the nitrates of strontium and lead give an example of another type. The amount of steam condensed is exactly the sum of the amounts required by the quantities of the respective salts separately. For $Sr(NO_3)_2$ $(t-T)=6{\cdot}53^\circ$ C., and for $Pb(NO_3)_2$ $(t-T)=3{\cdot}29$. In the mixture $0{\cdot}2\frac{Sr_2+Pb}{3}(NO_3)_2$ a constant value of $t-T=5{\cdot}98^\circ$ C. is observed for thirteen out of the twenty-five minutes that were required to dissolve all the salt. Therefore the saturation temperature of the mixture lies between those of the components. A third type is furnished by a mixture of the nitrates of strontium and barium. The elevation of boiling point is not as great as with $Sr(NO_3)_2$ alone, and the maximum temperature does not remain constant for even one minute out of the fifty minutes which were required to dissolve the fifth of a molecule used. The water required to dissolve this mixture is about 25 per cent. more than is required to dissolve the salts separately.

The nitrates of strontium, barium, and lead are isomorphous salts, and no doubt are capable of forming mixed crystals. The chlorides of potassium and sodium, though both crystallising in the same form, do not form mixed crystals, and therefore do not eliminate each other from solution.

These few remarks will show the extent of the subject, and also its great interest.

In order to provide a standard of comparison between the effect of dissolved salt and that of increased pressure upon the boiling temperature of water, we imagine a quantity of water in a shallow tank kept boiling by steam. The tank is of uniform depth of 1 centimetre, but it can expand laterally to accommodate condensed steam, the depth of the enlarged tank remaining uniformly 1 centimetre. In these circumstances the number expressing the weight, in grammes, of the water, expresses also its volume and its surface in cubic and square centimetres respectively. Let the initial quantity of water be W grammes, and let it be at the boiling temperature corresponding to the atmospheric pressure, A, in kilogrammes per square centimetre (k/c^2). If we dissolve a quantity, say one-fifth, of a molecule in grammes of a salt in this quantity of water, the temperature can be raised above the boiling temperature, T, of pure water under the atmospheric pressure, A, and if the quantity, W, of water is exactly sufficient to dissolve the fifth of a molecule salt at its boiling temperature, which is then the temperature of the boiling saturated solution of the salt under atmospheric pressure, A, which we represent by t_0, then the temperature of the boiling water has been raised from T to t_0. Now this effect could be produced without adding salt by increasing the load pressing upon the surface, W, of the water. It is already pressed by the weight of the atmosphere A

kilogr. per square centimetre, or a total weight of WA kilogr. In order to confine the water so as to admit of its temperature being raised from T to t, we must add to WA a certain additional load, b kilogrammes, which is found by consulting Regnault's tables connecting the temperature and tension of saturated steam. In these tables we find directly the tension, p_0, of saturated steam of temperature t_0, in millimetres of mercury, which is converted into a_0 in k/c^2. The excess of this above the atmospheric pressure multiplied by the surface, W, gives the extra load required :

$$b = \mathrm{W}(a_0 - \mathrm{A}),$$

and this quantity b kilogrammes is the mechanical equivalent of the fifth of a molecule of salt in so far as the raising of the boiling temperature of water, or the resistance to steam pressure, is concerned.

In the case of the boiling saturated solution of salt when steam is continued to be passed through and heat is lost, the solution is diluted by the addition of the condensed steam to the original quantity, W, of saturated water. This dilution, or addition of pure water to the saturated water, is accompanied by a fall of the temperature of ebullition, which is very rapid at first, but becomes slower as the quantity of condensed steam increases, tending ultimately towards the boiling temperature of pure water at atmospheric pressure.

The more concentrated the solution is, the more accentuated are the specific properties of the dissolved salt, and they are most pronounced in the saturated solution which approximates to the condition of the liquefied salt, as the dilute solution approximates to that of pure water. The specific nature of the dissolved salt shows itself first in the maximum temperature to which the solvent water can be raised under a given pressure, and then in the rate of fall of boiling temperature with dilution. Different salts behave differently in these respects.

The uniformity observed in the physical properties of very dilute solutions is due in part to our limited powers of perception, and to arithmetical necessity. In proportion as the number expressing the dilution becomes very great it tends to occupy the whole field of view, and, consequently, to obscure or obliterate the specific properties of the substance dissolved.

Similarly, in considering the trigonometrical functions of angles, if we limit our contemplation to very small angles, we can perceive no difference between the sine, the arc, and the tangent, yet the difference is none the less real on that account.

In our mechanical experiment the excess of pressure of the water at any moment above that of the atmosphere is $\frac{b}{\mathrm{W}_n}$, and when this is multiplied by the quantity of water present, W_n, the product is constantly b. Now b is the equivalent of the salt dissolved, therefore our mechanical experiment represents the case where the increase of steam tension, neutralised by the salt, is proportional to the quantity of the salt.

For convenience in reference, the values of the tension of saturated steam, at tem-

peratures from 90° C. to 120° C., expressed in kilogs. per sq. cm., are collected in Table VIII., page 572.

In Table IX. we have the case of 100 grms. water in the elastic tank. The atmospheric pressure is 735·5 mm., or 1 kilogr. per sq. cm. The depth of the tank, which can be enlarged laterally, is uniformly 1 cm., therefore its volume at the beginning is 100 cub. cms., and its surface is 100 sq. cms. The boiling temperature of pure water at a pressure of 1 k/cm². is 99·09° C. The tank is securely covered, and its temperature raised to 119·57° C., at which temperature the tension of saturated steam is exactly 2 kilogrs. per sq. cm. Neglecting, or allowing for, any thermal expansion of the tank and its contents, the area of it has remained the same, namely, 100 cm². Let the cover be loaded until its fastenings just become slack, then the surface of the water is pressed by the loaded cover and by the atmosphere ; the latter on a surface of 100 cm². amounts to 100 kilogrs., and the former makes up the difference between this weight and 200 kilogrs., because the pressure of the steam at 119·57° C. is 2 k/cm². Therefore, our extra load, b_0, is 100 kilogrs., and it may be looked on as the weight of the cover, which, like the tank, is supposed to be capable of lateral extension of its area without alteration of weight, keeping pace with the lateral increase of volume and area of the tank, while its contents are being increased by the condensation of steam.

Let the temperature of the water in the tank be now reduced to 118·03° C., at which temperature its vapour tension is 1·9 k/cm²., and let steam of this temperature be condensed in it. The volume of water in the tank increases while its area expands in the same ratio, until the weight pressing on its surface is at the rate of 1·9 k/cm²., when the steam will lift the cover and escape. The weight of the cover has remained constant (= 100 kilogs.), but the area exposed to the atmospheric pressure has increased. The resulting area and volume of water is given by the equation $1{\cdot}9W = 100 + W$, whence

$$W = \frac{100}{0{\cdot}9} = 111.$$

Again, let the temperature of the water in the tank be reduced to 116·29° C., at which temperature its vapour tension is 1·8 k/cm²., and let steam of this temperature be passed into it. It will be condensed until the volume and surface of the water have increased to such an extent that the total pressure on its surface is at the rate of 1·8 k/cm². As before, we find the value of W, when this point has been reached, to be $\frac{100}{0{\cdot}8} = 125$, and so on.

In the principal table the pressure is reduced by 0·1 k/cm². at a time, from 2·0 to 1·1 k/cm²., then by 0·01 k/cm². down to 1·01 k/cm²., and by 0·001 down to 1·001 k/cm². The value of W is inversely proportional to the difference of pressure, $\alpha - A$, and becomes infinite when $\alpha - A = 0$. On the other hand, it diminishes rapidly at high temperatures and pressures, and would become 0 when $\alpha - A = \infty$.

The table is carried upwards to $\alpha = 10$ k/cm²., and downwards to $\alpha = 1{\cdot}001$ k/cm². In these limits the value of W varies from 11·1 to 100,000.

The temperature of saturated steam rises at a slower rate than its tension. Hence in our mechanical experiment with pure water, when the area is increased by a given amount, and the pressure per unit of area is correspondingly reduced, the consequent fall of temperature depends on the actual temperature, and is proportional to the reciprocals of the figures representing the mean difference of tension per degree for the interval. In the following table we have values of W, each of which is the double of

W.	$a-$A.	$t-$T.	Diff.
		C.	C.
125	0·8	17·20°	
250	0·4	9·62°	7·58°
500	0·2	5·14°	4·48°
1,000	0·1	2·66°	2·48°
2,000	0·05	1·35°	1·31°
4,000	0·025	0·68°	0·67°
8,000	0·0125	0·34°	0·34°
16,000	0·00625	0·17°	0·17°

the one preceding it; the concentration is therefore halved in each case and corresponds exactly with the values of $a-$A, which give the excess of vapour tension above atmospheric pressure. Under $t-$T we have the corresponding excesses of temperature above that of water boiling at atmospheric pressure (99·09° C.). In the last column we have the differences of successive values of $(t-\mathrm{T})$. Did the value of $t-$T fall in exact proportion to the concentration, then each of these differences should be identical with the values of $t-$T opposite it, because the temperature interval should be halved. When the value of $t-$T falls below 0·68° C. the interval is apparently halved, and at temperatures within half a degree of that of water boiling at atmospheric pressure, the tension of saturated steam seems to vary proportionately with its temperature; but the differences which undoubtedly exist occur in places of decimals which are excluded from the table.

TABLE I.—*Particulars of Boiling Mixtures of Steam and NaCl at different Elevations.*

Locality, . . .	Edinburgh.	Edinburgh.	Edinburgh.	Zürich.	Fiesch.	Andermatt.	Pontresina.	Pontresina.
Date, 1897, . . .	21st Oct.	19th Oct.	15th Oct.	19th Aug.	16th Aug.	18th Aug.	12th Sept.	12th Sept.
Elevation in Metres, .	80	80	80	410	1054	1444	1820	1820
Temp. of boiling mixture, °C., $t_0°$	108·98	108·56	107·65	107·20	105·05	103·82	102·57	102·57
Temp. of boiling water, °C., . T	100·44	100·06	99·21	98·75	96·71	95·58	94·37	94·37
Difference, °C., $t_0 - T$	8·54	8·50	8·44	8·45	8·34	8·24	8·20	8·20
Tension of saturated steam at t_0, mm., . p_0	1039·0	1024·4	993·1	977·9	908·0	870·0	832·7	832·7
Atmospheric pressure, mm., . P	772·0	761·6	738·8	726·7	674·9	647·6	619·4	619·4
$p_0 - P$	268·0	262·8	254·3	251·2	233·1	222·4	213·3	213·3
$\frac{p - P}{p}$	0·2570	0·2565	0·2561	0·2569	0·2567	0·2556	0·2561	0·2561

Locality, . . .	Pontresina.	St Moritz.	St Moritz.	Eggischorn.	Julier.	Schafberg.	Schafberg.	Schafberg.
Date, 1897, . .	17th Sept.	28th Aug.	23rd Aug.	15th Aug.	25th Aug.	5th Sept.	31st Aug.	10th Sept.
Elevation in Metres, .	1820	1860	1860	2193	2244	2733	2733	2733
Temp. of boiling mixture, °C., $t_0°$	102·32	102·48	102·10	101·30	101·00	99·57	99·54	99·30
Temp. of boiling water, °C., . T	94·13	94·28	94·00	93·12	92·85	91·47	91·46	91·20
Difference, °C., $t_0 - T$	8·19	8·20	8·10	8·18	8·15	8·10	8·08	8·10
Tension of saturated steam at t_0, mm., . p_0	825·4	829·9	821·9	796·0	787·6	748·4	747·6	741·2
Atmospheric pressure, mm., . P	614·0	617·8	611·0	591·4	585·5	556·0	555·8	550·4
$p_0 - P$	211·4	212·1	210·9	204·6	202·1	192·4	191·8	190·8
$\frac{p - P}{p}$	0·2561	0·2556	0·2566	0·2570	0·2566	0·2571	0·2566	0·2574

TABLE II.—*Temperatures of a Boiling Mixture of Steam and Sodium Chloride at Barometric Pressures from* 550 *to* 790 *Millimetres.*

Atmospheric Pressure. P	Temperature of Saturated Steam of Tension P. T	Tension of Saturated Steam of Temperature t (1·345 P). p	Temperature of Boiling Mixture. t	Elevation of Boiling Point. $t-T$
Millimetres.	°C.	Millimetres.	°C.	°C.
790	101·09	1062·5	109·64	8·55
780	100·75	1049·05	109·27	8·54
770	100·37	1035·6	108·88	8·51
760	100·00	1022·15	108·50	8·50
750	99·63	1008·7	108·11	8·48
740	99·26	995·25	107·72	8·46
730	98·88	981·8	107·32	8·44
720	98·49	968·35	106·92	8·43
710	98·11	954·9	106·51	8·40
700	97·71	941·45	106·10	8·39
690	97·32	928·0	105·68	8·36
680	96·92	914·55	105·29	8·37
670	96·51	901·1	104·83	8·32
660	96·10	887·65	104·39	8·29
650	95·68	874·2	103·96	8·28
640	95·26	860·75	103·51	8·25
630	94·83	847·3	103·07	8·24
620	94·40	833·85	102·61	8·21
610	93·96	820·4	102·15	8·19
600	93·51	806·95	101·68	8·17
590	93·06	793·5	101·21	8·15
580	92·60	780·05	100·73	8·13
570	92·13	766·6	100·24	8·11
560	91·66	753·15	99·75	8·09
550	91·18	739·7	99·24	8·06

TABLE III.—*Particulars of Boiling Mixtures of Steam and various Salts.*

KCl.			NaCl.			NH_4Cl.			$BaCl_2.2H_2O$.			$KClO_3$.		
t	$t-T$	$\frac{p-P}{p}$	t	$t-T$	$\frac{p-P}{p}$	t	$t-T$	$\frac{p-P}{p}$	t	$t-T$	$\frac{p-P}{p}$	t	$t-T$	$\frac{p-P}{p}$
° C.	° C.		° C.	° C.		° C.	° C.		° C.	° C.		° C.	° C.	
108·84	8·40	0·2534	108·98	8·54	0·2570	113·76	14·43	0·3913	104·47	4·52	0·1476	104·08	3·80	0·1250
108·45	8·35	0·2515	108·56	8·50	0·2565									
107·50	8·29	0·2522				113·76	14·43	0·3913	104·47	4·52	0·1476	104·08	3·80	0·1250
			108·77	8·52	0·2567									
108·26	8·35	0·2520				107·92	13·52	0·3813	98·81	4·39	0·1478	97·73	3·30	0·1138
			107·65	8·44	0·2561	107·77	13·51	0·3813	98·46	4·37	0·1478	97·55	3·30	0·1138
102·24	7·87	0·2474	107·20	8·45	0·2569	107·77	13·51	0·3813						
102·23	7·89	0·2480							98·63	4·38	0·1478	97·64	3·30	0·1138
102·19	7·90	0 2482	107·42	8·45	0·2565	107·82	13·51	0·3813						
102·00	7·88	0·2479							95·70	4·23	0·1453	94·54	3·07	0·1080
101·90	7·90	0·2489	105·05	8·34	0·2567	107·79	13·49	0·3823	95·44	4·26	0·1465	94·54	3·06	0·1079
			103·82	8·24	0·2556	107·58	13·51	0·3817				94·25	3·05	0·1075
102·11	7·89	0·2481							95·57	4·24	0·1459			
			104·44	8·29	0·2562	107·68	13·50	0·3820				94·44	3·06	0·1078
100·63	7·78	0·2467												
99·12	7·65	0·2450	102·57	8·20	0·2561	106·14	13·29	0·3792						
98·98	7·69	0·2463	102·57	8·20	0·2561									
98·89	7·69	0·2463	102·48	8·20	0·2556	106·14	13·29	0·3792						
98·86	7·69	0·2464	102·32	8·19	0·2561									
			102·20	8·20	0·2566	104·49	13·02	0·3757						
98·96	7·68	0·2460				104·28	12·98	0·3752						
			102·43	8·20	0·2561	104·20	13·10	0·3757						
			101·30	8·18	0·2570	104·32	13·03	0·3755						
			101·00	8·15	0·2566									
			101·15	8·165	0·2568									
			99·57	8·10	0·2571									
			99·54	8·08	0·2566									
			99·30	8·10	0·2574									
			99·47	8·093	0·2570									

TABLE III. (*continued*).—*Particulars of Boiling Mixtures of Steam and various Salts.*

$NaNO_3$.			$Ba(NO_3)_2$.			$Sr(NO_3)_2$.			K_2SO_4.			$(NH_4)_2SO_4$.		
t	$t-T$	$\frac{p-P}{p}$	t	$t-T$	$\frac{p-P}{p}$	t	$t-T$	$\frac{p-P}{p}$	t	$t-T$	$\frac{p-P}{p}$	t	$t-T$	$\frac{p-P}{p}$
° C.	° C.		° C.	° C.		° C.	° C.		° C.	° C.		° C.	° C.	
112·45	18·04	0·4686	101·19	1·28	0·0445	106·52	6·52	0·2045	101·70	1·42	0·0493	108·08	7·80	0·2383
112·22	17·80	0·4643	101·19	1·28	0·0445				101·70	1·42	0·0493	107·04	7·64	0·2350
112·33	17·92	0·4664	95·55	1·13	0·0404				95·72	1·30	0·0467	107·56	7·72	0·2367
108·39	17·09	0·4576	95·37	1·15	0·0415				95·50	1·28	0 0460	101·80	7·39	0·2345
108·29	17·11	0·4581	95·46	1·14	0·0409				95·61	1·29	0·0463	101·70	7·46	0·2365
108·25	16·95	0·4550	92·59	1·12	0·0411				92·74	1·25	0·0458	101·75	7·425	0·2355
108·31	17·05	0·4569	92·59	1·12	0·0411				92·54	1·24	0·0454	98·56	7·26	0·2346
									92·64	1·24	0·0456	98·42	7·28	0·2354
												98·49	7·27	0·2350

TABLE IV.—*Temperature of Condensation of Steam on Salts and in their Brines.*

No., . . .	N.	1.			2.			3.			4.			5.		
Salt, . . .	rM.	0·2 KCl.			0·4 KCl.			0·2 KCl.			0·2 KCl.			0·2 KCl.		
Temp. of Saturated Steam, . .	T.	100·44° C.			100·10°–100·12° C.			99·21° C.			94·12°–94·13° C.			94·34°–94·35° C.		
	n	*t* – T	W	W(*t* – T)	*t* – T	W	W(*t* – T)	*t* – T	W	W(*t* – T)	*t* – T	W	W(*t* – T)	*t* – T	W	W(*t* – T)
	0	8·39	26·30	220·6	8·33	26·3	219·0	8·30	26·56	220·5	7·88	27·21	214·4	7·89	27·09	213·7
	1	8·28	25·91	214·5	8·22	26·14	214·9	8·10	26·76	216·8	7·68	27·25	209·3	7·66	27·76	212·6
	2	7·58	28·71	217·6	7·37	29·09	214·4	7·08	30·26	214·2	5·94	34·35	204·0	5·83	35·22	205·3
	3	6·53	32·61	212·9	6·85	30·93	212·4	6·06	34·46	208·8	4·93	40·35	198·9	4·81	41·58	200·0
	4	6·02	34·91	210·1	6·34	33·04	209·2	5·05	40·36	203·8	4·43	44·25	196·0	3·80	50·99	193·8
	5	5·54	37·31	206·6	5·86	35·33	206·8	4·54	44·26	201·1	3·92	49·25	193·1	2·79	67·63	188·8
	6	5·05	40·61	205·0	5·37	38·29	205·7	4·04	49·16	198·8	3·42	55·65	190·3	1·77	103·95	184·0
	7	4·56	44·31	202·0	4·87	41·63	202·7	3·53	55·36	195·6	2·91	64·45	187·4	.	.	.
	8	4·06	48·91	198·5	.	.	.	3·02	63·66	192·4	2·41	76·85	185·2	.	.	.
	9	3·56	54·91	195·9	.	.	.	2·51	75·76	190·2	.	.	.	.	.	.
	10	3·06	62·81	192·2	.	.	.	.	.	.	.	.	.	.	.	.
	11	2·58	73·31	189·1	.	.	.	.	.	.	.	.	.	.	.	.

No., . . .	N.	6.			7.			8.			9.			10.		
Salt, . . .	rM.	0·2 KCl.			0·2 KCl.			0·2 KCl.			0·2 NaCl.			0·4 NaCl.		
Temp. of Saturated Steam, . .	T.	94·37°–94·39° C.			91·18° C.			91·20° C.			100·44° C.			100·06° C.		
	n	*t* – T	W	W(*t* – T)	*t* – T	W	W(*t* – T)	*t* – T	W	W(*t* – T)	*t* – T	W	W(*t* – T)	*t* – T	W	W(*t* – T)
	0	7·87	26·58	209·2	7·69	26·70	204·0	7·69	27·12	208·6	8·54	29·85	254·9	8·48	59·88	253·8
	1	6·82	30·04	204·9	7·43	27·42	203·7	7·50	27·08	203·1	8·36	30·41	254·2	8·38	60·52	253·6
	2	5·79	34·64	200·6	6·72	30·11	202·3	6·69	30·41	203·4	7·53	33·31	250·8	7·49	66·92	250·6
	3	4·78	41·00	196·0	5·71	34·74	198·4	5·67	34·96	198·2	6·54	37·21	243·3	6·90	70·92	244·6
	4	3·26	58·18	189·7	5·20	37·75	196·3	4·66	41·57	193·7	5·99	39·91	239·0	6·41	75·32	241·4
	5	2·75	68·43	188·2	4·69	41·19	193·2	3·65	51·78	189·0	5·55	42·51	235·9	5·92	80·22	237·4
	6	.	.	.	.	.	.	2·64	69·50	183·5	5·06	45·61	230·8	5·43	86·52	234·9
	7	.	.	.	.	.	.	.	.	.	4·57	49·51	226·3	4·93	93·02	229·3
	8	.	.	.	.	.	.	.	.	.	4·07	54·21	220·5	.	.	.
	9	.	.	.	.	.	.	.	.	.	3·58	60·41	216·3	.	.	.
	10	.	.	.	.	.	.	.	.	.	3·08	68·41	210·7	.	.	.
	11	.	.	.	.	.	.	.	.	.	2·59	79·81	206·6	.	.	.

TABLE IV. (*continued*).—*Temperature of Condensation of Steam on Salts and in their Brines.*

No., . . .	N.	11.			12.			13.			14.			15.		
Salt, . . .	rM.	0·2 NaCl.			0·2 NaCl.			0·2 NaCl.			0·2 NaCl.			0·2 NaCl.		
Temp. of Saturated Steam, . .	T.	99·21° C.			94·13°–94·12° C.			94·37° C.			94·37°–94·34° C.			91·47° C.		
	n	*t* – T	W	W(*t* – T)	*t* – T	W	W(*t* – T)	*t* – T	W	W(*t* – T)	*t* – T	W	W(*t* – T)	*t* – T	W	W(*t* – T)
	0	8·44	29·84	252·4	8·19	29·50	241·6	8·21	29·21	239·8	8·21	28·65	235·2	8·11	.	.
	1	8·31	30·3	251·8	8·07	29·85	240·9	8·03	29·84	239·6	8·04	29·24	235·1	8·06	28·63	230·8
	2	7·19	34·4	247·3	6·96	33·65	234·2	7·13	33·44	238·4	7·14	32·83	234·4	7·10	32·88	233·4
	3	6·18	38·9	240·4	5·94	38·25	227·2	6·12	37·93	232·1	6·14	37·11	227·9	6·09	38·47	234·3
	4	5·16	44·8	231·2	4·93	44·30	218·4	5·10	43·72	223·0	5·12	43·04	220·4	.	.	.
	5	4·65	48·6	226·0	4·44	48·35	214·7	4·09	52·34	214·1	4·12	51·59	212·5	.	.	.
	6	4·15	53·4	221·6	3·93	53·45	210·1	3·08	66·13	203·7	.	.	.	.	.	.
	7	3·64	59·5	216·6	3·43	60·25	206·7	2·07	94·11	194·8	.	.	.	.	.	.
	8	3·14	67·5	212·0	2·92	69·05	201·6	.	.	.	.	.	.	.	.	.
	9	2·63	78·5	206·5	2·42	81·35	196·9	.	.	.	.	.	.	.	.	.
	10	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
	11	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
	12	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.

No., . . .	N.	16.			17.			18.			19.			20.		
Salt, . . .	rM.	0·2 NaCl.			0·2 NH_4Cl.			0·4 NH_4Cl.			0·4 NH_4Cl.			0·4 NH_4Cl.		
Temp. of Saturated Steam, . .	T.	91·20° C.			99·33°–99·38° C.			94·26° C.			94·26° C.			94·40°–94·41° C.		
	n	*t* – T	W	W(*t* – T)	*t* – T	W	W(*t* – T)	*t* – T	W	W(*t* – T)	*t* – T	W	W(*t* – T)	*t* – T	W	W(*t* – T)
	0	8·11	29·33	238·0	14·46	13·62	197·0	13·48	26·50	178·6	13·48	26·92	181·5	13·49	27·08	182·7
	1	7·90	30·02	237·2	14·12	13·54	191·2	13·24	27·24	180·3	13·24	27·47	181·8	12·44	29·66	184·4
	2	7·10	33·04	234·6	11·75	16·64	195·5	12·08	30·15	182·1	11·47	32·07	183·9	10·42	36·06	187·8
	3	6·09	37·66	229·4	9·78	19·84	194·0	11·07	33·35	184·6	9·96	37·30	185·7	9·41	40·06	188·5
	4	5·07	43·42	220·1	7·80	24·84	193·7	10·06	37·03	186·2	8·94	41·92	187·4	8·40	44·90	188·6
	5	4·06	52·20	211·9	6·80	28·34	192·7	9·05	41·34	187·0	7·93	47·15	186·9	7·39	49·62	183·3
	6	3·05	66·65	203·3	5·81	32·94	191·4	8·03	45·37	182·1	6·92	54·11	187·2	6·37	58·88	187·5
	7	.	.	.	4·83	39·04	188·6	7·02	53·57	188·0	5·91	63·04	186·3	5·35	69·30	185·3
	8	.	.	.	4·33	43·24	187·2	.	.	.	4·90	75·64	185·3	4·34	84·56	183·5
	9	.	.	.	3·81	48·54	184·9	.	.	.	.	.	.	3·33	109·10	181·6
	10	.	.	.	3·33	54·94	182·9	.	.	.	.	.	.	.	.	.
	11	.	.	.	2·83	63·94	181·0	.	.	.	.	.	.	.	.	.
	12	.	.	.	2·32	76·84	178·2	.	.	.	.	.	.	.	.	.

TABLE IV. (*continued*).—*Temperature of Condensation of Steam on Salts and in their Brines.*

No., . . .	N.	21.			22.			23.			24.			25.		
Salt, . . .	rM.	0·4 NH_4Cl.			0·1 $BaCl_2.2H_2O$.			0·2 $BaCl_2.2H_2O$.			0·1 $BaCl_2.2H_2O$.			0·2 $KClO_3$.		
Temp. of Saturated Steam, . .	T.	91·20° C.			99·95° C.			94·42° C.			91·18° C.			100·30° C.		
	n	*t* − T	W	W(*t* − T)	*t* − T	W	W(*t* − T)	*t* − T	W	W(*t* − T)	*t* − T	W	W(*t* − T)	*t* − T	W	W(*t* − T)
	0	12·96	13·95	180·8	4·51	32·01	288·8	4·38	73·7	322·8	4·25	36·6	310·9	3·78	40·10	151·6
	1	12·77	28·5	181·9	4·48	32·07	287·4	4·23	73·7	311·8	4·14	36·7	303·8	3·71	40·43	150·0
	2	10·93	33·8	184·6	4·10	34·97	287·0	3·62	85·6	309·9	3·66	41·4	303·0	3·45	44·63	154·0
	3	9·92	37·6	186·5	3·59	39·67	284·8	3·12	96·6	301·4	3·16	46·9	296·4	3·00	52·43	157·3
	4	8·90	42·0	186·9	3·08	45·67	281·2	2·71	108·5	294·0	2·65	54·3	287·8	2·50	63·73	159·3
	5	7·89	47·3	186·6	2·58	53·27	274·4	2·31	123·1	284·4	.	.	.	2·00	80·83	161·7
	6	6·88	54·2	186·4	2·07	64·97	273·0	2·00	138·7	277·4	.	.	.	1·51	111·13	167·8
	7	5·87	63·1	185·2	1·57	83·47	262·2	1·70	159·1	270·5	.	.	.	.	.	.
	8	4·86	75·9	184·4	.	.	.	1·40	186·8	261·5	.	.	.	.	.	.
	9	.	.	.	.	.	.	1·20	236·6	283·9	.	.	.	.	.	.

No., . . .	N.	26.			27.			28.			29.			30.		
Salt, . . .	rM.	0·2 $KClO_3$.			0·2 $KClO_3$.			0·2 $KClO_3$.			0·05 $Ba(NO_3)_2$.			0·2 $Ba(NO_3)_2$.		
Temp. of Saturated Steam, . .	T.	98·77°–98·79° C.			94·43° C.			91·20° C.			99·91° C.			94·42° C.		
	n	*t* − T	W	W(*t* − T)	*t* − T	W	W(*t* − T)	*t* − T	W	W(*t* − T)	*t* − T	W	W(*t* − T)	*t* − T	W	W(*t* − T)
	0	3·61	42·2	152·3	3·30	47·5	156·7	3·04	50·5	153·4	1·28	37·5	192·1	1·12	163·5	183·2
	1	3·55	42·2	149·8	3·22	47·0	151·3	2·93	50·55	148·6	1·22	39·88	194·6	1·08	169·3	182·8
	2	3·28	46·5	152·5	2·81	54·8	154·0	2·53	59·37	150·2	1·10	44·88	197·5	0·98	190·5	186·7
	3	2·97	52·0	154·4	2·41	64·8	156·2	2·03	75·50	153·3	1·00	50·08	200·3	0·88	215·0	189·2
	4	2·67	58·6	156·5	2·00	78·6	157·2	1·52	102·81	156·2	0·80	65·28	208·9	0·78	247·3	192·9
	5	2·37	67·0	158·8	1·70	93·8	159·5	.	.	.	0·70	74·88	209·7	0·68	288·9	196·5
	6	2·06	77·8	160·3	1·52	106·8	162·3	.	.	.	0·60	92·48	224·4	0·58	348·4	202·1
	7	1·86	87·1	162·0	1·29	125·8	162·3	.	.	.	.	.	.	0·48	419·6	201·4
	8	1·65	99·3	163·8	1·09	149·4	162·8	.	.	.	.	.	.	0·38	584·1	221·9
	9	1·43	114·3	163·5	.	.	.	.	.	.	.	.	.	.	.	.

TABLE IV. (*continued*).—*Temperature of Condensation of Steam on Salts and in their Brines.*

No., . . .	N.	31.			32.			33.			34.			35.		
Salt, . . .	rM.	0·2 $Sr(NO_3)_2$.			0·2 $Pb(NO_3)_2$.			0·1 K_2SO_4.			0·033 K_2SO_4.			0·05 K_2SO_4.		
Temp. of Saturated Steam, . .	T.	100·00° C.			99·60° C.			100·28° C.			94·42° C.			91·30° C.		
	n	*t* – T	W	W(*t* – T)	*t* – T	W	W(*t* – T)	*t* – T	W	W(*t* – T)	*t* – T	W	W(*t* – T)	*t* – T	W	W(*t* – T)
	0	6·53	42·8	279·3	3·29	50·00	164·6	1·38	73·0	201·5	1·28	.	.	1·23	.	.
	1	6·32	42·3	267 3	3·15	51·7	162·8	1·33	75·3	200·3	1·18	25·8	182·64	1·21	39·01	188·8
	2	5·44	49·7	270·4	2·94	56·5	166·1	1·21	84·1	203·5	0·98	30·5	179·3	0·81	61·56	199·5
	3	4·94	53·9	266·3	2·64	62·9	166·1	1·00	102·94	205·9	0·88	34·7	183·2	.	.	.
	4	4·45	59·0	262·6	2·44	69·0	168·4	.	.	.	0·78	38·4	179·7	.	.	.
	5	3·95	66·0	260·7	2 24	75·2	168·6	.	.	.	0·68	43·8	178·7	.	.	.
	6	3·46	74·6	258·1	2·03	83·2	168·9	.	.	.	0·58	51·6	179·6	.	.	.
	7	2·96	86·0	254·5	1·82	92·9	169·1	.	.	.	0·48	64·8	186·6	.	.	.
	8	2·47	102·3	252·7	1·62	106·1	171·9	.	.	.	.	.	.	.	.	.
	9	.	.	.	1·42	126·8	180·0	.	.	.	.	.	.	.	.	.
	10	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
	11	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
	12	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.

No., . . .	N.	36.			37.			38.			39.			40.		
Salt, . . .	rM.	0·1 $(NH_4)_2SO_4$.			0·4 $(NH_4)_2SO_4$.			0·4 $(NH_4)_2SO_4$.			0·2 $(NH_4)_2SO_4$.			0·2 $(NH_4)_2SO_4$.		
Temp. of Saturated Steam, . .	T.	99·40° C.			100·28° C.			94·24° C.			94·41° C.			91·15°–91·17° C.		
	n	*t* – T	W	W(*t* – T)	*t* – T	W	W(*t* – T)	*t* – T	W	W(*t* – T)	*t* – T	W	W(*t* – T)	*t* – T	W	W(*t* – T)
	0	7·63	12·64	192·88	7·81	50·7	197·9	7·46	51·9	193·6	7·38	25·8	190·1	7·25	26·2	190·1
	1	7·39	13·28	196·49	7·52	51·8	194·8	7·42	47·7	176·9	7·17	24·8	177·8	7·16	22·7	162·5
	2	5·97	16·18	191·14	6·93	56·2	194·7	6·94	55·2	191·5	6·36	29·5	187·6	5·91	31·5	186·2
	3	5·06	18·79	190·00	6·42	60·1	192·9	5·93	63·2	187·4	5·35	34·6	185·1	4·90	37·4	183·3
	4	4·05	22·94	185·81	5·93	64·8	192·1	.	.	.	4·34	41·7	181·0	3·88	46·4	180·0
	5	3·54	25·95	183·69	5·44	69·9	190·1	.	.	.	3·33	53·7	178·8	.	.	.
	6	3·04	29·68	180·44	4·94	76·2	188·2	.	.	.	2·32	75·4	174·9	.	.	.
	7	2·53	35·28	178·51	4·45	83·7	186·2	.	.	.	.	.	.	.	.	.
	8	2·02	43·78	176·87	3·95	93·5	184·7	.	.	.	.	.	.	.	.	.
	9	1·82	48·97	178·24	3·46	106·2	183·7	.	.	.	.	.	.	.	.	.
	10	1·62	55·19	178·82	.	.	.	.	.	.	.	.	.	.	.	.
	11	1·42	64·00	181·76	.	.	.	.	.	.	.	.	.	.	.	.
	12	1·21	75·62	182·99	.	.	.	.	.	.	.	.	.	.	.	.

TABLE IV. (*continued*).—*Temperature of Condensation of Steam on Salts and in their Brines.*

No., . . .	N.	41.			42.			43.			44.			45.		
Salt, . . .	rM.	0·2 $(NH_4)_2SO_4$.			0·2 $\frac{K+Na}{2}Cl$.			0·4 $\frac{K+Na}{2}Cl$.			0·4 $\frac{K+Na}{2}Cl$.			0·2 $\frac{K_2+Na_3}{5}Cl$.		
Temp. of Saturated Steam, . .	T.	91·30° C.			100·44° C.			100·13°–100·15° C.			94·26°–94·25° C.			100·18° C.		
	n	*t* − T	W	W(*t* − T)	*t* − T	W	W(*t* − T)	*t* − T	W	W(*t* − T)	*t* − T	W	W(*t* − T)	*t* − T	W	W(*t* − T)
	0	7·24	26·4	191·1	11·87	20·7	245·7	11·81	40·06	241·3	11·19	.	.	11·81	.	.
	1	6·98	26·2	182·7	11·53	20·95	241·6	11·46	41·98	240·5	10·89	41·9	228·2	10·58	23·0	243·3
	2	6·17	30·4	187·5	10·44	23·35	243·8	10·80	44·68	241·2	9·87	46·1	227·5	9·38	25·9	243·0
	3	5·16	35·7	184·1	9·01	26·35	237·4	9·86	48·38	238·0	9·17	49·2	225·6	8·39	28·6	239·0
	4	4·15	43·5	180·4	8·01	29·15	233·5	8·88	52·98	235·2	8·17	54·4	222·3	7·40	31·5	233·1
	5	3·04	57·9	176·0	7·04	32·25	227·0	8·38	55·68	233·4	7·65	57·6	220·3	6·41	35·5	227·6
	6	2·13	82·1	174·8	6·04	36·75	222·0	7·88	58·58	230·9	7·15	61·0	218·1	5·92	38·0	225·0
	7	.	.	.	5·55	39·45	218·9	7·39	61·78	228·3	6·64	65·1	216·1	5·43	40·8	221·5
	8	.	.	.	5·06	42·65	215·8	6·89	65·78	226·7	6·14	69·7	214·0	4·93	44·2	217·9
	9	.	.	.	4·57	46·45	212·3	6·39	70·08	223·9	5·62	75·2	211·3	4·44	48·2	214·0
	10	.	.	.	4·07	51·15	208·2	5·90	74·88	220·9	.	.	.	3·94	53·2	209·6
	11	.	.	.	3·58	57·25	205·0	5·41	80·88	218·8	.	.	.	3·45	59·7	206·0
	12	.	.	.	3·08	64·95	200·0	4·90	87·48	214·3	.	.	.	2·95	68·3	203·5
	13	.	.	.	2·59	75·85	196·5	.	.	.	.	.	.	2·46	80·5	198·0

No., . . .	N.	46.			47.			48.			49.			50.		
Salt, . . .	rM.	0·2 $\frac{K_3+Na_2}{5}Cl$.			0·2 $\frac{K+Na_2}{3}Cl$.			0·2 $\frac{K_2+Na}{3}Cl$.			0·2 $\frac{K+Na_3}{4}Cl$.			0·2 $\frac{K_3+Na}{4}Cl$.		
Temp. of Saturated Steam, . .	T.	100·18° C.			100·23° C.			100·23° C.			100·22° C.			100·17° C.		
	n	*t* − T	W	W(*t* − T)	*t* − T	W	W(*t* − T)	*t* − T	W	W(*t* − T)	*t* − T	W	W(*t* − T)	*t* − T	W	W(*t* − T)
	0	11·84	.	.	11·86	.	.	11·80	.	.	11·82	.	.	11·80	.	.
	1	10·80	22·15	239·2	9·98	24·59	245·4	10·32	22·43	231·5	9·56	25·7	245·7	9·66	23·56	227·6
	2	9·38	25·35	236·9	8·89	27·49	244·4	9·39	25·83	242·5	8·40	29·4	247·0	8·40	27·06	227·3
	3	8·39	27·85	233·7	7·91	30·29	239·6	7·91	28·63	226·5	7·41	32·1	237·9	7·50	30·16	226·2
	4	7·40	30·85	228·3	6·92	33·79	233·9	6·92	32·13	222·3	6·87	34·3	235·6	6·42	33·86	217·4
	5	6·41	34·85	223·4	6·36	36·29	230·9	5·93	36·63	217·2	6·42	36·4	233·7	5·93	36·46	216·2
	6	5·92	37·25	220·5	5·93	38·29	227·1	5·44	39·33	213·9	5·93	38·6	228·9	5·44	39·06	212·4
	7	5·43	40·25	218·6	5·38	41·69	224·3	4·94	42·63	210·4	5·44	41·3	224·7	4·94	42·46	209·8
	8	4·93	43·55	214·7	4·94	45·69	225·8	4·45	46·63	207·4	4·94	44·9	221·8	4·45	46·36	206·3
	9	4·44	47·55	211·1	4·45	48·49	215·8	3·95	51·43	203·0	4·45	48·8	217·2	3·95	51·36	202·8
	10	3·94	52·65	207·4	3·95	53·59	211·7	3·46	58·03	200·7	3·95	53·9	212·9	3·46	57·56	200·2
	11	3·45	59·15	204·1	3·46	60·19	208·3	2·96	66·43	196·5	3·46	59·3	205·8	2·96	66·16	195·8
	12	2·95	67·85	200·2	2·96	68·69	203·4	2·45	78·63	192·6	2·96	68·6	203·0	2·47	77·86	192·3
	13	2·46	80·35	197·7	2·47	80·59	199·1	.	.	.	2·47	80·5	198·9	.	.	.

TABLE IV. (*continued*).—*Temperature of Condensation of Steam on Salts and in their Brines.*

No., . . .	N.	51.			52.			53.			54.			55.		
Salt, . . .	rM.	$0{\cdot}2\,\frac{K+Na_4}{5}Cl.$			$0{\cdot}2\,\frac{K_4+Na}{5}Cl.$			$0{\cdot}2\,\frac{K+Na_2}{3}Cl.$			$0{\cdot}2\,\frac{K_2+Na}{3}Cl.$			$0{\cdot}2\,\frac{K+Na_3}{4}Cl.$		
Temp. of Saturated Steam, . .	T.	100·18° C.			100·18° C.			94·20° C.			94·20° C.			94·20° C.		
	n	$t-T$	W	$W(t-T)$	$t-T$	W	$W(t-T)$	$t-T$	W	$W(t-T)$	$t-T$	W	$W(t-T)$	$t-T$	W	$W(t-T)$
	0	11·80	.	.	11·78	.	.	11·19	.	.	11·18	.	.	11·17	.	.
	1	9·21	26·86	247·4	9·16	25·0	229·0	9·63	24·0	231·1	9·43	23·26	219·3	9·13	25·35	231·4
	2	8·38	29·36	246·0	8·39	26·75	224·4	8·62	26·2	225·8	8·42	25·74	216·7	8·11	27·75	225·1
	3	7·40	32·46	240·2	7·40	29·95	221·6	7·60	29·3	222·7	7·40	28·86	212·6	7·10	31·05	220·5
	4	6·41	36·46	233·7	6·41	33·95	217·6	6·59	33·1	218·1	6·39	32·66	208·7	6·08	35·25	214·3
	5	5·92	38·86	230·0	5·92	36·25	214·6	6·08	35·3	214·6	5·87	35·00	205·5	5·57	37·89	211·0
	6	5·43	41·86	227·2	5·43	39·15	212·5	5·57	37·9	211·1	5·37	37·94	203·7	5·07	41·01	207·9
	7	4·93	44·96	221·6	4·93	42·45	209·2	5·07	41·2	208·9	4·86	41·14	200·0	4·57	44·85	205·0
	8	4·44	50·26	223·2	4·44	46·25	205·3	4·56	44·9	204·7	4·36	45·34	193·1	4·06	49·35	200·4
	9	3·94	54·26	213·7	3·94	51·25	201·9	4·06	49·9	202·6	3·85	50·60	194·8	3·55	55·35	196·5
	10	3·45	60·66	209·3	3·45	57·45	198·0	.	.	.	.	.	.	3·05	63·30	193·1
	11	2·95	69·66	205·5	2·95	66·15	195·2	.	.	.	.	.	.	.	.	.
	12	2·46	82·16	202·1	2·46	78·25	192·5	.	.	.	.	.	.	.	.	.

No., . . .	N.	56.			57.			58.			59.			60.		
Salt, . . .	rM.	$0{\cdot}2\,\frac{K_3+Na}{4}Cl.$			$0{\cdot}2\,\frac{K+Na_4}{5}Cl.$			$0{\cdot}2\,\frac{K_4+Na}{5}Cl.$			$0{\cdot}4\,\frac{NaNH_4}{2}Cl.$			$0{\cdot}4\,\frac{NaNH_4}{2}Cl.$		
Temp. of Saturated Steam, . .	T.	94·17° C.			94·16°–94·14° C.			94·14°–94·13° C.			94·30° C.			94·30° C.		
	n	$t-T$	W	$W(t-T)$	$t-T$	W	$W(t-T)$	$t-T$	W	$W(t-T)$	$t-T$	W	$W(t-T)$	$t-T$	W	$W(t-T)$
	0	11·19	.	.	11·18	.	.	11·01	24·00	.	15·52	.	.	15·52	.	.
	1	8·95	24·33	217·7	8·87	25·90	229·7	8·27	24·00	198·5	10·60	41·6	220·5	10·57	40·7	215·1
	2	7·93	27·18	215·5	7·95	28·44	226·1	6·95	28·16	195·7	9·62	45·7	219·8	9·59	44·9	215·3
	3	6·92	30·63	212·0	6·94	31·16	216·2	5·94	32·16	191·0	8·63	50·3	217·0	8·60	49·3	212·0
	4	6·42	32·52	208·8	5·92	36·36	215·2	5·43	34·72	188·5	7·64	56·1	214·3	7·61	55·1	209·6
	5	5·90	35·07	206·9	5·42	39·16	212·2	4·93	37·76	186·1	6·65	63·4	210·8	6·62	62·0	205·2
	6	5·40	37·83	204·3	4·91	42·52	208·8	4·43	41·36	183·2	6·16	67·7	208·5	6·13	66·2	202·9
	7	4·89	41·22	201·6	4·41	46·36	204·4	3·92	46·32	181·6	5·66	72·7	205·7	5·63	71·2	200·4
	8	4·39	45·27	198·7	3·91	51·56	201·6	3·42	52·56	179·7	5·17	78·6	203·2	5·14	77·4	198·9
	9	3·88	50·28	195·1	3·41	57·96	196·7	2·91	60·80	176·9	.	.	.	.	.	.
	10	3·38	57·03	192·7	.	.	.	.	.	.	.	.	.	.	.	.
	11	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
	12	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.

TABLE IV. (*continued*).—*Temperature of Condensation of Steam on Salts and in their Brines.*

No., . .	N.	61.			62.			63.			64.		
Salt, . .	rM.	$0{\cdot}2\,\frac{2KCl+KClO_3}{3}$.			$0{\cdot}4\,\frac{3KCl+KClO_3}{4}$			$0{\cdot}2\,(KCl+KClO_3)$.			$0{\cdot}2\left(\frac{(NH_4)_2SO_4}{2}+NH_4Cl\right)$.		
Temp. of Saturated Steam, .	T.	100·28° C.			100·28° C.			94·10° C.			93·86°–93·85° C.		
	n	$t-T$	W	$W(t-T)$	$t-T$	W	$W(t-T)$	$t-T$	W	$W(t-T)$	$t-T$	W	$W(t-T)$
	0	9·82	.	.	9·87	.	.	8·98	.	.	15·49	.	.
	1	7·09	26·83	190·2	9·34	43·15	201·5	5·15	33·8	174·1	12·64	21·25	179·1
	2	6·37	29·93	190·6	8·39	47·65	199·9	4·66	37·3	173·8	10·98	25·45	186·3
	3	5·82	32·63	190·0	7·41	53·65	198·8	4·16	41·8	173·9	9·99	27·95	186·1
	4	5·22	36·23	189·1	6·92	57·25	198·1	3·65	47·3	172·6	9·01	30·95	185·9
	5	4·45	41·93	186·6	6·42	61·35	197·0	3·15	54·6	172·0	8·02	34·75	185·8
	6	3·95	46·93	185·4	5·93	66·15	196·1	2·64	65·1	171·8	7·04	39·55	185·6
	7	3·46	53·03	183·5	5·43	71·65	194·5	2·23	77·4	172·6	6·56	41·35	181·2
	8	2·96	61·43	181·8	4·94	78·55	194·0	.	.	.	6·06	45·55	184·0
	9	2·47	72·93	180·1	4·45	86·25	191·9	.	.	.	5·08	53·85	182·4
	10	.	.	.	3·95	96·85	191·3	.	.	.	4·59	58·90	180·2
	11	.	.	.	.	.	.	.	.	.	4·09	65·80	179·4

No., . .	N.	65.			66.			67.			68.		
Salt, . .	rM.	$0{\cdot}1\left(\frac{K_2SO_4}{2}+KCl\right)$.			$0{\cdot}1\left(\frac{K_2SO_4}{2}+NaCl\right)$.			$0{\cdot}05\ Ba(NO_3)_2+0{\cdot}2NaCl$.			$0{\cdot}05\ Ba(NO_3)_2+0{\cdot}1NaCl$.		
Temp. of Saturated Steam, .	T.	94·12° C.			93·83°–93·80° C.			94·10° C.			93·97°–93·95° C.		
	n	$t-T$	W	$W(t-T)$	$t-T$	W	$W(t-T)$	$t-T$	W	$W(t-T)$	$t-T$	W	$W(t-T)$
	0	7·93	.	.	10·32	.	.	9·75	.	.	9·72	.	.
	1	2·11	66·35	186·6	3·44	42·95	196·9	6·69	41·7	223·2	3·59	42·25	202·3
	2	2·01	70·35	188·5	3·03	48·05	194·1	6·18	44·7	221·0	3·09	48·65	199·9
	3	1·91	73·65	187·6	2·63	54·65	191·6	5·67	47·9	217·3	2·59	57·95	200·1
	4	1·80	77·05	184·9	2·43	58·95	190·6	5·17	52·6	217·5	2·09	70·85	197·5
	5	1·70	81·65	185·1	2·23	63·85	189·9	4·66	57·5	214·3	.	.	.
	6	.	.	.	2·04	69·95	190·3	4·16	64·0	213·0	.	.	.
	7	.	.	.	.	.	.	.	.	.	.	.	.
	8	.	.	.	.	.	.	.	.	.	.	.	.
	9	.	.	.	.	.	.	.	.	.	.	.	.
	10	.	.	.	.	.	.	.	.	.	.	.	.
	11	.	.	.	.	.	.	.	.	.	.	.	.

TABLE IV. (*continued*).—*Temperature of Condensation of Steam on Salts and in their Brines.*

No., . .	N.	69.			70.			71.			72.		
Salt, . .	rM.	$0{\cdot}1\frac{4BaCl_2.2H_2O+Ba(NO_3)_2}{5}$			$0{\cdot}1\frac{5BaCl_2.2H_2O+2Ba(NO_3)_2}{7}$			$0{\cdot}05(BaCl_2.2H_2O+Ba(NO_3)_2)$			$0{\cdot}4\ \frac{6(NH_4)_2SO_4+K_2SO_4}{7}$		
Temp. of Saturated Steam, .	T.	99·90° C.			99·98° C.			93·94°-93·91° C.			100·28° C.		
	n	$t-T$	W	$W(t-T)$	$t-T$	W	$W(t-T)$	$t-T$	W	$W(t-T)$	$t-T$	W	$W(t-T)$
	0	5·56	.	.	5·59	.	.	5·40	.	.	7·24	.	.
	1	4·95	25·2	259·4	5·28	22·9	241·8	2·32	47·7	221·4	5·99	66·15	198·1
	2	4·15	31·1	258·2	4·57	27·0	246·8	2·12	52·7	223·4	5·45	70·65	192·5
	3	3·64	35·7	259·8	4·07	30·6	249·3	1·91	57·7	220·4	4·96	77·25	191·6
	4	3·13	41·1	257·2	3·56	34·8	247·8	1·72	66·2	227·8	4·46	84·75	189·0
	5	2·63	48·5	255·2	3·06	40·7	249·1	1·61	69·8	226·8	3·97	94·55	192·7
	6	2·12	59·4	251·8	2·55	48·4	246·8	.	.	.	.	.	.
	7	1·62	76·5	247·8	2·04	59·7	243·6	.	.	.	.	.	.
	8	1·11	110·3	244·8	1·54	77·9	239·7	.	.	.	.	.	.
	9	.	.	.	.	.	.	.	.	.	.	.	.

No., . .	N.	73.			74.			75.			76.		
Salt, . .	rM.	$0{\cdot}1\ \left.\begin{matrix}K\\NH_4\end{matrix}\right\}SO_4$.			$0{\cdot}16\ Sr(NO_3)_2+$ $0{\cdot}0423Ba(NO_3)_2$.			$0{\cdot}1\ \frac{Sr_2+Ba}{3}(NO_3)_2$.			$0{\cdot}2\ \frac{3Pb+Ba}{4}(NO_3)_2$.		
Temp. of Saturated Steam, .	T.	94·13° C.			100·00° C.			99·60° C.			99·60° C.		
	n	$t-T$	W	$W(t-T)$	$t-T$	W	$W(t-T)$	$t-T$	W	$W(t-T)$	$t-T$	W	$W(t-T)$
	0	5·36	.	.	6·31	.	.	5·43	.	.	2·90	.	.
	1	2·31	39·25	181·4	3·02	79·7	237·9	2·23	50·1	222·4	2·42	68·6	166·0
	2	2·11	43·35	183·0	2·74	88·4	239·8	2·02	56·0	226·2	2·22	75·6	167·8
	3	1·90	47·65	181·0	2·47	99·2	242·5	1·72	66·1	227·4	2·01	84·8	170·4
	4	1·70	53·55	182·0	.	.	.	1·52	75·1	228·4	1·81	94·9	171·8
	5	1·60	56·95	182·2	.	.	.	1·32	88·2	232·8	1·61	110·6	178·1
	6	1·49	60·45	180·2	.	.	.	1·11	106·3	236·0	1·41	132·6	186·9
	7	1·39	65·15	181·0	.	.	.	1·01	119·8	242·0	.	.	.
	8	1·29	70·35	181·4	.	.	.	.	.	.	.	.	.
	9	1·19	77·35	184·0	.	.	.	.	.	.	.	.	.

TABLE IV. (*continued*).—*Temperature of Condensation of Steam on Salts and in their Brines.*

No., . .	N.	77.			78.			79.		
Salt, . .	rM.	$0{\cdot}1\ \frac{2Pb+Ba}{3}(NO_3)_2$.			$0{\cdot}2\ \frac{5Sr+4Pb}{9}(NO_3)_2$.			$0{\cdot}2\ \frac{2Sr+Pb}{3}(NO_3)_2$.		
Temp. of Saturated Steam, .	T.	99·62°–99·63° C.			100·29° C.			100·29° C.		
	n	*t* – T	W	W(*t* – T)	*t* – T	W	W(*t* – T)	*t* – T	W	W(*t* – T)
	0	2·89	.	.	5·93	.	.	5·98	.	.
	1	2·00	42·2	168·8	4·77	45·7	218·0	5·27	44·0	231·9
	2	1·70	51·3	174·4	4·16	52·3	221·7	4·46	51·4	229·2
	3	1·50	59·7	179·2	3·55	60·1	213·4	4·06	56·1	227·8
	4	1·30	71·4	185·6	3·05	69·5	212·0	3·55	63·7	226·1
	5	1·08	88·2	190·6	2·54	83·8	212·9	3·05	73·6	224·5
	6	0·98	100·3	196·6	2·33	90·7	211·3	2·54	88·2	224·0
	7	0·88	113·6	200·0	2·13	100·1	213·2	2·34	95·6	223·7
	8	.	.	.	1·93	111·0	214·2	2·13	104·7	223·0
	9	.	.	.	1·73	126·7	219·2	1·93	116·5	224·8
	10	.	.	.	.	.	.	1·73	132·4	229·0
	11	.	.	.	.	.	.	.	.	.

No., . .	N.	80.			81.			82.		
Salt, . .	rM.	0·2 $NaNO_3$.			0·5 $NaNO_3$.			0·5 $NaNO_3$.		
Temp. of Saturated Steam, .	T.	94·41° C.			91·18° C.			91·30° C.		
	n	*t* – T	W	W(*t* – T)	*t* – T	W	W(*t* – T)	*t* – T	W	W(*t* – T)
	0	17·93	8·90	159·5	16·91	21·9	148·6	16·89	24·50	164·7
	1	17·60	8·84	155·6	16·30	22·00	143·4	16·29	24·98	162·8
	2	14·96	10·80	161·6	14·21	26·7	151·8	13·92	30·8	171·5
	3	12·99	12·54	163·0	12·56	30·5	153·2	11·39	38·9	177·2
	4	11·01	14·97	164·8	10·91	35·4	154·5	10·25	41·9	171·7
	5	10·02	16·64	166·7	.	.	.	9·42	45·8	172·8
	6	8·77	19·02	166·8	.	.	.	8·43	51·0	172·0
	7	7·50	22·48	168·6	.	.	.	7·50	57·5	172·5
	8	6·48	26·13	169·3	.	.	.	.	.	.
	9	5·49	30·86	169·4	.	.	.	.	.	.
	10	4·47	38·27	171·0	.	.	.	.	.	.
	11	3·83	44·84	171·7	.	.	.	.	.	.

TABLE V.—*Temperature of Condensation of Steam in diluted Brines.*

No.,	N.	83.			84.			85.			86.		
Salt,	rM.	0·05 LiCl.			0·05 KCl.			0·05 NaCl.			0·05 $\frac{Na+K}{2}$ Cl.		
Temp. of Saturated Steam,	T.	99·88°-99·87° C.			99·75° C.			99·75° C.			99·42°-99·45° C.		
	n	$t-T$	W	W$(t-T)$	$t-T$	W	W$(t-T)$	$t-T$	W	W$(t-T)$	$t-T$	W	W$(t-T)$
	0	3·97	16·1	63·9	3·59	14·2	51·2	5·82	10·7	62·3	5·54	10·5	58·2
	1	2·95	19·8	58·4	2·58	19·0	49·0	3·79	15·0	56·8	4·03	13·4	54·0
	2	2·45	23·0	56·3	2·07	23·0	47·6	2·58	20·7	53·4	3·02	17·2	52·0
	3	1·94	27·4	53·2	1·57	29·9	47·0	2·07	24·7	51·1	2·51	20·1	50·5
	4	1·74	30·3	52·7	1·37	34·0	46·6	1·57	31·7	50·0	2·00	24·2	48·4
	5	1·54	33·6	51·7	1·16	39·9	46·3	1·37	36·1	49·5	1·70	28·2	48·0
	6	1·34	37·9	50·8	0·96	47·9	46·0	1·16	42·2	49·0	1·49	31·7	47·2
	7	1·13	43·8	49·5	0·76	60·3	45·8	0·96	51·4	49·3	1·29	36·3	46·8
	8	0·93	52·8	49·1	0·66	69·7	46·0	0·76	65·6	49·3	1·08	42·7	46·1
	9	0·84	59·2	49·7	0·56	83·0	46·5	0·66	77·9	51·4	0·98	46·8	45·9
	10	0·74	66·9	49·5	0·46	106·6	49·0	0·56	92·2	51·6	0·88	52·7	46·4
	11	0·64	77·2	49·4	.	.	.	0·46	113·7	52·3	0·78	59·3	46·3
	12	0·54	90·8	49·0	.	.	.	.	.	.	0·68	68·6	46·7
	13	0·44	112·6	49·5	.	.	.	.	.	.	0·58	81·8	47·5
	14	.	.	.	.	.	.	.	.	.	0·47	101·9	47·9
	15	.	.	.	.	.	.	.	.	.	0·36	141·1	50·8

No.,	N.	87.			88.			89.			90.		
Salt,	rM.	0·05 NH_4Cl.			0·05 RbCl.			0·05 CsCl.			0·05 $BaCl_2.2H_2O$.		
Temp. of Saturated Steam,	T.	99·46°-99·47° C.			99·51°-99·52° C.			99·51° C.			99·49°-99·50° C.		
	n	$t-T$	W	W$(t-T)$	$t-T$	W	W$(t-T)$	$t-T$	W	W$(t-T)$	$t-T$	W	W$(t-T)$
	0	4·49	9·1	40·9	5·05	10·1	51·0	5·55	9·5	52·7	2·54	27·1	68·8
	1	4·09	11·8	48·3	4·04	12·4	50·1	4·04	12·5	50·5	2·03	32·6	66·2
	2	3·08	15·3	47·2	3·03	15·8	47·9	3·03	15·8	47·9	1·83	35·7	65·3
	3	2·57	17·9	46·0	2·52	18·6	46·9	2·02	22·7	45·9	1·63	39·7	64·7
	4	2·06	21·8	44·9	2·01	22·7	45·6	1·61	28·2	45·4	1·43	44·7	63·9
	5	1·66	26·9	44·6	1·81	25·1	45·4	1·41	32·2	45·4	1·23	51·4	63·2
	6	1·46	30·3	44·2	1·61	28·0	45·1	1·20	37·2	44·6	1·02	60·7	61·9
	7	1·25	35·3	44·1	1·41	32·0	45·1	1·00	45·0	45·0	0·92	68·2	62·7
	8	1·05	42·2	44·3	1·20	37·3	44·8	0·90	49·8	44·8	0·82	75·7	62·1
	9	0·85	51·4	43·7	1·00	45·7	45·7	0·80	56·5	45·2	0·72	86·6	62·4
	10	0·74	64·8	.	0·90	49·9	44·9	0·70	63·9	44·7	0·62	101·7	63·6
	11	0·64	69·6	44·6	0·80	56·1	44·9	0·60	78·1	46·9	0·51	126·0	64·2
	12	0·54	82·0	44·3	0·70	64·4	45·1	0·50	96·8	48·4	.	.	.
	13	0·44	101·4	44·6	0·60	77·9	46·7	0·40	120·3	48·1	.	.	.
	14	.	.	.	0·49	95·1	46·6	.	.	.	.	.	.
	15	.	.	.	0·39	124·1	48·4	.	.	.	.	.	.

TABLE V. (*continued*)—*Temperature of Condensation of Steam in diluted Brines.*

No., . . .	N.	91.			92.			93.			94.		
Salt, . . .	rM.	0·05 KI.			0·05 KBr.			0·05 $KClO_3$.			0·05 KNO_3.		
Temp. of Saturated Steam, .	T.	99·59° C.			99·59° C.			99·31°-99·29° C.			99·57°-99·55° C.		
	n	*t* − T	W	W(*t* − T)	*t* − T	W	W(*t* − T)	*t* − T	W	W(*t* − T)	*t* − T	W	W(*t* − T)
	0	6·08	10·2	62·0	5·07	11·0	55·8	3·02	13·2	39·9	3·27	6·8	22·2
	1	4·56	12·8	54·4	3·55	14·8	52·5	2·52	16·2	40·8	2·56	14·8	37·9
	2	3·55	15·7	55·7	2·54	19·7	50·0	2·01	20·3	40·8	2·04	18·5	37·7
	3	2·54	20·7	52·6	2·03	24·2	49·1	1·71	24·4	41·7	1·54	25·2	37·8
	4	2·03	25·1	51·0	1·63	29·2	47·6	1·51	27·6	41·7	1·34	29·4	39·4
	5	1·53	32·4	49·6	1·43	33·0	47·2	1·31	32·3	42·3	1·13	35·0	39·6
	6	1·43	34·9	50·0	1·22	38·4	46·8	1·10	38·1	41·9	1·02	38·8	39·6
	7	1·22	39·7	48·4	1·02	45·6	46·5	1·00	41·7	41·7	0·92	42·6	39·2
	8	1·02	46·9	47·8	0·92	50·7	46·6	0·90	45·9	41·3	0·82	48·3	39·6
	9	0·92	51·6	47·5	0·82	56·5	46·3	0·80	51·6	41·3	0·72	55·6	40·0
	10	0·82	57·7	47·3	0·72	64·7	46·6	0·70	59·8	41·9	0·62	63·9	39·6
	11	0·72	65·2	46·4	0·62	75·1	46·6	0·61	69·6	42·5	0·53	78·4	41·6
	12	0·62	75·8	47·0	0·52	89·4	46·5	0·52	91·5	47·6	0·43	95·2	40·9
	13	0·52	90·8	47·2	0·42	116·1	48·8	0·42	108·9	45·7	0·35	137·4	48·1
	14	0·42	116·0	48·7	.	.	.	.	.	.	.	.	.

No., . .	N.	95.			96.			97.			98.		
Salt, . . .	rM.	0·05 $NaNO_3$.			0·05 $Sr(NO_3)_2$.			0·05 $Pb(NO_3)_2$.			0·05 $AgNO_3$.		
Temp. of Saturated Steam, .	T.	99·87° C.			99·30°-99·31°-99·30° C.			99·29° C.			99·87° C.		
	n	*t* − T	W	W(*t* − T)	*t* − T	W	W(*t* − T)	*t* − T	W	W(*t* − T)	*t* − T	W	W(*t* − T)
	0	4·08	11·4	46·4	5·06	14·7	74·4	2·34	18·1	42·4	3·06	10·1	30·9
	1	3·06	14·9	45·6	3·54	18·4	65·1	2·03	20·8	42·2	2·56	12·7	32·5
	2	2·56	17·9	45·8	3·04	21·1	64·1	1·83	23·5	43·0	2·05	16·6	34·0
	3	2·05	22·1	45·3	2·53	24·7	62·5	1·63	26·5	43·2	1·85	19·0	35·2
	4	1·85	24·5	45·3	2·02	30·5	61·6	1·43	30·6	43·7	1·65	21·6	35·6
	5	1·65	27·3	45·0	1·82	34·0	61·9	1·22	36·3	44·3	1·45	24·7	35·8
	6	1·45	31·0	44·9	1·61	37·9	61·0	1·02	44·2	45·1	1·24	29·7	36·8
	7	1·24	36·4	45·1	1·41	42·9	60·5	0·92	49·4	45·4	1·04	36·3	37·8
	8	1·04	43·4	45·1	1·20	49·9	59·9	0·82	57·0	46·7	0·94	40·5	38·1
	9	0·94	47·3	44·5	1·00	59·5	59·5	0·72	65·6	47·2	0·84	46·3	38·9
	10	0·84	54·1	45·4	0·90	66·0	59·4	0·62	79·6	49·4	0·74	52·7	39·0
	11	0·74	61·0	45·1	0·80	75·0	60·0	0·52	96·5	50·2	0·64	62·6	40·1
	12	0·64	70·4	45·0	0·71	84·4	59·9	.	.	.	0·54	76·1	41·1
	13	0·54	83·5	45·7	0·61	99·8	60·9	.	.	.	0·44	98·2	43·2
	14	0·44	106·1	46·7	.	.	.	.	.	.	.	.	.

TABLE VI.—*Values of* W($t-$T) *for even Temperatures taken from Curves.*

$t-$T.	Chlorides.						Potassium Salts.					Nitrates.					
	LiCl.	NaCl.	$\frac{NaK}{2}$Cl.	KCl.	CsCl.	NH_4Cl.	KI.	KBr.	$KClO_3$.	KCl.	KNO_3.	$NaNO_3$.	$Ba(NO_3)_2$.	$Sr(NO_3)_2$.	$Pb(NO_3)_2$.	$AgNO_3$.	$KClO_3$.
	W($t-$T).	W($t-$T).	W($t-$T).	W($t-$T).	W($t-$T).	W($t-$T).	W($t-$T).	W($t-$T).	W($t-$T).	W($t-$T).	W($t-$T).	W($t-$T).	W($t-$T).	W($t-$T).	W($t-$T).	W($t-$T).	W($t-$T).
°C																	
3·5	...	...	...	...	...	...	...	...	...	...	...	...	...	65·0	...	...	...
3·0	...	54·8	52·0	50·0	47·8	47·0	54·1	51·3	40·0	49·9	...	45·5	...	64·0	...	...	...
2·5	56·4	52·9	50·3	48·8	46·9	45·9	52·6	50·0	40·8	48·8	37·9	45·5	...	62·5	...	...	40·5
2·0	53·8	50·9	48·6	47·5	45·8	44·8	50·9	48·5	41·2	47·7	37·8	45·3	...	61·8	42·4	39·4	40·9
1·8	52·8	50·4	48·1	47·3	45·6	44·6	50·3	48·1	41·6	47·2	37·9	45·1	...	61·4	42·9	40·3	41·2
1·6	52·0	50·0	47·6	47·1	45·4	44·4	49·7	47·6	41·9	46·9	38·1	45·0	...	61·0	43·3	40·7	41·7
1·4	50·9	49·5	47·0	46·7	45·3	44·3	49·0	47·1	42·0	46·6	39·0	45·0	...	60·5	43·7	41·0	42·2
1·2	49·9	49·0	46·3	46·3	44·9	44·2	48·4	46·8	42·0	46·3	39·5	45·0	48·7	59·9	44·3	41·7	42·2
1·0	49·5	48·8	46·0	46·0	44·8	43·9	47·7	46·5	41·4	45·9	39·5	45·0	50·2	59·4	45·0	42·8	41·7
0·9	49·3	48·9	46·1	45·9	44·9	43·9	47·4	46·4	41·3	45·8	39·5	45·0	51·2	59·6	45·7	43·3	41·5
0·8	49·2	49·2	46·3	45·8	45·0	44·0	47·1	46·3	41·4	45·7	39·6	45·1	52·0	59·8	46·7	43·9	41·4
0·7	49·1	50·0	46·7	45·9	45·5	44·0	46·7	46·3	41·9	45·8	39·8	45·1	52·5	60·3	47·9	44·6	42·0
0·6	49·3	51·1	47·2	46·2	46·8	44·2	46·8	46·5	42·9	46·3	40·3	45·2	...	60·7	49·4	45·8	42·7
0·5	49·4	52·4	48·0	48·1	48·3	44·4	47·4	47·0	44·4	48·2	41·4	45·9	...	...	50·5	47·2	44·2
0·4	49·7	...	...	...	...	...	48·5	...	...	...	44·9	...	...	...	...	48·8	45·7

TABLE VII.—*Relations between relative Reduction of Vapour Tension and Dilution.*

No.,	N.	1.			2.			3.		
Salt,	rM.	0·2 KCl.			0·4 KCl.			0·2 KCl.		
	P.	772·0 mm.			762·7–763·3 mm.			738·8 mm.		
	n.	p mm.	$\frac{p-P}{p}$	$W\frac{p-P}{p}$	p mm.	$\frac{p-P}{p}$	$W\frac{p-P}{p}$	p mm.	$\frac{p-P}{p}$	$W\frac{p-P}{p}$
	0	1037·6	0·2560	6·733	1019·9	0·2522	6·633	988·4	0·2525	6·756
	1	.	.	.	.	.	.	.	.	.
	2	1005·6	0·2323	6·669	987·3	0·2272	6·609	947·6	0·2205	6·672
	3	970·2	0·2043	6·662	969·8	0·2132	6·592	915·0	0·1926	6·635
	4	953·4	0·1903	6·643	953·4	0·1994	6·588	883·5	0·1637	6·606
	5	937·7	0·1767	6·593	937·7	0·1860	6·571	867·9	0·1488	6·584
	6	922·0	0·1628	6·111	925·2	0·1750	6·701	852·8	0·1337	6·571
	7	906·4	0·1483	6·571	906·1	0·1576	6·561	837·6	0·1180	6·529
	8	890·9	0·1334	6·525	.	.	.	822·7	0·1020	6·493
	9	875·4	0·1181	6·485	.	.	.	808·0	0·0857	6·493
	10	860·4	0·1027	6·451	.	.	.	.	.	.
	11	845·9	0·0874	6·407	.	.	.	.	.	.

No.,	N.	4.			5.			7.		
Salt,	rM.	0·2 KCl.			0·2 KCl.			0·2 KCl.		
	P.	613·7–614·0 mm.			618·7 mm.			549·8 mm.		
	n.	p mm.	$\frac{p-P}{p}$	$W\frac{p-P}{p}$	p mm.	$\frac{p-P}{p}$	$W\frac{p-P}{p}$	p mm.	$\frac{p-P}{p}$	$W\frac{p-P}{p}$
	0	816·0	0·2479	6·732	822·5	0·2478	6·772	729·6	0·2464	6·611
	1	.	.	.	.	.	.	.	.	.
	2	761·9	0·1941	6·666	764·6	0·1909	6·722	704·5	0·2196	6·611
	3	734·8	0·1644	6·633	737·5	0·1611	6·697	679·0	0·1903	6·610
	4	721·7	0·1492	6·601	711·1	0·1299	6·630	666·6	0·1752	6·612
	5	708·6	0·1335	6·573	685·6	0·0976	6·601	654·3	0·1597	6·576
	6	695·9	0·1177	6·548	660·6	0·0635	6·600	.	.	.
	7	683·1	0·1011	6·515	.	.	.	.	.	.
	8	670·8	0·0846	6·501	.	.	.	.	.	.
	9	.	.	.	.	.	.	.	.	.
	10	.	.	.	.	.	.	.	.	.
	11	.	.	.	.	.	.	.	.	.

TABLE VII. (*continued*).—*Relations between relative Reduction of Vapour Tension and Dilution.*

No., ...	N.	8.			9.			12.		
Salt, ...	rM.	0·2 KCl.			0·2 NaCl.			0·2 NaCl.		
	P.	550·4 mm.			772·0 mm.			614·0-613·7 mm.		
	n.	p mm.	$\frac{p-P}{p}$	$W^{\frac{p-P}{p}}$	p mm.	$\frac{p-P}{p}$	$W^{\frac{p-P}{p}}$	p mm.	$\frac{p-P}{p}$	$W^{\frac{p-P}{p}}$
	0	730·3	0·2463	6·680	1038·9	0·2569	7·668	825·4	0·2561	7·555
	1	.	.	.	.	.	.	.	.	.
	2	704·5	0·2187	6·650	1003·9	0·2309	7·725	790·1	0·2229	7·500
	3	678·9	0·1894	6·619	970·4	0·2045	7·609	761·9	0·1941	7·424
	4	654·3	0·1588	6·599	952·4	0·1894	7·565	734·8	0·1644	7·183
	5	630·5	0·1270	6·574	938·0	0·1769	7·520	721·7	0·1496	7·233
	6	607·4	0·0938	6·518	922·4	0·1631	7·439	708·6	0·1339	7·157
	7	.	.	.	906·7	0·1485	7·352	695·9	0·1181	7·116
	8	.	.	.	891·2	0·1338	7·253	683·1	0·1016	7·015
	9	.	.	.	876·0	0·1187	7·171	670·8	0·0851	6·923
	10	.	.	.	860·9	0·1033	7·067	.	.	.
	11	.	.	.	846·3	0·0878	7·007	.	.	.

No., ...	N.	16.			17.			19.		
Salt, ...	rM.	0·2 NaCl.			0·2 NH_4Cl.			0·4 NH_4Cl.		
	P.	550·4 mm.			742·0-743·3 mm.			619·9 mm.		
	n.	p mm.	$\frac{p-P}{p}$	$W^{\frac{p-P}{p}}$	p mm.	$\frac{p-P}{p}$	$W^{\frac{p-P}{p}}$	p mm.	$\frac{p-P}{p}$	$W^{\frac{p-P}{p}}$
	0	741·5	0·2577	7·558	1220·1	0·3918	.	996·13	0·3807	.
	1	.	.	.	.	.	.	.	.	.
	2	715·0	0·2302	7·606	1115·1	0·3346	5·558	929·70	0·3364	5·394
	3	689·3	0·2015	7·588	1043·6	0·2890	5·734	882·23	0·3007	5·608
	4	664·2	0·1713	7·438	975·9	0·2394	5·937	851·31	0·2753	5·770
	5	640·1	0·1401	7·313	942·9	0·2127	6·028	821·57	0·2491	5·872
	6	616·7	0·1075	7·165	911·2	0·1853	6·114	792·64	0·2217	5·998
	7	.	.	.	880·7	0·1571	6·133	764·63	0·1932	6·089
	8	.	.	.	865·8	0·1423	6·153	737·47	0·1635	6·183
	9	.	.	.	850·4	0·1264	6·135	.	.	.
	10	.	.	.	836·2	0·1116	6·131	.	.	.
	11	.	.	.	821·6	0·0955	6·106	.	.	.
	12	.	.	.	807·4	0·0794	6·101	.	.	.

TABLE VII. (*continued*).—*Relations between relative Reduction of Vapour Tension and Dilution.*

No., ...	N.	21.			22.			23.		
Salt, ...	rM.	0·4 NH_4Cl.			0·1 $BaCl_2.2H_2O$.			0·2 $BaCl_2.2H_2O$.		
	P.	550·35 mm.			758·64 mm.			620·56 mm.		
	n.	p mm.	$\frac{p-P}{p}$	$W\frac{p-P}{p}$	p mm.	$\frac{p-P}{p}$	$W\frac{p-P}{p}$	p mm.	$\frac{p-P}{p}$	$W\frac{p-P}{p}$
	0	880·4	0·3782	.	889·7	0·1473	.	728·0	0·1475	.
	1	.	.	.	.	.	.	.	.	.
	2	819·8	0·3287	5·555	876·9	0·1349	9·435	708·3	0·1238	10·597
	3	790·9	0·3042	5·719	861·5	0·1194	9·473	695·6	0·1078	10·413
	4	762·7	0·2784	5·846	846·2	0·1035	9·454	685·3	0·0944	10·242
	5	735·6	0·2519	5·957	831·5	0·0877	9·343	675·4	0·0811	9·983
	6	709·3	0·2241	6·073	816·6	0·0710	9·226	667·8	0·0708	9·820
	7	683·8	0·1966	6·203	802·3	0·0545	9·103	660·6	0·0605	9·626
	8	659·1	0·1650	6·261	.	.	.	653·3	0·0502	9·377
	9	.	.	.	.	.	.	646·2	0·0396	9·369

No., ...	N.	24.			25.			27.		
Salt, ...	rM.	0·1 $BaCl_2.2H_2O$.			0·2 $KClO_3$.			0·2 $KClO_3$.		
	P.	549·94 mm.			768·2 mm.			620·8 mm.		
	n.	p mm.	$\frac{p-P}{p}$	$W\frac{p-P}{p}$	p mm.	$\frac{p-P}{p}$	$W\frac{p-P}{p}$	p mm.	$\frac{p-P}{p}$	$W\frac{p-P}{p}$
	0	644·07	.	.	879·4	0·1264	.	700·4	0·1136	.
	1	.	.	.	.	.	.	.	.	.
	2	630·25	0·1274	10·549	867·9	0·1149	5·128	688·1	0·0979	5·365
	3	618·73	0·1112	10·431	854·3	0·1008	5·285	678·1	0·0845	5·476
	4	607·17	0·0942	10·230	839·4	0·0848	5·404	668·1	0·0708	5·565
	5	.	.	.	824·8	0·0686	5·545	660·8	0·0605	5·675
	6	.	.	.	811·1	0·0529	5·879	656·5	0·0544	5·810
	7	.	.	.	.	.	.	650·9	0·0463	5·824
	8	.	.	.	.	.	.	646·2	0·0393	5·871

TABLE VII. (*continued*).—*Relations between relative Reduction of Vapour Tension and Dilution.*

No., ...	N.	28.			29.			30.		
Salt, ...	rM.	0·2 $KClO_3$.			0·05 $Ba(NO_3)_2$.			0·2 $Ba(NO_3)_2$.		
	P.	550·35 mm.			757·56 mm.			620·6 mm.		
	n.	p mm.	$\frac{p-P}{p}$	$W\frac{p-P}{p}$	p mm.	$\frac{p-P}{p}$	$W\frac{p-P}{p}$	p mm.	$\frac{p-P}{p}$	$W\frac{p-P}{p}$
	0	616·4	0·1072	.	792·9	0·0445	.	646·7	0·0403	.
	1	.	.	.	.	.	.	.	.	.
	2	604·9	0·0902	5·355	787·9	0·0385	6·911	643·4	0·0355	6·763
	3	593·8	0·0732	5·527	785·1	0·0350	7·011	641·0	0·0318	6·837
	4	582·6	0·0584	5·696	779·5	0·0281	7·337	638·6	0·0283	6·998
	5	.	.	.	776·7	0·0246	7·368	636·3	0·0247	7·136
	6	.	.	.	774·0	0·0212	7·927	634·0	0·0211	7·351
	7	.	.	.	.	.	.	631·6	0·0175	7·343
	8	.	.	.	.	.	.	629·3	0·0138	8·061

No., ...	N.	31.			32.			36.		
Salt, ...	rM.	0·2 $Sr(NO_3)_2$.			0·2 $Pb(NO_3)_2$.			0·1 $(NH_4)_2SO_4$.		
	P.	760·00 mm.			749·2 mm.			743·9 mm.		
	n.	p mm.	$\frac{p-P}{p}$	$W\frac{p-P}{p}$	p mm.	$\frac{p-P}{p}$	$W\frac{p-P}{p}$	p mm.	$\frac{p-P}{p}$	$W\frac{p-P}{p}$
	0	955·7	0·2048	8·376	842·0	0·1102	5·510	972·2	0·2348	5·941
	1	.	.	.	.	.	.	.	.	.
	2	920·5	0·1743	8·663	831·8	0·0993	5·610	918·2	0·1898	6·149
	3	904·6	0·1598	8·613	822·8	0·0895	5·629	889·7	0·1638	6·155
	4	889·4	0·1455	8·584	817·2	0·0832	5·741	858·8	0·1338	6·139
	5	873·9	0·1203	8·600	811·4	0·0766	5·760	843·5	0·1181	6·130
	6	859·1	0·1154	8·610	805·4	0·0698	5·807	828·9	0·1025	6·085
	7	844·1	0·0996	8·514	799·4	0·0628	5·834	814·0	0·0861	6·074
	8	823·7	0·0773	7·910	793·8	0·0562	5·963	799·4	0·0694	6·077
	9	.	.	.	788·1	0·0493	6·251	793·8	0·0628	6·149
	10	.	.	.	.	.	.	788·1	0·0561	6·191
	11	.	.	.	.	.	.	782·6	0·0494	6·323
	12	.	.	.	.	.	.	776·7	0·0422	6·381

TABLE VII. (*continued*).—*Relations between relative Reduction of Vapour Tension and Dilution.*

No., ...	N.	39.			41.			42.		
Salt, ...	rM.	0·2 $(NH_4)_2SO_4$.			0·2 $(NH_4)_2SO_4$.			0·2 $\frac{K+Na}{2}Cl$.		
	P.	620·33 mm.			552·43 mm.			772·0 mm.		
	n.	p mm.	$\frac{p-P}{p}$	$W\frac{p-P}{p}$	p mm.	$\frac{p-P}{p}$	$W\frac{p-P}{p}$	p mm.	$\frac{p-P}{p}$	$W\frac{p-P}{p}$
	0	810·0	0·2342	.	721·2	0·2345	.	1161·9	0·3356	6·947
	1	.	.	.	.	.	.	.	.	.
	2	781·2	0·2059	6·074	693·8	0·2038	6·196	1107·7	0·3031	7·077
	3	753·5	0·1769	6·121	668·8	0·1740	6·212	1055·7	0·2686	7·038
	4	726·7	0·1464	6·105	644·5	0·1429	6·316	1020·6	0·2436	7·101
	5	700·6	0·1146	6·154	618·7	0·1071	6·201	987·3	0·2181	7·034
	6	675·4	0·0816	6·153	598·2	0·0765	6·281	954·0	0·1908	7·012
	7	.	.	.	.	.	.	938·0	0·1770	6·983
	8	.	.	.	.	.	.	922·4	0·1631	6·956
	9	.	.	.	.	.	.	906·7	0·1485	6·898
	10	.	.	.	.	.	.	891·2	0·1337	6·839
	11	.	.	.	.	.	.	876·0	0·1187	6·795
	12	.	.	.	.	.	.	860·9	0·1032	6·702
	13	.	.	.	.	.	.	846·2	0·0877	6·652

No., ...	N.	44.			80.			82.		
Salt, ...	rM.	0·4 $\frac{K+Na}{2}Cl$.			0·2 $NaNO_3$.			0·5 $NaNO_3$.		
	P.	616·9–616·68 mm.			620·3 mm.			552·43 mm.		
	n.	p mm.	$\frac{p-P}{p}$	$W\frac{p-P}{p}$	p mm.	$\frac{p-P}{p}$	$W\frac{p-P}{p}$	p mm.	$\frac{p-P}{p}$	$W\frac{p-P}{p}$
	0	921·1	0·3302	.	1163·0	0·4666	.	1011·5	0·4539	.
	1	.	.	.	.	.	.	.	.	.
	2	879·4	0·2985	6·880	1052·9	0·4109	4·438	913·4	0·3952	4·869
	3	858·2	0·2812	6·917	984·6	0·3700	4·640	836·2	0·3394	5·281
	4	828·3	0·2554	6·947	919·8	0·3256	4·874	803·1	0·3122	5·232
	5	813·2	0·2416	6·958	888·7	0·3020	5·025	779·8	0·2916	5·342
	6	798·9	0·2281	6·957	850·7	0·2708	5·151	752·7	0·2661	5·428
	7	784·5	0·2139	6·962	813·4	0·2374	5·337	728·0	0·2412	5·548
	8	770·7	0·1998	6·963	784·8	0·2096	5·477	.	.	.
	9	756·5	0·1848	6·948	757·3	0·1809	5·582	.	.	.
	10	.	.	.	730·1	0·1504	5·756	.	.	.
	11	.	.	.	713·4	0·1305	5·872	.	.	.

TABLE VIII.—*Vapour Tension of Water in Kilogrammes per Square Centimetre for whole Degrees Centigrade.*

° C.	Kilogrammes per Square Centimetre.	Difference.	2nd Difference.
120	2·0275	...	...
119	1·9639	0·0636	...
118	1·9020	0·0619	0·0017
117	1·8417	0·0603	0·0016
116	1·7830	0·0587	0·0016
115	1·7258	0·0572	0·0015
114	1·6702	0·0556	0·0016
113	1·6160	0·0542	0·0014
112	1·5633	0·0527	0·0015
111	1·5120	0·0513	0·0014
110	1·4620	0·0500	0·0013
109	1·4135	0·0485	0·0015
108	1·3662	0·0473	0·0012
107	1·3203	0·0459	0·0014
106	1·2757	0·0446	0·0013
105	1·2323	0·0434	0·0012
104	1·1902	0·0421	0·0013
103	1·1492	0·0410	0·0011
102	1·1094	0·0398	0·0012
101	1·0708	0·0386	0·0012
100	1·0333	0·0375	0·0011
99	0·9970	0·0363	0·0012
98	0·9616	0·0354	0·0009
97	0·9273	0·0343	0·0011
96	0·8940	0·0333	0·0010
95	0·8617	0·0323	0·0010
94	0·8303	0·0314	0·0009
93	0·8000	0·0303	0·0011
92	0·7705	0·0295	0·0008
91	0·7420	0·0285	0·0010
90	0·7144	0·0276	0·0009

TABLE IX.—*Example of Use of Elastic Tank.*

$T = 99{\cdot}09°$ C. $A = 1{\cdot}000$ k/c². $W_0 = 100 \left\{ \begin{matrix} \text{gr.} \\ \text{c.c.} \\ \text{c}^2. \end{matrix} \right\}$ $a_0 = 2{\cdot}000$ k/c². $t_0 = 119{\cdot}57°$ C.

$b_0 = 100$ Kilogrammes. $\frac{b_0}{W} = a - A$.

$t - T$ ° C.	a k/c²	t ° C.	$(a - A)$ $\frac{b_0}{W}$	W	W$(t - T)$
79·79	10·0	178·88	9·0	11·1	886
75·29	9·0	174·38	8·0	12·5	941
70·37	8·0	169·46	7·0	14·3	1006
64·94	7·0	164·03	6·0	16·7	1084
58·85	6·0	157·94	5·0	20·0	1177
51·91	5·0	151·00	4·0	25·0	1298
43·73	4·0	142·82	3·0	33·3	1456
33·71	3·0	132·80	2·0	50·0	1686
27·63	2·5	126·72	1·5	66·7	1843
20·48	2·0	119·57	1·0	100·0	2048
18·94	1·9	118·03	0·9	111·0	2102
17·20	1·8	116·29	0·8	125	2150
15·44	1·7	114·53	0·7	143	2208
13·61	1·6	112·70	0·6	167	2272
11·67	1·5	110·76	0·5	200	2334
9·62	1·4	108·71	0·4	250	2405
7·45	1·3	106·54	0·3	333	2481
5·14	1·2	104·23	0·2	500	2570
2·66	1·1	101·75	0·1	1000	2660
2·40	1·09	101·49	0·09	1110	2664
2·14	1·08	101·23	0·08	1250	2675
1·88	1·07	100·97	0·07	1430	2688
1·62	1·06	100·71	0·06	1670	2705
1·35	1·05	100·44	0·05	2000	2700
1·08	1·04	100·17	0·04	2500	2700
0·82	1·03	99·91	0·03	3330	2730
0·54	1·02	99·63	0·02	5000	2700
0·27	1·01	99·36	0·01	10000	2700

XIX.—*On a Silurian Scorpion and some additional Eurypterid Remains from the Pentland Hills.* By Malcolm Laurie, B.A., D.Sc. (Plates I.–V.)

(Read 6th June 1898.)

In 1892 I communicated to this Society an account of a collection of Eurypterids from the Upper Silurian rocks of the Pentland Hills, in which I described new species and a new genus. Since that time I have had the opportunity of examining two other collections from the same locality. One of these was formed with the aid of a grant from the British Association. The fossiliferous bed—for these remains occur in a single thin bed of rock—was laid bare, and a considerable amount of it removed and split up. In this latter work I was fortunate to get the assistance of Mr John Henderson, the original discoverer of the locality, and I am glad to have this opportunity of recording my indebtedness to him. A fair number of specimens was thus procured, which not only threw considerable light on the structure of some of the already described forms, but gave evidence of some species as yet undescribed. More important than this collection, however, was the one formed by the late Mr Hardie of Bavelaw Castle, who had for many years collected among the Pentland Silurians. His collection was, on his death, acquired by the Edinburgh Museum of Science and Art, and Dr Traquair kindly put the collection in my hands for examination. From the size of the collection I expected some new and interesting results, and have been far from being disappointed. Seven new species of Eurypterids have come to light and one Scorpion. These, and the new facts about the already described species, form the subject of this paper. Some fragments of Ceratiocaris have also been found, but they do not seem to me sufficient to determine the species.

The horizon of the beds is Wenlock, so far as can be ascertained from the other fossils contained in it, and from the very scanty fauna of the neighbouring rocks. I had come to this conclusion from my own investigations, and am glad to find that the position assigned to it in the forthcoming memoir of the Geological Survey corresponds to my own idea on the subject.

The discovery of so many forms (10 in all) puts this locality far ahead of any other for Eurypterids. The total number hitherto described from the Silurian of Great Britain, apart from this deposit, is about 16. The only deposit approaching this for the richness of its Eurypterid fauna is the Water Lime in America, from which about 15 species have been described. Some five or six of these, however, are of very doubtful specific rank, being based either on small fragments or on slight differences in proportion. The latter is for Palæozoic fossils, very unsatisfactory when one considers how much compression and distortion they have undergone. The reason for the extraordinary abundance of these

forms in this particular bed along with masses of "Dictyocaris" and layers of structureless black matter must remain obscure. They must have existed both before and after the period marked by this particular bed, but so far as is known have with one exception left no trace in the surrounding rocks. The variety of forms—4 genera being represented—suggest that even in this bed we are far from the time when this group came into existence. On the other hand, no one, I think, will deny that *Drepanopterus bembicoides* is the least differentiated form known. The presence of a Scorpion indicates that the origin of that group is earlier than had been supposed, a conclusion which agrees quite well with the views I have published elsewhere as to the relation of the Scorpions and Eurypterids. That this Scorpion was an air-breather does not necessarily follow. The characters which mark it as a Scorpion may well have been developed before the terrestrial mode of life and consequent modification of the respiratory organs took place. Unfortunately, these respiratory organs are necessarily so delicate in texture that we know very little of their structure and arrangement in any of the fossil orthropoda, and, as in respect of many other interesting points in Palæontology, must wait for further evidence before we can pronounce an opinion.

I cannot let this opportunity pass without expressing the gratitude I owe to Dr Traquair for the opportunity of examining the Hardie Collection, and for the sympathy and encouragement which has helped me to surmount many difficulties. I am also indebted to Mr B. N. Peach for much assistance in the more difficult points which have arisen; assistance which only those who have worked with him can appreciate the value of.

The forms to be described come under six genera:—*Palæophonus*, *Slimonia*, *Stylonurus*, *Drepanopterus*, *Eurypterus*, and *Bembicosoma* (n. g.), and are dealt with in the above order. The new species are *Palæophonus loudonensis*, *Slimonia dubia*, *Stylonurus elegans*, *Drepanopterus lobatus*, *Drepanopterus bembicoides*, *Eurypterus scoticus*, *E. minor*, and *Bembicosoma pomphicus*.

Palæophonus loudonensis, n. sp. (Pl. I., fig. 1.)

Carapace long and narrow, median eyes on a double papilla very far forward; mesosomatic sclerites band-like; tail long; cheliceræ comparatively long and narrow.

The single specimen on which this species is founded is very obscure—so much so that it was long before I realised that it was anything more than a crushed Eurypterus. It had also been carelessly developed, so that the limbs for the most part are destroyed, and the tail entirely loses itself among some of the black layers which are so frequent in this Gutterford bed. The specimen shows the dorsal surface.

The carapace is 10 mm. long and 7 mm. wide, a very different proportion to *Palæophonus nuncius* (♀), in which the carapace is 7·5 mm. long and 8 mm. wide. The anterior margin is distinctly concave, and the sides almost parallel, but slightly convex. The posterior margin appears fairly straight. Situated about 1 mm.

from the anterior border is a double depression, which I take to mark the position of the median eyes. If they corresponded in size to these depressions they must have been very large. Their position far forward on the carapace corresponds to what is found in *P. nuncius* and *Proscorpius osborni* (Whitfield). In the middle line behind the eyes and some 3 mm. from the anterior margin is a small depression, the significance of which is unknown to me. There is a slight roughness at the left anterior corner of the carapace, which probably marks the position of the lateral eyes.

The six mesosomatic segments are wider than the carapace, *i.e.*, about 11 mm., but the lateral boundaries of this part of the body are very indistinct. In striking contrast to the length of the carapace is the shortness of these segments. The first measures only 2 mm., the next two 3 mm. each, and the remaining ones 4 mm. They are thus considerably shorter and wider than the corresponding sclerites in *P. nuncius*. Some obscure markings occur on them and seem to have some meaning.

The first segment is cut across at the corners by the basal joints of the last pair of legs. On the fifth segment is clearly seen a curved ridge cutting off the anterior right-hand corner. Indications of a similar structure are visible on the three preceding segments. That on segment two, one may take as the impression of the outline of the pectines, but the succeeding segments in recent scorpions bear no free appendages, but only the respiratory organs or "lung books." If this curved ridge is an indication, as I think probable, of the breathing organs, they must have been of a very different type to those in the modern Scorpions. The lung books are generally regarded as modifications of plate-like appendages, bearing gill lamellæ on their posterior surface. It does not follow that because this form agrees with the recent ones in respect of the body form and the arrangement of the prosomatic appendages that it had reached the same point of specialisation in respect of its respiratory organs. The terrestrial mode of life and consequent adaptation to air breathing may have come later. It is thus just possible that this curved ridge on the mesosomatic segments is the outline of a plate-like gill bearing appendage. It is interesting to compare this structure with fig. 1*a*, which is a reproduction of a figure of an embryo Scorpio fulvipes, published some years since (5). The mesosomatic segments in this are seen to be marked with a curved ridge corresponding more or less closely to that in *P. loudonensis*. At the time, I regarded this ridge as indicating the line of fusion of the edge of the abdominal appendage with the body, and I see no reason to change my view.

The first metasomatic segment is as usual a truncated cone. The breadth at the posterior margin is just equal to the length, *i.e.*, 4·5 mm. This segment is marked off from the sixth, as the sixth is from the fifth, by a well-defined ridge. This ridge marks the overlap of the two sclerites. The last five segments are even more obscure than the rest of the specimen. The tail, which they constitute, appears to have been narrow (2·5 mm.) and long, though the few segments remaining vary so much in length that it is difficult to estimate the length of the whole. The first segment (No. 8 of the body) measures 4 mm. in length, the next 6 mm. Beyond this the tail can be traced for

some distance, but finally gets itself lost in one of the black layers so common in this bed, and the details of the posterior segments and the telson cannot be made out. The tail segments appear to have been ornamented with longitudinal ridges possibly composed of a series of knobs.

The limbs are, as mentioned above, very imperfectly preserved. Careful following out of what was left has however yielded some results.

The cheliceræ are well shown and are remarkable for the large size of the pincers and the length of the basal portion. They project 4·5 mm. beyond the front of the carapace and the movable limb of the pincers measures 3 mm. The inner border of the fixed limb is crenulated, and probably the corresponding border of the movable limb was also. Compared with *Palæophonus nuncius*, the basal part of this limb is longer, while the width both of the basal part and the two limbs of the pincers is considerably less. They seem to agree more closely with the *Palæophonus* from Lesmahagow (7).

The second pair of appendages—Chelæ—are partially preserved on the left side—the right side showing only the barest traces of them. The segments agree in form with recent Scorpions so far as their outline can be ascertained. The fourth segment is much longer than in *P. nuncius*, and the hand seems to have been stouter. This last, however, is so badly preserved that it would be unwise to say anything definite about its form. Part of one of the walking legs is preserved. The proximal part is very indistinct, but the last three segments are visible. Unfortunately the extreme end is gone, so it is impossible to say whether these limbs ended in a single spine as in the other species of *Palæophonus*.

This specimen is the fourth scorpion from Silurian strata. The other three are *Palæophonus nuncius* of Thorell (9) from Gothland; a *Palæophonus* described by Peach (7) from the collection of the late Dr Hunter, Selkirk; and *Proscorpius osborni* of Whitfield (11) from the Lower Helderberg rocks of America. This present form is probably the oldest of the four, and this makes it the more to be regretted that so few details can be made out.

Slimonia.

Slimonia dubia, n. sp. Pl. I., figs. 2–3.

Carapace quadrangular; eyes at the anterior corners; body tapering gradually; telson oval, terminating in a long sharp spine.

The type of this species is a very badly preserved carapace (fig. 2), with parts of the first eleven segments attached. Enough can, however, be made out to place it as a *Slimonia*, as no other genus possesses the quadrangular carapace with eyes at the corners. The other specimen I have relegated to this species shows the body segments and telson of a somewhat larger individual than the type. The telson has a more elongated form than in *S. acuminata*, and the broadest part is not so far back. The

shape of the body differs from *S. acuminata* in tapering evenly the whole way down, instead of narrowing suddenly at the seventh segment into a cylindrical tail. This difference of body form is so marked that at first I was disposed to place these body segments and telson under *Pterygotus*, to which *Slimonia* is closely allied. *Slimonia acuminata* comes from the Lesmahagow beds, and belongs to a higher horizon than the Pentland beds. A telson indistinguishable from *S. acuminata* has also been found in the Pentlands in beds belonging to a higher horizon in the Esk Valley.

Stylonurus.

Stylonurus macrophthalmus. (Pl. I., figs. 4–7.)

Of this form the general shape is better shown in fig. 4 than in the type specimens described in my former paper. The median appendage of the genital operculum is characterised by extreme length, reaching down across four segments. It appears to have terminated in a point. The hind segments of the body are very characteristically furnished with epimera (figs. 5–6). These are not mere backward prolongations of the posterior angles of the segments, but project from the sides of each segment, being attached by a narrower peduncle, and expanding into a thin oval plate. The only form with epimera approaching these is *St. scoticus*, but in it they are not so well marked or so large in proportion.

Appendage III. (fig. 4) is moderately long and very stout, ending in a curved spine. The other joints probably bore spines also, as in *St. ornatus* (7).

Appendage IV. (fig. 7) is about twice the length of III., but not so broad. The distal joints are furnished with strong, curved spines—probably a pair to each segment, and the limb terminates in a spine.

Appendages V. and VI. do not differ much from the typical *Stylonurus* form. V. seems to have a small spine arising from the anterior side of the joint, between segments 4 and 5, while VI. shows the narrowing and elongation characteristic of *Stylonurus* to a marked degree.

Stylonurus ornatus. (Pl. I., fig. 8. Pl. II., figs. 10–12.)

A great deal has been added to our knowledge of this species. Only the general body form the metastoma, and portions of the last appendage and telson were described in my former paper. The greater part of the structure of all the six appendages has now been made out and the telson completed. In many specimens, the characteristic ornamentation is quite invisible, but I regard this as merely due to difference in the details of preservation.

Appendage I. (Pl. I., fig. 8).—The most anterior appendages are a small pair of chelicerae well shown in the figure. Only the two end joints forming the pincers are preserved, and the bases of the first somewhat broken. The proximal joint, probably the 2nd of the limb, is 10 mm. long, with an articular surface for the terminal joint half way

up on the outer side. Beyond this articular surface the joint continues as a triangular process which forms the inner ramus of the pincers. The outer margin of this—*i.e.*, that against which the terminal joint bites—is straight, and shows no sign of denticulation. It ends in a sharp point. The terminal joint forming the outer movable ramus of the pincers is 2 mm. longer than the inner ramus (*i.e.*, 7 mm.) and is curved, the convexity being toward the outer side. It also appears quite smooth on the margin. The basal joint of this appendage probably passed upwards at a sharp angle (or may even have been directed backwards) to its point of attachment. From their structure it would seem as though the cheliceræ could only bite by a single point.

Appendage II.—Slanting inwards and backwards from outside the cheliceræ and overlapping each other just behind their apices lie the ends of the 2nd pair of appendages. The form is somewhat indefinite, and they had probably a thin cuticle that on the right side shows two conical spines, while on the left side there are indications of spines and a considerable number of setæ. These appendages were probably tactile like the corresponding pair in Slimonia.

Appendage III. (fig. 10).—The gnathobase of this, I have not made out; in fact, only the four distal joints are known. They are all short and tapering, and the two proximal ones furnished each with a stout spine. Markings on the sides of these joints seem to be the points of attachment of other spines. This limb is very short when compared with the corresponding one in *St. elegans*.

Appendage IV. (fig. 11).—This is a reproduction of Appendage III. on a larger scale. The spines are longer and more delicate, and the terminal segment appears to bear several. A spine is seen arising from the side of the antepenultimate segment.

Appendage V.—The five distal segments of this limb are shown in fig. 12. They are less elongated and slightly broader than the corresponding segments in Appendage VI.

Appendage VI. (fig. 12) shows the great elongation of all the segments, especially the two proximal ones (2nd and 3rd). The length and narrowness of the 2nd joint is in marked contrast both to *St. macrophthalmus* and *St. elegans*.

The metastoma was incorrectly figured in my former paper (*loc. cit.*, Pl. I., fig. 7), the front margin being shown much too deeply cleft and simple. Fig. 8 shows it as it ought to be, and shows further that the front margin is crenulated.

St. elegans, n. sp. (Pl. II., figs. 13–15. Pl. III., fig. 19.)

Carapace long and narrow, tapering towards the front. Metastoma comparatively wide; 3rd and 4th appendages long and furnished with backwardly directed longitudinally ridged spines; 5th and 6th appendages with 2nd joint comparatively short, broad, and subconical; tail segments probably narrow and with sharply pointed epimera, and telson wide in proportion to last segment.

Five specimens certainly, and two probably, belong to this species. The type

specimen (Pl. II., fig. 13) shows about half the carapace, the metastoma, fragments of five body segments, and portions of appendages III. to VI. of the left side. Another specimen (Pl. III., fig. 19), mounted in the same slab of plaster of paris, shows appendages III., IV., V., and VI., and is probably the other side of the same animal as fig. 13, a portion of the carapace being gone. Two other specimens (one the very fragmentary reverse of the other) show parts of the carapace, and appendages III.–VI. of a much smaller individual. These two specimens have been useful in confirming certain points, but have added nothing to the information derived from the specimens, figured. The other two specimens show the posterior body segments, and I have ascribed them to this species, as they show certain differences from the corresponding portions of *St. ornatus*. Their place must be considered as only probable, however, till the discovery of fresh material. A somewhat broken carapace (fig. 14) belonging to a small individual is valuable as giving the anterior margin and the position of the eyes.

The carapace (Pl. II., fig. 13) is widest about one-third from the hind margin. In front of this it narrows, the anterior one-third being concave in outline. There is a narrow but well-marked border in the anterior third which narrows as it approaches the level of Appendage IV. Whether it continues beyond that point is doubtful, the cutline of the carapace being rather indicated than preserved behind this point. The anterior margin (fig. 14) is crenulated.

The metastoma is proportionately broad and short and slightly concave on its anterior and posterior margins. The ratio of width to length is 5 to 7, a marked contrast to *St. ornatus* in which the ratio is 5 to 11. The proportionate width of metastoma and carapace also differs markedly from *St. ornatus*, being in *St. elegans* as 5 to 11·5, and in *St. ornatus* as 1 to 8. The surface of the metastoma was covered with scattered projections.

The only appendages known are parts of III., IV., V., and VI., lying beyond the carapace. The gnathobases have left no impressions.

The third and fourth appendages are well furnished with spines along the posterior margin of the four distal joints. These spines are slightly curved and longitudinally striated and were probably arranged in a double row. Those towards the end of the limb are larger than those towards the base.

Appendage III.—Presuming that this appendage consisted of the typical number of joints (*i.e.* 7), portions of 3 and 4 are shown on the left side (fig. 13), and 3, 4, 5, 6, and probably the beginning of 7, on the right side along with a small fraction of 2 (fig. 19). Joint 3 is short, the length being only one-third greater than the width. There are apparently no spines on the posterior margin, but the distal end of the anterior margin is produced into a short spine (fig. 13). Joint 4 is twice the length of joint 3, and appears to expand at the distal end owing to the presence of spines on both anterior and posterior margins. The posterior margin bears a number of spines. Joint 5 tapers slightly and is three-fourths the length of joint 4. It is furnished with spines on the posterior surface. Joint 6 is nearly the same length and is well furnished with spines along the posterior

margin—at least two of these being considerably larger than the rest. Of the last joint almost nothing remains. Half a spine lying along the edge of the slab and a short process arising at its base are all that are left.

Appendage IV.—This limb (figs. 13 and 19) is somewhat longer and stouter than III. Joint 3 is longer in proportion than in the foregoing appendage, and seems free from spines. Joint 4 is the longest, and joint 5 the shortest in this leg, both being furnished with well-developed spines. Joint 6 is long and tapering, and bears a remarkably powerful spine at its distal extremity. The last joint is narrow and runs out into three spines, that near the base being the stoutest.

The 5th and 6th appendages have the elongated form characteristic of *Stylonurus*, each joint being somewhat narrower than the last. The proximal joints of them are, however, much wider and shorter in proportion than in *St. ornatus*.

The points of difference between this form and *St. ornatus* are very marked. The narrow carapace and wide metastoma would alone serve to distinguish the two forms, while the greater length of the 3rd and 4th appendages, especially the 3rd, afford equally good diagnostic characters. The proportion of the joints of the two posterior limbs, and especially the shortness and width of the proximal joints, also offer a point of difference. The only form I am acquainted with which resembles this species in the form of the carapace is *St. excelsior* (3) from the Upper Devonian (Catskill group) of New York. The appendages of this form do not, unfortunately, admit of comparison with those of *St. elegans*.

Drepanopterus.

Two new species come under this genus, and the point of chief generic importance in the light of these new forms would seem to be a negative one:—"Last pair of appendages neither expanded, as in *Eurypterus*, etc., nor excessively elongated, as in *Stylonurus*."

The genus unquestionably comes very near *Stylonurus* in some respects. The form of the carapace and the comparatively elongated last appendages in *D. Pentlandicus* are suggestive of stylonuroid affinities, but *D. bembicoides* is a very well marked form, easily separated from *Stylonurus*, and quite justifies the existence of the genus.

Drepanopterus lobatus, n. sp. (Pl. II., fig. 16. Pl. III., figs. 17, 18.)

Carapace almost semi-circular; body without marked differentiation of meso- and metasoma; last segment with posterior angles produced into ovate lobes; telson long.

I have ventured to found this species for the reception of three or four specimens, which are most definitely characterised by the form of the last segment.

The carapace (Pl. II., fig. 16) has a length of 15 mm. and a breadth at its widest point of 25 mm. The proportion in this specimen is probably not quite correct, as the

animal seems to have been squeezed out laterally and slightly telescoped into itself. The greatest width of the carapace is about one-third from the posterior margin. The position of the eyes is unfortunately not shown.

The body segments become progressively narrower and longer from before backwards. The last six at least have the posterior angles produced into processes. These are large on the 11th segment, and on the 12th form a pair of ovate lobes about the same length as the segment itself. The telson is long, and in one specimen (Pl. III., fig. 17) curved, but this is probably accidental. The end of the telson is not shown.

Some fragments of a carapace and body segments with the two last limbs attached (Pl. III., fig. 18), I am inclined for the present to regard as belonging to this species, and it is this specimen which has decided the generic position of the species. Apart from the limbs here shown, the general body shape of *D. lobatus* is more suggestive of a *Stylonurus*. The carapace shows little of the shape, but is covered with a scattered punctate marking. The body segments, portions of six of which are shown, are covered —especially the anterior part of each segment, with flattened scale markings. The greater part of appendages V. and VI. is shown, and these undoubtedly belong to a *Drepanopterus* though approaching the *Stylonurus* form more closely than *D. bembycoides*. The fifth appendage appears to have had a long spine arising from probably the 5th segment. The sixth appendage has the second and third joints considerably elongated, while the fourth, fifth, sixth, and seventh joints might belong to *D. bembicoides*. The measurements of the limbs are :—

	Appendage V.		Appendage VI.	
Segment.	Length.	Breadth.	Length.	Breadth.
3	?	9 mm.	19 mm.	6 mm.
4	12 mm.	6 mm.	7 mm.	6 mm.
5	16 mm.	4·5 mm.	9 mm.	5 mm.
6		2 mm.	6 mm.	3 mm.
7	5 mm.	1·2 mm.	6 mm.	2 mm.

This specimen, whether belonging to *D. lobatus* or not, is interesting as giving us an intermediate form between *D. bembicoides* and *Stylonurus*.

Drepanopterus bembycoides, n. sp. (Pl. III., figs. 20–21).

Surface smooth; carapace with comparatively straight anterior margin and sides, length = one-half breadth; body tapering regularly and ending in a stout conical tail spine; segments increasing in length towards posterior end; eyes inside a well-marked marginal band; limbs increasing in length from before backwards, last pair neither expanded as in *Eurypterus* nor elongated as in *Stylonurus*, ending in a slightly sickle-shaped subconical joint; spines wanting on the limbs; metastoma oval; genital plate with broad oval median process hardly extending beyond lateral portions.

The carapace, which, as stated above, is twice as broad as it is long, has a well-marked border about 5 mm. wide. This border is ornamented with fine reticulating lines.

Immediately inside the border and just behind the antero-lateral angle of the carapace are the comparatively small oval eyes. The metastoma (Pl. III., fig. 20) is oval, narrowing slightly towards the front, and with a small but deeply-cut notch in the anterior border.

The limbs are short and stout, formed of a series of sub-cylindrical joints of approximately the same length. Each joint is narrower than the one before it, and the distal end of each expands slightly. The last joint is subconical and slightly concave on the post. marg.

No spines have been observed on any of the appendages. The most marked feature of these limbs is the close resemblance of one to another and the absence of any marked specialisation. The increase in size from before backwards is an increase in both length and breadth, so that the proportions of the limbs remain unaltered.

The body segments are short, increasing slightly in length towards the posterior end as far as the 11th. The 12th is very much longer than any of the preceding ones, and has the shape of a truncated cone, from the posterior end of which arises the short conical telson. The tapering of the body is almost uniform from the hind margin of the carapace to the end of the telson. The tergites of the segments have their posterior angles produced into short spines.

The genital plate is short and the median process oval and scarcely projecting beyond the posterior margin of the plate.

This form is easily distinguished from *D. pentlandicus* by the smooth body surface, the less central position of the eyes, the markedly conical form of the body. The proportions of the carapace and indeed of the whole animal show a greater breadth in proportion to the length. It resembles *E. conicus* in general form, but is usually easily distinguished by the large submarginal eyes of conicus and the absence in it of the well-marked border to the carapace. The telson in *Conicus* is also much longer and slighter.

Taken as a whole, this form appears to me the most primitive Eurypterid known. The absence of differentiation of the body into meso- and metasomatic regions is shared by it with several Eurypterus forms, but to this must be added the comparatively slight development of the genital plate, and above all the legs. The almost entire absence (much more marked than in *D. pentlandicus*) of differentiation of the posterior pair, either as flattened paddles as in the majority of the Eurypterids, or as enormously elongated walking legs as in *Stylonurus*, is a very striking feature. It is unfortunate that in this, as in so many of these Pentland specimens, the form of the gnathobases cannot be made out.

I have termed it bembycoides from *βέμβιξ*, a peg-top.

Drepanopterus pentlandicus. (Pl. IV., fig. 22.)

Some good specimens of this species have come to light. The most instructive is the small individual figured in fig. 22. It is less distorted than the type specimen, and

probably approximates closely to the true form. The form of the carapace reminds one of *Stylonurus*, being distinctly horse-shoe shaped, with a well-marked marginal band. The eyes are situated rather far back, and show only a narrow semi-circular band. This, however, is probably merely the outer margin, and does not represent the size of the whole visual area. The body expands slightly down to the fourth segment, and then contracts regularly to the tail. The last segment is very long and conical like the corresponding one in *D. bembycoides*. The telson is remarkably large, being nearly half as long as the body, and is covered with the granular markings characteristic of this form.

Of the Appendages, No. IV. is comparatively short, stout, and sub-conical. V. is somewhat elongated, the 3rd joint being most markedly modified. The last joint is conical and slightly concave on the posterior margin. This form of terminal joint seems to occur in all the members of this genus.

The last pair of appendages are considerably elongated, without, however, reaching the specialisation shown in *Stylonurus*. They appear longer in proportion in this specimen than in the type, a difference which may be due to the different stage of growth of the two. None of the segments are particularly elongated, and the chief point of interest is the presence of short, coarse spines on the third and fourth joints. Spines are not common on this pair of limbs in the Eurypteridæ. A very large number of specimens of this species in the Hardie Collection have not yielded any further information of importance.

Eurypterus scoticus. (Pl. IV., figs. 23–25. Pl. V., fig. 26.)

Carapace conical, broadest at hind margin and narrowing forwards. Body progressively wider to the third segment, narrowing abruptly at the 7th; last five segments almost equal in width; dorsal surface with finely punctate markings. Appendages.—2nd pair short, provided with spines; 3rd, 4th, and 5th pairs long, the four distal joints armed with a double series of long, curved spines directed forwards; 6th with last segment (paddle) much narrower than penultimate. Gnathobases and metastoma unknown.

In my former paper I provisionally referred two fragments belonging to this species to *E. scorpioides* (Woodward), as there was not evidence enough to justify the foundation of a new species. A large number of fragments in the Hardie Collection have, however, enabled me to found a new species with some certainty.

Carapace.—The carapace (Pl. V., fig. 26) has a curved posterior margin and somewhat rounded posterior angles. Its width in this specimen at this, the broadest point, is about 100 mm. The lateral margin is concave, and narrows rapidly for the hind one-third of its length, for the other two-thirds it is convex, the two sides sloping in so that the anterior margin is scarcely one-third the width of the posterior. Nothing could be ascertained as to the organs lying below the carapace, or as to the presence or absence of eyes.

Body Segments.—The type specimen shows the inside of the tergites of the first four segments. The specimen does not, however, extend far enough to show the margins of any of these segments except the first. This segment is only half the length of the succeeding ones (*i.e.*, 9 mm.), and extends laterally some 15 mm. beyond the angles of the carapace, which would make the total width of the segment more than 130 mm. Of the other segments in this specimen nothing can be made out except that they are band like sclerites, measuring some 18 mm. from front to back. A small portion of the ventral surface is shown on a small island towards the right side. One of the specimens referred to in my former paper shows the ventral surface of all the body segments of a specimen, the total length of which, without the telson, must have been between 550 and 600 mm. (Pl. IV., fig. 24.)

Appendages.

1*st pair*. (Pl. V., fig. 26.)—The position of the cheliceræ can be made out on each side, the middle line commencing 10 mm. behind the anterior margin, and having a length of 12 mm., and a breadth of about 5 mm. at the base. They are somewhat conical in form, the apex of the cone being directed backwards. They agree in all essentials with the corresponding appendages in *E. scorpioides* and other forms.

2*nd pair*.—This appendage projects some 22 mm. beyond the anterior margin of the carapace on the right side. The number of joints cannot be ascertained. The last joint ends in a powerful pair of curved spines very broad in proportion to their length. The penultimate joint carries a spine on the inner side, and another rather doubtful spine lies immediately in front of the carapace.

3*rd*-5*th pairs*. (Pl. IV., fig. 23. Pl. V., fig. 26.)—The next three limbs resemble one another in being powerful, curved organs armed with a double series of curved spines along the anterior margin of the four distal joints. The terminal (7th) segment bears a pair of particularly powerful spines, and the strongest spines on the 5th and 6th joints appear always to arise from the distal angle; indeed, it is doubtful if these joints bear any spines beyond the one pair. The 4th joint bears, at all events, in the 3rd pair of appendages, three or four moderate sized spines. The spines themselves are smooth except when occasionally an extra large one shows signs of longitudinal striæ. The proportion of the joints to one another seems to be slightly different in the different pairs, but I have not been able to get any certainty as to which is which.

6*th pair*. (Pl. IV., fig. 23.)—Only one fragmentary specimen of this has been found. It probably belonged to an individual slightly larger than the type. The last two joints only are preserved, and of these the penultimate is a good deal broken. It (the penultimate joint) is very broad (40 mm.) and comparatively short (45 mm.), but presents no points of special interest. The terminal joint is oval in shape, comparatively long (63 mm.) and narrow (25 mm.). It narrows towards its point of articulation. The posterior border is marked along the distal third of its length by a series of four oblique

incisions. Of these the one nearest the apex is the best developed and marks off a distinct lobe. The other three are much smaller.

The triangular plate, arising as usual from the posterior portion of the distal end of the penultimate joint, is very large in this form. Its breadth at the base is 27 mm., and the length must have been about 35 mm. It is larger in proportion to the last joint than in any other Eurypterus with which I am acquainted.

I have ascribed a fragment of tail spine (Pl. IV., fig. 25) to this form. The chief reason for placing it here is that it does not seem to belong to either of the large species of *Stylonurus*. It had a triangular ridge running down the middle and occupying one third of the width.

In comparison with other species, *Eurypterus scoticus* seems most nearly allied to *E. scorpioides* (Woodward); *E. punctatus* (Salter); *E.* (*Eusarcus*) *scorpionis*, Grote; *E.* (*Echinognathus*) *clevelandi* (Walcott); *E. acrocephalus* (Semper). The last two may be disposed of in a few words. *E. clevelandi* was founded on a single limb, which in structure shows a close resemblance to the Eurypterids of this type, being furnished with long, curved spines. The number of spines is greater than in any of the usual forms, and it is possible that more than one limb may be present. Of *E. acrocephala* only the general shape of the carapace and body segments is known. These agree fairly well with *E. scoticus*, the carapace being conical, and the body widening in the first few segments and suddenly narrowing in the seventh. The remains of appendages in this Bohemian form are too slight and indistinct to admit of comparison.

E. punctatus of Salter resembles our form closely in some respects. The spine bearing legs are, on the whole, similar, but the spines are, according to the description, markedly striated. Further, the form of the swimming foot differs in the much greater length in proportion to breadth of the terminal joint, and in the greater breadth of the terminal joint in proportion to the penultimate one. *E. scorpioides* can be distinguished by the proportionately shorter spines on the 3rd—5th appendages, and by the less-marked width of the swimming foot as a whole. The terminal joint of this foot in *E. scorpioides* is also of the same width as the penultimate, while the triangular plate on the posterior side is very small. *Eurypterus obesus* resembles *E. scoticus* in the general body form, but the anterior appendages do not seem to be furnished with spines, while the last pair have a form more like *E. remipes*, the terminal joints being much expanded.

Eurypterus minor (Pl. V., figs. 27–29).

Carapace subquadrate, breadth slightly greater than length; eyes oval; body tapering; telson strong, triangular in section about equal in length to the last three segments; body surface granular.

This is a small form, the type specimen (fig. 27) measuring only 74 mm. in length. The specimen from which fig. 29 was drawn is somewhat larger, but wants the carapace. The carapace is widest at the posterior margin which is slightly concave. The sides

converge slightly towards the front and the anterior margin is comparatively straight. The eyes are well marked and oval in form, placed about equidistant from the lateral and anterior margins. There is a curious curved line arising from the posterior end of the eye (fig. 28). The body segments increase in length and decrease in breadth from before backwards, and are not marked by any special characters except the fine and sharply-cut granular markings. There appears to be no prolongation back of the posterior angles of any of the segments.

The telson is powerful in proportion to the animal, with a triangular section. The ridge which is on the dorsal surface is expanded at the proximal end (fig. 29) and forms a triangular, flattened area.

Only traces of the appendages are preserved, but the last pair seem to have been expanded.

This form differs from *E. lanceolatus* in the shape of the carapace and eyes and the squareness of the posterior segments which are conical in *lanceolatus*. The form of carapace and position of the eyes recall the arrangement in *E. Fischeri* (13) and its allies, but the eyes are more oval in form and the last body segment is not lobed.

Bembycosoma, gen. nov.

Carapace shaped like the ace of clubs, breadth greater than length, body conical short ; telson stout.

It is with much hesitation that I create this genus for the reception of an obscure form of which a number of mostly fragmentary specimens have come to light. My hesitation is not so much due to any doubt of the generic value of the form, but to the absence of sufficient information to supply a satisfactory diagnosis. Insufficiently described genera are the greatest curse of systematic zoology, and I should be sorry to add to their number. The one species known is :—

B. pomphicus, n. sp. (Pl. V., figs. 31 and 32.)

Characters, those of the genus and in addition a warty texture of skin. The cuticle was from its state of preservation thin and delicate, and the exact outlines are difficult to make out in many parts. The trilobed form of the carapace is only shown in one specimen (fig. 31), the others having a well-defined semi-circular form (fig. 32). The semi-circular outline of the carapace can be seen in the type specimen, and I am inclined to think that this is the true shape of the edge ; and that the lateral lobes are due to the flattening of dorso-lateral protuberances of the carapace. No trace of eyes has been made out, and though some obscure markings in some specimens probably indicate the gnathobases, I have not been able to interpret them. Any sign of limbs beyond the carapace is wanting.

The body tapers evenly down to the whole way and the telson continues the same

slope, being as wide at its base as the posterior margin of the last segment. The number of segments is not certain. I have never been able to count more than nine, but in fig. 32 the second segment is so large that it may represent two, and the posterior end is incomplete. The telson is 7 mm. wide at its base and seems to have been about 12 mm. long. The whole surface of the body is covered with the pimply structure from which the specific name has been derived, but the telson seems to have been smooth.

If the number of segments is really less than 12 this form would have to be removed from the Eurypterida and placed along with *Bellinurus* and *Prestwichia* among the *Xiphosura*. There is a resemblance in texture between this species and *Eurypterella ornata* described by MATTHEW (6) from the Lower Devonian of New Brunswick. According to his description, however, the carapace is triangular and segmented. It is just possible that his specimen showed only the body segments (he only figures 9 including the segmented carapace) and these broken away in front. I have not, unfortunately, had an opportunity of seeing the original specimen.

LIST OF PAPERS REFERRED TO.

(1) GROTE AND PITT, "New Crustacea from the Water Lime," *Bull. Buffalo Nat. Hist. Soc.*, vol. iii.
(2) HALL, "Natural History of New York," *Palæontology*, vol. iii.
(3) HALL AND CLARKE, "Natural History of New York," *Palæontology*, vol. vii.
(4) LAURIE, "Some Eurypterid Remains from the Upper Silurian Rocks of the Pentland Hills," *T. R. S. E.*, vol. xxxvii.
(5) LAURIE, "Some points in the development of *Scorpio fulvipes*," *Q. J. M. S.*, vol. xxxii.
(6) MATTHEW, "Remarkable Organisms of the Silurian and Devonian in S. New Brunswick," *T. R. S.*, Canada, vol. vi.
(7) PEACH, "Ancient air breathers," *Nature*, 1885.
(8) POHLMANN, "Additional notes on the Fauna of the Water Lime," *Bull. Buffalo, Nat. Hist. Soc.*, vols. iv. and v.
(9) THORELL AND LINDSTRÖM, "A Silurian Scorpion from Gothland," *Kongl. Svensk. Vetinck. Acad. Handl.*, vol 21.
(10) WALCOTT, "New Genus of the order Eurypterida from the Utica," *State Am. J. Science and Art*, vol. xxiii.
(11) WHITFIELD, "An American Scorpion," *Bull. Amer. Mus. Nat. Hist.*, vol. i.
(12) WOODWARD, "Monograph of the British fossil Crustacea, belonging to the order *Merostomata*," *Palæontographical Society*.
(13) SCHMIDT, *Mem. de l'Acad Imp. de St Pétersbourgh*, vol. xxxi.

DESCRIPTION OF PLATES.

(*Except where otherwise stated, the figures are natural size.*)

PLATE I.

Fig. 1. *Palæophonus loudonensis*, n. sp. × $\frac{2}{1}$
Fig. 1a. *Embryo of Scorpio fulvipes.* Magnified.
Fig. 2. *Slimonea dubia*, n. sp. Carapace and body segments.
Fig. 3. *Slimonea dubia.* Body segments and telson.
Fig. 4. *Stylonurus macrophthalmus.*

Fig. 5. *Stylonurus macrophthalmus.* Posterior segments showing epimera.
Fig. 6. *Stylonurus macrophthalmus.* Last two segments.
Fig. 7. *Stylonurus macrophthalmus.* Fourth appendage.
Fig. 8. *Stylonurus ornatus.* First and second appendages.

PLATE II.

Fig. 9. *Stylonurus ornatus.* Appendage 3.
Fig. 10. *Stylonurus ornatus.* Appendage 4.
Fig. 11. *Stylonurus ornatus.* Appendages 5 and 6.
Fig. 12. *Stylonurus elegans*, n. sp. Left side.
Fig. 13. *Stylonurus elegans.* Portion of carapace of a small specimen to show nature of margin.
Fig. 14. *Stylonurus elegans.* Posterior segments presumably belonging to this form.
Fig. 15. *Drepanopterus lobatus*, n. sp.
Fig. 16. *Drepanopterus lobatus.*

PLATE III.

Fig. 17. *Drepanopterus lobatus.* Fragment of specimen showing appendages and ornamentation.
Fig. 18. *Stylonurus elegans.* Right side.
Fig. 19. *Drepanopterus bembycoides*, n. sp.
Fig. 20. *Drepanopterus bembycoides.*
Fig. 21. *Drepanopterus pentlandicus.*

PLATE IV.

Fig. 22. *Eurypterus scoticus*, n. sp. Last two appendages.
Fig. 23. *Eurypterus scoticus.* Body segments × $\frac{1}{4}$.
Fig. 24. *Eurypterus scoticus.* Telson.
Fig. 25. *Eurypterus scoticus.* Type specimen.

PLATE V.

Fig. 26. *Eurypterus minor*, n. sp.
Fig. 27. *Eurypterus minor.* Carapace.
Fig. 28. *Eurypterus minor.* Posterior segments and telson of a somewhat larger specimen.
Fig. 29. *Eurypterus conicus.* Last segments and telson.
Fig. 30. *Bembicosoma pomphicus*, n. gen. and sp.
Fig. 31. *Bembicosoma pomphicus.*

CORRIGENDA.

On p. 590, Fig. 9 should be numbered Fig. 10, and so on throughout, the last fig. being No. 32. The figs. numbered 16, 21, and 25 appear in Plates III., IV., and V. respectively.

P. 583, lines 17 and 31, for "*bembycoides*" read "*bembicoides.*"

P. 588, for "*Bembycosoma*" read "*Bembicosoma.*"

Dr Laurie on a Silurian Scorpion Etc. — Plate I.

Fig. 1.

Fig. 1a

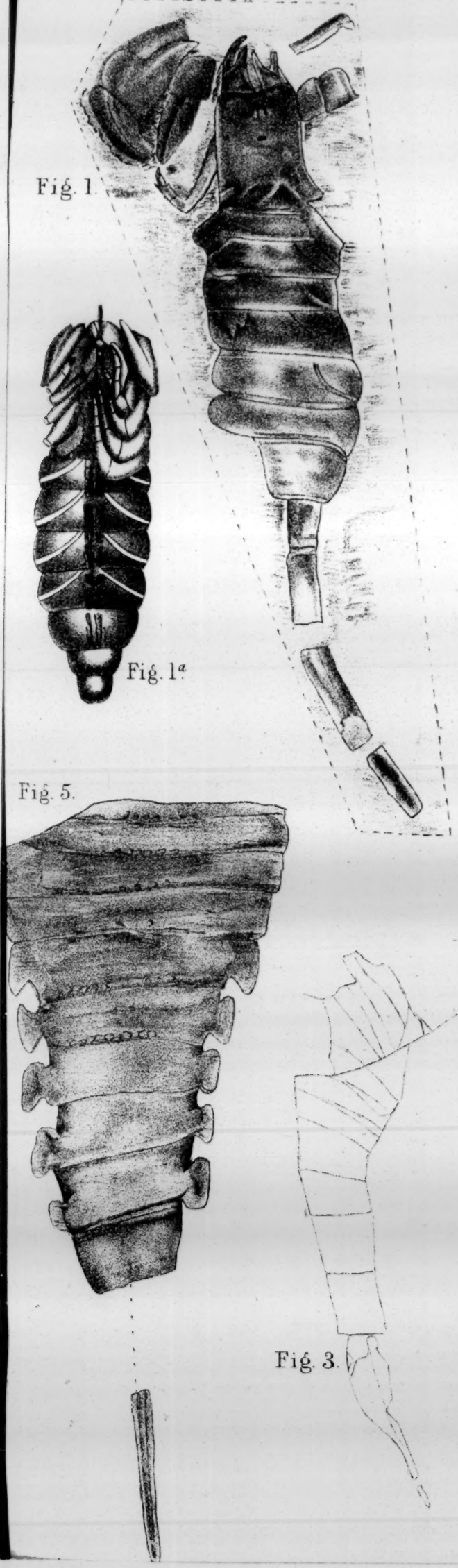


Fig. 5.

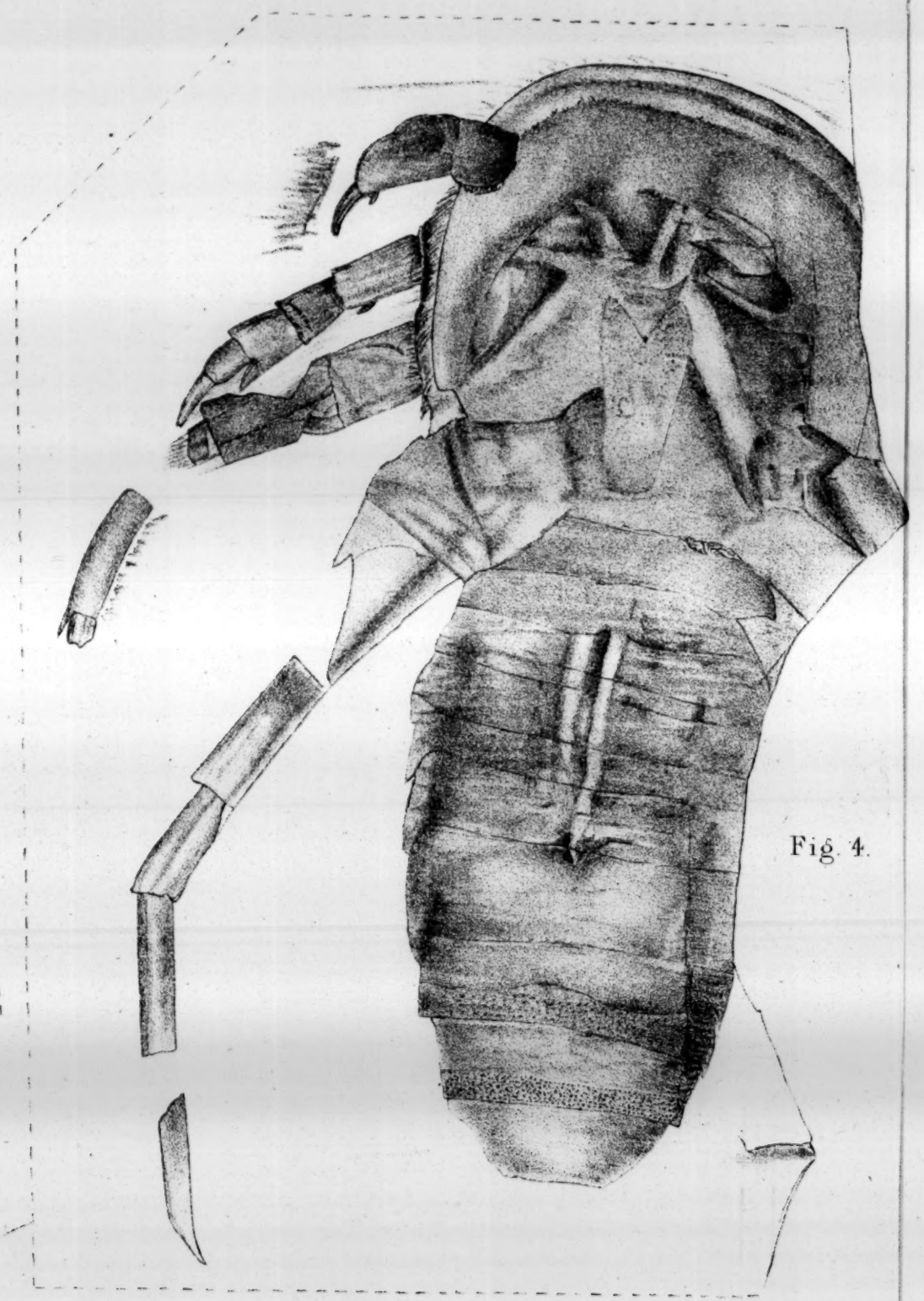


Fig. 4.



Fig. 3.



Fig. 2.

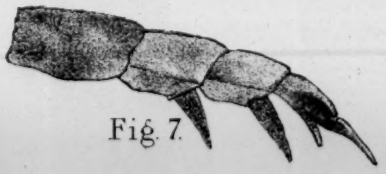


Fig. 7.



Fig. 6.



Fig. 8.

DR LAURIE ON A SILURIAN SCORPION ETC. — PLATE II.

Fig. 12.

Fig. 13.

15.

Fig. 16.

Fig. 10.

Fig. 11.

Dr Laurie on a Silurian Scorpion Etc. — Plate III.

Fig. 19.



Fig. 20.



Fig. 17.

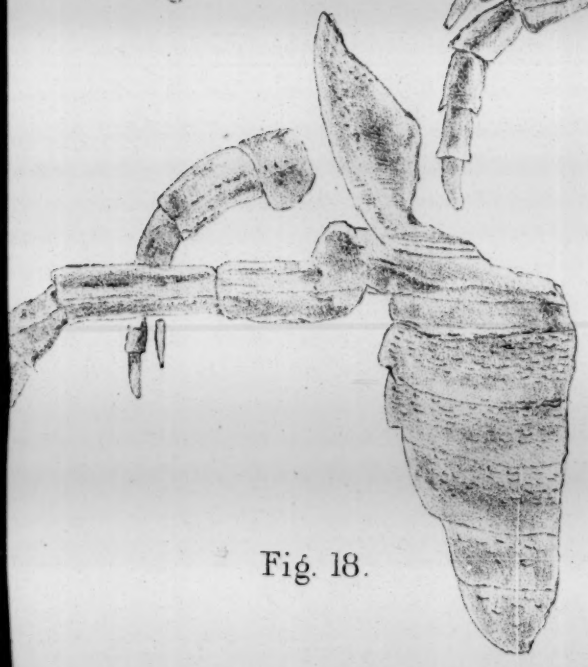


Fig. 18.

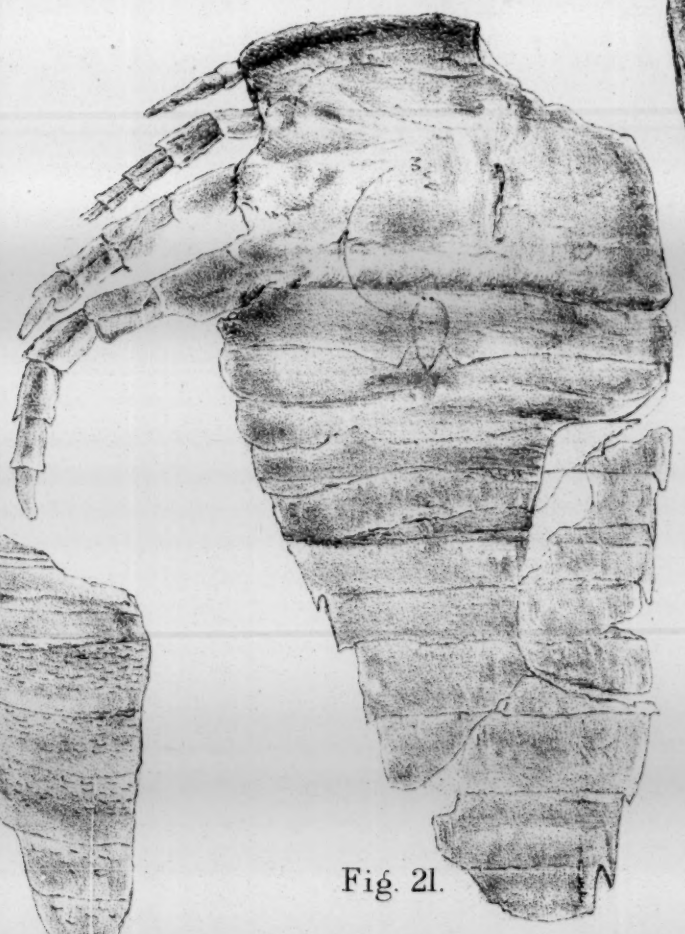


Fig. 21.

DR LAURIE ON A SILURIAN SCORPION ETC. — PLATE IV.

Fig. 22.

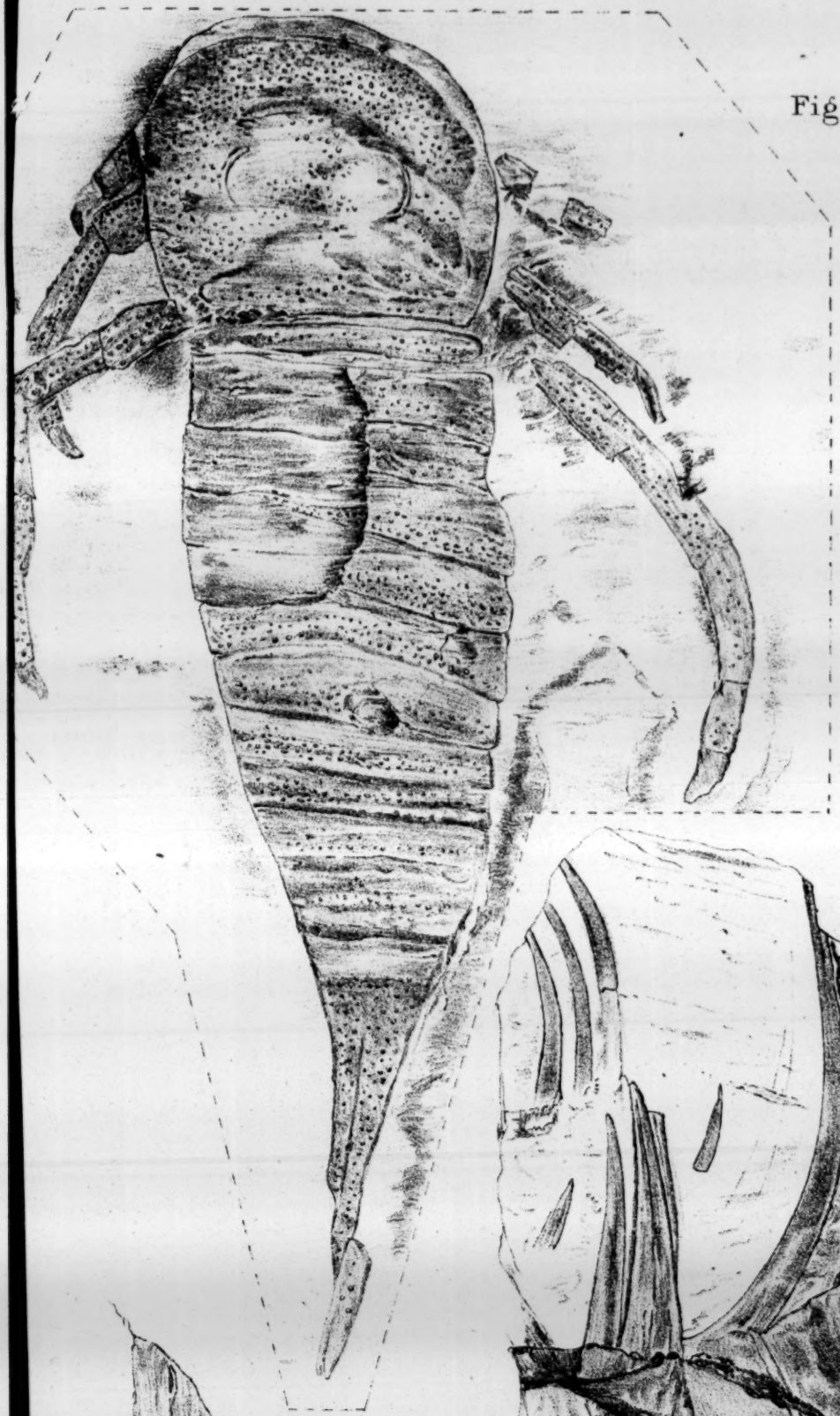


Fig. 24.

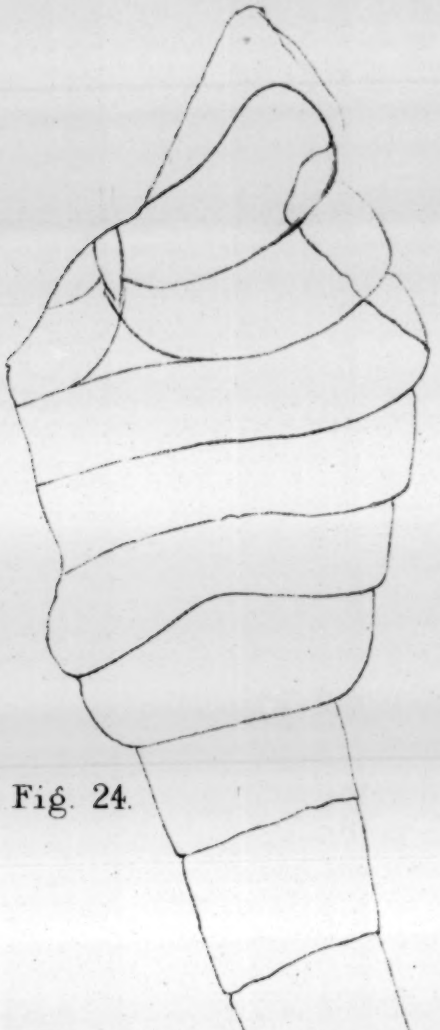




Fig. 25.



Fig. 23.

Dr Laurie on a Silurian Scorpion Etc. — Plate V.



Fig. 27.

Fig. 28.



Fig. 29.



Fig. 30.



Fig. 31.



Fig. 32.



Fig. 26.

XX.—*On a New Species of* Cephalaspis, *discovered by the Geological Survey of Scotland, in the Old Red Sandstone of Oban.*

By Ramsay H. Traquair, M.D., LL.D., F.R.S., Keeper of the Natural History Collections in the Museum of Science and Art, Edinburgh. (With a Plate.)

(Read 4th July 1898.)

In the autumn of last year Sir Archibald Geikie, F.R.S., Director-General of the Geological Survey of the United Kingdom, kindly placed in my hands for determination a number of specimens of *Cephalaspis*, collected by his officers in the Lower Old Red Sandstone of the neighbourhood of Oban. On examining them, I found that they all belonged to one species, which was, however, new to science.

Accordingly I drew up a brief diagnosis of this new form, which was included by Sir Archibald in his *Summary of Progress of the Geological Survey* for 1897, and it is now my privilege, with his sanction, to offer to this Society a more detailed description of the species, accompanied with figures.

Class, PISCES.
Sub-class, *OSTRACODERMI.*
Order, OSTEOSTRACI.
Family, CEPHALASPIDÆ.
Genus, *CEPHALASPIS, Agassiz*, 1835.

Cephalaspis Lornensis, Traquair.

Cephalaspis Lornensis, Traq. In the Director-General's *Summ. Progr. Geol. Survey* for 1897 (1898), p. 83.

Diagnosis.—Shield bluntly rounded in front; cornua *short*, considerably shorter than the occipital projection; eyes placed considerably *in front of the middle point* between the anterior margin and the posterior extremity of the shield.

The distinctive marks of this species are the short cornua, together with the anterior position of the orbits. This is illustrated by the accompanying restored outlines of the cephalic shield in *C. Lornensis* (fig. 2) and in the common *C. Lyelli* of the Lower Old Red of Forfarshire and Herefordshire (fig. 1).

As regards the cornua, the known species which most closely approaches *C. Lornensis* is *C. Murchisoni*, Egerton (fig. 3), from the Passage Beds (Downtonian) of

Herefordshire. Here, however, the cornua may be said to be obsolete (or undeveloped), while these projections in the Argyllshire species, though short, are yet quite distinct.

Description.—Fig. 1 in Plate represents a head nearly perfect in its contour, and more than usually uncompressed. It measures $1\frac{7}{8}$ inch in length, by about the same in greatest breadth, allowing for a slight deficiency of the cornual region of the right side. The upper aspect of the shield shows a flat margin, about $\frac{1}{4}$ inch in breadth, extending

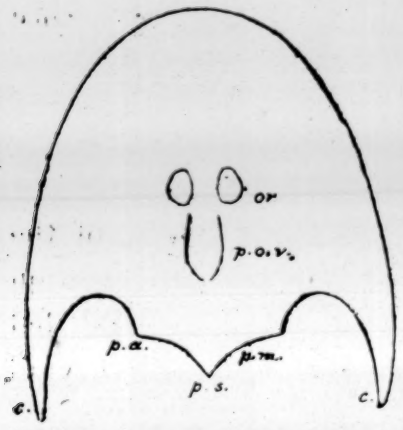


FIG. 1. — Outline of the cranial shield of *Cephalaspis Lyelli*, Ag.

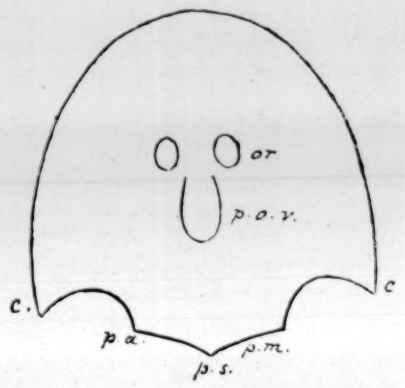


FIG. 2.—Of *C. Lornensis*, Traq.



FIG. 3.—Of *C. Murchisoni*, Eg.

or., orbit; *p.o.v.*, post-orbital valley; *c.*, cornu; *p.s.*, posterior spine; *p.m.*, posterior margin; *p.a.*, posterior angle.

from cornu to cornu round the front, within which the surface is convexly arched. Anteriorly the contour of the head is bluntly rounded; posteriorly the left cornu, which is completely shown, is short and bluntly pointed, and does not extend as far back as the posterior median point of the shield, nor even as far back as the posterior lateral angle.* The eyes are placed considerably in front of the middle of the shield, for the distance from the prominence of the posterior margin to a point midway between the orbits exceeds that from the same point to the front by $\frac{5}{16}$ of an inch, or about one-sixth part of the entire length of the head. Each of the oval orbits measures $\frac{3}{16}$ inch in length, and the two are rather less than the same distance apart. The antorbital prominences and valley are well marked, as is likewise the post-orbital valley, the latter being succeeded by a ridge ending in the posterior spine, which is, however, not prominent, and forms rather an obtusely angular point.

The substance of the test is present pretty nearly all over, and in front of the orbits indications of the external ornament are seen, apparently consisting of a very minute granulation.

Fig. 2 in Plate represents another nearly perfect cephalic shield, which is, however, compressed into a condition of nearly absolute flatness. In size, in proportion, and in contour, it closely agrees with the one just described. But the greater part of the bony substance being removed, the impressions of the radiating blood-vessels of the lower aspect are very distinctly exhibited.

* In some specimens the cornu extends back as far as the posterior lateral angle, but in none does it go as far as the posterior median point of the shield.

Fig. 3 in Plate presents us with a specimen in which not only the head but also a portion of the body is seen, though in both cases only in impression. The contour of the shield is well exhibited, and its proportions agree in all essentials with those of the specimens described above. The impressions of the inner surface of the lateral body scutes show a sort of corrugated appearance, which does not, however, in my opinion, indicate that these inner surfaces were not smooth as usual.

In fig. 4 of the same Plate we have an entire specimen lying on its side, but, unfortunately, not very well preserved. Only a portion of the impression of the cephalic shield, not including the orbits, is here seen, but it shows the peculiar short cornu (*c.*) on left side. We have also in this specimen, on the left side, the impression of the peculiar obtuse flap-like organ (*c.o.*) which passes back from the head between the cornu and the posterior angle of the shield in all the species of this genus where the specimens are sufficiently entire to show it. Of the body scutes, only the inner surfaces of some, and the impressions of the outer surfaces of others, are seen, whence it appears that they were smooth inside and minutely granulated outside. Some remains of the dorsal fin are exhibited opposite the point marked *d.* in the figure, and as regards the caudal it is present but not expanded.

An imperfect cranial shield, not figured, must, judging from its proportions, have been originally 4 inches in length, and its dimensions must have consequently exceeded those of any other British species except *C. Salweyi*, Eg., and *C. magnifica*, Traq. This specimen, consisting only of part of the right side of a shield, is interesting as showing the tesselation of the middle layer, and also the impression of that strange marginal strap-like internal space which lies close to and parallel with the outer rim, beginning close to the cornu behind and extending nearly to the middle line in front. This space has long been known in *Eukeraspis pustuliferas*,* in which it is subdivided into six loculi; but I had not observed it in *Cephalaspis* until some time ago Mr Smith Woodward pointed it out to me in specimens of *C. Murchisoni* in the British Museum.

Geological Position and Locality.—All the specimens as yet known were collected by the Geological Survey of Scotland in the Lower Old Red Sandstone of the neighbourhood of Oban. One specimen has also turned up in the island of Kerrera.

EXPLANATION OF PLATE.

Fig. 1. Detached cranial shield of *Cephalaspis Lornensis*, Traq., from a cliff on the roadside near Carrick Villa, Oban.

Fig. 2. A similar shield, much compressed, from the railway cutting near Dalintart, about one mile from Oban.

Fig. 3. A specimen, showing the head and a portion of the body, from the same locality as that represented in fig. 1.

Fig. 4. A specimen, nearly entire, lying on its side, showing the cornu (*c.*); cornual flap (*c.o.*); and remains of the dorsal fin (*d.*). From the shore 100 yards west of Gallanach Lodge, 3 miles south-west of Oban.

* Lankester, *Brit. Cephalaspideans, Pal. Soc.*, 1870, p. 58; Smith Woodward, *Cat. Foss. Fishes Brit. Mus.*, pt. ii. p. 193.

D.r R. H. TRAQUAIR ON CEPHALASPIS LORNENSIS.

Fig. 1.

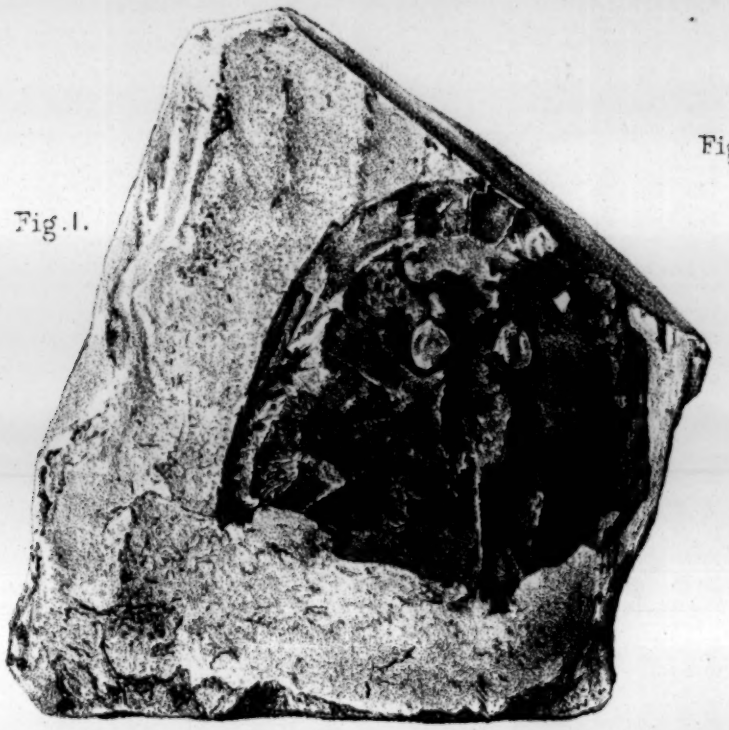


Fig. 3.

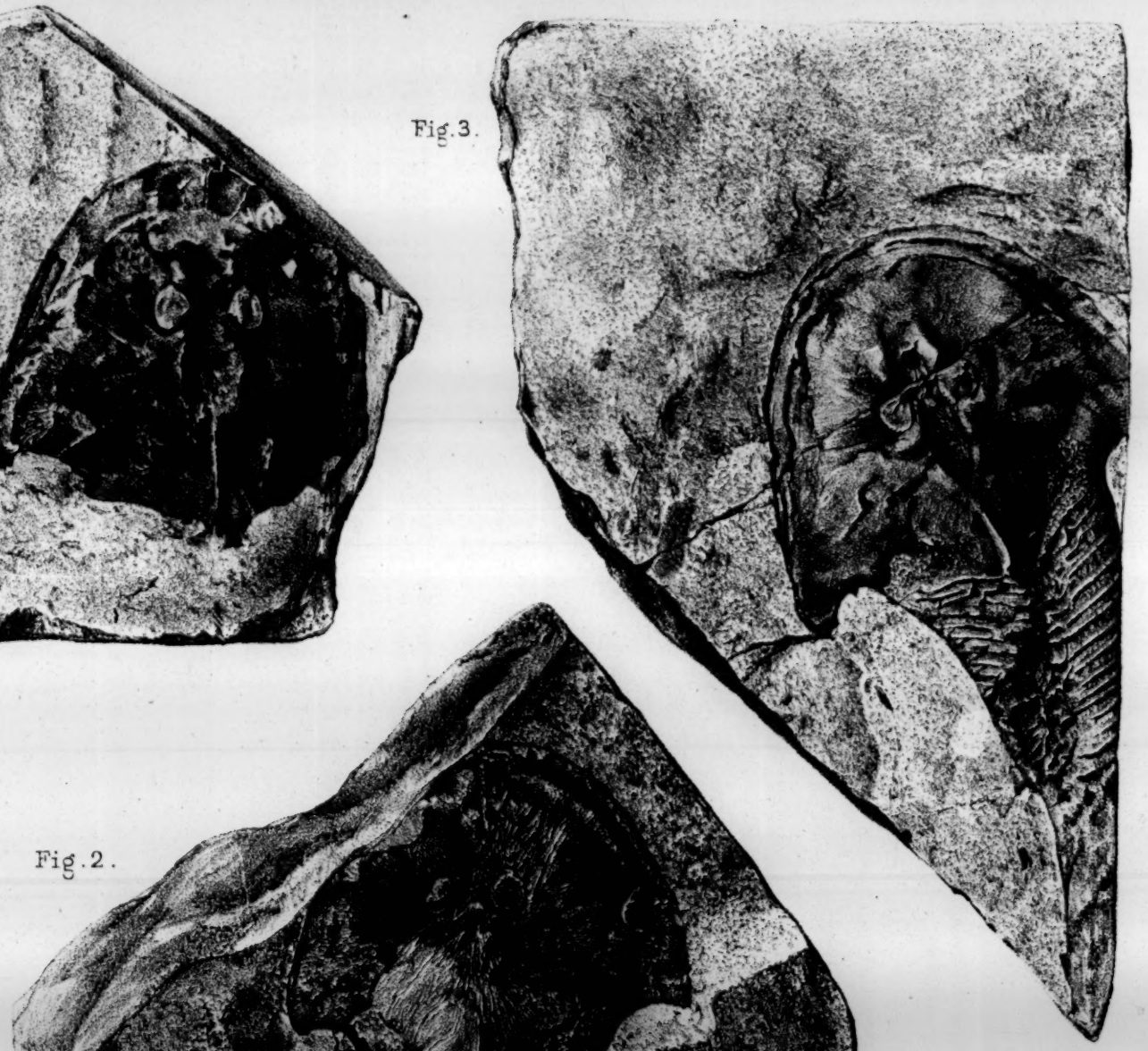


Fig. 2.



Fig. 4.

XXI.—*On* Thelodus Pagei, *Powrie, sp. from the Old Red Sandstone of Forfarshire.*

By Ramsay H. Traquair, M.D., LL.D., F.R.S., Keeper of the Natural History Collections in the Museum of Science and Art, Edinburgh. (With a Plate.)

(Read 4th July 1898.)

The remarkable fossil fish which forms the subject of the present communication belonged originally to the late Mr Powrie of Reswallie, and was acquired, along with the rest of his collection, by the Edinburgh Museum in 1892.

The specimen was obtained by Mr Powrie in a quarry of the Lower Old Red Sandstone at Turin Hill, near Forfar.

In his monograph on the *Fossil Cephalaspidæ of Great Britain*, published in 1870, (p. 41), Professor E. Ray Lankester refers in the following terms to the specimen in question :—

"A very remarkable specimen of shagreen-like structure has been discovered by Mr Powrie in Forfarshire, in beds which have furnished *Cephalaspis*. As I have not been able to assign it to Cephalaspidian fishes, though it may possibly be connected with them, I only allude to it here. It consists of a surface covered with minute spinous tubercles, the whole having the appearance of a fossilised piece of shagreen, and its shape is more or less that of a Cephalaspid, with head and body. The spinous tubercles on some species of *Cephalaspis* suggested the idea that this might belong to the lower surface of one of these fishes, covering in the ventral aspect of the head and the ventral series of scales; being detached, as it were, in this part of the animal, but adhering closely to the deeper aponeurotic layers of the exoskeleton in the superior parts, and forming the characteristic tuberculated surface of the shield and scales. There is, however, no proof of its connection with these fishes, and it seems more probable that it belongs to some early representative of the Sharks and Rays (representative, perhaps, only in the character of its dermal ossifications) than that it is the ventral surface of a Cephalaspid."

In the same year, however, Mr Powrie himself published a paper in which he named the fish in question *Cephalopterus Pagei*, and gave likewise a rough pen and ink drawing of it.* His description of the species is as follows :—

"Head and anterior portion of body very depressed, sub-elliptical, and truncated posteriorly; pectoral fins forming its outer posterior angles, which are rounded; posterior

* "On the Earliest Known Vestiges of Vertebrate Life; being a Description of the Fish Remains of the Old Red Sandstone of Forfarshire," *Trans. Geol. Soc. Edin.*, vol. i. pp. 284–301. Paper read 16th April 1869, published 1870.

portion of body short, narrow, triangular, scarcely half the width of anterior portion, rapidly narrowing to tail, which is long and linear; fins membranous, covered with very small scales in parallel rows; no demarcation between body and commencement of pectorals; caudal fin large, entirely below; scales small, somewhat elliptical in form, umbonate, smooth, inner surface having an elliptical indentation in centre. Branchiæ, consisting of exposed arches, arranged seven or eight on each side of, and at right angles to, a ridge running longitudinally along the centre of the under side of the disk, and occupying about one-half its length. Length about fourteen inches."

Regarding its affinities, Mr POWRIE thus expressed himself:—

"The peculiar form of the anterior portion of the body and pectoral fin, the long linear tail, and, if I am correct in considering the peculiar markings on the under side of the head as the branchial apparatus, it would appear decidedly placoid, and strangely allied to modern Rays; but at the same time the scales, although small, and somewhat like those of the Acanthodian fishes, are more ganoid in character than those of the latter family."

In the second part of Mr A. SMITH WOODWARD'S *Catalogue of the Fossil Fishes in the British Museum*, published in 1895, p. 200, the fish is simply mentioned as "Another supposed ally of the Cephalaspidæ," there being no remains of it in the London collection.

In 1894 I myself made a brief allusion to the so-called *Cephalopterus*, in treating of the remarkable plates from the Old Red Sandstone, known as *Psammosteus*, which are now usually considered to have belonged to some sort of "armoured Sharks." Speaking of the external sculpture of these *Psammosteus* plates, I remarked:—

"The minute stellate tubercles which form the ornament of the outer layer are clearly shagreen-bodies, which have coalesced, as very similarly shaped bodies, still disunited, may be seen to form the dermal covering in POWRIE'S *Cephalopterus Pagei* from the Lower Old Red of Forfarshire, the subjacent thickness of the plate being formed in a deeper layer of the skin." And as the name *Cephalopterus* had been pre-occupied as early as 1809, I proposed, in a footnote, to re-name the genus *Turinia*, from the locality whence the specimen was derived.*

It was not, however, till I became acquainted with a remarkable series of Upper Silurian fishes discovered in Lanarkshire by the Geological Survey of Scotland, and which, having been placed in my hands for description by Sir ARCHIBALD GEIKIE, will form the subject of a subsequent paper, that I discovered the special importance of this hitherto problematic fish from the Forfarshire Old Red, as well as the fact that the name *Turinia*, which I had given to it, must share the same fate as *Cephalopterus*, though for a different reason.

From their squamation, it became clear that some of these Silurian fishes were referable to *Thelodus* of AGASSIZ, a genus previously known only by detached scales

* "The Extinct Vertebrate Animals of the Moray Firth Area," published in HARVIE BROWN and BUCKLEY'S "*Vertebrate Fauna of the Moray Basin*," vol. ii., Edinburgh, 1896, p. 262.

from the Silurian of England and of Russia,* and, on more perfect examples being found, it turned out that their contour of body was identical with that of the *Cephalopterus* or *Turinia*. This led to a critical re-examination of the Forfarshire specimen, which, first convincing me of its close alliance with *Thelodus*, left me finally no choice but to merge it in the same genus.

Thelodus parvidens was the name given by AGASSIZ in 1839 to certain minute bodies, which are abundant in the well-known Ludlow "Bone Bed," on the supposition that they were fish-teeth resembling those of *Lepidotus*. These bodies, "small as grains of fine sand," are of a somewhat stud-like form, that is to say, constricted below the top, which is smooth and flat, or slightly convex, while in the centre of the under surface of the base is seen the opening of an internal pulp-cavity.†

That these bodies were not, however, teeth, but *scales*, or "the earthy grains or shagreen of the skin of large cartilaginous fishes," was maintained by M'COY, not only because of the abundance with which they sometimes occur over patches of rock, but also on account of their microscopic structure. He suggested that they were in all probability the shagreen granules of the same fish as bore the spines known as *Onchus tenuistriatus*, Ag., fragments of which are common in the same "Bone Bed."‡

MURCHISON, in his *Siluria* (1854, p. 238), adopted M'COY's views as to the correlation of *Onchus* and *Thelodus* in the following words :—

"*Onchus tenuistriatus* and *Murchisoni* are the bony fish defences such as were possessed by many placoid fishes of the old rocks. . . . The small cushion-like bodies, called formerly *Thelodus parvidens* by AGASSIZ, and which occur in myriads in the stratum, often forming large portions of its thin layers, are certainly the granules of the skin, or shagreen of one or other of these two common species."

In his well-known work on the *Silurian Fishes of Russia*,§ PANDER instituted the family Cœlolepidæ for various minute scales, or shagreen-bodies, from the limestones of Oesel, and of which he made three genera,—viz., *Cœlolepis*, *Pachylepis*, and *Nostolepis*, which were to be distinguished by the relative size, or, in the last case, by the absence, of the opening on the under surface of the base of the scale. Recognising the close resemblance of his *Pachylepis* to *Thelodus* of AGASSIZ, PANDER stated his readiness to withdraw the former name, if the identity of the two should afterwards come to be fully proved; though in that case he proposed to change *Thelodus* into *Thelolepis*. I do not find any indication of PANDER having believed in the correlation of the *Onchus* spines with the minute scales which he included under the term Cœlolepidæ.

However, Prof. von ZITTEL, writing in 1887, placed *Thelodus* in the sub-order

* Detached scales have also occurred in the Upper Devonian of Russia, and have been described and figured by Rohon as *Thelolepis* (=*Thelodus*) *Tulensis*.

† In MURCHISON'S *Silurian System*, vol. ii., 1839, p. 606, pl. iv., figs. 34–36.

‡ *Qu. Journ. Geol. Soc.*, vol. ix. (1853), p. 14; *British Palaeozoic Fossils*, p. 576.

§ C. H. PANDER, *Monographie der Fossilen Fische des Silurischen Systems der Russisch-baltischen Gouvernements*, St Petersburg, 1856.

Squalidæ or Sharks, and says of it and of PANDER's *Pachylepis*, that they "wahrscheinlich zu *Onchus* gehören."*

Mr SMITH WOODWARD in his *Catalogue*, part ii. (1891), p. 156, places the Cœlolepidæ as "Incertæ sedis" at the end of the Elasmobranchii. Like PANDER, Mr WOODWARD makes no remark as to these scales having belonged to the same fishes which bore the *Onchus* spines.

Professor J. V. ROHON, in his second paper on the "Upper Silurian Fishes of Oesel," published in 1893,† adopts PANDER's Cœlolepidæ, which he places in the sub-class Selachii, order Plagiostomi, and sub-order Squaloidei. They are, he says, "ausgestorbene Selachier, deren Haut mit Placoidschuppen bedeckt war," adding also that "zu diesen Schuppen gehören wahrscheinlich auch Zähne und Flossenstacheln (*Monopleurodus*, *Onchus*)." ROHON, in fact, adopts M'COY's opinion, that the *Thelodus* scales and *Onchus* spines belonged together, adding also the small teeth named *Monopleurodus* by PANDER, as probably appertaining to the same group of fishes. He also describes minutely the microscopic structure of the Cœlolepid scales, comparing them in this respect with those of the extinct Acanthodei, and of the living Selachii. His conclusion is that the Cœlolepid scales are genuine "Placoidschuppen," but, as they in many respects show a simpler condition of histological structure, they must be regarded as placoid scales in a lower stage of development.

We have seen that ZITTEL, in 1887, placed the Cœlolepidæ among the Squalidæ, but in 1895 he is more disposed to refer them to the Acanthodei. Treating of the last-named group, he says :—

"Die grosse Uebereinstimmung der *Climatius*-stacheln mit gewissen als *Onchus* bezeichneten Ichthyodorulithen lässt es überaus wahrscheinlich erscheinen dass wenigstens ein Theil der letztgenannten im oberen Silur und Devon verbreiteten Reste von Acanthodiden herrühren. Höchst wahrscheinlich gehören auch die vierseitigen Chagrinschuppen welche von AGASSIZ unter dem Namen *Thelodus* aus dem obern Silur von England, von PANDER unter der Bezeichnung *Cœlolepis*, *Thelolepis*, *Pachylepis*, und *Nostolepis* aus dem oberen Silur von Oesel beschrieben wurden." ‡

To sum up, then, the current opinions concerning the Cœlolepidæ, it would seem that these scales are now looked upon either as *incertæ sedis*, or as having belonged to true *Squali* or to *Acanthodei*, and that the notion is also entertained by many that the defensive spines known as *Onchus* formed part of the dermal armature of the same creatures.

It is the object of this paper to show, by means of a re-examination of the Lower Old Red Fish called by POWRIE *Cephalopterus Pagei*, that, though probably of Elasmobranch ancestry, the Cœlolepidæ cannot be classed either as Squali or as Acanthodei, and that it is not among *them* that the ancient possessors of the *Onchus* spines must be looked for.

* *Handbuch der Palæontologie* (Palæozoologie), vol. iii. p. 64.
† *Die Obersilurischen Fische von Oesel*, ii. Theil ; *Mem. Acad. Imp. Sc. de St Petersbourg*, t. xli., No. 5, 1893.
‡ *Grundzüge der Palæontologie*, Munich and Leipzig, 1895, p. 530.

Fig. 1 on the accompanying Plate represents the specimen in question, one-fourth smaller than the natural size. The entire length is 12 inches, the breadth at the widest part, namely, between the two lateral fin-like expansions, is 6 inches.

The fish, as it lies before us, may be described as consisting of two divisions, anterior and posterior. The former of these, broad and expanded, may be said to be approximately trapezoidal in shape, the narrow side, which is also convex anteriorly, being in front, the broad one behind, while the opposite lateral margins diverge as they pass backwards; and we note further that the angles are rounded off. Or, in other words, the anterior vertically compressed and laterally expanded portion forms at each posterior rounded angle a fin-like flap, the contour of which is sharply marked off behind from the tail, but is continuous anteriorly with the outline of the head, which is also rounded in front.

These flap-like expansions were by POWRIE considered to be pectoral fins comparable with those of the Rays, and for my own part I cannot see how we can interpret them otherwise than as lateral fin-folds. If that interpretation is correct, we have here a very interesting point in connection with the much discussed question of the morphology of the paired limbs in vertebrates.

The surface contained within the boundaries of this expanded anterior part of the fish is gently convex. About the middle, however, there is a slight longitudinal depression about $2\frac{1}{2}$ inches in length, and reaching to $1\frac{1}{2}$ inch from the front, at the bottom of which, and extending for its whole length, there is a feeble bar-like elevation or rounded ridge. From this central longitudinal ridge there pass out on each side eight transverse bar-like elevations of a similar character, the more posterior of which tend more and more to a backward direction, so that the last pair show a contour like that of a V widely open and reversed in position.

I have not the slightest doubt that POWRIE was right in interpreting this arrangement of ridges and intervening grooves as indications of a branchial apparatus, but it is quite another thing to describe the "branchiæ" as "consisting of exposed arches," or to deduce from the presence of those markings that we have the *ventral* aspect of the fish before us. We see neither branchiæ nor branchial openings in the specimen, and the ridges and grooves alluded to are, to my mind, simply indications of the presence of a cartilaginous branchial skeleton in the living fish, but which of course is lost in the fossil. That the exposed aspect of the specimen is dorsal and not ventral, is, as we shall see further on, conclusively shown by the position of the lower lobe of the heterocercal caudal fin.

It seems, therefore, certain that this expanded anterior part contains much more than the *head*,—namely, the anterior portion of the body as well,—and in this manner the lappet-like expansions which form its right and left posterior angles may well be interpreted as lateral fin-folds. Their general appearance also suggests an analogy with the cornual flaps of *Cephalaspis*, which were originally considered by LANKESTER as the equivalents of pectoral fins.

At this stage we may take notice of the fact that the head affords us no traces whatever of teeth or of jaws. Where the mouth was is not evident, and we are as little informed as to the position of the eyes.

The tail is $2\frac{1}{2}$ inches broad at its origin, and becoming rapidly narrower, extends back for $6\frac{1}{2}$ inches where it is broken off. Unlike the anterior part of the animal the tail appears laterally compressed, but nothing can be seen in its structure but a mass of the shagreen-bodies to be presently described. Before this tail is cut off by the edge of the stone, part of a median fin is seen passing off obliquely on the aspect opposite to that from which we view the fish as a whole. As there can be no doubt that this is the lower lobe of a heterocercal caudal, it follows that the aspect of the body, which is exposed in the fossil, is the dorsal, and not the ventral as supposed by POWRIE.

There is no evidence in this specimen of the presence of ventral, dorsal, or anal fins.

The only actual structures which are preserved in the specimen, are the scales or shagreen-bodies with which the entire surface of the fossil is closely covered, from the front of the head to the broken off extremity of the tail. They are mostly of an oval form, as seen in figs. 2–4, though they are more nearly round at the anterior margin. Their size is also pretty uniform, the long diameter measuring $\frac{1}{27}$ to $\frac{1}{30}$ inch, but on the lateral fin-expansions they are considerably smaller. The outer surface (fig. 2) is brilliantly ganoid, slightly convex, and distinctly nicked or crenulated round the margin; the lower surface (fig. 3) is also convex, and shows in the middle a comparatively large hole or opening passing into the interior. Most of the scales in this specimen are, however, broken through horizontally, as seen in fig. 4, and here the internal cavity, into which the hole in the base passes, is very obvious and of considerable size.

To complete our knowledge of the configuration of these scales, it is, however, necessary to turn to a second specimen, from Canterland, Kincardineshire, consisting only of a mass of the shagreen about 2 inches square, referable, I believe, to the same species but to a larger and older individual. The scales are larger, their long diameters often measuring $\frac{1}{16}$ inch, and their form is frequently more irregular as seen in figs. 5–8.

In fig. 5 three scales are represented, magnified eight diameters, one of which, of a peculiarly narrow shape, is seen from below, while the other two are seen *from the side*. Here we see that the smooth rounded basal portion is separated by a slight constriction from the upper part or crown, and that the crenulations of the margin of the latter are continued down the "neck" as a sort of vertical fluting as far as the upper boundary of the base. Fig. 8 represents the lower surface of a scale similarly magnified from which we see that it is strongly convex and provided with a very small opening leading into the interior, but in many cases this hole is quite imperceptible as in the two lower scales of the group of four shown in fig. 6; in this figure we also see that the convexity of the base may be in places obtusely

angulated instead of being regularly curved. In this specimen it is also to be noted that where the scales are broken horizontally the internal cavity is reduced to a mere narrow cleft as seen in fig. 7.

The extreme softness of the scales in both of these specimens renders it impossible to extract or isolate any one of them as they crumble away at once if any attempt is made to develop them with a needle or other sharp instrument; nevertheless, the examples which I have figured just as they lie in the stone afford us a very good idea of their configuration. Their friable condition is also adverse to their being examined microscopically. Seen with a powerful platyscopic hand lens, the substance of the scales, where broken across, is white pearly and homogeneous in texture, some traces of concentric lines being occasionally observable as in the fractured example shown in fig. 4.

A consideration, however, of the macroscopic character of these dermal appendages leaves no doubt that the fish before us belongs to the family Cœlolepidæ of PANDER, and that it throws a startling light upon the bodily configuration of fishes, of which, as aforesaid, we have hitherto known nothing but the detached scales. This is clearly shown by the general form of the scales, by the internal pulp cavity, as well as by the opening in the lower surface of the base, which in those of the entire specimen is pretty large (fig. 3), while in those of the shagreen-fragment it is very small as in fig. 8, or even apparently absent as in figs. 5 and 6.

There is also as little doubt that the genus *Thelodus* itself (*Thelolepis*, PANDER, ROHON) is the one to which this species from the Forfarshire Old Red is referable. ROHON'S suppression of *Nostolepis*, PANDER, is here clearly justified by the scales of the shagreen-fragment to which I have just alluded, in many of which no basal opening can be seen, while in others it is present. And if I am right in referring this fragment to the same species as the entire fish represented in fig. 1, then a question comes up as to the validity of the genus *Cœlolepis*.

For the essential difference between *Cœlolepis* and *Thelodus* is the larger size of the pulp cavity and basal opening with a feebler development of the base in the former, while in the latter the case is reversed. To quote ROHON'S words :—

> "Placoidschuppen mit weiter Pulpahöhle und rudimentärer Basalplatte . . *Cœlolepis.*
> Placoidschuppen mit kleiner oder rudimentärer Pulpahöhle und wohl entwickelter Basalplatte . . . . . *Thelolepis.*"

It is to be observed, however, that in the scales of the entire and presumably younger specimen of *Th. Pagei* (figs. 1–4) the pulp cavity and basal opening are proportionally larger than in the scale of the shagreen-fragment (figs. 5–8) which evidently belonged to an older and larger fish. Here the pulp cavity and basal opening have been evidently reduced by the progress of calcification advancing with age; and it may well be asked—may not the supposed distinction between *Cœlolepis* and *Thelodus* be entirely of this nature? I, for my part, am inclined to reply in the affirmative.

The recognition of POWRIE'S "*Cephalopterus*" *Pagei* as a veritable species of the genus *Thelodus* does indeed throw quite a new light upon the ancient family of Cœlolepidæ.

In the first place it absolutely disposes of the supposition that the fin-spines known as *Onchus*, AGASSIZ, or the teeth known as *Monopleurodus*, PANDER, and *Ancistrodus*, ROHON, had anything to do with this family. *Thelodus Pagei* shows neither teeth nor fin-spines, and in these particulars the Cœlolepidæ, to be described in my forthcoming "Report" on the Upper Silurian fishes collected by the Geological Survey of Scotland in Lanarkshire, perfectly agree. The general form of the body likewise, as well as the absence of recognisable jaws and teeth are features decidedly against the retention of *Thelodus* in the order Selachii at all.

In the second place, these specimens render it equally certain that *Thelodus* is not referable to the Acanthodei.

In my next paper I shall endeavour to show that a series of forms connects *Thelodus* with the Heterostracous division of the Ostracodermi, and that the Cœlolepidæ in fact belong to that order. Nevertheless, the nature of their dermal covering clearly points to an Elasmobranch affinity, and to the idea that through them the Heterostraci have had a common origin with the primitive Elasmobranchs.*

EXPLANATION OF PLATE.

Figs. 2–8 enlarged eight diameters.

Fig. 1. *Thelodus Pagei* (POWRIE); reduced one-third. From the Lower Old Red Sandstone, Turin Hill, Forfarshire. (Powrie Collection, Edinburgh Museum of Science and Art.)

Fig. 2. Outer surface of scales from near the front; magnified eight diameters.

Fig. 3. Under surface of a scale, from a wax impression, showing a basal opening of considerable size.

Fig. 4. A scale broken through horizontally, showing the internal pulp cavity.

Fig. 5. Three scales from a patch of shagreen belonging to *Thelodus Pagei* but from Canterland, Kincardineshire. The two lower scales seen from the side; the upper one, from below. (Powrie Collection.)

Fig. 6. Four scales from the same patch seen from below; in two the basal opening is very small; in two it is apparently absent.

Fig. 7. Scale from the same patch, broken through horizontally, showing the pulp cavity reduced to a narrow chink.

Fig. 8. Scale from the same patch, seen from below, showing a small basal opening.

* It may, however, here be noted that Dr O. M. REIS of Munich has already expressed the opinion regarding the Pteraspidæ, the only family hitherto by common consent included in the Heterostraci, that they, along with the Psammosteidæ, "eine einheitliche Degenerationsgruppe der Elasmobranchier bilden," *Geognostische Jahreshefte*, vi. p. 64. See also the same author in SCHWALBE'S *Morphologische Arbeiten*, vi. p. 213.

Dr R. H. TRAQUAIR ON THELODUS PAGEI.

Fig. 1. $\frac{3}{4}$



Fig. 8. $\frac{8}{1}$



Fig. 7. $\frac{8}{1}$



Fig. 6. $\frac{8}{1}$

Fig. 5. $\frac{8}{1}$



Fig. 4. $\frac{8}{1}$

Fig. 3. $\frac{8}{1}$

Fig. 2. $\frac{8}{1}$

XXII.—*The Emblem of the Crab in Relation to the sign Cancer.*

By D'Arcy Wentworth Thompson, C.B.

(Read 16th May 1898.)

This paper deals with a leisure-time study of mine, from which I once before drew the subject of an essay for this Society. The representations of animal form in Ancient Art interest me chiefly in so far as they present to us a problem involving an unsolved question as to the whole spirit and motive with which such representations were designed. We may see expressed in them merely a tendency to portray familiar objects in appropriate associations, to use the fish, or the crab, or the cuttle-fish as the device of an habitation of fishermen or resort of mariners, or to employ those creatures and others—the ox, the sheep, and the goat, the lion, the wolf, and the eagle—as the more fanciful appanage of divinities whose imaginary attributes were the attributes of the sea, the plain, the mountain or the sky. But we err, in my opinion, if we fail to recognise in this antiquated symbolism a deeper intention, whose spirit is rather Oriental than Western, archaic than modern. I believe these representations for the most part to have been associated with specific notions or beliefs of astronomical or astrological science, of religion or superstition. And though the origin of such ideas may be hidden from us, and the meaning of the symbols often obscure, yet I think we may discern that the emblems were grouped in a very artificial and curious way, that these conventional but much varied collocations correspond in a singular and precise degree with natural groupings of constellations that are similarly figured and designated, and that the divinities with whom the emblems are associated had themselves a corresponding relation to the Signs of Heaven, where they had their places according to the doctrines of astrology, or which marked their festivals in the astronomic system of the sacred calendar. I have chosen the Crab for the subject of this essay; how are we to decide whether this Crab was portrayed merely as a familiar object to the fisherman, a valued product of the settlement, a natural emblem of the ocean, a symbol of the symbolised god or goddess of the sea, or whether, on the other hand, it recalled and recorded allusions to the celestial Crab, in whose sign astrology taught that one deity was exalted and another domiciled, and whose conjunction with the Sun marked a great epoch in the year? We may reasonably regard it as proved that the Crab which

artists fashioned or story-tellers told of was indeed the stellar one, if we can show it to be, for instance, associated on coins and in story with the Moon that was domiciled, with Jupiter * that was "exalted," and with Mercury, that in the scheme of distribution of the twelve greater gods was located, in the same sign; with the Hydra, the Crow and the Cup, that lie under or near it in the heavens, the Lion and the Virgin that stand beside it, and next again, the Dog that rises with it, the Eagle, the Fish, and the Sea-monster that set as it rises, or the Horse and the Altar that rise as it sets. I propose, in this paper, to set forth what we know of the Crab in relation to the sign Cancer, as it is presented to us in classical myth and emblem, and to deal with the references made by classical authors to the sign, both such as are descriptive merely, and such as are symbolical or mythological in their interpretation.

The sign Cancer was in classical times, as it no longer is, though we still speak of it as if it were, the sign of the summer solstice: aestus erat, mediusque dies: solisque vapore Concava littorei fervebant brachia Cancri (Ovid, *Met.* x. 126). It was the goal of the Sun's yearly journey,—Cancer ad ardentem fulgens in cardine metam, Quum Phœbus summis revocatus curribus ambit, Articulum mundi retinet, lucesque reflectit (Manil., iv. 162), and a bye-word for the heats of summer, ausus in ardentem ripas attollere Cancrum (Lucan, x. 287), nam quis ad exustam Cancro torrente Syenen, Ibit (Lucan, viii. 851). According to the Calendar of the Fasti, the Sun entered Cancer on June 17th or 19th (xv.–xiii. Kal. Jul.) and a week later is marked as the date of the *solstitium*, Sol abit e geminis et Cancri signa rubescunt (*F.*, iii. 727). The former of the two dates is that used by Pliny, solstitium peragi in octava parte Cancri, et octavo Kalendas Julii diximus. Magnus hic anni cardo, magna res mundi (*N.H.*, xviii. 68 †). Avienus tells us that Meton reckoned Cancer the first of months, Sed primaeva Meton exordia sumsit ab anno Torreret rutilo cum Phœbus sidere Cancrum (*Aratea*, v. 46).

The sign Cancer was the "domicile" of the Moon, and though we need not profess to explain thereby this location of her domicile, yet we may remember that the full Moon stood in Cancer, opposite to the Sun, at the time of the winter solstice, of the longest nights. I think that she stood here, at the northern boundary of her course, and overpowering the winter darkness and the lesser lights of the brightest stars (all of them stars surrounding her in her position there) in the opening passage of Claudian's Ode to the new Consuls of the year :—

* I am favoured by Dr Burgess with the following note :—Pushya, the Hindu Asterism corresponding to a large portion of Cancer (or long. 93° 20′ to 106° 40′ in A.D. 500), from the root *pūsh*, "nourish, thrive," is also called *Tishya* "auspicious" and *sidhya* "prosperous." Its divinity is Bṛihaspati (Jupiter), the priest and teacher of the gods. Its three principal stars are δ Cancri (108° 42′) and γ, δ, Cancri, with the nebulous cluster Præsepe between. It is represented as an arrow and also as a *crescent*.

† In the Calendar of Constantine (*Uranol.* Petav., p. 112) the Sun enters Cancer on June 15 (xvii. Kal. Jul.), and the solstice is placed on the 25th. Ptolemy fixes the Sun's entry into the sign on June 18th, and the solstice on the 26th.

Haud secus ac tacitam Luna regnante per Arcton
Sidereæ cedunt acies, cum fratre retuso
Aemulus adversis flagraverit ignibus orbis.
Tunc jubar Arcturi languet, tunc fulva Leonis
Ira perit. Plaustro jam raro intermicat Arctos
Indignata tegi, jam caligantibus armis
Debilis Orion dextram miratur inermem.

The Crab is associated with the Lunar crescent on various coins, *e.g.*, of Consentia and Terina, both towns of the Bruttii, and with the head of Selene in the zodiacal series of Antoninus Pius. It figures more conspicuously in the symbolism of the Ephesian Diana, on whose images it is frequently seen as the pendant or centre-piece of a necklace, lying on the bosom of the goddess (*cf.* Cl. Menetreius, *Symb. Dian. Ephes.*, Romæ, 1657), where it not only figures (with its horns curved crescent-wise) as a lunar emblem of a lunar goddess and in suggestion of the goddess's astrological house, but may perhaps remind us also that Cancer, in the astrological distribution of the body, was guardian of the breast, pectusque locatum Sub Cancro est (Manil., ii. 459). According to Menetreius (p. 26), apud Bruttios peculiari et insigni coronamento Dianæ caput Cancri testa ornatum, ut in eorum nummis palam est; and precisely such a headdress is figured on a Bruttian coin by Imhoof-Blumer and Keller (pl. viii. 5), though the goddess is there wrongly identified with Amphitrite;—a similar headdress occurs on a female figure (identified as Thalassa) on coins of Corycus, etc. (*ibid.* p. 51, Head, *Hist. Numm.*, p. 602). The crab also occurs together with the head of Artemis on coins of Massilia (Saussaye, pl. i. 6–10; Head, p. 7; *cf.* also Macrob., *S. Sc.*, i. 17, 21). Menetreius suggests also, I think with reason, that it was not without allusion to lunar influences over generation and birth that Plato, and those whom he followed or who followed him, made Cancer the celestial gateway by which the souls of men enter the world (*cf.* Macrob., *S. Sc.*, i. 12; Porph., *de Antro N.*, vi.; Clem. Alex., *Strom.*, v. p. 675).

It was at Ephesus that the marvellous cavern of serpents was that the crabs slew, ἢ πάντως ἂν ἀπολώλασιν οἱ περὶ τὸν χῶρον ἐκ πολλοῦ, εἰ μὴ φύσει τινὶ ἀποῤῥήτῳ περιειληφότες οἱ προειρημένοι κάρκινοι τῆς λίμνης τὰ χέιλη, καὶ ἀνείργοντες, εἶτα εἰρηναῖα ἀπέφαινον τὰ ἐν τῷ τόπῳ πάντα (Ael., *N.H.*, xvi. 38). We find similar myths alluded to in Pliny (*N.H.*, xxxii. 19), Thrasyllus auctor est, nihil aeque adversari serpentibus quam cancros, suesque percussas hoc pabulo sibi mederi. Quum sol sit in cancro, torqueri serpentes. It is an old and common legend that crabs are in better condition or worse as the Moon waxes and wanes.

It is not as a zodiacal sign, but simply as a constellation that the Crab figures in the second labour of Hercules. In the more ancient period to which the legend appertains, Leo was (or was still thought of as) the sign of the summer solstice, and it was with the slaying of the Nemæan Lion that the hero's task began. It was while Hercules fought the Hydra, in the second of his labours, that the Crab, coming to the aid of the monster, bit the hero in the foot (Panyasis of Halicarnassus, *ap. Apollodor.* iv. 2):—hic dicitur

Junonis beneficio inter astra collocatus, quod cum Herculem contra Hydram lernaeam constitisset, ex palude pedem ejus mordicus arripuisset; qua de re Herculem permotum, eum interfecisse (HYGIN., *P.A.*, ii. 23; *cf.* SCHOL., in *C. German. Aratea, Cataster.*, p. 245; MANIL., ii. 33). This was while the Sun was in his second zodiacal sign, the sign Virgo, which is placed in part over the long body of the constellation Hydra, and the latter, indeed, rises together with the sign, as far as that portion of its body on which is placed the Cup, *ἀντέλλει δ' Ὕδρη μὲν ἐπὶ πλέον ἄχρι παρ' αὐτόν* Κρητῆρα (ARAT., 602, *cf.* HYGIN., iii. 24). The sign Virgo was identified by some with Ceres (HYGIN., ii. 25, *cf.* *C.I.G.*, vii. 759, etc., etc.*), and was represented as the seat of Ceres in the Roman partition of the signs among the twelve greater gods†; and we find another indication of the link between Virgo and Demeter on the one hand, and the fabled Hydra on the other, in the celebration of the mysteries of Demeter at Lerna, *καὶ τελετὴν Λερναίᾳ ἄγουσιν ἐνταῦθα Δήμητρι* (PAUS., ii. 36, 7).

We can trace an association with the Crab in the case, also, of the Babylonian Hercules, Gilgames, the Γίλγαμος of AELIAN. (xii. 21); on a certain cylinder (LAYARD, *Culte de Mithra*, pl. xxxiv. 7), he "appears triumphantly bearing a crab (*Cancer*) at the end of a stick over his left shoulder, whilst his right hand holds two fishes" (R. BROWN, jun., "The Celest. Eq. of Aratos," *Tr.* ix. *Brit. Or. Congr.*, 1892, p. 461). Mr ROBERT BROWN has told us also how, in the Euphratean epic, Gilgames slays, under the sign of the Crab, the tyrant Khumbaba, the Κύμβαβος of LUCIAN (*de Syr. D.*, xix., R. BROWN, jun., *Semitic Influences*, p. 178).

In the classical sequence of the story, which begins with Leo, the sign Cancer corresponds to the last of the Hero's labours, though in the legend the Crab itself does not figure in the twelfth scene; but the great epic had many variants, and we need not assume that the Crab on the cylinder of Gilgames refers to the same episode as that of the second labour, in which Cancer appears as a paranatellon and not as the dominant sign. On coins of Croton, of Cos, and in the later coinage of Agrigentum, Hercules and the Crab figure on opposite sides of coins that depict for us no details of a story: the allusion may possibly be simply to the circumstance that as Cancer rises, Engonasis, the kneeling Hercules, sets, "Cancer exoriens obscurat . . . caput cum reliquo corpore ad umbilicum ejus qui Engonasis vocatur" (HYGIN., *Poet. Astr.*, iv. 12).

For the understanding of most of the other representations of the Crab on coins, and of the emblems that are shown in conjunction with it, it is necessary to know the relative positions, not only of the constellations adjacent to Cancer, but also of those that rise or set in conjunction with or in opposition to it.

According to MARTIANUS CAPELLA (viii. 283 G), Oriente cancro, occidunt Corona Ariadnes et austrini Piscis pars dimidia: Ophiuchus à pedibus ad usque humeros, Serpensque quem detinet, praeter fauces caputque totum: Boötis etiam medietas.

* See also BÉRARD, *Cultes Arcadiens*, p. 180.

† *Cf.* MANIL., ii. 442; also, *int. al.*, PETERSEN, *Das Zwölfgöttersystem der Gr. und R.*, Hamburg, 1853; Berlin, 1870.

Oriuntur verò Orion totus, Eridanique principium, et in linguâ Caniculæ lucida Stella. The table given by Hyginus is nearly identical, save that he speaks of Orionis corpus ad zonam, et Eridanus totus (*Poet. Astr.*, iv. 12).

In such tables there is often a certain measure of discrepancy, which may be in many cases explained by the different domiciles of the writers, or the different dates at which they, or those from whom they took their statement, made their observations. There is, for example, much conflict of statement over the relation of the Dog-star to the Crab: in this case the difficulty arises partly from the uncertain meaning of the word *Canicula*, which some would interpret as meaning Sirius itself, and some as Procyon, the lesser Dog; and, secondly, from confusion between the so-called *cosmical* and *heliacal* risings of the star, in other words, between the point of the ecliptic together with which it actually rises, and the point at which the sun rises when the star is first seen on the eastern horizon in the morning twilight. SERVIUS (ad G. i. 218) is right when he says: Canis paranatellon est Cancri, id est cum eo oritur; but the "heliacal" rising of Sirius was not with Cancer but with Leo: and in the table of MANILIUS (v. 206), it is with the Lion and not with the Crab that the Dog-star is said to rise. The matter is elaborately discussed by PETAVIUS (*Uranol.*, var. diss., vii., capp. i.–iii.).

Of all the combinations of coin-emblems in which the Crab takes part, the most striking is that afforded by the coinage of Agrigentum, where it is in the main associated with, or rather opposed to, the Eagle (for it is very seldom, indeed, that both appear on the same side of the coin), each with various other configurated emblems.

As the Crab rises, the Eagle is visible on the extreme verge of the western sky. I have elsewhere argued (*Trans. Roy. Soc. Edin.*, xxxviii. p. 187, 1895; *Gloss. of Gk. Birds*, p. 8) that it is this fact, or the solstitial date coincident with it, that is symbolised or depicted by the association of the Crab and the Eagle on the Agrigentine coins. But there is another circumstance that must also be borne in mind, namely, that Cancer was the "house" of the "exaltation" of Jupiter (*cf.* SALMAS., *Ann. Clim.*, p. 620, and MANETHO, ii. 370, ed. DIDOT, *γηθοῖ δ' ἂν Φαέθων ἐν Καρκίνῳ, οὕνεκεν αὐτοῦ ὑψοῦται*), and that probably the two facts were together present to the mind, and together depicted in allegory by the skill, of the designer of the coins. On certain coins of the same city, of somewhat later date, the Crab is absent, and the Eagle is opposed to the head of Zeus. The Eagle and Crab are opposed in the same fashion on coins of Motya, a hint, if further hint be needed, that there was more than a local allusion in the symbolism of the Agrigentine coins.

The imagery of the Agrigentine coins deserves to be dealt with more fully under the head of the constellation Aquila, but we may note that on these coins we also find the Crab associated with a Horse, a sea-monster or "Pistrix," with Scylla and her Dogs, and with a Fish. If the horse be Pegasus, that constellation is in truth setting with the Eagle as Cancer rises: if the Pistrix be the great constellation of the sea-monster or Cetus, for which, indeed, *Pistrix* was but another name (HYGIN., iii. 30), that constellation also is setting or beginning to set as Cancer rises, haec cadit ex orto cancro et leone (HYGIN.,

l.c.): Scylla is but a variant phase of the Moon, as the watery goddess, and the two Dogs, Procyon and Sirius, are rising as Cancer rises. The Fish is not rare in association on coins with Cancer. Sometimes, but rarely, there are two fishes, usually there is but one; and that one is, as a rule, large and depicted as a Tunny: we have it on coins of Agrigentum, but most typically on those of Cyzicus. The Tunny is in all probability the Piscis Australis, the "great fish" of ERATOSTHENES (xxxviii.) and of GERMANICUS (xxxviii.), which contains the brilliant star Fomalhaut, "the mouth of the fish," and which, as we have already seen, sets at the rising of Cancer,—hic occidit oriente cancro, exoritur autem cum piscibus (HYGIN., iii. 40). On the coins of Cyzicus, the Tunny is associated with many other emblems, all or nearly all of them being, as I believe, capable of interpretation as constellations obviously related to that of the Southern Fish: we may merely note here that one of these associated emblems is the Ram, together with which sign THEON expressly states that the latter part of the constellation rises.

We have now discussed, very briefly and imperfectly, the Crab in connection with its related constellations, and, also, as a zodiacal sign, in connection with Luna, of which it is the domicile, and of Jupiter, of whom it is the exaltation. It comes also into relation with Mercury, of whom it is the seat in the order of distribution among the signs of the twelve gods: and we have the Crab as a symbol associated with the head of Hermes on a coin of Ænus (HEAD, *op. cit.*, p. 213). I am more than tempted to see in this collocation, a connection with that coincident rising of Sirius, the Star of Thoth or Hermes, with which we have already dealt: and Mercury is also the patron of the middle Decan of the sign, as Sirius was the middle Decan * or Dynast † in the corresponding table of Eratosthenes (*cf.* MARSH, *Can. Chron.*, p. 18; DUPUIS, *op. cit.*, x. p. 136), who calls it *filius genæ*, a paraphrase, plainly oriental, of the simpler description, splendida Stella, quæ *in maxillâ* Canis cernitur. It was in relation to the same god, the patron of traffic and gain, that Cancer was said by the astrologers to confer an aptitude for commerce, merce peregrina fortunam ferre per urbes (MANIL., iv. 167, *cf. ib.*, 381, etc.). The dog-headed Mercury or Thoth or Anubis is familiar to us, but we may pause to note that the Lunar Hecate was sometimes also Dog-headed (*cf.* HESYCH. in voc. Ἑκάτης ἄγαλμα). The sun's entry into Cancer was the signal for a festival of Pallas, sol abit e Geminis et Cancri signa rubescunt: Cœpit Aventina Pallas in arce coli (OVID, *F.*, vi. 727); and on coins of Cumae, Pallas is again associated with the Crab.

The sign Cancer is a dark and inconspicuous sign, with no brilliant stars, "lumina desunt Cancro" (MANIL., ii. 259); the horoscope of Cancer was gloomy or νεφελοειδής, and those born under it were prone to blindness, "lumina deficient partus, geminamque creatis mortem fata dabunt" (MANIL., iv. 534).

Of particular stars in the constellation Cancer, the most important are those known as the ὄνοι or *aselli* (γ. δ), of which the southern one stands on the ecliptic, and the star-cluster known as Φάτνη or Praesepe. They are first mentioned by ARATUS (v. 892):

* *Firmicus Maternus Math.*, ii. 4; *Salmasii Plin. Ex.*, p. 460 f.

† *Cf.* AESCH., *Ag.*, λαμπρούς δυνάστας, ἐμπρέποντας αἰθέρι.

Σκέπτεο καὶ Φάτνην. ἡ μέν τ' ὀλίγη εἰκυῖα
ἀχλύϊ βοῤῥαίη ὑπὸ Καρκίνῳ ἠγηλάξει·
ἀμφὶ δέ μιν δύο λεπτὰ φαεινόμενοι φορέονται
ἀστέρες, οὔτε τι πολλὸν ἀπήοροι οὐτε μάλ' ἐγγύς,
ἀλλ' ὅσσον τε μάλιστα πυγούσιον οἰήσασθαι,
εἷς μὲν πὰρ βορέαο· νότῳ δ' ἐπικέκλιται ἄλλος.
καὶ τοὶ μὲν καλέονται Ὄνοι, μέσση δέ τε Φάτνη,
ἥτε καὶ ἐξαπίνης πάντῃ Διὸς ἐνδιόωντος
γίνετ' ἄφαντος ὅλη· τοὶ δ' ἀμφοτέρωθεν ἰόντες
ἀστέρες ἀλλήλων αὐτοσχεδὸν ἰνδάλλονται·
οὐκ ὀλίγῳ χειμῶνι τότε κλύζονται ἄρουραι:

cf. PLIN., xviii. 55, "Sunt in signo Cancri duæ stellæ parvæ, Aselli appellatæ, exiguum inter illas spatium obtinente nubeculâ, quam Præsepia appellant." As ARATUS speaks of Præsepe as ἀχλύς, so ERATOSTHENES calls it νεφέλιον, and PTOLEMY νεφελοειδὴς συστροφή (*cf.* IDELER, *Sternnamen*, p. 161). There is a difficult passage in MANILIUS (v. 174), "Nunc Cancro vicina canam, cui parte sinistra Consurgunt Iugulæ," where, according to Scaliger and others, Iugulæ stand for Aselli. But, according to IDELER, there is no basis for this assumption, and we must interpret Iugulæ or Iugula, here as elsewhere, to mean Orion, which constellation, as we have already seen, rises with Cancer: *cf.* VARRO (*L. L.*, vii. 3), "Iugula signum est quod Accius appellat Oriona." The same name occurs in PLAUTUS (*Amphit.*, I. i. 119), "nec Iugulæ, neque Vesperugo,* neque Vergiliæ occidunt."

HYGINUS (II. 23), THEON, LACTANTIUS (I. 21), and others, relate various fables in connection with these Aselli. They were set in the Heavens by Bacchus, whom they had transported to the temple of Zeus at Dodona, across the Thesprotian marsh,† when, rendered mad by Juno, he was fleeing for sanctuary and cure; while, according to ERATOSTHENES (*ap.* HYGIN., *l.c.*), these were two of the asses that carried Bacchus, Vulcan, Silenus (*cf.* OVID, *F.*, iii. 748, etc.) and the Satyrs, and that by their braying put to flight the giants with whom Jupiter was at war. We observe that Jupiter, whom we have seen to be intimately associated with the sign Cancer, figures in both versions of the legend, and that the defeat of the giants and the triumph of Bacchus alike correspond to the midsummer turning-point of the Sun: the Aselli marked the precise point of the tropic about the time of the Building of the City. Priapus (*cf.* OVID, *F.*, vi. 320, etc.) figures in another version of the story (HYGIN., *l.c.*).‡

The Ass is represented on coins sometimes in association with emblems that we have seen depicted with the Crab, and sometimes with others that we have not hitherto met with, but that stand, like the rest, in close relation to the sign Cancer.

* Vesperugo is the planet Venus, Hesperus, or Vespertinus: *cf.* VITRUV., ix. 4, "Veneris stella, post occasum solis apparens in Caelo, Vesperugo vocitatur, aliis autem temporibus eum antecurrens, et oriens ante lucem, Lucifer appellatur.

† *Cf.* AVIEN., *Arat.*, 383, "qua duro concava dorso Tegmina curvantur, geminos micat ardor in aures; Hos dixere asinos ortos Thesprotide terra, Et sidus, Lernæe, tuum."

‡ For other references to the mythology of the Ass in relation to the sign Cancer, see DUPUIS, *Orig. de tous les cultes*, i. p. 377, v. p. 104, ed. 1835, and other authorities there quoted.

It is opposed to the Tunny on certain coins of Cyzicus, as is the Crab on others: on certain coins of Mende it comes in relation with the lunar Crescent and with the Dog. It is also figured with an Altar and Cup on coins of Timæa, and with the Crow on those of Mende. The Crow and Cup lie near by, on the folds of Hydra,

μέσση δὲ σπείρῃ Κρητήρ, πυμάτῃ δ' ἐπίκειται
εἴδωλον Κόρακος σπείρην κόπτοντι ἐοικός

(ARAT., 448); and the Altar, which rises with Capricorn (HYG., iii. 38), is therefore rising as Cancer sets. On the coins of Mende the Ass is often in relation to the Vine, recalling the legends of Bacchus that have been related above. It is important to note that the coinage of Mende, in which the Ass is especially conspicuous, was very ancient, *circa* B.C. 500, and that at this early period the Aselli were very important stars, being still very near indeed, and much nearer than any other conspicuous stars, to the precise point of the tropic.

In the Calendar of the Fasti, the 8th June (vi Id. Jun.) is marked "Asinus coronatur, Ara Priapi" (*cf.* OVID, *F.*, vi. 345). This was, as near as may be, in the time of Ovid, the precise date of the cosmical rising of the Aselli, the date, that is to say, when the Sun crossed them on the ecliptic.

Præsepe is mentioned by CICERO (*Prognost. ap. Priscian.*), "Ast autem tenui quæ candent lumina Phatnæ." It tells us for the present no mythological story, but the identification of it with the Indian 'Pushya,' *i.e.*, 'foam,' and with the Pers. 'Avra-k' (? *ἀφρός*), by Mr R. BROWN (*Euphrat. Stellar Researches*, v. p. 22) may open a new field of conjecture. The same writer quotes from the *Bundahis* (vii. 1), "The star Tistar [Sirius] was in Cancer . . . . . in the subdivision they call Avrak"; that is to say, we have here in the Persian a relation noted between the Dog-star and the Crab, as paranatellons of one another, just as we have found it already in the verses of MANILIUS.

We may abbreviate and summarise, as follows, the chief coincidences that have been related above:—

1. Cancer was *domus Lunæ*, and the Crab is associated with the Moon on coins of Consentia, Terina, etc., with the lunar Diana of the Ephesians, and with various other images of the lunar goddess.
2. Cancer was *exaltatio Jovis*, and the Crab is peculiarly associated with the Bird of Jove in the coinage of Agrigentum, while the Aselli, individual stars of the same constellation, are mythologically associated with the same god.
3. Cancer was *sedes Mercurii*, and the Crab is figured with the head of Hermes on coins of Aenus.
4. Cancer rose with Sirius, and there are dog-headed representations of both Mercury-Anubis and Luna-Hecate.

5. Cancer marked the date of a festival of Pallas, and the Crab is figured with Pallas on coins of Cumæ.
6. Cancer is constellated with Hydra, simultaneously with part of which Virgo rose, and the Crab is associated with Hydra in the legend of Hercules, and hence with Lerna when the rites of the Virgo cœlestis were performed.
7. Cancer is in the neighbourhood of Corvus and Crater, and on coins of Mende the Ass is figured with the Raven and Cup.
8. Cancer rose opposite to Aquila and Delphinus which set soon afterwards, and moreover set precisely as the Dolphin rose; the Crab or the Ass is associated with the Eagle on coins of Agrigentum and Motya, and with the Dolphin on coins of Motya and Argolis.
9. Cancer rose as Pegasus set, and the Crab is figured with the Horse on coins of Agrigentum.
10. Cancer rose as Cetus or Pistrix set, and the Crab is figured with the Sea-monster on coins of Agrigentum.
11. Cancer rose as the Southern Fish set, and the Crab and Ass are figured with the Fish on coins of Agrigentum and Cyzicus.

Very few indeed of the numismatic emblems and symbols that are found associated with the Crab have been omitted from the foregoing discussion. Of the few others that occur the most remarkable is the Toad, on coins of Venusia. It would be a very beautiful coincidence of emblem and appellation if we could show this Toad to be the symbol of that lunar Hecate that in an ancient invocation or incantation is addressed as φρουνή, the 'she-toad,'* and that some would identify with the Egyptian Hek-t, the frog-headed Heka, the frog-goddess Hiquit or Hequit, the goddess of maternity and the divine watcher over the birth of the Sun.†

* *Vide* F. Legge, "A Coptic Spell of the Second Century," *Proc. Soc. Bibl. Archæol.*, May 1897.
† *Cf.* R. Brown, jun., *Semitic Influence in Hellenic Mythology*, 1898, p. 158.

XXIII.—*The Development of the Müllerian Ducts of Reptiles.*

By Gregg Wilson, D.Sc.
Communicated by Professor J. C. Ewart, F.R.S. (With Two Plates.)

(Read 5th July 1897.)

We have already accounts of the development of the Müllerian ducts of reptiles from Braun (1), Mihalkovics (2), Hoffmann (3), and Wiedersheim (4). All of these writers agree in saying that the Müllerian ducts arise quite independently of the segmental ducts; but in view of recent attempts to show that in the other groups of vertebrates there are forms in which the Müllerian ducts are derived from the segmental ducts, it seemed expedient to reconsider this conclusion,* and I have accordingly examined the development of the ducts in *Crocodilus biporcatus*, *Chelone viridis*, and *Lacerta agilis*.

As regards the derivation of the Müllerian ducts from cœlomic epithelium, I can only confirm the above-mentioned authorities; but I am able to supplement what they say as to the first foundation and the formation of the anterior end. I find that the anterior end is formed in reptiles, very much as in amphibians, by a modification of the cœlomic epithelium in the region of the pronephros. Backward growth proceeds independently alike of the segmental duct and of the epithelium of the body-cavity posterior to the *ostium abdominale*.

Braun examined all stages of the development of the ducts in *Lacerta agilis*, *Anguis fragilis*, and *Tropidonotus natrix*, and controlled his results by observations on *Coronella lævis*. He found essential uniformity in all.

In *Anguis fragilis* he described two anterior diverticula of the cœlom, separated by a bridge of connective tissue. These are, of course, lined by peritoneal epithelium, and Braun remarked that this, at one place on the *pleura costalis*, near its transition to the *pleura visceralis*, shows an incomprehensible thickening—*eine mir völlig unverständliche Verdickung*. Braun made no suggestion as to the significance of this thickening, and did not attempt to connect it with the origination of the Müllerian duct. He described the first Anlage of this as further back, where the diverticula open into the cœlom proper, and the mesonephros projects freely into the body-cavity. Here he described the peritoneal epithelium as being thickened and raised on a longitudinal ridge that has appeared on the excretory organ. Most anteriorly this ridge is almost quite ventral, but it passes gradually in a spiral till it comes to lie dorsal to the mesonephros. It then accompanies the Wolffian duct to the posterior end of the cœlom. In the thickened epithelium at the anterior end of this ridge an invagination appears, and the funnel so formed grows back along the ridge. Braun is emphatic as to the independent

* Burger (5) suggests that probably the Müllerian ducts of reptiles are derived from the segmental ducts. He even makes the statement that there is no evidence of independent origin.

backward growth of the foundation. He had apparently expected to find some such origin of the Müllerian ducts as SEMPER had described in the Elasmobranchs; for he says his results, though so simple, had cost him much trouble, inasmuch as he had expected them to be quite different. He also mentions that he carefully examined the tip of the foundation to determine if it showed any communication with the peritoneal epithelium such as WALDEYER had previously found in the chick; but he saw no evidence of this. The peritoneal epithelium and the growing duct appeared sharply marked off from one another, except in some specimens that had been cut obliquely, or that had suffered otherwise in the course of preparation.

MIHALKOVICS confirmed the observations of BRAUN. He examined the adder and lizard, and found that at the proximal end of the Wolffian body, and lateral to it, the cœlomic epithelium becomes cylindric over a triangular area. The edges of the distal end of this area rise till they meet, and form a closed tube or funnel. This runs to a fine point, which grows back in a previously formed *Tubenfalte*. MIHALKOVICS compared the early foundation to a slipper set on end: the elongated oval opening representing the mouth, while the closed part of the shoe corresponds to the part of the duct formed by the union of the lateral edges of the foundation. MIHALKOVICS denied that the development of the Müllerian ducts of reptiles was at all dependent on the mesonephric duct. He believed that growth backwards was effected by increase of the cells at the tip of the foundation.

HOFFMANN'S general concurrence in the views of BRAUN and MIHALKOVICS is the more remarkable because of his published work on the ducts of *Rana* and *Triton*, in both of which he described at least partial derivation of the Müllerian ducts from the segmental ducts. In the lizards he mentions a relatively high cylinder-epithelium, lying ventral to the cœlom in the region of the pronephros. Further on he describes the epithelium ventral to the pronephros, as distinguished from that of the lateral and median parts by the high form of its cells; and he adds that it is this patch of epithelium that forms the foundation of the Müllerian duct. The heightened epithelium passes laterally till it comes into close proximity to the segmental duct, and can be followed back along this duct to the cloaca—at least, in female specimens. It is by invagination of the high cylindric epithelium of the pronephros that the *ostium abdominale* of the Müllerian duct is formed. HOFFMANN concludes that in the case of the lizards further development is independent of the segmental duct; but, at the same time, he says he is unable to determine whether the growth backwards of the Müllerian duct is also independent of the thickened band of epithelium that lies immediately external to it.

WIEDERSHEIM investigated the development of the Müllerian ducts in Crocodilus and Chelone. He agrees, in the main, with previous workers on the reptiles, but finds that in Chelone the backward growth of the Müllerian duct foundation is dependent on proliferation of the cells of the thickened epithelium outside the duct.

The Müllerian Ducts of the Crocodile.

I shall now describe the various stages examined by me in *Crocodilus biporcatus*.

The first of these (A) is found in a specimen of 10 mm. in extreme length. In this the cœlom is simple in the region of the pronephros.* There are, however, lateral ridges along the alimentary canal, indicating the line of the usual secondary connectionwith the somatopleure (fig. 1, *l.r.*). These ridges gradually become lower as we follow them towards the posterior end, and finally are lost in the mesentery.

The first section through the pronephros shows a nephrostome, one tubule, and the anterior end of the glomus. The nephrostome has already lost connection with the pronephros, so that the funnel is quite isolated; while, posterior to the nephrostome, the pronephros has degenerated so far that through several sections it ceases to appear. It is noteworthy, however, that the heightened epithelium of the nephrostome extends out on all sides, and passes back towards the posterior end as a distinct thickened band. This unites the anterior nephrostomes, and passes gradually into the ordinary epithelium of the cœlom.

The next stage that I have is got in a specimen (B) 12 mm. long. Anterior to the excretory organ and the glomus, we see in transverse section three divisions of the cœlom; and in the dorsal diverticula formed by secondary fusion of the alimentary canal with the body-wall, we find again distinct thickenings of the epithelium. These, at first, are ventral in position (fig. 2, *m.d.*); but, anterior to the beginning of the glomus, they lie on the lateral and dorsal walls of the diverticula (fig. 3). The lateral anterior part of the embryonic excretory organ found in A has disappeared, and with it its nephrostomes; but a few sections further back than the anterior of the glomus, the remainder of the embryonic excretory organ is met with, and just external to this the above-mentioned thickening runs along the lateral wall of the cœlom. It is not, however, only on the lateral wall immediately external to the pronephros that the thickening is found passing back, for, opposite the anterior end of the mesonephros, the thickening, which is at first simple, passes back as two distinct bands, one of which lies close to the mesonephros, while the other is quite ventral to the diverticulum (fig. 4). The lateral wall shows an intermediate area of ordinary cœlomic epithelium. A little further back, where the dorsal diverticulum opens into the cœlom proper, the ventral thickening is altogether separated from the dorsal one (fig. 5). It gradually becomes less conspicuous, and passes into the ordinary epithelium of the cœlom.

The occurrence of this band recalls the ventral extension of the Müllerian ducts in elasmobranchs, and the temporary ventral growth of the Müllerian duct foundation in Rana.

The more dorsal backward prolongation of the lateral plate narrows gradually as it passes back. It comes to lie on a bridge of tissue that is between the cardinal vein and the mesonephros; and at the same time it becomes grooved. External to the thickening,

* Identified by Wiedersheim.

a second furrow appears, indenting the lateral wall of the cœlom, and as it gets deeper, cutting off the Müllerian duct foundation from the lateral wall (fig. 6). Further back, the band of thickened epithelium passes on to the mesonephros. Some twenty-five sections—about 150 μ—posterior to the passing over of the Anlage to the excretory organ, the grooved thickening closes to form a tube. Through thirty sections more this remains open; then the Müllerian duct foundation abruptly ends, and its tip or growing-point appears, independent alike of the contiguous cœlomic epithelium and of the segmental duct that lies just internal to the Müllerian duct. The segmental duct, it is true, shows an obliquity in a few of the sections near the termination of the Müllerian duct, and the obliquely cut side nearest to the Müllerian duct Anlage might, at first sight, be supposed to show a process of budding; but careful examination makes it clear that the Müllerian and segmental ducts are well marked off from one another, and the staining differences emphasise this distinction. The obliquity of the sections of the segmental duct in the neighbourhood of the growing point of the Müllerian duct is to be explained by the intrusion of the Müllerian duct causing the segmental duct to deviate from its first position close to the cœlomic epithelium.

Outside the Müllerian duct there is a distinct thickening of the cœlomic epithelium, and this is specially noticeable in the region immediately posterior to the growing tip of the duct; but the thickening does not appear to take part in the formation of the duct proper, though it may contribute to the formation of sheathing tissue.

It thus appears that the simple epithelial thickening found in the region of the pronephros in specimen A is represented in B by a similar lateral plate with two posterior continuations, the one ventral, and passing into the ordinary cœlomic epithelium, the other dorsal, passing into the already tubular Müllerian duct. It is, of course, open to anyone to assert that there may be a missing intermediate stage between A and B, showing trace of the derivation of some of the cells of the Müllerian duct from the segmental duct. I can only say that the anterior of the Müllerian duct is clearly derived from the cœlomic epithelium, and that at such an early stage as is represented in B there is no evidence of any other derivation; while growth backwards is distinctly not dependent on any budding from the segmental duct.

The two new facts that I am able to add from my examination of stages A and B to what WIEDERSHEIM has written on the subject are:—(1) That the anterior and first foundations of the Müllerian ducts are thickened areas of the cœlomic epithelium in the region of the pronephros; and (2) that there is a distinct ventral development of this foundation, comparable to the temporary ventral development of the Müllerian duct in Rana.

Both of these facts are made clearer by the study of longitudinal horizontal sections. Fig. 7 represents such a section through an embryo that is somewhat further advanced than B. The open *ostium abdominale* is seen, and stretching away in front of it, and anterior to the mesonephros, is the thickened plate of epithelium that has already been described as the first foundation of the Müllerian duct. Fig. 8 shows a more ventral

section of the same series. In it the thickened plate is seen yet further forward in the lateral diverticulum of the cœlom. Figs. 9 and 10, which represent still more ventral sections, show the ventral extension of the plate: in the former, the thickening appears still dorsal and internal, but projecting towards the lateral wall of the cœlom; in the latter, the seventy-fourth section more ventral than fig. 9, the end of the ventral extension of the anterior plate is seen raised on a more prominent ridge, just anterior to the liver.

My third stage (C), found in a specimen of probably less than 20 mm. in length, shows a modification of the anterior thickened epithelial plate, that is interesting as being similar to what is found in amphibia (6). The plate is undermined by a diverticulum of the cœlom; and by the splitting of the plate the foundation is partly divided into two areas, the one of which is connected with the ventral extension of the plate, while the other leads directly back to the Müllerian duct.

The pronephros of C has degenerated completely, but along the anterior cœlomic diverticulum there is still a distinct band of thickened epithelium. This spreads out to form a broad plate in the region just in front of the mesonephros. Under this plate a slight diverticulum of the cœlom is seen (*d.*, fig. 11), and examination of the posterior margin of the plate reveals the fact that the plate has been partially divided by the breaking down of the slight partition between the old diverticulum and the new one. The inner end of the plate remains projecting laterally between the united diverticula, while the outer end projects slightly from the lateral wall (*r.*, fig. 11). From the fact that the external remnant of the plate continues to extend into the body-cavity through a considerable number of sections, it is evident that the anterior plate formerly extended much further.

My next specimen (D) is 19 mm. in length, and illustrates even better than the last one the division of the anterior foundation.

The anterior diverticula expand posteriorly, and along the lateral epithelium, but nearly dorsal, there appears a distinct narrow band of thickened epithelium. Where the glomus first appears, projecting slightly forwards into the diverticula, the ridge spreads out to form a plate, and at the same time the diverticula open into the main division of the cœlom. The anterior part of the plate has apparently been largely obliterated by the secondary attachment of the *radix mesenterii* to the lateral body-wall. The plate is at this stage divided at the anterior end of the mesonephros, and, as in C, the two posterior (ventral and dorsal) continuations of the thickened plate pass back, separated by a deep channel. The dorsal division hangs free into the cœlom, which has undermined it from its ventral edge; and it only gradually passes into connection with the mesonephros. The ventral division is found to pass the base of the lung, and ends just anterior to the beginning of the liver.

I have examined specimens up to 75 mm. in length, and all of them show similar relations of parts. In all, there is the anterior lateral plate. It is usually slightly undermined, and is divided posteriorly to form two bands, one of which leads to the

Müllerian duct, while the other ends close to the anterior attachment of the liver. It is noteworthy that the ventral extension of the anterior plate passes posteriorly across the secondary bridge, connecting the alimentary tract and the lateral body-wall, and comes to project freely into the cœlom from near the base of the lung.

The Müllerian Ducts of Chelone.

As I mentioned above, the development of the Müllerian ducts of Chelone has been investigated by Professor Wiedersheim. He describes, in a 13 mm. embryo, a well-marked proliferation of the peritoneal epithelium on the side of the anterior region of the mesonephros. This extends to the parietal peritoneum. The formerly smooth surface has become rough and uneven, and in numerous places involution of the epithelium is visible, and proliferation inwards. In an embryo of 21 mm. the inequalities have, for the most part, disappeared, and in their place there is a rapidly increasing fold hanging into the body-cavity. This fold, according to Wiedersheim, hangs down the side of the mesonephros, is applied to it, and fuses with it; and so the *ostium abdominale* is formed. Three sections further towards the posterior the duct stops, and is followed by a solid pointed rod, such as occurs at the growing tip of the foundation in the crocodile. As in the case of the crocodile, this solid rod lies in the immediate neighbourhood of the mesonephric duct and of the *Tubenfalte*. In the rod, a few sections posterior to the ostium, a slight lumen appears.

As to backward growth, Wiedersheim asserts that there is not merely appositional growth, such as is generally described for Amniota. He figures two sections, showing proliferation inwards of the thickened epithelium of the *Tubenfalte*; and he believes that the cœlomic epithelium thus contributes directly to the backward growth of the foundation.

By the kindness of Professor Wiedersheim, in whose laboratory much of the work recorded in this paper was done, I have had an opportunity of examining a number of series of sections of embryos of Chelone; and I shall now state the facts that I have observed.

In an embryo (A) of 13 mm. in length, there is a lateral thickening of the cœlomic epithelium lining the anterior diverticulum, and extending on to the excretory organ, along which it passes, as described by Wiedersheim.

A specimen (B), 21 mm. long, shows the lateral thickening to be undermined anteriorly (fig. 12). Further back, the plate is divided, as in the case of the crocodile, by the breaking down of the partition between the two diverticula (fig. 13). One remnant of the thickening projects into the cœlom from the ventral body-wall; another is intimately connected with the mesonephros (fig. 14). In the posterior extension of the plate that passes along the mesonephros there is a funnel-like pit (fig. 15). This conical funnel is the *ostium abdominale tubæ*. It is quite distinct from the mesonephric duct, and its inner pointed end, which extends a very short distance towards the posterior, is quite independent of the epithelial thickening that continues back external to the mesonephric duct.

The next stage is found in an embryo (C) of 25 mm. In this, as in B, the lateral thickened area of the cœlomic epithelium is undermined; and ventral and dorsal extensions of it pass towards the posterior. Here, also, the ostium lies into the side of the mesonephros. The funnel turns towards the posterior, and its lumen may be followed back through several sections. Further back there is a solid rod of cells in direct continuation of the tube, but this extends only through a few sections. On the right side the tip of the rod is quite distinct from the segmental duct, beside which, however, it lies. On the other side I had, at first, difficulty in distinguishing the tip of the Müllerian duct from the obliquely cut segmental duct, which, from being somewhat remote from the cœlomic epithelium, passes close up to it, just behind the Müllerian duct. Fig. 16 shows the blunt point of the latter lying close to the segmental duct. Fig. 17 shows the next section with no remainder of the Müllerian duct, but with the segmental duct filling the space immediately posterior to the end of the Müllerian duct. Here, as in other cases, we must assume that the growing tip of the Müllerian duct thrusts itself between the segmental duct and the cœlomic epithelium, and causes a shifting of the position of the former. Fig. 18 bears out this view by showing the altered position of the segmental duct in the second section, posterior to the end of the Müllerian duct. It shows, at the same time, the inward proliferation of the epithelial cells in this neighbourhood, described by Professor WIEDERSHEIM; but I would point out that these are probably destined to take part in the formation of the sheath of the Müllerian duct, rather than the duct itself; for the posterior end of the duct is absolutely distinct from the thickened epithelial band (fig. 16).

A further stage is met with in an embryo (D) of 23 mm., in which the Müllerian ducts are considerably better developed. The relations of parts at the anterior end of the foundation are similar to those described in C. The *ostium abdominale* lies, as in other specimens, on the side of the mesonephros, at its anterior end; the open funnel passes inwards and back towards the posterior through five or six sections; then a rod-like continuation is met with in several sections, behind which a lumen is again met with; and after a few sections the foundation ends. In this specimen, again, the tip of the Müllerian duct lies on the mesonephric duct.

The last specimen (E) that I have to describe is of unknown length. It is considerably larger than D, and shows the relations of the various parts more distinctly than any other stage that I have.

A few sections after the beginning of the excretory organ, the lateral diverticulum is met with. Projecting into this from the side of the excretory organ is the thickened epithelial plate, in which, further back, the *ostium abdominale* appears. Just outside the excretory organ, on the lateral wall of the cœlom, there is a remnant of the original anterior plate, and it projects towards the thickening on the excretory organ. In more posterior sections the ventral thickening may be traced passing back, and extending far round ventralwards.

The *ostium abdominale* leads to a solid foundation, which is found through many

sections before a part is reached in which a lumen occurs. The lumen is distinct till within a few sections of the end of the foundation. The solid rod that forms the tip of the foundation lies external to the segmental duct, touching it, but independent of it, and remote from the cœlomic epithelium. In the section immediately posterior to that which contains the last cells of the rod-like apex there is no appearance of budding of the cells of the mesonephric duct, such as one would expect if growth backwards of the Müllerian duct were dependent on it; and there is no indication of any contribution of cells from the cœlomic epithelium to add to the growing tip.

It thus appears clear that in this specimen growth backwards is by apical proliferation; and I conclude that, as in the formation of the anterior end, so in the manner of growth backwards there is essential agreement between Chelone and Crocodilus.

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(2) Mihalkovics, "Untersuchungen über die Entwickelung des Harn- und Geschlechtsapparates der Amnioten," *Intern. Monatssch. für Anat. und Hist.*, Bd. ii.

(3) Hoffmann, "Zur Entwicklungsgeschichte der Urogenitalorgane bei den Reptilien," *Zeit. f. wiss. Zool.*, Bd. xlviii., 1889.

(4) Wiedersheim, "Ueb. die Entwickelung des Urogenitalapparates bei Crocodilen und Schildkröten," *Arch. f. mikr. Anat.*, Bd. xxxvi.

(5) Burger, "De Ontwikkeling van de Müller'sche Gang bij de Eend en de Bergeend," *Tijdsch. d. Med. Dierk. Vereen.* (2), iv.

(6) Gregg Wilson, "The Development of the Müll. Ducts of Amphibia," *Trans. Roy. Soc. Ed.*, 1896.

DESCRIPTION OF FIGURES.

Fig. 1. Transverse section through the pronephric region of crocodile of 10 mm. in length. *l.r.* is the lateral ridge that in older stages forms the secondary connection between the alimentary canal and the body-wall.

Fig. 2. Section across crocodile 12 mm. in length, showing thickened epithelium (*m.d.*) on the ventral wall of the cœlomic diverticulum, anterior to the glomus.

Fig. 3. Section through the same embryo further back, and showing an extensive lateral epithelial thickening on a level with the anterior end of the glomus.

Figs. 4 and 5. Sections through the same embryo further back, showing the two posterior extensions of the pronephric thickened plate.

Fig. 6. Section through the same embryo, showing the grooved Müllerian duct foundation in relation to the mesonephros.

Figs. 7, 8, 9, 10. Sections from a series through an embryo further advanced than stage B, and showing the lateral and ventral extension of the Müllerian duct foundation.

Fig. 11. Transverse section through a crocodile embryo of about 20 mm., showing undermining of the anterior lateral plate of thickened epithelium.

Figs. 12, 13, and 14. Sections through embryo of Chelone of 21 mm., showing undermining of anterior lateral plate.

Fig. 15. Section through the same embryo, showing the *ostium abdominale*.

Fig. 16. Section through Chelone embryo of 25 mm., showing the end of the Müllerian duct in close proximity to the segmental duct.

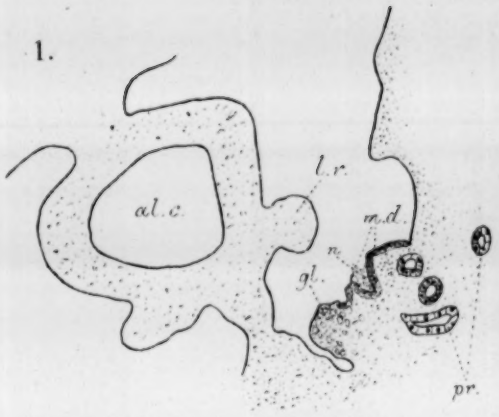
Fig. 17. Next section posterior to that shown in fig. 16, to show the segmental duct passing obliquely behind the Müllerian duct.

Fig. 18. Next section to that illustrated in fig. 17. The segmental duct is seen in the position occupied further forward by the Müllerian duct.

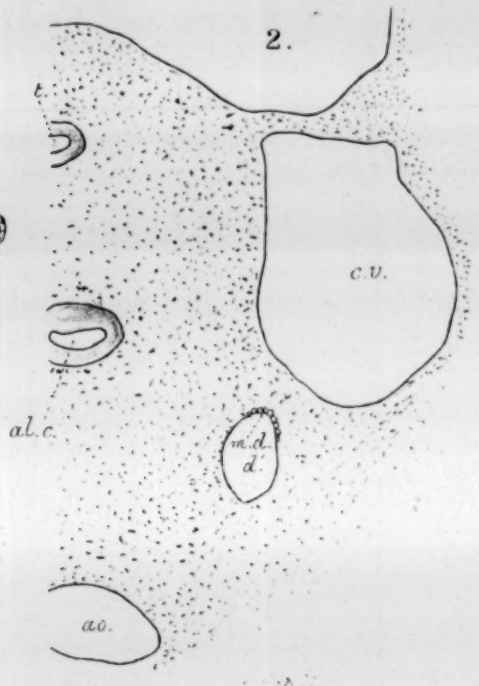
al.c. = alimentary canal.
ao. = aorta.
br. = bronchus.
c. = cœlom.
ch. = notochord.
c.v. = cardinal vein.
d. = diverticulum.
gl. = glomus.
li. = liver.
lu. = lung.
m.d. = Müllerian duct or its foundation.
mes. = mesonephros.
n. = nephrostome.
o. = *ostium abd. tub.*
pr. = pronephros.
r = remnant of lateral plate.
s.d. = segmental duct.
tr. = trachea.

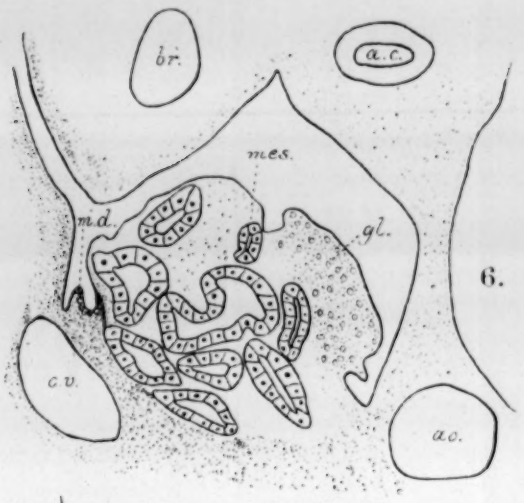
Dr Gregg Wilson: Müllerian Ducts of Reptiles — Plate I

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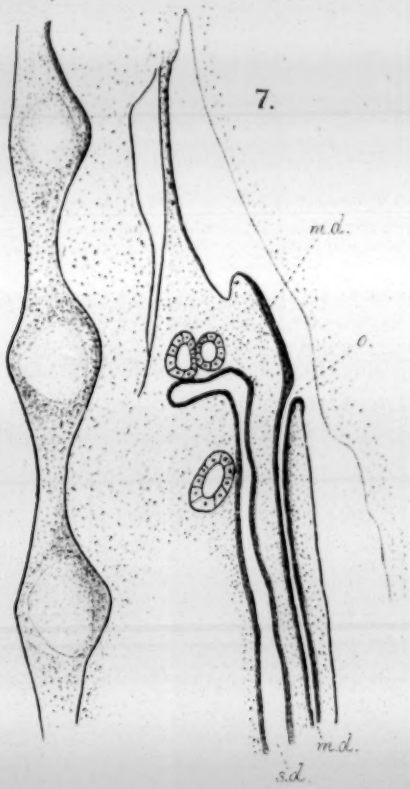




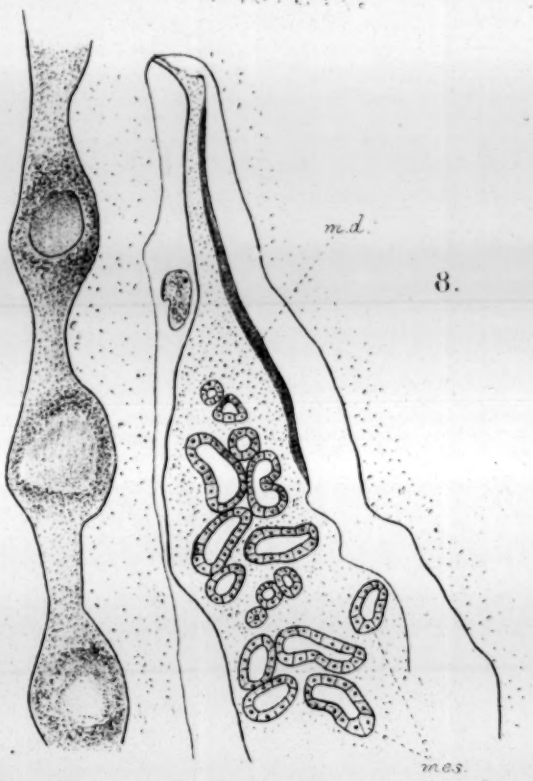
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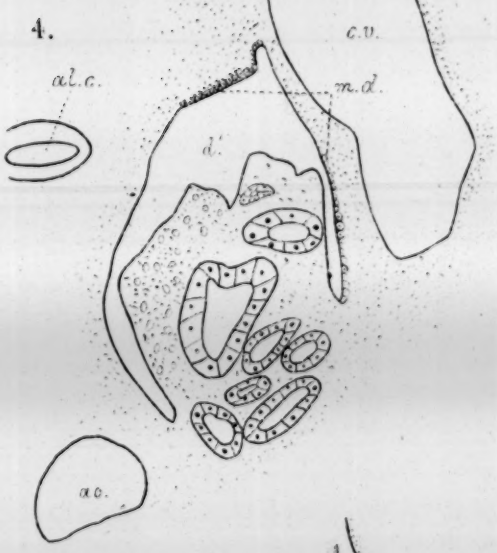


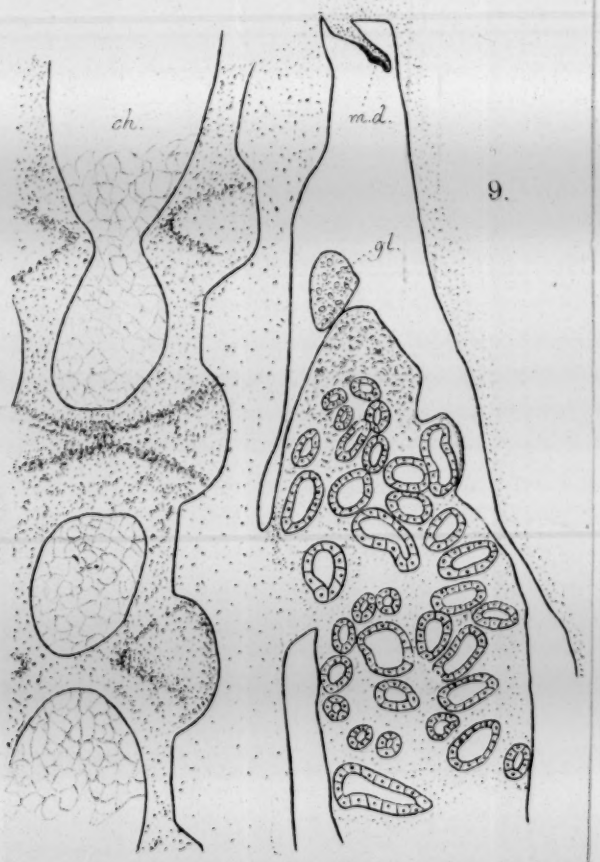
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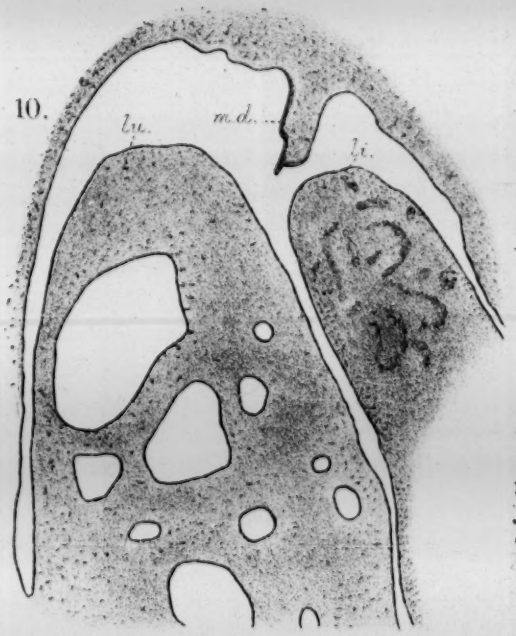
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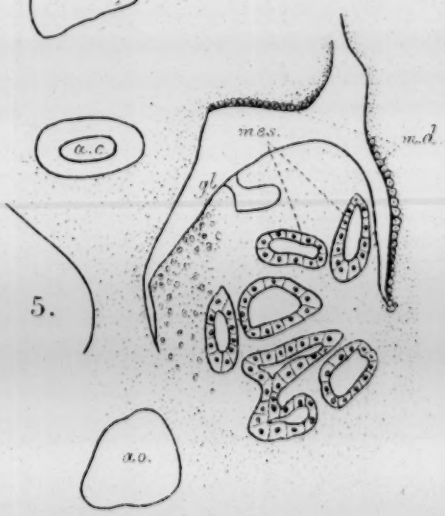




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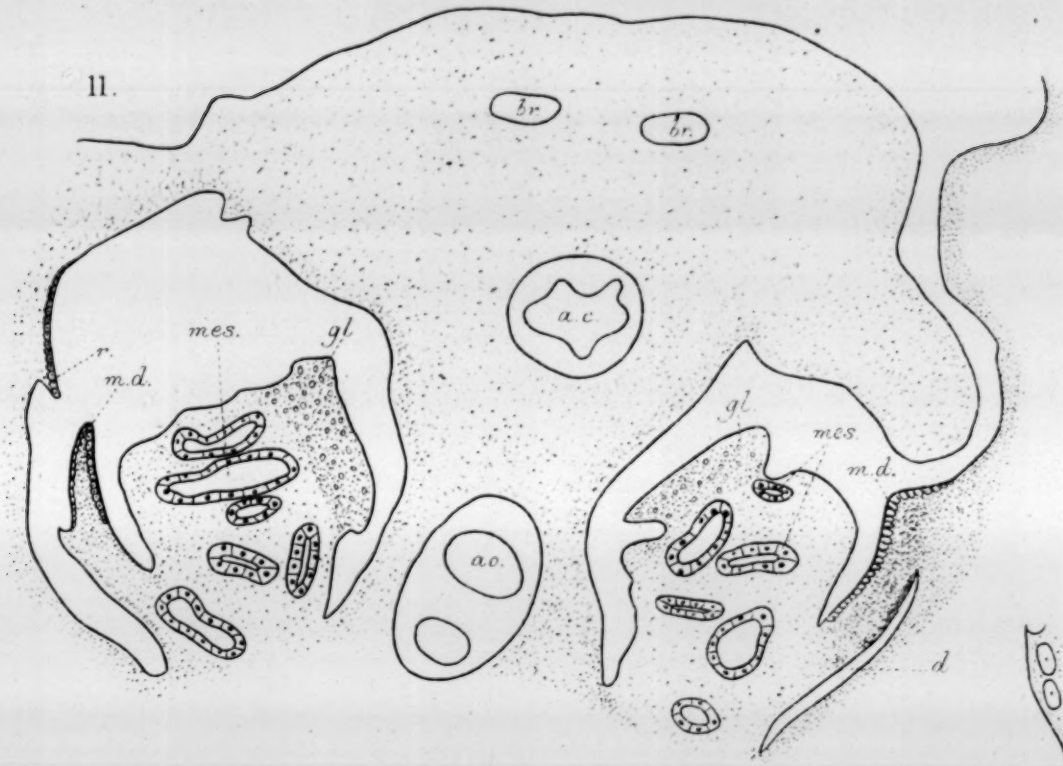
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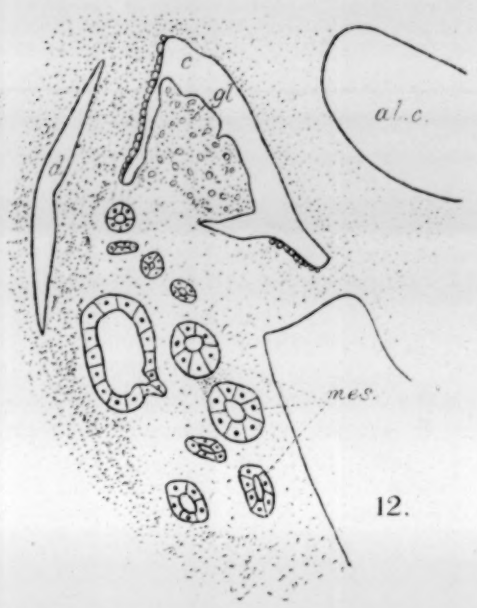
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Dr Gregg Wilson: Müllerian Ducts of Reptiles — Plate II.

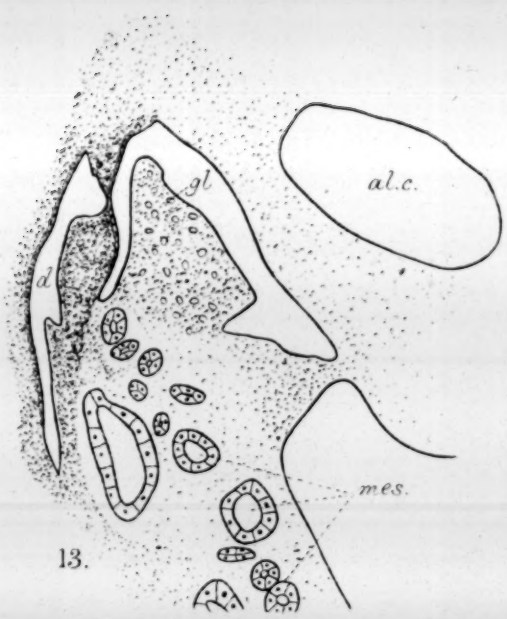
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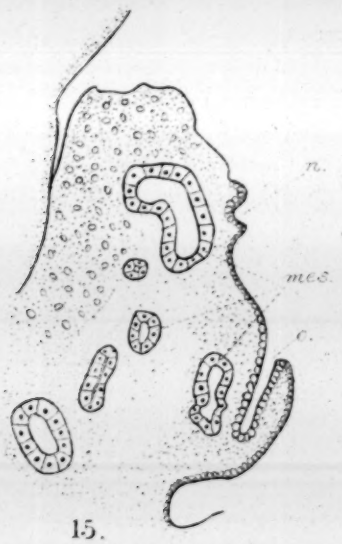
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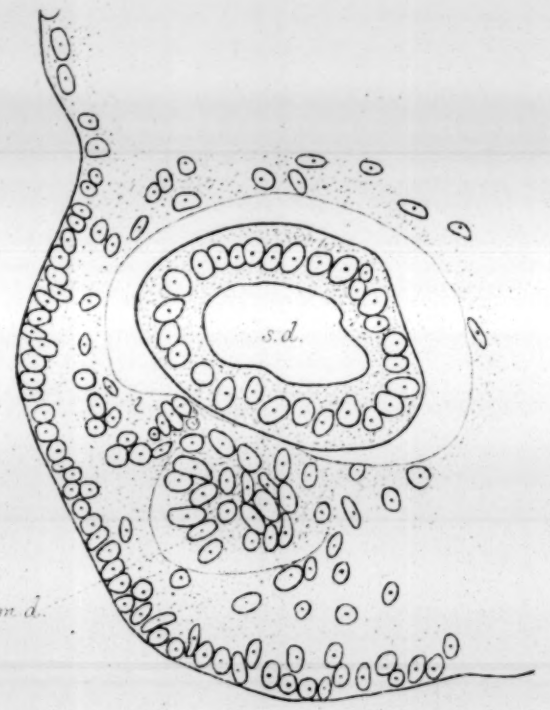
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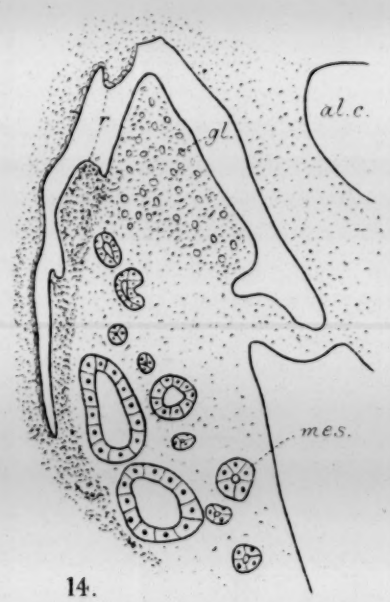
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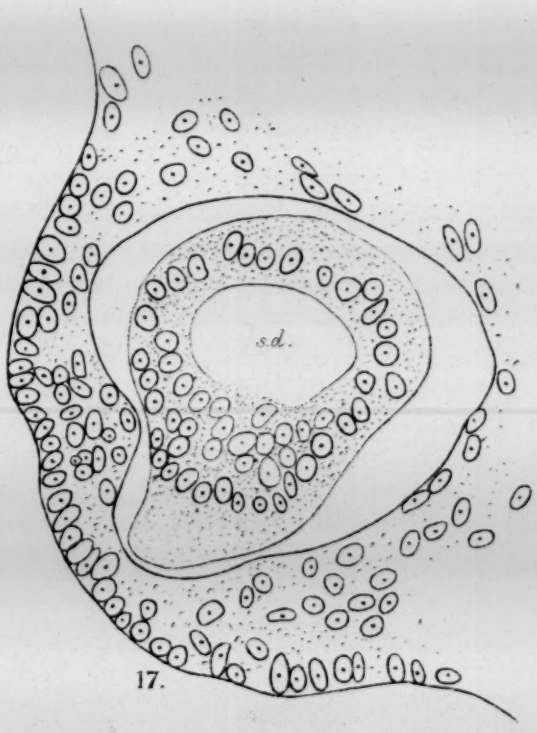
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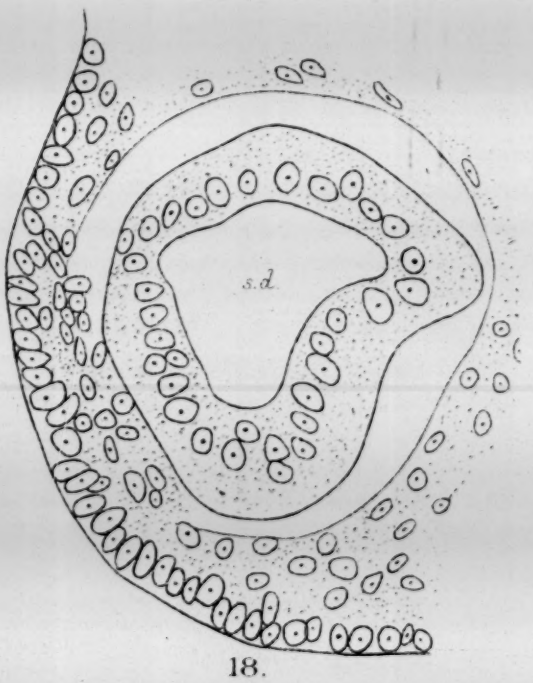
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XXIV.—*On a Development of a Determinant of the* mnth *Order.*

By Thomas Muir, LL.D.

(Read 20th March 1899.)

(1) The theorem in question concerns the finding of an expression for a determinant of composite order in terms of compound determinants or permanents of lower order; or, more definitely, the expression of a determinant of the mn^{th} order as an aggregate of determinants or permanents of the mth order, each of whose elements is a determinant of the nth order.

(2) Taking the simplest possible case, viz., where the given determinant is of the 4th order, we have by Laplace's expansion-theorem

$$\begin{vmatrix} a_1 & a_2 & a_3 & a_4 \\ b_1 & b_2 & b_3 & b_4 \\ c_1 & c_2 & c_3 & c_4 \\ d_1 & d_2 & d_3 & d_4 \end{vmatrix} = |a_1b_2| \cdot |c_3d_4| - |a_1c_2| \cdot |b_3d_4| + |a_1d_2| \cdot |b_3c_4| + |b_1c_2| \cdot |a_3d_4| - |b_1d_2| \cdot |a_3c_4| + |c_1d_2| \cdot |a_3b_4|.$$

Now the sum of the first and last terms here is clearly equal to the permanent

$$\overset{+}{\begin{vmatrix} |a_1b_2| & |a_3b_4| \\ |c_1d_2| & |c_3d_4| \end{vmatrix}}.$$

the sum of the second and fifth equal to

$$-\overset{+}{\begin{vmatrix} |a_1c_2| & |a_3c_4| \\ |b_1d_2| & |b_3d_4| \end{vmatrix}},$$

and the sum of the third and fourth equal to

$$\overset{+}{\begin{vmatrix} |a_1d_2| & |a_3d_4| \\ |b_1c_2| & |b_3c_4| \end{vmatrix}};$$

so that we have

$$a_1b_2c_3d_4 = \overset{+}{\left|\, |a_1b_2| \;\; |c_3d_4| \,\right|} - \overset{+}{\left|\, |a_1c_2| \;\; |b_3d_4| \,\right|} + \overset{+}{\left|\, |a_1d_2| \;\; |b_3c_4| \,\right|}.$$

(3) The next case, viz., where the given determinant is of the 6th order, admits of two forms of development, the compound determinants in the one being of the 3rd order with elements of the 2nd order, and in the other of the 2nd order with elements of the 3rd order.

Taking the latter first, we have, as before

$$\begin{aligned}|\,a_1b_2c_3d_4e_5f_6\,| = {} & |\,a_1b_2c_3\,|\cdot|\,d_4e_5f_6\,| - |\,a_1b_2d_3\,|\cdot|\,c_4e_5f_6\,| + |\,a_1b_2e_3\,|\cdot|\,c_4d_5f_6\,| \\ & - |\,a_1b_2f_3\,|\cdot|\,c_4d_5e_6\,| + |\,a_1c_2d_3\,|\cdot|\,b_4e_5f_6\,| - |\,a_1c_2e_3\,|\cdot|\,b_4d_5f_6\,| \\ & + |\,a_1c_2f_3\,|\cdot|\,b_4d_5e_6\,| + |\,a_1d_2e_3\,|\cdot|\,b_4c_5f_6\,| - |\,a_1d_2f_3\,|\cdot|\,b_4c_5e_6\,| \\ & + |\,a_1e_2f_3\,|\cdot|\,b_4c_5d_6\,| - |\,b_1c_2d_3\,|\cdot|\,a_4e_5f_6\,| + |\,b_1c_2e_3\,|\cdot|\,a_4d_5f_6\,| \\ & - |\,b_1c_2f_3\,|\cdot|\,a_4d_5e_6\,| - |\,b_1d_2e_3\,|\cdot|\,a_4c_5f_6\,| + |\,b_1d_2f_3\,|\cdot|\,a_4c_5e_6\,| \\ & - |\,b_1e_2f_3\,|\cdot|\,a_4c_5d_6\,| + |\,c_1d_2e_3\,|\cdot|\,a_4b_5f_6\,| - |\,c_1d_2f_3\,|\cdot|\,a_4b_5e_6\,| \\ & + |\,c_1e_2f_3\,|\cdot\ a_4b_5d_6\,| - |\,d_1e_2f_3\,|\cdot|\,a_4b_5c_6\,|\,; \end{aligned}$$

and as the sum of the first and last terms of this development is equal to

$$\begin{vmatrix} |\,a_1b_2c_3\,| & |\,a_4b_5c_6\,| \\ |\,d_1e_2f_3\,| & |\,d_4e_5f_6\,| \end{vmatrix},$$

the sum of the second from the beginning and the second from the end equal to

$$-\begin{vmatrix} |\,a_1b_2d_3\,| & |\,a_4b_5d_6\,| \\ |\,c_1e_2f_3\,| & |\,c_4e_5f_6\,| \end{vmatrix},$$

and so on, there results the identity

$$|\,a_1b_2c_3d_4e_5f_6\,| = \sum \begin{vmatrix} |\,a_1b_2c_3\,| & d_4e_5f_6\,| \end{vmatrix},$$

there being ten terms on the right included under the sign of summation, and the sign preceding each being the same as the sign of that particular term of the original determinant which is brought into prominence by the notation employed.

Taking next the expansion of $|\,a_1b_2c_3d_4e_5f_6\,|$ in terms of minors of the 2nd order, we should obtain 90 terms of the form

$$|\,a_1b_2\,|\cdot|\,c_3d_4\,|\cdot|\,e_5f_6\,|\,;$$

and these we should find capable of being collected into 15 sets of 6, with each set expressible as a permanent of the 3rd order, the identity reached being

$$|\,a_1b_2c_3d_4e_5f_6\,| = \sum \begin{vmatrix} |\,a_1b_2\,| & |\,a_3b_4\,| & |\,a_5b_6\,| \\ |\,c_1d_2\,| & |\,c_3d_4\,| & |\,c_5d_6\,| \\ |\,e_1f_2\,| & |\,e_3f_4\,| & |\,e_5f_6\,| \end{vmatrix}^{+}_{+}.$$

(4) Towards the establishment of the theorem in all its generality the first step necessary is to prove the following :—

If the rows of a determinant of the $(\mathrm{mn})^{th}$ *order be separated by horizontal lines into* n *sets of* m *rows each, and the columns be similarly divided, the result may be viewed*

as roughly representing a compound determinant of the nth *order, each of whose elements is a determinant of the* m^{th} *order and a minor of the original determinant, and each term of the compound determinant thus arising will produce* $(m!)^n$ *terms, differing at most only in sign from terms of the original determinant.*

The original determinant being denoted by

$$| a_{11}\, a_{22}\, a_{33} \ldots\ldots a_{mn,mn} |,$$

the compound determinant referred to will be

$$\left| \begin{array}{cccc} a_{1,1} & a_{1,2} & \ldots & a_{1,m} \\ a_{2,1} & a_{2,2} & \ldots & a_{2,m} \\ \ldots\ldots & \ldots\ldots & \ldots\ldots & \ldots\ldots \\ a_{m,1} & a_{m,2} & \ldots & a_{m,m} \end{array} \right|, \quad \left| \begin{array}{cccc} a_{1,m+1} & a_{1,m+2} & \ldots & a_{1,2m} \\ a_{2,m+1} & a_{2,m+2} & \ldots & a_{2,2m} \\ \ldots\ldots & \ldots\ldots & \ldots\ldots & \ldots\ldots \\ a_{m,m+1} & a_{m,m+2} & \ldots & a_{m,2m} \end{array} \right|, \quad \ldots\ldots\ldots$$

$$\left| \begin{array}{cccc} a_{m+1,1} & a_{m+1,2} & \ldots & a_{m+1,m} \\ a_{m+2,1} & a_{m+2,2} & \ldots & a_{m+2,m} \\ \ldots\ldots & \ldots\ldots & \ldots\ldots & \ldots\ldots \\ a_{2m,1} & a_{2m,2} & \ldots & a_{2m,m} \end{array} \right|, \quad \left| \begin{array}{cccc} a_{m+1,m+1} & a_{m+1,m+2} & \ldots & a_{m+1,2m} \\ a_{m+2,m+1} & a_{m+2,m+2} & \ldots & a_{m+2,2m} \\ \ldots\ldots & \ldots\ldots & \ldots\ldots & \ldots\ldots \\ a_{2m,m+1} & a_{2m,m+2} & \ldots & a_{2m,2m} \end{array} \right|, \quad \ldots\ldots\ldots$$

$$\ldots\ldots\ldots

(5) In the preceding the mn rows of the original determinant were separated in the simplest way into n sets of m rows each, but it is clear that if they had been separated in a different way into n sets of m rows each, the same mode of reasoning would have led to the same result. Now the number of different ways of breaking up mn things into n sets of m each is

$$\frac{C_{mn,m} \cdot C_{mn-m,m} \cdot C_{mn-2m,m} \cdot \cdots\cdots C_{m,m}}{n\,!},$$

or (what is the same thing, since $C_{rs} = \frac{r}{s} C_{r-1,s-1}$),

$$C_{mn-1,m-1} \cdot C_{mn-m-1,m-1} \cdot C_{mn-2m-1,m-1} \cdot \cdots\cdots$$

i.e.,
$$\frac{\frac{(mn)\,!}{m\,!(mn-m)\,!} \cdot \frac{(mn-m)\,!}{m\,!(mn-2m)\,!} \cdot \frac{(mn-2m)\,!}{m\,!(mn-3m)\,!} \cdot \cdots\cdots \frac{m}{m\,!\,0}}{n\,!}$$

i.e.,
$$\frac{(mn)\,!}{(m\,!)^n\, n\,!}.$$

If, therefore, we form this number of different compound determinants, the total number of terms of the original determinant which we shall thus obtain is

$$\frac{(mn)\,!}{(m\,!)^n n\,!} \times n\,!\,(m\,!)^n$$

i.e. $(mn)\,!$

which is exactly the full number of terms in the original determinant. It is consequently manifest that the terms of the original determinant can be represented, so far as magnitude is concerned, by a sum of compound determinants obtainable in the manner indicated.

(6) As the consideration of the question of sign necessitates the use of a theorem regarding inverted-pairs, it is desirable to digress for a moment in order to enunciate and prove this theorem. It is to the following effect :—

If, in the natural series of integers 1, 2, 3, ... *a group of* m *consecutive members, taken in any order, be passed backwards over* r *groups of* s *consecutive members each, the members of each group being arranged in any order, the number of additional inverted-pairs in the permutation thus obtained is* mrs.

This is self-evident, for the number of members passed over is clearly rs, and each of the m members to which precedence has been given will thus give rise to rs inverted-pairs, making mrs inverted-pairs in all.

When the number of members in the group moved is the same as the number in each of the groups passed over,—that is, when $s = m$,—the number of additional inverted-pairs is, of course, rm^2. Consequently, when m is even, the number of additional inverted-

pairs will be always even whatever r may be; and, when m is odd, the number of additional inverted-pairs will be odd or even according as r is odd or even. From this there is immediately deduced the following theorem :—

If the first mn *integers be separated into* n *groups of* m *each, say the groups* A_1, A_2, A_3, ..., A_n, *so that*

A_1 *stands for any permutation of* 1, 2, 3, ..., m
A_2 ... ... m+1, m+2, ..., 2m
.....
A_n ... ... mn−m+1, mn−m+2, ..., mn,

then, when m *is even, any one of the* n! *permutations of* $A_1A_2A_3 \ldots A_n$ *has an even number of inverted-pairs of the* mn *integers more than the standard permutation* $A_1A_2A_3 \ldots A_n$ *has; and, when* m *is odd, has an odd or even number of inverted-pairs more, according as the suffixes of the* A's *have an odd or even number of inverted-pairs.*

(7) Returning now to the consideration of the terms of the original determinant of the $(mn)^{th}$ order, which are obtainable from the term $M_{hk}M_{pq}M_{rs}\ldots$ of the compound determinant, we see that, in order to tell the sign in any case, we have to take into account the sign of each of the contributing terms in its own minor, and, in addition to this, the sign which ought to precede $M_{hk}M_{pq}M_{rs}\ldots$ Now, we have just seen that when m is even, this sign must be a + in every case, and that when m is odd its sign must be the sign which it bears in the compound determinant to which it belongs. This is equivalent to saying that when m is even the compound determinants with which we started should be all viewed as permanents, and that this change is not necessary when m is odd.

(8) When the given determinant of the $(mn)^{th}$ order is an alternant, interesting changes are possible in connection with each term of the development.

Thus in the case of the simplest form of alternant of the 4th order we have

$$|a^0b^1c^2d^3| = \begin{vmatrix}+\\ |a^0b^1| & |a^2b^3|\\ |c^0d^1| & |c^2d^3|\end{vmatrix}^{+} - \begin{vmatrix}+\\ |a^0c^1| & |a^2c^3|\\ |b^0d^1| & |b^2d^3|\end{vmatrix}^{+} + \begin{vmatrix}+\\ |a^0d^1| & |a^2d^3|\\ |b^0c^1| & |b^2c^3|\end{vmatrix}^{+},$$

$$= \begin{vmatrix}+\\ b-a & a^2b^3-a^3b^2\\ d-c & c^2d^3-c^3d^2\end{vmatrix}^{+} - \ldots\ldots\ldots,$$

$$= (b-a)(d-c).\begin{vmatrix}+\\ 1 & a^2b^2\\ 1 & c^2d^2\end{vmatrix}^{+} - \ldots\ldots\ldots,$$

which agrees with a result obtained by JACOBI in his paper, "De functionibus alternantibus" (*Crelle's Journ.*, xxii. p. 363).

In the case of the simplest alternant of the 6th order we have—

$$|a^0b^1c^2d^3e^4f^5| = \sum \begin{vmatrix} |a^0b^1c^2| & |a^3b^4c^5| \\ |d^0e^1f^2| & |d^3e^4f^5| \end{vmatrix},$$

$$= \sum |a^0b^1c^2| \cdot |d^0e^1f^2| \cdot \begin{vmatrix} 1 & a^3b^3c^3 \\ 1 & d^3e^3f^3 \end{vmatrix},$$

$$= \sum |a^0b^1c^2 \cdot |d^0e^1f^2| \cdot \begin{vmatrix} (abc)^0 & (def)^3 \end{vmatrix},$$

where it should be noted that each term of the development consists of three factors, each of which is a simple alternant, and each of which is, therefore, expressible as a product of binomial factors. Thus the specimen term here given after Σ is equal to

$$(c-b)(c-a)(b-a)(f-e)(f-d)(e-d)(d^3e^3f^3-a^3b^3c^3).$$

Similarly we have

$$|a^0b^1c^2d^3e^4f^5g^6h^7i^8| = \sum |a^0b^1c^2| \, |d^0e^1f^2| \, |g^0h^1i^2| \cdot \begin{vmatrix} (abc)^0 & (def)^3 & (ghi)^6 \end{vmatrix},$$

where each term under the sign of summation is expressible as a product of twelve binomial factors, the first, for example, being equal to

$$(c-b)(c-a)(b-a)(f-e)(f-d)(e-d)(i-h)(i-g)(h-g)$$
$$\times (g^3h^3i^3-d^3e^3f^3)(g^3h^3i^3-a^3b^3c^3)(d^3e^3f^3-a^3b^3c^3).$$

XXV.—*On the Rimes in the Authentic Poems of William Dunbar.*

By Henry Bellyse Baildon, M.A. Cantab., F.R.S.E.

(Read April 3, 1899.)

Introduction.

It may well be thought that, in a field that has been so carefully reaped and garnered and gleaned by so many learned workers as have the works of the great Scottish poet, William Dunbar, there remained nothing still to be accomplished. Where such erudite students of Scottish literature as Laing, Small, Gregor, and Æneas Mackay, and such an illustrious scholar as Professor Schipper have laboured, and where even the poet's metrical forms have been the subject of careful investigation by Mr M'Neill, it might be thought alike vain and presumptuous to attempt to follow. Yet it so happens, nevertheless, that there has never been a thorough investigation made of Dunbar's rimes with a view of throwing light on the phonology or, in more popular phrase, the pronunciation of his day. And yet, perhaps, no more suitable, interesting, and instructive subject could be found for such treatment than just this same William Dunbar.

In the first place, Dunbar is nothing if not a conscientious artist,—a man with a thorough appreciation of the value of technique, and with an excellent ear both for metric and phonetic effects. Without such an ear a man cannot be a poetic artist of the first rank, and it is marvellous how these gifts secure a man immortality, even when his thought is neither important nor original. And, on the other hand, the want of, or the occasional neglect to use, these gifts threaten the immortality of some of the greatest names. In Wordsworth, for instance, how often are we jarred by the toneless, musicless quality of his lines! and in Byron's dramas how are we repelled by the harsh, dry *timbre* of his blank verse! Even the least cultured are sensitive on this point, as we gather from the felicitous cadence and clang of popular proverbs and sayings, and other evidences of the sensuous pleasure given to the young and uneducated by song and verse. At any rate, no one can read Dunbar at all without feeling convinced that he took a real pleasure,—perhaps the purest pleasure he had in his grumbling, mendicant existence (even as perhaps did his predecessor and model, the scamp Villon)—in the rhythm and sonorous melody of his verses; and, at a time in his disreputable early career, when he would have unscrupulously robbed hen-roosts or pocketed spoons, or used the pulpit, like Chaucer's Pardoner, for the most sordid ends, he was in his art a purist of the first water. This quality is naturally an invaluable one for our purpose; and, if we except his very latest verse, where the instruction of his flock and not the production of poetry is his main object, one may safely rely on the conscientious work

manship of Dunbar. So that, when we find a really bad rhyme in his poems, we may be well-nigh certain that the text is corrupt and that Dunbar never uttered the poem for complete in such a form.

If one is asked for absolute proof of this conviction, it is not easy to produce one, except on general grounds. So I would put the matter in the form of two propositions, neither of which can reasonably be doubted. The first is that Dunbar's *technical accomplishments* as a verse-writer were such, as witness his mastery of a great variety of metrical forms and his marvellous command of rimes (as shown, for instance, in the two concluding stanzas of his part of the "Flyting," where he has no less than thirty-two rimes, some of them dissyllabic, in each stanza), that he could never really have been at a loss to find a correct rime; and so, if he used them, must have done so from pure carelessness, which is clearly not his characteristic. In the second place, we are really not able to *convict* Dunbar of a *palpably bad* rime, because, in the instances where he is *apparently* guilty, there are usually indications of corruption in the text. I do not, of course, maintain that all Dunbar's rimes are *exact* and *perfect*, a statement which would probably be true of no poet that ever wrote. Indeed, anyone who has given any attention to verse-effects must know that an *occasional imperfection* gives a curious charm in the hands of a master. But it must be slight, and cannot extend to discord in *consonantal* sound. I cite just two cases of apparently false rimes. The first is in Dunbar's "Dirge to the King at Stirling," in the couplet (41. 9, 10):—

"O ȝe eremeitis and hankersaidilis
That takis your pennance at your tablis."

Even if my conjecture of *hanker saiblis* = black anchorites be not correct, *saidlis* = *cells*, *hermitages* makes no sense, and it would be too unjust to Dunbar to suppose that he wrote both a bad rime and nonsense at the same time.

The second instance is perhaps not quite so conclusive: still there is every appearance of some error having crept into the text. It occurs in the courtly and beautiful poem, "To the Princess Margaret on her arrival at Holyrood" in the following couplet:—

"Rejoysing from the sone beme
Welcum of Scotland to be Quene."

On this it may be said (see Professor SCHIPPER's note, p. 92) that there is *some* corruption of the text here, so that if we are not convinced that *beme* is *wrong*, we have no certainty that it is *right*; and I venture the conjecture that the original word was *schene*, which gives a rather better sense, as though the queen came from the sunshine of the sunny south—sunny as compared with Scotland. Further, even supposing the reading "beme" to be correct, the poem is not *provably* Dunbar's, so that in any case he cannot be clearly *convicted* of a false rime in this instance.

And, apart even from these arguments, I must really claim the right of an *expert* in poetry, in poetical technique, and in literary criticism generally [being a critic and verse-writer myself of at least twenty-five years standing] to speak with authority on

the literary and technical qualities of Dunbar's verse; and when it is borne in mind that my own high opinion of Dunbar's technical excellence and conscientious workmanship, if independently formed, coincides with that of such unimpeachable authorities as Sheriff MACKAY, Professor SCHIPPER, and others, it would surely be a *reductio ad absurdum* of scientific scepticism to ask for further *proof*.

If we only had a fair copy of Dunbar's original MS. a mass of uncertainties would disappear. For we can seldom be quite certain whether the spelling is (1) Dunbar's own, or (2) that of his own time, or (3) the Scotch and not the (4) English spelling. These uncertainties are introduced in the case of MSS. by the scribe, or in that of print by the compositor. Now in those times those persons do not seem to have been either very careful or conscientious or skilful; and that in the chaotic state of ME. spelling complicated with the almost as chaotic MSc. spelling, and possibly also with attempts at compromise between the two, creates elements of difficulty enough.

There is fortunately one other reliable and, within certain limits, fixed foothold for us in this examination, considered as an independent investigation, and that is the pronunciation of modern Scotch (NSc.), with which, owing to my long residence in Scotland, I may claim some familiarity. And the Scotch with which I am most familiar is, fortunately, locally Dunbar's Scotch, not BURNS's west country Scotch, but the Lowland, one may almost say Lothian Scotch of SCOTT and STEVENSON, who is held by those competent to judge to write particularly good Scotch, and who makes the attempt to write phonetically, especially in the volume entitled "Underwoods," where he gives a "Table of Common Scottish Vowel Sounds," which appears to me easily intelligible and quite accurate. For more scientific and historic treatment of these sounds one must naturally have recourse to the works of SWEET, ELLIS, and MURRAY in English, and to such modern investigators as Professor LUICK in Germany, and the Dissertations of Dr CURTIS, "On the Middle Scotch Romana Clariodus" (Anglia, vols. xvi, xvii), and of Dr GERKEN, on "Die Sprache des Bishofs Douglas von Dunkeld" (Strassburg, Karl J. Trübner, 1898).

While one can know how a Scottish word is pronounced to-day in the very district and city where Dunbar wrote and spoke, there remain still two elements to be supplied before the problem of fixing the phonology of Dunbar and his time can be attempted. The first is a knowledge of the original sounds of the words, *i.e.*, the pronunciation of the vowels, diphthongs, and consonants in the language from which his vocabulary is derived, viz., Anglo-Saxon (Old English, OE.), Old French (OF.), Old Norse (ON.), Latin (Lat.), Old Irish (OI.), Dutch (Du.), etc., and the second that of the corresponding sounds in contemporary Middle English (ME.). Now the former, thanks to the labours of such scholars as SIEVERS, KLUGE, SWEET, SKEAT and others, are fortunately fairly well established, and affords us in this inquiry our first point of attachment so to speak. If we then, so to speak, make fast one end of our line to the original sound and lead the other to the corresponding modern Scotch sound, we know then that the path of this sound through its successive changes must pass through these two points, and

we know also that it will not usually deviate very far from what we may call the "lin of least resistance" between the two positions, so that, if we suppose this to be represented by a straight line joining the two points, we may, to continue the metaphor, draw our line tight, and have thus approximately fixed the path of the sound change. Then it becomes the function of phonetics to show what this "line of least resistance" really is. And the principle on which this is to be determined is obviously one of the place and mouth-position in which the two sounds are formed and uttered, together with the postulate that sounds not only do not suddenly travel from a certain mouth-place or mouth-position to another far removed or widely different, but must of necessity, when these have become far separated, have passed through some, if not all, the intermediate stages. Such changes, for example, as now distinguish the English pronunciation of vowels from the Germanic, Italian, or Continental sounds for the same vowels, *cannot* have taken place *per saltum*, but *must* have passed through intermediate stages. Our use of the vowel *i*, for instance, as equivalent to Ger. *ai*, *cannot* have come in suddenly, because it involves too great a change in the mouth-place in which the sound is formed. And the same is true of the other principal changes in our vowel and consonantal sounds. Still it is not to be assumed that because a sound does not change *suddenly* it may not change *quickly*, for the intermediate position may be one of, so to speak, unstable equilibrium ; it may be more difficult or less agreeable than the two positions between which it mediates.

These, of course, are the commonplaces of phonetics, of which, however, it is useful to be reminded in connection with this investigation.

The other element of which I have spoken, viz., the contemporary pronunciation of the corresponding words in ME., does not unfortunately rest on so secure a basis, there being still room, in spite of all the labours of the eminent scholars already mentioned and of such recent investigators as LUICK, for doubt, not so much perhaps as to the path taken by sound-changes, as to the time they took place and especially the moment of their complete transmutation to the present accepted pronunciation. Hence arises the possibility that this inquiry, taken together with such an admirable and thorough investigation as that of Dr F. J. CURTIS "On the Rimes of the Middle Scotch Romance Clariodus" (Anglia, vols. xvi and xvii), to which I am infinitely indebted for its excellent method, not to speak of its valuable results, may reflect some light on the Middle English of Dunbar's time. All the more, naturally, is this the case, in that Dunbar often writes what must be called ME. and not MSc., just as BURNS wrote in standard eighteenth century English, as well as in the Ayrshire dialect of that time.

As Dr CURTIS's article "On the Rimes of the Middle Scottish Romance Clariodus" treats that work exactly on the same lines as I propose to treat Dunbar, and treats also of much the same period, it will not be necessary to handle the results with the same elaboration as Dr CURTIS has done. My results must of necessity either agree or disagree with his, and in neither case is it likely that any elaborate argument will be

necessary. I did not, indeed, undertake the investigation with this idea of making any important discoveries, but, firstly, with the modest ambition of rounding off the work done on Dunbar by the Scottish Text Society and by the edition of Professor SCHIPPER, by the compilation of a Rime-Index, which had not before been exhaustively made. I have used Professor SCHIPPER's edition, as embodying the results of previous workers, along with his own, and my references are to page and line of his work. I have confined myself strictly to poems which seem undoubtedly authentic, because I thought it possible that this might furnish a criterion by which some of the doubtful poems might ultimately be tested. But as that involves a separate rime-index for these doubtful poems, which I have not been able yet to prepare, this criterion cannot at present be easily applied, except to quite short pieces. I do not propose in the present instance to apply it, but the index will be essential to whoever wishes to employ such a criterion. Another use of the index will be to assist in clearing up doubtful readings where rimes are involved, for, as I have already said, Dunbar is so conscientious an artist that we can rely on the purity of his rimes to an even unusual extent, certainly more than in Clariodus, who has a formidable number of false rimes (CURTIS, § 554) and very probably more than in Douglas. And on this account I have reckoned all words riming with the same refrain-word as riming with each other.

The plan of the index, following that of Dr CURTIS, is to classify the rimes under the original vowel sounds in OE. or ON. so that OF. and Latin and other words of non-Germanic origin only appear in so far as they rime with these Germanic words.

I wish to take this opportunity of very cordially thanking Professor KARL LUICK of the University of Graz for his kind assistance in seeing this work through the press, and in making many very valuable suggestions. At the same time, I take the full responsibility for any faults the work may still possess.

[RIME-INDEX.

RIME-INDEX ACCORDING TO GERMANIC VOWEL-SOUNDS.

A-

§ 1. Not followed by g or w, rimes with

a) itself.
forsaik : taik, 83, 52. 253, 1. baik : saik, 72, 35.
schaik : quaik, 150, 9. tail (OE. talu) : nychtingale, 347, 29.
gaip : laip, 132, 100.
gaittis : schaittis, 85, 8. lakkis : takkis, 77, 5.
undertaker : balletmaker, 246, 87.
name : schame, 85, 5, etc., lame : schame, 87, 53.
schame : came (s) 86, 39.
same : lame, 126, 17. sam : gam, 202, 11.
cravis : wavis, 281, 56. crave : forgaif, 365, 145.

b) OE. ǣ.
mak : bak, 202, 17. brak, 377, 79. spak. 79, 1.
nychtingail : small, 350, 114.
gait : lait, 316, 26. crave, graif, 365, 145.
said : maid, 125, 46. pak : bak, 269, 58.

c) OE. æ + g.
fare (= mien) : fair (adj.) 111, 225.
phane : brain : fane, 266, 83.

d) OE. ā.
have : laif, 214, 90., etc., gaip : raip, 84, 75. 83, 61.
aip : graip : saip : 206, 6. cair : rair, 225, 114.
nychtingale : hale : 370, 34.
lame : schame : hame, 87, 53.
name : hame, 86, 18. schame : hame, 86, 17.
fair (v) : sair : thair, 71, 19. wair (s) lair, 194, 76.
„ (v) : mair, 71, 17. fare (s) : mair, 100, 222.
cair : mair, 308, 44. rair, 225, 114.

e) OF. or Lat. a.
haif : raif : 214, 90.
gaip : chaip : 84, 76.
gaittis : debaittis : estaittis : 85, 8.
„ : stait, 316, 28.
schame : fame, 85, 4.
name : fame, 85, 7.
schame : clame, 86, 32.
„ : proclaime, 87, 67, etc.
name : lame : proclaime : 87, 53.
„ : shame : blame, 87, 60.
„ : „ : defame, 85, 11.
fare (s) : repair, 111, 223.
spair : foreclair, 301, 67.
„ : preclair, 301, 67.
nychtingaile : vaill, 347, 28.

f) OF. al or au, Lat.—al.
haif : saiff, 214, 90.
knaiff : saiff, 205, 43.
have : saiff, 266, 98.
nychtingaile : oriental, 370, 26.
„ : scale, 370, 28.

g) OF. ai.
nychtingaille : travaill, 347, 31. 370, 34.
„ : faill, 348, 35.
„ : awail, 351, 17.
„ : battale, 370, 34.

h) OF. ei.
phane : pane, 266, 95.
haif : persaif, 279, 17.

§ 2. It is, I think, impossible to gainsay Dr CURTIS's conclusion that *a* in open syllables was *in most instances* already lengthened, or in some more strictly speaking half-long, in the usage of Dunbar; and in these instances had already an ę̆-sound, and that this lengthening and change of sound is usually indicated by the spelling *ai* instead of the ME. *a* + *cons* + *e* which Dunbar or his scribes and printers also use. There are, however, apparent exceptions, and particularly when *a* is followed by *k*. Thus such rimes as *lakkis* : *takkis*, *mak* : *bak*, *brak* : *spak*, taken together with their NSc. pronunciation, which is undoubtedly ă in the modern Lothian and most other Scotch dialects, point to the survival of the *a*-sound in these words. But again the rimes and spelling *forsaik* : *taik*, *baik* : *saik*, show *taik* to have sometimes the *e*-sound, as was probably also the case with *mak* when spelt *maik* (see CURTIS, § 5). It seems as though the *k* had a tendency to check the lengthening (see CURTIS, § 6) [GERKEN, § I. 4], and thus to prevent the change to *e*, it being, of course, almost an axiom that a long vowel changes much more readily than a short one. The other consonants, especially

the liquids, encourage lengthening and vowel-change, so that we find no such instances in their case. I am inclined also to think that Dunbar still distinguished between ā and ă, though probably only in quantity, the latter being probably only half-long with him. I gather this from the fact of the *comparative* infrequency of rimes between ā and ă, and that most even of these are before *r*, a consonant which has an undoubted tendency to give length to a syllable. So that *fair*, *fare*, *cair*, etc. may be regarded as full long and thus form perfect rimes for *sair*, *mair*, etc.*

On the whole, the evidence seems in consonance with the view of Professor KLUGE (P.G. 1. 877), that the change from *a* to *e* was completed early in Scotland, as it appears to be well established in Dunbar's time.

According to Professor LUICK (*Archiv für das Studium der neueren Sprachen und Litteraturen*, Band cii. Heft 1–2, pp. 52 and 63) the originally short vowels had begun to lengthen to a sort of half-long quantity in the Northumbrian dialect as early as the second half of the tenth century. But how long this would take to spread to the Lothians, if it spread in this manner at all, it is impossible to say.

§ 3. A + G rimes with

a) itself.
lawis : sawis, 317, 36.
(?) flawis (OE. fla3u) : lawis, 317, 39.
b) OE. ā + w.
draw : knaw, 276, 32.
,, : blaw, 166, 226.
c) OE. a or ea + ll.
draw : staw, 276, 32.
draw : waw, 166, 228.
,, : aw (NE. all), 166, 226.
law : waw, 317, 38.
sawis : wawis, 317, 39.
d) Lat. au.
lawis : cawis, 317, 37.
sawis : ,, 317, 36.

The rimes in b) confirm the lengthening of *ău* (from OE. *ag*) in MSc. (CURTIS, § 27).

§ 4. A + W

a) with OE. a + g (see § 3).

b) with OE. eall.
staw : knaw : 276, 33.

Although, as we shall have reason to observe later on, the influence of the Scotch *w* on a following vowel seems to be different from that of the English *w*, its influence on a preceding is the same, *i.e.*, just the same as an *u*, giving an *au*-sound (see § 46).

A :

1. Followed by Nasals.

§ 5. (a) followed by -nd rimes with

a) itself.
landis : bandis : strandis, 104, 57. strandis, 271, 14.
land : stand : hand, 191, 5.
landit : blandit (NE. flattered) : handit, 262, 76.
band : hand : strand : stand, 376, 34.
brand : understand, 351, 1.
brand : land, 203, 9.
b) OF. and & ant.
stand : garland, 193, 45.
upaland : garland, 203, 19.
,, : variand, 328, 41.
handis : gyandis (OF. geant), 126, 21.
c) Sc pres : part : in – and.
understand : brand : kindilland, 351, 1.
hand : serwand, 239, 4. etc.
d) ON. önd.
landis : wandis (ON. vöndr), 104, 63.

* I am inclined to maintain this position in the face of Dr GERKEN's remark which seems to bear on it [§ 6. 3], to the effect that one must not ascribe any influence to *r* in preserving the quantity of the *ā*. It is quite possible that Dr GERKEN has ample materials to prove his point, but he does not produce them in his thesis.

There is here no sign of the dropping of the *d*, as is so common in NSc. in certain words, especially *hand* and *stand*, as in the phrase to "*stan' yer han'*" = *stɒn yər hɒ̄n* = to pay your share of the reckoning. In other words, such as *strand*, *band*, *bland*, and even *land*, the *d* is often pronounced. Nor am I so clear as Dr GERKEN appears to be as to the length of the vowel. The *syllable* is doubtless to be reckoned long, but that is due rather to the consonants than the vowel.

§ 6. (β) a + nt. rimes with

a) Fr. ant. stant : novaunt, 88, 9.

b) wantoun : dantoun : pantoun, 124, 24.

§ 7. (γ) a + ng rimes with

a) itself.
gang : wrang : lang : rang, 71, 29.
„ : amang : hang : strang, 170, 274.
lang : amang, 305, 32.
„ : strang : stang (s) (NE. sting) : fang (s) (OE. fang), 380, 10.

amang : dang : (praet. of ding, ON. dengja) : owt-sprang, 225, 111. wrang, 256, 3.
wrang : lang : gang : rang, 71, 27.
„ : amang : fang (v) (OE. fôn) : sang : belang : wrang, 257, 3.

Dunbar,* like Douglas [GERKEN, § 1. 3)], writes these rimes consistently with an *a* (and not, like Clariodus, with *o*), just as BURNS and modern writers of Scotch dialect do.

§ 8. (δ) a + n or nn rimes

a) with itself.
man : wan : 132, 106.
„ : dirry dan (?) : 40, 60.
„ : than 78, 27.
„ : gan : 197, 164. began, 235, 11.
„ : kan (s) : 197, 173.
„ : can, 230, 29. 235, 3.

man : swan, 235, 19.
„ : ran, 236, 23. 347, 27.
„ : St. An, 236, 31.
„ : clan, 261, 32.
than : wan, 235, 15.
„ : began, 235, 11.
„ : can, 235, 3.

§ 9. (ε) a + nk

a) with self or OE., etc., o :

ronk : bonk : donk : thonk, 106, 99.

This spelling with *o* appears also in *ronk*, *donk*, and *slonk* in Douglas [GERKEN, § 1. 3)].

§ 10. (η) a + m or mb rimes

a) with itself.
lam (lamb) : rame, 126, 17. 35, 3. am, 201, 3. sam, 202, 11.

lam : dam (?) 202, 15. dram † (?) 202, 23.
came (OE. camb) : schame : name, 86, 39.

As remarked by Dr GERKEN in regard to Douglas, the vowel in *lam* (=lamb) remains short in spite of the combination *mb*, which has a lengthening influence. But this is not surprising, considering how readily, especially in Scotland, the *b* was dropped.

* When I say Dunbar I mean, of course, his scribe or printer, and so in the case of the others.

† This word *dram* (*cf.* CURTIS, § 16) seems also connected with *drumlie* (NSc. = turbid, dark) and very possibly with *doldrums* through Icel. *draums*, Gen. from *draumr* = melancholy, a dream (OE. drēam) with which *dram* may be directly connected. The connection is not difficult to trace, as a person in a dreamy mood or deeply sunk in thought has usually a serious, even melancholy, expression.

§ 11. 2. A : Followed by other consonants rimes with

a) itself.
clappit : happit, 276, 19.
b) OE. æ :
agast : fast, 136, 34.
gather : father, 264, 37.
„ : hadder, 266, 85.
clappit : trappit, 276, 19.
c) ME. a : wrack : frack, 282, 76.
d) OF. or Lat. a.
ass : pass : 331, 5. terge : lerge : berge : chairge, 108, 183.
ass : vanitas, 387, 8.
last (OE. latost) : past, 43, 79.
e) OE. ea & eo.
at : ȝet, 114, 17.
rax (OE. raxan = to reach) : wax : 377, 66.
„ : sax, 377, 71.
f) Latin e.
last (latost) : est, 368, 46.

§ 12. There seems little to be observed on the *ă* in closed syllables in addition to what Dr CURTIS has already said.

The word *man*, which in NSc. has usually the pronunciation *mon*, appears in Dunbar's time to be still *man*, and I have noticed a curious distinction in the Lothian dialect, in that any one using the word man for *husband* says "man" (Ger. mann), but in using the word generally = NE. *man*, says *mon*. The spelling of the rimes *ronk* : *bonk* : *donk* : *thonk* suggest, although they cannot be said to prove, the presence of an open o-sound in these words, and this is rather supported by the fact that a similar spelling occurs in Douglas (see above, § 9). If it be maintained that *terge* : *lerge* : etc. is an instance of the northern *er* + *cons* = *ar* + *cons*, it can be replied that this is not the case in Scottish, when the following consonant is palatal, and indeed is only clearly provable in the case of *er* + *k*, as surviving in the modern *clark* for *clerk*. The NSc. pronunciation of *large* and *charge* = *lę̄rge*, *chę̄rge*, also bears out my view (see also § 30).

Ā-

Final ā or ā + h.

§ 13. 1. (*α*) written with a, rimes with

a) itself.
Sua : twa, 205, 45. therfra, 263, 9. ga : sla, (ON.ā) 195, 106.
gais : tais, 269, 54. gais : strais, (ON.ā) 176, 342.
b) Lat. a. Maria : bla, 373, 30. fra, 373, 22. fa, 373, 38.
c) āp.
clais : gais, 140, 21.

§ 14. (*β*) written with o, rimes with

a) itself.
ago : wo, 264, 21. also : ro, 96, 78. fo : wo, 42, 34. 280, 4. fo : tuo, 118, 1. mo, 280, 2. fro : wo : go, 197, 166.
b) Lat. ō.
Apollo : sepulchro : go : so : wo : fo : mo, 382, 2 etc.

The spelling with *o* is with Dunbar, as with Clariodus, commoner than with *α*, though not to the same degree, judging from what Dr CURTIS says in his article (§ 38), and is consistently carried out.

§ 15. 1. ā + n, rimes with

(*α*) (spelt with a)

a) itself, allane : gane, 38, 26. tane (NE. taken) 290, 93. ane, 38, 26. ane : nane, 37, 5.
b) OF. ain, Lat. an. stane : soverayne, 371, 64. trayne, 371, 66. suffragane, 371, 68.
c) OF. ei. pane : flane, 265, 59.

§ 16. (β) (spelt with o) rimes

a) with itself.
stone : allone, 42, 17. allone : none, 348, 46. tone (NE. taken), 350, 102.
bone : allone, 350, 110. allone : gone, 244, 63.

b) Latin ōn
stone : dispone, 42, 49.
annone : postpone, 240, 28.

2. ā + m rimes with

a) OF. a and ai. hame : clame, 229. 1.

There is here also a clear distinction kept up between *o* and *a* spellings. The rimes with Fr. *ein* and *ain*, together with the spelling of *trayne*, *grayne*, etc., and the fact that *suffragane* is sometimes spelt *-ene* in the rime syllable, point, I think, to the fact that *ā* + *m* or *n* is already *ē*, or some intermediate sound that comes very near it. *Tane* = *taken* occurs once as *tone*, riming with *allone*, and this, along with its appearance in Clariodus and in Douglas, clearly shows it was an accepted form of the word, however derived. Dr CURTIS explains it as formed on a false analogy from apparently similar forms in which ME. *ō* corresponded to MSc. *ā* [see CURTIS, § 23. GERKEN, § 20, 2)]. The occurrence of *tone* here seems quite to dispose of BRANDL's argument (Anzfd. A., 10, 333).

§ 17. 3. ā + w rimes with

a) itself.
thrawis : crawis, 176, 345.
blawis : ,, 225, 89.
raw (NE. row (s)) : haw (OE. haga), 173, 309.

b) OE. ǣ + w
crawis : mawis, 225, 89.

c) OE. ēaw, knawin : schawin (sc*ēa*wian), 355, 13.

d) a − + g (see § 3).

e) eall.
raw (s) : aw (NE. all), 173, 309.
blaw : waw, 166, 229.
raw : gaw (OE. gealla) : aw : haw, 173, 306.
sawlis (OE. sawol) rimes with Pawlis (Paul's) and brawlis (brawls).

This form *schawin* from *sc*ēa*wian* has been treated at great length by Dr CURTIS (§ 288, etc.); but the simple explanation of SWEET (HES., § 680) that the *e*-element was absorbed by the preceding consonantal sound commends itself more to me, especially when we remember that this preceding sound was palatalised by the influence of the *e*. It is even possible that, by a reversal of the process by which we get *tone*, the Scottish scribes have written *schaw* to represent ME. *schow*.

There seems little to notice here, unless it be the marked tendency in Scottish to go ahead of English in the dropping of the consonantal *l*-sound. I do not know whether this has been fully accounted for, but it may have been encouraged by French influence, which was strong in Scotland about this time.

§ 18. 4. ā before r or r + consonant rimes with

a) itself.
mair : sair : 71, 17. etc.
more : sore, 370, 49. hore (NE. old age), 371, 59.
mair : swair (NE. swore), 84, 86.

b) OE. ă (see § 1).

c) OE. ǣg.
ar (= oar) : fare (adj.) (NE. fair) 89, 28.
fair : mair : 74, 5.

d) OE. ae + r.
mair : hair : 193, 48.
,, : war (v) (NE. were), 256, 10.

mair : thair (ðaer), 71, 15. 194, 68.
e) OE. er.
mair : demair (NE. deemer), 308, 42.
„ : Thesaurair, 230, 20.
f) OE. o. soir (OE. sār) : befoir, 43, 86. lord : scorde, 376, 55.
evermoir : befoir, 118, 20.
moir : befoir, 370, 49.
forelore : moir : befoir, 370, 49.
g) OF. air or ar and Lat. ar.
air : mair, 193, 36. are : compare, 89, 87. preclare, 89, 26.
mair : compair, 84, 87. repair, 74, 2. 111, 223. squair, 193, 48.
hair (OE. hâr, NE. old age) : squair, 106, 111. 193, 44.
h) OF. or Lat. o.
lordis (OE. hlāford) : discordis : recordis, 227, 11.
lord : remord, 269, 42.
„ : accord, 315, 22.
„ : corde, 376, 52.
„ : deforde, 376, 53.

The MSc. *quhair* and *thair* do not come from the forms *hwār* and *þār*, but from *hwār* and *þār* [GERKEN, § 6. 6)]. The modern Scotch is not only often *whēr* and *thēr*, but even *whīr* and *thīr*. *Whīr* may be due to preceding *w* (see § 46), and *thīr* may have been influenced in turn by it. On the other hand, we have *whaur*, *whar*, and *whare* (BURNS), showing, in the first instance at least, the English w-influence, as we may call it (see § 46).

There can, I think, be no doubt that $\bar{a}+r=air$ is already sounded in MSc. *ēr* (LUICK, Untersuchungen, §§ 235, 260). For the spelling *ai* = *e* see Morsbach Gr., § 136, Anm. 3 f., and LUICK, Untersuch., §§ 359–61.

§ 19. For ā + ht = OE. āwiht, nāwiht see ōht (§ 93).

§ 20. ā + st rimes with

a) OE. ēo almoist : loist, 76, 61.
b) OF. *o* and Lat. o. almoist : indoist : coist, 76, 61.

I take the syllable *moist* in *almoist* to come from OE. *māst*, as neither this nor the Scottish *maist* can come from OE. *mæst*.

Æ-

§ 21. 1. Not followed by g, rimes with

a) itself, satt : fatt : that, 73, 6.

Æ :

§ 22. rimes with

a) itself, gaif : haif, 359, 25. blek : fek : sek (NE. sack), 116, 85.
b) OE. æ.
mast : blast, 136, 27. edder : ledder, 177, 368.
c) OE. ā (see § 18).
mad : sad, 304, 11. gaif : laif, 287, 46. weir : beir, 129, 33.
d) OE. a – (see § 1).
e) OF. ei. haif : persaif, 279, 17. dais : prais, 205, 46.
f) OF. au, haif : saiff, 214, 90. 235, 18.
g) OE. ĕ. bak : quhattrak (NE. what reck), 140, 30.
feddir (OE. ĕ) : eddir (OE. ae), 283, 8.
brak : frak, 111, 241.
neck : blek : 82, 34. hed (had) : sted (s) 126, 11.
h) OF. or Lat. ĕ.
blek : effek, 82, 31. weir (NE. war) : prisoneir, 116, 110.
k) OF. air – ar.
bair (adj.) : repair, 171, 281.
l) OE. eo war (OE. (ge) waer) : far, 36, 50. ar, 350, 93.
m) OE. a : (see § 11).
n) OF. a.
past : fast, 81, 2. 118, 18.
o) OE. eaht. ON. ātr.
rawchtir : lawchtir : slawchtir, 222, 37.
p) OE. æ + g. (see § 23).

This is a very curious instance in which *rafter* (OE. *ræfter*) has been confused with words from OE. *eaht*, and where the scribe or printer has conformed it to the spelling of the other words, while it seems more likely that *lawchter* and *slawchter* had the *f*-sound, like NE. *laughter*, than that *rafter* had a guttural spirant sound.

§ 23. Æ + G rimes with

a) itself and æg.
day : lay, 93, 8. 104, 48. snaill : taill, 70, 10.
lay : clay (OE. æ), 242, 5.
b) with eg. or ecg.
day : tway : say, 73, 1.
day : play, 126, 32.
lay : away, 100, 186.
c) OF. ei and ai
lay : affray : 100, 184.
lay : pray (prey), 104, 54.
lay : array, 242, 2.
day : pray, 104, 54. 126, 34.
day : abbay, 314, 9.
day : may, 347, 20.
,, : array, 347, 23.
they (or ON. ei) pray : display : 109, 177.
fair : air, 106, 115. 368, 33.
d) OE. āe or ā, + r.
fair : hair, 106, 14.
,, : sair, 375, 21.
e) OE. ae (not followed by g).
fair : bair, 376, 23.
f) OE. a – (see § 1).
g) OE. a (see § 18).
h) OF. or Lat. a fair : repair, 74, 1. 111, 223.
,, : preclare, 89, 26. 92, 2.
,, : compair, 89, 31.
,, : declair, 283, 10.
,, : etc., etc.
k) with OE. āe or ON. ei.
haill : taill : snaill, 70, 10.

For discussion of *æ + g* see under *e + g*, § 32; CURTIS, § 135; and GERKEN, § 6. 5).

Ē- sounds in ME. and MSc.

§ 24. E- sounds in MSc.

a) (1) OE. ēa *e.g.* rēad = NE. red.
(2) OE e – lengthened in ME. *e.g.* stede.
(3) ON. æ, *e.g.* saete (NE. seat).
(4) OF. e = (1) Lat. a + l or + n, *e.g.*, natural (2) Lat. ĕ, ĭ, æ or Grmc. in position.
(5) OF. ei, eĭ.

§ 25.

β) (1) WS. āe Angl. ē Grmc. āe. WGrmc ā *e.g.* dāed Angl. dēd, NE. deed.
(2) OE. āe (i – mutation of ą̄) māenan (to mean).
(3) AF. e = OF. ee + ch, *e.g.*, preechier (to preach).
(4) Lat. e Greek *η*.

§ 26.

γ) (1) WGrmc. ē (including ĕ final which was lengthened), *e.g.*, slēp (prt = slept), hē, (prn).
(2) OE. ĕ + ld, etc., *e.g.*, feld (field).
(3) OE. ē Northmb. īe (i mut. of ō), *e.g.*, swēte (sweet).
(4) Angl. ē WS. ie. y (i mut. ēa ēo), *e.g.*, hēran, hieran, -hyran (hear).
(5) OE. ēo = (I.) Grmc. eu, *e.g.*, dēop, (II.) contracted from ĕ ĭ y̆ + a, u, *e.g.* seon (to see) from sehan, (III.) eo before (Grmc. e or i) lengthening consonants, *e.g.*, leornian (to learn).
(6) OF. e = (I.) Lat. a, *e.g.*, cler (clear), (II.) Lat. ē or Gk. n Lat. ae and Gk. ai, Lat oe, Gk. oi and Lat. ĕ in open syllables, *e.g.*, ME. procede, tragedie, repete.
(7) AF. e = (I.) OF. ie., *e.g.*, grief, (II.) OF. ue = Lat. o in open syllables, *e.g.*, buef (beef).

Table of *α*, *β*, *γ* rimes.

§ 26a. *α* and *β* rimes.

heidis : beidis : leidis, 79, 17. deid : leid, 131, 84.
steidis : deidis (dāed), 246, 93 reid (red), leid, 131, 87.
deid (ēa) : deid (dāed), 363, 103. speiche : streiche, leiche, 308, 31.
leill (OF. leal) : deill : heill : 37, 6. weill : seill, 42, 60.
speir : feir (fear), 194, 96.

§ 27. *α* and *γ* rimes. See also LUICK, Untersuch., §§ 356 and 358.

speiris : weiris : effeiris : 98, 126.
leif (OE. lêâf) : cleif, 232, 7. mischief : 259, 10.
leif : breif, 232, 6. mischief : beleif, 262, 83.
,, : reif, 260, 33.
weill : quheill, 218, 13. skeillis : quheillis, 176, 359.
* leik (OE. ēa) : breik (OE. ê), 167, 240. eik : meik, 271, 35.
† ȝeir : cheir : 33, 4. 246, 110. and cleir, 231, 5.
,, : neir : compeir, 96, 72.
speir : deir, 99, 165.
steir : deir, 119, 48. 137, 52.

* *leik*, *eik* from late Anglian lēc, ēc (Sievers, § 163).

† Angl. ȝēr, for WS. ȝēr.

§ 28. β and γ.

deid (dǣd) forbid (e͞o), 118, 11.
feir (fear) : cheir, 97, 94. feir : cheir, 97, 96. 246, 101.
sweir (adj : cheir, 34, 19. feir : presoneir : 115, 46.
freiris : leiris, 80, 46.
deid : speid, 245, 81. and remeid, 363, 101.
,, : meid, 250, 2. weid : neid, 237, 12. 315, 13.
deidis : neidis, 250, 4.
,, : procedis, 281, 41.
,, : briedis, 281, 42.
,, : posseid, 296, 15.
deid : dreid, 245, 82.
leidis : remeid, 131, 90.
meidis : reidis, 104, 55.
weidis : ,, 104, 58.
upspreidis : ,, 104, 59.
shreidis : ,, 104, 62.
seid : speid, 221, 24.

§ 29. α, β and γ.

steid : reid (rǣdan) : remeid : pleid, 75, 38, etc. | weill : deill : feill : 75, 26.

This table amply proves that Dunbar rimed these three classes of sounds frequently together, so that they must to his ear have been very similar, and this can hardly mean anything else than that $\bar{e}$ had in MSc. already become $\bar{\imath}$, at least before certain consonants. This is an important distinction, especially as we notice that in Dunbar there are only six consonantal in the final letters of these rimes, viz., *d*, *ch*, *f*, *k*, *ll*, and *r*. In Clariodus there are also *v*, *m*, *st*, *t*, *s*, and *p*. The most striking point here is the complete absence from both of the nasal *n*, and the frequency of *d* and *r*. One is almost driven to the conclusion that, while before these last two consonants at least *e* had already the *i* quality, it had still before *n* and possibly before *m* an *e*-sound.

E-

§ 30. 1. Not before g rimes with

a) itself.
heris (v) : deris (hurts) 270, 73.
beir (v) : speir, 194, 84.
,, (s) (NE. bear) : spear, 194, 92.
b) OE. ea͞ (see § 55 and following).
c) OE. e͞a.
beir : eir, 115, 41.
mere : eir, 79, 6.
forbeir : eir, 251, 34 and 32.
meit (OE. mete) : threit, 251, 11.
d) OE. ē, y͞, ie͞, and eo=γ rimes.
speir : heir : deir, 99, 163. 107, 132.
e) OE. WS. æ͞ Angl. ē.
meir (NE. mare) : weir (NE. war), 79, 3. eit : bleit (v) (NE. bleat) : quheit (NE. wheat), 175, 332.
speiris : weiris, 98, 130.
speir : feir, 194, 88. beir (bear), 194, 92.
f) OF. e.
speiris : effeiris 98, 130. 137, 64. smellis : excellis, 358, 12.
g) OE. a͞e (see § 50).
eit : sweit (NE. sweat) : 175, 330.

The *e* in *beir* (v) *beir* (s) = (*bear*), *speir* and *deir* and also in *meit* and *eit* appears to be lengthened and to have *in these cases* an ī-sound (see CURTIS, § 131), with whom I am inclined to agree in spite of all said by Dr GERKEN, § 10. 4). We must again, however, notice that with the exception of *weit* : *threit*, and *eit* : *sweit*, all these rimes end in *r*, a confirmation, it would seem, of the lightening effect of the Scotch *r* already noticed in § 29, and confirmed, to my thinking, by such cases as *whīr* and *thīr* (*where* and *there*), *awīr* for *aware*, which occur as often in NSc. In this connection it must be borne in mind that there are in England and Scotland at least three different *r*s, the South English *trilled r*, with up-turned point of the tongue, the North English *burr* or *slurred guttural r*, and the Scotch *rolled r*, made, like the German, with *down-turned*

tongue-tip. The first tends to lose its *trill* and become almost a mere *breathing*, as in London and South England generally, the second produces the North English *a+r+cons*, from *e+r+cons*, while the Scotch seems to *lighten* the preceding vowel-sound.

§ 31. 2. E+G or CG rimes with

a) self.
way : play, 196, 132. 222, 25.
say : away, 114, 21.
b) ae+g. (see § 23).
c) ē+g. say : tway, 73, 2.
d) ON. ei.
wey (weigh) : suey (sway), 246, 104.
aye : play, 196, 146.
e) OF. ai.
way : verey, 82, 7. pley : gay, 75, 39.
pley : assay, 110, 205 etc. forwayit : assayit, 110, 204.
f) OF. ei.
play : pray, 126, 34. pley : array, 196, 136. 107, 127.
pley : affray, 196, 147. pley : deray, 234, 14.
away : affrey, 100, 187.
g) OF. e or ee.
pley : lufraye (livery), 196, 140.

§ 32. An extremely interesting and difficult question demands discussion here, viz., the history of the three sounds, OE. *æ+g* and *e+g* and ON. *ei*. Dr CURTIS has treated the question at great length and very thoroughly, and has arrived at a conclusion different from that formerly accepted. Dr GERKEN [§ 6. 5)], while admitting some force in Dr Curtis's arguments, states the objections which can be raised against his view, and appears to lean to the earlier theory. What we may call the received doctrine represents the line of development thus :—

$$\left.\begin{array}{l}\text{OE. æ+g>ai}\\ \text{OE. e+g}\\ \text{ON. ei}\end{array}\right\}\ \text{ei}\ \Big\}\ \text{ai}>\bar{a}>\bar{e}$$

Dr CURTIS represents it thus :—

$$\left.\begin{array}{l}\text{OE. æ+g>ai}\\ \text{OE. e+g}\\ \text{ON. ei}\end{array}\right\}\ \text{ei}\ \Big\}\ \text{ei}>\bar{e}$$

At the first glance Dr CURTIS's theory has the advantage of greater simplicity, and, what is more, it is from the phonetic stand-point much more credible, as it follows what I have called "the line of least resistance." The other theory labours under the disadvantage of being circuitous and not following this line, for the change from *ai* to *a* is not from the back towards the front, but from front to back, *i.e.*, in the reverse direction to that which it takes in the subsequent stage in passing from *ā* to *ē* (CURTIS, § 150). Indeed, the direct transition from *ā* to *ē* seems in itself improbable, so that we should have to insert an intermediate diphthong, *ai*, which would be very like a *reductio ad absurdum* of the theory. But we must now examine the objections to Dr CURTIS's view as collected together by Dr GERKEN (§ 6, 5). See also LUICK, Untersuch., § 282, etc.

The first is "that *ai* and *ei* are often, especially in the North, interchanged in writing, and that mostly in favour of *ai* (see SWEET, HES., § 706; TEN BRINK, Chaucer's Sprache, § 40)." "According to Dr Curtis's hypothesis," continues Dr GERKEN, "one

should expect the contrary." The reply to this is, I think, perfectly simple, *i.e.*, that *ei* was already used to express a different sound, viz. *ī*, and hence *ai* was written to avoid confusion. His second point, that the transition from *ei* to *ai* occurs in South England, though seemingly a point in favour of the received view, is not necessarily hostile, still less fatal to CURTIS's theory. Indeed, one might almost agree, from the very different fortunes of sounds in the north and south, that the probabilities are rather in favour of a difference than of a similarity. Much the same is true of Dr GERKEN's third argument from the occurrence of the change from *ei* to *ā*; and, moreover, the instances given do not seem to indicate a general law. His fourth objection would only be a serious one if we were certain that the other sounds riming with *ai* could not *possibly* have also come to the *ē* stage so early, a position in so difficult a question rather dangerous to assert. Nor does his fifth objection, somewhat loosely stated as it is, appear very formidable, especially when we remember that poets are often scrupulous that their rimes should appear correct to the eye as well as to the ear, and also that the stringency with which purity of riming is maintained differs very much from time to time, and also with the individuality of the writer. To say the least of it, one would have to know exactly who the poets referred to in the fifteenth and sixteenth centuries respectively were, and to what extent the one employed rimes which the other did not, before a serious argument could be grounded on these lines. His sixth objection also cannot be called very weighty, being of what one may call a negative character, as there may be some other reason than pronunciation that withheld the Scottish scribes from conforming to the English fashion of spelling. Still we must give Dr GERKEN all credit for having marshalled a number of ingenious objections to Dr CURTIS's theory, objections which it is to be hoped he, Dr CURTIS, will himself find occasion to deal with a fulness not possible in the present instance.

It must also be borne in mind that to all theories in all subjects, not capable of exact demonstration, there will always be many possible, often seemingly formidable, objections. It is here a question between two rival theories, for both of which there is much to be said, and against which much can also be said. There seems to me more fundamental objections to the elder theory than to Dr CURTIS's, and therefore, so far as I can judge, Dr CURTIS's theory "holds the field," just as the Darwinian theory and the Wave-theory of light hold the field in their respective sciences.

E:

= WS., *ie.*, after palatal.

§ 33. 1. Before ld rimes with

a) itself.
scheld : feld, 134. 7. 195, 114.
feild : scheld, 195, 115.
weld : feld : scheld, 207, 19.

b) OE y.
scheld : beild, 294, 61.

c) OE. ēo.
field : beheld, 98, 127.

d) OF. e.
feild : peiled, 276, 37.

The vowel here seems already lengthened.

§ 34. 2. Before n and n + cons. rimes with

a) OE. or ON. e.
end : wend : kend : spend, 75, 44. etc., strenth : lenth, 377, 65.
henis : menis, 171, 284. schrenkit : blenkit. 136, 28.
went : bent : schent, 107, 145.
hen : pen : men, 171, 284.

b) OE. ē shortened.
ken : ten, 75, 10.
den : ten : 36, 66.
ten : fen, 166, 220.
„ : men, 165, 218.

c) OE. eo.
? gend (ȝeond) : kend : end : wend, 70, 1.

d) OF. or Lat. e.
defend : end, 75, 43.
pen : den, 36, 69. men : pen, 165, 218. 171, 282. fen, 166, 221.
defend : kend, 75, 43. and 317, 42.
amend : send, 139, 40. and 317, 44.
„ : wend, 317, 44.
„ : spend, 317, 43.
wrennis : pennis, 98, 121.
schent : went : instrument : omnipotent : bent, 107, 145.
bendit : descendit, 150, 6.
blent (blinded) : omnipotent, 294, 75.
„ (blended) : hardiment, 135, 19.
content : went, 130, 57.
„ : schent, 334, 8,
„ : lent, 334, 28, and many more.

e) OE. ae.
ken : quhen, 75, 10.
men : „ , 75, 7.

f) OE. a.
wrennis : crennis (OE. cran, NE. crane), 98, 113.

§ 35. Before other consonants, rimes with

a) itself.
webbis : ebbis, 135, 16. derne, terne, 369, 7.
bellis : dwellis, 89, 44. 196, 127.
gekkis : neckis, 128, 17. dwell : well, 70, 8.
sell, dwell, 352, 25. „ : hell, 386, 4. and many such and all words in -nes.
fed : bed : wed, 77, 13.
rest : nest : 86, 42. best : rest : 113, 2. 235, 10.
mirthfullest : nest : rest, 101, 3. nett : gett, 257, 17. sett : forget, 357, 63.

b) with OE. u dialectically e.
ferry : Canterberry, 241, 38.

c) OE. y or i.
bred : fed : gled, 177, 365. dwell : kell (NE. kiln), 70, 4. dwell : well : kell (OE. cyln), 70. 2.
sped : bled : gled, 224, 78. sark : clerk, 176, 351, 315, 19.
clerk : merk (NE. darkness), 176, 349. 315, 19.
rest : list, 326, 1.

d) with OE. ēo and ON. ja.
hell : befell, 138, 106. dwell : fell, 42, 50. 43, 64. fell (ON. fjall), 70, 2.
tell : fell, 356, 37. fell : well, 372, 5.

e) OF. and Lat. e.
bellis : excellis, 89, 45. dwell : Gabriell, 43, 74.
dwellis : „ , 89, 45, etc.
„ : mellis, 208, 6. and many more.
dress : heaviness, etc., 111, 226.
distress : merriness, 41, 5. 196, 50.
incres : lustiness, 345, 40.
drest : rest : nest, 86, 42.
„ : west : 136, 37.
weir (NE. war) : beir (v), 138, 96. forsweir, 138, 90.
molest : rest : nest, 86, 42.
revest : rest : mirthfullest : ness : 86, 42.
best : test : 147, 7. and many more.
nett : gett : dett, 257, 16.
sett : pet : 179, 375.
nek : effek, 82, 32, and suspect, correct, etc.
derne : superne, etc., numerous similar words, 369, 3.
tern : „ „ „ „ 369, 7.

f) OF. a.
purchass : largeness, etc., 349, 82.
dwellis : bellis : kellis (Fr. cale = head-dress), 89, 42.

g) OE. ea.
clarkis : sparkis, 315, 18. 102, 21.

h) OE. awe, contracted to a before rk.
clarkis : larkis, 102, 25.

i) ON. ö.
clarkis : barkis, 102, 27.

k) ON. a.
rest : kest, 378, 108

l) OE. ēa.
mercyless : gentilnes, 117, 1.

m) OE. ǣ.
best : lest, 336, 13, and 28.
bedding : spredding, 175, 334.

n) OE. æ̆.
neck : blek : fek : sek, 315, 82.
„ „ 82, 34.
sted : hed, 126, 11.
feddir : eddir, 283, 8.

o) ON. ja, e.
well : fell (mountain), 70, 2.

§ 36. The short vowel *e* in closed syllables exhibits little or no change, and when it rimes with an originally long vowel, it points to a shortening of the long vowel, as in the termination *-less*, and not to a lengthening of the short one. Peculiar to the North of

England and South of Scotland is the pronunciation of *erk* = *ark*, which survives in the modern pronunciation of *clerk* = *clark*. But *er* before other consonants has not in Scotland the tendency, as in some parts of England, to become *ar*. What seems most probable is that *clerk* then, as now, and as was the case with so many words in Dunbar's time, had two pronunciations, of which the poet took advantage, just as in modern English poetry no one hesitates to avail oneself of the two pronunciations of *again* = *agen* or *agēn*.

§ 37. The rimes *nek* : *effek* : etc., instance a looseness of speech, a tendency to speak with as little trouble as possible, which is a correlative indication of that tendency to rapid change we have already noted in MSc. We have it exemplified in the loss of the consonantal *l*, and in the loss of *d* and *b* after *n* and *m* respectively, as in *hinner* for *hinder*, *wunner* for *wonder*, *lam* for *lamb*, etc. How far such a tendency can go is shown in the case of some Polynesian languages which have almost or entirely lost their consonantal sounds; and how far this might have gone in Scotland, but for the invention of printing and consequent spread of education, it is impossible to say. But the fact that these conservative factors were already in Dunbar's time beginning to work is to me a valid argument that we should not expect the same rate of change which had brought MSc. ahead of ME. to continue. And when we consider that not long after, in the time of Knox, education became more general and of better quality in Scotland than in England, and continued so, to a marked extent, up to the time of the adoption of the School-Board system in England, and indeed even later, if not up to the present day, we clearly have the reason why Scotch, which at one time ran ahead of English, should soon after the time of Dunbar begin to lag, and should, at the present day, be behind English in the sense of not yet having adopted such innovations of the present century as the lengthening of the *a* in *pass*, *grass*, etc. These factors, together with the retention of the guttural spirant in Scotch, have tended to keep *h* in its original place, while in nearly every other dialect of English it became and remains unstable, a condition it was tending towards in Dunbar's time, as witness such spelling as *hable*, *habound*, etc.

§ 38. We have here no instance of what Dr GERKEN finds in Douglas of the riming of *ĕnd* with *éond* (GERKEN, § 8, 1). In modern Scotch the word *friend* (*fréond*) is nearly always *friin*, which is BURNS' pronunciation when he writes it *as a Scotch word "frien,"* as he rimes it with *green*, *e'en*, *seen* (Epistle to John Lapraik, verse 1). When he writes it *friend*, he appears to use it as an English word, and rimes it with, for example, *end* (Epistle to R. Graham); so I think we must regard the pronunciation *frend* as an Anglicism.

$\bar{E}$, mutation of ō, rimes with

§ 39.

a) itself.
sweit : feit, 35, 9. 358, 7.
speid : meid, 352, 33.
quene : kene, 284, 17. 117, 11.
quene : wene, 352, 46.
„ : grene, 105, 76. 353, 88, etc., etc.
grene : kene, 107, 137. 110, 199.
„ : wene, 213, 62.

demiss : semiss, 308, 37.
b) Angl. ē = WS. ie later y.
meid : neid, 250, 2. speid : neid, 255, 39.
demiss : temiss, 308, 37. quene : schene, 92, 7, etc., etc.
demiss : temiss, 308, 37. kene : „ 107, 140. 110, 200.
flemit : ȝemit, 381, 39. grene : „ 103, 45, etc., etc.
c) OE. ēo.
breid (OE. brēdan) : leid (NE. lied praet. of OE leōgan), 169, 266.
quene : fyftene, 352, 28.
„ : betwene, 352, 34. 370, 47.
„ : besene : sene, 95, 43, etc., etc.
kene : sene, 97, 88. 107, 143.
„ : bene, 117, 15. 352, 11.
quhein : „ 316, 28.
quene : „ 92, 4. 105, 73.
grene : sene, 103, 43. etc., etc.
„ : bene, 105, 77.
quene : teyne, 307, 47.
beseik : meik, 42, 23.
breikis (s) : cheikis, 268. 23.
d) ON. œ.
flemit : temit, 381. 36.
e) Angl. ē = WS. æ and Grmc. ā. (see § 47).
g) OE. æ mut. of ā = Grmc. ai.
kene : mene, 117, 12.
breid : weid, 170, 271.
speid : meid : dreid, 352, 31.
„ : seid, 221, 24.
„ : weid, 221, 24.
„ : deid (deed), 245, 81.
quene : clene, 352, 16.
„ : mene, 353, 64. 370, 47.
wene : ene (evening), 70, 9.
shene : clene, 225, 106.
h) OE. ea + gutt.
quene : ene (eyes), 353, 58. 370, 39, etc.
grene : „ 110, 203.
kene : „ 110, 203. 367, 47.
i) OE. ĕ – see § 30.
speid : leid, 221, 16.
steid (place), meid, 250, 2.
beit : meit, 87, 65.
feill (NE. feel) : weill, 75, 25.
k) OE. ī.
(?) bedene : quene, 370, 41.
l) OE. ĕ : feill (OE. ê) : weill, 75, 25.
m) OF. e and ie.
breiding : posseding : exceding, 364, 122.
quene : sustene, 352, 28.
„ : contene, 352, 40.
„ : prevene : obtene, 353, 70, and 82.
„ : splene, 351, 6, etc., etc.
grene : „ 94, 12. 106, 106.
„ : serene, 106, 108.
„ : sustene, 110, 202. 376, 45.
„ : „ 294, 66.
quene : serene, 92, 11. and 370, 37.
„ : amene, 370, 37.
beit : discret, 87, 66.
flemit : redemit, 381, 36.
feir : maneir, 106, 95.
„ : chevallier, 105, 153.
breid : excede, 377, 73.
kene : sustene, 110, 202. 376, 45.
„ : serene, 292, 3.
„ : splene, 352, 12.
feir (company) : cheir, 106, 94. 108, 150.
„ : cleir, 106, 98. etc.
n) OF. ai.
beit : treit, 87, 64.
o) OF. i.
feir : pleseir, 105, 92.
p) OE. ea. speid : breid (NE. bread), 254, 36.*

§ 40. These rimes seem to confirm the view already stated that *ē* had already *in certain positions*, i.e., before *r* and *d*, and, probably, before *t*, *k*, *ll*, *v*, and *f* become *i̯* (or something very close to it), as we may judge from Dunbar's rhyming the three classes of sounds *α*, *β*, *γ* frequently together before these consonants (see § 29). But as already remarked, the absence of such rhymes before *n* and their rarity before *m* may give us just pause before ascribing to *ē* before *n* (and perhaps *m*) an *i* sound at this early period. One reason for this is the almost universal avoidance of the *ei* spelling, and observance of the *ene* form. This by itself might be accounted for on graphic grounds, but there is another reason for supposing that the spelling *ene* could not always mean an *i̯* sound, viz., the spelling and rimes of a word like *suffragene* = *suffragane* (NE. *suffragan*), which rimes with 'twane,' 100, 173, and with *meridiane*, *souerayne*, *trayne*, etc., 371, 68. Now, if our supposition that *āne* has already an *ē* sound be correct, in order to make *suffragene* a good rime for the other words, or a correct spelling of the word at all, we must suppose *ene* to indicate at anyrate an *e*

* I follow Professor SCHIPPER in taking *breid* to mean *bread*, but it may mean *breadth* (OE. brǣdu).

sound of some kind, and not an *i*-sound. It is not, indeed, at all necessary to suppose that *ene* and *ane* indicated identical sounds, for there is always, even in good modern verse, a tendency to a certain licence with such terminal syllables, which have only a secondary accent. Thus a modern poet will rime a word like *infinite* with *light*, or with *it*, and expect the reader to pronounce the word accordingly. But if *ene* already meant an *i*-sound, it could not be used as anything like equivalent to *ane*. My position then is this, that before *n*, and probably *me*, the OE. *ē* had, so to speak, lagged behind *e*+*r*, etc., and thus had still in all probability an *e*-sound, and that later, through the influence of the other words from OE. *e*-sound, it was brought into line with them. A confirmation of this view is to be found in the fact that Dunbar rimes words like *quene*, *grene*, etc., etc. with *sustene* and *contene*, from OF. *e*, which never acquire the *i* sound at all. My position is then to some extent intermediate between that of Dr CURTIS (§ 129) and Dr GERKEN (§ X. 1).

§ 41. We have 92, 15, a false rime between *beme* and *quene*, which shows that the passage is corrupt (see Introduction).

General OE. or ON. ē = WGrmc. ē, and ĕ, (lengthened).

§ 42. Not before g, rimes with

a) itself.
he : me, 82, 8. etc.
me : ȝe, 233, 33.
„ : we, 290, 100.
b) ē mut. of o. See § 39.
c) Ang. ē. = WS. ǣ (see § 47).
d) OE. eō.
ȝe : fle, 139, 15.
„ : se, 140, 23.
„ : hie, 139, 3.
„ : thre, 140, 27.
he : see, 125, 49.
„ : fre, 214, 73.
me : tre, 82, 13.
he : fe, 214, 77.
me : le, 82, 23.
„ : fre, 82, 28.
heir : steir, 34, 15.
„ : deir, 77, 20.
me : thre, 83, 63. etc., etc.
„ : se, 121, 21, etc., etc.
„ : fle, 233, 232. 87, 35.
she : se (see) : 97, 103.
the : kne, 97, 100.
e) Ang ē = WS. ēa + gutt (see § 49).
he : E (NE. eye), 214, 79.
f) Ang ē = WS. ǣ WGrmc. ā (see § 47).
g) ON. œ (see § 95).
ȝe : slie, ON. slœgr, 140, 31.
he : „ 204, 27.
me : „ 285, 8. 288, 40.
h) OF. e.
ȝe : dignitie : agilitie, 139, 7.
„ : supple : cumtre, 140, 39.
heir : prisoneir, 114, 16. 115, 64, and numerous others.
heir : cleir, 140, 38.
„ : cheir, 33, 3. 196, 131, and others.
me : cuntrie, 83, 48. 287, 55.
leir : frere, 80, 46.
seir : speir (sphere), 368, 12.
feir (OE. gefêr) : cheir, 97, 94.
j) ON. eyja.
de : we, 77, 7.
the : de, 295, 79.

Such rimes as ȝe : hie, ȝe : slie, dignitie, agilitie, me : countrie, seem also to point to an ī sound for final *e*, but not conclusively.

§ 43. 2) Before g, rimes with

a) =OE. ecg.
say : sway, 73, 2.

Angl. ē = WS. īe.

§ 44. Rimes with

a) itself.
heir (v) : deir, 99, 163. 107, 132.
steiris : heiris, 267, 4.
b) general OE. and W. Ger. ē.
steill : weill, 75, 25. 137, 78.
heir : deir, 77, 20.
c) OE. ē = mut. of ō (see § 39).
skeilis (NSc. skeel, OFris. skeel) : heillis, 176, 356.
d) OE. ēo.
skeilis : quheillis (NE. wheel), 176, 356.
e) OE. ăe.
ten : quhen, 75, 11.
f) Ang. ē = WS. ēa + gutt (see § 49).
g) OE. e.
ten : ken. 75, 10.
„ : men, 74, 7.
fen, 166, 220.
h) OE. y. steir (NE. stir) : deir, 119, 49. 137, 53.
i) OF. ei.
steir (steer) : heir, 34, 11.
k) OF. e and ie.
steir (v) (NE. steer) : cheir, 33, 3. 217, 11.
„ : perseveir, 33, 7.
„ : maneir, 217, 12.

It is evident that *ten* had already undergone shortening, perhaps from the frequent use of *tenfold.*

The word *steir* (NE. *stir*, OE. *styrian*) is in Dunbar's time already long, and that is another indication of the lengthening influence of the *Scotch r.*

Angl. ē = WS. ǣ WGrmc. ā.

§ 45. 1) followed by r, rimes with

a) itself.
ȝeir : sweir (OE. swǽr = heavy, 34, 19.
ȝeir : feir (NE. fear), 246, 105.
b) OE. ēa.
geir : eir, 251, 32.
c) OE. e. See § 33, etc.
d) General OE. or ON. and W.Gmc. ē (see § 42).
e) OE. ea, feir (NE. fear) : geir (OE. gearae), 254, 31.
f) Angl. ē = WS. īe.
ȝeir : steir, 34, 15. sweir : steir (v) (NE. steer), 34, 19.
ȝeir : heir (v), 231, 1.
g) OE. a.
thair : fair (OE. faran), 71, 19.
h) OE. ā.
war (were) : mair : sair, 256, 10.
hair : mair, 193, 40.
thair : mair : sair, 71, 15, etc.
„ : lair, 194, 76.
„ : evirmair, 221, 18.
i) ON. ǣ, ȝeir : feir (ON. faerr = sound), 244, 51.
k) OE. æ + g. See above § 3.
l) OF. e. ie.
ȝeir : perseveir, 33, 7. sweir : cheir : perseveir, 34, 19.
„ : presoneir, 113, 1. 114, 8.
„ : cleir, 231, 2.
„ : appeir, 244, 52.
m) OF. a.
thair : repair, 74, 2.
o) OF. ai.
hair : air, 106, 114, 193, 48.
thair : air, 96, 66.
p) OF. ei.
ȝeir : heir, 34, 11.

§ 46. Following Dr CURTIS, I have taken *thair* as derived from þǣr, but it seems more likely that, like *mair*, *sair*, etc., it represents an original *ā* in the form þār, as is well recorded in OE. I think, however, with Dr CURTIS, that *thair* has probably a double sound, viz. *ē* and *ī*, as I have often heard the latter sound in modern Scotch dialect, and my impression is that the two pronunciations may be heard in the same individual according to the position of the word. This second form, *thīr*, is another instance of the influence of *r* on the preceding vowel, an influence Dr GERKEN so persistently denies, but which must, I say, be very obvious to anyone who has heard much Scotch spoken. The word *aware*, for example, is often pronounced in Scotch *awīr*. This may also be influenced by the *w*, as I have pointed out, in words like '*swie*' (NE.

sway, ME. *sweye*) = *swing*. There seems, indeed, to be a difference between the English and Scotch *w*, as the latter has not the same influence on a following *a* as the English. It would thus seem as though the Scotch *w* lacked the vowel element, or had a different (lighter) vowel element from the English.*

§ 47. 2. not before r rimes with

a) itself.
weidis : meidis, 104, 55. deid (OE. ǣ = NE. deed) : dreid, 245, 82. 296, 13.
upspreidis : meidis, 104, 59. threidis : meidis, 104, 59. deid : weid, 221, 4.
sweit (NE. sweat) : bleit (bleat), quheit (OE. hwaete), 117, 330.
weid : dreid, 239, 11. reid : seid : weid (OE. wǣd), 221, 12. gredy : dredy, 372, 57.
red : cled (see NED.), 269, 43.
speiche : leiche (OE. laece), 308, 34. ever : never, 118, 4.

b) OE. ā̆e, mut. of ā (see § 50).

c) OE. e̊ā.
weid (OE. wā̆ed) : neid, 170, 268. leidis : heidis (NE. head), 79, 17.
dreid : neid, 239, 15. deid (NE. deed) : deid (NE. dead), 363, 103.

d) OE. ē, mut. of ō (see § 47).
meit (adj.) : feit, 149, 14.
speid : weid : reed : leid : ȝeid : seid, 221, 4. dreid : speid, 352, 31. meit (OE. gemǣte) : feit, 139, 13. sueit, 353, 75.

e) OE. e̊ō.
meidis (OE. æ̂) : reidis (OE. hre̊ōd) : weidis : upspreidis, 104, 55.
ever : levir, 138, 95. deid : forbid, 118, 12.

f) OE. e :
spredding : bedding, 175, 334.
leist : beste, 96, 71. speiche : streiche (OE. strec), 308, 31.
deill (ǣ) : weill, 75, 25. seill (OE. æ̂sl) : weill, 42, 61.

g) OE. ae :
agast : mast, 136, 27.

h) Fr. or Lat. e.
meter : discreter, 202, 9.
unhelit : conselit (NE. conceal), 355, 23. glemis : breimis (OF. bresme), 103, 32.
leid : remeid, 131, 8. reid (rædan) : pleid : remede : 75, 37.
ever : dissever, 118, 5.
never : „ 211, 22.
deid : remeid, 363, 101. posseid, 296, 15. proceid, 281, 41.
lene (NE. adj. = lean) : refrene, 73, 20.
leist : beist, 96, 71.
leve : escheve : greve, 244, 45.

i) Fr. a.
agast : trespast, 36, 31. past, 81, 1. 118, 18.
last : past, 43, 79.

k) OE. e – .
bleit : eit, 175, 330.
reid : steid, 75, 41. leidis : beidis (OE. (ge)bedu = NE. bead), 79, 17. deidis : steidis (OE. stede), 246, 93. reid (OE. rædan) : steidis, 75, 41.

l) Gen. OE., etc., ē.
deidis : meidis (OE. mêd), 250, 4.

m) OE. ā.
deid : manheid, 296, 12. womanheid, 118, 11.

n) OE. i.
evin (OE. æfen) : schrevin, 128, 7.

Angl. ē = WS. e̊ā.

§ 48. before gutturals, rimes with

a) OE. eo.
bene : ene (eyes), 353, 58.
ene : sene, 352, 52. 314, 8.
„ fyftene, 352, 34.
„ : between, 352, 34.
hie : be, 295, 95.
e (eye) : be, 171, 290.
„ : he, 214, 75. the, 395, 79.
„ : ble, 171, 293.
„ : fe, 214, 79.

b) OE. ē mut. of ō.
ene : grene, 93, 9. 110, 206.
eyne : quene, 370, 39, 43. ene : shene, 110, 203. 225, 107.
ene : kene, 110, 203.

c) Gen. OE. ē.
hie : ȝe, 139, 3. E : me, 83, 43.
„ : me, 84, 83. 289, 75.

d) O. Fr. or Lat. e.
E : degree, 96, 87. nativite, 294, 74.
hie : strenuite, 295, 94. E. ene ; splene, 94, 12.
„ : qualitie, 84, 88.

e) OE. ā̆e
ene : clene, 225, 106.

f) ON. ø̆y.
de : e, 171, 292. 295, 77.

* When one forms the lips into the position for pronouncing *w* (or *wh*), and then blows or whistles, one produces either an *u*-sound, as in the exclamation "*Whew!*" or an *i*-sound, as in whistling a high note, and this may be the difference between the English *w*, which gives from *a*, *au*, and the Scotch, which gives from *e* an *i*-sound.

§ 49. There is nothing in this or the foregoing lists of OE. *e* sounds to gainsay the contention that *e* was already, in certain positions, in MSc = *ī*, and the spellings *hie*, *qualitie*, etc., bear out this view.

$\overline{\text{AE}}$. (mut. of OE. ā. Grmc. ai).

§ 50. 1, not followed by r or w, rimes with

a) itself.
heill : deill (v), 37, 2.
sweit (sweat) : bleit : quheit, 175, 332.
b) Angl. ē = WS. ǣ, WGrmc. ā.
breid : weid, 315, 12.
reid : leid (OE. lǣden = Latin = learning), 221, 16.
c) OE. ēa.
glemis : stremis : bemis, 102, 28. levit : berevit, 288, 61.
d) OE. gen. ē, ene (ēfen) : wene (ē), 70, 9.
deill : feill (ē,) 75, 26.
e) OE. ē mut. of ō.
mene (v) (NE. mean) : kene, 117, 12.
meyne (v = moan, lament) : mene : schene : greyne, 122, 39. quene, 370, 47.
f) Angl. ē — WS. īe.
deill : steill, 75, 28.
g) OE. ēo.
mene (moan) : bene, 353, 64. meyne (= moan), 370, 47.
glemis : lemis, 102, 29. lene, bene, 73, 23.
mene : bene, 117, 15. mene : sene, 315, 13. 123, 36.
seis : theis, 194, 69. kneis, 194, 86.
see : thre, 194, 85.
f) OF. and Lat. e.
les : progress : posses : incres, 87, 51.
„ : dress, excess, 131, 93.
„ : express, 75, 45.
teiche : preiche, 239, 13. 240, 26.
„ : fleiche, 240, 26.
g) Fr. a and ea.
maist : haist, 356, 46.
deill : leill, 37, 4.
beill : „ 37, 3.
h) Fr. ei.
lene : fene, 73, 8. 245, 68.
„ : disdene, 73, 13.
„ : strene, 74, 28.
j) Fr. ai.
lene : complene, 73, 5.

§ 51. 2) before r, rimes with

a) OF. e and ie.
leiris : Cordilleris freiris, 80, 45.
feir : cheir, 246, 101.
b) OE. ea.
fear : geir, 254, 31.
c) Gen. OE. ē.
leir : heir, 325, 21.
d) OF. ei.
sweir : heir, 34, 11.

There is nothing in these lists but what confirms the conclusions of § 40.

§ 52. 3, before w, rimes with

a) OE. ēo + w.
slewth : rewth : trewth, 237, 16 (see Curtis, 350, Gerken, § IX. 1).

ON. Æ

§ 53. rimes with

a) OE. ea.
thrall : all, 359, 28.

ON. EI

§ 54. rimes with

a) OE. e + g (see § 32).
ay : say, 114, 20. away, 114, 21. play, 196, 146.
b) OE. œ + g (see § 32).
ay : lay, 114, 23. day, 212, 39.
c) OF. ei.
ay : affrey, 196, 145. they : pray, 109, 180.
d) OF. ai.
ay : assay, 349, 82. they : array, 109, 180.

EA

after sc. and g.

§ 55. 1, after sc, g, in open syllables not followed by g, rimes with

a) OE. a – schame : name, 85, 5. etc. etc. (see § 1).

§ 56. 2, before g, schaw (OE. sceaga), rimes with a + g schawis : lawis, 97, 104.

§ 57. WS. ea + r + cons. *α*) before palatal cons. rimes with

a) OE. e.
sparkis : clarkis : 102, 24. sark, 315, 19.
merkis : sparkis, 315, 18.
b) OE. eo.
sparkis, werkis, 315, 17.
c) ON. o.
sparkis : barkis, (s), 102, 24.
d) OE. – awe.
sparkis : larkis, 102, 25 (see § 35.)

§ 58. *β*) before other consonants does not occur.

WS. eal + cons.

§ 59. *α*) eal or eall – rimes with

a) itself. aw : waw, 44, 11. haw, 173, 309. gaw, 173, 311.
all : fall, 89, 20. 334, 9. all : sall, 281, 54.
„ : wall, 115, 59. 371, 73. fall : hall : 371, 83.
„ : hall, 371, 77.
b) ON. all. all : call, 281, 52.
c) OE. œl. all : small, 35, 18. 89, 46.
fall : „ 35, 20, 334, 27.
hall : „ 41, 13.
d) ON. æll. thrall : all, 359, 28.
e) OE. a + g, waw : draw, 166, 226.
f) Fr. or Lat. al. All : clericall : naturall : imperiall : „ royal : victoriall : orygnal, 88, 8, etc.
Fr. or Lat. al. All : celestiall : terrestrial : 112, 257.
„ stall : cardinall, 265, 50.
wall : cardinall, 361, 75. imperiall, etc. etc.
g) Fr. ail.
h) OE. $\overline{aw}$ and eaw.
raw : aw, 173, 308.
knaw : staw (stall), 276, 31.
waw : blaw, 166, 229.
k) ON. $\bar{o}$ll.
wall : ball, 371, 79.
hall : „ „ „

§ 60. *β*) WS. eal + d., rimes with

a) itself.
awld : bawld, 352, 9. cawld, 352, 7.
fawd : hawd, 209, 37.
b) OE. o. cawld : bawld : fold (OE. fōlde = ground), 96, 65.
c) OF. au. fawd (OE. feald = – fold) : hawd (healdan) : frawd, 209, 37.
d) ON. ald : tald : ȝald : scald : cald : bald : 275, 6. ȝald : stald, 276, 28.
e) OE. a + g. gnawd : tald, 277, 58.

§ 61. In modern Scotch the vocalisation of *ll* is complete, and it is quite possible that in Dunbar's time it was optional, as is shown by the inconsequence of the spelling. But the vocalisation of *l* in *ld* in NSc. is not complete, and seems to remain very much as in Dunbar's time, *i.e.*, it may be pronounced either way.

Dr GERKEN [§ 3, 2)] says that Douglas sounded the *l* before *t* as a consonant, and cites the rime salt (OE. sealt) : *exalt*. But this seems to me no proof, as *exalt* might also be pronounced then, as now, with vocalised *l*.

$\overline{EA}$

§ 62. = Grmc. au. not followed by w or gutt., rimes

a) with itself.
deid (dead) reid (red) 131, 84. heid, 243, 21.
reid : heid : 95, 50.
b) OE. e- see § 30.
deid (death) : steid (place), 379, 136. grete (great) : etc., 310, 21.
c) ON. ae. see § 53 a.
d) Angl. ē WS. $\overline{æ}$ see § 47.
e) OE. $\overline{æ}$ mut. of ā (see § 50).
f) OE. e : merciless : gentilness : lustiness, 117, 1.
g) Gen. OE. etc. ē. heir : neir (NE. near), 330, 35.
h) OF. e. deid : remeid, 131, 84. 363, 10. Est : est (Lat.) 367, 6.
reid : „ 131, 90. les : expres, 175, 45.
les : incres, 87, 50. 132, 96. progres : posses, 87, 55. dress, 131, 93. excess, 131, 96.
i) OF. ei. deid (NE. death) : feid (NE. feud), 363, 100.

§ 63. $\overline{EA}$. + w., rimes with

a) itself. shewis : schrewis : few is, 252, 51.

b) OE. ā + w. Schawin : knawin, 355, 13.

EO-

§ 64. rimes with

a) self. ellevin : hevin, 139, 1.
hevin : sevin, 43, 65. 71. 26, etc.
b) OE. e – hevin : uneven, 246, 96.
sevin : „ „ „
„ : stevyne, 371, 54.
c) Angl. ē WS. i and ȳ.
hevin : allevin (?), 240, 1.
sevin : „ (?), „ , 2.
d) ON. e :
sevin : nevyne, 371, 58.
e) OE. $\overline{ea}$ + g (see § 56).
fe : E, 214, 79.
f) OF. or Lat. u. schute : refute, 109, 181.
g) OF. ou.
schute : rebute, 109, 181.
h) OF. ui.
schute : pursuite, 109, 182.
j) ON. ō.
schute : rute, 109, 184.

§ 65. *Allevin*, 240,1 is derived by JAMIESON from OE. *ālēfan*, but there is the difficulty that it is a weak verb; but it is not impossible that it was formed on analogy. If once the form occurred, the fact that it is one of the very few rimes to *hevin*, *sevin* and *elevin*, would encourage the use of it, and the form might thus become fixed. A possible derivation from the OF. *allever*, which might give a similar sense of allow or admit, has somewhat the same objection; but perhaps a loan word admits more readily of false inflexion, if, indeed, it can be called false, than a native word. It might be a form of *ellevin* (eleven), but that it makes *absolutely no sense* in this passage.

EO :

§ 66. 1. before r + cons. rimes with

a) itself. carvit : starvit, 222, 21.
b) OE. ea. Angl. a (r + cons.) (see § 57).
c) OE. i or y.
werkis : merkis, 315, 19. dirk : kirk : mirk, 86, 15.
wirk : „ „ 86, 20.
d) OE. or ON. e
werkis : sarkis, 315, 19.
e) OF. or Lat. e
carvit : starvit : desarvit, 222, 21.
f) OF. or Lat. a.

§ 67. It seems more probable that *carvit*, *starvit*, *desarvit* were pronounced with *e* sound, as is usual in NSc., than that they had the North E. *a* sound.

§ 68. 2. not before r + con., rimes with

a) with itself, wilk : milk : silk, 86, 22.

b) with OE. i. milk : wilk : silk : ilk, 86, 22.

$\overline{EO}$

(1) = Gmc. eu. (2) contraction from ĕ. ĭ. ў., with following a or u through dropping of intermediate h or j.

§ 69. 1. not followed by w or g, rimes with

a) itself.
he : see : 124, 35, etc., etc. bee, 248, 8. kne, 261, 61.
„ : ble, 171, 293. kne : se, 97, 100, etc., etc.
threis : kneis, 173, 300.
fe : fre, 214, 73.
kne : thre, 194, 85.
b) OE. $\overline{eo}$ + gutt (see § 71).
be : hie, 295, 90. he : dre, 354, 2.
theis : kneis, 194, 70. fle : se, 196, 128.
c) general OE. or ON. ē (see § 42).
be : he, 251, 23. se : the, 97, 103.
kne : the, 97, 103. se : me, 124, 19, etc., etc.
fre : he, 214, 75. tre : me, 82, 13, etc., etc.
d) OE. ē, Angl. ē, mut of ō (see § 39).
e) Angl. ē WS. $\overline{æ}$ (see § 45).
f) OE. ae mut of ā (see § 50).
g) Angl. ē = WS. $\overline{æ}$ (see § 45).

i) OE. ā. See § 20.
k) OE. ēa + g. fe : E. (NE. eye), 214, 79.
l) OE. or ON. ē (see § 42).
m) ON. See § 96.
n) OE. īk or ON. ig.
sie (see) : bysselye, 124, 40.
o) OF. or Lat. e., or earlier ie.
r) be : petie, 119, 41. scurrilite, 154, 58.
,, : importunitie, 245, 76. 249, 23 and others.
se : gree, 123, 5. quantite, 196, 153.
OF. ieu. se : parde, 195, 120.
s) OF. i. be : supple, 250, 8.
tyred (OE. tēorian) : requyred : expyred, 251, 17.
tyre : inspyre, 266, 93.
t) OE. ǣ (see § 50).
seis : kneis : 194, 69, and 86.
u) ON. eyj.
de : be, 172, 295. 251, 33.
,, : se, 195, 128.
v) ON. œ (see § 95).

This list, taken along with some of the foregoing, seems to confirm the idea that MSc. final *e* was already an *i* sound.

§ 70.—EÓ. + W. = Grmc. iuj, iw.

a) with itself.
blew : new : hew, 94, 16. new : flew (praet.), 251, 37.
tnew : hew : rew, 117, 10. 345, 37. hew : grew, 135, 24 (see under uw, § 103).
b) OE. ū (see § 104).
c) OE. ǣw (see § 53).
d) OF. and Lat. u. trow : Jesu, 361, 59.
e) OF. ü, hew : vertew, 345, 34.
f) Fr. eu, ieu, hew : persew, 117, 6. 347, 12.
trew : rew : ,, 262, 82.
g) Fr. ive. new : eschew (OF. eschever, eschiver), 221, 9.
h) OE. ūll. ȝow : fow, 38, 18.

§ 70a.—EÓ. + g or h, rimes with

a) OE. ēo, without guttural.
dre (dreogan) : be, 354, 2. fe (feoh) : fre, 214, 73.
theis : kneis, 194, 70. hie (hēoh) : he, 295, 95.
b) OE. ǣ, seis : theis, 194, 69.
c) OE. and gen. ē, le (leōgan) : me, 82, 23. fe : he, 214, 75. hie : me, 84, 83. 289, 75. ȝe, 139, 3.
d) OF. e.
hie : strennite, 295, 94. le (leōgan) : gre, 204, 33.
e) OE. ea + g. fe : E (NE. eye), 214, 77.

I-

§ 71. not followed by c or g, rimes with

a) itself. schittin : wittin, 174, 323.
b) OF. or Lat. i, hidder : consider, 43, 84.
c) OE. e. beschittin, sittin, 137, 70.

Into the interesting controversy over the fortunes of OE. *i* and *y* in open syllables so elaborately treated by Dr Curtis (§ 361, etc.), and by Professor Luick (§ 381, etc., §§ 515 and 530), and carefully summed up by Dr Gerken, I do not propose to enter at any length, especially as Dunbar's meagre rime-list above throws no fresh light on the subject. As to the quality of the OE. ĭ-sound, is not Dr Curtis (following Ellis, EEP., p. 105) going rather far in asserting that this sound is *in all cases* not a pure *i*-sound, but partakes of the *e* quality, as in NE? Is it not more likely that the case in Anglo-Saxon was, as in modern German, that the sound varied according to the adjacent consonants? I found this remark on what I have observed in teaching German students English, viz., on a tendency they have to introduce the pure *i*-sound in certain words, but not in others. I have often had to correct them for introducing it into a word like *with*, but have never noticed it in words like *in*, *sin*, *wind*, etc. I have unfortunately made no notes on this point, but I think there is no doubt that this sound tends to vary according to the following consonant. In modern English we have practically no pure ĭ-sound, but in Scotch we have it in words like *wi* (with), *pity*, *city*, etc., though the last two words have often a long vowel. Better examples would be the forms *pitiful*,

citizen, which a Scotchman will usually pronounce with a pure ɤ-sound in the first syllable.

I:

§ 72. not followed by ld, nd, ng, ht, g or c, rimes with

a) itself.
is : miss, 75, 23. this, 75, 22.
his : iwis, 335, 32.
is : bliss, 42, 36.
is : his, 335, 32.
widdy : smydy, 223, 56.
will : still, 115, 45.
bit : spit, 137, 54. upsit, 137, 51.
it : wit, 283, 4. 249, 28.
billie : illwillie : quhillylillie, 39, 31.
chitterlilling : rilling : schilling, 178, 371.
win : skin, 36, 60. 233, 21.
„ : within, 233, 22.
„ : ryn, 310, 9.
„ : pin, 317, 34.
drink : wink, 211, 16.
innys : schinnis, 202, 13.
begin : in, 43, 89.
drink : sink, 279, 13.
grippis : lippis, 207, 18.
schippis : lippis, 206, 3.
kirk : stirk, 261, 66.
will : thrill, 325, 12.
b) OE. y (later i or ii) (see § 105, §§ 107 and 109).
c) OE. e. (?) flint : hint (NE. hent), 224, 80.
unblist : brist (NE. burst), 138, 99.
d) ON. ȳ (shortened) (see § 111).
e) Fr. or Latin e.
win : gin, 317, 34.
stink : ink, 150, 10.
f) Fr. or Lat. i.
grippis : ecclippis, 151, 14.
mist : solist, 75, 27.
clippis : lippis, 193, 55. 206, 5.
unblist : resist, 138, 99.
bit : spit : quit, 137, 54.
wit : it : unquit, 249, 41.
g) Fr. or Lat. u.
widdy (NE. halter) : smydy : study, 223, 48.
will : still : bill (Lat. Bulla), 115, 42.
h) OE. eo (see § 66).
i) ON. i.
still : will : till, 115, 44.
will : ill, 239, 17. 325, 13.
will : till, „ „
still : ill, 325, 15.
k) Du. i.
stink : clink, 151, 16.
skippis : lippis, 206, 8.

There is not much calling for remark in this rime-list. That there is a tendency, however, as shown still in NSc. for ĕ and ĭ to be assimilated or interchanged, cannot be doubted, and before n + consonant the *i* prevails, especially before *ng* and *nk*, as in NE. in the words *English*, *engine*, etc., and in *ink*. It seems most probable that in Dunbar's time the pronunciation varied, as in modern Scotch dialect. The Irish dialect of to-day leans strongly to the *i*, as in *stringth* for *strength*. *Study* had probably a short *ü* sound = NSc. *ui* as in *guid*, a sound, when quickly spoken, not easily distinguished from ɤ. The words *bit* and *grip* I have classed according to the Anglian forms *bit* and *grip*, and not the WS. *bite* and *gripe*. We have in NE. the corresponding forms *bit*, *grip*, *bite*, *gripe*. (See *Archiv.*, cii. 73.)

§ 73. 2. I : before nd rimes with

a) itself.
behind : fynd : bind : blind, 87, 72.
behind : lind, 174, 324. wind, 194, 67.
„ : „ : bind : find, 192, 12.

§ 74. It is remarkable how sharply Dunbar discriminates between *i* + *nd* and *y* + *nd*. The reason seems to be that *y* + *nd* had been lengthened to a diphthong, since it rimes with *ȳ* + *nd*; and *i* + *nd* was not generally lengthened as in ME. The lengthening of *behind* for Scotch *ahint*, the only word in the list that has a long sound in NSc., must have taken place later through English influence (see under Y.). This view is accepted and, I think, rendered still more probable, in spite of all that is said on the other side by Heuser (Anglia, XIX. 404) and by Dr Gerken [§ 13, 2)]. (See also § 108.)

§ 75. 3. i+c rimes with

a) ON. ȳ.
I : sky, 192, 15.
b) OF. i or ie.
I : cry, 118, 7. 192, 27.
I : bellamy, 192, 26.
I : Lombardy, 195, 102.
c) OF. e.
I : army, 195, 101.
d) OE. or ON. ī.
I : by, 279, 24.
purspyk : lyk, 177, 366. 307, 12. tyk (ON. tik), 177, 366.
e) OE. ȳ+g.
I : dry, 96, 70.
f) ON. i+k.
flicker : sicker : wicker : bickarr, 212, 41.

§ 76. 4. i+g rimes with

a) itself.
dughtý : worthý, 292, 20. wry (?) 172, 303.
b) OF. i and ie.
dughtý : worthý : cry, 292, 20.
style (stile) : vyll, 86, 38.
purspyk : heretyk : lunatyk, 179, 375.
nyne : devyne, 42, 31.
c) OE. ē = WS īe.
worthy : nye, 90, 52.
d) ON. i+g.
worthy : prudently, 90, 50.
e) OE. i.
styll (NE. stile) : wyll, 86, 38.

§ 77. i+ld. rimes with

a) OE. ȳ.
wyld : fyld, 167, 237
b) OF. i.
begyled : wyld, 167, 237.
c) Gk. υ.
styled : wyld, 167, 237.

It is difficult to say whether the long *i* and *y* in MSc. represented a diphthongal sound at this time, or merely a long *i* sound. But the occasional use of such rimes as *styll*, which is now and was, probably, on account of the absorbed *g*, also then long, with the undoubted short *wyll* (unless *wyll* be also long, being NE. *wile*, *device*, and not NE. *will*), might incline us to the idea that the sound was not yet diphthongal, because in that case the rime would be a bad one, and not merely an imperfect one, like in quality, but differing in quantity. Before *ld* we have the usual lengthening of the vowel.

§ 78. ON. i+g. (terminations ly and lie) rimes with

a) OE. ig. (see 76).
womanly : ly, 243, 29.
b) OE. ī.
hairtfully : by, 72, 36.
lustily : „ 104, 53.
wretchitly : „ 251, 21.
c) OE. ī+c.
I : womanly, 279, 23.
d) OE. ēo. (see 69).
besalie : mischeifaislie : see, 118, 17.
e) OE. æ. (see § 21.)
f) OF. i and ie.
lustely : fantesy, 104, 49.
wretchitly : fy, 251, 24.
womanly : cry, 279, 27. malady : veraly : hestely, 261, 59.
h) OF. iue? or Spanish guia.
guye : prudently, 90, 53.
gy : womanly, 279, 11.
i) ON. i.
airly : harbry, 71, 14.
k) OE. ȳ+g.
airly : dry, 71, 16.
hairtfully : dry, 72, 37.
l) ON. ȳ.
lustely : sky, 104, 50.
m) OE. ī+g. wy (OE. wiga) : womanly, 279, 11.
n) OE. ȳ.
womanly : ky (NE. cattle), 279, 11.

§ 79. i+ng.

a) with itself *in all participles and verbal nouns*, which also rime with Fr. ign and ring : sing, 94, 30. 215, 105.
ring : thing, 215, 113. bring, 215, 111.
spring : nothing : inbring, 108, 161.
b) Fr. ign.
bening : sing : thing, 368, 28.
ring, 94, 33. sing, 94, 30, etc., etc.
c) O.Nhumb. i—WS. eo.
ȝing : spring, 99, 151. 108, 154.
ȝing : sing, 111, 245. thing, 370, 17. thring, 370, 17.
d) OE. e+ng.
ring : sprynge (dance), 215, 19.
e) OF. egn.
inbring : ryng (reign), 390, 19.

Words in ight : icht : yght : ycht.

§ 80. The vowel spelt i or ȳ has in these words various origins.

1. OE. ie, i, < eo = Grmc. i, ë : knight, right, bright, wight (person), sight, plight, fight.
2. OE. i < ea. (Northmb., æ). night : might (sb and vb).
3. OE. i = Lat. i. dight.
4. OE. eo (with later palatalisation) = Grmc. ī. light (adj. and adv.).
5. OE. ī. alight vb.
6. OE. ē : Ang. ē mut. of ēo ēa : light (vb.), hight.
7. OE. ēo (with later palatisation) = Grmc. iu : light, (sb.).
8. OE. y mut. of u : flight.
9. ONthmb, e in reduplic. pret. hight.
10. ON. ī. wight (adv. ON. vīgr).

§ 81. Dunbar rimes these sounds only among themselves, with one remarkable exception. In the poem "In Honour of the City of London" he rimes (89, 35) *knyght* and *white*, and this seems to me to point to the fact that in England, or at least in London, these words had already (1501) like sounds. That is to say, the guttural spirant has disappeared in England, but not in Scotland. But it is curious that the one exception given by Dr CURTIS as occurring in Clariodus, is *plicht* and *quhyte* (white). Yet *white* can have had no exceptional pronunciation, seeing how consistently and frequently it rimes with pure *īt* sounds.

§ 82. Present participles and verbal nouns in *ing* rime only among themselves with OE. words in *ing*, and with the Fr. *ign* adjectival endings as *benign* and *malign* (see § 79).

§ 83. Pres : part in *and* (see § 5).

OE. or ON. Ī. (including ī + g)

§ 84. rimes with

a) itself.
rydand : bydand, 218, 33.
abyde : besyde, 196, 159.
byd : besyde, 126, 29. 330, 37.
betyd : syd, 177, 365.
wyd : besyde, 330, 36.
glyde : syde, 137, 67.
blyth : wryth, 196, 121.
„ : swyth, 304, 9.
syne (siþþen) : myne, 214, 81.
beschitten : sitten, 137, 70.
bakbytting : flytting, 151, 22.
swyne : myne, 214, 85.
Ryne : wyne, 42, 53.
wyiss (adj.) : wpryse, 234, 11.
wyiss (sb.) syss, 106, 101.
syss : ryp, 106, 101.
gryss (NE. grice, ON. grīss) : unwyse, 319, 59.
dryfer : ryver : ȝadswyver, 178, 374.
smyle (NE. smile, deriv. uncertain) (?) : quhyle, 94, 36.
yle : (?) quhyle, 265, 61.
wyd : byd, 330, 36.
ryd : slyd : abyd, 125, 4.
stryd : syd, 194, 75.
Fyf (Fifeshire) : lyf : wyf, 197, 161.
lyfe : swyfe, 83, 67.
„ : wyf, 223, 43.
thryff : fyve : belyff, 246, 95.
smyle : (?) wyle : quhyle : myle, 110, 224.
glyde : wyde, 225, 112. besyde, 225, 116.
„ : tyde : ryd, 226, 128.
dryf : lyve, 385, 47.
fyve : schryve, 358, 9. 359, 18.
knyfe : lyfe, 83, 74.
lyfe : ryfe, 83, 66. 377, 91.
henwyfe : lyfe, 71, 24.
lyfe : schryve, 1359, 23.
„ : fyve, 359, 21.
ryfe : swyve, 83, 67.
friendlyk : tyk, 200, 13.
lyk : shryke : stryke, 225, 87.
ryne (NE. fountain, source) : schyne, 370, 12.

b) OE. i + g or c (see § 75, 76).

c) OE. or ON. ȳ (see § 112).
abyde : bryde, 196, 158.
besyd : bryde, 198, 157.
schyne : ryne, 370, 12.
wy (man) : sky, 219, 43.
myne : tyne, 82, 22.
wynis : tynis, 80, 31.

d) Fr. i, ie, or Lat. i.
abyde : provide, 44. 6.
ryce (NE. brushwood) : pryce : cuvatyce, etc., 310, 31.
belyve : discryve, 43, 71.

wrytting : indytting, 227, 15.
wynis : dynis, 80, 32.
gryce (see above) : nyce, 273, 65.
paradyis : wyiss (sb), 41, 3.
service : „ 255, 2.
wyiss (sb) : benefice : deviss, 106, 104. 304, 2.
syss (OE. si) : ryss : cryis, suppryis, 366, 2.
ryiss (v) : benefyiss, 280, 24.
„ : cryis : suppryis, 367, 5.
abydit : decydit : devydit, 383, 29.
syd : wyd : crucifyed, 377, 95.
henwyfe : stryfe (OF. estrif), 71, 25.
knyfe : lyfe : stryfe, 83, 72.
„ : stryfe, 129, 31.
wylis : ilis, 208, 17.
Syne (NE. since) : inclyne, 375, 6.
wyne : ingyne : propyne : fyne, 301, 63.
byd : homicyd, 222, 33.
„ : provyd, 266, 81.
depryf : dryf, 138, 91.
arryve : fyve, 385, 47.
lyve : wyve : laxatyve, 223, 41.
„ : stryfe, 83, 69. 71, 23.
myne : fyne, 82, 21.
schyne : devyne : regyne : rosyne : femynyne, 369, 2.
„ : Lucyne : matutyne : crystalyne : fine, 101, 1.
wyiss (adj.) : devyiss : suppryiss, 76, 67.
„ : survyiss : cryis, 249, 24.
„ : cuvatyce, 311, 40. ryss : dispyss (NE. despise), 269, 63. 280, 22.
ryne (ON. rīn) : devyne : regyne : rosyne : feminyne, 370, 12.
abydit : gydit, 383, 31.
wylis : begylis, 208, 16.
tyd : gyd, 326, 6. 327, 30.
gyis : wyiss (sb), 106, 103.
wyd : gyd, 252, 56.
syd : „ 327, 14.
ryd : „ 344, 13.
e) Fr. ei.
wyiss (adj.) : pryis (v), 76, 68.
f) OF. e
wyle : quhyle : myle : syle (OF. *celier*), 110, 217.
grysse : unwyse : dyce : pryce, 319, 56.
g) Gk. *v*.
wyle : style, 241, 41.

§ 85. There seems to be no reason for treating the endings *-ine*, *-ite*, *-ive*, *-ice*, etc., separately. Dunbar uses them sparingly, probably despising, good artist as he was, this cheap method of finding rhymes. But when he does use them, he seems to make no distinction between them and the other long $\bar{\imath}$ or $\bar{y}$ sounds.

With respect to the pronunciation of the $\bar{y}$ and $\bar{\imath}$ at this time, it is, as I have already said, difficult to determine it. But in this connection one may ask whether the diphthong was ever quite the same in South English and Scottish. In South NE., and in the American pronunciation, the diphthong is a distinct *ai*, the first element being like the *a* in *father*. In the NSc., the *a*-element is not so marked, and the diphthong is perhaps almost *ei*. In Irish brogue the first element has more the *o* quality and approximates to *oi*, which appears to have been the pronunciation of DRYDEN's time, from the riming of *join*, *shine*, etc., etc.

O-

§ 86. rimes with

a) itself.
befoir : thairfoir, 259, 4. forloir, 259, 2. score, 135, 14.
beforne : borne, 269, 46. „ : forschoir, 329, 8.
„ : forlorne, 382, 16.
schorne : „ 194, 82.
mainsworne : worne, 319, 64.
borne : forlorne, 381, 6.
worne : „ 382, 22.
b) OE. o :
mainssworne : corne, 319, 61.
forborne : horn, 94, 35.
toforrow : morrow, 100, 189.
forlorne : thorne, 383, 30.
c) OE. ū sowp : howp, 74, 27.
d) OE. ā (see § 18).
e) Lat. o.
before : decore : glore : restore : memore : aurore : implore : adore : indeflore : desore, 370, 49.
f) OF. a.
borne : worne : scorne, 176, 343.
mainsworne : „ 319, 64.
g) OF. o rose : inclois, 99, 156. suppois, 236, 22.

O:

§ 87. 1. before ld.

a) OE. ea. gold : behold : old : told, 89, 34.

§ 88. This rime, like knyght : white, is from the poem "In Honour of the City of London" (see above), and is clearly meant to be read in English fashion, as in the characteristic Scotch pronunciation, the *l* in *gold* is dropped and the word pronounced *ou* (gowd, goud), and is so written by BURNS in 16 out of 36 times he uses the noun or adjective.

§ 89. 2. Before r + cons.

a) OE. o.
morrow : borrow : sorrow, 73, 160.
„ : sorrow, 94, 29.
morn : thorne, 383, 38.
corse (= cross) : horse, 175, 337.
corne : morne, 319, 62.
corss (= corpse) : horss, 131, 80.

b) OF. and Lat. o.
forthir, 137, 55.
hornis : unicornis, 97, 109.
c) OF. a.
hornis : skornis, 97, 107.
horne : scorne, 176, 343.
corn : „ 319, 64.

§ 90. O + ht rimes with

a) itself.
For o + nk see § 9.

b) ō + ht (see § 93).

O before other consonants rimes in all cases with

a) itself, and with
OF. and Lat. o.
croce : force : voce : indoce, 379, 2.
fox : mokkis, 209, 45.
cot : note, 260, 16.
throt : „ 260, 19.
b) ON. o.
rockis : fox, 209, 47.
god : tod, 36, 41. 80, 37.
cok : dok, 179, 376.
god : od, 82, 39.
c) OE. ō.
oft : soft, 217, 8.
d) OE. eo.
throtť : schot, 130, 64.
e) OF. ou.
word : bourde, 202, 5. 245, 84.

The rimes *rokkis* : *mokkis* : *fox*, 209, 45, is a good proof, if one were required, of the silence of the *i* in plurals and in the third person singular in *is*, when it suits the poet's purpose. In the interior of his lines, Dunbar avails himself perpetually of this licence of either treating this as a distinct syllable or eliding the *i*, just as in modern English verse most poets make free with the termination *-ed*, and pronounce or elide the *e* as it suits them, with certain exceptions, of course.

$\overline{U}$-

§ 91. 1. not followed by ht or g rimes with

a) itself.
ado : to, 75, 36. 356, 51.
don : mone : sone : soon, 211, 10. 219, 47.
bruke : shuke : luke : tuke, 111, 228.
kuke : tuke : bruke, 223, 60.
uder : muder : tuder, 73, 14.
udder : fudder, 130, 61.
gud : vud (mad), 45, 16. 196, 142. etc.
„ : Hude : , 196, 141.
„ : rude, 197, 170.
dois : (?) russ (praise), 260, 37.
vud : hude, 196, 141.
woid (mad) : gud, 272, 48.
„ : blude : ganestude, 377, 60.
do : to, 73, 22. 340, 37.
sone (sun) : undone, 116, 89.
wouke : schuke : luke : buke, 114, 34.
tuke : nuke : ruke, 132, 111.
blude : gud, 121, 4.
„ : guid, 278, 10.
b) OE. ŏ.
toc : smowk (OE. smoca), 241, 43.
nuke : tuke : ruke : smoke, 133, 120.
c) OE. ŭ (see § 97).

d) OE. ū (see § 104).
e) Fr. i.
uther (OE. ōþer) : consydder, 356, 35.
f) Fr. oe, ue, etc.
scules : dulis (OFr. doel, duel, deul, duil), 316, 23.
g) Fr. or Lat. o.
sculis : fullis, 316, 24. bosum : ebriosum, 212, 33.
h) OF. or Lat. u, Fr. ü.
moune : Joun (NE. June), 86, 30. tone, 86, 29.
none : tone, 233, 19.
gud : blude : pulchritud, 121, 2.
gud : blude : pulchritud, 92, 5.
cuill : muill, 277, 61. stud : mansuetude, 94, 15.
russ (ON. hrōsa, NE. praise) : dois : refuss, 260, 36.
dukis : ruikis, 226, 119.
gud : conclude, 279, 22.
„ : selsitude, 121, 7.
j) OF. oi.
rois : rejoiss, 99, 158.
k) OF. oui.
sculis : trulis (?), 315, 22.

§ 92. The numerous rimes of ō with itself, the spelling with oi and u as though identical in sound, point to the change of ō to an *ü*-sound = NSc. *ui* in *guid*, *bluid*, &c. The spelling *oi* no doubt indicates a modification (*Umlaut*) of the vowel, *i.e.*, of the disrounded (*entrundet*) ō, which—not giving a pure *u*-sound, still converging towards it—may, without straining, be held to give us approximately an *ü*-sound. The whole history of the passage of the OE. ō into a sound which eventually coincided with Fr. ü, and is represented by the NSc. *ui*, is one of great interest and difficulty. Dr Curtis has treated it with great learning, and at great length, in his *Dissertation on Clariodus* (§ 459, etc., and §§ 468–482), and in the main his conclusions are to be accepted. But at once the most concise and the clearest treatment of the point seems to me to be that of Professor Luick (*Untersuch.*, § 123, etc.). Dr Gerken prefers the purely phonetic explanation of Professor Luick to Dr Curtis's suggestion of French influence, and no doubt it is the more satisfactory, but still it is necessarily of the nature of an hypothesis, while French influence was an historical fact, probably traceable in other phenomena of the kind, as I have suggested in the matter of the Scotch dropping of the *l*-sound. But, I ask, is there any reason to suppose the factors in this problem mutually exclusive? may they not *both* be valid? Professor Luick derives *smowk* from OE. *smuca* (Untersuch., § 469).

§ 93. 2. ō + ht rimes with
a) itself.
thocht : brocht, 115, 58. 248, 2.
thocht, 330, 5. 335, 4. ocht : nocht, 115, 58.
b) OE. ŏht.
thocht : wrocht, 138, 101. 248, 2.
„ : flocht (?), 335, 2.

After the very thorough manner in which Dr Curtis has treated the development of *o* + *ht* and *a* + *ht* in Scotland, there remains little to be said. But it is interesting to note that my table of rimes gives almost the same results as the list of Clariodus, in that only *ŏht*-sounds are rimed together. I obtain this result, as does Dr Curtis, by deriving *ocht* and *nocht* from the forms *ōwiht*, *nō-wiht*, and this, of course, goes to confirm Dr Curtis's position that *oht* developed no parasitical *u* in MSc. The vowel in MSc. seems to be short, otherwise it would surely have shown more tendency to get mixed up with words in *aucht* from *æ̆ht* and *aht*, for the difference in the quality is not very great. This difference persists in NSc. where words like *bocht*, *thocht*, etc., and *aucht* (eight), *straught* (straight), etc., are chiefly distinguished by longer quantity of

the latter group. *Taught*, again, in NSc. has usually the pronunciation *tāht*, as has, curiously enough, *dochter* = *dāhter*.

§ 94. 3. ō + g rimes with

a) OE. ū.
swownes : rownes, 45, 13.

ON. Œ

§ 95. rimes with

a) OE. gen. ē.
slie : he, 204, 27.
me, 285, 8. 288, 40. ȝe, 140, 31.
b) OE. ēō.
slie : be, 154, 63.
c) OF. e-.
slie : scurrilite, 154, 58.
„ : supple : cuntre, 140, 31, etc.
d) ea + ht.
slicht : micht : wicht : sicht, 122, 18.

ON. ΘY

§ 96. rimes with

a) OE. gen. ē. de : we, 77, 7.
the : de, 295, 79.
b) OF. ie and e. petie : de, 119, 28.
nativite : „ 294, 74.
c) OE. ēō. de : be, 171, 90.
de : ble, 171, 293.
d) OE. ēa + g. de : e (*ēage*), 171, 292. 295, 77.
e) Lat. e. de : me (Lat.), 285, 12.

U-

§ 97. 1) not followed by g, rimes with

a) itself.
spur : dure (NE. door), 281, 38.
thunnir : scunnir (OE. scunian), 136, 34.
b) OE. and ON. u : .
owk (NE. week) : bowk (NE. bulk, origin uncertain), 38, 25.
cum : dum, 249, 26. 136, 31.
c) OE. ō.
abone : sone, 239. 9.
lufe : behufe, 348, 56.
dure : fure, 282, 78.
spur : „ 280, 13.
duke (NE. duck) : luke, 257, 6. 272, 46.
dukis : cluikis : ruikis, 226, 119.
bruik : rebuik, cluik, 225, 86.
d) Fr. o, Lat. u = ME. ū.
abone : redoun, 243, 17.
e) Fr. ü or Lat. ū.
dure : assure : demure : future : obscure : azur, 218, 48.
abone : toun, 243, 20.
spur : pure : injure, etc., etc., 280, 13.
f) OE. ū. (see § 103, etc.).
g) Lat u.
cum : sum (Lat.), 249, 27.
„ : tuum „ 282, 84.
h) ?
somer : skomer (NE. micturate (of a dog)), 168, 251.

2) followed by g, rimes with

a) with OE. ū.
fowlis : owlis, 98, 22.
cowll : fowle (adj.), 268, 28.
cowll : oule, 208, 12.
b) Dan. ū (?)
fowlis : scowlis, 226, 121.

§ 98. It is clear that *ŭ* in open syllables was lengthened, and also that it already coalesced with *ō* in the same position. I have followed Dr Curtis in using the sign *ȳ* for the sound of the ME. *u*, which differs from the pure *u*-sound, as in German *gut*, etc.; but I cannot agree with his definition of the sound, as *mid-back-wide-round*, as it appears to me not to be a *back* sound, but *mixed*, i.e. *high-mixed-wide-round*, or even high-mixed *narrow-round*, that is to say, the U or *uh* of Prof. Luick's Table (Untersuch., § 21). This view seems to me to have the advantage of making the mixture of this sound with *o* (Luick *oh*), or their coincidence in an intermediate sound highly probable, as *oh* is *mid-mixed*, and thus immediately below either *u* or *uh* in the mouth-position. It is also consistent with the general tendency in English to throw forward

the mouth-position of sounds generally. I throw out this suggestion for what it may be worth, but into the general question of the history of the *u*-sound, as also the *i*-sound, as treated so fully by Prof. LUICK ("Untersuchungen," § 381, and following) and by Dr CURTIS (§ 361, etc.), I shall not at present enter further, as I could not hope to add anything to their results without an examination of a mass of material outside of Dunbar, which is impossible in the time at my disposal. Professor LUICK calls this sound *ü-artig*, that is, a kind of ü-sound, and I have merely suggested identifying it with the *u* or *uh* of his table (Untersuch., § 19), whose mouth-position lies immediately behind the regular *ü*-sound. As for *cŭm*, see *Arch.*, cii. 57.

§ 99. 1. U : +nd, u+nd, rimes with

a) itself.
under : hunder, 204, 23. sounder, 247, 13.
stound : dround : wound, 378, 103.
b) Fr. and Lat. o.
woundit : soundit, 380, 26.
dround : sound, 171, 286.
found : pound, 245, 75.

One is inclined here to agree with Dr GERKEN that Dr CURTIS's attempt to establish two classes of rimes here is not successful. The simplest explanation is, and that is borne out by the spelling *sounder*, that *under* and *hunder* have still an *u*-sound, if shorter in quantity than *dround*, *stound*, etc., which retain their *ū*-sound in NSc. *Pound*, again in NSc., is usually *pən*, and this shortening is probably connected with the loss of the final *d*, and consequent shortening before final *n*.

§ 100. 2. u+ng

a) with itself.
tung : sung, 325, 17. ȝoung, 357, 64.

§ 101. 3. +m or mb, rimes with

a) itself.
dum : sum, 249, 26.
b) with u– (see § 97).
c) OE. ū (see § 104).

§ 102. 4. before ll, rimes with

a) OE. $\overline{eo}$ + w.
fow : ȝow, 38, 17.
b) OE. ū + w.
fow : trow, 38, 20.
5. before other consonants.
a) with itself. pursis : cursis, 80, 39.
b) OF. ou.
pursis : cursis : tursis, 80, 38.
pursis : „ 231, 18.
c) ON. ū (see § 104).

$\overline{U}$- (OE. and ON.)

§ 103. 1.) followed by w, rimes with

a) OE. $\overline{eo}$ + w. trow : ȝow, 38, 17.

§ 104. 2. not followed by w, rimes with

a) itself.
hous : mowss, 36. 53. crowss (NSc. crouse), 36. 52.
rowmes (OE. rūm = rent, property) : thoumbes, 231, 6.
lowd : prowd : schroud, 274, 3.
oule : defowll, 177, 364.
fowle, 208, 8. 224, 73.
brown : toun, 310, 25.
sowp : cowp (NE. cask), 300, 45.
bouris : schowris, 102, 14.
b) OE. ul : growf (ON. agrūfu) : wowf (OE. wulf), 36, 57.
c) OE. ŭ + g (see § 97).
d) OE. ō + g (see § 94).
f) OE. ū :
ws : thuss, 42, 27.

sowk : owk, 38, 24.
g) Lat. ō Fr. ou.
us : marvelluss, 77, 15. 42, 27.
bouris : houris : cullouris : flouris, 102, 11.
boure : doure, 200, 1.
schouris : flouris : houris, 93, 2. 102, 14.
„ : cullouris, 102, 13.
„ : touris, 376, 7.
cowhuby (?) : ruby, 154, 62.
toun : doun, 294, 52.
sour : hour, 71, 28.
schow : Jow, 375, 9.
how : Jesu, 378, 102.
roumes : thoumbes : soumes (Lat. summa, Fr. somme), 231, 10. 311, 37.
roun : goun, 281, 36.
toun : „ 281, 38.
schouris : devouris, 367, 5.
crownis : bownis, 34, 17.
toun : soun, 42, 21. 192, 28.
„ : renoun, 85, 1. 192, 20.
„ : crowne, 191, 32. roun (round), 281, 37.
„ : reformatioun : regioun, etc., 85, 2.
h) ON. ū.
sowk : bowk, 38, 35.
toun : boun, 42, 44. 87, 59.
j) ON. au.
sowp : stowp, 74, 26.
k) ON. ō.
sour : clour (ON. klōr), 71, 30.

That the words in the foregoing list have still in NSc. the same sound, viz. *ū*, and have never, as in NE., had an *ow* sound, is another instance of the similarity in pronunciation between the Scotch of Dunbar's time and modern broad Scotch.

Y-

§ 105. 1. not before g, rimes with

a) OE. or ON. i.
king : thing, 245, 66. 368, 31. ring, 78, 29.
rimes also with participles and verbal subs. in *-ing*.
b) ONrthmb : i = WS. eo.
king : ȝing, 370, 19.
c) OF. or Lat. i –.
king : ding, 296, 8. 370, 15. making, 370, 17.
„ : syng. 370, 15 and 23. bening, 368, 28.
d) Angl. ē : WS. īe, steir (NE. stir) : deir, 119, 49. 137, 53.
e) OF. e and ie, steir (stir) : presoneir, 115, 80.
f) OF. ei. steir : heir, 378, 123.

§ 106. 2, before g.

a) OE. or ON. ȳ – (see § 112).
b) ON. lig. OE. lik.
by (buy) : hairtfully : humly, 368, 21.

§ 107. 1. Y : (OE. and ON.) before nd.

a) itself.
kynd : mynd, 153, 50. 239, 18.
b) OE. ȳ.
kynd : mynd : strynd, 154, 56.
kynd : strynd, 192, 29.
c) OF. or Lat. i.
kynd : mynd : dyned, 153, 50. 239, 18.
kynd : Ynd, 192, 31. 265, 66.

§ 108. In MSc. the history of the *y*-sound + nd is distinguished from that of ī in that y is lengthened, but *i* is not. In ME. *both* are lengthened. Dunbar, as we see, *never* rimes these two sounds together. In NSc. the *i* remains short in all the words that occur as rimes in Dunbar except *behind*, which is an English form of the Scotch *ahint* (see above, § 74, and GERKEN, § 13).

§ 109. 2. Y :—before other consonants, rimes with

a) itself.
fill : kill (kiln), 83, 59.
dynt : stynt, 224, 76.
myrtle : girtle, 194, 79.
(?) gorge-millaris : cartfillaris, 268, 25.
millaris : pudding-fillaris, 321, 79.
thin : kin, 254, 17.
sin : „ 316, 33.
mirry : wirry, 237, 24.
mynting : stynting, 150, 5.
b) OE. i : or ON. i : or ON. i shortened.
lift (heaven, sky) : rift, 193, 50.

fyll : ill : still, 126, 7.
kill : ill, 83, 56.
hill : averill (?), 173, 313.
will : thrill, 1325, 12.
din : skin : win, 36, 59.
thin : kin : win, 254, 17.
think : stink, 150, 10.
giddy : widdy : smydy, 223, 44.
thin : within : skin, 277, 56.
synnis : begynnis, 128, 19.
hippis : lippis, 193, 53, etc. etc.
list : unblist, 138, 99.
dynt : flynt, 224, 80.
stynt : „ 224, 80.
win : tin, 316, 33.
mirk : kirk, 86, 16.
kiss : is : miss, 75, 20.
„ : miss, 365, 145.
hippis : slippis, 124, 41.

c) OE. e :
dynt : stynt : hint, 224, 88.
mynting (OE. myntan) : stynting : hynting (OE. hentan), 150, 4.
list : brist, 138, 102.
d) OF. or Lat. é or ie.
list : resist, 138, 105.
by (buy) : mercy, 368, 22.
millaris : pudding-fillaris : gillaris : tutivillaris, 321, 76.
e) OF. or Lat. e.
sin : gin, 317, 34.
think : ynk, 150, 12.
mirry : wirry : chirry, 237, 21.
f) OE. y. (see § 112).
g) OE. eo.
lift : clift, 193, 49.
mirk : dirk (dark), 86, 15.
h) ON. io.
myrth : girth : firth, 194, 79.

§ 110. *Averill* is given by JAMIESON as a diminutive from *aver* = an old cart-horse, a meaning which fits *admirably* the passage in which it stands, but as *averill* in this sense does not occur elsewhere, it has been suggested that the word is *haverel* (from *haver*, to talk nonsense), and that it means here simply a *worthless, foolish* person. I incline, on literary grounds, to the first supposition, especially as Dunbar was quite capable of *coining* such a word for himself, as in the "Brash of Wooing" and "The Flyting," where words occur which are nowhere else to be found.

In the c) rimes we have again, as in the case of $i+nt$, rimes with $e+nt$ (see § 72), but in neither case are they quite conclusive.

ON. Y :

§ 111. rimes with

a) with ON. ī :
lift : thrift, 219, 48.

$\overline{\text{Y}}$

§ 112. (OE. and ON.) rimes with

a) itself.
hyd : bryd, 173, 317.
myce : lyce, 310, 23.
b) OE. or ON. ī.
ky : by, 278, 3.
hyde (v), glyde, 226, 128.
pryd : wyd : besyd, 82, 11.
lyte : wryt : quhyte, 105, 4.
pryd : syd, 128, 22.
„ : betyd, 177, 362.
skyiss : wyiss, 106, 107.
hyd : glyde, 136, 44.
„ : syd, 174, 319. 377, 57.
„ : byd, 222, 34.
myce : lyce : ryce, 310, 23.
myce : wyice, 311, 39.
lyce : unwyse, 319, 59. gryss (young pig, ON.), 319, 57.
dry : by, 72, 38. harbry, 71, 14.
kyth : blyth : wryth, 196, 121.
blyth : kyth : swyth, 304, 7.
hiddill : riddill (NE. sieve), 213, 53.
c) OE. ĭ and i + g.
fyld : wyld, 166, 236.
fyll (NE. defile) : styll (NE. stile) : wyll, 86, 37.
d) Fr. i. or ie.
hyd : homecyd, 222, 35. gyd, 344, 12.
myce : lyce : myce, 311, 35. gyd : pryd, 328, 38.
lyce : pryce, 319, 58.
firit : atyrit : desyrit : inspirit, 122, 31.
fyre : impyre : desyr, 219, 36.
e) OE. y + g.
sky : dry, 96, 70.
f) OE. i + c.
sky : I, 192. 13. dry : I, 174, 326.

i) Fr. ui. gyiss : skiyss, 128, 11.
fyld : begyld, 167, 236.
k) OE. lik : ON. lig.
dry : airly : prevely : hairtfully, 72, 38.

l) OF. e.
lyce : dyce, 319, 56.
m) Gk. ῡ.
fyld : styled, 167, 239.

§ 113. In the rimes *fyld* : *wyld*, 166, 236, we have a long vowel, owing to the lengthening of *ī* in *wilde* before *ld*. With regard to the pronunciation of *ȳ* at this time, see under *ī*. That the two sounds were regarded as identical is amply proved in this rime-list.

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LIST OF ABBREVIATIONS.

OE.	= Old English (Anglo-Saxon).	*AF.*	= Anglo-French.
OF.	= Old French.	*NED.*	= New English Dictionary.
Lat.	= Latin.	*WS.*	= West Saxon.
PG.	= Paul's Grundriss.	*Angl.*	= Anglian.
NE.	= New English.	*Northmb.*	= Northumbrian.
OFris.	= Old Frisian.	*Gen.*	= General.
MSc.	= Middle Scottish.	*Gk.*	= Greek.
ON.	= Old Norse.	*Gr.*	= Grammar.
ME.	= Middle English.	*Prt.*	= Preterite.
Du.	= Dutch.	*Prn.*	= Pronoun.
NSc.	= New Scottish.	*S.* or *Sb.*	= Substantive.
Fr.	= French.	*V.* or *Vb.*	= Verb.
Icel.	= Icelandic.	*Adj.*	= Adjective.
Dan.	= Danish.	*Adv.*	= Adverb.
Gen.	= Genitive.	*EEP.*	= Early English Pronunciation, Ellis.
Ger.	= Modern German.	*Cons.*	= Consonant.
HES.	= History of English Sounds (Sweet).	*Gutt.*	= Guttural.
Swed.	= Swedish.	*Palat.*	= Palatal.
Grmc.	= Germanic.	*Mut.*	= Mutation.

The sign > signifies that the sound preceding it is changed into the one following it, as ai > ā = the diphthong ai passes into an ā sound. An accent over a capital letter means the vowel is long, thus: Á means *ā*. A stroke - after the letter = in open syllables; a colon (:) = in closed syllables. A colon between words in the lists means they rime together. In the figures the first number gives the page and the second the line of Professor Schipper's edition of Dunbar.

XXVI.—*On the Eliminant of a Set of General Ternary Quadrics.*

By Thomas Muir, LL.D.

(Read June 19, 1899.)

(1) The process of dialytic elimination was first applied to a set of general ternary quadrics by Sylvester in 1841,* his method being to form from the three quadrics u_1, u_2, u_3 ten cubics, viz. :—

$$xu_1, yu_1, zu_1\,;\; xu_2, yu_2, zu_2\,;\; xu_3, yu_3, zu_3\,;\; \mathrm{J}(u_1,u_2,u_3),$$

and then eliminate dialytically the ten quantities

$$x^3, y^3, z^3\,;\; x^2y, y^2z, z^2x\,;\; xy^2, yz^2, zx^2\,;\; xyz.$$

The actual work of obtaining the eliminant as a determinant of the 10th order he did not perform; and, indeed, with the notation which he used the labour would have been very irksome.

(2) In the same paper he also dealt with three simple special cases of the problem, but proceeded in a totally different way, the eliminant in each case being obtained as a determinant of the 6th order. Here his aim was to obtain from the three quadrics other three, and then eliminate dialytically the six quantities

$$x^2, y^2, z^2\,;\; yz, zx, xy.$$

The mode of obtaining the three subsidiary quadrics varied in each case, and the possibility of finding such a triad in *every* case was not discussed.

(3) Eighteen years afterwards, when the first edition of Salmon's *Modern Higher Algebra* came to be published, Sylvester's general method was not given, but another was announced in the following words†:—

"We can now express as a determinant the eliminant of three equations, each of the second degree. For their Jacobian is of the third degree, and therefore its differentials are of the second. We have thus three new equations of the second degree, which will be also satisfied by any system of values common to the given equations. From the six equations, then, u, v, w, $\partial\mathrm{J}/\partial x$, $\partial\mathrm{J}/\partial y$, $\partial\mathrm{J}/\partial z$, we can eliminate the six quantities x^2, y^2, z^2, yz, zx, xy, and so form the determinant required."

* Sylvester, J. J., "Examples of the Dialytic Method of Elimination as applied to Ternary Systems of Equations," *Cambridge Math. Journ.*, ii. pp. 232–236.

† Salmon, G., *Lessons introductory to the Modern Higher Algebra* (1859), p. 38, § 55. [The paragraph appears unaltered in all the subsequent editions.]

(4) At first sight this might appear to be a generalisation of SYLVESTER's three special cases, but a little closer inspection suffices to show that the subsidiary quadrics obtained by differentiating the Jacobian are quite different in form from those to which SYLVESTER was led. Thus, to take the simplest of the special cases, when

$$\left.\begin{aligned} u_1 &= Ay^2 - 2C'xy + Bx^2 \\ u_2 &= Bz^2 - 2A'yz + Cy^2 \\ u_3 &= Cx^2 - 2B'zx + Az^2 \end{aligned}\right\}$$

we readily find

$$J = (Bx - C'y)(Cy - A'z)(Az - B'x) + (Bz - A'y)(Cx - B'z)(Ay - C'x),$$

and

$$\begin{aligned} \frac{\partial J}{\partial x} &= C(C'B' - AA')y^2 + B(B'C' - AA')z^2 \\ &\quad + 2(ABC - A'B'C')yz + 2B(A'B' - CC')zx + 2C(C'A' - BB')xy, \\ \frac{\partial J}{\partial y} &= A(A'C' - BB')z^2 + \ldots \\ \frac{\partial J}{\partial z} &= B(B'A' - CC')x^2 + \ldots \end{aligned}$$

Now instead of these lengthy and complicated subsidiary quadrics those which SYLVESTER used were

$$A'x^2 + Ayz - B'xy - C'zx,$$

and two others like it.*

An equally marked discrepancy is observed when the like examination is made of the two other special cases, the advantage in all three being on SYLVESTER's side. An inquiry is thus suggested as to whether there be not a general method, simpler than that given by SALMON, and leading in the special cases mentioned to SYLVESTER's results.

(5) The probability of this question being answerable in the affirmative seemed to be increased by a consideration of the results obtained in a recent investigation† of the first two cases; for it was then found that not only was there a considerable variety of suitable subsidiary quadrics available, but that in the second case those used by SYLVESTER, comparatively simple though they were, were far from being the simplest possible.

The main object of the present paper is to settle the point thus raised.

* The connection between $\partial J/\partial x$ and the corresponding quadric used by SYLVESTER is

$$\frac{\partial J}{\partial x} + CA'u_1 - B'C'u_2 + A'Bu_3 = 2BC\{A'x^2 + Ayz - B'xy - C'zx\}.$$

† MUIR, T., "A Problem of Sylvester's in Elimination," *Proc. Roy. Soc. Edin.*, xx. pp. 300–305; CAYLEY, A., "Note on Dr Muir's paper, 'A Problem, etc.,'" *ibid.*, xx. pp. 306–308; MUIR, T., "Further Note on 'A Problem, etc.,'" *ibid.*, xx. pp. 371–382; MUIR, T., "On the Eliminant of a Set of Ternary Quadrics," *ibid.*, xxi. pp. 220–234.

(6) The first step necessary for the purpose in view is to go through the work of the two methods—SYLVESTER's and SALMON's—as applied to a triad of perfectly general ternary quadrics, so as to have the resulting eliminants actually before us.

Let then

$$u_1 = \begin{vmatrix} x & y & z \\ a_1 & h_1 & g_1 \\ h_1 & b_1 & f_1 \\ g_1 & f_1 & c_1 \end{vmatrix} \begin{matrix} \\ x \\ y \\ z \end{matrix}, \qquad u_2 = \begin{vmatrix} x & y & z \\ a_2 & h_2 & g_2 \\ h_2 & b_2 & f_2 \\ g_2 & f_2 & c_2 \end{vmatrix} \begin{matrix} \\ x \\ y \\ z \end{matrix}, \qquad u_3 = \begin{vmatrix} x & y & z \\ a_3 & h_3 & g_3 \\ h_3 & b_3 & f_3 \\ g_3 & f_3 & c_3 \end{vmatrix} \begin{matrix} \\ x \\ y \\ z \end{matrix},$$

so that

$$\mathrm{J}(u_1, u_2, u_3) = \begin{vmatrix} a_1x+h_1y+g_1z & h_1x+b_1y+f_1z & g_1x+f_1y+c_1z \\ a_2x+h_2y+g_2z & h_2x+b_2y+f_2z & g_2x+f_2y+c_2z \\ a_3x+h_3y+g_3z & h_3x+b_3y+f_3z & g_3x+f_3y+c_3z \end{vmatrix},$$

$$\begin{aligned} = {} & |a_1h_2g_3|x^3 + |h_1b_2f_3|y^3 + |g_1f_2c_3|z^3 \\ & + \{|a_1h_2f_3|+|a_1b_2g_3|\}x^2y + \{|h_1b_2g_3|+|a_1b_2f_3|\}xy^2 \\ & + \{|h_1b_2c_3|+|g_1b_2f_3|\}y^2z + \{|g_1b_2c_3|+|h_1f_2c_3|\}yz^2 \\ & + \{|a_1f_2c_3|+|g_1h_2c_3|\}z^2x + \{|a_1h_2c_3|+|a_1f_2g_3|\}zx^2 \\ & + \{|a_1b_2c_3|+2|f_1g_2h_3|\}xyz, \end{aligned}$$

and

$$\begin{aligned} \frac{\partial \mathrm{J}}{\partial x} = {} & 3|a_1h_2g_3|\cdot x^2 + \{|a_1b_2f_3|+|h_1b_2g_3|\}y^2 + \{|a_1f_2c_3|+|g_1h_2c_3|\}z^2 \\ & + \{|a_1b_2c_3|+2|f_1g_2h_3|\}\cdot yz + 2\{|a_1f_2g_3|+|a_1h_2c_3|\}\cdot zx \\ & + 2\{|a_1b_2g_3|+|a_1h_2f_3|\}\cdot xy. \end{aligned}$$

Following SYLVESTER's method we thus have the eliminant in the form

$$\begin{vmatrix} a_1 & . & . & 2h_1 & . & c_1 & b_1 & . & 2g_1 & 2f_1 \\ . & b_1 & . & a_1 & 2f_1 & . & 2h_1 & c_1 & . & 2g_1 \\ . & . & c_1 & . & b_1 & 2g_1 & . & 2f_1 & a_1 & 2h_1 \\ a_2 & . & . & 2h_2 & . & c_2 & b_2 & . & 2g_2 & 2f_2 \\ . & b_2 & . & a_2 & 2f_2 & . & 2h_2 & c_2 & . & 2g_2 \\ . & . & c_2 & . & b_2 & 2g_2 & . & 2f_2 & a_2 & 2h_2 \\ a_3 & . & . & 2h_3 & . & c_3 & b_3 & . & 2g_3 & 2f_3 \\ . & b_3 & . & a_3 & 2f_3 & . & 2h_3 & c_3 & . & 2g_3 \\ . & . & c_3 & . & b_3 & 2g_3 & . & 2f_3 & a_3 & 2h_3 \\ |a_1h_2g_3| & |h_1b_2f_3| & |g_1f_2c_3| & +\begin{matrix}|a_1h_2f_3| \\ |a_1b_2g_3|\end{matrix} & +\begin{matrix}|h_1b_2c_3| \\ |g_1b_2f_3|\end{matrix} & +\begin{matrix}|a_1f_2c_3| \\ |g_1h_2c_3|\end{matrix} & +\begin{matrix}|h_1b_2g_3| \\ |a_1b_2f_3|\end{matrix} & +\begin{matrix}|g_1b_2c_3| \\ |h_1f_2c_3|\end{matrix} & +\begin{matrix}|a_1h_2c_3| \\ |a_1f_2g_3|\end{matrix} & \begin{matrix}+ \\ 2\end{matrix}\begin{matrix}|a_1b_2c_3| \\ |f_1g_2h_3|\end{matrix} \end{vmatrix},$$

and following SALMON's method we have it in the form

$$\begin{vmatrix} a_1 & b_1 & c_1 & 2f_1 & 2g_1 & 2h_1 \\ a_2 & b_2 & c_2 & 2f_2 & 2g_2 & 2h_2 \\ a_3 & b_3 & c_3 & 2f_3 & 2g_3 & 2h_3 \\ 3|a_1h_2g_3| & |a_1b_2f_3|+|h_1b_2g_3| & |a_1f_2c_3|+|g_1h_2c_3| & |a_1b_2c_3|+2|f_1g_2h_3| & 2|a_1f_2g_3|+2|a_1h_2c_3| & 2|a_1b_2g_3|+2|a_1h_2f_3| \\ |a_1h_2f_3|+|a_1b_2g_3| & 3|h_1b_2f_3| & |g_1b_2c_3|+|h_1f_2c_3| & 2|h_1b_2c_3|+2|g_1b_2f_3| & |a_1b_2c_3|+2|f_1g_2h_3| & 2|h_1b_2g_3|+2|a_1b_2f_3| \\ |a_1h_2c_3|+|a_1f_2g_3| & |h_1b_2c_3|+|g_1b_2f_3| & 3|g_1f_2c_3| & 2|g_1b_2c_3|+2|h_1f_2c_3| & 2|a_1f_2c_3|+2|g_1h_2c_3| & |a_1b_2c_3|+2|f_1g_2h_3| \end{vmatrix}.$$

(7) Now, as has been already explained, it is the introduction of the differential coefficients of the Jacobian which seems to give unnecessary complexity to SALMON's form, and what we have therefore to seek for is three quadrics of a simpler character to take their place. The further suggestion then arises that in a similar manner SYLVESTER's form may be more complicated than is necessary, and that possibly there is a cubic more suitable than the Jacobian to complete our set of ten.

Success has attended both quests, there having been obtained an eliminant of the 6th order simpler than SALMON's, an eliminant of the 10th order simpler than SYLVESTER's, a new eliminant of the 7th order, and a full elucidation of the relationships connecting them all.

(8) The three quadrics for the simplification of SALMON's form have been obtained as follows:—

Multiplying the first given quadric

$$a_1x^2 + b_1y^2 + c_1z^2 + 2f_1yz + 2g_1zx + 2h_1xy$$

by $|b_2c_3|$, the second by $-|b_1c_3|$, and the third by $|b_1c_2|$, and adding, we have

$$|a_1b_2c_3|\cdot x^2 + 2|f_1b_2c_3|\cdot yz + 2|g_1b_2c_3|\cdot zx + 2|h_1b_2c_3|\cdot xy.$$

Acting similarly with the multipliers $|f_2c_3|$, $-|f_1c_3|$, $|f_1c_2|$, we have

$$|a_1f_2c_3|\cdot x^2 + |b_1f_2c_3|\cdot y^2 + 2|g_1f_2c_3|\cdot zx + 2|h_1f_2c_3|\cdot xy.$$

Now in each of these two deduced quadrics there is only one term free of x, and it so happens that the coefficient of this term in the one case is exactly -2 times what it is in the other. We are thus easily able to eliminate the two terms. In fact, multiplying the former of the deduced quadrics by y and the latter by $2z$, and adding, we have, after division by x, the new quadric

$$\begin{aligned} &0\cdot x^2 + 2|h_1b_2c_3|\cdot y^2 + 4|g_1f_2c_3|\cdot z^2 \\ &\quad + \{2|g_1b_2c_3| + 4|h_1f_2c_3|\}\cdot yz + 2|a_1f_2c_3|\cdot zx + |a_1b_2c_3|\cdot xy. \end{aligned}$$

Proceeding with the given quadrics u_2, u_3, u_1 as we have just dealt with u_1, u_2, u_3, we obtain in like manner

$$\begin{aligned} &4|h_1g_2a_3|\cdot x^2 + 0\cdot y^2 + 2|f_1c_2a_3|\cdot z^2 \\ &\quad + |a_1b_2c_3|\cdot yz + \{2|h_1c_2a_3| + 4|f_1g_2a_3|\}\cdot zx + 2|b_1g_2a_3|\cdot xy; \end{aligned}$$

and similarly from u_3, u_1, u_2

$$\begin{aligned} &2|g_1a_2b_3|\cdot x^2 + 4|f_1h_2b_3|\cdot y^2 + 0\cdot z^2 \\ &\quad + 2|c_1h_2b_3|\cdot yz + |a_1b_2c_3|\cdot zx + \{2|f_1a_2b_3| + 4|g_1h_2b_3|\}\cdot xy. \end{aligned}$$

These new quadrics, from their mode of formation, are seen to be not mere aggre-

gates of multiples of the given quadrics: consequently they can be used along with the latter for the purposes of dialytic elimination. The simplified eliminant of the 6th order thus obtained is therefore

$$\begin{vmatrix} a_1 & b_1 & c_1 & 2f_1 & 2g_1 & 2h_1 \\ a_2 & b_2 & c_2 & 2f_2 & 2g_2 & 2h_2 \\ a_3 & b_3 & c_3 & 2f_3 & 2g_3 & 2h_3 \\ . & 2|b_1c_2h_3| & -4|c_1f_2g_3| & 2|b_1c_2g_3|+4|c_1h_2f_3| & 2|c_1a_2f_3| & |a_1b_2c_3| \\ -4|a_1g_2h_3| & . & 2|c_1a_2f_3| & |a_1b_2c_3| & 2|c_1a_2h_3|+4|a_1f_2g_3| & 2|a_1b_2g_3| \\ 2|a_1b_2g_3| & -4|b_1h_2f_3| & . & 2|b_1c_2h_3| & |a_1b_2c_3| & 2|a_1b_2f_3|+4|b_1g_2h_3| \end{vmatrix}.$$

(9) The cubic required, in place of the Jacobian, for the simplification of SYLVESTER's eliminant of the 10th order, has been obtained as follows :—

Multiplying the three given quadrics by $|f_2g_3|$, $-|f_1g_3|$, $|f_1g_2|$ respectively, and adding, we have

$$|a_1f_2g_3|\cdot x^2 - |b_1g_2f_3|\cdot y^2 + |c_1f_2g_3|\cdot z^2 + 2|f_1g_2h_3|\cdot xy.$$

Comparing this with the first of the three quadrics derived in the preceding section, viz.,

$$2|b_1c_2h_3|\cdot y^2 - 4|c_1f_2g_3|\cdot z^2 + \{2|b_1c_2g_3|+4|c_1h_2f_3|\}\cdot yz + 2|c_1a_2f_3|\cdot zx + |a_1b_2c_3|\cdot xy,$$

we readily see that if we multiply the latter by z and the former by $4z$, and add, we shall obtain a cubic free of terms in x^3, y^3, z^3, x^2y, xy^2, viz., the cubic

$$\begin{aligned} &\{2|b_1c_2h_3| - 4|b_1g_2f_3|\}\cdot y^2z + 2|c_1a_2f_3|\cdot z^2x \\ &+ \{2|b_1c_2g_3|+4|c_1h_2f_3|\}\cdot yz^2 + 4|a_1f_2g_3|\cdot zx^2 + \{|a_1b_2c_3|+8|f_1g_2h_3|\}\cdot xyz. \end{aligned}$$

This is exceedingly simple as compared with the Jacobian, but it fails in the matter of symmetry. Proceeding therefore again to the original triad of quadrics, and multiplying them by $|b_2g_3|$, $-|b_1g_3|$, $|b_1g_2|$ respectively, we obtain after addition a quadric with no term in y^2, viz.,

$$|a_1b_2g_3|\cdot x^2 - |b_1c_2g_3|\cdot z^2 + 2|b_1g_2f_3|\cdot yz + 2|b_1g_2h_3|\cdot xy.$$

The multiple of this by $2y$, if added to the cubic just obtained, gives us all that can be desired, viz., the cubic

$$\begin{aligned} &2|a_1b_2g_3|\cdot x^2y + 2|b_1c_2h_3|\cdot y^2z + 2|c_1a_2f_3|\cdot z^2x \\ &+ 4|b_1g_2h_3|\cdot xy^2 + 4|c_1h_2f_3|\cdot yz^2 + 4|a_1f_2g_3|\cdot zx^2 \\ &+ \{|a_1\ b_2c_3|+8|f_1g_2h_3|\}\cdot xyz. \end{aligned}$$

From the mode of obtaining it we see that if we call it M_3, and denote the first of the three derived quadrics of § 8 by M_2, we have

$$M_2\cdot z + 2|u_1b_2g_3|\cdot y + 4|u_1f_2g_3|\cdot z = M_3.$$

Now as $\mathbf{M}_2 z$ is not an aggregate of multiples of the given quadrics, and as $2|u_1 b_2 g_3| \cdot y + 4|u_1 f_2 g_3| \cdot z$ is such an aggregate, it follows that the new cubic is in this respect similar to the new quadric. The simplified eliminant of the 10th order thus obtained is therefore

$$\begin{vmatrix}
a_1 & . & . & 2h_1 & . & c_1 & b_1 & . & 2g_1 & 2f_1 \\
. & b_1 & . & a_1 & 2f_1 & . & 2h_1 & c_1 & . & 2g_1 \\
. & . & c_1 & . & b_1 & 2g_1 & . & 2f_1 & a_1 & 2h_1 \\
a_2 & . & . & 2h_2 & . & c_2 & b_2 & . & 2g_2 & 2f_2 \\
. & b_2 & . & a_2 & 2f_2 & . & 2h_2 & c_2 & . & 2g_2 \\
. & . & c_2 & . & b_2 & 2g_2 & . & 2f_2 & a_2 & 2h_2 \\
a_3 & . & . & 2h_3 & . & c_3 & b_3 & . & 2g_3 & 2f_3 \\
. & b_3 & . & a_3 & 2f_3 & . & 2h_3 & c_3 & . & 2g_3 \\
. & . & c_3 & . & b_3 & 2g_3 & . & 2f_3 & a_3 & 2h_3 \\
. & . & . & 2|a_1 b_2 g_3| & 2|h_1 b_2 c_3| & 2|a_1 f_2 c_3| & 4|h_1 b_2 g_3| & 4|h_1 f_2 c_3| & 4|a_1 f_2 g_3| & \begin{matrix}|a_1 b_2 c_3| \\ +8|f_1 g_2 h_3|\end{matrix}
\end{vmatrix}.$$

(10) This is not all, however, for the simplified form thus reached immediately suggests a further simplification by reason of the absence of terms in x^3, y^3, z^3 from the cubic which has been used to replace the Jacobian.

Multiplying each element of the 9th row by c_2 and subtracting c_3 times the corresponding element of the 6th row, and operating similarly in five other instances, we can transform the determinant into one of the 7th order, viz.,

$$\frac{1}{a_2 b_2 c_2} \begin{vmatrix}
2|h_1 a_1| & . & |c_2 a_1| & |b_2 a_1| & . & 2|g_2 a_1| & 2|f_2 a_1| \\
|a_2 b_1| & 2|f_2 b_1| & . & 2|h_2 b_1| & |c_2 b_1| & . & 2|g_2 b_1| \\
. & |b_2 c_1| & 2|g_2 c_1| & . & 2|f_2 c_1| & |a_2 c_1| & 2|h_2 c_1| \\
2|h_3 a_2| & . & |c_3 a_2| & |b_3 a_2| & . & 2|g_3 a_2| & 2|f_3 a_2| \\
|a_3 b_2| & 2|f_3 b_2| & . & 2|h_3 b_2| & |c_3 b_2| & . & 2|g_3 a_2| \\
. & |b_3 c_2| & 2|g_3 c_2| & . & 2|f_3 c_2| & |a_3 c_2| & 2|h_3 a_2| \\
2|a_1 b_2 g_3| & 2|h_1 b_2 c_3| & 2|a_1 f_2 c_3| & 4|h_1 b_2 g_3| & 4|h_1 f_2 c_3| & 4|a_1 f_2 g_3| & |a_1 b_2 c_3| + 8|f_1 g_2 h_3|
\end{vmatrix}.$$

(11) Before proceeding to a comparison of the various derived quadrics and cubics which may be found useful in obtaining an expression for the eliminant, it is desirable to note that, as the given quadrics are obtainable one from another by performing simultaneously the cyclical changes,



any single derived equation not symmetrical with respect to this system of cyclical changes will readily give rise to two others. Thus, when the quadric $\mathbf{M}_2$ was obtained in § 8, the two still awanting necessitated no additional work. On the other hand, the cubic $\mathbf{M}_3$ of § 9 is unique, being simply twice reproduced by the cyclical change.

In the case of expressions of the latter kind, a great saving of trouble is effected by using the symbol of cyclical summation, $\overset{\circ}{\Sigma}$. Thus M_3 may be written in the form

$$2\overset{\circ}{\Sigma}|a_1b_2g_3|\cdot x^2y + 4\overset{\circ}{\Sigma}|b_1g_2h_3|\cdot xy^2 + \{|a_1b_2c_3|+8|f_1g_2h_3|\}\cdot xyz,$$

and the Jacobian in the form

$$-\overset{\circ}{\Sigma}|a_1g_2h_3|\cdot x^3 + \overset{\circ}{\Sigma}\{|a_1b_2g_3|-|a_1f_2h_3|\}\cdot x^2y + \overset{\circ}{\Sigma}\{|a_1b_2f_3|+|b_1g_2h_3|\}\cdot xy^2 + \{|a_1b_2c_3|+2|f_1g_2h_3|\}\cdot xyz.$$

(12) Further, as the twenty determinants of the 3rd order formed by taking every possible set of three columns out of the six,

$$\begin{matrix} a_1 & b_1 & c_1 & f_1 & g_1 & h_1 \\ a_2 & b_2 & c_2 & f_2 & g_2 & h_2 \\ a_3 & b_3 & c_3 & f_3 & g_3 & h_3, \end{matrix}$$

will be often in evidence, it will be convenient to have temporarily a short notation for them; and as they, like all other expressions connected with the investigation, appear in cycles of three (when not cyclically symmetrical), it is desirable that this should not be lost sight of in choosing the symbolism. Now the twenty, by virtue of the cyclical change, break themselves up into the following groups—

$$\begin{matrix} |a_1b_2c_3|, & & & \\ |a_1b_2f_3|, & |b_1c_2g_3|, & |c_1a_2h_3|, & \\ |a_1b_2g_3|, & |b_1c_2h_3|, & |c_1a_2f_3|, & \\ |a_1b_2h_3|, & |b_1c_2f_3|, & |c_1a_2g_3|, & \\ |a_1f_2g_3|, & |b_1g_2h_3|, & |c_1h_2f_3|, & \\ |a_1f_2h_3|, & |b_1g_2f_3|, & |c_1h_2g_3|, & \\ |a_1g_2h_3|, & |b_1h_2f_3|, & |c_1f_2g_3|, & \\ |f_1g_2h_3|, & & & \end{matrix}$$

and may therefore be fitly denoted as follows:—

$$\begin{matrix} |a_1b_2c_3| & & & \text{by } [0] \\ |f_1g_2h_3| & & & \text{by } [0'] \\ |a_1g_2h_3|, & |b_1h_2f_3|, & |c_1f_2g_3| & \text{by } [1], [2], [3] \\ |a_1b_2g_3|, & |b_1c_2h_3|, & |c_1a_2f_3| & \text{by } [4], [5], [6] \\ |a_1f_2h_3|, & |b_1g_2f_3|, & |c_1h_2g_3| & \text{by } [4'], [5'], [6'] \\ |a_1b_2f_3|, & |b_1c_2g_3|, & |c_1a_2h_3| & \text{by } [7], [8], [9] \\ |b_1g_2h_3|, & |c_1h_2f_3|, & |a_1f_2g_3| & \text{by } [7'], [8'], [9'] \\ |a_1b_2h_3|, & |b_1c_2f_3|, & |c_1a_2g_3| & \text{by } [10], [11], [12]. \end{matrix}$$

The reason for repeating the numbers [4], [5], [6], [7], [8], [9], with an added dash,

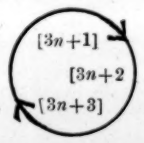
will appear later. Meanwhile it should be noted that the passing from one determinant to another by means of six of the cycles above given, say from

$$\begin{vmatrix} a_1 & g_1 & h_1 \\ a_2 & g_2 & h_2 \\ a_3 & g_3 & h_3 \end{vmatrix} \text{ to } \begin{vmatrix} b_2 & h_2 & f_2 \\ b_3 & h_3 & f_3 \\ b_1 & h_1 & f_1 \end{vmatrix},$$

is, on account of the identity of $|b_2h_3f_1|$ and $|b_1h_2f_3|$, much more readily effected by using only two cycles, viz.,



the subscripts 1, 2, 3 being considered invariable. The notation employed for the members of the six derived cycles of determinants enables us to remember them as one, viz.,



(13) With these preliminaries, let us now return to the consideration of the subsidiary quadrics necessary for obtaining the eliminant of the 6th order.

The process employed in § 8 for obtaining M_2 is given by the equation

$$|u_1b_2c_3|\cdot y + 2|u_1f_2c_3|\cdot z = x\cdot M_2,$$

from which we deduce

$$|u_1 \quad b_2y+2f_2z \quad c_3| = x\cdot M_2;$$

or, what is the same thing,

$$\begin{vmatrix} a_1x^2+b_1y^2+c_1z^2+2f_1yz+2g_1zx+2h_1xy & b_1y+2f_1z & c_1 \\ a_2x^2+b_2y^2+c_2z^2+2f_2yz+2g_2zx+2h_2xy & b_2y+2f_2z & c_2 \\ a_3x^2+b_3y^2+c_3z^2+2f_3yz+2g_3zx+2h^3xy & b_3y+2f_3z & c_3 \end{vmatrix} = x\,M_2.$$

Simplifying the determinant by diminishing each element of the first column by y times the corresponding element of the second column, and by z^2 times the corresponding element of the third column, we have

$$\begin{vmatrix} a_1x^2+2g_1zx+2h_1xy & b_1y+2f_1z & c_1 \\ a_2x^2+2g_2zx+2h_2xy & b_2y+2f_2z & c_2 \\ a_3x^2+2g_3zx+2h_3xy & b_3y+2f_3z & c_3 \end{vmatrix} = xM_2$$

and therefore

$$\mathrm{M}_2 = \begin{vmatrix} a_1x+2g_1z+2h_1y & b_1y+2f_1z & c_1 \\ a_2x+2g_2z+2h_2y & b_2y+2f_2z & c_2 \\ a_3x+2g_3z+2h_3y & b_3y+2f_3z & c_3 \end{vmatrix}.$$

Now this determinant is what is got by writing the three given quadrics in the form

$$\left.\begin{aligned} (a_1x+2g_1z+2h_1y)x &+ (b_1y+2f_1z)y &+ c_1z^2 &= 0 \\ (a_2x+2g_2z+2h_2y)x &+ (b_2y+2f_2z)y &+ c_2z^2 &= 0 \\ (a_3x+2g_3z+2h_3y)x &+ (b_3y+2f_3z)y &+ c_3z^2 &= 0 \end{aligned}\right\}$$

and eliminating dialytically x, y, z^2. Consequently we have the following simple rule for finding M_2 and its fellow subsidiary quadrics :—

Separate each of the given quadrics into three parts, viz., (1) *a part containing all the terms which have* x *for a factor*, (2) *a part containing all the remaining terms which have* y *for a factor*, (3) *the part having* z^2 *for a factor: and then eliminate dialytically* x, y, z^2.

(14) From this it is also suggested to obtain other subsidiary quadrics by varying the mode of partitioning the three originals preparatory to eliminating x, y, z^2. Doing this it will be found that the possible modes are three in number, viz.,

$$\begin{aligned} &(a_1x+2g_1z+2h_1y)\cdot x &+ &(b_1y+2f_1z\quad\quad)\cdot y &+ c_1\cdot z^2 \\ &(a_1x+2g_1z\quad\quad)\cdot x &+ &(b_1y+2f_1z+2h_1x)\cdot y &+ c_1\cdot z^2 \\ &(a_1x+2g_1z+h_1y)\cdot x &+ &(b_1y+2f_1z+h_1x)\cdot y &+ c_1\cdot z^2 \end{aligned}$$

The first of these, as we have seen, gives

$$2[5]\cdot y^2 - 4[3]\cdot z^2 + \{2[8]+4[8']\}\cdot yz + 2[6]\cdot zx + [0]\cdot xy,$$

the second gives

$$2[9]\cdot x^2 - 4[3]\cdot z^2 + 2[8]\cdot yz + \{2[6]-4[6']\}\, zx + [0]\cdot xy,$$

and the third gives

$$2[9]\cdot x^2 + [5]\cdot y^2 - 4[3]\cdot z^2 + \{2[8]+2[8']\}\cdot yz + \{2[6]-2[6']\}\cdot zx + [0]\cdot xy.$$

Of these the last may be left out of account, as being half the sum of the two others. The other new form, however, is as simple and as useful as that previously obtained: and, taking the two others derived from it by the cyclical change, we have an alternative form of the eliminant of the 6th order, viz.,

$$\begin{vmatrix} a_1 & b_1 & c_1 & 2f_1 & 2g_1 & 2h_1 \\ a_2 & b_2 & c_2 & 2f_2 & 2g_2 & 2h_2 \\ a_3 & b_3 & c_3 & 2f_3 & 2g_3 & 2h_3 \\ \cdot & -4[2] & 2[8] & 2[5]-4[5'] & [0] & 2[7] \\ 2[9] & \cdot & -4[3] & 2[8] & 2[6]-4[6'] & [0] \\ -4[1] & 2[7] & \cdot & [0] & 2[9] & 2[4]-4[4'] \end{vmatrix}.$$

On looking back it will be seen that this is what would be obtained on changing the rule of § 13 so as to read 'y' for 'x' and 'x' for 'y.' Indeed, as there is no reason for treating x differently from y, an alternative form of the eliminant might thus have been anticipated. Both forms are secured by putting 'x or y' for 'x, and 'y or x' for 'y' in the said rule.

(15) It might be expected that by subtracting the second derived quadric from the first, we should obtain a better form than either. It will be found, however, that though the quadric so deduced is simpler, it is absolutely useless for the purpose in view. The result in fact is

$$-[9]\cdot x^2 + [5]\cdot y^2 + 2[8']\cdot yz + 2[6']\cdot zx,$$

or

$$-|c_1a_2h_3|\cdot x^2 + |b_1c_2h_3|\cdot y^2 + 2|c_1h_2f_3|\cdot yz + 2|c_1h_2g_3|\cdot zx,$$

which is easily seen to be an aggregate of multiples of the original quadrics, viz., the aggregate representable by

$$u_1\ c_2\ h_3|.$$

It thus appears that only one independent triad of derived quadrics is in this way obtainable, but that two of the interdependent triads seem equally advantageous for the purpose in view.

(16) The question now arises as to the connection of the differential-quotients of the Jacobian with either of the triads here proposed to displace them.

From § 6 we have

$$\frac{\partial J}{\partial x} = -3[1]\cdot x^2 + \{[7]+[7']\}\cdot y^2 + \{[6]-[6']\}\cdot z^2 + \{[0]+2[0']\}\cdot yz + 2\{[9]+[9']\}\cdot zx + 2\{[4]-[4']\}\,xy,$$

and the second member of the first of the said two triads is

$$-4[1]\cdot x^2 + 2[6]\cdot z^2 + [0]\cdot yz + \{2[9]+4[9']\}\cdot zx + 2[4]\cdot xy;$$

so that the difference between them is

$$[1]\cdot x^2 + \{[7]+[7']\}\cdot y^2 - \{[6]+[6']\}\cdot z^2 + 2[0']\cdot yz - 2[9']\cdot zx - 2[4']\cdot xy.$$

Now this difference, as might be expected, is found to be an aggregate of multiples of the original quadrics. In fact, taking the aggregates

$$|u_1g_2h_3| \quad \text{i.e.,} \quad [1]\cdot x^2 + [7']\cdot y^2 - [6']\cdot z^2 + 2[0']\cdot yz,$$
$$|u_1a_2f_3| \quad \text{i.e.,} \quad -[7]\cdot y^2 + [6]\cdot z^2 + 2[9']\cdot zx + 2[4']\cdot xy,$$

we see that their difference is identical with the previous difference. The result arrived at thus is

$$\frac{\partial \mathrm{J}}{\partial x} = \mathrm{M}_2 + |u_1 g_2 h_3| - |u_1 a_2 f_3|,$$

it being understood, of course, that M_2 here stands for a particular member of a triad of quadrics, one of which has been so denoted.

(17) If we do not discriminate as to form, the number of aggregates of multiples of the original quadrics is infinite. Keeping, however, to the form which has turned up repeatedly in what precedes we have only fifteen to reckon with, this being the number of pairs of letters choosable from the six, a, b, c, f, g, h. The fifteen must, of course, be separable into five triads, and it is best so to arrange them. They may therefore be conveniently tabulated as follows:—

	x^2	y^2	z^2	yz	zx	xy
$\|u_1 b_2 c_3\|$	[0]			2[11]	2[8]	2[5]
$\|u_1 c_2 a_3\|$		[0]		2[6]	2[12]	2[9]
$\|u_1 a_2 b_3\|$			[0]	2[7]	2[4]	2[10]
$\|u_1 b_2 f_3\|$	[7]		−[11]		−2[5′]	−2[2]
$\|u_1 c_2 g_3\|$	−[12]	[8]		−2[3]		−2[6′]
$\|u_1 a_2 h_3\|$		−[10]	[9]	−2[4′]	−2[1]	
$\|u_1 b_2 g_3\|$	[4]		−[8]	2[5′]		2[7′]
$\|u_1 c_2 h_3\|$	−[9]	[5]		2[8′]	2[6′]	
$\|u_1 a_2 f_3\|$		−[7]	[6]		2[9′]	2[4′]
$\|u_1 b_2 h_3\|$	[10]		−[5]	2[2]	−2[7′]	
$\|u_1 c_2 f_3\|$	−[6]	[11]			2[3]	−2[8′]
$\|u_1 a_2 g_3\|$		−[4]	[12]	−2[9′]		2[1]
$\|u_1 f_2 g_3\|$	[9′]	−[5′]	[3]			2[0′]
$\|u_1 g_2 h_3\|$	[1]	[7′]	−[6′]	2[0′]		
$\|u_1 h_2 f_3\|$	−[4′]	[2]	[8′]		2[0′]	

Unlike these, which cannot be taken along with u_1, u_2, u_3 for purposes o elimination, we have, as above pointed out, the single triad—

	x^2	y^2	z^2	yz	zx	xy
$\lvert u_1 \quad b_2y+2f_2z \quad c_3 \rvert \div x$		2[5]	−4[3]	2[8]+4[8′]	2[6]	[0]
$\lvert u_1 \quad c_2z+2g_2x \quad a_3 \rvert \div y$	−4[1]		2[6]	[0]	2[9]+4[9′]	2[4]
$\lvert u_1 \quad a_2x+2h_2y \quad b_3 \rvert \div z$	2[4]	−4[2]		2[5]	[0]	2[7]+4[7′]

If, however, from the eighteen quadrics here tabulated two or more be taken, one at least being taken from the second table, and an aggregate of multiples of these be formed, the resulting quadric, if not symmetrical with regard to the cyclical change, will be one of a triad which also may be used along with u_1, u_2, u_3 for the purpose of dialytically eliminating x^2, y^2, z^2, yz, zx, xy. In this way, as has been seen, the triad of quadrics which are differential-quotients of the Jacobian may be found, and the triad of § 14.

Notwithstanding the large number of possible aggregates of the kind here referred to, it does not appear that anything simpler than the triads of §§ 8 and 14 can be found.

(18) As an illustrative example, let us take SYLVESTER's special case, where

$$\left.\begin{aligned} u_1 &= \mathrm{A}x^2 + ayz + bzx + cxy \\ u_2 &= \mathrm{M}y^2 + lyz + mzx + nxy \\ u_3 &= \mathrm{R}z^2 + pyz + qzx + rxy \end{aligned}\right\}.$$

In accordance with the rule of § 13 we write the equations in the form

$$\left.\begin{aligned} (\mathrm{A}x+bz+cy)\cdot x + az\cdot y &= 0 \\ (mz+ny)\cdot x + (\mathrm{M}y+lz)\cdot y &= 0 \\ (qz+ry)\cdot x + pz\cdot y + \mathrm{R}\cdot z^2 &= 0 \end{aligned}\right\},$$

and eliminating x, y, z^2 we obtain

$$\begin{vmatrix} \mathrm{A}x+bz+cy & az \\ mz+ny & \mathrm{M}y+lz \end{vmatrix} = 0,$$

or

$$c\mathrm{M}\cdot y^2 + (bl-am)\cdot z^2 + (cl-an+b\mathrm{M})\cdot yz - \mathrm{A}l\cdot zx + \mathrm{AM}\cdot xy = 0.$$

Instead of this SYLVESTER obtained *

$$\begin{aligned} (ra-cp-c\mathrm{M})\cdot y^2 &- (b\mathrm{R}+bl-am)\cdot z^2 \\ &+ (aq-b\mathrm{M}-bp-c\mathrm{R}-cl+an)\cdot yz - (\mathrm{AR}+\mathrm{A}l)\cdot zx - (\mathrm{A}p+\mathrm{AM})\cdot xy; \end{aligned}$$

and NANSON, following SALMON's method, obtains † something still more forbidding, viz.:—

$$\begin{aligned} &- 3\mathrm{A}(mr-gn)\cdot x^2 + \mathrm{M}(2\mathrm{A}p+gc-br)\cdot y^2 + \mathrm{R}(2\mathrm{A}l+bn-mc)\cdot z^2 \\ &+ \left\{ \begin{vmatrix} a & b & c \\ l & m & n \\ p & q & r \end{vmatrix} + 4\mathrm{AMR} \right\}\cdot yz + 2\mathrm{A}(2\mathrm{R}n+lg-pm)\cdot zx + 2\mathrm{A}(2\mathrm{M}g+pc-lr)\cdot xy. \end{aligned}$$

* SYLVESTER, J. J., "Examples of the Dialytic Method, etc." *Cambridge Math. Journ.*, ii. p. 233.

† NANSON, E. J., "On the Eliminant of a Set of Quadrics, Ternary or Quaternary," *Proc. Roy. Soc. Edin.*, xxii. p. 354.

(19) Leaving now the eliminant of the 6th order let us return to the problem of finding the cubic, M_3, to be used in place of the Jacobian in forming the eliminant of the 10th order.

As we have seen

$$M_3 = M_2 \cdot z + 2|u_1 b_2 g_3| \cdot y + 4|u_1 f_2 g_3| \cdot z.$$

Now the sum of the last two terms on the right is

$$\begin{aligned} &= |u_1 \quad b_2 y \quad 2g_3| + |u_1 \quad 2f_2 z \quad 2g_3|, \\ &= |u_1 \quad b_2 y + 2f_2 z \quad 2g_3|; \end{aligned}$$

and from § 13

$$\begin{aligned} M_2 \cdot z &= |u_1 \quad b_2 y + 2f_2 z \quad c_3 \left|\frac{z}{x}\right. \\ &= |u_1 \quad b_2 y + 2f_2 z \quad c_3 z x^{-1}|. \end{aligned}$$

Consequently we have

$$M_3 = |u_1 \quad b_2 y + 2f_2 z \quad c_3 z x^{-1} + 2g_3|.$$

Now diminishing each element of the first column by y times the corresponding element of the second column and by zx times the corresponding element of the third column we obtain

$$M_3 = |a_1 x^2 + 2h_1 xy \qquad b_2 y + 2f_2 z \qquad c_3 z x^{-1} + 2g_3|,$$

and thence

$$M_3 = \begin{vmatrix} a_1 x + 2h_1 y & b_1 y + 2f_1 z & c_1 z + 2g_1 x \\ a_2 x + 2h_2 y & b_2 y + 2f_2 z & c_2 z + 2g_2 x \\ a_3 x + 2h_3 y & b_3 y + 2f_3 z & c_3 z + 2g_3 x \end{vmatrix}.$$

But this determinant is what is got by writing the three given equations in the form

$$\begin{aligned} (a_1 x + 2h_1 y) \cdot x + (b_1 y + 2f_1 z) \cdot y + (c_1 z + 2g_1 x) \cdot z &= 0 \\ (a_2 x + 2h_2 y) \cdot x + (b_2 y + 2f_2 z) \cdot y + (c_2 z + 2g_2 x) \cdot z &= 0 \\ (a_3 x + 2h_3 y) \cdot x + (b_3 y + 2f_3 z) \cdot y + (c_3 z + 2g_3 x) \cdot z &= 0 \end{aligned}$$

and eliminating dialytically x, y, z. Consequently we have the following simple rule for finding M_3 :—

Separate each of the given quadrics into three parts, viz., (1) *a part containing the terms which have* x *for a factor and do not involve* z, (2) *a part containing the terms which have* y *for a factor and do not involve* x, (3) *a part containing the terms which have* z *for a factor and do not involve* y : *and then eliminate dialytically* x, y, z.

(20) It is thus seen that M_3 and J resemble each other in being both obtainable by dialytically eliminating the first powers of x, y, z from the three given quadrics, just as M_2 and its fellow resemble each other in being obtainable by the elimination of x, y, z^2.

The other cubics of the family to which M_3 and J belong arise from different modes of partitioning a general quadric into three terms having x, y, z respectively for factors ; and on trial it will be found that these modes are eighteen in number, viz.,

three of the form

$$(a_1x+2g_1z+2h_1y)\cdot x \;+\; (b_1y+f_1z)\cdot y \;+\; (c_1z+f_1y)\cdot z :$$

three of the form

$$(a_1x+2g_1z+h_1y)\cdot x \;+\; (b_1y+f_1z+h_1x)\cdot y \;+\; (c_1z+f_1y)\cdot z :$$

three of the form

$$(a_1x+g_1z+2h_1y)\cdot x \;+\; (b_1y+f_1z)\cdot y \;+\; (c_1z+f_1y+g_1z)\cdot z :$$

three of the form

$$(a_1x+2g_1z+h_1y)\cdot x \;+\; (b_1y+h_1x)\cdot y \;+\; (c_1z+2f_1y)\cdot z :$$

three of the form

$$(a_1x+g_1z+2h_1y)\cdot x \;+\; (b_1y+2f_1z)\cdot y \;+\; (c_1z+g_1x)\cdot z :$$

one of the form

$$(a_1x+2h_1y)\cdot x \;+\; (b_1y+2f_1z)\cdot y \;+\; (c_1z+2g_1x)\cdot z :$$

one of the form

$$(a_1x+2h_1y)\cdot x \;+\; (b_1y+2f_1z)\cdot y \;+\; (c_1z+2g_1x)\cdot z :$$

and one of the form

$$(a_1x+h_1y+g_1z)\cdot x \;+\; (b_1y+f_1z+h_1x)\cdot y \;+\; (c_1z+g_1x+f_1y)\cdot z .$$

Each of the last three is unique, by reason of being invariant to the cyclical change. It is the last of all which gives rise to the Jacobian; and this, strange to say, is the lengthiest cubic in the whole series, as many as 17 of the 20 determinants of § 12 appearing in it.

The second from the end gives rise to M_3. The third from the end originates a cubic equally simple with M_3; so that, exactly as in the case of the eliminant of the 6th order, we have two alternative forms. On turning to the rule of § 19, it will be seen that from the three parts of each of the given quadrics we excluded z, x, y respectively when we might equally reasonably have excluded y, z, x: it is this exclusion of y, z, x from the three parts respectively which gives rise to the alternative form.

(21) Out of the 18 partitions there arise only 8 cubics which are invariant to the cyclical change; for, in five different instances, it is necessary to use three of the partitions to secure one such result.

Thus the first partition

$$(a_1x+2g_1z+2h_1y)\cdot x \;+\; (b_1y+f_1z)\cdot y \;+\; (c_1z+f_1y)\cdot z$$

gives the irregular cubic

$$\begin{aligned} -\,2[2]\cdot y^3 \;-\; 2[3]\cdot z^3 \;+\; &\{2[5]-2[5']\}\cdot y^2z \;+\; [6]\cdot z^2x \\ &+\; [7]\cdot xy^2 \;+\; \{2[8]+2[8']\}\cdot yz^2 \\ &+\; [0]\cdot xyz\,; \end{aligned}$$

but, on performing on this the cyclical changes and adding, we obtain a cubic which, like J and M_3, is cyclically symmetrical, viz.,

$$-\overset{\circ}{\Sigma}4[1]\cdot x^3 + \overset{\circ}{\Sigma}\{3[4]-2[4']\}\cdot x^2y + \overset{\circ}{\Sigma}\{3[7]+2[7']\}\cdot xy^2 + 3[0]\cdot xyz.$$

The eight cubics of this kind are—

	x^3	y^3	z^3	x^2y	y^2z	z^2x	xy^2	yz^2	zx^2	xyz
A	$-4[1]$	$-4[2]$	$-4[3]$	$3[4]-2[4']$	$3[5]-2[5']$	$3[6]-2[6']$	$3[7]+2[7']$	$3[8]+2[8']$	$3[9]+2[9']$	$3[0]$
B	$-3[1]$	$-3[2]$	$-3[3]$	$2[4]-5[4']$	$2[5]-5[5']$	$2[6]-5[6']$	$4[7]+[7']$	$4[8]+[8']$	$4[9]+[9']$	$3[0]+6[0']$
B′	$-3[1]$	$-3[2]$	$-3[3]$	$4[4]-[4']$	$4[5]-[5']$	$4[6]-[6']$	$2[7]+5[7']$	$2[8]+5[8']$	$2[9]+5[9']$	$3[0]+6[0']$
C	$-2[1]$	$-2[2]$	$-2[3]$	$[4]-8[4']$	$[5]-8[5']$	$[6]-8[6']$	$5[7]$	$5[8]$	$5[9]$	$3[0]+12[0']$
C′	$-2[1]$	$-2[2]$	$-2[3]$	$5[4]$	$5[5]$	$5[6]$	$[7]+8[7']$	$[8]+8[8']$	$[9]+8[9']$	$3[0]+12[0']$
M_3				$-4[4']$	$-4[5']$	$-4[6']$	$2[7]$	$2[8]$	$2[9]$	$[0]+8[0']$
M'_3				$2[4]$	$2[5]$	$2[6]$	$4[7']$	$4[8']$	$4[9']$	$[0]+8[0']$
J	$-[1]$	$-[2]$	$-[3]$	$[4]-[4']$	$[5]-[5']$	$[6]-[6']$	$[7]+[7']$	$[8]+[8']$	$[9]+[9']$	$[0]+2[0']$

On account of the cyclo-symmetry and the fact that the determinant in the first column is always [1], in the second column always [2], and so on, the cubics may be denoted shortly by the coefficients of the determinants only. The list would then stand as follows :—

A	$(-4, 3-2, 3+2, 3+0)$
B	$(-3, 2-5, 4+1, 3+6)$
B′	$(-3, 4-1, 2+5, 3+6)$
C	$(-2, 1-8, 5+0, 3+12)$
C′	$(-2, 5+0, 1+8, 3+12)$
M_3	$(0, 0-4, 2+0, 1+8)$
M'_3	$(0, 2+0, 0+4, 1+8)$
J	$(-1, 1-1, 1+1, 1+2)$.

(22) Of course, any seven of these are derivable from the remaining one with the help of aggregates of multiples of the original quadrics. As a matter of fact, only two such aggregates are necessary, viz.,

$$|u_1g_2h_3|\cdot x + |u_1h_2f_3|\cdot y + |u_1f_2g_3|\cdot z$$

which will be found to be equal to

$$\overset{\circ}{\Sigma}[1]\cdot x^3 - \overset{\circ}{\Sigma}[4']\cdot x^2y + \overset{\circ}{\Sigma}[7']\cdot xy^2 + 2[0]\cdot xyz,$$

or, in the abridged notation just explained,

$$(1,\ 0-1,\ 0+1,\ 0+6);$$

and

$$|u_1a_2f_3|\cdot x\ +\ |u_1b_2g_3|\cdot y\ +\ |u_1c_2h_3|\cdot z$$

which will be found to be equal to

$$(0,\ 1+2,\ -1+2,\ 0+0).$$

Calling these two aggregates σ and σ', so that

$$\sigma-\sigma' = (1,\ -1-3,\ 1-1,\ 0+6),$$

we have

$$\begin{array}{lll} B'-2\sigma' = B, & A+\sigma-\sigma' = B, & C+2\sigma-\sigma' = 3M_3; \\ C'-4\sigma' = C, & B+\sigma-\sigma' = C, & \\ M'_3-2\sigma' = M_3, & J+\sigma-\sigma' = M_3, & \end{array}$$

and consequently, if it were desired to have all the cubics expressed in terms of M_3, we should have

$$\begin{array}{l} A = 3M_3 + 3\sigma' - 4\sigma, \\ B = 3M_3 + 2\sigma' - 3\sigma, \\ B' = 3M_3 + 4\sigma' - 3\sigma, \\ C = 3M_3 + \sigma' - 2\sigma, \\ C' = 3M_3 + 5\sigma' - 2\sigma, \\ M'_3 = M_3 + 2\sigma', \\ J = M_3 + \sigma' - \sigma. \end{array}$$

(23) As an illustrative example, let us again take SYLVESTER's second case, where, as we have seen,

$$\begin{array}{l} u_1 = Ax^2+ayz+bzx+cxy, \\ u_2 = My^2+lyz+mzx+nxy, \\ u_3 = Rz^2+pyz+qzx+rxy. \end{array}$$

Following closely the rule of § 19 we write the equations in the form

$$\left.\begin{array}{rrrr} (Ax+cy)\cdot x + & az\cdot y + & bx\cdot z & = 0 \\ ny\cdot x + & (My+lz)\cdot y + & mx\cdot z & = 0 \\ ry\cdot x + & pz\cdot y + & (Rz+qx)\cdot z & = 0 \end{array}\right\}$$

and eliminating x, y, z we obtain

$$\begin{vmatrix} Ax+cy & az & bx \\ ny & My+lz & mx \\ ry & pz & Rz+qx \end{vmatrix}$$

or

$$\begin{array}{l} AMq\cdot x^2y + MRc\cdot y^2z + RAl\cdot z^2x \\ \quad + M(qc-br)\cdot xy^2 + R(cl-na)\cdot yz^2 + A(lq-pm)\cdot x^2z + (\Delta+AMR)\cdot xyz. \end{array}$$

Instead of this, Professor NANSON, following SYLVESTER's method, obtains

$$\begin{aligned}
&- \mathrm{A}(mr-qn)\cdot x^3 - \mathrm{M}(ra-cp)\cdot y^3 - \mathrm{R}(am-lb)\cdot z^3 \\
&+ \mathrm{A}(2\mathrm{M}q+pn-lr)\cdot x^2y + \mathrm{M}(2\mathrm{R}c+bp-qa)\cdot y^2z + \mathrm{R}(2\mathrm{A}l+nb-cm)\cdot z^2x \\
&+ \mathrm{M}(2\mathrm{A}p+qc-br)\cdot xy^2 + \mathrm{R}(2\mathrm{M}b+cl-na)\cdot yz^2 + \mathrm{A}(2\mathrm{R}n+lq-pn)\cdot xz^2 \\
&+ (\Delta+4\mathrm{AMR})\,xyz.
\end{aligned}$$

Simple, however, though M_3 in this case is as compared with J, there is still a simpler cubic obtainable by subtracting

$$\mathrm{M}qy\cdot u_1 + \mathrm{R}cz\cdot u_2 + \mathrm{A}lx\cdot u_3$$

from M_3, viz., the cubic

$$\begin{aligned}
&- \mathrm{A}lr\cdot x^2y - \mathrm{M}qa\cdot y^2z - \mathrm{R}cm\cdot z^2x \\
&- \mathrm{M}br\cdot xy^2 - \mathrm{R}na\cdot yz^2 - \mathrm{A}pm\cdot xz^2 + (\Delta+\Delta'-amr-\mathrm{AMR})\cdot xyz
\end{aligned}$$

where

$$\Delta = \begin{vmatrix} a & b & c \\ l & m & n \\ p & q & r \end{vmatrix} \quad \text{and} \quad \Delta' = \begin{vmatrix} \mathrm{A} & b & c \\ l & \mathrm{M} & n \\ p & q & \mathrm{R} \end{vmatrix}.$$

(24) The derived quadrics needed for SALMON's eliminant being differential-quotients of the derived cubic which is needed for SYLVESTER's eliminant, it is of interest to inquire whether any one of the other cubics of § 20 is such that a differential-quotient of it will vanish for the same set of values as the original quadrics.

The answer to this may be formulated as follows:—

If any one of the derived cubics which vanish for the same set of values as the original quadrics have in its determinant form one column composed of the halved differential-quotients of the said quadrics, the differential-quotient of the cubic with respect to the same variable will also vanish for that set of values.

Let the column be the first, thus implying differentiation with respect to x; and let the cubic in question be

$$\begin{vmatrix} \frac{1}{2}\frac{\partial u_1}{\partial x} & \lambda_1 & \mu_1 \\ \frac{1}{2}\frac{\partial u_2}{\partial x} & \lambda_2 & \mu_2 \\ \frac{1}{2}\frac{\partial u_3}{\partial x} & \lambda_3 & \mu_3 \end{vmatrix} \quad \text{or K say.}$$

Then, from the mode in which such cubics are obtained, we have

$$\left.\begin{aligned}
\tfrac{1}{2}\tfrac{\partial u_1}{\partial x}\cdot x + \lambda_1\cdot y + \mu_1\cdot z &= u_1 \\
\tfrac{1}{2}\tfrac{\partial u_2}{\partial x}\cdot x + \lambda_2\cdot y + \mu_2\cdot z &= u_2 \\
\tfrac{1}{2}\tfrac{\partial u_3}{\partial x}\cdot x + \lambda_3\cdot y + \mu_2\cdot z &= u_3
\end{aligned}\right\},$$

and therefore

$$xK = \begin{vmatrix} u_1 & \lambda_1 & \mu_1 \\ u_2 & \lambda_2 & \mu_2 \\ u_3 & \lambda_3 & \mu_3 \end{vmatrix},$$

and therefore, by the theorem regarding the differentiation of a determinant,

$$x\frac{\partial K}{\partial x} + K = \begin{vmatrix} \frac{\partial u_1}{\partial x} & \lambda_1 & \mu_1 \\ \frac{\partial u_2}{\partial x} & \lambda_2 & \mu_2 \\ \frac{\partial u_3}{\partial x} & \lambda_3 & \mu_3 \end{vmatrix} + \begin{vmatrix} u_1 & \frac{\partial \lambda_1}{\partial x} & \mu_1 \\ u_2 & \frac{\partial \lambda_2}{\partial x} & \mu_1 \\ u_3 & \frac{\partial \lambda_3}{\partial x} & \mu_3 \end{vmatrix} + \begin{vmatrix} u_1 & \lambda_1 & \frac{\partial \mu_1}{\partial x} \\ u_2 & \lambda_2 & \frac{\partial \mu_2}{\partial x} \\ u_3 & \lambda_3 & \frac{\partial \mu_3}{\partial x} \end{vmatrix},$$

$$= 2K + \begin{vmatrix} u_1 & \frac{\partial \lambda_2}{\partial x} & \mu_3 \end{vmatrix} + \begin{vmatrix} u_1 & \lambda_2 & \frac{\partial u_3}{\partial x} \end{vmatrix},$$

from which it is manifest that any set of values which causes u_1, u_2, u_3, K to vanish will cause $\partial K/\partial x$ to vanish also.

Had the chosen column been the second, the same mode of reasoning would have sufficed to show that the differential-quotient with respect to y would have vanished; and similarly in the case of the third column. The theorem is thus established.

(25) There are seven such cubics among the eighteen of § 20, those of the second and third triads, and the last one of all—the Jacobian, in the case of which the theorem is triply applicable.

As an example, the first of the third triad may be taken. In its determinant form it is

$$\begin{vmatrix} a_1x+g_1z+2h_1y & b_1y+f_1z & c_1z+f_1y+g_1x \\ a_2x+g_2z+2h_2y & b_2y+f_2z & c_2z+f_2y+g_2x \\ a_3x+g_3z+2h_3y & b_3y+f_3z & c_3z+f_3y+g_3x \end{vmatrix},$$

and arranged according to powers of the variables it is

$$\begin{aligned} &- 2[2]\cdot y^3 - [3]\cdot z^3 \\ &+ [4]\cdot x^2y + \{2[5]-[5']\}\cdot y^2z + [6]\cdot z^2x \\ &+ \{[7]+2[7']\}\cdot xy^2 + \{[8]+2[8']\}\cdot yz^2 + [9']\cdot zx^2 \\ &+ \{[0]+2[0']\}\cdot xyz. \end{aligned}$$

Differentiating with respect to z, on account of the fact that it is the third column of the determinant which is composed of differential-quotients, we have

$$\begin{aligned} &[9']\cdot x^2 + \{2[5]-[5']\}\cdot y^2 - 3[3]\cdot z^2 \\ &+ \{2[8]+4[8']\}\cdot yz + 2[6]\cdot zx + \{[0]+2[0']\}\cdot xy, \end{aligned}$$

—a quadric which, with the two others obtained from it by the cyclical change, might be used to form an eliminant of the 6th order. In practice, however, it would not be desirable so to use it, as, although it is at least as simple as $\partial J/\partial x$, it is much more cumbrous than M_2; in fact it will be found to be M_2 increased by an aggregate of multiples of the original quadrics, viz., the aggregate $|u_1 f_2 g_3|$.

XXVII.—*On the Restoration of Co-ordinated Movements after Nerve Section.*

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(Read April 3, 1899.)

From the point of view of its function, a nerve fibre is a conductor of nervous impulses, and as such is the path of communication between two structures, the one situated in the central nervous system, and the other in the periphery. In the mixed nerve, such as the sciatic, the nerve fibres are distinguished as afferent or as efferent, according as they conduct impulses originating at the periphery, and received by a cell in the central nervous system, or *vice versâ*. It has long since been shown that nerve fibres are capable of conducting impulses in either direction, but normally, from their anatomical connections, the individual nerve fibres are conductors for impulses only in the one or in the other direction. This is proved by the Wallerian method of investigation, as on severance of the posterior spinal root distal to the ganglion only certain fibres degenerate and the conductivity of the nerve only for afferent impulses is lost, while the severance of the anterior root is followed by the degeneration of the remainder with loss of functions depending on efferent impulses.

But the conception of nerve fibre and its terminals carries us further, and leads to the view that a particular nerve fibre is concerned only with the conduction of impulses from a particular point, *e.g.*, from a particular sensory area of the skin, or, on the other hand, that it is concerned with the passage of impulses from a particular central cell to particular end-organs. The individual nerve fibres are thus viewed not as common conductors for impulses originating from different points, but each as a specific conductor for impulses starting in a particular cell, and received at the other end also in a particular cell. Thus a transverse section of a mixed nerve with its innumerable nerve fibres might, if knowledge were sufficiently exact, have each of the latter named according to its origin or distribution.

It is true that the passage of the nerve fibres between the peripheral organ and the brain is not necessarily continuous, but may be interrupted by the intervention of cells ; but in the modern conception of say the motor neuron the cell body situated in the cornua of the cord connects directly with the peripheral organ by means of its neuraxon, while the cell body receives its stimuli from another definite cell, situated it may be in the cortex, by means of the neuraxon of the latter. The individual nerve fibres in a nerve must, however, be regarded as paths for the conduction of impulses each from a definite point to a definite point.

A nerve is thus, both from the point of view of its morphology and of its physiology, a highly specialised structure, and it is, therefore, not surprising that the study of its repair after division should lead to problems of great difficulty.

Two methods of repair after division have been described, namely, that by first intention, and that by regeneration of the peripheral segment. In the former case, it is supposed that after the division of the fibre the two ends speedily become connected again, so as to restore the fibre as before the division. After such a process of reunion, the peripheral segment is still the same structure which before the division conducted the impulses. This view, then, carries with it the supposition that when the nerve fibres heal by this process, the Wallerian degeneration does not take place, that after the section the peripheral end becomes again connected with its trophic centres before the advent of degeneration, so that by the early restoration of the influence of the centres the degeneration is avoided. The evidence which is advanced by those who regard this as a possibility is the early return of function, but before such a process can be accepted as possible, it would be necessary to have anatomical proof from microscopic preparations of the peripheral end of a recently divided and reunited nerve, that the peripheral segments of the fibres present the normal characters of adult medullated fibres, and that degeneration remains are absent. This is what I have been unable to find satisfactorily in the papers of those authors who claim that healing by first intention is possible, the evidence relied on being chiefly that of early return of function.

In the other process of repair of a nerve, after destruction of the peripheral segment by Wallerian degeneration, that segment is regenerated, and the continuity of the nerve to its end-organs thus restored. On the process by which this is effected there has been much difference of opinion. That described by RANVIER of outgrowth from the central segment of processes from the old axis-cylinders, which continue their growth till they reach the end-organs, is the view which has been most widely accepted, while the other view is that simultaneously with the degeneration, new fibres are formed in the peripheral segment independently of the central segment, and that these become connected with the old fibres of the central segment. The process described in the former view necessarily requires for its accomplishment a considerable period of time, a period which must be longer the farther the section is from the peripheral terminations of the nerve. It is the acceptance of this view which gave origin and support to the view of healing by first intention, as something had to be supposed to explain cases of early return of function after nerve section. On this view, and on the view of development of the nerve fibres advanced by KÖLLIKER and by HIS, is founded the modern conception of the 'neuron.' According to the second view, it is not necessary to suppose a lengthened period of time, or a different period of time, according as the section is made near or far removed from the peripheral terminals, until the nerve becomes again a functional structure.

Both from clinical and from experimental experience, it is established that early return of function can take place, but there are certain fallacies which should be guarded

against in judging of the value of early return or of late return of function after section. Thus, where early return of function occurs after nerve section, it is necessary to remember the possibility of a vicarious nerve supply, and where late recovery takes place this is not conclusive evidence in favour of the view of RANVIER, as the reunion of the nerve at first may be inadequate to permit of function being carried on through the segment of reunion. Thus failure of the wound to heal without suppuration will lead to delay or even to failure of reunion. Delay of returning function may, therefore, mean only inadequate reunion of the nerve, and early return may mean vicarious nerve supply.

But there are certain cases in which very early return of function occurs after suture, which preclude the possibility of vicarious nerve supply, and which cannot be explained as cases of healing by first intention without degeneration. I refer to cases in which reunion has failed after the section, and in which, after a period of time more or less prolonged, an operation is undertaken for the secondary reunion of the central and peripheral ends. In such cases no vicarious return of function has taken place, and yet, after reunion at the secondary operation, the function, which for a long period of time has been in abeyance, speedily returns. Such return of function can only be explained as due to the restoration by the secondary operation of the conductivity of the nerve, and the speed with which function returns implies the view that the peripheral segment, though separated from its centres, has in the interval formed new fibres in a condition, when united to the central segments, to conduct impulses. But what is remarkable about such cases is that not only does sensation return in the parts which were formerly insensitive, but, though at first indefinite, within a remarkably short period of time localisation becomes perfect. Thus, in cases which I have published * in which the nerves had been completely divided, and had not reunited, and in which a secondary operation was followed by return of sensation in a few days, localisation was found by repeated examination to be well established by a month. It is easy to understand, on the supposition of independent regeneration of the peripheral segment, how function might return in a very short time after reunion; but the speedy or even remote perfection of localisation is not what we should expect from our conception of the functional differentiation of the nerve fibres. How is it possible, immediately after division of a nerve, or more so in a secondary operation for repair of a non-united nerve, in bringing the ends together by suture, that the central ends of the individual nerve fibres can be brought to lie opposite their proper peripheral ends? We should certainly expect that considerable confusion of localisation of sensation and of co-ordination of movements should result from unavoidable imperfect coaptation. This difficulty is common to all the views of repair of divided nerves. In healing by first intention, if it exists, and in healing by regeneration of nerve fibres by independent formation in the peripheral segment, it is scarcely possible that the nerve ends will be so accurately coapted that all the corresponding nerve fibre segments will be brought into apposition. In the view of RANVIER the position is no better, for there the sheaths of SCHWANN of the old

* *Phil. Trans.*, Series B., vol. clxxxviii. (1897), p. 257.

fibres, which are to act as the guides for the growth of the new fibres to the terminal organs, can scarcely be supposed to be brought by the suture into line with their proper central segments. Also, if we take the view held by VANLAIR and others, that the new sprouting fibres simply run in the endoneurial spaces, how can the new fibres be guided to their appropriate terminal organs?

From these considerations I have been led to inquire if a difference in the resulting return of function will obtain between reunion with the most accurate coaptation, and reunion with the most extreme displacement of the one segment with reference to the other. In two dogs I divided the sciatic nerve at the level of the trochanter, and after rotating the peripheral segment to the extent of a semicircle, reunited it to the central end.* To insure that this displacement was accurately carried out, the suture was placed before the division of the nerve. The nerve having been raised from its bed on the point of the finger, was transfixed by the needle threaded with chromicised catgut. The needle was then carried through below the nerve, and the nerve again transfixed at a lower level in the same direction as before. Next, the nerve was divided between the two points of transfixion, and the suture tied, the result being that the peripheral segment was rotated with reference to the central segment through a semicircle. Thus the two segments were coapted so that the fibres of one side in the central segment were brought into line with those of the opposite side in the peripheral segment, only the fibres in the centre of the nerve being approximately correctly coapted. In a third experiment, to serve as a control to the first two, the same nerve was divided at the same point and accurately coapted, the suture being placed before the division, so as to insure accuracy of coaptation in the normal position.

Experiment I.—On 19th January 1898, a collie bitch, aged 15 months, having been anæsthetised by means of a subcutaneous injection of 0·5 gram. sulphate of morphia followed by inhalation of ether, the hair was shaved from a considerable area around the trochanter and tuber ischii of the left side. The skin was then thoroughly scrubbed with soap and warm water, next with turpentine, then with alcohol, and finally with 5 per cent. carbolic lotion. The precaution taken to prevent contamination of the wound during the operation was to fix round the body a jaconette sheet with an oval aperture in its centre, sufficiently large to expose only the field of operation. The instruments, dressings, ligatures, etc., to be used had all been previously sterilised. The sciatic nerve was exposed as it passes between the trochanter and the tuber ischii. The nerve held up on the point of the finger, was transfixed by a flat needle threaded with chromicised catgut. The needle having been drawn through, was passed underneath the nerve, and the nerve again transfixed parallel to the first transfixion and in the same direction, but about 0·5 cm. lower down, and drawn through, leaving the suture in place. The nerve was then transversely divided between the two points of transfixion, and the suture tied. The result of the two ends having

* I am indebted to Professor M'Kendrick for suggesting to me this form of experiment.

been thus transfixed by the catgut in the same direction was, of course, that on tying the knot the two ends of the nerve could be brought into apposition only after rotation on their long axes with reference to each other through a semicircle. Thus the fibres running down the external side of the central segment were brought into apposition with the fibres on the internal side of the peripheral segment, and *vice versâ*, the displacement of the ends of the fibres being thus made as extreme as compatible with coaptation of the two ends.

The subcutaneous tissue and the skin were then separately united by sutures, dressings applied, and the whole limb fixed by plaster of Paris bandages, commencing above the claws, passing up the limb and round the body, a thick double copper wire moulded to the parts being incorporated in the bandages. The effect of the morphia was to keep the dog in narcosis sufficiently long to allow of complete drying of the plaster of Paris without breakage.

At the end of the second day the animal was able to stand on the splint. There was no sensation on pricking the foot with a needle, except over the region on the inner side of the paw supplied by the internal saphenous nerve.

By the end of the third day the dog was walking freely about, using the splint as a support, but dragging the paw along the ground with the dorsal surface down. The foot was, therefore, bandaged so as to keep the plantar surface down in order to prevent further excoriation of the dorsal surface.

At the end of the seventh day, on removing the supporting bandage from the paw, the dog was able to walk across the room, placing its paw with the plantar surface down. On passing a projecting board on the floor the foot was turned back, and the animal rested on the dorsal surface, but at the next step the foot was restored to the normal position, with the plantar surface down. On pricking the skin with a needle the animal gave no sign of pain, although it turned round and commenced vigorously to lick the paw.

The first undoubted sign of returning sensation was obtained on the tenth day; for on pricking the formerly insensitive part of the foot, the animal at once withdrew the limb with signs of pain.

At the end of the fourteenth day the splint was removed, and the wound was found healed, and the stitches were removed. The animal then walked well on the unsupported leg, always placing the plantar surface of the paw correctly, although it threw the leg a little outwards in a movement of circumduction at each step, but it was not markedly lame. Occasionally it missed a step, carrying the leg forward with a hop.

On the following day the sound leg was held up without supporting the weight of the body, and it was found that the animal was able to walk, supporting the hind part of its body entirely on the affected leg. Occasionally the foot was placed with the dorsal surface down, and to give it support a figure-of-eight bandage was again applied to the paw to hold it in the extended position.

At the nineteenth day the dog was running about without any trace of limping,

and while running about, only once or twice the paw was observed to be turned over on the dorsal surface by the toes scraping the ground, but on these few occasions the normal position was regained at the next step. The position of the leg in standing, walking, and running, was the same for the affected as for the other leg, and the degree of voluntary extension and flexion at the tibio-tarsal articulation appeared to be the same in the two legs. Only slight atrophy was noted in the muscles, no hair had fallen out, the claws were normal, and two small erosions on the dorsum of the paw, caused by scraping along the floor during the first day that the animal was walking about, were now healed.

The animal distinctly now felt the prick of a needle in the paw, as, when this was done, it always looked sharply round, and occasionally tried to withdraw the limb, at times attempted to bite, and always turned and licked the paw. In testing the sensation of the paw, sensation on the inside was not taken as evidence of returning function, as this part, as already mentioned, was sensitive from the earliest examination, being innervated by twigs of the saphenous nerve. The difficulty of ascertaining the return of sensation is illustrated by the fact that when pricked on the sound leg or paw the animal often gave no sign.

On the twenty-first day sensation was considerably improved, as during the examination of the paw the animal's head had to be held, as it frequently snapped when the foot was pricked.

An attempt was made to ascertain the state of localisation of sensation by blindfolding the animal and attaching a bulldog forceps to the limb, but it was futile, as the animal's first endeavours were always to remove the bandage from its eyes.

On the thirty-second day the animal was photographed in the standing posture assumed by itself (Plate I. fig. 1).

On the fifty-fourth day, the dog having been again anæsthetised by chloroform and ether, after having a subcutaneous injection of morphia sulphate, the skull was trephined over the region of the crucial sulcus on both sides, and this region of the brain exposed. Stimulation of the centre for the hind limb on the post-crucial gyrus by a faradic current just to be felt on the tip of the tongue gave no reaction, but when the current was slightly increased, the normal reaction was induced, being the same movement which obtained in the right leg by stimulation of the centre on the left hemisphere. The movement induced was advancement of the leg as in walking, and no other movement could be obtained until the current was further increased, when general convulsions of the body ensued.

The seat of suture of the nerve was next exposed, and the nerve was found united, there being a neuroma at the seat of reunion. The torsion of the nerve produced at the operation was clearly maintained. The peripheral segment showed this torsion commencing about 3·5 cm. from the seat of reunion.

Stimulation with a very weak interrupted current induced contractions, when the electrodes were placed above the seat of reunion, on the neuroma, and distal to the

neuroma. The animal was then killed while still under the influence of the anæsthetic, and various portions of the reunited nerve removed for microscopic examination.

On excising a segment, including the seat of reunion, the proximal and distal transverse sections exposed showed that the nerve at this level consisted mainly of two fasciculi, one large and the other small. On comparing the proximal and distal ends of the segment, it was ascertained that the position of the large fasciculus exposed at the one end corresponded with the position of the small bundle exposed at the other end, and *vice versâ*, thus proving that the nerve had reunited in the position which was given it at the operation.

Experiment II.—On 13th April 1898, a collie bitch, aged 1 year, having been anæsthetised by means of a hypodermic injection of 0·5 gram. sulphate of morphia, followed by inhalation of ether and chloroform, and the same antiseptic precautions having been taken as before, the same operative procedure was carried out as in the case of Experiment I., *i.e.*, the left sciatic nerve was divided at the level of the trochanter, and the ends sutured together with carbolised catgut, with the central and peripheral segments rotated on their long axes in opposite directions, with reference to each other to the extent of a semicircle. The fascia and skin were separately sutured, and the hip, knee, and ankle joints immobilised by means of plaster of Paris bandages.

On the following day the animal was making no attempt to walk on the splint. There was no sign given of sensation on pricking the paw with a needle, except over the small area on the inner aspect of the paw from twigs of the saphenous nerve. The paw was now fixed with a figure-of-eight bandage to keep the foot from being turned over with the dorsal surface down, and thus becoming abraded in the event of the animal making attempts to walk.

At the end of the second day the dog was sitting up resting on the splint. When pulled along by its collar it walked on the splint, but dragged the paw along the ground dorsal surface down.

On the following day the animal was walking freely about, but when the foot was unsupported by the bandage, it always was dragged dorsal surface down.

This state lasted till the end of the seventh day, when it was found that, while no evidence of returning sensation could be obtained, the animal had now regained some use of the leg, as it walked well, and with the paw left unsupported, placed it correctly plantar surface down on the ground. Occasionally, the toes catching on the ground, the paw was turned over and rested on the dorsal surface, but after being scraped along the ground in this position for two or three steps, was again voluntarily replaced in the correct position.

At the eighth day the plaster of Paris splint was removed, and the wound was found completely healed, and the stitches were removed. The leg was somewhat atrophied, but there was no other trophic change. The animal used the leg in walking, not, however, resting its whole weight on the limb, but as a rule it placed the paw

correctly plantar surface down, readjusting it when it turned over, as it occasionally did, by the toes scraping the ground.

Improvement in walking advanced day by day, the foot turning over on to the dorsal surface less and less, until, when examined at the end of the twenty-first day, the animal constantly walked on the leg, always placing the plantar surface down, and supporting the entire weight of the hind part of the body on the affected leg, when the sound hind leg was held up. Sensation was now returning, as, when the portion of the paw formerly insensitive was pricked with a needle, the animal withdrew the limb.

At the thirtieth day the complete recovery of the use of the leg was maintained, and the muscles were much increased in bulk. The dog was therefore killed, and the seat of section exposed. The nerve was found reunited, the seat of reunion presenting a slight swelling on the central and peripheral ends, united by a short segment having the diameter of the normal nerve. The torsion given at the operation was clearly visible, and stimulation of the nerve above and below the seat of reunion caused contractions of the muscles.

Experiment III.—On 13th April 1898, a collie bitch, aged 6 months, was anæsthetised by a hypodermic injection of 0·4 gram. sulphate of morphia, followed by inhalation of chloroform. The same antiseptic precautions as before having been taken, the left sciatic nerve was exposed at the level of the trochanter, divided, and accurately reunited in the normal position with carbolised catgut. The accuracy of adjustment was secured by placing the suture before the division, the suture having been carried through the nerve at two points, 0·5 cm. apart, passing in opposite directions. After section, therefore, the tying of the catgut suture brought the two ends together as nearly as possible in their normal relationship to each other. The limb was then immobilised in the same manner as in the other experiments.

On the following day sensation tested in the paw was found to be absent, except over the area supplied by the saphenous nerve.

On the third day the animal was walking on the splint, but dragging the paw along dorsal surface down.

This condition lasted till the seventh day, when the animal was found to walk with the plantar surface down. The paw, however, frequently turned over with the dorsal surface down, and was then dragged, and did so till the foot was passively again placed in the normal position, the animal apparently having no voluntary power to readjust it.

On the eighth day the plaster of Paris splint was removed, and the animal was found to be able to use the leg for walking, not, however, resting its full weight on it. It walked with the plantar surface down, but frequently the paw turned back on the dorsal surface, but now the animal had regained sufficient power to enable it to voluntarily readjust the abnormal position. The wound was found quite healed, and the stitches were removed. There was no return of sensation as far as could be ascertained.

Improvement in the muscular power and control advanced, and on the eleventh day the dog was able to run about freely on the leg, only occasionally getting on to the dorsal surface of the paw, and then voluntarily readjusting the position after one or two steps. There was distinct evidence of returning sensation, as, when pricked over the formerly anæsthetic region of the paw and leg, the animal whined and withdrew the limb. When pricked on the paw, while standing, it at once withdrew the leg with flexion at the tibio-tarsal joint.

The dog by the fourteenth day was running about apparently quite recovered, and never treading on the dorsal surface of the paw. The only differences to be found in the two legs were the less keen sensation and the slight atrophy of the muscles.

On the forty-ninth day, all improvement having been maintained, the animal was killed, and the seat of section having been exposed, the nerve was found united, a neuroma having been developed at the seat of section. Stimulation with a weak current above, below, and on the seat of section caused muscular contraction. The muscles were apparently normal, though somewhat less bulky than those of the opposite limb.

Comparison of Experiments.

The conditions of these three experiments coincided so closely that a comparison between them is justifiable. All the dogs were of the same kind, and all were young. The same nerve in all was divided at the same point, and the healing of the wound in all was satisfactory, in no case becoming septic. In Experiments I. and II., however, the peripheral was sutured to the central segment after having been rotated on its long axis to the extent of a semicircle, while in Experiment III. the reunion was made in the normal position. The last was thus a control experiment, intended to ascertain if any important differences could be detected in the course of recovery of function in the two cases, viz.: in the case in which the corresponding central and peripheral segments of each of the nerve fibres were brought to lie as closely as possible opposite each other, and in that in which these two segments were displaced from each other as far as compatible with suture of the nerve stem.

During the first six days the course was the same in all three. Thus, after a day or two, the dogs were up and walking about on their splints, but with no return of function, the paw being simply dragged along the ground dorsal surface down.

In all three the seventh day brought the first improvement; for on this day all were able to walk with the plantar surface down. In all the dogs the paw was on this day frequently turned over while walking, bringing the dorsal surface into contact with the ground; but in Exp. I. and II. the animal had the power voluntarily to readjust the abnormal position of the paw, and again place it plantar surface down, while in Exp. III. no voluntary power in this respect was shown, and the foot

had to be passively replaced in the normal position. No sign as yet of returning sensation could be got by pricking the affected parts with a needle.

On the eighth day the plaster of Paris splint was removed in the case of Exp. II. and III., and in both cases the animal was able to use the leg for walking without any support, flexing and extending the leg at the tibio-tarsal joint, and placing the plantar surface of the paw correctly; and now voluntary power to readjust the paw when it turned over, had been regained in the case of Exp. III.

The first sign of returning sensation, viz., withdrawal of the limb on pricking the paw with a needle, was shown in Exp. I. at the tenth day, and in Exp. III. at the eleventh day, while Exp. II. showed no sign till the twenty-first day; but the evidence as to the date of return of sensation was very unsatisfactory, as the animals were of such a docile nature that they often allowed the sound leg to be pricked without making any sign. There is, therefore, no doubt that during the early days of returning function, the animals would not be likely to show signs of sensation if this was less keen than normally.

At the fourteenth day the splint was removed in the case of Exp. I., and the animal used the leg perfectly in walking and in running, always placing the plantar surface down; and in Exp. II. the foot very seldom turned over on the dorsal surface, and was then quickly readjusted, and in Exp. III. the same progress was made.

By the twenty-first day all had recovered not only the use, but also the power, of the leg so perfectly that the animals appeared to be perfectly normal, only close inspection showing that the affected limb was somewhat less bulky than that of the opposite side.

Microscopic Examination.

Methods.

Immediately after the death of the animals portions of the nerves were removed for histological examination. The portions removed were—a portion including the seat of reunion, portions from the central segment, various portions from the peripheral segment and its branches, muscular and cutaneous, and a portion from the opposite sciatic at the level of the section. A number of glass rods with the two ends bent over in the form of rings were at hand, and the portions removed were each stretched between the two rings of one of these glass rods. The glass rod is, I think, preferable to wood or cork for stretching, as by its use there is no possibility of contamination of the fixing agent. The portions of nerve fixed to the glass rods were then placed in Müller's fluid, in which, with frequent renewals of the fluid, they remained for eight or ten weeks. They were then prepared for paraffin, imbedded, and longitudinal sections of the seat of reunion, and longitudinal and transverse sections

of the remaining portions prepared with the rocking microtome. The sections were mounted in series.

The chief method of staining employed was that recommended by STROEBE;* but, as I had formerly some difficulty in getting adequate staining of the axis-cylinder by this method, although closely following the directions given in STROEBE'S paper, I give here the steps of the process which I have followed, and by which fairly good differentiation of the axis-cylinder has been obtained. The deviations from the method are principally those of the time allowed for each step.

1. After removal of the paraffin, the slide bearing the series of sections is washed for a few minutes in running water, so as to displace the alcohol, and then placed in a tube containing freshly-prepared saturated aqueous solution of anilin blue (GRÜBLER'S for STROEBE'S stain), in which it remains for twenty-four hours.

2. It is then washed in running water for some minutes.

3. It is next placed in a dish containing a solution prepared by adding 15 drops of a 1 per cent. solution of caustic potash in absolute alcohol to 12 c.c. absolute alcohol. The dish is rocked from side to side, and the blue stain is extracted from the sections, and this process is continued until the sections have a blanched blue appearance, usually about one minute or less. Formerly I allowed the sections to remain in this destaining liquid too long, waiting for the appearance of the clear brown-red colour described by STROEBE. That tint never appeared, and the sections, when examined, were found to be over destained, and the axis-cylinders therefore badly differentiated. I found afterwards that with thicker sections cut in celloidin, the sections did assume a brownish-red tint in the destain, while the stain was removed in brownish-red clouds; but with the extremely thin sections cut by the rocking microtome, it is only at the moment that the sections are placed in the liquid that a slight brownish-red cloud appears, after which the blue of the section simply continues to fade. The obvious explanation of the difference is that with the thicker sections the brownish-red altered stain is not so easily washed out by the alcohol, and, remaining entangled in the tissue, gives the section the brownish-red tint, while with thinner sections the altered stain is at once washed out by the alcohol, leaving the bluish tint unaffected.

4. The slide is then washed in running water for some minutes, when the intensity of the blue is somewhat restored.

5. It is then placed in a tube containing anilin-water-safranin solution, in which it remains for half an hour.

6. It is next rinsed in water, dehydrated rapidly in absolute alcohol, and treated with oil of cloves. This is allowed to remain on till the red colour begins to fade, but before the bluish colour reappears, when the oil is drained off.

7. The clove oil is then washed off with xylol, and the sections mounted in balsam.

Much depends on stopping the extraction of the safranin by the oil of cloves at

* Ziegler's *Beiträge*, 1893, vol. xiii. p. 160.

the right moment, as if allowed to act too short, the axis-cylinders appear red, and if too long, the colour is extracted from the myelin.

An addition to the method, which gives very beautiful results in some cases, is that of rinsing the slide after removal from the safranin solution for a few seconds in a very dilute aqueous solution of eosin. With this addition all parts of the section are stained red, except the axis-cylinders, which stand out in blue, and the clear differentiation thus obtained between the young axis-cylinder and connective tissue fibres is of much advantage. The objection, however, is that the eosin is apt to stain also the axis-cylinders so intensely as to be incapable of removal by the oil of cloves, unless the immersion in eosin is very short, and possibly in all cases some of the finer axis-cylinders are so stained.

Histological Characters.

Central Segments (Plate III. fig. 4).—The central segments of all three, down to about 5 mm. from the seat of reunion, present the characters of a normal sciatic nerve. Comparison with sections made from the opposite sciatic, brings out only one difference, namely, that the nerve is thicker than the normal sciatic, and that this increase in bulk is due not to any numerical increase of nerve fibres, not to any hypertrophy of the fibres themselves, nor to overgrowth of connective tissue, but to the fact that the individual nerve fibres are separated by wider spaces than in the normal nerve. The increase in thickness is thus due to distension of the lymphatic spaces surrounding the nerve fibres. At a distance of about 5 mm. from the termination of the central segments, the alteration in character of the nerve commences; for here many of the medullated nerve fibres are seen to cease abruptly, often in bulbous ends, and their place to be occupied by a bundle of young nerve fibres. As the termination of the segment is further approached, this replacement of old fibres by new becomes more and more complete, until, finally, no adult medullated fibres are present, and the entire fasciculus is composed of new-formed nerve fibres. The only difference between the three central segments is that of the stage of development; for, while in Exp. I. no distinct traces of degeneration products remain, in Exp. II. there are distinct remains of the old myelin situated between the young nerve fibres, and the same is also the case, but to a much less extent, in the central segment from Exp. III.

Peripheral Segments (Plate III. fig. 5).—The peripheral segments of all three agree in presenting full regeneration of young nerve fibres, and this is the case throughout the entire nerve and its branches, muscular and cutaneous. All were examined, both close to the seat of reunion and far removed from it, without showing any adult nerve fibres. In Exp. I. the nerve close to the seat of reunion presents many young fibres, which are well developed, each presenting a clearly defined axis-cylinder, traceable for considerable distances in the section, a well-marked but thin medullated sheath, and a delicate

sheath of SCHWANN, having at intervals spindle-shaped nuclei encroaching on the medullated sheath. Such fibres are, however, very much smaller than the medullated nerve fibres of the central segment. Many other fibres are present, which show less distinct characters, and are finer, but in which the axis-cylinder is equally distinct, but with less prominent medullated sheath; while in other cases the axis-cylinder is so indistinct as to be recognisable with difficulty, and only so in parts of the fibre, the parts in which it is not recognisable looking very like connective tissue fibres. At this level of the segment there are a few bulky leucocytes present, but no distinct remains of the degenerated old fibres. In portions of the branches of this nerve from the middle of the leg the most striking difference is that there are present remains of the degenerated old fibres—balls of myelin arranged in columns one behind the other, together with many leucocytes loaded with degeneration products. But between these degeneration remains are present abundant young nerve fibres, either singly or in bundles.

In Exp. II. the peripheral segment presents also an abundant supply of young nerve fibres, but being at the thirtieth day of development, the stage is not the same as in Exp. I., which is at the fifty-fourth day of development. The essential differences are that the young nerve fibres do not show the medullated sheath so distinctly as do the furthest advanced fibres in Exp. I., and that degeneration remains are still present close to the seat of reunion, lying between the young fibres, while in the sections prepared from the terminal divisions of the nerve the degeneration remains are still more abundantly present, with the young fibres still finer and less distinct.

In Exp. III., at the forty-ninth day of development, the young nerve fibres are not quite so far advanced as in Exp. I., and some distinct remains of old degenerated fibres are still present, not only in the terminal divisions of the nerve, but also close to the seat of reunion.

The Nerve Cicatrices (Plate II. figs. 2 and 3).—The appearances presented in each of the three cases show no important differences. The fasciculi of the central and peripheral segments pass into the cicatrix, the nerve fibres diverging to some extent, the fibres of the central segment being, as in the peripheral segment, in every case new-formed fibres. The continuity of these fibres across the cicatrix is not traceable, although at some points, especially in the sections from Exp. I., it is clear that the fibres do in reality pass right across, but that they are not traceable simply from the fact that they pass out of the plane of the section. This is shown in the middle line of the section, in which fibres may be traced for a considerable distance, until they suddenly terminate, their further course being contained in other sections of the series. But fibres which for some distance have run parallel, but which drop out of the section when traced centralwards, continue their course into the peripheral segment. In the greater part of the cicatrix the fibres evidently run a very tortuous course, as they are seen in the sections cut in all directions, some showing for short distances in longitudinal section, while others appear as small clusters of transversely cut fibres. The entire cicatrix has

a fairly dense appearance, being made up of a tangle of fibres. Much of this tangle is composed of undoubted young nerve fibres, the stain bringing out the axis-cylinders, but in addition there are present undoubted connective tissue fibres, and also fibres of which it is doubtful to which category they belong. The appearances presented are, in short, those of a neuroma, and this description may be applied to all three cases.

Deductions from Experiments.

The results of these experiments show that in the dog there is, in the case of section of the sciatic nerve, no important differences in the return of function whether the two ends of the nerve are sutured so as to bring the corresponding segments of the fibres into accurate apposition, or whether, by rotation of the peripheral segment, the ends of nerve fibres which do not correspond are brought into apposition. Provided that the two ends of the divided nerve are brought into apposition, whether in the old relationship or in a new, reunion of the nerve follows, and the normal function returns. This return of function, also, not only is qualitatively the same in the two cases, but the time taken for the first evidence of restoration is also the same, namely, seven days, and the further improvement, which rapidly follows, runs essentially the same course in both cases. Thus voluntary co-ordinated movements of the hind limb were regained as soon in Exp. I. and II. as in Exp. III. This is the more striking in a nerve like the sciatic, which not only is distributed to so many different structures, but which is composed of nerve fibres which supply antergic muscles, or muscles with opposing functions. Thus, when the animals commenced to walk on their splints, the paw was always dragged along the ground with the dorsal surface down, but on the seventh day, the foot being placed with the plantar surface down indicated a return of function in the extensor muscles; and it is noteworthy that in the cases of Exp. I. and II. there was more control over the extensor muscles than in the case of Exp. III. at the seventh day, as in the former cases the animals had the power voluntarily to readjust the paw when it turned over, a power not shown in the latter case till the day following.

Such early return of function suggests the probability of the nerve having united by first intention, but as stated above such a view implies that Wallerian degeneration had not taken place, having been prevented by the early reunion of the peripheral segment to its trophic centres, and that the nerve fibres of the peripheral segment in the reunited nerve are the identical fibres which existed before the section. That this was not the case is proved by the microscopic examination; for not only are no adult nerve fibres found in the peripheral segment, their place being taken by young nerve fibres (Plate III. fig. 5), but in all cases evidence of degeneration of the old fibres is abundantly present.

Vicarious nerve supply has often been suggested as an explanation of early return of function, but in the experiments this possibility is quite excluded from the extensive distribution of the sciatic. Thus the section of the sciatic at the level of the trochanter

certainly paralysed all the muscles below the knee-joint, and the recovery of voluntary flexion and extension of the paw, and at the tibio-tarsal articulation, implies regained conductivity of the sciatic, as there is no other nerve entering the leg which could possibly have taken up its function. The prompt muscular responses elicited by stimulation of the nerve above, below, and on the cicatrix, also prove that the muscles were still innervated through this nerve.

The early return of function is adequately explained by the view of regeneration of nerves which I have already elsewhere * advocated, namely, that the young nerve fibres are formed in the peripheral segment simultaneously with the degeneration process, and that they become connected with those of the central segment by means of young fibres formed in the cicatrix from cellular elements which have migrated into the cicatrix from both ends of the nerve, the cellular elements in question having been derived by proliferation from those of the sheath of SCHWANN.

The uniform time at which function commenced to return, and the uniform rate with which improvement advanced are, I think, explained by the fact that in all cases the wounds healed without becoming septic, a result due not only to the strictest antiseptic precautions, but also to the complete immobilisation of the entire limb while the process of healing was in progress. A nerve may certainly unite, and conductivity ultimately be restored, even although the wound becomes septic, a fact of which I have recently obtained clinical proof, but the return of conductivity is in such cases uncertain; it may never return, but if it does so, is much delayed. But if the wound runs an aseptic course, the experiments would seem to show that return of function takes place uniformly, and equally so whether the two ends are united in their old relationship or not.

The uniform manner in which return of co-ordinated function was established in the experiments might be hypothetically explained in different ways. Thus it might be supposed that in the cicatrix in Exp. I. and II., which formed between the two divided ends of the nerve, crossings of the young nerve fibres were effected so as to place the corresponding nerve fibre segments in connection. This view need not imply such an improbable thing as inherent faculty or instinct in the nerve fibres to grow towards their appropriate peripheral segments, but might possibly be brought about in a way more in accord with natural processes. Thus, at the earliest stage of development of nerve fibres in the cicatrix, when the spindle-cells derived from the immigrated neuroblasts were making connections one with the other, it is conceivable that the nervous impulses would at first pass into the network of spindles in the cicatrix, and be communicated to the fibres of the peripheral segment diffusely, the impulse passing along a single central fibre being diffused through many or all of the nerve fibres in the peripheral end. On such a supposition it is possible that the old path in the peripheral segment would be the easiest for the passage of the impulses, and that the connections in the cicatrix which established the passage between the corresponding ends of the

* *Loc. cit.*

fibres would, therefore, from being the most selected route, become more rapidly developed, structural development thus following on functional activity. Thus in the cicatrix paths of connection between corresponding nerve fibre segments would be established. On such a supposition, it would be expected that the microscopic appearances would be different, according as the two nerve segments were accurately coapted or not, as in the former case the connecting nerve fibres might be expected to follow an approximately direct course in the cicatrix, and in the latter to follow a crossed course—to show, in other words, a decussation. But the microscopic appearances in the three cicatrices examined, as stated above, presented no very marked differences, in each case showing throughout its greater part much convolution of the young nerve fibres. The microscopic appearances, however, though they do not prove that corresponding nerve fibre segments become connected, do not disprove it, as in Exp. III. the convoluted course exhibited by the nerve fibres does not necessarily preclude the possibility of the corresponding fibres connecting, although by a tortuous path. It is, therefore, impossible to state from the above experiments whether or not the return of co-ordinated movements was the result of the re-establishment of the old paths for nervous impulses, although the histological appearances render the possibility extremely doubtful.

The restoration of function might be explained in a different way. Thus it is possible, owing to the intimate admixture of the nerve fibres in the nerve stem, that, despite the altered relationship of the two segments, yet fibres were brought into juxtaposition which innervated muscular fibres in close proximity, *i.e.*, muscular fibres belonging to the same muscle or group of muscles, and that therefore the resulting inco-ordination was so slight as to be without functional importance. Such a supposition would of necessity leave much to the operation of chance, and would mean in a nerve like the sciatic, with its extensive distribution to antergic muscles, that the functional result would be the balance between the number of fibres distributed to flexor muscles and the number to extensor muscles which happened to be brought into apposition. It is not probable that the admixture of nerve fibres in the nerve stem, and the relationship in which the two segments are placed in different cases of suture, are so uniform that this balance would be identical in different cases, and consequently on such a view a difference of development of restoration of function would be expected, not only in different cases of experiments such as I. and II., but a marked difference in development of function would be expected between cases of accurate and of inaccurate coaptation. The supposition is therefore an unlikely one, as not only did Exp. I. and II. agree in the dates of returning co-ordination and in following a practically identical course, but the same also occurred as between Exp. I. and II. and Exp. III.

The third hypothesis which may be brought forward to explain the results is, that although non-corresponding segments of the nerve fibres were made to connect by the mode of suturing the nerve, the nerve centres, brought by the altered paths into connection with new peripheral endings, were capable of speedily altering their

characters, and of supplying the required nervous impulses in perfect co-ordination. This hypothesis implies a capacity of the organism to undergo a profound change in the functions of the central nervous system. It implies that the centres controlling different groups of muscles can interchange their functions, and carry on co-ordinated movements as before. This question has frequently been tested by means of nerve crossing, or the division of two neighbouring nerves and cross union of the ends by suture, to ascertain if under the new conditions the animal would regain its lost functions. A review of the literature of this subject shows that opinions as to the possibility differ. Thus, of the more recent contributors to this subject, RAWA,* STEFANI,† and CUNNINGHAM,‡ the first two affirm the possibility, while the last named denies it. I have conducted some experiments of this kind, and the results, although not yet ready for publication, lead me to adopt the views of RAWA and of STEFANI. The microscopic appearances, as already mentioned, from the extreme degree of convolution of the fibres in the nerve cicatrix, do not give a satisfactory answer to this question, all that can be taken as evidence being that the appearances are much the same, whether accurate union was effected or union after twisting of the peripheral segment. Seeing that function was restored as soon in the one case as in the other, whatever the explanation may be, it may be taken as a practical result of the experiments that in reuniting a divided nerve, it is not necessary to take care that the two segments are brought together in their old relationship, but only to see that the two segments are brought into apposition, no matter in what relationship.

I conclude from the results recorded :—

1st. That after section and immediate coaptation of a nerve, restoration of conductivity and of voluntary function may be effected in a few days.

2nd. That this early restoration of conductivity need not be the result of reunion of the old nerve fibres, *i.e.*, reunion by so-called first intention or without Wallerian degeneration, but may be the result of regeneration of young nerve fibres in the peripheral segment.

3rd. That voluntary co-ordinated movements are regained equally soon, whether the two ends of the divided nerve are united as accurately as possible, so as to bring the corresponding ends of the nerve fibres into contact as nearly as possible, or whether previous to reunion the peripheral segment is twisted so that, when united to the central segment, non-corresponding ends of the nerve fibres are brought into contact.

4th. That in the latter case the microscopic examination of the seat of reunion leaves it doubtful whether the restoration of function is due to the re-establishment of the old paths by decussation in the nerve cicatrix, or to the reunion of ends of nerve fibres which do not correspond, but which happen to be brought into apposition.

5th. That in suturing a divided nerve no trouble need be taken to secure that

* *Archiv für Physiologie*, 1885, S. 296.
† *Archiv für Physiologie*, 1886, S. 488.
‡ *American Journal of Physiology*, vol. i., 1898, p. 239.

coaptation of the two segments is effected in the old relationship, the simple approximation of the two ends, no matter in what relationship, being all that is required.

In conclusion, I have to express my thanks to Professor M'KENDRICK and to Professor YOUNG, in whose laboratory the histological work was done, for much valued counsel in connection with this research, and to Principal M'CALL, of the Glasgow Veterinary College, who granted every facility for conducting experiments.

EXPLANATION OF PLATES.

PLATE I.

Fig. 1. *Experiment I.* Division of left Sciatic Nerve, rotation through a semicircle of peripheral segment, and suture in that abnormal relationship of the two segments. Shows the dog thirty-one days after the operation. The left hind leg is in the position in which the dog itself placed it. The normal positions of the paw and at the tibio-tarsal articulation are exhibited. The cicatrix of the operation wound is indicated in the area which was shaved before operation, and on which, when the photograph was taken, the hair had not yet grown.

PLATE II.

Fig. 2. *Experiment I.* Longitudinal Section of the nerve cicatrix fifty-four days after section. A fasciculus of nerve fibres of the central segment is shown at the upper part, and the corresponding fasciculus of the peripheral segment on the opposite side at the lower part of the section. In the middle line of the section the fibres are almost traceable across the cicatrix, but in other parts the structure is that of a neuroma, and the course of the nerve fibres, therefore, not traceable.

Zeiss, Obj. a. $\frac{12}{1}$.

Fig. 3. *Experiment I.* Portion of same longitudinal section from which fig. 2 was prepared. Shows the neuromatous structure displayed in a large part of the cicatrix, in which the continuity of the nerve fibres across the cicatrix is not traceable. Shows some fibres longitudinally, but not traceable for any great distance, and others cut transversely.

Zeiss, Obj. D. $\frac{300}{1}$.

PLATE III.

Fig. 4. *Experiment I.* Longitudinal section of central segment of reunited sciatic close to the cicatrix. Shows adult medullated nerve fibres.

Zeiss, Obj. D. $\frac{300}{1}$.

Fig. 5. *Experiment I.* Longitudinal section of peripheral segment of reunited sciatic close to cicatrix. Shows young nerve fibres, and no adult medullated fibres, demonstrating that restoration of conductivity of the nerve was the result not of union by "first intention," but of regeneration of the peripheral segment. The same structure is exhibited also in the terminal divisions of the nerve.

Zeiss, Obj. D. $\frac{300}{1}$.

Dr R. KENNEDY on the Restoration of Co-ordinated Movements after Nerve Section.—PLATE I.



FIG. 1.

Dr R. KENNEDY on the Restoration of Co-ordinated Movements after Nerve Section.—PLATE II.

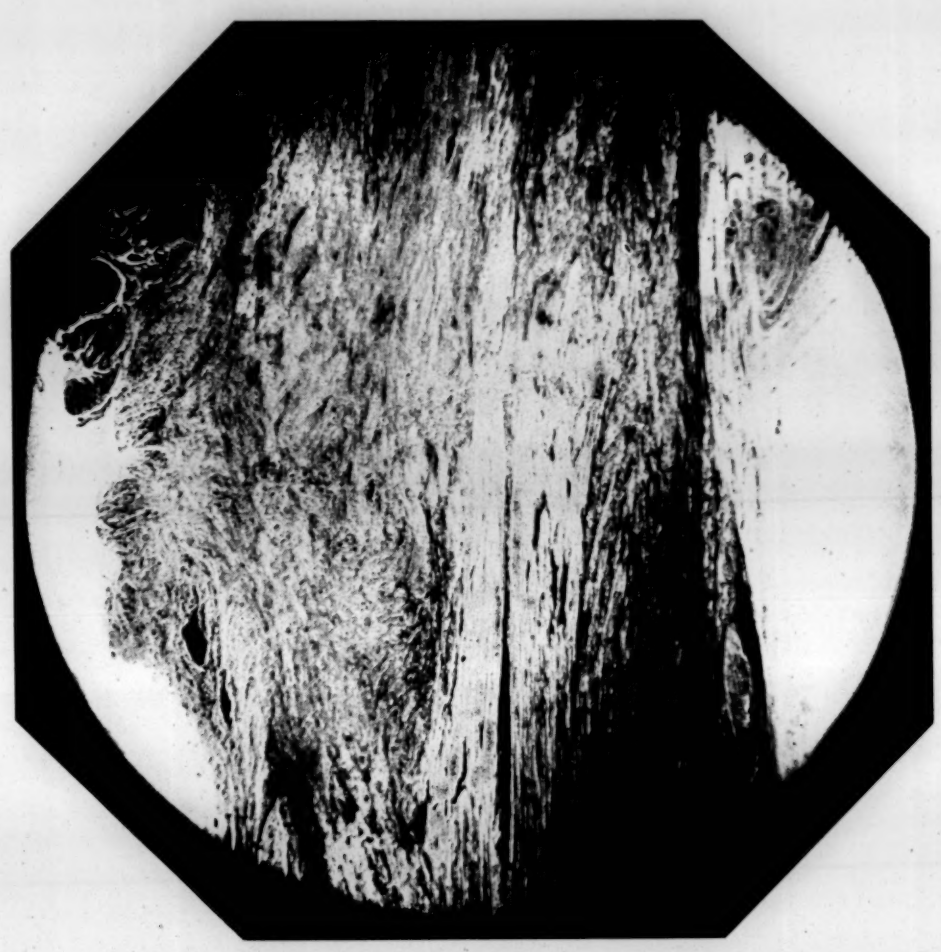


FIG. 2.

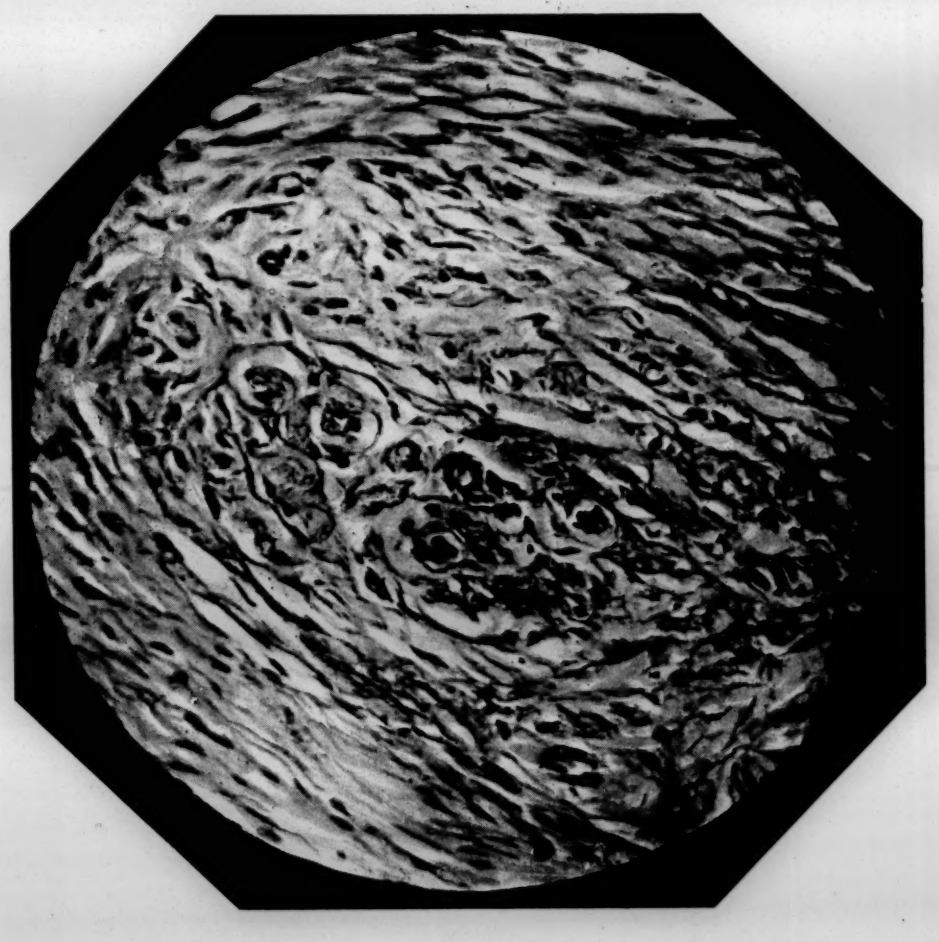


FIG. 3.

Dr R. KENNEDY on the Restoration of Co-ordinated Movements after Nerve Section.—PLATE III.

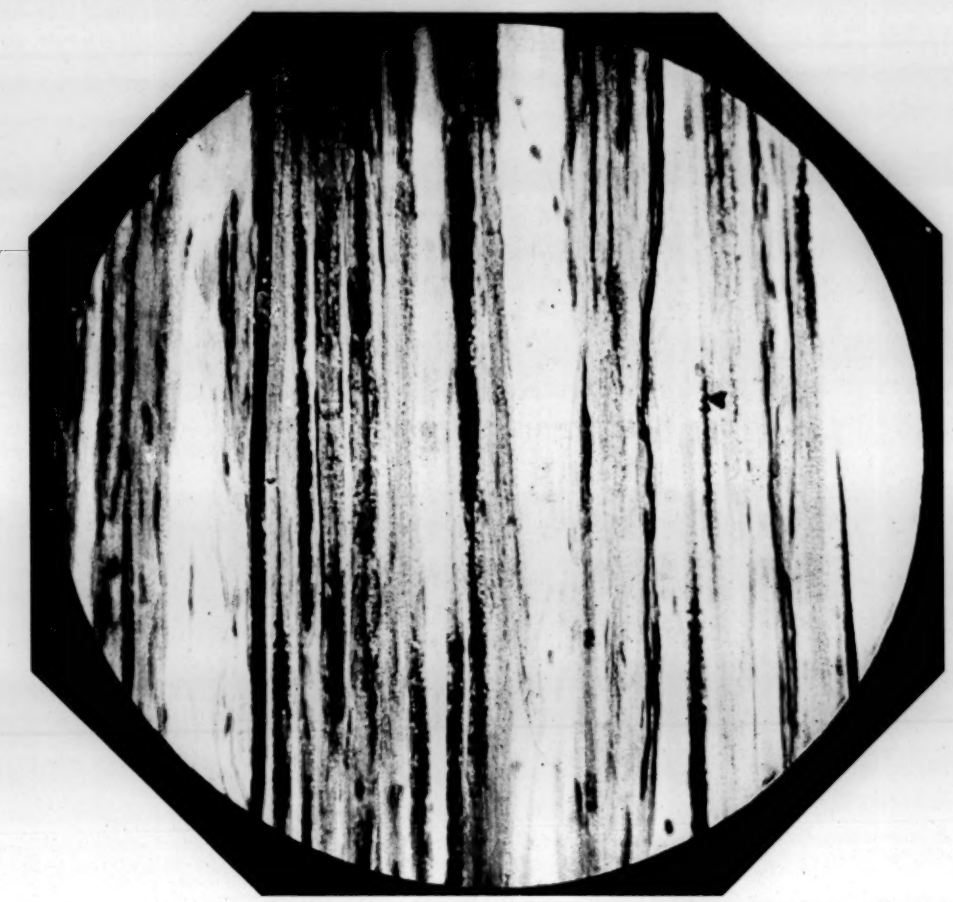


FIG. 4.

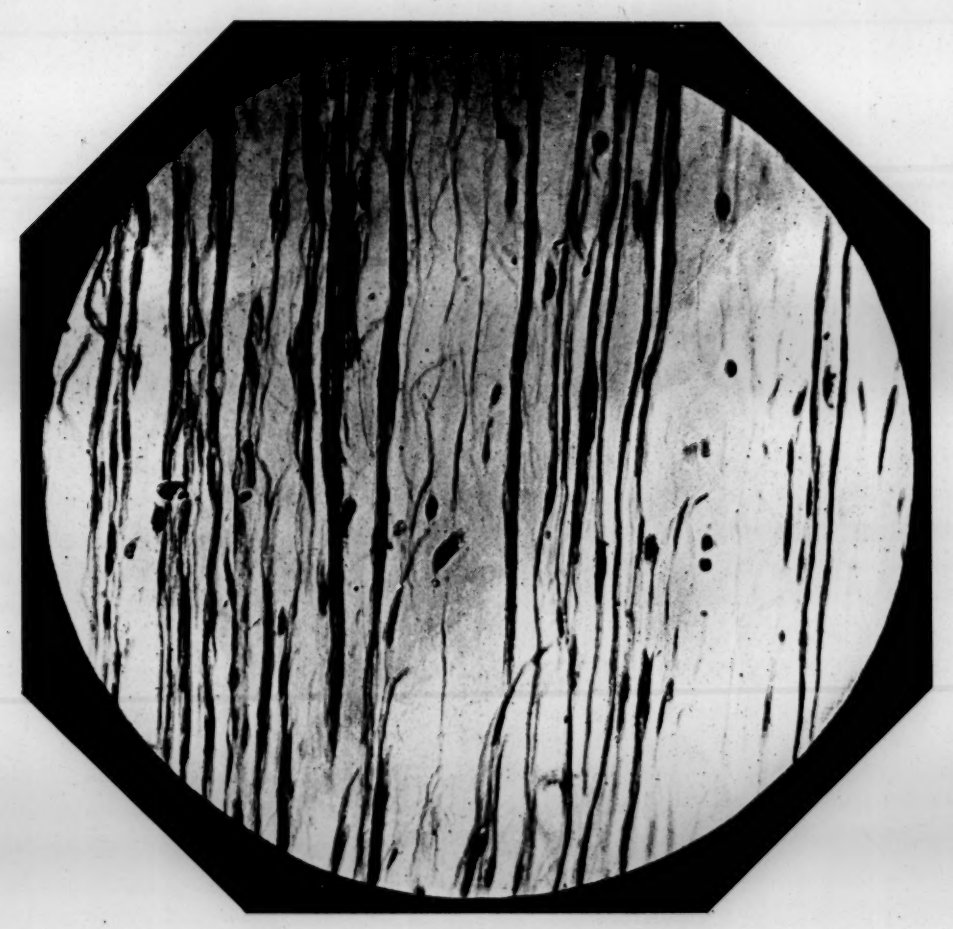


FIG. 5.

XXVIII.—*Contributions to the Craniology of the People of the Empire of India.* Part I. *The Hill Tribes of the North-East Frontier and the People of Burma.* By Professor Sir Wm. Turner, M.B., D.C.L., F.R.S. (With Three Plates.)

(Read July 3, 1899.)

For a number of years I have been collecting specimens and conducting an investigation into the craniological characters of the native inhabitants of our great Indian Empire, and several hundred skulls have now been under examination, and almost all have been measured. The sources to which I have been indebted for material are in part the collection of crania belonging to the Henderson Trustees, long known as the Edinburgh Phrenological Museum, and now deposited by the Trustees in the Anatomical Museum of the University; in part, a few specimens belonging to the University collected by my predecessors in office; in part, the valuable series of Indian crania belonging to the Indian Museum, Calcutta, which through the intercession of Dr John Anderson, F.R.S., the former Director, the Trustees of that Museum, with great liberality, most courteously permitted me to have the loan of for purposes of study; and lastly, a number of crania which have been forwarded to me by friends and former pupils, engaged in the public service in India, to whom I take this opportunity of expressing my indebtedness for the valuable material which I have received from them.

Owing to the number of specimens and the wide range of country from which they have been derived, I have thought it advisable to depart from my original intention of including in one memoir my observations on the whole series of crania, and in preference, to arrange and publish them in groups, based on the geographical distribution of the people.

The skulls described in this, the first part of my memoir, are sixty-four in number, and include specimens from the hill tribes of the North-east frontier of India and from Burma. For purposes of comparison I have also given tables of measurements of skulls from China and Siam.

Hill Tribes.

Before I commence the description of the skulls of the Hillmen, it may be well to preface the anatomical details with some reference to the localities from which the crania were obtained, as well as the names which have been given to the places and to the people who dwell in them.

In entering on the consideration of the savage and barbarous tribes who inhabit the wide range of mountainous country which lies south and east of the river Brahmaputra and Assam on the one hand, and north and west of Burma on the other, we are confronted by differences in the nomenclature employed by those who have explored

this extensive region, and have written descriptions of its inhabitants. Travellers who have approached the hills from the side of India have applied to the places and people such names as the natives of Bengal have been in the habit of using, whilst those who have entered from the Burmese frontier have employed Burmese names to designate the same tribes and localities. As regards the Hillmen themselves, as they usually neither recognise nor pay allegiance to any central authority, they do not apparently possess race or tribal names, but call themselves after the village, or group of villages, in which they live; or after the petty chief who for the time being exercises authority over them. In some villages no chief appears to be recognised, and the government is a democracy in which all the men are on an equality. The want of a common tribal name is also accentuated by the fact that in adjoining hill ranges the language in use possesses such dialectic differences that the words employed are often mutually unintelligible—a condition which is probably due to the state of constant feud in which the people live, so that they have had but little intercommunication with each other, except as enemies.

The name by which the Hillmen on the north-east frontier first became known to Europeans was that of Kookie, which is a Bengalee word for highlander, and is also written Kuki or Cúcì. As Kookie it appears in a letter addressed in 1777 by the Chief of Chittagong to Warren Hastings.* In 1778 the Honourable Robert Lindsay, who was Collector at Sythet, speaks of the hill people as Kukis.† He describes them as living more in the style of the brute creation than other savages that he had seen. Their habitations were on spreading trees to defend them from beasts of prey; their food was wild honey and the fruits of the forest. The form Cúcì was used by Mr John Rawlins‡ in 1790 in his description of the mountaineers of Tipra (Tipperah), to the east of Bengal, and it was also employed by Mr J. Rennel in 1800 to designate the same people. §

Mr John Macrae, surgeon at Chittagong, writing in 1801, ‖ states that the Kookies or Lunctas, who live in the mountains north-east of Chittagong, are active mountaineers, but not tall. The face, he says, is like that of eastern Asiatics, broad and round; the nose is flat, the eye small. The men go naked, hence the term Luncta, though the chiefs wear a black loin cloth, and the women an apron. The chiefs bring the hair forward and tie it in a bunch to overshade the forehead, whilst the other Kookies wear it loose over the shoulders.

Colonel Lewin, who acted for many years as Deputy Commissioner in the Chittagong district, and who also accompanied the Lushai Expedition of 1871–72, uses the term Lhoosai or Lushai as equivalent to Kookie, and states that it is derived from "Lu," signifying head, and "sha," to cut, from the practice of decapitating their enemies. In

* Quoted in the *Report* on the Hill Tracts of Chittagong, by Deputy Commissioner T. H. Lewin. Calcutta, 1869.
† *The Thackerays in India*, by Sir W. W. Hunter. London, 1897.
‡ *Asiatic Researches*, 1790, vol. ii. p. 187.
§ Quoted in Deputy Commissioner Lewin's Report, p. 109.
‖ *Asiatic Researches*, 1801, vol. vii. p. 185.

one passage he says that these people are named Lankhé by the Burmese.* He arranges the people occupying these hill tracts, into the Khyoungtha, children of the river, and the Toungtha, or children of the hills. These words, he says, are both Arracanese. The Khyoungtha conform to Buddhist customs, and he considers them to be of pure Arracanese origin. The Toungtha are, he believes, the aboriginal people, and under this name he includes the Tipperah tribes, the Kumi, Mroos, Khyengs, Bungees, Pankhos, Shendoos, and the Lushais or Kookies with their offshoots. In his introductory remarks LEWIN states (p. 33) that the general physique of the hill tribes is strongly Mongolian: broad faces, flat nose with no perceptible bridge; eyes narrow and set obliquely; high cheek bones, no beard or moustache, stature about 5 ft. 6 in. In his special description of the Lushais he says, however, that they differ entirely from the other hill tribes of Burman or Arracanese origin, in that their faces bear no marks of Tartar or Mongolian descent; their complexion is swarthy; the height of the men is about 5 ft. 8 in., that of the women 5 ft. 4 in. In his subsequent book, *The Fly on the Wheel*, written after he had penetrated some distance amongst the Lushai hill tracts, as a member of the military expedition of 1871–72, he repeats the statement that the features did not have the Mongolian type, but were more like Portuguese half-castes. The hair, he says, is black, and fastened in a knot on the nape of the neck.

Colonel WOODTHORPE, R.E., who was also a member of the Lushai expedition of 1871–72, gives an account of the people.† He states that they were of three tribes—Lushais, Paités or Soktés, and Pois. Both sexes were well made and muscular; the average stature of the men was 5 ft. 6 in., that of the women 5 ft. 4 in. The colour of the skin was every shade of brown, but the Pois were fairer than is usual with hillmen. The cheek bones were high and prominent, the face broad, the lips thick, the nose usually *retroussé*, with wide nostrils; though in the higher classes the nose was sometimes thin and aquiline and with small nostrils, and the lips were thin. The eyes were small and almond shaped; the beard and moustache were scanty. The tribes differed in their mode of wearing the hair. The Lushai men part it in the middle, smooth it on each side, bind it in a knot at the nape of the neck, and secure it by a copper or steel pin. The Sokté men do not part it, but wear it short and standing out around the forehead; sometimes the hair is twisted into a tail behind. The Poi men part the hair across the head from ear to ear; that in front of the parting is drawn forwards into a high double knot on the forehead and fastened by a comb; that behind the parting hangs in wavy curls over the back and shoulders. The dress is a long sheet of cotton cloth. The women sometimes dilate the lobe of the ear with a disc of baked clay.

In Mr E. A. GAIT's Report on the Census of Assam‡ it is said that the tribes variously

* See his *Report* on the Hill Tribes of Chittagong, 1869, already quoted, and his book, *A Fly on the Wheel*, London, 1884. Possibly Lankhé is a modified form of the word Luncta used by Mr John Macrae.

† "The Lushai Expedition," 1871–72, in *United Service Institution Journal*

‡ *Census of Assam*, 1891.

known as Kuki, Lushai, Poi, etc., are closely allied. They are all of the Mongolian type, being a short, squat, muscular people, but effeminate in appearance. Mr BAKER gives in the Report the height of a Kuki measured by him as 4 ft. 11½ in. The return made in the census of Assam, 1891, of the tribes designated as Kukis and Lushais was 60,652 of both sexes.

In 1828 Lieut. T. A. TRANT gave an account * of the Khyen tribe inhabiting the Yuma Mountains between Ava and Arracan. He states that they differed in several respects from the Burmese: their faces were flatter and not so regular, and the girls tattooed the face. The men wore a black cloth, striped red and white, over the shoulders, a black cloth round the loins, and occasionally a black jacket; the women wore a black petticoat reaching to the knees.

Major G. E. FRYER describes by the name of Khyengs† tribes extensively distributed in the western mountains of Burma from 18° to 21° N. lat. The people who came under his observation belonged to the Sandoway district, Arracan. The Khyengs, he says, regard the Shendoos (Chins), Khumis and Lungkhes (Lunctas) as of the same race as themselves, and the tradition is that they came from the sources of the Kyendweng (Chendwin) river. Major FRYER gives some interesting facts on their physical characteristics. The average height of twenty-five men was 65·2 inches, and their weight was 110 lbs.; the average height of twenty-five women was 57·4 inches, and their weight 94 lbs. The colour of the skin corresponded with No. 28, and that of the eyes with No. 1 of Broca's Tables; the hair was black, though some women had reddish-brown patches on the crown of the head. The faces of the women were tattooed. The heads of a number of men and women were measured, and the mean length in the men is given as 7·5 inches, the mean parietal breadth 5·5 inches; interzygomatic breadth 5·3 inches. The corresponding dimensions in the women were 6·8, 5·0, and 5·2 inches. The length-breadth index of the head, calculated from these data, gave 73·3 for the men, and 73·5 for the women; so that both sexes were distinctly dolichocephalic. As to clothing, the men wear a loin-cloth, passed between the thighs with an end hanging down in front and behind, whilst the women wear a loose blouse reaching to the knee. As regards the practice of wearing the breech-cloth tucked between the legs like a dog's tail, LEWIN states that the Kúmi are called by the Arracanese, Khivé mi, dog-men, though he thinks that the name may also refer to the practice of eating dog for food.

LEWIN, FRYER, and other writers make reference to tribes situated to the east of the Lushai hill-tracts by the name of Shendoos or Shendús. Little that was definite was known about them until the annexation of Upper Burma brought our Government officials into contact with the wild mountain tribes living to the east of the Koladyne river. These tribes were known to the Burmese as Chins. The Chin hill-tracts lie between the Koladyne river and the Chinduri river, and the ranges extend northwards

* *Asiatic Researches*, vol. xvi. p. 261.
† *Journal Asiatic Soc.*, Bengal, 1875, vol. xliv. part i. p. 39.

beyond latitude 24°. Owing to depredations committed by the Chins it was found necessary to organise an expedition against them in 1889–90.

Surgeon-Lieut.-Col. A. S. Reid has published an interesting account of the expedition, along with maps of the Lushai and Chin hill-tracts.* He regards the Koladyne river as separating the Lushais on the west from the Chins to the east, and he considers that the Burmese word Chin should replace the name Shendú given to these people by those who approached their hills from the Indian frontier.

Whilst exhibiting differences in dialect and dress, Dr Reid regards the Lushais and Chins as practically one race. The men, he says, are well built, with strong limbs and good figures. The average height is about 5 ft. 6 in., though individuals approach 6 ft. Like the Lushais, the northern Chins gather the hair in a knot on the nape of the neck, but the tribe of Baungshes wear it on the forehead. The Soktés, again, have it short, and outstanding like the tresses of Medusa. The mode of dressing the hair accords with Colonel Woodthorpe's description. The Chinmen have a small loin-cloth, and a large shawl or blanket thrown loosely over the shoulders; the clothes of the chiefs are in coloured patterns. A haversack of hairy skin is worn on the right side, suspended by a strap from the left shoulder. The women wear a dark cloth jacket and skirt; the latter is sometimes woven in coloured patterns.

The tribes which inhabit the Kachin Hills on the borders of Upper Burma and Yunnan are often called Kachins or Kakhyens, though a more appropriate name is Chingpaw or Singpho. They have been described both by Dr John Anderson† and Mr E. C. S. George.‡ Their ancestral home was apparently the head waters of the Irrawaddy, and they are probably offshoots of the same race as gave origin to the Chins. The men are said to average 5 ft. 4 in. in height, and the women are three or four inches shorter. The oblique eyes widely separated, high cheek-bones, colour of skin from a brunette almost to black, point to their Mongolian affinities. The nose, however, varies from aquiline to a broad, squat projection on the face. The hair varies between black and brown; the eyes between dark and light brown.

South-east of Assam and north-west of Burma, and in proximity to the state of Manipur, are ranges of hills which lie between 25° and 28° latitude and 93° to 97° longitude. Our knowledge of the tribes inhabiting them is largely due to Captain Butler,§ Colonel Woodthorpe,‖ Mr G. H. Damant,¶ Dr Brown,** and General Sir James Johnstone.†† The principal tribes inhabiting these mountains are called Nágás,

* *Chin-Lushai Land.* Calcutta, 1893.

† *Expedition to Western Yunnan*, Calcutta, 1871.

‡ Appendix to *Census of Burma*, 1892.

§ *Journal Asiatic Soc.*, Bengal, 1875, vol. xliv. part i. p. 307.

‖ *Journal Anthropol. Inst.*, 1882, vol. xi. pp. 56, 196.

¶ *Journal Royal Asiatic Soc.*, 1880, vol. xii.

** *Statistical Account of the Native State of Manipur*, 1873.

†† *Experiences in Manipur and the Nágá Hills.* London, 1896.

An excellent account of the social structure, religion, myths, dances and songs, cultivation, trade and war of the Nágás has been compiled by Miss Gertrude M. Godden from the above and other authorities. It is published in the *Journal Anthropological Inst.*, vol. xxvi., Nov. 1896, and vol. xxvii., Nov. 1897.

or naked, from their scanty clothing.* This name is said by WOODTHORPE to be foreign, and not recognised by the natives themselves. The Nágás are divided into two groups, the kilted Nágás or Angamis, and the non-kilted or Kutcha Nágás. General JOHNSTONE states that Cacharees—people resembling those settled in Cachar—and Kukis are also found in the Nágá Hills. The Kukis came from the south, and are doubtless the same as the Lushais already referred to in the earlier part of this chapter. JOHNSTONE states that they are readily distinguished from the Nágás. The Kuki men are mostly copper-coloured, often with good features; the women are frequently fair, and wear the hair in a long, thick plait down the back.

WOODTHORPE describes the Lhota tribe of the non-kilted Nágás as of square build; eyes small, oblique; face flat; cheek-bones high; complexion dirty sallow; countenance sullen. The hair is cut short or shaved, except a large basin-shaped patch on the crown, where it is two or three inches long and combed down. The tribes living in the hills bordering the Sibsagor district are fair as to colour; the men shave the head except a long tuft from crown to forehead. The tribes in the Jaipur district show every shade of brown in the complexion; the hair is shaved just above the ears, the remainder being drawn back from the forehead and tied behind in a knot, through which strips of horn are passed; some have a small moustache, but few a beard. The Rengmahs wear a wooden tail, $1\frac{1}{2}$ foot long, attached to the small of the back. The non-kilted Nágás go either quite naked, or the men wear a waist-cloth drawn tightly between the legs, and the women a waist-cloth or short petticoat; some tribes also wear a long bright blue cloth. Tattooing is commonly practised.

The Angamis, or kilted Nágás, are taller than the non-kilted tribes, their average height is from 5 ft. 8 in. to 6 ft. They are also more muscular and more courageous. They have small features; in some cases aquiline, in others flat noses; high cheek-bones; colour in different shades of brown, seldom very dark, and the eastern tribes are fairer than the west; eyes set slightly obliquely. Hair is generally straight, but never frizzly. In youth it is cut short or shaven, except one long tuft from the crown; in adolescence it is about three inches long, brushed down all round, but with the long lock at the back usually worn in a knot bound round with cotton. The lobes of the ears are pierced and decorated. The men wear kilts of cotton cloth, decorated with cowries when on the warpath, and long blue and yellow cloths across the breast and shoulders. General JOHNSTONE says that they wear tails of wood, decorated with goats' hair dyed red. The women are tall for the sex, comparatively fair, with a ruddy glow in the cheeks, well-made, and active. They wear a petticoat, and a cloth around the shoulders.

Mr A. W. DAVIS, Deputy Commissioner of the Nágá Hills district, has also given an account of the Angami and some of the other tribes of Nágás in the Report on the

* These people are not to be confounded with a sect of religious mendicants also called Nágás; or with totemistic sections of several castes in Bengal named after Nág, snake. See Mr H. H. Risley's *The Tribes and Castes of Bengal*, Ethnographic Glossary, vol. ii. p. 120, Calcutta, 1891.

Census of Assam, 1891. As many as 102,857 Nágás belonging to different tribes were living in that year in the province of Assam.

The skulls from the Nágá Hills, which Surgeon-Lieut.-Col. WRIGHT has presented me with, belonged to the Tonkal tribe, about seventy miles north-east of Manipur. General JOHNSTONE speaks of visits which he paid to the Tankhool village of Chingsow, to the north-east of Manipur, which is probably of the same tribe as that named Tonkal by Colonel WRIGHT. Both of these authorities speak of Nágá villages in this district as having been raided by Kukis. Sir JAMES JOHNSTONE describes the people as having a fine physique, equal to that of the Angami; but they went mostly naked.

Lushai Hillmen. TABLE I.*

In 1890, my former assistant and pupil, now Surgeon-Captain D. MACBETH MOIR, who was engaged in a military expedition against the Lushais, forwarded to me a skull (H in Table) which was dug up in the process of constructing Fort Tregear, built in the loop made by the Koladyne river in the South Lushai hill-tracts, a few miles to the north of the Blue Mountain. The country visited by the expedition lies between 92° and 94° longitude and 22° and 24° latitude, and consists of a succession of steep hills and deep narrow ravines. Some of the hills attain a height of 9000 feet, and many of the villages are from 4000 to 5000 feet above the sea-level. In the following year Dr MOIR sent me a skull (I in Table) which had been found in the bed of the Koladyne river, immediately to the north of Fort Tregear. He believed it to be the skull of a Lushai who, when returning to a village on the Don Mountain, from a village on the Aitur Mountain, was drowned in crossing the river. The two skulls were found within fifteen miles from each other. Dr MOIR states that the Lushais place the severed heads of their enemies on posts, but do not impale the skull.

In 1891 I received from a former pupil, Surgeon-Captain H. B. MELVILLE, at that time civil surgeon stationed at Fort Aijal in the North Lushai Hills, the skull of a Lushai warrior who had sustained a sword-cut in the left temporal region during a skirmish. The edges of the cut were sharp and somewhat splintered, and the injury had doubtless been the cause of death (G in Table).

Through the kindness of my friend Professor CUNNINGHAM of Trinity College, Dublin, I have had the opportunity of examining two Lushai skulls in his museum. One was procured in 1892 by Dr MALCOLM MOORE. It was dug up in the floor of a hut in Poi Boi, a village of the North Lushai people, situated a little to the north-east of Fort Aijal. The dead are said to be buried in the huts of their relatives. The other specimen was obtained in the village of Ramree in the South Lushai Hills, by Assistant-

* In this and the succeeding Tables the letters E. U. A. M. mean Edinburgh University Anatomical Museum; H. T. the Museum of the Henderson Trust; T. C. D. the Museum of Trinity College, Dublin. The cubic capacity has been taken by the method which I described in my *Challenger Report* on Human Crania, part xxix., 1884, to which I may also refer for an explanation of the greater number of the measurements employed in the Tables. The terms chamæprosopic (low faced) and leptoprosopic (high faced) are adopted from Professor Kollmann's memoirs.

Surgeon V. L. Watts, who was quartered at Fort Lungley, about fifteen miles to the west of Fort Tregear. In digging it up the left side of the face was injured.

The skulls had all reached adult life, but one was aged. Four were presumably men and one a woman. The North Lushai skull, from the Poi Boi village, was metopic.

Three of the crania were elongated and ovoid, though the metopic skull was broader in proportion to the length than the two others. H was somewhat ridged and roof-like in the sagitto-parietal region, whilst the others were more flattened. G and H were dolichocephalic, but the metopic skull was mesaticephalic. In G and in the metopic skull the height was less than the breadth, but in H the reverse was seen. None of the skulls was akrocephalic. In G, immediately behind the coronal suture, a shallow transverse constriction, such as is produced by wearing a head-band during infancy, was seen; this skull was cryptozygous, the two others were phænozygous. In these skulls the glabella and supra-orbital ridges were feeble, and the forehead was almost vertical; the cranial vault was fairly arched in the fronto-parietal region. In H the curve in the parieto-occipital region was gradual, and ended in a remarkably elongated inion, which formed the projecting occipital pole of the cranium. In the other two skulls the parieto-occipital slope was shorter and more abrupt, and the occipital squama projected behind the inion. In these skulls the parietal bones, from the obelion to the lambda, were flattened. The mastoid processes and temporal curved lines were moderate in two skulls, but in H the temporal lines were strongly marked behind, and approached to within 34 mm. of the sagittal suture. Owing to the occipital squama in H being remarkably small both vertically and transversely, it measured only 43 mm. from lambda to inion, and was only 55 mm. wide. As the temporal lines joined the lambdoidal suture only 34 mm. from the inion, three definite areas were marked in this region, viz., a mesial, between the two temporal ridges, and a right and left lateral, extending from the temporal ridge to the mastoid-temporal. The nuchal impressions in the occipital bone were strongly marked.

In these crania, the occipital arc was the shortest, the frontal was the longest in G and H, but in the metopic skull the parietal was much the longest. All three specimens rested behind on the cerebellar part of the occiput. The mean interzygomatic diameter was 127·6.

In all three the bridge of the nose was faintly concave, and the nasal bones projected so slightly that the face was flattened in the nasal region, and in H the nasals were short and narrow. The fronto-nasal suture was not depressed; the nasal spine of the superior maxillæ was moderate, and the incisive surface of the upper jaw was marked off from the floor of the nose by a definite ridge. In the metopic skull the nasal index was leptorhine, in the others mesorhine. In G the upper jaw was slightly prognathic, in H and in the metopic skull, orthognathic; in all, the incisive and canine fossæ were moderate in depth. The orbits, though wider than high, were megaseme in G and in the metopic skull, but mesoseme in H. The palate was much broader than long in these

Table I.

Chin and Lushai Skulls.

	Chins.						Lushais.				
	Edinburgh University Anatomical Museum.						North Lushai Hills, Poi Boi.	North Lushai Hills, Fort Aijal.	South Lushai Hills, Fort Tregear.	South Lushai Hills, Kola-dyne River.	South Lushai Hills, Ramree
	Jiddim.					Klungroa.	Metopic.				
Collection,	A.	B.	C.	D.	E.	F.	T.C.D.	E.U.A.M. G.	E.U.A.M. H.	E.U.A.M. I.	T.C.D.
Age,	Ad.	Ad.	Ad.	Ad.	Ad.	Ad.	Ad.	Ad.	Ad.	Ad.	Aged.
Sex,	M.	M.	M.	M.	M.	F.	M.	M.	M.	M.	F.
Cubic capacity,	1270	1310	1330	1290	1375	1200	1405	1390	1480	1330	1285
Glabello-occipital length,	174	181	177	173	183	173	176	181	188	169	170
Basi-bregmatic height,	131	128	127	136	131	126	132	128	136	132	127
Vertical Index,	*75·3*	*70·7*	*71·8*	*78·6*	*71·6*	*72·8*	*75·0*	*70·7*	*72·3*	*78·1*	*74·7*
Minimum frontal diameter,	95	95	84	85	94	87	90	91	88	91	90
Stephanic diameter,	104	106	108	105	110	107	107	116	105	109	107
Asterionic „	104	100	105	99	104	105	107	103	113	107	109
Greatest parieto-squamous breadth,	131s.	129s.	137s.	134p.	130s.	134s.	136s.	135p.	131s.	136s.	145
Cephalic Index,	*75·3*	*71·3*	*77·4*	*77·5*	*71·0*	*77·5*	*77·3*	*74·6*	*69·7*	*80·5*	*85·3*
Horizontal circumference,	490	505	496	480	518	490	506	510	512	490	504
Frontal longitudinal arc,	121	122	126	128	137	115	121	140	135	123	119
Parietal „ „	117	132	114	120	119	105	133	129	134	125	118
Occipital „ „	107	120	123	117	115	122	109	114	121	104	110
Total „ „	345	374	363	365	371	342	363	383	390	352	347
Vertical transverse arc,	282	281	297	295	286	286	295	305	300	292	282
Length of foramen magnum,	36	31	35	32	31	34	32	30	35	37	34
Basi-nasal length,	99	98	93	93	98	94	94	91	106	100	95
Basi-alveolar length,	94	91	99	94	96	87	88	95	103	101	89
Gnathic Index,	*94·9*	*92·9*	*106·5*	*101·1*	*98·*	*92·6*	*93·6*	*104·4*	*97·2*	*101·*	*93·7*
Interzygomatic breadth,	139	130	124	119	133	117	127	127	129	132	
Intermalar „	126	121	112	109	124	106	113	118	120	118	
Nasio-mental length,	109	111	110	107	109	105	114	118	121	112	
Nasio-mental complete facial Index,	*78·4*	*85·3*	*88·7*	*89·9*	*81·9*	*89·7*	*89·7*	*92·9*	*93·7*	*84·8*	
Nasio-alveolar length,	67	62	64	62	66	63	68	69	74	63	71ap.
Maxillary upper facial Index,	*48·1*	*47·6*	*51·6*	*52·1*	*49·6*	*53·8*	*53·5*	*54·3*	*57·3*	*47·7*	
Nasal height,	49	48	46	47	51	45	52	50	53	45	53
Nasal width,	23	25	25	25	27	24	23	26	27	25	26
Nasal Index,	*46·9*	*52·1*	*54·3*	*53·2*	*52·9*	*53·3*	*44·2*	*52·*	*50·9*	*55·5*	*49·1*
Orbital width,	42	41	36	36	40	35	37	37	40	36	40
Orbital height,	38	32	35	34	37	31	36	33	34	32	35
Orbital Index,	*90·5*	*78·*	*97·2*	*94·4*	*92·5*	*88·6*	*97·3*	*89·2*	*85·*	*88·9*	*87·5*
Palato-maxillary length,	51	48	51	52	52	45	50	54	55	55	
Palato-maxillary breadth,	64	64	60	57	62	58	63	63	70	65	
Palato-maxillary Index,	*125·4*	*133·3*	*117·6*	*109·6*	*119·2*	*128·8*	*126·*	*116·6*	*127·2*	*118·1*	
Lower jaw. Symphysial height,	31	31	25	27	30	27	31	33	32	33	
Lower jaw. Coronoid „	70	63	51	66	58	51	56	64	60	60	
Lower jaw. Condyloid „	69	68	60	69	61	54	59	65	62	61	
Lower jaw. Gonio-symphysial length,	86	86	84	90	82	80	81	88	88	91	
Lower jaw. Inter-gonial width,	105	94	88	91	100	88	97	...	98	99	
Lower jaw. Breadth of ascending ramus,	32	36	29	38	31	30	35	35	36	36	

skulls, and the index was brachyuranic. The teeth were not decayed; they were not stained, and were partially flattened on the crowns from use. The mean nasio-mental length was 117·7, which is high for that diameter; the mean complete facial index was 92·1, and the mean upper facial index was 55·0; both indices were leptoprosopic, and the face was high in relation to the width. In their cubic capacity all three crania were mesocephalic. Each skull had small Wormian bones in the lambdoidal suture. G had a small left epipteric bone, and each orbit showed the rare variety of the superior maxilla, giving rise from its orbital plate to a broad process, which joined the frontal and separated the os planum of the ethmoid from the lachrymal.* The metopic skull had a large epipteric bone on each side and broad ecto-pterygoid plates.

The Ramree skull and I, both of which came from the South Lushai Hills, were in absolute length much shorter than those above described, and as I was about equal to and the other much exceeded them in breadth, they were distinctly brachycephalic. The outline in the norma verticalis was not elongated, but was broadly ovoid. The vertex sloped downwards to the parietal eminences, which were prominent. The vertical index was less than the cephalic. Both crania were phænozygous.

The glabella and supra-orbital ridges were scarcely marked; the forehead was nearly vertical and full; the nasal bridge was flattened, and the nasal bones in one were short and narrow, in the other longer and broader. The occipito-parietal slope was steep in I, in which this region was not symmetrical and was twisted to the left, probably from artificial pressure in infancy. The occipital arc was the shortest in each skull; in I the parietal arc, in the other the frontal arc was somewhat the longer. In the male the interzygomatic diameter was 132 mm.

The upper jaw was moderately projecting, mesognathic in I, but orthognathic in the other; the nose was platyrhine in I, mesorhine in the other; the orbital index was high up in the mesoseme group. The face in I was chamæprosopic in both its complete and its maxillary index. In capacity both crania were microcephalic, and the one with the smaller capacity was that of a woman. In I a small Wormian bone was in the lambdoidal and another in the left parieto-mastoid suture, whilst the parieto-sphenoid suture was broad. In the other specimen, both parieto-mastoid sutures contained sutural bones, and the right pterion had an epipteric bone. There were no unusual ossifications at the base of the cranium, and the sutures of the vault were comparatively simple.

Two skulls of Lushais, obtained during the expedition of 1871–72, have been catalogued by Dr Barnard Davis in the *Supplement* to his *Thesaurus Craniorum*. In one the length-breadth index was 73, in the other 76; in both the height exceeded the breadth, and the mean interzygomatic diameter was 127 mm. Data are not given for determining the proportions of the height and width of the nose and the degree of projection of the upper jaw. Obviously these skulls had a dolichocephalic character. In

* Some years ago I described and figured an example of this rare variety in the skull of a Bushman (*Challenger Reports*, part xxix. p. 12, pl. 1, fig. 4, 1884), and I have recently seen it in the skull of a Papuan from New Guinea (*Proc. Roy. Soc. Edin.*, 3rd July 1899).

the tables of anthropological measurements published by Mr H. H. RISLEY,* seventeen 'Kukis,' natives of Rangamati in the Chittagong Hills, showed in their head measurements a mean cephalic index 76·2, and a mean nasal index 85. In the living person the nose is mesorhine. The customary deduction of two units from the cephalic index in the living head would place the same index in the skull at 74·2, *i.e.*, in the dolichocephalic group. The average stature of the people measured was 5 ft. $1\frac{3}{4}$ in. (1566 mm.).

Chin Hillmen. TABLE I.

In 1891 I received from Surgeon-Captain C. L. WILLIAMS a skull which, whilst acting in a surveying expedition, he had picked up in a graveyard within a quarter of a mile of Jiddim, the former capital of the Kankow country.† He states that it is the custom to dry a recent corpse over a fire for some days and afterwards in the sun for many months before it is buried beneath a stone. The skull cannot be that of a captive Burman, as the Kankows impale all captive heads on poles, and the skulls consequently have a large hole in the vertex. The Kankows are a wild tribe living in the mountains north of Burma, reaching almost as far north as lat. 24°, and westwards to the Lushai Hills. Dr WILLIAMS writes that, as compared with the Burmese, the forehead is higher, the nose less sunken, the malar bones less prominent, the lips less thick, and the chin more marked. They are a brave, hardy race of warriors and hunters, with good muscular development.

In 1894 Surgeon-Captain D. H. GRAVES sent me some skulls, which he had collected in the village graveyard at Jiddim, now the chief post for a regiment in the North Chin Hills. Up to three years prior to his visit it had been the largest village of a tribe which he names Nwengal. Dr GRAVES writes that he understands it is the custom when a member of the tribe dies to expose the body to the weather until it is decomposed. The skull is then placed along with others in an earthenware pot, which is buried. Dr GRAVES found two of these pots containing six skulls, four of which he was so good as to send me. In 1893 I also received a woman's skull collected by Surgeon-Captain GRAVES in the village of Klungroa, situated in the South Chin Hills, about sixteen miles to the south-west of Haka, between lat. 22° and 23°. She is said to have been killed by falling into a tiger trap.

The measurements of these skulls are given in Table I. E is the specimen collected by Dr C. L. WILLIAMS, the others are from Surgeon-Captain GRAVES. They were all adult. Five were presumably men, and one, F, a woman.

Norma Verticalis.—In this aspect two skulls, viz., B and E, were seen to be elongated and ovoid, so that in their proportions they were distinctly dolichocephalic, whilst A only slightly exceeded the dolichocephalic index. The three others were relatively

* *Tribes and Castes of Bengal*, vol. i. p. 204, Calcutta, 1891.

† See for an account of the Kankow campaign, *Chin Lushai Land*, by Surgeon-Lieutenant-Colonel Reid, I.M.S., p. 67, Calcutta, 1893. In the large map in this work the name apparently of this village, some miles to the north of Fort White, is printed Tiddim.

wider in the parietal region, and had a somewhat higher length-breadth index, which placed them in the lower term of the mesaticephalic group. In these three, C, D, and F, the parietal tubera projected, so that the outline of the skull approached the pentagonal or coffin shape. There was only a slight tendency to the formation of a sagittal ridge, and the slope outwards from it to the parietal eminences was not steep. One cranium was phænozygous; the rest were cryptozygous.

Norma Lateralis.—None of the skulls had a very prominent glabella or supra-orbital ridge, though in A they were more distinct than in the other crania; in A the frontal bone also showed a somewhat shelf-like projection immediately above the external orbital process; in this skull also the forehead was more receding than in the other specimens, in which indeed it approached to the vertical. The vault of the cranium was fairly well arched in the parieto-frontal region, and sloped backwards and downwards in the parieto-occipital region, somewhat more gently in B than in the other specimens. The occipital squama projected behind the inion; there was no appearance of parieto-occipital flattening, though D showed a want of symmetry in that region. The skulls rested behind on the cerebellar part of the occiput. The nasal bones had a concave bridge, and projected so slightly that the face was flattened in the nasal region; the fronto-nasal suture was not depressed. The nasal spine of the superior maxillæ was feeble in some specimens, and in no case strong; a moderate ridge marked the separation of the incisive part of the upper jaw from the floor of the nose. The incisive and canine fossæ were moderate in depth. C and D were more prognathic than the other skulls. As a rule the orbits were high in proportion to their width, but B had a low orbital index. In C, D, and F the nasal index was moderately platyrhine, in A leptorhine, in the rest mesorhine. The teeth had to a large extent been lost, and of those that remained many were worn down and stained. The palate showed no unusual arching. The mastoid processes, temporal and occipital ridges, were moderate. The sutures were not obliterated in any of the crania, though in some, fusion of the bones had begun. Small Wormian bones were present in the lambdoidal suture in three skulls, and in D the suprainial part of the occipital squama had ossified as a distinct inter-parietal bone. All the skulls, with one exception, had an epipteric bone either on the one or on both sides; the parieto-sphenoid suture, when present, was usually narrow. The upper part of the coronal suture and the anterior end of the sagittal suture were almost devoid of denticulations. No skull had an exostosis in the auditory meatus, neither was a third condyle or paramastoid process present. No skull was metopic. The skull D showed a hole in the coronal suture 25 mm. to the right side of the sagittal suture. The hole measured 6 mm. by 4 mm., and the bone around it had a smooth bevelled margin, whilst the surface of the parietal bone behind it was abraded; the appearance led one to think that during life the skull had been injured, probably by the cut of a sword.

The six skulls from the Chin Hills form a homogeneous group, and in their dimensions and relative proportions may appropriately be classed together.

In the glabello-occipital length the crania ranged from a maximum of 183 mm. to a

minimum of 173, and the mean length of the series was 176·8 mm. In their parieto-squamous breadth the maximum was 137 mm., the minimum was 129, and the mean was 132·5 mm. The mean length-breadth index of the group was 75·0. Three skulls had the index either 77·5 or 77·4, which placed them in that division of the mesaticephalic group which approached closer to the dolichocephalic than the brachycephalic standard. No skull was brachycephalic. Both in numerical proportion and in general shape these Chin crania may be regarded either as distinctly dolichocephalic or as approximating to that group.

In basi-bregmatic height the crania ranged from a maximum 136 mm. to a minimum 126, and the mean was 129·8 mm. The mean length-height (vertical) index was 73·4, so that the skulls belong to the group with a moderate vertical index, which I have named metriocephalic.* In D and E the height slightly exceeded the breadth; in A they were equal; in the remaining three the breadth was more than the height.

The mean stephanic diameter, 106·6 mm., exceeded the mean asterionic diameter, 102·8 mm., and the mean minimum frontal diameter was 90 mm. The bizygomatic diameter, with a mean 127 mm., ranged from 117 to 139 mm., and invariably exceeded the intermalar diameter.

The occipital longitudinal arc in four skulls was less than either the frontal or parietal, but in F it was greater than either of these, and in C it was greater than the parietal. In five crania the frontal arc exceeded the parietal, and in B the parietal was the longer of the two.

The nasio-mental length of the entire face ranged from 105 to 111 mm., with a mean of 108·5 mm. The complete facial index ranged from 78·4 to 89·9, and gave a mean of 85·6, so that the skulls fall into the chamæprosopic or low-faced group, not a single specimen was leptoprosopic. As regards the maxillary facial index the range was from 47·6 to 53·8, and the mean was 50·4; they were therefore leptoprosopic in the proportions of the upper face.

In four of the six skulls the basi-nasal diameter exceeded the basi-alveolar. The gnathic index ranged from 92·6 to 106·5, and the mean was 97·6; the majority were orthognathous or mesognathous, though C was prognathous.

The nasal index ranged from 46·9 to 54·3, and the mean of the series was 52·1, *i.e.*, mesorhine; individually, however, A was leptorhine, C, D, and F were platyrhine, and only two were mesorhine. The orbital index ranged from 78 to 97·2, and the mean was 90·2; the orbits therefore were generally megaseme, B only being microseme. The palato-maxillary index ranged from 109·6 to 133·3, and only one specimen was below 115; the mean was 122·3, which placed the palate well into the brachyuranic group.

The cubic capacity of the cranium in the five men ranged from 1270 to 1375 c.c.; thus there was only a small range of variation amongst them, and the mean, 1315 c.c., was distinctly microcephalic. The capacity of the skull in the specimen which I have regarded as a woman was 1200 c.c.

* *Challenger Reports*, part xxix. p. 5, 1884.

Although I have described the crania from the Lushai hill-tracts as a group separate from those collected in the hills occupied by the Chins, yet as the peoples known by these names, if not one race, have close affinities with each other, it will be instructive to look at the two series together.

Of the eleven skulls under observation four had a length-breadth index below 75, five were between 75 and 77·5, and two from the South Lushai hill-tracts were above 80; the mean of the series was 76·1. If the two brachycephalic crania are excluded the mean of the rest is 74·6, so that the skulls are in the main dolichocephalic, or approximating thereto in their numerical index as well as in their general form. In three of the skulls the length-height index was slightly above the cephalic, in one they were equal, but the mean vertical index of the series was 73·78; on the whole, therefore, in these skulls the breadth exceeded the height. The mean stephanic diameter was 107·6, whilst the mean minimum frontal breadth was only 90.

If we take the figures suggested by Sir WILLIAM H. FLOWER* as limiting the three divisions of the gnathic index, two skulls were prognathous, three were mesognathous, the rest orthognathous; and as the mean of the eleven crania was 97·8, orthognathism is apparently a preponderating character.

As the lower jaw was present in ten specimens the complete facial index was obtained. In only one skull was it below 80, in seven between 80 and 90, in two above 90; the mean of the series was 87·5, which places them in the chamæprosopic or low-faced group of Kollmann. The upper facial or maxillary index is on the average 51·5.

The width of the anterior nares was moderate in relation to the height of the nose, the nasal index was leptorhine in only two specimens, in four it was platyrhine, in the others mesorhine; the mean of the eleven crania was 51·3, *i.e.*, mesorhine; the bridge of the nose was concave and feeble above and tilted forward below, but the face must have been flattened in this region. The height of the orbit was considerable in relation to the breadth, and the mean index was 89·9, *i.e.*, megaseme. The palato-maxillary breadth was great in relation to the length, and the mean index was 122, so that the skulls were in the brachyuranic group; no specimen was dolichuranic.

The mean cubic capacity of the crania of nine men was 1353 c.c., which places them on the confines of the microcephalic and mesocephalic groups.

To summarise the characters of the crania of the natives of the Lushai-Chin hills, one may say that in the main they are dolichocephalic: as a rule the breadth of the cranium exceeds the height; the upper jaw is orthognathic; the face is low, chamæprosopic; the nasal width is moderate in relation to the height, mesorhine; the height of the orbit approximates to the breadth, and the index is megaseme; the palato-maxillary breadth is wide in relation to the length, brachyuranic; and the cranial capacity is moderate.

* *Catalogue* of the Museum of the Royal College of Surgeons, p. 252. 1879.

Tonkal Nágás. TABLE II.

In 1893 a box reached me from Surgeon-Lieutenant-Colonel F. W. WRIGHT, D.S.O., containing eight skulls which he had collected in the house of a Tonkal Nágá, in the upper village of Hwining, situated about 6000 feet above the sea-level in the hills some forty miles north-east of Manipur. The occasion which led to an expedition being sent into the hills was a raid by the "Kukis" on the Nágá village of Swemi, situated some 7000 feet above sea-level, and about 70 miles north-east of Manipur. The people of Hwining, although themselves Nágás, had joined the Kukis in the raid on villages of their own tribe.

Dr WRIGHT also wrote a most interesting letter, in which he informed me that there are two villages at Hwining, an upper and a lower, built on the crest of a spur running from about south-west to north-east, and at the south-west end is the upper village. The villages are separated by about half a mile of uneven ground, and their inhabitants used to fight with each other, and take each other's heads. As it is not the custom of the Tonkal Nágás to preserve the heads of friends and relatives, but to bury their dead close to their houses, the skulls collected had evidently been those of persons murdered or killed in battle, and afterwards preserved. Dr WRIGHT found these skulls fixed as trophies to a board on the wall of the front room facing the entrance to a house. He believes them to be the skulls of Tonkal Nágás, as Hwining is surrounded by Tonkal villages, with which it was, and indeed in some instances is, still at feud; possibly they are skulls of the Nágás of the lower village of Hwining. The head of a woman is as much prized as that of a man, for as women do not go far away from their homes, the captor requires to approach close to the hostile village, and puts himself therefore into greater danger in order to secure the head.

From the very instructive account of the Nágás with which Dr WRIGHT has favoured me I make the following extract:—

"The hills north-east of Manipur range in height from 3000 to 7000 feet. They are clothed with forests, and abound in game. The human inhabitants are Nágás and Kukis. Both are savage tribes, and go about nearly naked, but the women are more clothed than the men. They make clearings in the forests and grow crops of rice, Indian corn, etc., and from the rice they make a fermented liquor called 'Zoo,' which is not unlike a rough kind of cider. The Nágás are the indigenous natives, and reside permanently in one place, and live in huts on the tops of the hills, where they can command a view of the approach of their enemies. The Kukis have immigrated from the south from the hills between Manipur and Burma. They are not settled in their habits, but make from time to time new clearings, so that they are very destructive to the forests, and raid the Nágá villages and kill the inhabitants. Both Nágás and Kukis eat the flesh of pigs and other animals. It is said that a Nágá gives a good meal of rice to a dog, then kills and roasts it, and makes a meal of dog, stomach and rice.

Neither Nágás nor Kukis drink milk, which they look upon as an excrement.* Their native weapons are bows, spears and poisoned arrows; the poison is said to be aconite. They are now using guns, and employ urine and fæces in the manufacture of gunpowder. They are demon worshippers. They seem to have slaves, and in both the Nágá and Kuki villages there are head-men or village elders, though in theory all the men are equal. Both Nágás and Kukis make very good coolies, but the Nágá is preferred, as he is both cheerful and enduring."

"In the Nágá houses the wall of the front room facing the entrance is decorated with the heads and bones of the animals killed for food and in the chase. Heads or horns of the Sambre deer, mithan buffalo, pig, barking deer, bear, dog, porcupine, and capricorn were recognised. Outside the entrance of the house of a head-man a small grove of dead trees is sometimes seen. Each tree signifies a big feast, the trees being set up as monuments of the head-man's hospitality. They are also used incidentally for the growth of orchids. The Kukis do not set up monuments of dead trees, but they fix trophies of the skulls and horns of animals at the entrance to their houses.† A Kuki warrior therefore can point to the human skulls in his house as evidence of his cunning and bravery as a head hunter, and to the crania of the large mammals as testifying to his success in the chase and to his hospitality."

"The Nágás shave the head, but leave a crest of hair in the middle of the crown from front to back, which ends in a lock hanging down behind. The Kukis do not shave the head. Neither they nor the Nágás have hair on the face. The Tonkal Nágás wear a ring made of bone, or ivory, or porcelain, around the middle of the penis, and it appears to be a mark of bad manners to appear without the ring."

When the expedition occupied the Kuki village of Mougham some recent scalps were noticed on a tree near the chief's house in the highest part of the village. On examining them more closely they were seen to consist not only of the scalp but of part of the skull, the top of which had been cut off and the bone pierced with a spear. They were trophies of the raid on the Nágá village of Swemi. The Political Agent told Dr WRIGHT that in the Nágá villages the young men sleep together in a house of their own, but he is not sure if a similar arrangement is provided for the young women, though he thinks that it is so.‡

The skulls of the Tonkal Nágás were all from adults, though one was aged, and in two specimens the upper wisdoms were not erupted. Six were without

* Miss Mary H. Kingsley (*Travels in West Africa*, p. 451, London, 1897) states that the West Coast Africans have a horror of the idea of drinking milk, and hold it as a filthy habit.

† In some of the Pacific Islands, as in the Solomon group, human skulls and those of pigs, dogs, and dugongs are preserved in and around the Tambu house, and the practice of preserving and decorating the skulls of relatives and enemies alongside of the skulls of animals prevails extensively in New Guinea.

‡ The custom of providing a separate sleeping house in each village for all the unmarried girls and another for all the young men prevails generally amongst the races to the north-east and south of Assam (S. E. Peal in *Journal Asiatic Soc.*, Bengal, vol. lii. part ii., 1883). A similar practice also exists amongst the Khonds, a hill tribe in the Indian peninsula (R. W. Frazer, *Silent Gods and Sun-Steeped Lands*, London, 1895). It is also the custom with some of the tribes in New Guinea and other islands in Polynesia.

doubt those of men, one a woman, and one was more doubtful, though most probably a man.

Each of these skulls was enclosed in an open basket-work frame of split cane. In the greater number two parallel bands of cane were bent antero-posteriorly and mesially around the base of the skull to the occiput, vertex, forehead and face, including the lower jaw. These longitudinal bands were intersected and knotted to a band which passed around the skull in its vertical transverse circumference. A vertical transverse band of cane had been passed below the angles of the lower jaw and was secured to the zygomata. A decorative feature in each orbit consisted of a strip of cane rolled once or twice around the interior of the chamber near the facial orifice; so as, when seen at a short distance, to simulate an eye. The skulls had been dried with the scalp on, but the hair had been removed. In three specimens the base of the skull had been partially broken away, doubtless to assist in the extraction of the brain, so that the determination of the capacity of these crania was only approximative. The heads had been exposed to smoke, and were more or less blackened. The scalp and the basket-work had to be removed in order to examine the crania and take the measurements; but the basket-work was subsequently replaced.

Norma Verticalis.—From this aspect the series of skulls did not present a uniform appearance. The woman's and four men's, D, E, F, G, were elongated and more or less ovoid, with vertical sides and a tendency to a sagittal ridge, from which the skull sloped rapidly downwards and outwards to the parietal eminences; in E, F, and H the crania had an "ill-filled" character. In the other three male skulls, A, B, C, the breadth was proportionately greater in relation to the length, so that the form was not so elongated an ovoid as in the other specimens; the vertex also had not the same tendency to be ridged, and the slope outwards to the parietal eminences was not so steep. One skull was phænozygous, but in the majority the zygomata were concealed in the vertex view; the condition in G could not be ascertained, owing to the zygomata being broken, but from the wide stephanic diameter it would probably have been cryptozygous.

Norma Lateralis.—In none of the skulls were the glabella and supra-orbital ridges very prominent, and they were best marked in the skull A, which was metopic. The forehead was almost vertical; the arch of the vault was moderate, and the slope backwards into the occipital region was as a rule gentle, and in A, B, and C, that is, in the more brachycephalic crania, the occipital squama projected in all behind the inion; there was no sign of parieto-occipital flattening. As a rule the skull rested behind on the cerebellar part of the occiput, and in five of the skulls the parietal arc was somewhat longer than the frontal. In all, the occipital arc was less than the frontal, and in only one specimen did it exceed the parietal. The face was flattened in the nasal region, and the osseous bridge of the nose was slightly concave and not projecting. The nasal bones were relatively narrow, the fronto-nasal suture was not depressed. The nasal spine of the superior maxillæ was faint; a fairly-defined ridge demarcated the incisive

Table II.

Tonkal Nágás, Hwining. Nepal.

	Edinburgh University Anatomical Museum.									
	Metopic.									Par-buttia. Gurung. Nepal.
Collection,	A.	B.	C.	D.	E.	F.	G.	H.		
Age,	Ad.	Ad.	Ad.	Ad.	Ad.	Ad.	Ad.	Ad.		Ad.
Sex,	M.	M.	M.	M.	M.	M.	M.	F.		M.
Cubic capacity,	1565	1455	1520	1520 ap.	1395 ap.	1455	1600 ap.	1250		1655
Glabello-occipital length,	188	171	177	182	180	183	186	174		168
Basi-bregmatic height,	137	136	136	...	...	137	138	132 ap.		144
Vertical Index,	*72·9*	*79·5*	*76·8*	...	...	*74·9*	*74·2*	*75·9*		*85·7*
Minimum frontal diameter,	103	93	98	97	88	97	97	85		97
Stephanic diameter,	111	111	109	110	110	107	120	102		121
Asterionic ,,	110	97	109	107	115	106	120	108		109
Greatest parieto-squamous breadth,	145	140	145	132	135	134	140	130		152s.
Cephalic Index,	*77·1*	*81·9*	*81·9*	*72·5*	*75·*	*73·2*	*75·3*	*74·7*		*90·5*
Horizontal circumference,	535	500	516	510	505	514	528	488		507
Frontal longitudinal arc,	134	120	125	122	134	133	134	120		137
Parietal ,, ,,	120	130	128	127	133	131	138	121		128
Occipital ,, ,,	125	110	109	121	115	116	...	...		108
Total ,, ,,	379	360	362	370	382	380	...	...		373
Vertical transverse arc,	302	308	301	301	302	298	306	291		325
Length of foramen magnum,	35	37	37	...	...	32	...	...		36
Basi nasal length,	106	96	101	...	...	102	100	...		97
Basi-alveolar length,	99	93	87	...	...	100	93	...		90
Gnathic Index,	*93·4*	*96·9*	*86·1*	...	...	*98·*	*93·*	...		*92·8*
Interzygomatic breadth,	146	131	137	...	127	140	131	122		141
Intermalar ,,	132	121	123	120 ap.	116	124	114	115		122
Nasio-mental length,	119	118	114 ap.	107	115	117	111	115		
Nasio-mental complete facial Index,	*81·5*	*90·*	*83·1*	...	*90·5*	*83·5*	*84·7*	*94·2*		
Nasio-alveolar length,	71	72	72	67	67	70	66	68		46
Maxillary upper facial Index,	*48·6*	*54·9*	*52·5*	...	*52·7*	*50·*	*50·3*	*55·7*		*46·8*
Nasal height,	53	53	57	58	50	54	50	50		51
Nasal width,	28	26	25	27	26	29	26	24		21
Nasal Index,	*52·8*	*49·1*	*43·8*	*46·5*	*52·*	*53·7*	*52·*	*48·*		*41·2*
Orbital width,	42	37	42	40	36	37	38	33		39
Orbital height,	36	35	38	37	35	34	36	30		36
Orbital Index,	*85·7*	*94·6*	*90·5*	*92·5*	*97·2*	*91·9*	*94·7*	*90·9*		*92·3*
Palato-maxillary length,	51	55	46 ap.	52	48	56	46	52		51
Palato-maxillary breadth,	67	66	59 ap.	63	65	67	66	69		68
Palato-maxillary Index,	*131·3*	*120·*	*128·2*	*122·1*	*135·4*	*119·6*	*143·4*	*132·6*		*133·3*
Lower jaw. Symphysial height,	28	28	33	30	30	31	28	32		
Lower jaw. Coronoid ,,	62	66	67	57	60	61	57 ap.	56		
Lower jaw. Condyloid ,,	64	68	65	65	58	64	59	58		
Lower jaw. Gonio-symphysial length,	84	74	89	88	92	92	82	88		
Lower jaw. Inter-gonial width,	97	90	105	...	107	102	...	100		
Lower jaw. Breadth of ascending ramus,	36	36	35	36	40	36	30	33		

part of the upper jaw from the floor of the nose; the canine and incisor fossæ were moderate in depth, though in the aged skull they were deeper. The jaws were not prognathic; the orbits were high in proportion to their width. The teeth were deeply stained, and as a rule free from decay, though in the older skulls they showed evidence of wear, and in the aged specimen they had almost all been shed and the sockets absorbed. The sutures in the aged skull were almost obliterated, and in some of the other crania they were also disappearing. The mastoid processes were moderate, the temporal and occipital ridges were fairly marked. The palate was arched and horseshoe shaped. The external meatus was free from exostoses. No third condyle or para-mastoid process was seen, but in one specimen each external pterygoid plate sent a spur-like process backwards which did not reach the spine of the sphenoid. In one skull the infra-orbital suture was seen.

In three specimens small Wormian bones were in the lambdoidal suture. The breadth of the parieto-sphenoid suture varied from 3 to 12 mm. In the right pterion of two specimens, F and G, an epipteric bone was seen, and in the left pterion of both of these skulls a tongue-shaped process of the squamous temporal articulated with the frontal; in the skull F this process was so broad as to separate the ali-sphenoid from the parietal by an interval of 17 mm.

The eight skulls of the Tonkal Nágás varied in maximum length from 171 to 188 mm., with a mean of 180 mm. In their greatest breadth the range was from 130 in the woman to 145 mm. in the broadest-headed man, and the mean was 137·6. The mean cephalic index of the series was 76·4, *i.e.*, mesaticephalic; two of the crania were brachycephalic, four were dolichocephalic, and the remaining two were in the lower half of the mesaticephalic group.

The crania ranged in basi-bregmatic height from 132 to 138 mm., with the mean 136 mm., and the mean vertical index was 75·7, which is moderately high. In two specimens the vertical index was slightly above the cephalic, but the opposite condition was the rule.

The mean stephanic diameter, 110 mm., slightly exceeded the mean asterionic, 109 mm., and both were considerably higher than the mean minimum frontal diameter 94·7 mm. The bizygomatic diameter, with a mean of 133·4 mm., ranged from 122 to 146 mm. In each skull it invariably exceeded the intermalar diameter.

The mean complete facial index was 86·7, *i.e.*, chamæprosopic, whilst the proportions of the upper face gave an index 52, or leptoprosopic. In the five skulls in which the dimensions could be taken the basi-nasal diameter exceeded the basi-alveolar, and the mean relative index, 93·5, was orthognathous.

In the nasal index two skulls were leptorhine, five were mesorhine, and only one was platyrhine; the mean index of the series was 49·7, or mesorhine. The mean orbital index was 92·2; the orbit, except in one skull, was megaseme, and with no great difference between the breadth and height. The mean palato-maxillary index was 128·9, and every skull was brachyuranic.

The seven male skulls had a mean internal capacity 1501 c.c., whilst the single woman's skull was only 1250 c.c.

Up to this time very few examples of the skulls of the natives of the Nágá Hills have been deposited in Museums. The specimens sent home by Surgeon-Lieutenant-Colonel WRIGHT form therefore an important addition to the material collected for the investigation of their cranial characters. In the Barnard Davis collection, now in the Museum of the Royal College of Surgeons of England, are three Nágá crania;* and a fourth specimen from Ninu, in the Patkoi Mountains, has subsequently been acquired by the College. These, together with a fifth specimen, collected by Colonel WOODTHORPE in the Patkoi Mountains, have been described by Professor G. D. THANE,† who looks upon three as those of men and two those of women. They are all adult, but not aged. Two were decorated: one with wire passed through the orbits and zygomata, which supported fragments of shell as well as some small bells; the other having rings of thick wire placed through the zygomatic arches, orbits and nasal cavities.

Both in Professor THANE'S series and in mine the skulls had a certain smoothness of surface, owing to the muscular ridges and processes possessing no special prominence, and the forehead was almost vertical. His specimens were, however, shorter than mine, for though the mean height and breadth were almost identical in the two series, the mean length of THANE'S specimens was 4 mm. less than in mine. In both sets the mean cephalic index was mesaticephalic; but in THANE'S series owing to the diminished length it was 78·1, being 1·7 higher than in mine; taking both series together the mean cephalic index in the thirteen Nágá skulls was 77. The mean vertical index in THANE'S specimens was 78·4, which was appreciably higher than in mine, and the mean of both series was 76·9, so that the mean breadth very slightly exceeded the mean height in the two groups. The crania may be regarded as hypsicephalic.

In Professor THANE'S series the mean gnathic index was 98·6, but in mine it was much lower, 93·5: the mean of both series was 96, *i.e.*, orthognathous. In his specimens the mean nasal index was 53·3, in mine 49·7, but the mean of the two was 51·1, *i.e.*, mesorhine: the anterior nares therefore are moderately wide in relation to the height. In his crania the mean orbital index was 88·5, in the higher term of the mesoseme series; but in mine they were definitely megaseme, so that in the people generally we may say that the height of the orbit approaches its width. In both series of skulls the palate was wide in relation to its length, and the index was brachyuranic.

In THANE'S specimens the mean interzygomatic diameter was 129·7, but in mine it was 133·4, and as five of my skulls exceeded in this dimension the mean of his collection, it follows that they had greater breadth in the facio-zygomatic region.

The three male skulls in Professor THANE'S series ranged in their cubic capacity from 1300 to 1400 c.c., with a mean of 1377 c.c., whilst the mean capacity of the two women was 1237 c.c. In my series, only one skull apparently was that of a woman

* *Thesaurus Craniorum*, p. 173; and *Supplement*, p. 88.

† *Journal of the Anthropological Institute*, vol. xi. p. 215, 1882.

with a capacity of 1250 c.c., whilst the mean of the seven men was 1501 c.c., which is much above the average of savage or barbarous people, corresponding indeed to the European mean. If THANE's males are, however, computed along with my series of males, the mean capacity is reduced to 1464 c.c., a measurement which is also high for a tribe of savages.

In the preceding narrative it will have been noticed that explorers in the hill ranges occupied by the Lushais (Kukis) and Nágás have recognised differences in the physical characters of these people. Sir JAMES JOHNSTONE, for example, definitely states that they are readily distinguishable from each other. There is, however, a general consensus that their narrow oblique eyes, flat broad faces, high cheek bones, flat noses, skin of various shades of brown, inclining sometimes to copper colour, long straight black hair, and scanty beard and moustache, are Mongolian characters. Colonel LEWIN, however, in both his works asserts that the Lushais do not exhibit the Mongolian type of feature, and he compares them with Portuguese half-castes. WOODTHORPE speaks of some of the Angami Nágás as having aquiline features and a complexion so fair that the cheeks show a ruddy glow.

It would seem, therefore, whilst the Mongolian type of feature prevails, that departures from that type do occur with sufficient frequency to be noticeable. The study of the skulls proves that they also possess some diversities of character. Though the majority of specimens in the Chin-Lushai group and in the Nágás were dolichocephalic or approximated thereto, in both the Lushais and Nagas two distinctly brachycephalic crania were met with, though in the series of Chins 77·5 was the highest index of breadth. Both groups, however, were alike in the absence of a marked projection of the upper jaw: in both, the face was wide in relation to its height, and the complete index was chamæprosopic; the nose was not prominent, and the mean nasal index in both groups was mesorhine and the orbital index was megaseme. Their facial characters were therefore closely allied, and testify to a corresponding physiognomy. As regards the breadth of the face, the mean interzygomatic diameter of ten Lushai-Chin skulls was 127·7 mm., and that of seven Nágás was 133 mm., as compared with 130·6, the mean of the same diameter in thirteen Chinese crania in the collection, and 131·5, the mean of four Siamese skulls. The Nágás, therefore, in absolute width of face surpassed the Chinese and Siamese which I have measured. In the Nágás the mean capacity of the crania was distinctly higher than in the Chin-Lushai series.

As the best marked Mongolian races are either definitely brachycephalic or in the higher terms of the mesaticephalic group, it is interesting to note that these hill tribes, with a prevailing type of Mongolian feature, possessed crania in which brachycephalism is the exception, and where the customary form of skull is dolichocephalic or approximating thereto. It would seem, therefore, that the Mongolian character of face is not necessarily associated with only one type of cranium.

Nepal. TABLE II.

More than thirty years ago the late Sir JOHN BROWN, of the Indian Medical Service, presented to the Anatomical Museum of the University, a skull without the lower jaw, which he had found in the valley of Nepal. He believed it to be that of a Gurung or Magar, and it is marked, apparently in his own handwriting, Parbuttia, which signifies hillman. Surgeon-Lieut.-Colonel REID states* that the Gurungs and Magars occupy the country to the west of the Nepal valley. They are, he says, short and powerful men of Mongolian cast of features, with broad flat faces and oblique eyes. They form the Gurkha regiments in the British army in India.

The skull is obviously that of a man not thirty years of age, for the upper wisdom teeth were not erupted. The sutures were unossified and comparatively simple. The squamous-temporals were small, but the ali-sphenoids were wide, and each had a broad articulation with the parietal at the pterion. The mastoids and the temporal and occipital ridges were feeble, and there were no unusual ossifications.

In the *norma verticalis* the breadth of the cranium approximated to the length. The parieto-occipital region was almost vertical, flattened and unsymmetrical, the flattened surface being directed to the right. Sir JOHN BROWN ascribed the shape of the skull behind to the mother, as she carried her infant, having kept this aspect of the head pressed against some part of her person. The vertex was not ridge-like, the parietal and frontal eminences were distinct, the parieto-squamous region bulged laterally. The length-breadth index was 90·5, and the skull was hyper-brachycephalic. The height was materially less than the breadth, notwithstanding that the basi-bregmatic diameter was as high as 144 mm. The skull was cryptozygous.

In the *norma lateralis* the glabella and supra-orbital ridges were seen to be feeble, the forehead was lofty and not very receding. The frontal longitudinal arc was much the longest and the occipital the shortest. The bridge of the nose was almost straight, sharp, and moderately projecting, and there was scarcely any fronto-nasal depression. The nasal spine of the superior maxillæ was distinct, and a sharp ridge separated the floor of the nose from the incisive region. The nasal index was markedly leptorhine. The interzygomatic diameter was 141 mm., so that the face was unusually wide. The orbital index was strongly megaseme. The upper jaw was not prognathic. The palate was not highly arched, and as its breadth materially exceeded the length it was highly brachyuranic. The internal capacity was 1655 c.c. and the skull was megacephalic. In its brachycephalic form and proportions, in the breadth being less than the height, the flattened nasal region, the broad face, the slight forward projection of the upper jaw, megaseme orbit, and brachyuranic palate, the cranium exhibited well defined Mongolian characters.

Sir R. OWEN has given the measurement of a skull of an adult male Gurung† in

* *Chin-Lushai Land*, p. 72. 1893.

† Owen, *Rep. Brit. Assoc.*, 1859, p. 100.

the British Museum, the length of which was 7 inches and the breadth 5 in. 8 lines: the length-breadth index may be regarded as 81·4. Other crania from Nepal had different proportions. From the measurements which he has recorded of two Magar skulls it is probable that in this race the crania are dolichocephalic. A skull from Nepal, figured by MM. DE QUATREFAGES and HAMY,* plate lxii., is elongated in form, and with a length-breadth index 75·5. Dr BARNARD DAVIS catalogues, *Thesaurus Craniorum*, p. 158, seven crania from Nepal, which he names Khas. The length-breadth index varied in them from 73 to 78, and gave a mean 75·7. The skulls were either dolichocephalic or mesaticephalic. In the anthropological tables compiled by Mr H. H. RISLEY† the mean cephalic index in 28 living Gurungs is stated to be 81·6, and the nasal index in the same persons was 78·5. The heads were brachycephalic, and the nose was mesorhine. The average stature was 5 ft. $2\frac{3}{4}$ in. (159·8 mm.). It would appear, therefore, that the people of Nepal are not a homogeneous race. A strong Mongolian element, however, exists in that country, as is shown both in the skulls and heads of the Gurungs which have been measured.

BURMA.

The inhabitants of Burma consist in the main of the people termed Burmese, but intermingled with them are representatives, sometimes in considerable numbers, of other tribes and races. The Burmese proper are in all probability of the same stock as the Himalaya-Tibetan people, offshoots of which race migrated, it is believed, in a south-easterly direction until they reached Burma. How far the country was populated by aborigines, prior to and at the time of the invasion, it is impossible to say. It is, however, thought that the district forming the delta of the Irrawaddy was occupied by a people named Môns or Talaings, whose descendants remain more or less commingled with the Tibeto-Burmese stock. The Burmese proper, according to the census return for 1891, were 9,000,000, whilst the Talaings were not quite 1,000,000 in number.‡

Partly on the confines of and partly within the Burmese territory are other races, which in their respective districts modify the population. To the east are the Shan states; to the northward are Manipur and the Nágá hills; to the north-west the Lushai-Chin hill ranges, the people of which were described in an earlier chapter of this memoir; and to the east of Lower Burma are the Karens, who constitute an important element in the population.

The Shans, according to the census returns for 1891, were about 180,000 in number in Upper Burma, and about 108,000 in Lower Burma. The Chins, under which term the census report includes apparently also the Kukis (Lushais) and Nágás, were 206,000.

* *Crania Ethnica*, p. 416.

† *Tribes and Castes of Bengal*, Calcutta, vol. i. pp. 232 and 220. 1891.

‡ The above figures are compiled from the Census of 1891, *Report on Burma*, prepared by Mr H. L. Eales, the Provincial Superintendent, Rangoon, 1892.

The Karens numbered about 1,000,000. In addition to these races, natives of India, Malays, Chinese and Europeans were also represented.

The Burmese proper are people of moderate stature. In the lists which accompanied the valuable series of crania of prisoners who had died in the jail at Insein, for which I am indebted to Surgeon-Major BELL, the stature of each person is given in feet and inches. They were all men. The mean stature was 5 ft. $2\frac{3}{4}$ in. The tallest man, Nga Aung Myat, a native of Yebouk, was 5 ft. 7 in., and the shortest, Nga Pe, a native of Sharsayboo, was 4 ft. $9\frac{1}{2}$ in.; whilst another, Nga Pu, born at Aungmyingain, was 4 ft. 11 in. Seven measured from 5 ft. 5 in. to 5 ft. 6 in., and the others were between 5 ft. and 5 ft. 4 in. The Burmese men are thick-set, muscular, and active. The skin in the higher classes is a light olive-brown, but a darker brown in those people who are much exposed to the sun. The hair is black and straight, abundant on the head, but scanty on the face. The face itself is broad and flattish, the nostrils are usually spread out laterally and the nose is short. The eyes are wide asunder and inclined to be oblique and almond-shaped. The lips are not thick and projecting as in the negro.

The Karens consist of three divisions,* the Pghos (Pwos), who are found along the sea-board of Tenasserim from Moulmein to Tavoy and Mergin; the Chghaws (Sgau), who occupy the hills and jungles of the lower part of the Irrawaddy river, in the district of Henzada on the right bank, and those of Prome and Shwegyin on the left bank, as far east as the Salween river. The Bghai (Bwi) division are found in the Toungoo hill-tracts which lie to the east of Prome. Mr SMEATON says that the Karens are short in stature, but broad and muscular. A Karen man from the Toungoo district who died in the jail at Insein, and whose skull was presented to me by Major BELL, was 5 ft. $1\frac{3}{4}$ in. high. The skin is naturally fair, like that of the Chinese, and the features of those of pure blood are, according to Mr SMEATON, Caucasian in type. The hair is black and straight; the eyes are black, though in the north brownish hair and hazel eyes are sometimes found. It is difficult to give the original home of the Karens. The prevailing opinion, however, is that they left the borders of Tibet and passed through Western China on their way to Burma.

The Shans (Htai or Tai, to employ their own name), on the eastern frontier of Burma, are divided into the Chinese Shans, the Salween Shans and the Siamese Shans. They form a number of tribes, which occupy the hill-ranges, elevated plateaus and valleys of the extensive tract of country in which they dwell.† They present differences in their physical characters in different districts. Dr ANDERSON states that the Shans dwelling in the valleys have the sallow tint of the Chinese, usually with red cheeks, dark brown eyes, black hair, face generally rather short, broad and flat, cheek bones prominent, a faint obliquity and contraction of the outer angle of the eyelids as in the Chinese. The

* *The Loyal Karens of Burma*, by D. M'Kenzie Smeaton. London, 1887.

† The Shan country has been visited by many travellers. The works that I have consulted are Dr John Anderson's *Expedition to Western Yunan*, 1871; *Report on Administration of Shan States for* 1889-90 *and* 1892-93, by J. G. Scott; *Census of Burma*, 1891; Colonel Woodthorpe in *Journ. Anthrop. Inst.*, August 1896, vol. xxvi. p. 13; *From Tonquin to India*, by Prince Henri d'Orleans, 1898.

nose is well formed, not so broad and depressed as in the Burmese, and the bridge is usually prominent, almost aquiline. In the higher ranks the features are, he says, decidedly Tartar. The Hill Shans (Poloungs) have darker skins and are shorter than the Shans of the valleys, the average height of the valley men being 5 ft. 8 in. or less. The Chinese Shans are described as resembling Laplanders in their squat figures, broad, short, round, flat faces, and prominent cheek bones. Like the Nágás, they do not drink milk.

Mr Scott, in his account of the Keing Tung Shans, says that in stature and complexion they do not differ materially from the Western Shans. The nose, though small, is straight and not flattened out or button-shaped, and without a bridge, as in the people west of the Salween river. Of the hill races the Kwi are short in stature, and grow the hair to its full length. The Leu tribe, again, cut the hair short except a short tail. He speaks of a tribe as the wild Wās, who treat the hair like the Leus; whose skins are as dark as negroes or negrittos, and who go naked or nearly naked. They decorate their villages with the skulls of animals, as well as with human skulls, for the people are head-hunters. The wild Wa country is a little to the south of 23° lat., and a little to the east of 19° long.

As a rule the Shans are civilised. They are Buddhists, and although not so prominent a political power as they were some centuries ago, they are organised into principalities. They are agriculturists and traders, weavers, dyers and expert workers in metals. They are properly clothed, and construct houses, monasteries and temples. Notwithstanding the differences observed amongst the tribes, it is obvious that the Mongolian cast of features is the prevailing type. They have Chinese affinities in both physical characters and language, and it seems probable that they have migrated from Western China.

The Southern or Siamese Shans have both a political and philological affinity to the kingdom of Siam. The form Siam is a corruption of the French method of writing Shan or Scian, and the original monosyllabic term has been converted by them into a word of two syllables.*

I have had the opportunity of examining forty-four skulls collected in different parts of Burma, almost the whole of which are in the University Museum.

In 1889 my friend and former assistant, Surgeon-Major Wm. B. Bannerman, who was attached to the military expedition to Upper Burma, presented me with the skulls of two Dacoits.† The one, an old man, was the leader of a band in the Ye-U district, and was shot by the military police at Mugan; his head was brought into the village of Ye-U for identification in August 1888. The other, named Pau-dun, was hanged for murder at Ye-U in June of the same year. Dr Bannerman states that the people in the Ye-U district have, as a rule, the bridge of the nose flattened with the point turned up, and with wide nostrils. The eyes have the Mongolian cast, the cheeks are broad, the hair is black, long and straight, the skin yellow, and with scarcely any hair on the face

* *Report on Census of Burma*, 1891, p. 201. Rangoon, 1892.

† The Dacoits were the disbanded troops of King Thebaw's army. They were not hillmen, but Burmese.

except a lanky moustache. They are muscular, active, and under the average height of Europeans. The religion is Buddhist. From personal observations on infants and young children, Dr BANNERMAN has seen no evidence of modification from artificial pressure of the skull.

Another skull from Upper Burma, obtained at Mahlaing, Meiktila district, and said to be that of a Dacoit, was presented by Dr GEOFFREY H. PRANCE.

In the summer of 1895 I received from my friend and former assistant, Surgeon-Major G. J. H. BELL, a box containing the crania of sixteen men who had died in the central jail, of which he is the superintendent, at Insein, in Lower Burma. In 1897 the same gentleman forwarded to me a series of twenty skulls from this prison. The skulls were accompanied by explanatory lists, from which it appeared that thirty-two were Burmese, one was a Karen, one a Shan, and one a Mohammedan from Ralum, Akyab. Another, a Hindoo from the Coromandel coast, is not included in the following description. The name, jail number, sex, age, height, birthplace, crime for which imprisoned, and cause of death were given in the lists. To each specimen was appended a metal plate stamped with the jail number, the period of imprisonment, etc., which, I understand, it is customary for each criminal to wear suspended with a string around the neck. All the Burmese names have the prefix Nga,* a term employed by a superior when addressing one of much inferior social status. In more than one instance the cranial and dental characters did not correspond with the age of the person having the jail number specified in the lists, so that either the criminal had mis-stated his age, or the attendant employed to clean the specimens had not been sufficiently careful to attach the proper metal plate to the skull.

Early in 1896 I received from Surgeon-Captain J. M. CRAWFORD the skull of Nga Pota, æt. 32, a Burmese prisoner who had died in 1895 of phthisis in the jail at Benares when under Dr CRAWFORD'S charge.

In March 1897 Miss VIOLET G. S. ADAMS presented to the Museum two skulls which had been dug up in an old cemetery in Upper Burma. They had the appearance of buried bones which had lost much of their organic matter. One, an adult, had female characters; the other was a male somewhat advanced in life.

In the collection of the Henderson Trust, now in the University Museum, is a skull, No. 158, presented in 1827 by Mr GEORGE LYON, who procured it from Ava proper in Upper Burma. A second specimen, No. 159 in the same collection, is also said to be from Burma, but the precise locality is not stated.

Through the courtesy of Professor D. J. CUNNINGHAM I have been able to examine the skull in the museum under his charge of a Shan, Nga To, from the Insein jail.

In the following description I have arranged and compared with each other in Part I. thirty-seven skulls which were marked Burmese by the collectors.† The

* In the Abor Miri group of the Tibeto-Assam languages, Ngá is the personal pronoun (see Report on *Census of Assam*, 1891, p. 183).

† Shan Gyi and San Min from the Insein jail were both catalogued as Burmese; their measurements are given in Table VI.

Burmese crania from the prison of Insein are those of men. They are mostly in the prime of life, although three present marks of age, and one is said to be only eighteen years old. The other Burmese crania are also of the male sex; one is an old man, one is said to be twenty-one years of age, the other three are adults.

Part I. Tables III., IV., V., VI.

The skulls in this series gave, without doubt, a fair representation of the type met with amongst the male natives of Burma.

Norma Verticalis.—When arranged side by side on a table and examined from the *norma verticalis*, this series of skulls from Burma could be arranged in two more or less clearly defined groups. The one, which I shall designate Group A, included skulls, generally of a rounded form, and usually unsymmetrical in the parieto-occipital region, which, both from this character and from the steep vertical direction of the region in some of the specimens, gave evidence of the production of parieto-occipital flattening by artificial pressure applied during infancy. The unsymmetrical flattened surface in some specimens was directed obliquely to the right, in others obliquely to the left. In this group were a large proportion of the crania from the Insein jail, and five skulls not from that prison. All of these crania were brachycephalic, and several of them, as may be seen from the Tables, were hyper-brachycephalic. With three exceptions the vertex was not ridged in the sagittal region, nor did the vault slope rapidly downwards and outwards from the mesial suture to the parietal eminences. The curve of the vault in the vertical transverse direction from one parietal eminence to the other was not steep, and the skulls had generally a well-filled character.

The other Group, B, consisted of the remainder of the skulls from the jail at Insein. These had a more elongated form than those in Group A when examined from the *norma verticalis.* They did not show a definite want of symmetry in the parieto-occipital region, which, with one or two exceptions, was not so flattened and steep as in Group A, but sloped more gradually downwards and backwards into the occipital squama. As a rule these skulls did not reach the brachycephalic index, and they were usually longer than those in Group A. Two were dolichocephalic and elongated: one of these, San Min, with a length-breadth index 74, was said to be from the Southern Shan States, though marked Burman in the list sent along with the Insein skulls; the other, San Kun, with an index 74·9, was from the district of Monyo. In ten crania the cephalic index ranged from 75·3 to 79·5. In several the parietal eminences were prominent. Except in five crania there was no definite ridge in the sagittal line, and the slope outwards from it, as well as the curvature of the vault to the parietal eminences, was much the same as in Group A. As a rule the crania were cryptozygous both in A and B, but in some specimens in Group B the zygomatic arches could be distinctly seen from the *norma verticalis.*

Norma Lateralis.—In a few of the crania in both Groups A and B the glabella and supra-orbital ridges were moderately projecting; in others these ridges were so slight

as to be scarcely noticeable; but in none was the projection very strong. In one from the Insein jail an old depressed fracture was seen in the left frontal region just above the orbit; in two others from the same prison the frontal bone had been broken, and in a fourth the frontal and parietals had been extensively fractured during life. As a rule the forehead receded no more than one is accustomed to see in well-formed male skulls. The cranial vault was usually fairly well arched, and the parieto-occipital region showed the characters already described. In thirteen specimens the skulls rested behind on the tips of the mastoids, in the remainder on the cerebellar part of the occiput. In all the crania, with three exceptions, the occipital longitudinal arc was the shortest, and in most instances it was considerably below either the frontal or parietal. In twelve crania the parietal arc exceeded the frontal, and in three they were equal. The osseous bridge of the nose was often elongated, moderately projecting at its tip, and its outline was slightly concave. In the specimens with the projecting glabella the fronto-nasal suture was somewhat depressed, but the face did not show a marked flattening in the nasal region. The nasal spine of the superior maxillæ was, as a rule, only moderate, but in some skulls it was more strongly marked. A distinct ridge of demarcation separated the incisive region from the floor of the nose. In many of the crania the incisive region of the upper jaw was almost vertical, in others it projected slightly forward; it was exceptional to see a marked amount of alveolar prognathism. In some specimens the incisive and canine fossæ were deep. The orbits showed much variation in the relations of height and width.

In many of the crania the crowns of the teeth were flattened and much stained with betel-chewing. The palate was moderately arched; the mastoid processes, temporal and occipital ridges were not strong, as a rule, but in only a few specimens was the inion projecting. In a few of the crania the sutures were in process of obliteration, two skulls were metopic, the lambdoidal suture was usually free from Wormian bones, and in only two specimens were they numerous. The parieto-sphenoid articulation in the pterion was, as a rule, broad. Three skulls had an epipteric bone on one side, in one on both sides, and in two crania the squamous temporal articulated with the frontal on one side. No skull had an exostosis in the auditory meatus, but the left tympanic plate in one was much thickened at its free outer edge. In two skulls the external pterygoid plate was broadened backwards, but did not quite reach the spine of the sphenoid, so that the osseous boundary of a pterygo-spinous foramen was not completed. No skull had a third condyle, and in none was a para-mastoid process present, although in a few specimens the jugal process was tuberculated; an infra-orbital suture was occasionally seen. Variations from the normal ossification in this series of crania were therefore not common. As a rule the sutures of the cranial vault were simple in their denticulations.

The examination of the series of thirty-seven male skulls, and the study of their absolute and relative dimensions in certain diameters, as expressed in the tables of measurement, have given the following results.

TABLE III.

Burmese, from Insein Prison.

Province and Name.	Prome.					Tharrawaddy.					Hanthawaddy.			
	Maung.	Pyaw.	Shwe Hman.	Lu Ga Le.	Shwe Htun.	Po Nwe.	Shwe Gaung.	San Min.	Kwe Yoe.	Shwe Noe.	Ngwe Thee.	Po Tsan.	Kywet Oh.	Kya Huit.
Age,	23	Aged.	Aged.	18	40	29	30	32	23	72	20	30	33	Ad.
Sex,	M.	M.	M.	M.	M.	M.	M.	M.	M.	M.	M.	M.	M.	M.
Cubic capacity,	...	1460	...	...	1290	...	...	1820	1270	1340	1540	...	...	1440
Glabello-occipital length,	177	166	171	171	169	186	170	184	159	164	178	168	185	171
Basi-bregmatic height,	141	142	135	127	130	139	133	144	128	138	138	141	135	131
Vertical Index,	*79·7*	*85·5*	*78·9*	*74·3*	*76·9*	*74·7*	*78·2*	*78·3*	*80·5*	*84·1*	*77·5*	*83·9*	*73·0*	*76·6*
Minimum frontal diameter,	101	94	88	93	87	93	87	96	89	92	96	98	99	92
Stephanic diameter,	...	113	92	105	103	109	109	120	110	111	112	106	114	112
Asterionic ,,	105	97	101	106	101	105	108	114	105	103	117	104	112	103
Greatest parieto-squamous breadth,	143	150s.	134p.	140	140	140	142	153	142	139	148	145	147	141s.
Cephalic Index,	*80·8*	*90·4*	*78·4*	*81·9*	*82·8*	*75·3*	*83·5*	*83·2*	*89·3*	*84·8*	*83·1*	*86·3*	*79·5*	*82·5*
Horizontal circumference,	...	503	487	499	485	520	495	536	475	484	515	491	529	495
Frontal longitudinal arc,	134	129	130	126	121	137	129	135	121	127	128	111	130	122
Parietal ,, ,,	140	123	128	115	124	133	120	137	104	118	106	119	134	127
Occipital ,, ,,	111	109	100	112	100	123	111	125	98	111	128	112	121	109
Total ,, ,,	385	361	358	353	345	393	360	397	323	356	362	342	385	358
Vertical transverse arc,	317	323	298	300	290	308	294	328	302	306	312	304	312	303
Length of foramen magnum,	34	32	33	36	35	33	35	37	38	33	38	36	33	35
Basi-nasal length,	96	103	96	97	97	99	95	99	98	97	104	107	100	96
Basi-alveolar length,	97	102	93	98	93	97	88	100	93	...	98	101	101	91
Gnathic Index,	*101·0*	*99·*	*96·9*	*101·0*	*95·9*	*98·0*	*92·6*	*101·0*	*94·9*	...	*94·2*	*94·4*	*101·0*	*94·8*
Interzygomatic breadth,	137	141	128	134	134	132	130	135	127	132	138	144	138	129
Intermalar ,,	123	124	122	120	118	117	115	121	112	121	124	128	127	119
Nasio-mental length,	117	122	117	112	117	120	118	114	113	103	115	114	120ap	106
Nasio-mental complete facial Index,	*85·4*	*86·5*	*91·4*	*83·5*	*87·3*	*90·9*	*90·7*	*84·4*	*88·3*	*78·0*	*83·3*	*72·2*	*86·5*	*82·1*
Nasio-alveolar length,	72	72	73	67	70	71	74	69	67	...	71	73	75	62
Maxillary upper facial Index,	*52·5*	*51·*	*57·*	*50·*	*52·2*	*53·7*	*56·9*	*51·1*	*52·7*	...	*51·4*	*50·6*	*54·3*	*48·*
Nasal height,	55	54	54	53	54	52	53	51	53	48	56	58	50	48
Nasal width,	22	26	27	25	25	26	21	27	22	25	23	28	26	23
Nasal Index,	*40·0*	*48·1*	*50·*	*47·2*	*46·3*	*50·0*	*39·6*	*52·9*	*41·5*	*52·1*	*41·1*	*48·3*	*52·0*	*47·9*
Orbital width,	43	42	39	38	39	39	37	38	40	38	41	42	43	39
Orbital height,	33	37	32	36	34	36	36	34	38	32	35	33	35	30
Orbital Index,	*76·7*	*88·1*	*82·*	*94·7*	*87·2*	*92·3*	*97·3*	*89·5*	*95·0*	*84·2*	*85·4*	*78·6*	*81·4*	*76·9*
Palato-maxillary length,	56	57	57	51	53	54	52	57	49	...	51	65	59	50
Palato-maxillary breadth,	63	70	61	67	64	65	66	66	55	64	64	69	70	61
Palato-maxillary Index,	*112·2*	*122·8*	*107·*	*131·3*	*120·*	*120·3*	*126·9*	*115·7*	*112·2*	...	*125·5*	*106·1*	*118·6*	*122·*
Lower jaw. Symphysial height,	34	35	30	29	31	36	32	31	29	30	31	30	33	30
Lower jaw. Coronoid ,,	63	64	65	62	64	65	62	68	55	60	65	68	66	51
Lower jaw. Condyloid ,,	65	64	65	60	63	68	65	63	60	60	63	67	69	53
Lower jaw. Gonio-symphysial length,	86	89	85	88	89	93	86	90	92	87	90	94	95	80
Lower jaw. Inter-gonial width,	97	97	97	94	95	105	104	93	94	98	97	109	111	104
Lower jaw. Breadth of ascending ramus,	35	37	36	39	36	36	29	40	35	36	37	38	40	31

Table IV.

Burmese, from Insein Prison.

Name, with Place or Province.	Pyn Win, Zikeywa.	Lon Htaw, Zibyngon.	Pe, Sharsayboo.	Lu Gyi, Monyo.	San Kun, Monyo.	Noo, Aleywa.	Chut, Yaykayon.	Tun Yan, Mahathaman.	Shwe In, Sagaing.	ShweByaung, Pakokko.	Tun Tha, Goumyindan.	Pu, Aungmyingain.	Tun U, Myanaung.	Aung Myat, Yebouk.	Kyouk Lon, Sakangyi.
												Metopic.			
Age,	61	35	Ad.	52	Ad.	52	56	57	60	44	29	Ad.	33	30	Adult.
Sex,	M.	M.	M.	M.	M.	M.	M.	M.	M.	M.	M.	M.	M.	M.	M.
Cubic capacity,	1240	1235	1480	...	1240	1670	1350	...	1350	1480	1340	1330	1445	1445	1315
Glabello-occipital length,	173	168	167	170	179	179	179	185	172	179	161	168	183	170	167
Basi-bregmatic height,	132	133	131	139	130	139	141	145	132	136	136	131	140	136	130
Vertical Index,	*76·3*	*79·2*	*78·4*	*81·8*	*72·6*	*77·7*	*78·8*	*78·4*	*76·7*	*76·0*	*84·5*	*78·*	*76·5*	*80·*	*77·8*
Minimum frontal diameter,	93	94	91	92	91	98	95	98	92	94	95	91	88	93	90
Stephanic diameter,	105	108	112	109	102	121	108	113	113	111	110	101	102	117	111
Asterionic ,,	106	105	104	111	107	112	119	114	107	111	111	105	100	104	107
Greatest parieto-squamous breadth,	136s.	133s.	147s.	145s.	134s.	151s.	139s.	140	140	142	146s.	132s.	143s.	147p.	142s.
Cephalic Index,	*78·6*	*79·2*	*88·*	*85·3*	*74·9*	*84·4*	*77·7*	*75·7*	*81·4*	*79·3*	*90·7*	*78·6*	*78·1*	*86·5*	*85·*
Horizontal circumference,	493	478	502	498	494	527	510	518	500	516	484	485	505	503	489
Frontal longitudinal arc,	121	129	126	129	124	135	129	137	123	129	129	125	133	131	128
Parietal ,, ,,	127	118	127	118	124	140	121	126	123	129	103	127	130	126	115
Occipital ,, ,,	107	99	109	105	106	113	115	124	112	113	106	99	126	112	113
Total ,, ,,	355	346	362	352	354	388	365	387	358	371	338	351	389	369	356
Vertical transverse arc,	290	291	311	310	280	325	301	311	300	301	306	291	305	317	304
Length of foramen magnum,	35	36	35	35	29	39	33	37	35	36	35	33	31	29	33
Basi-nasal length,	96	94	89	103	109	98	106	101	95	102	99	95	101	95	93
Basi-alveolar length,	92	95	108	93	111	96	106	93	96	102	96	98	96	92	97
Gnathic Index,	*95·8*	*101·1*	*121·3*	*90·3*	*101·8*	*98·*	*100·*	*92·1*	*101·1*	*100·*	*97·*	*103·2*	*95·*	*96·8*	*104·3*
Interzygomatic breadth,	125	130	133	132	136	137	138	137	132	139	142	130	130	131	128
Intermalar ,,	117	121	120	121	127	127	126	126	115	126	131	118	119	118	113
Nasio-mental length,	108	106	116	117	122	122	118	122ap	112	120	113	109	120	116	115
Nasio-mental complete facial Index,	*86·4*	*81·5*	*87·2*	*88·6*	*89·6*	*89·*	*85·5*	*89·0*	*84·8*	*86·3*	*79·5*	*83·8*	*92·3*	*88·5*	*89·8*
Nasio-alveolar length,	66	63	68	71	71	75	69	74	65	74	68	64	73	69	66
Maxillary upper facial Index,	*52·8*	*48·4*	*51·1*	*53·7*	*52·2*	*54·7*	*50·*	*54·0*	*49·2*	*53·2*	*47·8*	*49·2*	*56·1*	*52·6*	*51·5*
Nasal height,	52	49	48	52	54	56	53	53	50	55	51	49	55	48	48
Nasal width,	29	26	24	25	32	27	27	27	26	25	26	23	25	24	23
Nasal Index,	*55·8*	*53·1*	*50·*	*48·2*	*59·1*	*48 2*	*50·9*	*50·9*	*52·0*	*45·5*	*51·*	*46·9*	*45·5*	*50·*	*47·9*
Orbital width,	38	36	40	37	40	40	39	43	40	39	40	38	40	37	36
Orbital height,	34	32	35	35	34	37	34	32	32	33	35	31	30	33	30
Orbital Index,	*89·5*	*88·9*	*87·5*	*94·6*	*85·*	*92·5*	*87·2*	*74·4*	*80·0*	*84·6*	*87·5*	*81·6*	*75·*	*89·2*	*83·3*
Palato-maxillary length,	50	53	50	49	60	57	55	50	54	62	54	53	55	54	54
Palato-maxillary breadth,	59	60	64	66	72	71	70	68	...	68	67	64	68	64	62
Palato-maxillary Index,	*118·*	*113·2*	*128·*	*134·6*	*120·*	*124·5*	*127·2*	*136·0*	...	*109·6*	*124·*	*120·7*	*123·6*	*118·5*	*114·8*
Lower jaw. Symphysial height,	28	28	36	29	33	32	36	38	27	35	30	40	32	33	34
Lower jaw. Coronoid ,,	63	55	61	64	62	61	70	62	65	71	63	62	60	66	65
Lower jaw. Condyloid ,,	66	64	64	64	67	64	73	66	66	74	68	67	69	67	61
Lower jaw. Gonio-symphysial length,	90	86	83	86	94	90	94	80	90	91	94	89	79	85	84
Lower jaw. Inter-gonial width,	93	105	103	100	102	115	106	98	98	97	110	83	88	93	99
Lower jaw. Breadth of ascending ramus,	38	37	32	35	40	32	44	37	36	33	40	40	34	34	33

TABLE V.

Burmese.

Name or Native Place.	Ava Proper.	Saung. Ava.	Upper Burma, Mahlaing, Meiktila District.	Paudun. Ye-U.	Mugan. Ye-U.	Nga Pota.
Collection,	H.T. 158	Insein.	E.U.A.M.	E.U.A.M.	E.U.A.M.	E.U.A.M.
Age,	Ad.	53	Ad.	21	Aged.	32
Sex,	M.	M.	M.	M.	M.	M.
Cubic capacity,	1248	1330	1300	1405	1160	1600
Glabello-occipital length,	158	172	173	178	163	176
Basi-bregmatic height,	131	139	132	131	127	140
Vertical Index,	*82·9*	*80·8*	*76·3*	*73·6*	*77·9*	*79·5*
Minimum frontal diameter,	92	92	96	92	93	97
Stephanic diameter,	112	109	110	109	105	114
Asterionic „	106	109	104	108	105	111
Greatest parieto-squamous breadth,	141s.	139	139s.	143s.	140s.	147s.
Cephalic Index,	*89·2*	*80·8*	*80·3*	*80·3*	*85·9*	*83·5*
Horizontal circumference,	481	486	500	515	482	515
Frontal longitudinal arc,	118	123	129	130	124	132
Parietal „ „	112	115	117	125	117	129
Occipital „ „	100	106	107	108	102	107
Total „ „	330	344	353	363	343	368
Vertical transverse arc,	305	298	297	306	293	314
Length of foramen magnum,	37	38	33	36	33	38
Basi-nasal length,	97	102	99	100	91	102
Basi-alveolar length,	98	103	100	95	...	96
Gnathic Index,	*101·*	*101·0*	*101·*	*95·*	...	*94·1*
Interzygomatic breadth,	135	139	134	131	130	138
Intermalar „	123	125	125	119	121	123
Nasio-mental length,	...	116	110	130	...	...
Nasio-mental complete facial Index,	...	*83·4*	*82·*	*99·2*	...	...
Nasio-alveolar length,	71	69	63	74	...	70
Maxillary upper facial Index,	*52·5*	*49·6*	*47·*	*56·4*	...	*50·7*
Nasal height,	52	52	49	53	46	56
Nasal width,	26	24	26	22	25	28
Nasal Index,	*50·*	*46·0*	*53·1*	*41·5*	*54·3*	*50·*
Orbital width,	40	39	41	40	36	40
Orbital height,	31	33	31	33	34	33
Orbital Index,	*77·5*	*84·6*	*75·6*	*82·5*	*94·4*	*82·5*
Palato-maxillary length,	53	56	55	53	...	61
Palato-maxillary breadth,	68	63	66	61	...	64
Palato-maxillary Index,	*128·3*	*112·2*	*120·*	*115·*	...	*125·5*
Lower jaw. Symphysial height,	...	29	31	37	...	...
Lower jaw. Coronoid „	...	63	74	60	54	...
Lower jaw. Condyloid „	...	64	64	62	61	...
Lower jaw. Gonio-symphysial length,	...	88	93	88	82	...
Lower jaw. Inter-gonial width,	...	104	100	86	94	...
Lower jaw. Breadth of ascending ramus,	...	42	38	40	35	...

In the glabello-occipital length the crania ranged from a maximum 186 mm. to a minimum 158 mm., and the mean of the series was 172·8 mm. In their parieto-squamous breadth the maximum was 153 mm., the minimum 132 mm., and the mean 141·7 mm. The mean length-breadth (cephalic) index was 82·1, which placed the series well into the brachycephalic group. In only two crania was this index below 75, and of the ten specimens which were mesaticephalic eight were above 77·5, *i.e.*, nearer to the brachycephalic than to the dolichocephalic standard. On the other hand eight specimens had a cephalic index of 85 or upwards, and two of these were above 90, so that a sensible proportion were hyper-brachycephalic. Both as regards the numerical index and the configuration of the cranium generally, there can be no doubt that the customary form of the Burmese skull is brachycephalic. The few exceptional specimens which had an elongated shape and an index either dolichocephalic or approximating thereto, are probably to be regarded as affiliated to the people with dolichocephalic skulls described in the earlier paragraphs in Part II.

In the basi-bregmatic height the crania ranged from a maximum of 145 mm. to a minimum of 127 mm., and the mean was 135·1 mm. The mean length-height (vertical) index was 78·2, which placed the series in the group of skulls termed akrocephalic or hypsicephalic, *i.e.*, with a high vertical index. But notwithstanding this relatively high index, in only three specimens did the vertical index slightly exceed the cephalic, and in two others they were equal. That the breadth of the skull is greater than the height is therefore a character which prevails in the Burmese skull.

The mean stephanic diameter, 109·2 mm., slightly exceeded the mean asterionic diameter, 106·8 mm., and the mean minimum frontal diameter was 93·1 mm. The bizygomatic diameter with a mean of 133·7 mm. ranged from 125 to 144 mm., and in each skull it invariably exceeded the intermalar.

The measurements made for the purpose of determining the length and breadth of the face gave the following results:—In thirty-five skulls the lower jaw was present, and the complete nasio-mental diameter, which ranged from 103 to 130 mm., had a mean length of 115·7 mm.; in its relation to the bizygomatic diameter the resulting index was in the mean 86·3, which places the crania in the chamæprosopic or low-faced group of Kollmann. In only six specimens did the index exceed 90, so as to bring these crania into the leptoprosopic division. In these skulls the upper facial index gave a different result, for although it had a range from 47 to 57, the mean was 52, which places the face generally in the leptoprosopic or high upper face group, and no fewer than twenty-six of these crania came into this category. The vertical diameter of the lower jaw in the mental region does not therefore contribute proportionally to the length of the face in the same measure as the vertical diameter of the superior maxilla.

In eighteen skulls the basi-nasal diameter was greater than the basi-alveolar, in thirteen it was slightly less, in one materially less, and in two they were equal. The mean gnathic index, calculated on the relations of these two diameters, was 98·9, which

shows how nearly equal they were in their mean relative proportions, so that they fall into the mesognathic group. It was exceptional to see a marked degree of alveolar prognathism.

The mean nasal index was 48·6, thus on the average the nasal height was something more than twice the width; though in the individual specimens the index ranged from 40·0 to 59·1. They came collectively just within the mesorhine group, but five specimens had the index above 53, *i.e.*, were platyrhine, and fourteen were leptorhine. The mean orbital index was 85·0, though in individual orbits it ranged from 73·2 to 97·3; the skulls collectively came within the mesoseme group, though ten were megaseme and fourteen were microseme. The mean palato-maxillary index was 119·7, and the range was from 106·8 to 136·0. In twenty-four specimens the index was 115 and upwards; they were brachyuranic, and showed a wide palato-alveolar diameter in relation to the length.

As regards the cubic capacity it must be remembered that all the skulls were males. The mean of twenty-eight specimens capable of being measured was 1388 c.c., which places them in the mesocephalic group. One skull had a capacity of only 1160 c.c.; two were 1600 and 1670 c.c. respectively, and one had the remarkably high capacity 1820 c.c.; but these were exceptional, and the usual capacity ranged from 1240 to 1450 c.c.

To sum up, the Burmese proper are brachycephalic; as a rule the cranial breadth is greater than the height; the face is low, chamæprosopic; the upper jaw is moderately projecting, mesognathic; the nasal width is moderate in relation to the height; the orbits vary in their dimensions, but the mean is mesoseme; the palato-alveolar arch is wide in relation to the length; the cranial capacity is moderate.

Part II. Table VI.

In this part are included the description of some skulls from Burma, which apparently belonged to tribes that form distinct elements in the population, and which may very properly be considered apart from those which belonged to the customary type of the people. With one exception, they were all apparently men.

H. T., No. 159 (Table VI.), referred to on page 728, though catalogued by the Henderson Trust as a Burmese skull, is not associated with any definite locality, and on this account and from its special character it has not been included in the preceding description. In the proportion of length and breadth it was distinctly dolichocephalic (72·2), and its outline in the *norma verticalis* was so elongated that it presented a striking contrast to the usual brachycephalic Burmese cranium. It was keeled in the anterior half of the sagittal region, from which the parietals sloped downwards to their eminences, below which the side walls of the skull were almost vertical. It differed also from the customary type of the Burmese skulls in having its basi-bregmatic height and vertical index considerably higher than its greatest breadth and cephalic index. The skull was phænozygous. The forehead was narrow, but was

almost vertical. The glabella and supra-orbital ridges were feeble. The nasal bridge was concave, depressed above and slightly projecting below; the anterior nares were wide, and the nasal index was distinctly platyrhine. The nasal spine of the superior maxillæ was moderate, and an imperfect ridge separated the incisive region from the floor of the nose. The absence of the lower jaw prevented the proportions of the entire face from being taken, but the upper face was leptoprosopic. Some small Wormian bones were in the lambdoidal suture, and there was a large left epipteric bone. The prognathism of the upper jaw was well marked; the breadth of the orbit was materially greater than the height, and the index was microseme. The combination of the most important of these characters caused the skull to differ from the type described in Part I., so that it does not possess the customary features of a Burmese skull.

The two skulls obtained from an old cemetery in upper Burma also differed materially in character from the brachycephalic crania sent to me from the Insein jail. They were both distinctly dolichocephalic both in form and measurements, and in each specimen the height exceeded the breadth. In this respect they corresponded with the skull 159 above described in the collection of the Henderson Trust. They did not, however, possess the prognathic condition of the upper jaw, which was a feature in that specimen. Although the nasal bones were not projecting, the proportions of the nose were not platyrhine. As the two dimensions of the orbit were more nearly on an equality, the orbital index was higher than in 159. The breadth of the palato-maxillary arch, in relation to the length, was not so great. In the male skull there was a small inter-parietal bone, and in the female, Wormian bones were in the lambdoidal suture. In one pterion in the female the ali-sphenoid had a very slight articulation with the parietal, in the other they were separated by a process continuous with the squamous temporal.

It is obvious that a certain admixture with the brachycephalic Burmese of a race or races with dolichocephalic proportions of the skull is to be found in Burma. It is possible that they may be the descendants of the aboriginal people, or be those of persons, or the descendants of persons, who had migrated into Burma from the hill districts at present inhabited by a dolichocephalic race.

One of the skulls from Insein, marked Erinia, was from Ralum, Akyab, in the northern part of Burma, south of Chittagong, where the people are for the most part Mahommedans. It was that of a man, said to be seventy years of age, whose height was 5 ft. 6 in. The condition of the sutures and the state of the teeth proved it to be that of a person who had passed middle life. The skull was hyper-brachycephalic, with a vertical parieto-occipital region, which pointed to artificial flattening during infancy. The height of the cranium was considerably less than the breadth. The skull was cryptozygous. The glabella and supra-orbital ridges were well marked, and the forehead sloped gently backwards and upwards. The nasal bridge was moderate in length, slightly concave, and somewhat depressed at the root; the nasal index was platyrhine. The upper jaw was not prognathous, the incisive region was short, but separated from

TABLE VI.

Karen, Shan, etc.

Name, or Native Place or Province.	Burma, no Locality.	Here. Toungoo. Karen.		Erinia. Akyab.		Ko Nanda. Shan.	To, Shan State, Yunnan.	Shan Gyi, Tharrawaddy.	San Min, Southern Shan States.	Old Cemetery, Upper Burma.	Old Cemetery, Upper Burma.
Collection,	H.T.159	Insein.		Insein.		Insein.	T.C.D.	Insein.	Insein.	E.U.A.M.	E.U.A.M.
Age,	Ad.	23		70		40	27	55	24	Aged.	Ad.
Sex,	M.	M.		M.		M.	M.	M.	M.	M.	F.
Cubic capacity,	1345	1420		1410		...	1510	1360	1380	...	1270
Glabello-occipital length,	180	175		167		186	178	175	181	185	181
Basi-bregmatic height,	136	134		136		147	146	135	133	138	135
Vertical Index,	*75·6*	*76·6*		*81·4*		*79·0*	*82·0*	*77·1*	*73·5*	*74·6*	*74·6*
Minimum frontal diameter,	91	85		92		96	94	96	89	89	93
Stephanic diameter,	101	106		116		110	107	108	107	108	109
Asterionic „	103	105		115		113	109	101	106	105	102
Greatest parieto-squamous breadth,	130p.	141		150		150	140	140	134	135	129
Cephalic Index,	*72·2*	*80·6*		*89·8*		*80·6*	*78·7*	*80·0*	*74·0*	*73·*	*71·3*
Horizontal circumference,	504	498		501		538	505	506	502	518	502
Frontal longitudinal arc,	132	128		127		133	117	120	122	123	121
Parietal „ „	120	118		118		140	135	140	124	253	120
Occipital „ „	124	116		110		124	122	110	114		125
Total „ „	376	362		355		397	374	370	360	376	366
Vertical transverse arc,	302	300		317		320	306	301	292	308	296
Length of foramen magnum,	35	32		32		35	34	31	36	33	34
Basi-nasal length,	97	97		106		102	107	103	102	103	100
Basi-alveolar length,	105	97		104		101	100	109ap	105	96	99
Gnathic Index,	*108·2*	*100·*		*98·1*		*99·0*	*93·5*	*105·8*	*102·9*	*93·2*	*99·0*
Interzygomatic breadth,	130	121		140		140	141	138	125	...	...
Intermalar „	119	109		125		123	118	129	112	...	...
Nasio-mental length,	...	109		113		129	113	120	115	...	114
Nasio-mental complete facial Index,	...	*90·*		*80·7*		*92·*	*80·1*	*87·*	*92·0*	...	...
Nasio-alveolar length,	70	67		65		74	67	72	69	72	66
Maxillary upper facial Index,	*53·8*	*55·3*		*46·4*		*52·8*	*47·5*	*52·*	*55·2*	...	...
Nasal height,	52	53		52		55	54	52	53	56	50
Nasal width,	29	26		29		24	26	25	24	28	22
Nasal Index,	*55·8*	*49·1*		*55·8*		*43·6*	*48·1*	*48·2*	*45·3*	*50·*	*44·*
Orbital width,	38	36		38		38	41	41	38	41	39
Orbital height,	28	30		33		31	34	30	33	34	31
Orbital Index,	*73·7*	*83·3*		*86·8*		*81·6*	*82·9*	*73·2*	*86·8*	*82·9*	*79·5*
Palato-maxillary length,	59	50		52		58	53	58ap	58	...	55
Palato-maxillary breadth,	68	61		...		68	...	65	62	...	63
Palato-maxillary Index,	*115·2*	*122·*		...		*117·2*	...	*112·*	*106·8*	...	*114·5*
Lower jaw. Symphysial height,	...	24		31		38	30	37	35	30	34
Lower jaw. Coronoid „	...	54		60		65	65	68	71	63	60
Lower jaw. Condyloid „	...	57		66		68	63	71	69	68	66
Lower jaw. Gonio-symphysial length,	...	80		90		85	88	99	95	83	81
Lower jaw. Inter-gonial width,	...	96		107		105	95	105	88	98	87
Lower jaw. Breadth of ascending ramus,	...	30		43		38	36	47	40	35	33

the floor of the nose by a long ridge; the nasal spine was moderate. The orbital index was mesoseme. The interzygomatic breadth, 140 mm., was a feature in the face, and both the entire facial and upper facial indices were chamæprosopic. The cranial capacity was moderate, 1410 c.c. Owing to the extensive senile obliteration of the sutures, nothing can be said as to Wormian or epipteric bones.

The skull shows no material difference from the Burmese type, so that although a Mahommedan in religion he was probably of the Burmese race.

The skull marked Karen from the jail at Insein was that of a man named Here, aged twenty-three, 5 ft. $1\frac{3}{4}$ in. in height. In the relation of length to breadth it was brachycephalic, and the vertical index was distinctly below the length-breadth index. The outline in the *norma verticalis* was broadly ovoid, and the parieto-occipital slope was not so steep as to suggest artificial flattening in that region. The cranium was moderately capacious, and contained 1420 c.c. The skull was cryptozygous. The forehead was full, sloping moderately backwards, and the glabella and supra-orbital ridges projected very slightly. The nasal bridge was elongated, concave, not depressed at the root, and slightly projecting below; the nose in its proportions was mesorhine. The nasal spine of the superior maxillæ was small, and the incisive region was continued into the floor of the nose by a smooth surface. The basi-nasal and basi-alveolar diameters were equal and the upper jaw was mesognathous. In its dimensions the orbit was microseme. The entire face in the relations of length and breadth was chamæprosopic, but the upper face was leptoprosopic. The interzygomatic breadth, 121 mm., was relatively small. The ossification of the cranium was normal.

So far as a single skull can enable one to express an opinion on the cranial characters of a people, it would appear that the Karens are a brachycephalic race. This view of the proportion of the breadth to the length of the cranium is borne out by two male skulls marked Karen, the measurements of which are recorded in Sir Wm. Flower's Catalogue of the Museum of the College of Surgeons. In one the length-breadth index was 82·9, in the other 79·2. It should be stated that in both of these the height of the cranium exceeded the breadth. The mean gnathic index was 98·5.

The collection from Insein contained the skull of a man marked Shan, named Ko Nanda, whose height is given as 5 ft. 5 in. His death was caused by a fracture of the skull. I have also had the opportunity of examining the skull of another Shan named Nga To, said to be twenty-seven years of age, a native of Yunnan, now in Professor Cunningham's Museum. These skulls differed from each other in some particulars. Nanda was brachycephalic, 80·6, without artificial parieto-occipital flattening; Nga To, again, was in the higher term of the mesaticephalic series. In Nanda the height of the cranium was less than the breadth, but in To the height materially exceeded the breadth. In both skulls the basi-nasal length exceeded the basi-alveolar, and there was no prognathism. In Nanda the glabella and supra-orbital ridges were feeble, and in To moderately projecting; the nasal bridge was concave, elongated, not depressed at the root, and projecting slightly forward below. The nasal

region was generally flattened. In neither specimen was the nose platyrhine. In Nanda the nasal spine of the superior maxillæ was strong, the incisive fossa was deep and was separated from the floor of the nose by a ridge. In Nanda the index of the entire face was leptoprosopic; in To it was chamæprosopic, and a similar proportion was seen in the upper facial index; but both specimens had great interzygomatic diameter. In both crania the orbital index was mesoseme. As regards the cubic capacity of the crania, Nanda was so much injured that the cubage of the skull could not be taken, but the capacity of To was 1510 c.c.*

From the relations of length to breadth in the two Shan crania there can be little doubt that these people are in the main brachycephalic, as might have been expected from their Siamese and Chinese affinities.

For purposes of comparison I may refer to four adult male skulls in the Anatomical Museum of the University, which belong to the collection formed by Dr R. Broom. They are from Bangkok; three are undoubted Siamese, whilst the one lettered A in Table VII. is said to be probably a cross between a Malay and a Siamese.† Their measurements are given in the Table.

All the crania were brachycephalic, both in their general form and numerical proportion; and in three the flattened parieto-occipital region showed evidence of artificial pressure applied during infancy. In each specimen the height was not equal to the breadth. In three specimens the frontal longitudinal arc was longer than either the parietal or occipital. The glabella and supra-orbital ridges were not prominent, and the forehead only slightly receded. The nasal bones had so small a degree of projection that the face was flattened in that region, and the nasal index was mesorhine. The nasal spine of the superior maxillæ was well marked, and the incisive region of the upper jaw was differentiated from the floor of the nose by a ridge. In one specimen the jaw was orthognathic; the others showed to the eye a degree of alveolar prognathism greater than was indicated by the gnathic index. Although in one specimen the complete facial index was 92·8, in the others the face was low, chamæprosopic, a condition which was obviously due to the breadth between the zygomata. The orbital index was variable, and in only two crania the orbits could be regarded as round or megaseme. The palato-alveolar region was either mesuranic or brachyuranic. The mean cubic capacity of the four skulls was 1332 c.c. The teeth were stained with betel-chewing. In two specimens an epipteric bone was present, in one there were two small Wormian bones. One had flat occipital condyles, which were not associated with a third condyle. The palate was highly arched, and the lower jaw was well developed.

In the Barnard Davis Collection, now in the Museum of the Royal College of

* From the name, Shan Gyi, of one of the men from the jail at Insein (Table VI.), it is possible that he may have been a Shan. It is to be observed that his skull was also brachycephalic. Another skull, that of San Min (Table VI.), described as from the Southern Shan States, was distinctly dolichocephalic, index 74, so that it differed from both the Burmese and Shan type of cranium, and probably belonged to a foreign race.

† A fifth adult specimen is in the collection, but as it has been deformed, apparently from hydrocephalus, the measurements have not been given. Its internal capacity was 1930 c.c.

Table VII.

Siamese.

	Assam. Cross between Siamese and Malay.	Warng.			Metopic, Hydrocephalic.
Collection (Dr R. Broom),	A.	B.	C.	D.	
Age,	Ad.	31	Ad.	Ad.	Ad.
Sex,	M.	M.	M.	M.	M.
Cubic capacity,	1330	1270	1330	1400	1930
Glabello-occipital length,	168	162	166	173	Size and
Basi-bregmatic height,	138	131	135	138	proportions
Vertical Index,	*82·1*	*80·9*	*81·3*	*79·8*	abnormal.
Minimum frontal diameter,	88	92	98	95	
Stephanic diameter,	113	114	111	115	
Asterionic „	108	100	108	105	
Greatest parieto-squamous breadth,	139	137s.	143s.	144s.	
Cephalic Index,	*82·7*	*84·6*	*86·1*	*83·2*	
Horizontal circumference,	494	484	490	502	
Frontal longitudinal arc,	135	127	120	132	
Parietal „ „	116	123	129	117	
Occipital „ „	114	98	95	107	
Total „ „	365	348	344	356	
Vertical transverse arc,	305	305	305	311	
Length of foramen magnum,	30	35	36	34	
Basi-nasal length,	98	94	101	101	
Basi-alveolar length,	92	93	103	101	
Gnathic Index,	*93·9*	*98·9*	*102·*	*100·*	
Interzygomatic breadth,	128	126	134	138	
Intermalar „	118	117	124	125	
Nasio-mental length,	113	117	118	118	
Nasio-mental complete facial Index,	*88·2*	*92·8*	*88·*	*85·5*	
Nasio-alveolar length,	67	67	66	69	
Maxillary upper facial Index,	*52·3*	*53·1*	*49·2*	*50·*	
Nasal height,	53	52	50	52	
Nasal width,	26	25	26	26	
Nasal Index,	*49·1*	*48·2*	*52·*	*50·*	
Orbital width,	37	40	39	41	
Orbital height,	34	33	31	37	
Orbital Index,	*91·9*	*82·5*	*79·5*	*90·2*	
Palato-maxillary length,	53	52	55	58	
Palato-maxillary breadth,	62	62	66	63	
Palato-maxillary Index,	*116·9*	*119·2*	*120·*	*108·6*	
Lower jaw. Symphysial height,	26	28	31	31	
Lower jaw. Coronoid „	63	68	68	70	
Lower jaw. Condyloid „	67	68	71	68	
Lower jaw. Gonio-symphysial length,	88	89	95	90	
Lower jaw. Inter-gonial width,	91	104	99	108	
Lower jaw. Breadth of ascending ramus,	39	40	44	39	

Surgeons of England, are several skulls from Siam, which are catalogued by the name Thai.* Six of the crania ranged in their length-breadth index from 80 to 89, and the mean was 85; they were distinctly brachycephalic. A seventh specimen was dolichocephalic, index 73, which Dr Davis ascribes to the sides of the coronal suture having been obliterated: an explanation which does not appear to me to be satisfactory. There can be no doubt that the normal shape of the Siamese skull is brachycephalic.

The University Museum also contains a collection of crania ascribed to natives of China. With the greater number the history supports the view that they are undoubted Chinese, but two or three specimens are uncertain. They are all adults; eleven are male, two female. Their measurements are given in Table VIII.

I do not intend to give a detailed description of this series of skulls. I may, however, state that one skull obtained at Chusan was dolichocephalic (index 74·3), seven were brachycephalic, five were mesaticephalic. Of the latter three had the cephalic index above 77·5; the remaining two, with the index 76·9, had a doubtful history, and, as well as one from Chusan, were possibly not true Chinese. Even if we include all the specimens the cephalic index works out with a mean 81·2, and if the doubtful specimens be excluded, it is a little higher, and the mean of the entire series is brachycephalic. The breadth of the cranium exceeded the height in all but three specimens.

As the lower jaw had not been preserved in the majority of the crania, the complete facial index could only be obtained in three skulls, which were low-faced, chamæprosopic. I have compared these crania, as regards their interzygomatic breadth, with the corresponding dimension in neighbouring Mongolian people, whose skulls approximate in general magnitude, and also with the Esquimaux. From the appended list it will be seen that in this diameter the Chinese face has a less transverse diameter than the Burmese, Shans, Nágás and Esquimaux, though somewhat greater than in the small number of Siamese under examination.

	Number of Skulls.	Sex.	Mean Interzyg. Diam.
Chinese, . . .	11	M.	132·5
Siamese, . . .	4	M.	131·5
Burmese, . . .	38	M.	133·7
Shans, ...	2	M.	140·5
Chin-Lushais, . . .	9	M.	128·8
Nágás, ...	6	M.	135·3
Esquimaux, . . .	18	M.	138·0

So far as the degree of prognathism can be determined by the measurements from which the gnathic index is computed the skulls generally were orthognathous, but three were mesognathous. The only prognathic skull was the one found at Chusan with a length-breadth index of 74·3, an additional reason therefore for regarding it as not a genuine Chinaman. As a rule the nose was either mesorhine or leptorhine. Four specimens were platyrhine, and the Chusan skull was in this category. In six crania

* *Thesaurus Craniorum*, p. 174. The mean interzygomatic diameter of these crania was 132 mm.

Table VIII.

Chinese.

Collection,	E.U.A.M.*	E.U.A.M.†	E.U.A.M.	E.U.A.M.‡	E.U.A.M. Hong Kong.	H.T. 161	H.T. 163	H.T. 165	H.T. 169	H.T. 170 Chusan.	H.T. 494	H.T. 162	H.T. 523
Age,	Ad.	Ad.	Ad.	Ad.	Ad.	Aged.	Ad.	Ad.	Aged.	Ad.	Ad.	Ad.	Aged.
Sex,	M.	M.	M.	M.	M.	M.	M.	M.	M.	M.	M.	F.	F.
Cubic capacity,	1320	...	1370	1400	1240	1335	1590	1540	1300	1330	1340	1280	1140
Glabello-occipital length,	175	170	182	175	166	167	179	168	170	179	168	167	173
Basi-bregmatic height,	129	136ap.	129	141	134	136	144	141	137	126	135	129	125
Vertical height,	*73·7*	*80·*	*70·9*	*80·6*	*80·7*	*81·4*	*80·4*	*83·9*	*80·6*	*70·4*	*80·4*	*77·2*	*72·3*
Minimum frontal diameter,	94	100	95	90	90	93	92	91	92	86	95	86	89
Stephanic diameter,	109	116	105	107	100	105	119	113	112	112	116	103	103
Asterionic ,,	114	108	108	111	100	107	106	111	116	108	104	99	109
Greatest parieto-squamous breadth,	143	148	140	138	133	143	142	150s.	143s.	133	139	132	133
Cephalic Index,	*81·7*	*87·1*	*76·9*	*78·9*	*80·1*	*85·6*	*79·3*	*89·3*	*84·1*	*74·3*	*82·7*	*79·*	*76·9*
Horizontal circumference,	505	...	510	500	479	490	512	503	500	493	495	478	485
Frontal longitudinal arc,	122	128	127	130	117	118	133	128	133	115	122	117	124
Parietal ,, ,,	123	130	131	130	127	120	134	130	113	114	122	116	115
Occipital ,, ,,	121	114	109	117	104	102	115	110	124	130	113	116	102
Total ,, ,,	366	372	367	377	348	340	382	368	370	359	357	349	341
Vertical transverse arc,	297	...	289	313	295	300	315	317	310	295	298	285	291
Length of foramen magnum,	30	...	32	36	32	35	37	36	30	36	35	31	31
Basi-nasal length,	93	...	99	93	98	106	99	94	95	96	94	93	101
Basi-alveolar length,	90	...	100	84	91	98	94	93	96	103	90	86	94
Gnathic Index,	*96·8*	...	*101·*	*90·3*	*92·9*	*92·5*	*94·9*	*98·9*	*101·1*	*107·3*	*95·7*	*92·5*	*93·1*
Interzygomatic breadth,	132	139	134	128	126	141	134	132	124	132	136	121	119
Intermalar ,,	117	126	122	118	116	129	120	115	113	117	121	110	105
Nasio-mental length,	116	113	...	...	114	...	...	...	...	...	...	...	...
Nasio-mental complete facial Index,	*87·8*	*81·2*	...	...	*90·4*	...	...	...	...	...	...	...	...
Nasio-alveolar length,	73	66	...	71	70	70	72	66	66	70	67	61	61
Maxillary upper facial Index,	*55·3*	*47·4*	...	*55·4*	*55·5*	*49·6*	*53·7*	*50·*	*53·2*	*53·*	*49·2*	*50·4*	*51·2*
Nasal height,	51	50	47	51	52	56	56	53	49	52	54	48	46
Nasal width,	24	24	26	24	25	26	24	28	26	28	26	24	25
Nasal Index,	*47·1*	*48·*	*55·3*	*47·1*	*48·2*	*46·4*	*42·8*	*52·8*	*53·1*	*53·5*	*48·1*	*50·*	*54·3*
Orbital width,	38	38	43	36	40	41	37	35	37	37	36	33	38
Orbital height,	34	29	33	33	33	35	33	35	35	31	31	32	32
Orbital Index,	*89·5*	*76·3*	*76·7*	*91·7*	*82·5*	*85·4*	*89·2*	*100·*	*94·6*	*83·8*	*86·1*	*97·*	*84·2*
Palato-maxillary length,	52	53	52ap.	48	47	50	56	52	53	57	52	45	50
Palato-maxillary breadth,	67	67	57	62	60	65	64	63	59	67	66	59	53
Palato-maxillary Index,	*128·8*	*126·4*	*109·6*	*129·*	*127·6*	*130·*	*114·2*	*121·*	*111·3*	*117·5*	*126·9*	*131·1*	*106·*
Lower jaw. Symphysial height,	39	37	...	...	31								
Lower jaw. Coronoid ,,	64	65	...	...	59								
Lower jaw. Condyloid ,,	55	75	...	...	56								
Lower jaw. Gonio-symphysial length,	89	92	...	...	86								
Lower jaw. Inter-gonial width,	92	95	...	...	97								
Lower jaw. Breadth of ascending ramus,	33	37	...	...	34								

* With skeleton—presented by G. D. Hutchison, Esq. † Presented by Dr More Reid. ‡ Presented by Professor Greenfield.

the orbit was rounded (megaseme), but in four the transverse diameter so much exceeded the vertical as to place them in the microseme group. In nine specimens the palato-alveolar arch was horseshoe-shaped, brachyuranic; in only two skulls it was elongated so as to be dolichuranic.

In the Chinese the mean cranial capacity of the males was 1376·5 c.c. They approximate closely, therefore, to the Burmese and Siamese in the volume of the cranial cavity.

Since I began, about thirty-five years ago, to collect human crania for purposes of anthropological study, I have endeavoured, as far as possible, to obtain for each skull or group of skulls, a statement of the locality where the specimen was obtained, and of the conditions under which it was got. In a large majority I have found it possible to acquire these particulars, and to speak therefore with some precision of the specimens. When I have resorted to the older collections to which I have had access, not unfrequently I have found a skull catalogued under some general designation, such as from Australia, from India, or from Ceylon, without any attempt being made to specify the exact locality. Such specimens, of course, have not the same value in determining the distribution of the two great groups of dolichocephali and brachycephali.

In all cases, however, the conservator of a museum is dependent on the accuracy of the original collector, and the care with which the specimens have been marked. The series of crania described in this memoir have, with few exceptions, been gathered by members of the medical profession, who have carefully labelled them and given me an account of the locality, and the conditions under which they were collected. We may rely therefore on the specimens as representing, so far as they go, the crania of the people inhabiting the regions in which they were obtained.

It will have been noticed that from time to time in the course of the description, I have referred to the occurrence of crania, brachycephalic in form and proportions, in districts where the skulls are usually dolichocephalic, and conversely of skulls, dolichocephalic in form and proportions, being found in districts where brachycephalic crania are the customary type. The question may, therefore, be very properly considered, in how far the contrasted forms of skulls which we designate by the terms dolichocephalic and brachycephalic, are to be regarded as two distinct race types, or merely extremes found in the same race, graded into each other by a series of intermediate forms. If the latter proposition be correct they would lose the value which has been assigned to them, since the time of Anders Retzius, as important guides in the classification of races. In employing these terms it should be understood that I recognise with Broca and the later school of craniologists a mesaticephalic (mesocephalic) group, as interposed between the more extreme brachycephalic and dolichocephalic forms, and that, to enable a comparison to be made between my observations and those of craniologists generally, the arbitrary numerical division into dolichocephali, with the length-

breadth index below 75, mesaticephali, index from 75 to 80, and brachycephali, index 80 and upwards, has been employed in this memoir. It is obvious that those mesaticephalic skulls which have the length-breadth index below 77·5 approach nearer to the dolichocephali, whilst those with this index above 77·5 approximate to the brachycephali. Thus a skull with the index at or near 76 or 77 is in its form essentially dolichocephalic; whilst one with an index at or near 78 or 79 is essentially brachycephalic, though not falling numerically into this category.

To assist one in determining the value of these classificatory characters as expressing racial distinctions, one should strive to obtain a sufficient number of skulls of a given race, and determine, both by inspection of their form and by actual measurement, how far they fall exclusively either into the brachycephalic or the dolichocephalic group, or present an admixture of both groups, or possess the form and proportion, termed mesaticephalic, *i.e.*, intermediate to the two extremes. One ought not, however, to attach, as is sometimes done, too exclusive an importance in the determination of race characters to the differences expressed by the terms dolichocephalic and brachycephalic; as if those races were necessarily allied to each other, which on the one hand had in common dolichocephalic skulls, or, on the other, heads brachycephalic in form and proportions.* Retzius himself emphasised also the necessity of the study of the relative projection of the upper jaw, and employed the terms orthognathic and prognathic in his classification of races in accordance with their skull and head-forms. Since his time the relation between the length and breadth of the nose, the breadth and height of the orbit, the breadth and length of the palato-alveolar arch, the breadth and height of the face, the breadth and height of the box of the cranium, as well as its cubic capacity, have all attracted attention. The value of cranial characters as a basis for the classification of races depends therefore upon a comparison not only of the relative length and breadth of the skull or head, but of several other characters. When, with but a slight range of variation, the majority of these characters correspond in a particular tribe or people, they may then properly be considered as the cranial and head characters of the race, and be of value for purposes of classification.

It is not easy at the present time to find a race so pure that the possibility of an intermixture with another race may not at some previous period in the history of the race or the locality have taken place. In using this term 'intermixture' one should understand that it may cover one or other of two conditions. Either it may be produced by the cohabitation of parents of different races, whose offspring would therefore be a half or mixed breed. Or by the residence side by side either, in the same

* The question of the signification of brachycephaly and dolichocephaly has been discussed in a recent memoir by Dr A. B. Meyer of Dresden, "On the Distribution of the Negritos in the Philippine Islands and elsewhere," and he has arrived at the conclusion that they are not necessarily to be looked upon as constant factors in the determination of racial features. He regards the Negritos and Papuans to be of one race, notwithstanding the differences in the form of the skull and in the stature; so that in his view considerable variability may exist in the physical characters of the same race.

village or in adjacent villages, of individuals or families of, say, two different races, one of which may have reached the place either as captives in war, or as invaders, and the other may represent the aboriginal inhabitants. Skulls collected in such a district would be therefore those of distinct races, and might possess very different forms and proportions, although cohabitation and the production of a mixed breed would also doubtless give rise to a people in which the individuality of the parent types would be lost.

There are, however, certain parts of the globe where, from the climatic conditions, or the geographical position, an almost perfect isolation of the people is possible, and where one may expect to find the race as nearly as possible in its purity.

It is customary, for instance, to speak of the Esquimaux as a dolichocephalic race, and numerous skulls have been measured and recorded in evidence of this character. For my present purpose I may refer to the specimens enumerated and measured in Sir Wm. Flower's catalogue,* where the mean cephalic index of twenty-seven crania was 72. Twenty-five of these crania ranged in the length-breadth index from 66·1, the minimum, to 76·6, the maximum, but two specimens were respectively 78·1 and 78·7, *i.e.*, in the higher term of the mesaticephalic group. It is to be noted that both of these were from the eastern side in proximity to Baffin's Bay, where the possibility of the production of a half-breed by intercrossing with a brachycephalic Dane is not unlikely to have occurred.

In the Anatomical Museum of the University of Edinburgh are twenty-two adult Esquimaux crania collected at various places from Greenland to Behring Straits. Eighteen of these had a mean length-breadth index 71·4, and the range was from 69·3 to 75·7; they may all be regarded as essentially dolichocephalic. The remaining four specimens presented different proportions, for the length-breadth indices ranged from 76·2 to 87, so that three were mesaticephalic and one hyperbrachycephalic. A special interest is to be attached to these four crania, as they belonged to the western division of the Esquimaux, and were collected by the late Mr John Simpson,† Surgeon to H.M.S. *Plover*, at Point Barrow, Kotzebue Sound, on the American side of Behring Straits. From Mr Simpson's description communication takes place yearly with the Asiatic coast by boats, which cross the Straits after midsummer, and an active trade is carried on between the Esquimaux and the Asiatics. Opportunities are therefore given for an intermixture of the brachycephalic people of Northern Asia with the dolichocephalic Esquimaux, and in this manner a crossing of the two races and the production of half-breed children could without difficulty arise; or some of the Asiatics might, and it is probable do, stay and cohabit with the Esqui-

* Museum of the Royal College of Surgeons of England, 1879.

† See an excellent description of the locality and people by Mr John Simpson in the *Nautical Magazine*, vol. xxiii. p. 639, 1854. A fifth specimen from the same locality was dolichocephalic, with a length-breadth index 72·7. It is included in the eighteen crania referred to in the text.

maux and be adopted as members of the tribe. One may therefore legitimately draw the conclusion that, as regards the Esquimaux, the occurrence of a brachycephalic cranium or of skulls in the higher terms of the mesaticephalic group may be accounted for by the introduction from without of another race possessing brachycephalic proportions, and not by the evolution within the dolichocephalic Esquimaux of a brachycephalic type.

As regards certain of the other leading characters of the adult crania, it is to be observed that in the dolichocephalic Esquimaux, with few exceptions, the height of the cranium was greater than the breadth; the nasal region was narrow and elongated and well within the leptorhine index, with the exception of one specimen which was mesorhine. The mean gnathic index was 99·5, mesognathous; one specimen only was prognathous; the index variation between 94, the lower, and 104·6, the highest, was 10·6; and twelve out of sixteen specimens ranged only from 97·3 to 101. The skulls therefore showed in these relations a remarkable constancy of type, in harmony with the uniformity in the proportion of the length to the breadth of the cranium.

Another race, from its geographical isolation, and from the number of specimens which I have collected, may also appropriately be considered. I refer to the aborigines of Australia. Several travellers have expressed the opinion that the natives conform to one pattern as regards features, colour of skin, hair and mental characters. The University Museum contains seventy-one adult crania of these people. In almost every instance the locality where the skull was got is known, and the series is representative of all parts of the great island, except the central region. Sixty-nine skulls ranged in their length-breadth index from 61·5 (a specimen elongated from scaphocephaly) to 71·1, and their mean index was 70·2; they were all dolichocephalic both in form and proportion. Of the remaining two skulls, one, a female from West Victoria, had a cephalic index, 77·9; the other a male, from the Thomson River, Queensland, had an index 77·4; both, therefore, were mesaticephalic. Although brachycephalic Malays do, it is said, visit the west coast, and brachycephalic Polynesians may possibly have visited the east coast of Australia, yet in the large series of skulls now before me not a single brachycephalic specimen occurred. There is no evidence therefore of an evolution within the dolichocephalic Australians, or even of the intrusion from without, of a brachycephalic type. As regards the proportions of other parts of the skull, the platyrhine nasal index, dolichuranic palate, upper jaw either markedly prognathic or mesognathic, and the microcephalic brain cavity are characters which, conjoined with the dolichocephalic cranium, constitute race features of the aboriginal Australians. The relation of the breadth to the height of the cranium is not, as I pointed out in my *Challenger Report* (Part xxix., 1884), constant in the different tribes; for whilst in South Australia, and in some other localities along the southern seaboard, a considerable proportion of the crania possess the basi-bregmatic diameter distinctly below the greatest breadth, in other parts of the island it is altogether exceptional to meet with a skull in which the height is less than the breadth.

From the geographical relations of the hill-tracts in North-Eastern India, occupied by a dolichocephalic people, to the surrounding countries, where the prevailing type of skull is brachycephalic, it seems more reasonable to conclude that the occurrence of exceptional specimens in a district is due to an intermixture of races possessing different head-forms, rather than to the evolution of a new type, on the one hand, in a dolichocephalic race, or, on the other, in a brachycephalic race,—the more so when it is kept in mind that tradition and history point to these countries as having during many centuries been occupied by successive waves of invading people.

EXPLANATION OF PLATES I.–III.

The figures in these plates are reproductions of photographs kindly taken for me by Mr W. E. Carnegie Dickson, B.Sc.

FIG. 1. Profile of Skull of Lushai from the north hill tracts. G in Table I.
„ 2. Front view of the same skull.
„ 3. Profile of skull of Chin. B in Table I.
„ 4. Front view of the same skull.
„ 5. Profile of skull of Nágá. F in Table II.
„ 6. Front view of the same skull.
„ 7. Profile of Gurung skull from Nepal. Table II.
„ 8. Front view of the same skull.
„ 9. Profile of Siamese skull. C, Table VII.
„ 10. Profile of Burmese skull, Tun Tha. Table IV.
„ 11. Front view of the same skull.
„ 12. Profile of Burmese skull, Paudun. Table V.
„ 13. Front view of same skull.
„ 14. Profile of a Burmese skull from an old cemetery, Upper Burma. Table VI.

Sir WILLIAM TURNER on "Craniology of People of India."—PLATE I.



FIG. 1.—Lushai.



FIG. 2.—Lushai.



FIG. 3.—Chin.



FIG. 4.—Chin.

Sir WILLIAM TURNER on "Craniology of People of India."—PLATE II



FIG. 5.—Nágá.



FIG. 6.—Nágá.



FIG. 9.—Siamese.

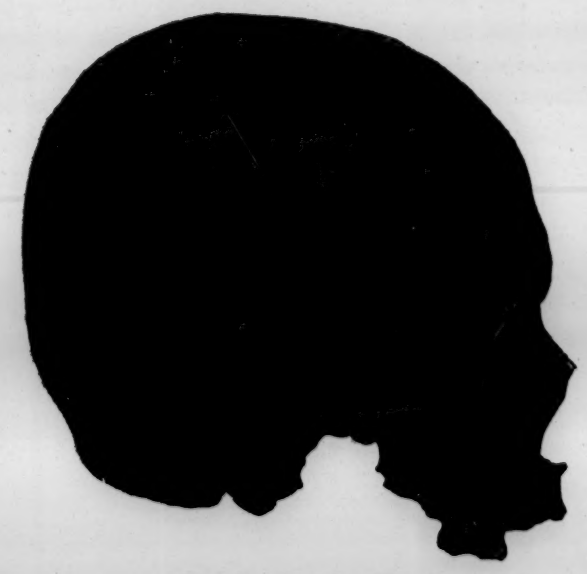


FIG. 7.—Gurung, Nepal.



FIG. 8.—Gurung, Nepal.

Sir WILLIAM TURNER on "Craniology of People of India."—PLATE III.



FIG. 10.—Burmese.



FIG. 11.—Burmese.



FIG. 14.—Upper Burma.



FIG. 12.—Burmese.



FIG. 13.—Burmese.

XXIX.—*On the Development and Morphology of the Marsupial Shoulder Girdle.*

By R. Broom, M.D., B.Sc. Communicated by Professor Sir Wm. Turner.

(With Two Plates.)

(Read January 9, 1899.)

The various structures in the marsupial shoulder girdle are so unquestionably homologous with similar structures in the girdle of the human subject and of the Eutheria generally, that there has never been any difficulty in interpreting the parts in terms of human anatomy; but when the elements are compared with those in what we may consider to have been the ancestral condition, such as is exemplified in the shoulder girdle of the Monotremes, or of the Anomodonts, or in the more distantly related conditions met with in modern Reptiles and Amphibians, we are confronted with quite a number of difficulties, to which the most varied solutions have been applied by different morphologists. The chief difficulty in the way of comparing the typical mammalian girdle with that of the lower forms, is that in the girdle of the higher mammals certain structures are rudimentary, and, further, that the forms that have hitherto been examined have been so far removed from the ancestral stock that even a study of the early developmental conditions fails to give any clues that are more satisfactory than those afforded by comparative anatomy. Though it has been suspected that further results would be obtained from the study of the early condition in Marsupials and Monotremes, the embryos of these groups that have hitherto been studied have been too old to yield much further evidence than that obtained by a study of the adult forms.

In studying the conditions in marsupial embryos much earlier than any that had hitherto been examined, I have been fortunate in discovering one or two facts which throw considerable light on the changes which have taken place in the course of the modification of the highly specialised shoulder girdle of the Anomodonts into the simple and more rudimentary condition met with in the higher mammals.

Some time ago I published a short paper (1) recording the occurrence in the newly-born Trichosurus of a well-developed coracoid which articulates with the sternum. Since then I have traced the course of development from the very early and rudimentary condition met with in an intra-uterine embryo of Trichosurus of 8·5 mm., onward till the adult condition is reached.

Before entering on a discussion of the various views that have been advanced regarding the morphology of the various elements, it will be well to describe the course of development of the structures in the marsupial. In the following descriptions I have made use of the terminology which seems to me the most satisfactory, reserving for the latter part of the paper the reasons for the various opinions.

The Shoulder Girdle in an 8·5 mm. Intra-uterine Embryo of *Trichosurus vulpecula* (fig. 1).

In this early embryo, which I have elsewhere (*Proc. Linn. Soc., N.S.W.*, 1898) figured and described in some detail, the skeleton is as yet but very imperfectly chondrified. The anterior limbs, though fairly well formed, and having the digits well marked, are still widely apart, owing to the skeletal elements of the chest-wall not having yet met in front of the heart. In the hind limbs there are as yet no indications of digits.

The shoulder girdle is in a very interesting stage of development, but certain parts of its structure, from their imperfect state of development, can only be clearly understood when compared with those in a more advanced stage.

The scapula is fairly well chondrified and moderately thick. Its long axis points upwards and somewhat forwards, and, as will be seen from the position of the first rib in the figure (fig. 1), the upper border of the scapula lies wholly in the cervical region. The upper half of the scapula is moderately flat, and there is no trace of a spine. From the anterior border of the scapula, near the point of union of the middle with the lower third, there passes outwards and slightly forwards and downwards a moderately thick, rounded, cartilaginous acromion process. After passing outwards for a distance about equal to the width of the scapula at its middle, it curves downwards, slightly backwards, and inwards, for a distance about equal to the length of the outward passing portion. This downward passing portion, unlike the other, is entirely mesenchymatous. At its lower extremity it meets on its inner side the outer end of the developing clavicle.

The glenoid cavity is comparatively shallow, and looks outwards and slightly backwards. Though the cartilage forming the lower part of the cavity is quite continuous with the scapula, there can be very little doubt but that it belongs, if not entirely, at least mainly, to the coracoid. In the figure a dotted line indicates what appears to be the limit of the two elements. As the shoulder girdle is not chondrified much beyond the region of the glenoid cavity, the further tracing of the elements in their mesenchymatous condition would be a matter of some little difficulty, were we in ignorance of the latter stages of development. As it is, most of the structures can be followed even in their pre-cartilaginous condition with moderate certainty. The cartilaginous portion of the coracoid ends in a rounded process which points almost directly backwards; but though the cartilage here ends, the element is manifestly continued as a very well marked mesenchymatous structure, which, on passing backwards, spreads out like a fan, and becomes continued into the mesenchymatous anterior portion of the first rib, and also into the less clearly differentiated sternum. When viewed in the light of the more developed condition as found at birth, there seems little doubt but that not only is the portion of cartilage forming the lower half of the glenoid cavity coracoid, but also the whole of its mesenchymatous fan-like continuation, with the exception of so much as

may be taken to represent the first rib and the sternum. In the figure a dotted line indicates what is probably the limit of the mesenchymatous portion of the coracoid.

In front of the coracoid, and between it and the developing clavicle, is a very thin and feebly developed continuous sheet of mesenchymatous cells. It is best developed at its anterior end, but it can be traced throughout the whole space between the clavicle and the coracoid, with the exception of the extreme anterior part of the coracoid. Near the anterior border of this sheet, and about its middle, is a small foramen through which passes a minute blood-vessel, and apparently a nerve. The degree of development of this structure is so slight that it is probably well not to place too much weight on its importance, but as it represents almost exactly the area occupied by the precoracoid (epicoracoid of most authors) in the Monotremes, it seems to me probable that it represents that structure. Its anterior part gives rise to the coraco-clavicular ligament, and if its morphology be here correctly interpreted, it of course follows that this ligament in the higher mammals and man is the homologue, and apparently the sole representative of the precoracoid of the Anomodonts.

The clavicle can be very distinctly traced, though as yet it is but very imperfectly ossified. Where ossification has not commenced, as in almost the whole lower half, the structure of the developing clavicle differs but little from that of mesenchyme, which is about to give rise to cartilage. There is, however, no trace of cartilage at this stage, nor for long after, in the clavicle. Whether any distinct histological difference can be made out between the mesenchymatous cells which give rise to cartilage and the very similar cells which give rise to bone, I am not aware, but, at any rate, in their further evolution, the cells become quite distinct. In the case of developing cartilage, the nuclei become more rounded, slightly enlarged, and more translucent, and become fairly uniformly surrounded by the hyaline substance. In the development of the clavicle, on the other hand, the nuclei do not increase in size, and instead of becoming rounder they become more oval, and develop angular processes. The little osteoblasts then arrange themselves in little groups of twos or threes, and the little bony spicules form along the side of the little groups. So far, then, as the Marsupial is concerned, it may be definitely stated that the clavicle is a pure membrane bone in whose early development cartilage plays no part. At the part where the clavicle meets the acromion, the mesenchyme of the acromion may be said to merge into the mesenchyme of the clavicle, but to assume that therefore the clavicle has a similar basis to the acromion, as has been done by HOFFMANN, seems to me unjustifiable.

The Shoulder Girdle in a 10 mm. Intra-uterine Embryo of *Trichosurus vulpecula* (fig. 2).

In this embryo, though the development of the shoulder girdle is not much in advance of the condition in the previous embryo, there are some interesting changes. The skeletal elements of the chest-wall very nearly meet in front of the heart, and as the figure of this girdle is, like the previous one, a true lateral view, much of the apparent difference in the ventral part of the girdle is due to foreshortening brought about by this further development.

The scapula is not very much further developed than in the previous stage. Its upper part is, however, broader, and the borders which were there mesenchymatous are now cartilaginous. The broadening out of the upper part is still proceeding; the cartilaginous borders being continued on for some distance in a semi-cartilaginous condition. The lower part of the scapula is considerably better developed, and the scapular part of the glenoid cavity looks almost directly downwards. The acromion very much resembles that in the earlier stage, and is not yet completely chondrified, though the lower part is much more clearly defined.

The coracoid is only but slightly better developed than in the previous stage, and but little more than the glenoid portion is chondrified. If we assume that the inner half of the glenoid is formed by the coracoid, and also the knob at its anterior end, part of which gives attachment to the long head of the biceps muscle, the cartilaginous part of the coracoid may then be described as an irregular oblong structure attached to the scapula along the inner side of the glenoid cavity, and forming part of the cavity. At its anterior end it is moderately thick, but at its posterior end, after passing the glenoid region, it rapidly tapers away. Though the cartilage here ends, the coracoid is continued as a well marked mesenchymatous structure to the anterior border of the first rib and to the sternum. In the earlier stage this mesenchymatous portion is moderately flat, and its margins ill-defined, but the structure is here fairly thick and much narrower. As the first rib is fairly well chondrified, the posterior border of the mesenchymatous coracoid is readily defined. The limits of the coracoid, where it meets the sternum, can only be guessed at by comparison with the more perfectly developed later stages. In the figure (fig. 2) a dotted line indicates the probable limits of the coracoid.

Of the thin, imperfectly developed structure seen in front of the coracoid in the previous stage, and which is believed to represent the precoracoid, only the anterior part which forms the coraco-clavicular ligament can be distinctly made out.

The clavicle is considerably better ossified than in the previous stage, bony spicules being present to the inner end. The inner end is considerably thicker than the outer, and now fits in between the lower end of the coracoid and the anterior end of the sternum.

The omo-sternum is not yet distinctly differentiated from the sternum proper.

The Shoulder Girdle of a Newly-born Embryo of *Trichosurus vulpecula* (14·8 mm. Greatest Length), (figs. 3, 5, 10, and 20).

It was in the shoulder girdle of the newly-born Trichosurus that the interesting development of the marsupial coracoid was first noticed, and it is remarkable that it is here that the development is most perfect, for whereas before birth chondrification is incomplete, almost immediately after birth degeneration begins.

The scapula is an irregular triangular plate lying in a plane parallel to median plane, and with its long axis directed upwards and forwards. Its upper border is markedly convex, the posterior concave, and the anterior moderately straight. The outer surface of the plate is almost perfectly flat, there being no trace of a cartilaginous spine. From the anterior border of the scapula, near the union of the middle with the lower third, arises the well-developed cartilaginous acromion. It passes outwards as a moderately round process, and then forms a more flattened plate, which passes downwards and slightly inwards to articulate with the clavicle. Where the outward part of the acromion meets the downward part, a sort of cartilaginous knuckle is formed. From this point upwards and forwards, along the outer surface of the scapula in its anterior part, passes a distinct membranous structure, which divides the supra-spinatus and infra-spinatus muscles. It may be compared to a membranous flap whose posterior surface is somewhat concave, owing to the outer border partly overlapping the infra-spinatus muscle. It is this membranous structure which ossifies to form the spine of the scapula along with the acromion. Though it is not as yet ossified, it is on the point of ossifying, and osteoblasts and osteogenic fibres can be distinctly made out. Though the projecting part is very thin, it rests on a well-formed base, which on section is somewhat triangular, thus giving the membranous spine a firm attachment to the scapula. In connection with the occurrence in Pareiasaurus of a distinct 'epiclavicle,' or, as it has been termed by Gegenbaur (2), a 'cleithrum,' Seeley (3) has already pointed out the probability of the mammalian scapular spine representing this structure, and of its thus being morphologically a distinct element. The fact that in the Marsupial, and most probably all other mammals, the spine, apart from the acromion, is not developed from cartilage, affords very strong confirmation of the correctness of Seeley's opinion. In the acromion being itself a cartilaginous structure, it is manifestly not a part of the cleithrum, but a development of the scapula proper.

The coracoid is very well developed, and may be regarded as formed of an upper part which articulates with the lower end of the scapula and forms part of the glenoid cavity, and a lower part which articulates with the sternum and the first rib; the two parts being united by a slightly constricted intermediate portion. The outer surface of the upper part is mostly hollowed out as if forming part of the glenoid cavity, but probably only a part of the hollowed surface forms an articular surface for the humerus. To a little ridge on the lower edge of this part of the coracoid, and a little behind the plane of the front of the glenoid cavity, is attached the long head of the biceps. The lower

part of the coracoid lies in a downward, backward, and inward direction from the upper part. It is somewhat round and bulbous. At its lower and inner side its cartilage is continued into that of the sternum, while further back it rests on the first rib. On the outer side of the posterior part of the coracoid near its upper part is the point of attachment of the coracoid head of the biceps. The intermediate portion of the coracoid is moderately flat, its outer surface looking downwards and outwards.

The clavicle is now fairly well developed. The outer part is somewhat slender, while the inner two-thirds is moderately stout. Its posterior part is pointed, and passes into the angle formed by the coracoid and the sternum. This part of the clavicle is hollowed out on its anterior and inner side to receive the well-developed omo-sternum.

The omo-sternum is a small, irregularly oval cartilage which lies between the sternum and the clavicle, and in a deep depression in the latter. It is quite distinctly differentiated from the sternum proper.

The Shoulder Girdle in a 17 mm. Mammary Fœtus of *Trichosurus vulpecula* (figs. 4, 7, 8, and 11).

The shoulder girdle at this stage, though it resembles the previous stage in being still attached to the sternum, shows a considerable advance in many of its characters.

The scapula differs from the earlier stages in that the long axis passes upwards and slightly backwards. The posterior angle is developed to a much greater degree, and extends well upwards and backwards. The acromion is of large size. It arises from the outer side of the anterior scapular border, and passes first forwards and slightly downwards and outwards. It then broadens out and passes downwards and slightly forwards and inwards to meet the outer end of the clavicle. From the upper side of the upper part of the acromion the non-cartilaginous part of the spine extends upwards along the anterior part of the outer side of the scapula to about the beginning of the upper third of the blade. This little element—the supposed homologue of the cleithrum —is now a well-developed bony plate. It must be admitted, however, that at this stage it is not an independent bone, as it is continuous, at its attachment with the scapula, with the delicate osseous film which is now developing on the surface of the scapula; and it is probable that the non-cartilaginous spine is at no time a distinct bone, as ossification sets in in connection with the perichondrium apparently about the same time as ossification begins in the structure dividing the supra-spinatus and infra-spinatus muscles. The ossification of the scapula is as yet a pure ectostosis, the cartilage being as yet unossified, though the cells are considerably increased in size.

The coracoid is of much the same absolute size as in the previous stage, and is thus considerably smaller relatively. Its glenoid portion differs but little from that at birth, but there is considerable difference in the portion which articulates with the sternum. At birth this lower and posterior portion is irregularly pear-shaped. Here the knob-

like lower part is of an elongated oval shape, and is undergoing a peculiar sort of degeneration. The axis of this elongated oval portion points in an upward and backward direction, and degeneration of the cartilaginous cells can be detected throughout its whole extent except along its outer and upper side. The hyaline substance becomes largely absorbed, giving rise to a sort of cellular cartilage, and around the surface of this degenerated cartilage a further degeneration is seen to be taking place in that what remains of the cartilaginous matrix is becoming completely absorbed, the cells apparently going to form connective tissue. In the ventral view of the girdle (fig. 11) a dotted line indicates the limits of the cartilages in which degeneration is not taking place.

The clavicle is well ossified, and, as seen in figs. 4 and 11, the inner part is considerably stouter than the outer. The inner third has a well-marked concavity on its anterior side, in which lies the omo-sternum, the cartilage of which, however, is separated from the bone by a thick layer of connective tissue of a peculiar structure. Surrounding the osseous tissue of the clavicle throughout its whole length, but most developed at the ends, is a fairly thick covering of large connective tissue cells. These cells have no doubt a slight superficial resemblance to cartilage, but they are certainly not cartilage cells. They are irregularly oval and taper at their apices, and are only separated from each other by a thin layer of connective tissue substance. They are apparently imperfectly differentiated osteoblasts. Cells scarcely differing in appearance are found surrounding the bony plates of the developing dentary, maxillary, or pre-maxillary elements, but in these bones the surrounding layer of cells is much thinner relatively than in case of the clavicle. In the case of the tympanic bone, however, in many forms the surrounding layer of cells is proportionately very thick, and a condition is presented almost exactly like that found in connection with the marsupial clavicle.

Later Development of the Shoulder Girdle in Trichosurus.

The degeneration of the inner end of the coracoid continues, and the coracoid becomes detached from the sternum when the embryo attains to about 20 or 21 mm.—greatest length, apparently. In an embryo of 23 mm. the inner end of the coracoid is quite free, and is about one-sixth of a millimetre distant from its former point of attachment. To the lateral portion of the pre-sternum there is still attached a small portion of degenerating cartilage—apparently the inner part of the bulbous portion of the coracoid, whose degeneration permits the detachment of the shoulder girdle from the sternum. Though this coracoidal fragment apparently completely disappears, its presence is interesting, in view of the occurrence in a number of the higher mammals of similarly situated elements, and it confirms the correctness of the view that these fragments represent the rudimentary sternal end of the coracoid. The coracoid process is pointed, and extends for about ·3 of a millimetre behind the point of attachment of the coracoid head of the biceps. This posterior portion is degenerating, and it would seem as if the

limit of the degeneration was determined by the muscular attachment. The clavicle is very well ossified, but there is still no cartilage in connection with either end. The omo-sternum is of smaller proportional size than in the earlier stages. At this stage the whole shoulder girdle comes more to the front, so that the clavicles, the sternum, and the coracoid processes lie practically all in one plane.

In a Trichosurus embryo of 37 mm. greatest length the whole shoulder girdle differs but little from that of the adult. The scapula is directed upwards and backwards. The spine is very large, and divides the outer surface into two deep fossæ. The acromion resembles that of the adult in shape and proportions. The coracoid process is quite small, being absolutely scarcely larger than in the 23 mm. fœtus, and is far removed from the sternum. The clavicle is directed from its inner attachment outwards, forwards, and slightly downwards. At this stage it shows a distinctly new feature—in that cartilage is now found at both the outer and inner ends. This cartilage is evidently a secondary development of the large cells which, in the earlier stages, tip the bone. It would seem as though the cells at the ends of the bone develop more rapidly than the rate at which the bone encroaches on them, and that hence the more peripheral cells become chondrified.

Parker (4) describes the shoulder girdle of a young specimen measuring 8 inches from snout to root of tail. It differs but little from that of the earlier stages. Both ends of the clavicle are tipped with cartilage—that of the outer end being called the 'meso-scapular segment'; the inner, the 'unossified part of the præcoracoid.' The coracoid is small and ossified by a single endosteal centre.

The latest stage of the developing girdle in my possession is that of a young one two-thirds grown. In it the coracoid only shows one centre of ossification. Whether any further centre of ossification may be found as the animal approaches its full size I am unable to say, but if a second centre exists it must be very small.

The Shoulder Girdle in a 16 mm. Mammary Fœtus of *Pseudochirus peregrinus* (figs. 6 and 12).

This embryo of the Ring-tailed Phalanger is in a stage of development about equal to that of the 17 mm. Trichosurus. The shoulder girdles of the two genera, however, present a number of very dissimilar features.

The scapula is almost quite flat. The acromion springs from the anterior border a little above the glenoid cavity. It passes outwards and downwards as a thick, rounded process; then, becoming flattened antero-posteriorly, it passes downwards and inwards as a broad, somewhat flattened plate to meet the outer end of the clavicle. The glenoid cavity looks almost directly downwards.

The coracoid probably forms the inner and anterior third of the glenoid cavity. The anterior part of the coracoid is an irregular cuboid structure projecting inwards from the anterior border of the glenoid cavity, and giving attachment to the long head

of the biceps. From the posterior part of the inner end of the main part of the coracoid an irregular quadrangular process passes backwards and downwards to meet the lateral process of the pre-sternum. Though the tip of this coracoid process rests on the lateral process of the sternum, the two structures are quite distinct and separated by a few fibres of connective tissue. It will be noticed that this mode of attachment of the coracoid to the sternum differs considerably from that of Trichosurus. Here, one would almost be inclined to think that there never had been any structural continuity. If in an earlier stage the cartilage of the coracoid is continuous with that of the sternum, the separation of the two elements can hardly have been by the degeneration of a part of the coracoid, as in Trichosurus, and the appearances would rather suggest that the structures had split apart without loss to either of the elements.

The clavicle is shorter and stouter than in Trichosurus embryos. The outer half is considerably flattened, being oval on section. The inner half is much stouter, and towards its inner end becomes hollowed on the under and inner side to accommodate the small omo-sternum. At both ends of the clavicle there is a distinct cap of large irregular cells which are becoming cartilage cells, though true cartilage can scarcely, as yet, be said to exist.

The omo-sternum is comparatively small, and its anterior end is ill defined. It is specially interesting in that it is structurally continuous with the true sternum.

In fig. 6 an anterior view is shown of the right shoulder girdle with the clavicle removed. In fig. 12 is shown a ventral view of the same arch, with the sternum, omo-sternum, and base of first rib.

The Shoulder Girdle in a Newly-born Embryo of *Petrogale penicillata* (21 mm. Greatest Length), (figs. 9 and 13).

The Rock-wallaby differs so little in its anatomical structure from the true Kangaroos that this embryo may safely be taken as the type of the Macropodidæ; and though some differences will likely be found in the girdle of the Rat-kangaroos, it is probable that the condition in true Kangaroos will be found to agree closely with that in the present embryo.

The axis of the scapula is directed upwards and well backwards, and the blade of the scapula is larger in comparison with the size of the glenoid cavity than in phalangers. The acromion arises from the outer side of the anterior border a little distance above the glenoid cavity. It passes forwards and slightly outwards as a stout process which is oval on section. After passing forwards for some distance it passes downwards and inwards to meet the clavicle—this lower part being somewhat flattened. There is a tract of ossification representing the beginning of the spine, and presumably homologous with the cleithrum of the primitive reptiles. With the exception of this low

ridge and commencing ectostoses radiating from its margins over a small part of the surface of the cartilaginous scapula, both the scapula and coracoid are entirely cartilaginous.

The coracoid has a very peculiar structure. It consists of an upper and a lower part, with a connecting neck. The upper part presumably forms the anterior third of the glenoid cavity. In both Trichosurus and Pseudochirus the coracoid part of the glenoid cavity is formed by an excavation of the coracoid cartilage, but here, in addition to the simple excavation, the cavity is enlarged by a well-developed outstanding rim which projects downwards from the coracoid and forms the inner margin of the margin, somewhat after the style of the well-marked rim in Ornithorhynchus. From the glenoid cavity the coracoid passes almost directly inwards for a short distance, and then passes downwards and slightly backwards as a comparatively narrow neck, which is continued into the large inferior part of the coracoid. A very deep groove is formed between the coracoid rim of the glenoid cavity and the descending neck, and to the anterior part of this groove is attached the long head of the biceps. The inferior part of the coracoid is a long, broad, flattened structure which passes from the lower end of the neck backwards and slightly outwards. Near its middle it adjoins the sternum, and is structurally continuous with it. Posteriorly it rests on the first rib, but is distinct from it.

The clavicle is unusually flat. In its outer third it is slender and rounded, but in its inner two-thirds it is almost quite flat. From the anterior rounded portion a ridge is continued for a short distance along the outer border. The posterior end passes between the coracoid and the sternum, and rests on the upper side of the sternum. Round the whole outer third of the clavicle is a thick coating of large cells, the more central of which are cartilaginous. A similar coating is found round the inner end, and here again many of the cells are cartilaginous.

I fail to detect any trace of an omo-sternum.

The Morphology of the Scapular Borders.

In all marsupials, and practically all the higher mammals, the scapula is formed on one and the same plan, and all the variations met with are due to different degrees of development of the different regions. According to Flower (5), "the scapula may be considered as essentially an elongated rod or bar of bone," which usually has "three projecting plates or ridges arranged round the longitudinal axis, and three surfaces or fossæ bounded by these." This is also the view taken by Parker (4), who has given the following special names to the projecting plates:—The præscapula, mesoscapula, and postscapula. While such a manner of looking at the scapula is very convenient for the higher forms, it leads to confusion when the lower forms are considered. In almost all reptiles, the scapula, instead of being made up of an anterior, a posterior, and a median plate, has but two borders, an anterior and a posterior, and even in the monotremes

there are only two well-marked borders. The endeavours which have been made to find the homologues of the higher mammalian scapular borders in the lower forms have led to very different conclusions.

Most of the discussion concerning the morphology of the scapular borders has been in connection with the scapula in the monotremes. In both Ornithorhynchus and Echidna it is a flattened bone which bears a superficial resemblance to the eutherian scapula; but though at first sight the two borders look as though they might correspond with the anterior and posterior borders of the typical mammalian scapula, yet the muscular attachments are so dissimilar that the majority of anatomists have been led to the opinion that neither of the seemingly similar borders are really homologous.

The whole subject has recently been very fully and most ably discussed by Wilson and M'Kay (6). In 1893 M'Kay (7) published an exhaustive account of the morphology of the muscles of the shoulder girdle of the monotremes, and in connection with the same work, he and Wilson were led to examine the current views on the homology of the borders and fossæ of scapula of the monotremes. The border, which is usually spoken of as the anterior, but which is really directed as much outwards as forwards, is shown to agree as regards all its muscular attachments and relations with the "mesoscapula" of the higher mammals, and also in its giving attachment to the clavicle. This view has been held by Owen (8), Mivart (9), Parker (4), Flower (5), and Howes (10), and there seems very good reason for believing it to be correct. Concerning the homology of the posterior border, opinion has been much divided, and the facts are doubtless less unequivocal. There is a well-marked ridge on the outer side of the scapula of Echidna, and a corresponding though slightly less marked ridge on that of Ornithorhynchus, situated above the glenoid cavity, which gives attachment to the triceps muscle, and which divides the origins of the infra-spinatus and subscapularis muscles. This ridge is held by Wilson and M'Kay to be the true homologue of the posterior border of the eutherian scapula—a view which had previously been advocated by Mivart and Flower. Owen, Parker, and most other anatomists, on the other hand, have held that notwithstanding the differences in their muscular relations, the actual posterior border in the monotremes is the morphological posterior border. These differences of opinion are, however, slight in comparison with the differences which have arisen in the endeavour to find in the monotremes the homologue of the anterior or præscapular border of the higher forms. On the inner side of the scapula in Ornithorhynchus is a small ridge which represents the anterior border of the origin of the subscapularis muscle, and in front of which arise the omo-hyoid and the minute supra-spinatus. This ridge was regarded by Owen as the morphological anterior border, and the same view is also held by Wilson and M'Kay. Flower, whose opinion apparently has been mainly influenced by the condition in Echidna, where the supra-spinatus muscle is of large size and occupies a very large part of the inner surface of the scapula, and where the subscapularis arises almost entirely from the

outer side of the bone, regards the actual posterior border as the true homologue of the anterior border in the higher mammals; while PARKER's opinion is that in the monotremes "there is no præscapula."

The leading views that have been held as to the homologues in the monotremes of the borders of the eutherian scapula are shown in the following table:—

	Præscapular Border.	Mesoscapular Border.	Postscapular Border.
OWEN (1847) ...	Ridge on inner side of scapula in Ornithorhynchus	Anterior border	Posterior border
PARKER (1868) ...	Absent	Anterior border	Posterior border
FLOWER (1870) ...	Posterior border	Anterior border	Tricipital ridge
BRÜHL (11) (1875) ...	Anterior border	Tricipital ridge	Posterior border
WILSON & M'KAY (1893)	Ridge on inner side of scapula in Ornithorhynchus	Anterior border	Tricipital ridge

It will be seen that the differences of view have arisen owing to the differences of opinion as to what value is to be placed on the positions of the muscular attachments. In considering the scapula of the monotremes in reference to the muscular attachments, it is important to keep in mind that in neither Echidna nor in Ornithorhynchus does the scapula lie, as in the higher mammals, by the side of the chest, and in an antero-posterior plane, but that in both the plane of the scapula is oblique, the posterior border looking about as much inwards as backwards, and the anterior border being directed mainly outwards. Furthermore, the long axis of the scapula, instead of, as in the higher forms, pointing upwards and backwards, points upwards and forwards —the scapulæ embracing the neck well in front of the chest. When this different attitude of the scapula is considered, it seems to me a more natural explanation of the peculiarities, that the muscles had to alter their attachments, than that the bone had become developed in new axes.

It may be regarded as moderately certain that the scapula of the higher mammals is not derived from that of the monotremes, but it is not difficult to derive the monotreme type from what may be regarded as the ancestral mammalian type. Let us consider the structure of the scapula in this primitive type. We are familiar with the structure of the shoulder girdle in Pareiasaurus and Dicynodon, and though probably neither of these genera is an ancestral type, their scapulæ are so similar that it seems probable that in the Theromorphine types, from which the mammals are derived, the scapula differed but little from that of the forms already known. In these types the scapula is a flattened, elongated blade, with but two distinct borders, an anterior and a posterior. There seems good reason to believe that the long axis of the scapula was directed upwards and slightly forwards. SEELEY, however, it must be admitted, opposes this view, and in his (12) restorations of Rhopalodon and Deutero-

saurus and of other forms turns the long axis of the scapula almost directly backwards. As may be seen in the almost perfect skeleton of Pareiasaurus in the British Museum, if this be the case the interclavicle has to lie almost at right angles to the axis of the body—a rather improbable condition. Moreover, if the inner borders of the precoracoid and coracoid of Dicynodon be placed in contact with an imaginary antero-posteriorly directed interclavicle, the scapula will be found to incline forwards. The imperfect remains of two specimens of Kistecephalus, in the British Museum, show that in that genus at least, the scapula almost certainly was directed somewhat forwards. The early developmental condition of the scapula in the marsupial also seems to confirm the view that in the early types this was the direction of the scapular axis. In Pareiasaurus the anterior border of the scapula is supported by a distinct cleithrum or epiclavicle, as is most probably also the case in Kistecephalus and Dicynodon. In the early development of the monotremes it seems probable that the anterior part of the scapula will be found to develop like the spine in the marsupial, from a membranous basis, and if this be the case it will mean that the monotremes, like the lower Theromorpha, have the anterior border of true scapula extended by the addition of a membrane-bone element, the cleithrum, but with the cleithrum becoming co-ossified with the scapula, as is probably the case in the higher Theromorphine genus Cynognathus (14). In the marsupial and eutherian type the change which has taken place is, that the cleithrum has become shifted from the anterior border of the cartilaginous scapula to its outer side. The marsupial scapula in its early condition scarcely differs from that of the adult monotreme, except in the greater development of the acromion. There is an acromion process situated on the anterior border, and no spine on the outer surface of the scapula, and it is probable that if the scapula of the fœtal marsupial were compared with that of a very young monotreme before the development of the supposed cleithral portion, the resemblance would be greater. If this comparison be correct, it will follow, that though the spine in the marsupial and the eutheria is the homologue of the actual anterior scapular border in the monotreme, the anterior border of the marsupial scapula is the homologue of the anterior cartilaginous portion of the scapula in the monotreme. Whether the study of the early development will prove that the anterior part of the cartilaginous scapula is coincident with the ridge on the inner side of the scapula in Ornithorhynchus, which is believed by Owen and Wilson and M'Kay to be the morphological anterior border, remains to be seen.

How the muscles may have arranged themselves in the Theromorpha it is difficult to tell, but it may be regarded as fairly certain, that no muscle could have passed from the inner side of the scapula to the humerus in front; and the peculiar development of the supra-spinatus muscle in the monotremes is probably due to the attitude of the scapula, for if this muscle existed in Cynognathus, it must have been extremely minute.

The question as to whether the posterior border of the monotreme scapula is really the homologue of the posterior border in the higher forms, or, as held by Wilson and

M'Kay, a new development, is more difficult of solution. If the question is to be decided by muscular attachments, no doubt the evidence would favour the homology of the typical mammalian posterior border with the little ridge on the outer side of the scapula of the monotremes. If, however, we admit the possibility of the plane of the scapula becoming partly rotated, and this we unquestionably find in some of the higher mammals (*e.g.*, Chrysochloris, where the præscapular border is carried inwards), it will be seen that such a rotation would compel certain muscles to take up new attachments. It can hardly be denied that the anterior border of the scapula in the monotreme is the homologue of the anterior border in the Theromorph, and yet while this border in the ancestral forms is directed forwards, in the monotremes it looks more outwards than forwards. And if the anterior border has thus been rotated outwards it seems but reasonable to suppose that the posterior border has been correspondingly rotated inwards. In both monotremes, the change has probably been brought about by the digging habits of the genera. If we now suppose this change to have taken place, it will be seen that the triceps muscle must of necessity take up a point of attachment on the outer side of the scapula, as, were it to continue attached to the posterior border, the action of the muscle would be hampered. The inward rotation of the posterior border also of necessity in Echidna brings almost the whole of the subscapularis muscle to the outer side, as it would be practically impossible for the muscle to work round the posterior border in its present attitude. In Ornithorhynchus, owing to the upper and posterior part of the scapula being almost sickle-shaped, a large portion of the subscapularis muscle is enabled to retain its original attachment and to work clear of the lower part of the posterior border. The muscles not connected with the arm would be less affected by the altered attitude of the blade, and we find that the serratus magnus and rhomboideus practically retain their original attachments.

It will thus be seen that there is good reason for believing both the monotreme and the marsupial types of scapula to have been derived from the primitive Theromorphine type; the monotreme type being formed by the coalescence of the cleithrum to the morphological anterior border of the scapula, complicated by a slight rotation of the plane of the blade on its long axis; the marsupial type, on the other hand, being formed by the cleithrum coalescing to the scapula on its outer surface, leaving the anterior border of the scapula free, and thus giving rise to a new fossa—that between the cleithrum or spine and the anterior scapular border. In the Cetacea we have apparently a scapula in which, though the acromion is well developed, there is no true spine, the infraspinous fossa extending to the anterior border of the blade, and the supra-spinatus muscle occupying a little hollow on the inner side of the acromion.

It is, of course, not improbable that the cleithrum element may be completely lost in the monotremes. If the study of the early development prove this to be so, the actual anterior border must be the morphological anterior border.

The Morphology of the Coracoid.

Until comparatively recently there seemed little doubt in the minds of the majority of morphologists that the rudimentary coracoid process of the higher mammals was the true homologue of the larger and posterior of the two coracoidal elements in the monotremes, and of the well-developed bone which unites the scapula with the sternum in birds.

In 1887, Howes (10) published a paper on the morphology of the mammalian coracoid, in which he very ably discusses the whole question, and gives reasons for doubting the correctness of the currently accepted views. The coracoid of the rabbit and some other mammals he has shown to be developed from two distinct ossifications, which he holds to be probably homologous with the two coracoidal elements of the monotreme. The smaller of the two ossifications which forms part of the articular surface for the head of the humerus he regards as the homologue of the posterior element in the monotreme usually regarded as the coracoid, while the larger ossification which forms the coracoid process he believes to be the representative of the 'epicoracoid' of the monotremes. His chief reasons for this determination are that in the rabbit the ossification which gives rise to the coracoid process lies anterior to that which forms part of the glenoid cavity, and also that this larger ossification, like the 'epicoracoid' in the monotremes, takes no part in the formation of the glenoid surface. The 'epicoracoid' of the monotremes he regards as the homologue of the anterior coracoidal element in Dicynodon, but does not regard it as homologous with the precoracoid of the Amphibia. His reason for this latter opinion is that the true precoracoid is believed to become absorbed by the clavicle in the mammalia and most of the higher vertebrates.

In 1893, he published (14) a second paper, giving the results of his further researches in the development of the mammalian coracoid process, and also dealing with the terminology. As Howes holds that the two coracoidal elements of the monotreme are together homologous to the single coracoid of the bird or the true coracoid of the Amphibian, he proposes calling the posterior coracoidal element the 'metacoracoid,' as suggested by Lydekker, and to retain the name 'epicoracoid' for the anterior element.

In this same year, Lydekker (15), by a different argument, came to a conclusion somewhat similar to, though not identical with, that of Howes. By comparing the shoulder girdle of the sloth with that of Dicynodon he comes to the conclusion that the coracoid of the sloth is the homologue of the anterior coracoidal element in the Anomodont, from their having somewhat similar relations; and consequently, that the mammalian coracoid process is homologous with the 'epicoracoid' of the monotremes.

If we compare the early condition of the marsupial coracoid with that of the Monotreme or Theromorph there seems no difficulty in homologising the cartilaginous element with the posterior of the two coracoidal elements in the lower forms. In the monotreme it is the posterior element which forms with the scapula the glenoid cavity,

and it is this same element which serves to fix the shoulder girdle by articulating with the sternum immediately in front of the first rib. In the fœtal marsupial the relations and functions of the cartilaginous element are precisely similar, and I am aware of no fact or argument which seems to me to be opposed to this element being the true homologue of the posterior of the two coracoidal elements in the monotreme.

If the cartilaginous ventral element in the fœtal marsupial be not the homologue of the posterior element in the monotreme, it must either represent a fusion of the coracoid and precoracoid, or be the homologue of the precoracoid. As no scientist, so far as I am aware, holds the view that the mammalian coracoid process represents two elements, it will be unnecessary to discuss this alternative. Though HOWES holds that both coracoidal elements are present in the typical eutherian shoulder girdle, he admits that the ossification which forms the coracoid process is a single element, and there is no doubt that the cartilaginous process in the fœtal marsupial represents this one element. The remaining alternative, that the process represents the precoracoid ('epicoracoid'), is the view held by both HOWES and LYDEKKER. HOWES, as we have seen, was led to this view by the discovery of a second centre of ossification which he took for the true coracoid. If we assume the well-developed cartilaginous ventral element in the fœtal marsupial to be the precoracoid, any rudiment of the true coracoid which might be present we would expect to find on the posterior side of the well-developed element, and, of course, on the inner side of the glenoid cavity; but the little ossification which HOWES believes to represent the coracoid lies on the outer side of the glenoid cavity. If HOWES' view be correct, the following most improbable changes must have taken place in order to have the mammalian shoulder girdle derived from that of the Theromorph:—1st, the great reduction of the element which is most useful in uniting the scapula with the sternum; 2nd, its exact place and uses being taken up by the precoracoid; and 3rd, the rudimentary true coracoid being shifted from the inner side of the glenoid cavity to the outer. As I can see no reason for believing that any of these changes has taken place, and many reasons for believing that such remarkable changes have not taken place, I feel compelled to accept the view of the majority of earlier writers, that the coracoid process of the marsupials and all higher mammals is the true homologue of the posterior of the two elements in the monotremes and theromorphs.

In LYDEKKER'S comparison of the shoulder girdle of the sloth with that of Dicynodon, he concludes that the coracoid of the sloth is the homologue of the precoracoid ('epicoracoid') in Dicynodon, because of "each articulating with the lower border of the front of the scapula, from which they are partially separated by a foramen, and each entering into the formation of the glenoid cavity." But if the shoulder girdle of Plesiosaurus or of Cryptoclidus be compared with that of Dicynodon it will be seen that the very same argument would serve to prove that the element which articulates with the scapula in these Sauropterygians was a precoracoid—a view which could hardly be accepted.

Assuming, then, that the main coracoid ossification, and that which forms the coracoid process, is the homologue of the coracoid of the lower forms, in what light are we to regard the secondary and more rudimentary ossification? From its position on the outer side of the glenoid cavity it might be taken to represent the precoracoid, but such a view does not seem probable for a variety of reasons. Its development is inconstant; when present it always, I believe, forms part of the glenoid surface; and it is always late in developing. That it is merely an epiphysis with no special morphological significance seems to me to be the most natural interpretation of the structure. In Echidna during the development of the coracoid two epiphyses form—one on the posterior end, and a minute one, which however may not be constant, between the coracoid and the scapula, at the posterior edge of the glenoid cavity. Though neither of these epiphyses is homologous with the small ossification in the rabbit, the condition in Echidna is interesting as showing that epiphyses may develop in connection with the coracoid, in cases where it is practically impossible that the ossifications can have any important morphological significance.

In the mammalia during the development of almost all the cartilage bones epiphyses are to be found. The reason for the formation of these epiphyses is probably not the same in all cases, and yet, as a general rule, it may be stated that epiphyses form wherever, for one reason or another, a considerable portion of cartilage remains, till comparatively late in development, beyond the region of the central ossification. In the case of the long bones the ends probably remain cartilaginous for a considerable time, to afford a greater elasticity to the skeleton during the development of the bone. But in a number of other cases, epiphyses seem to be formed to accomplish the ossification of tracts which from some peculiarity of their structure are not readily ossified from the main centre. Thus almost all bony processes are provided with epiphyses. As instances of this sort in the human subject we may take the epiphyses of the tubercle of the rib, of the anterior inferior spine of the ilium, or of the acromion. Of a similar nature would seem to be the epiphysis on the outer side of the glenoid cavity in the rabbit and many other of the higher mammals. The reason for its development may have been as follows:—The coracoid process has apparently been preserved in the higher mammals, not to increase the glenoid surface, but because it gives an attachment to the biceps and coraco-brachialis muscles, which is specially advantageous in that these muscles are kept out clear from the working of the joint, and from pressing on the subscapularis. The pull of these muscles, in course of time, has brought the centre of ossification of the coracoid into the line of strain, as is well seen in the coracoid of, say, the young baboon; and the centre of ossification being thus brought somewhat back from the glenoid cavity, a portion of cartilage is left on the anterior wall of the cavity to be secondarily ossified as an epiphysis.

We may next consider the significance of the anterior of the two coracoidal elements in the monotremes. This element which in the monotremes has usually been referred to as the 'epicoracoid,' is admitted by practically every one to be the homologue of the

anterior of the two coracoidal elements in Dicynodon and other allied reptiles; and that this conclusion is correct can hardly be doubted. What has given rise, however, to much debate is whether this is the homologue of the precoracoid of the Amphibia, or a new element secondarily developed. It is developed as a large and independent element in probably the whole of the Theromorpha, but the condition is best known in Dicynodon, Pareiasaurus, and Procolophon.

The tracing of the coracoidal elements back to the condition found in the Amphibia is rendered especially difficult owing to the fact that in the Labyrinthodontia the shoulder girdle was very imperfectly ossified. If, however, the girdles of modern amphibia be examined it will be found that in practically every case there is a division of the ventral portion into an anterior and posterior part—the anterior of which has generally been referred to as the precoracoid. Among the Urodela this division of the coracoidal region is most marked in Proteus and Menobranchus, while in Salamandra, though the differentiation of the ventral cartilage into its two elements is less marked, there are, according to PARKER (4), distinct precoracoid and coracoid ossifications.

In the Anura a distinct precoracoid and coracoid are almost always present, though the ventral portion of the arch is here relatively much smaller than in the Urodela. The coracoid and scapula are invariably well ossified by endostoses, but the precoracoid bar as a rule remains largely cartilaginous, though supported in front by an ectostosis. Whether this bony splint is, as believed by PARKER, a true precoracoid ossification, or, as held by the majority of comparative anatomists, including GEGENBAUR (16), WIEDERSHEIM (17), and HOWES (10), a true clavicle, is a point which is difficult to decide. Of the two views that of PARKER seems to me the more probable. That the clavicular plates of the Labyrinthodonts which remain quite distinct in most reptiles, should in the much more nearly related frogs be represented by bony splints intimately connected with the precoracoid bars seems to me unlikely; and, moreover, the fact that in the Anura other elements usually ossified by endostoses are found to be ossified by ectostoses would seem to confirm the opinion that the split developed on the surface of the precoracoid cartilage is really part of the precoracoid. The difference in the mode of ossification of the two coracoidal elements might have been brought about through the precoracoid, owing to its being protected by the superficial dermal plates remaining in a cartilaginous state long after the coracoid had become ossified.

Apart, however, from any difference of opinion as to whether a clavicle is present in the Anura, it will be seen that in all the existing amphibia with limbs the ventral part of the shoulder girdle is made up of a distinct precoracoid and coracoid. When the most primitive known reptile is examined it is found that the ventral part of the shoulder arch is here also divided into two elements, and I fail to see any valid reason why the two elements in such an amphibian-like form as Pareiasaurus are not to be regarded as the homologues of the two ventral elements in the living amphibians.

The only argument, so far as I am aware, that has been urged against the homology of the amphibian precoracoid with the anterior coracoid element in the Theromorpha

and the monotremes is, that there is believed to be evidence of the amphibian precoracoid being absorbed by the clavicle. This view is held by the majority of present comparative anatomists, though opposed by SEELEY. It has apparently originated in the belief that the mammalian clavicle is developed from a cartilaginous basis. In 1864, GEGENBAUR (18) discovered in connection with the early developing human clavicle, cells which he believed to be cartilage cells. GÖTTE (19), in 1877, and HOFFMANN (20), in 1879, confirmed GEGENBAUR'S view. From these observations it has been assumed that the clavicle has a cartilaginous basis, and further, that this basis represents the amphibian precoracoid. It seems rather remarkable that these conclusions should have been so readily accepted, seeing that none of the observers have clearly demonstrated the presence of true cartilage in the clavicular region at a stage antecedent to the development of bone. In the development of the marsupial clavicle I have shown that not only is there no cartilaginous basis, but that what cartilage appears at the ends of the clavicle is only developed at a late stage in development, and is probably of no more morphological significance than is the cartilage found in some mammals in connection with the premaxillary bone. It is very probable that in many Eutheria the secondary cartilage at the ends of the clavicle develops at a much earlier stage than in the marsupial, but I very much doubt whether in any mammal cartilage is present till a considerable time after the bony clavicle is well formed. In a human embryo of seven weeks, though the clavicle is well ossified, I can find no cartilage at either end; and even in the mole, where cartilage is so well developed in the later stages as to lead PARKER to believe that the short clavicle represented the clavicle + coracoid, the clavicle develops as a pure membrane bone. In the lizard (*Mabuia sulcata*) the clavicle is also a pure membrane bone, and is developed in almost exactly the same manner as in the marsupial.

It will thus be seen that there is good reason for believing that the precoracoid has not been absorbed by the clavicle; and if this be so, there is left no reason for doubting the homology of the amphibian precoracoid with the anterior coracoidal element in the theromorphs and monotremes.

In most reptilian orders the precoracoid is either quite rudimentary or lost. It is found as a rudiment in some birds, e.g., *Apteryx*, *Struthio*, *Casuarius*, and possibly also in some lizards. In both the Chelonia and the Sauropterygia a well-developed osseous bar passes from the scapula forwards and inwards, and which has generally been regarded as the precoracoid. ANDREWS (21) has shown, however, that in the Plesiosaurians the structure is a true portion of the scapula, and there is no satisfactory evidence that the anterior bar in the turtles is anything more.

Another element in connection with the shoulder girdle that has given rise to a difference of opinion is the little cartilage situated at the inner end of the clavicle. By PARKER it was named the 'omo-sternum,' and was held to be the homologue of the median element situated in front of the precoracoid in the frog. By GEGENBAUR it was formerly held to be a 'lateral epi-sternum,' but recently he has referred to it as

the 'præclavium.' From the development of the structure in Trichosurus and Pseudochirus there seems little doubt that the element is a true sternal structure, apparently homologous with the anterior part of the sternum in Ornithorhynchus. Whether the homology can be carried to the anterior element in the frog is less clear, but at present PARKER'S name seems much the more appropriate.

CONCLUSION.

In conclusion, the following may be regarded as a brief outline of the course of the evolution of the marsupial shoulder girdle from that of the primitive terrestrial vertebrates.

In the Labyrinthodonts we have a shoulder girdle mainly cartilaginous, but supported by strong exoskeletal plates. Not improbably the endoskeletal apparatus resembled that of Proteus, only the scapula being ossified. The exoskeleton consisted of a large median plate and two lateral plates, corresponding to the interclavicle and clavicles of higher forms, and a more delicate element—the cleithrum—adjoining each lateral plate. When in Permian times the terrestrial reptiles came to be developed from the amphibian forms, the chief changes that took place in the shoulder girdle were that the endoskeletal elements became well ossified, and the exoskeletal elements became more closely united with the other structures and less distinctly dermal bones. Such a girdle is seen in Pareiasaurus, where the five exoskeletal elements of the Labyrinthodont are still distinct elements, but where the scapula, coracoid, and precoracoid are well ossified.

In the further evolution in the mammalian line the cleithrum became lost through becoming co-ossified with the scapula, forming its spine. Cynognathus probably illustrates this stage.

In the next stage a change, similar to that found in the monotremes, took place in the ventral part of the girdle, the precoracoid becoming reduced.

And finally, in connection with the many changes which gave to the mammalia their typical characteristics, and of which the development of a soft flexible skin only protected by hair probably was the most important, the coracoid became detached from the sternum and much reduced in size to give greater freedom of movement to the limbs. With this change the interclavicle would become lost to admit of the clavicles becoming moveable.

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REFERENCES TO FIGURES.

General—*ac.*, acromion; *cl.*, clavicle; *cr.*, coracoid; *ep.*, ep^1., ep^2., epiphyses; *gl.*, glenoid cavity; *ost.*, omo-sternum; *pcr.*, precoracoid; r^1., r^2., 1st and 2nd ribs; *sc.*, scapula; *sp.*, the rudimentary non-cartilaginous spine = cleithrum; *st.*, sternum.

Plate I.

(Figures 1 to 13, except fig. 8, are reconstructions from series of sections.)

Fig. 1. Outer view of shoulder girdle of 8·5 mm. Trichosurus embryo, × 34. The dotted portions are mesenchymatous.

Fig. 2. Outer view of shoulder girdle of 10 mm. Trichosurus embryo, × 34. The dotted portions are mesenchymatous.

Fig. 3. Outer view of shoulder girdle of 14·8 mm. Trichosurus embryo, × 27. The clavicle has been removed to show the omo-sternum.

Fig. 4. Outer view of shoulder girdle of 17 mm. Trichosurus embryo, × 21. The limit of commencing ossification is shown by interrupted lines.

Fig. 5. Ventro-lateral view of shoulder girdle of 14·8 mm. Trichosurus embryo, × 27.

Fig. 6. Anterior view of shoulder girdle of 16 mm. Pseudochirus embryo, × 18.

Fig. 7. Anterior view of shoulder girdle of 17 mm. Trichosurus embryo, × 20.

Fig. 8. Trans. vert. sect. of shoulder girdle of 17 mm. Trichosurus embryo, × 20. The dark shading indicates the region where the cartilage is about to ossify; the medium shading, normal hyaline cartilage; the light shading, the portion of coracoid cartilage undergoing degeneration.

Fig. 9. Anterior view of shoulder girdle of 21 mm. Petrogale embryo, × 17.

Plate II.

Fig. 10. Ventral view of shoulder girdle of 14·8 mm. Trichosurus embryo, × 34.

Fig. 11. Ventral view of shoulder girdle of 17 mm. Trichosurus embryo, × 28.

Fig. 12. Ventral view of shoulder girdle of 16 mm. Pseudochirus embryo, × 30.

Fig. 13. Ventral view of shoulder girdle of 21 mm. Petrogale embryo, × 28.

Figs. 14–22. A series of specimens illustrating the phylogeny of the coracoidal elements. Variously magnified and reduced.

Fig. 14. Coracoid and Precoracoid of the Common Toad (from nature).

Fig. 15. Coracoid and Precoracoid of Procolophon (modified after Seeley).

Fig. 16. Coracoid and Precoracoid of Dicynodon (modified after Seeley and Lydekker).

Fig. 17. Coracoid and Precoracoid of Gomphognathus (restored from fragments in Brit. Mus.).

Fig. 18. Coracoid and Precoracoid of Ornithorhynchus (from nature).

Fig. 19. Coracoid and Precoracoid of Echidna (from nature).

Fig. 20. Coracoid of Embryo Trichosurus (reconstructed from sections).

Fig. 21. Coracoid of nearly adult Pseudochirus (from nature).

Fig. 22. Coracoid and Epiphysis of Cape Hare (from nature).

Broom on Marsupial Shoulder Girdle.

Plate I.

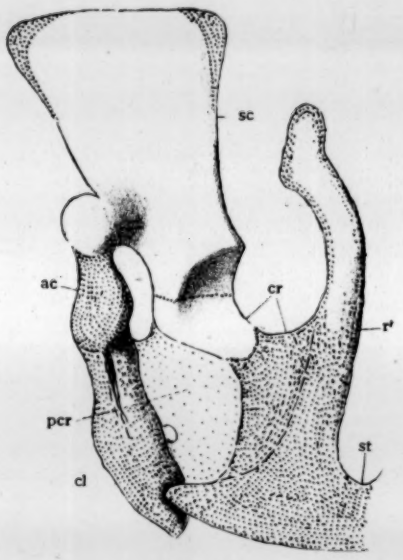


Fig. 1.

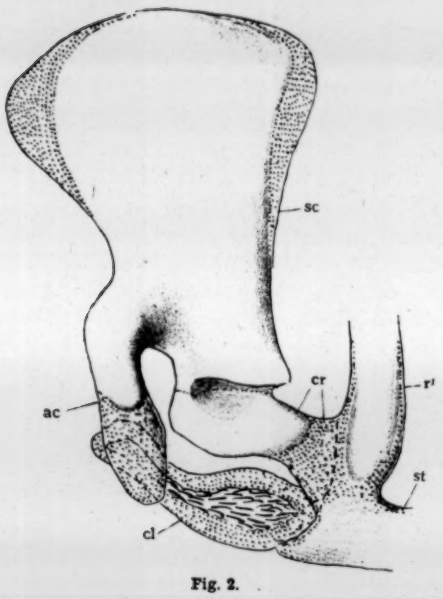


Fig. 2.

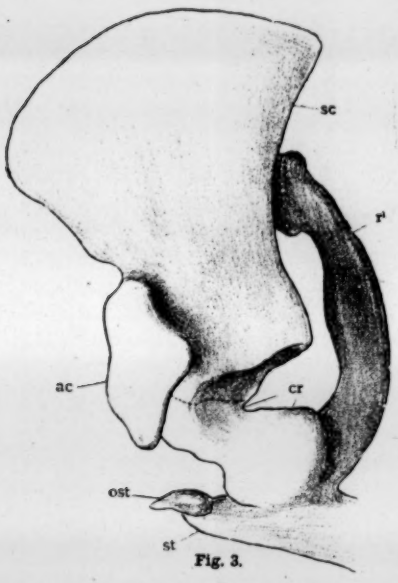


Fig. 3.

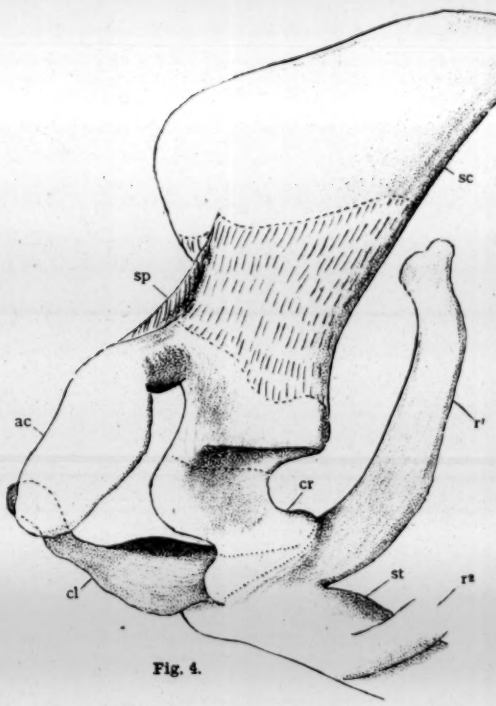


Fig. 4.

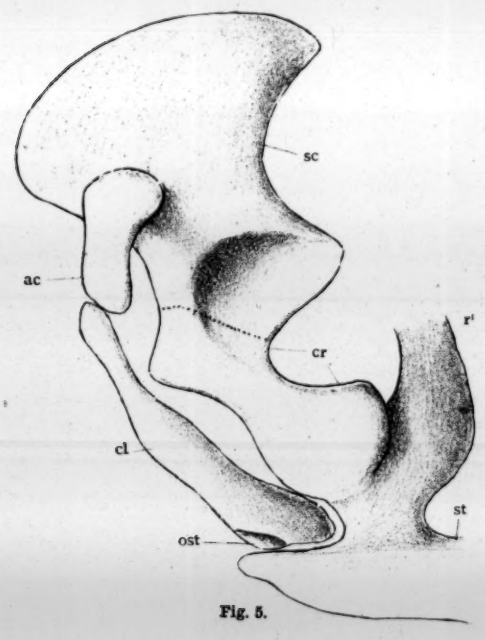


Fig. 5.

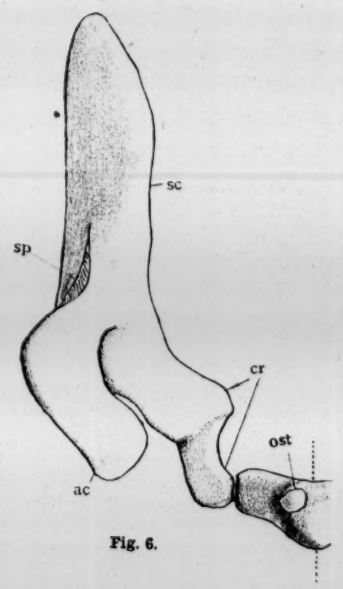


Fig. 6.

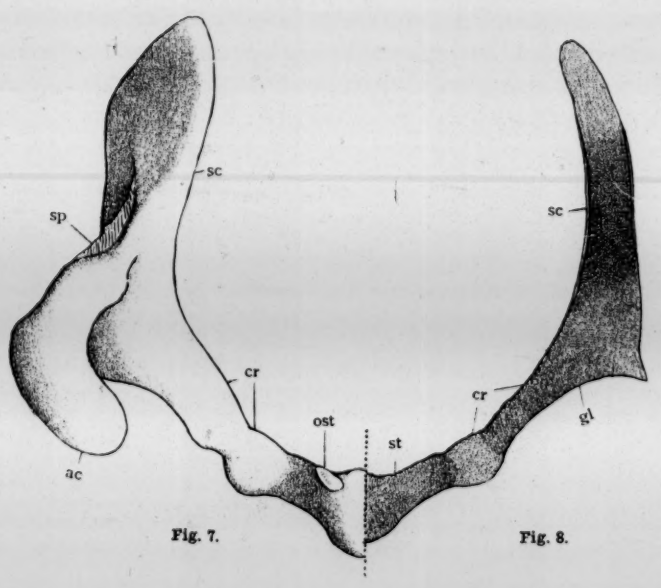


Fig. 7.

Fig. 8.

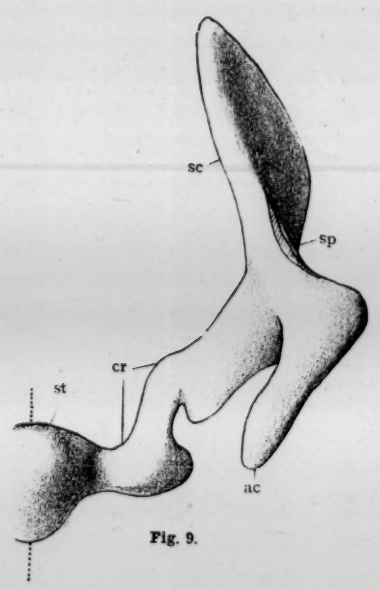


Fig. 9.

Broom on Marsupial Shoulder Girdle.

Plate II.

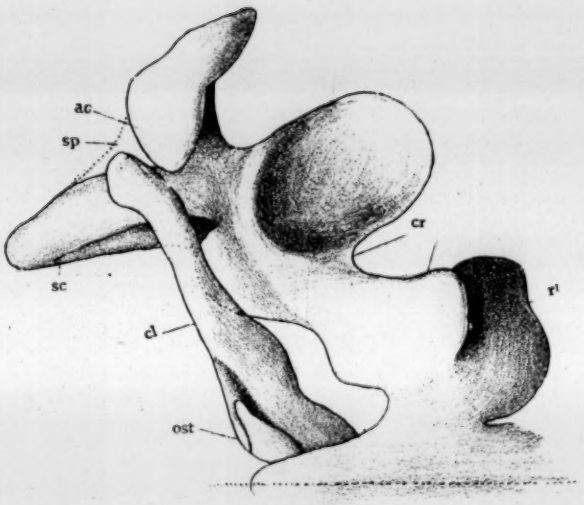


Fig. 10.

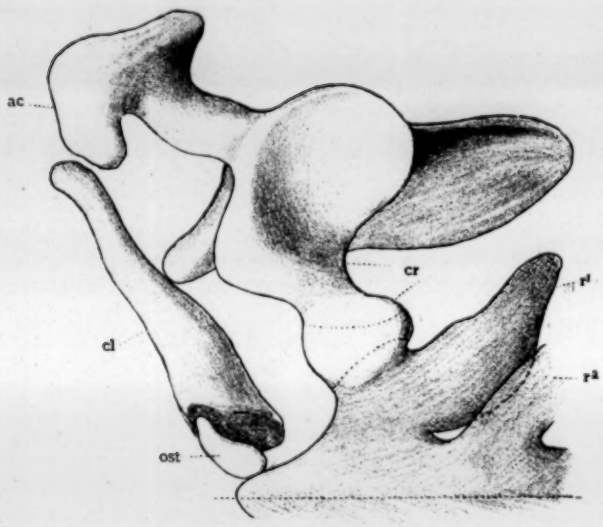


Fig. 11.

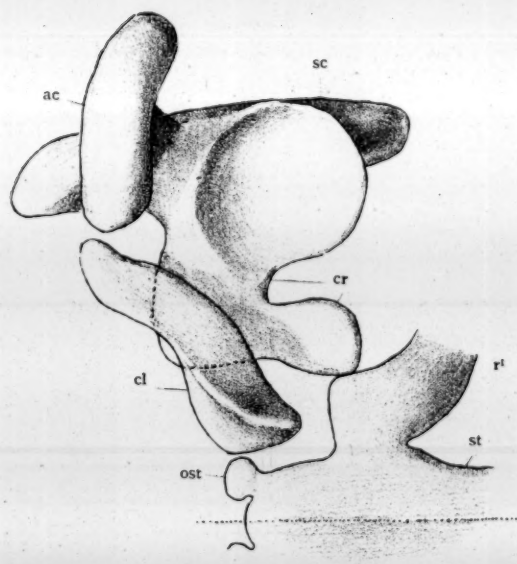


Fig. 12.

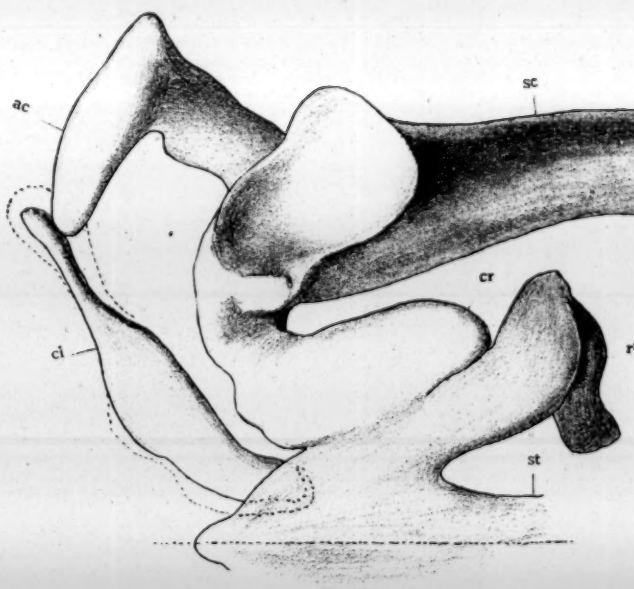


Fig. 13.



Fig. 14.

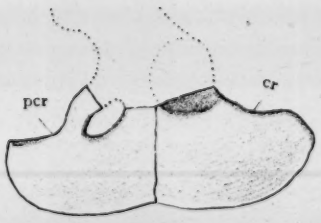


Fig. 15.

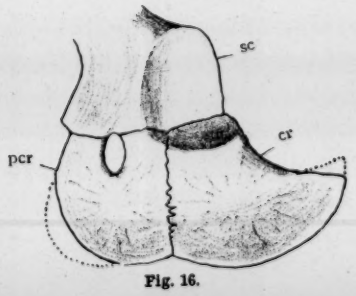


Fig. 16.

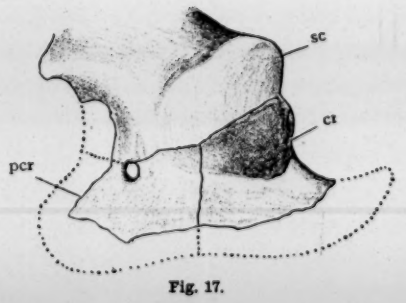


Fig. 17.

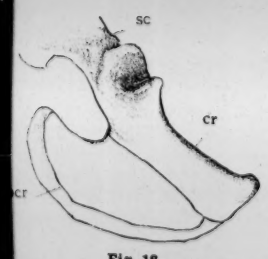


Fig. 18.

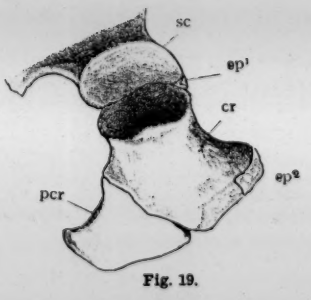


Fig. 19.

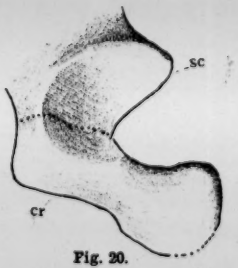


Fig. 20.

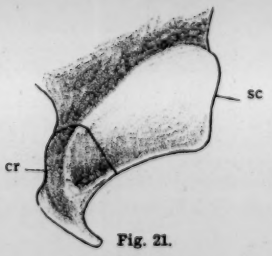


Fig. 21.

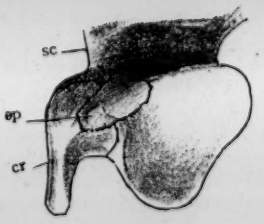


Fig. 22.

ERRATA FOR PROF. LITTLE'S PAPER.

p. 778. 6th last line, *for* 253 *read* 233.
„ 5th „ „ 339 „ 319.

The laps referred to below are parts of the *external* boundary of the figure, and are supposed to be described *with* the sun. The *first* specification of a lap refers to the *side* of the figure, the second to its position on the side.

Plate I.	8th col., last row.	Right-hand upper lap should cross *over*.
Plate III.	4th „ 6th „	Upper lap should go under, over, *over*, *under*, over, etc.
	10th „ 9th „	Upper right-hand lap should cross *over*.
	Last „ 2nd last row.	Right-hand middle lap should cross *under*.

BRITISH
17 MY 1900
MUSEUM

XXX.—*Non-Alternate* ± *Knots.* By Professor C. N. Little, Ph.D. Communicated by Professor Tait. (With Three Plates.)

(Read July 3, 1899.)

1. The following paper is a contribution to the theory of non-alternate ± knots, together with a census of these knots for Order Ten; that is, all the knots are given which have in *reduced* form just ten crossings, and in which the thread does not proceed alternately over and under.

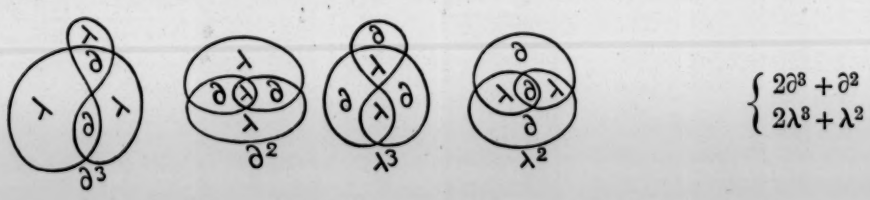
The census was begun in the fall of '93, and carried so far that the forms were drawn. The matter was then laid aside and taken up anew in the spring of '99.

2. Having postulated an endless one-dimensional continuum which may change its length and form in any way, subject only to the condition that it can never have a double point, and consequently no one portion can be made to break through another, I understand by a knot *a continuum which can not be brought to a circular form.*

3. The above definition makes of a knot a purely mathematical concept. As physical approximations for the continuum may be taken:—(a) a vortex filament of frictionless ether; (b) a flexible, extensible thread. It is convenient and can lead to no error, although keeping to the mathematical conception, nevertheless to speak of the continuum as a thread.

4. Through the work of Listing,* Tait,† Kirkman,‡ and the writer,§ but pre-

* "Vorstudien zur Topologie," *Göttinger Studien*, 1847, pp. 859–866. To the kind courtesy of Professors Felix Klein and P. Stäckel, for which I here express my appreciation, I owe the opportunity to examine the topological *Nachlasse* of Gauss and Listing. The former will appear in the forthcoming Bd. VIII., *Gesammelte Werke*, and must not be commented upon in advance of publication. The latter contains among the drawings of reduced knots not figured in the "Vorstudien," a sheet bearing date March 18, 1849, on which are the following forms marked as equivalent:—



$$\begin{cases} 2\partial^3+\partial^2 \\ 2\lambda^3+\lambda^2 \end{cases}$$

The interest of this series lies in the fact that it shows that Professor Listing fifty years ago recognised the amphicheiral character of this knot.

† These *Transactions*, vol. xxviii. pp. 145–190; vol. xxxii. pp. 327–342; *ibid.*, pp. 493–506. See also *Collected Scientific Papers*, vol. i. pp. 273–346.

‡ These *Transactions*, vol. xxxii. pp. 281–309; *ibid.*, pp. 483–491.

§ *Trans. Connecticut Academy*, vol. vii. pp. 1–17; these *Transactions*, vol. xxxvi. pp. 253–255.

eminently of Professor TAIT, the theory of the *alternate* knot is well understood. It may be useful to recapitulate the main points of this theory:

(*a*) The knot is *reduced* (projected with fewest crossings) if no one of the compartments into which its projection divides the plane is opposite itself.

(*b*) Reduced forms of the same knot divide the plane into two sets of vertically opposite parts, with a *constant* number of parts in each set. This gives a convenient basis for classification.

(*c*) It is a simple problem in the circular arrangement of letters to determine from a given reduced form all reduced forms of the knot. Hence it is easy to say as to two given forms whether or not they are projections of the same knot.

(*d*) Simple methods are known by which *all* the knots of a given *order* (minimum number of crossings) can be found.

(*e*) The theory of amphicheiral knots.

5. It is quite the contrary with the non-alternates. They constitute an almost untouched field, bristling with difficulties. In these *Transactions*, vol. xxxv. part ii. p. 664, I published a census of these knots for Orders Eight and Nine; for brevity I shall refer to this paper under the letter A in brackets.

6. It is there stated, [A] § 8, *There is no reduced non-alternate ± knot of fewer crossings than eight.* I proceed to give formal proof*:—

In class II., order n, all knots are obviously alternate.

In class III., order n, the leading partition (set of compartments with smaller number of parts) has three parts, say A, B, C. If any one of the connections (AB), (BC), (CA) is null, the form is not a knot. If any one is a single crossing the knot is alternate, [A] § 3. If two of them have each two crossings there must be a link, that is, at least two threads. Hence there must be for these connections at least 3, 3, 2 crossings respectively, and a non-alternate knot of class III. must be at least an eightfold.

Consider class IV., order n. Here there are in the leading partition four parts, A, B, C, D, and these have six connections.

First, let any one, say (AB), be null. Then in order that the form may be the projection of a reduced knot, (AC), (AD), (BC), (BD) must exist. There are two cases:—

(*a*) Let (CD) also be null. If now any one of the existing connections be a single crossing, say λ, then all of the crossings of the other connections will also be λ, by [A] § 3, and the form alternate. But if not, the form is at least elevenfold, since only one connection can consist of an even number of crossings, else the form would be a link.

(*b*) Let (CD) exist. We now have D joined to C by B, by A, and by the one or more crossings of (CD). If either (DB) or (CB), and at the same time either (DA) or (CA), are single crossings, the form is alternate. If (DB) and (CB), or (DA) and (CA), are

* The classification of alternate knots according to the number of parts in the projection, in that set of vertically opposite compartments which has the smaller number, does not answer for non-alternates, since the same knot can be projected in forms belonging to different classes. Later, in § 9, a new basis of classification will be proposed. In this and the following sections, however, the term class has the old signification.

both even, the form is a link. Hence to have a non-alternate knot, A (or B) must furnish at least five crossings and the remaining three connections at least one each. The knot is at least an eightfold.

Second, let all six connections exist. If each is a single crossing the form is a link of three threads. If one connection has two crossings, and the other five one each, the form has two threads. In all other cases in this class the form is at least an eightfold.

Class V. and higher classes do not exist in orders lower than Order Eight. This completes the proof of the theorem that there exists no non-alternate ± knot with fewer than eight crossings.

7. *No reduced, non-alternate knot with three consecutive overs has fewer than eleven crossings.*

Since in classes II., III., and IV., order n, it is always possible to go from any part of the leading partition to any other part of the same partition with not more than two crossings, no *sequence of three* (three consecutive overs) can exist in these classes. Hence no reduced eightfold knot with a sequence of three can exist.

All other ninefolds of class V. can be derived from the ninefold 35, below, by erasing crossings at some of the existing connections of the leading partition and adding the crossings erased at other connections. Although it requires three crossings by every path to go from A to E, nevertheless the form as it stands cannot have three consecutive overs, for this would require that we should have three consecutive overs on a closed circuit of five crossings; but this is impossible, since, in order to have in a reduced knot three consecutive overs on a closed circuit, the circuit must have at least seven crossings. If (BC), (CD), or (DB) were erased it would be possible to go from A to E with two crossings. If an A (or E) crossing be erased, say (AB), we have a link, and for knot must add the crossing erased to (BC), (EC), (BD), or (ED). But this will leave D and C connected by the 2-gon A and the crossing (DC). By the transposition of these it becomes possible to go from A to E with two crossings. Hence no reduced ninefold knot can have a sequence of three.



All class V. tenfolds are to be had from the ninefold 35 by the process above described, except that the number of crossings added must exceed by one the number erased. If none be erased and one added, we have either a link or the tenfold 87, which can have no sequence of three, from A to E, except upon a five-crossing circuit. As above, we may not erase (BC), (CD), or (DB). If (BA) be erased and two crossing added in any way to (BC), (EC), (BD), (ED), we again have, after transposition of 2-gon A and the crossing (CD), only two portions of the thread between A and E. If one crossing be added to any one of the four mentioned connections and the other to (DA), (CA), or (DC), A can still be brought into the same position with respect to E. If the second crossing be added to (CE) or (DE) there will be two threads. Hence no class V. tenfold can have a three sequence.

Lastly, suppose a class VI. form to have the sequence of three of fig. 1. The parts A, B, C, D, a, b, c, d must all be distinct, and A, B, a, b can not be 2-gons, or the form would be reducible. Nor can the latter be 3-gons. For, suppose any one, as A, to be a 3-gon. If, now, the thread be shifted from the position fig. 2 to that of fig. 3, the part A is lost to the leading partition and the form comes under class V., which, as has just been shown, can have no form with three consecutive overs. Hence we have the portion of the class VI. tenfold shown in fig. 4. A moment's consideration shows that the parts marked are all distinct. Of the three pairs of adjacent parts C, c; D, d; F, f, only one in each can be a 2-gon. Hence to complete the form at least two additional crossings must be introduced, so that the form becomes at least an elevenfold. Still less is it possible for A, B, a, or b to have more sides than four. Hence no class VI. tenfold can have a three sequence.

C	a	B	d
c	A	b	D

FIG. 1.

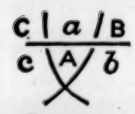


FIG. 2.

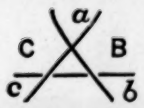


FIG. 3.

	F	f	
C	a	B	d
c	A	b	D

FIG 4.

This completes the proof of the theorem that no reduced non-alternate knot with three consecutive overs has fewer than eleven crossings.

It is easy to verify these theorems by inspection of the plates of alternate forms.

An irreducible elevenfold knot form with three consecutive overs is shown at B, the last in Pl. III. As alternate it is No. 39 of my census.*

8. TWIST.—Let the direction of the moving point, continuously tracing the projection of a knot be recorded at each crossing by marking the thread with arrows. Crossings are of two kinds, as shown in the figs. 5 and 6. I call the first a *twist* of $+\pi$, the second a *twist* of $-\pi$.

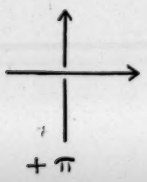


FIG. 5.

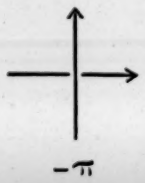


FIG. 6.

Theorem.—*The total twist of a reduced knot is constant for all forms in which the knot can be projected.* The proof is very simple. The twist of the crossings is not altered by any of the transformations permissible to alternate forms, since these consist of rotations of a portion of the knot through an angle of π about an axis in the plane of the knot projection. This does not change the relation of the arrows to a crossing nor a λ crossing to a γ, or *vice versa*. In the changes peculiar to non-alternate forms the thread is shifted from one portion of the knot to another, so as to alter the position of two consecutive overs (or unders). Either the twist of the two crossings is unchanged, or else the twist of the two crossings was originally unlike, and in both crossings it is reversed.

9. Knots may be classified according to twist. The non-alternate tenfolds will be so classified in the following census.

10. An amphicheiral knot is one that can be distorted into its own perversion; and

* *Trans. R. S. E.*, vol. xxxvi. pl. I. (39. $D_1 . \mu$.)

the twist of the perversion of a form is the negative of the twist of the original; hence *the twist of an amphicheiral must be zero*, since it must at the same time change sign and be constant.

Census of Tenfold Non-Alternates.

11. The basis of a non-alternate census of any order is the corresponding census of alternates of the same order, since projections of the former, looked at as plane curves with double points, must be included in the projections of the latter, also so regarded.

12. It will be remembered that two censuses* of tenfold alternates were made independently. These agreed in classes III. and V. The one discrepancy in class IV. was due to an oversight of Professor TAIT, and by him corrected before his plates were printed. There were four discrepancies in class VI., for all of which I was responsible. The substantial agreement of these results, obtained by quite different methods, leaves no doubt that we have all the tenfold alternates without error or omission.

Because of their greater elegance, I have used Professor TAIT's drawings upon which to mark the crossings; the numbers given in the following sections and on the plates are those of the knots of his census

13. The alternate tenfold knots which can in projection also be the projection of non-alternate knots are the following:—

(a) Those in which two parts have three necessarily alternate connections:—

2	39	57	75	115
5	42	63	100	116
18	47	64	103	118
23	50	69	109	123
36	56	70	114	

The two or three forms of each of these were at once drawn and marked. For example, 123 has two 3-gons connected by the alternate connections 2-gon, 2^2 coil and 2^4 coil. Any one of these connections may be γ (or λ) and the other two λ (or γ), giving six forms—the three found on the plates in knots: I, class I.; I, class III.; and I, class VI., and their perversions.

(b) More complicated:—

6	27	38	86	95
8	28	44	87	96
9	29	46	88	97
10	30	54	89	98
12	31	59	90	99
13	32	82	91	110
17	33	83	92	111
25	34	84	93	112
26	35	85	94	113

* *Trans. R. S. E.*, vol. xxxii.; *Trans. Conn. Acad.*, vol. vii.

14. For the latter, tables of all possible crossings were made out, as described in [A] § 6. To secure accuracy these tables were written out twice, once by myself and again later under my direction.* The limitation of § 7 keeps these tables in bounds. Nevertheless a considerable number of the forms given are reducible, and these must in the after work be detected and excluded. This was invariably done by so distorting the form that a sequence of three overs occurred.

15. Upon these tables also the *twist* of each form was computed.

16. From the tables the crossings were marked upon the tracings, giving all possible *forms* of non-alternate knots.

Forms are regarded as distinct only when compartments are dissimilar, or when the direct connections of the compartments are dissimilar in the number or character of the crossings. The tables lead to a certain number of forms equivalent to others already given, and differing only in the circumstance that the form had been rotated through an angle π about an axis of symmetry in the plane of the projection. These were excluded from the plates. In the case of forms which as alternates are amphicheiral, two equivalent forms occur which differ in that the amplexum partition of one is made the non-amplexum partition of the other. Such duplicates are also omitted.

17. The task of finding the knots from the knot forms was exceedingly laborious, and one that I should not have been able to accomplish except for the constant check given by the twist.

The process was as follows :—The twenty-four knots of which the forms obtained in (α) § 13 are projections were easily found. Every other form was examined to see if it could be distorted so as to have two parts with three alternate connections. If this could be done, its proper place in these twenty-four knots was at once known. Where this was found to be impossible, the form was distorted into a form of which the knot was known. In the end it was necessary to see that all permissible distortions of the forms of each knot gave only forms of the same knot. The conditions attendant upon this process are very numerous; I must repeat the caution previously given that until a simple test is found which shall distinguish the forms of one knot from those of every other in the same class, it may happen that non-alternate knots regarded as distinct may be in reality the same.

18. The identification of a given tenfold non-alternate by means of the census may, in a particular instance, require some labour. In the first place it must be ascertained that the given knot is reduced. For this, unfortunately, no simple test is as yet known. Next observe the twist, to determine the class in which the knot will be found. Then find the number of the corresponding alternate knot and compare with the list of § 19. If this number is to be found in more than one knot of the class, a closer comparison with the forms of the plate must be made. In case of apparent failure to find the form, the limitations of § 16 must be kept in mind.

* By Dr H. F. Blichfeldt, now instructor, at the time a student in Stanford University, for whose care my thanks are due.

The Tenfold Alternate ± Knots.

19. In the following list the forms of § 13 (a) are put at the beginning of the knots in which they occur. Other forms are given in numerical order; this is not the case upon the plates.

Class I.—Twist 0. Six knots, and forty-six numbered forms.

I.—70, 116, 123, 26, 33, 88, 94, 94, 97, 99, 112, 113, 113.
II.—18, 57, 75, 17, 17, 34, 44, 54, 93, 93, 95, 111.
III.—2, 63, 8, 10, 25, 25, 84, 85, 89, 90.
IV.—5, 50, 64, 6, 6, 8, 46.
V.—25, 28, 87.
VI.—59.

Class II.—Twist 2π. Nine knots, eighty-four numbered forms.

I.—56, 115, 26, 27, 38, 38, 88, 88, 96, 97, 112.
II.—2, 63, 114, 10, 25, 30, 44, 83, 83, 85, 90, 92, 92.
III.—23, 69, 34, 92, 95, 98.
IV.—39, 42, 47, 12, 12, 13, 13, 29, 82, 84, 84, 86, 86, 86, 86, 89.
V.—42, 47, 12, 12, 13, 28, 28, 29, 29, 30, 82, 82, 82, 84, 84, 85, 86, 86, 87, 89, 91.
VI.—39, 42, 12, 13, 84, 84.
VII.—31, 32, 38, 88, 96, 112.
VIII.—34, 89, 91.
IX.—91, 98, 110.

Class III.—Twist 4π. Ten knots, ninety-eight numbered forms.

I.—36, 70, 123, 33, 94, 99, 113.
II.—23, 69, 118, 25, 28, 34, 82, 85, 87, 91, 91, 93, 98, 110, 110, 111.
III.—56, 100, 115, 26, 31, 32, 33, 88, 94, 96, 112.
IV.—57, 75, 109, 54, 17, 93, 111.
V.—50, 64, 103, 6, 8, 25, 25, 28, 30, 44, 46, 85, 87, 90, 90, 92, 92, 93, 95, 98, 110, 110, 111.
VI.—28, 29, 93.
VII.—26, 27, 32, 33, 35, 59, 94, 96, 97, 97, 113.
VIII.—26, 27, 27, 88, 94, 96.
IX.—6, 8, 9, 10, 44, 59, 95.
X.—10, 25, 83, 83, 92, 92, 98.

Class IV.—Twist 6π. Eight knots, sixty-five numbered forms.

I.—36, 70, 116, 97, 99, 99, 113.
II.—18, 57, 109, 17, 54, 54, 95.
III.—5, 50, 103, 6, 8, 25, 30, 30, 44, 46, 46, 54, 85, 85, 90, 95, 98, 111.
IV.—28, 82, 85.
V.—10, 25, 28, 29, 85, 87, 89, 89, 91, 110.
VI.—8, 9, 10, 30, 44, 46, 59, 83, 85, 89, 111.
VII.—31, 32.
VIII.—33, 33, 35, 59, 96, 97, 99.

Class V.—Twist 8π. Two knots, ten numbered forms.

I.—63, 114, 10, 29, 83, 86, 89.
II.—27, 32, 96.

Class VI.—Twist 10π. Eight knots, fifty numbered forms.

I.—36, 116, 123, 26, 31, 33, 38, 88, 94, 97, 99, 112, 113.
II.—23, 118, 25, 87, 92, 98, 110.
III.—100, 115, 27, 32, 96.
IV.—18, 75, 109, 8, 17, 28, 34, 54, 85, 93, 95, 111.
V.—5, 64, 103, 6, 30, 44, 46.
VI.—10, 29, 83, 89.
VII.—9.
VIII.—35.

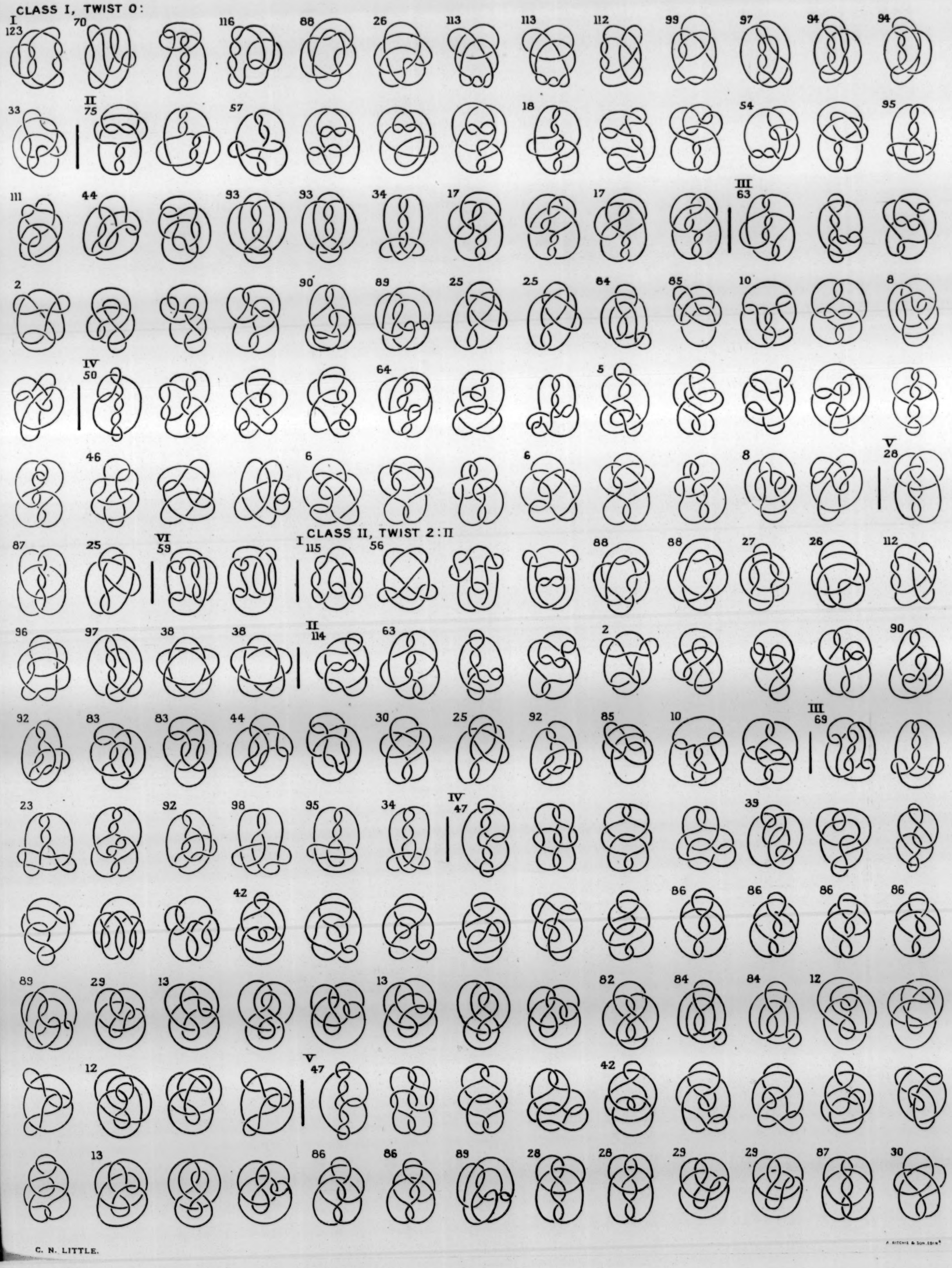
20. It is interesting to note that in order ten as in order eight no amphicheiral non-alternate knots have made their appearance. The numbers of knots must be doubled, therefore, to include perversions.

To the 253 alternate knots previously described this census adds 86 non-alternate, making 339 reduced knots of ten crossings.

These new knots are shown upon Plates I., II., III., the perversions being obtained as usual by holding the plates before a mirror and looking at the images. In knot II of class II., Plate I., § 16 should have excluded the second form of 83. The first 59 in knot IX of class III. on Plate II. is also manifestly superfluous.

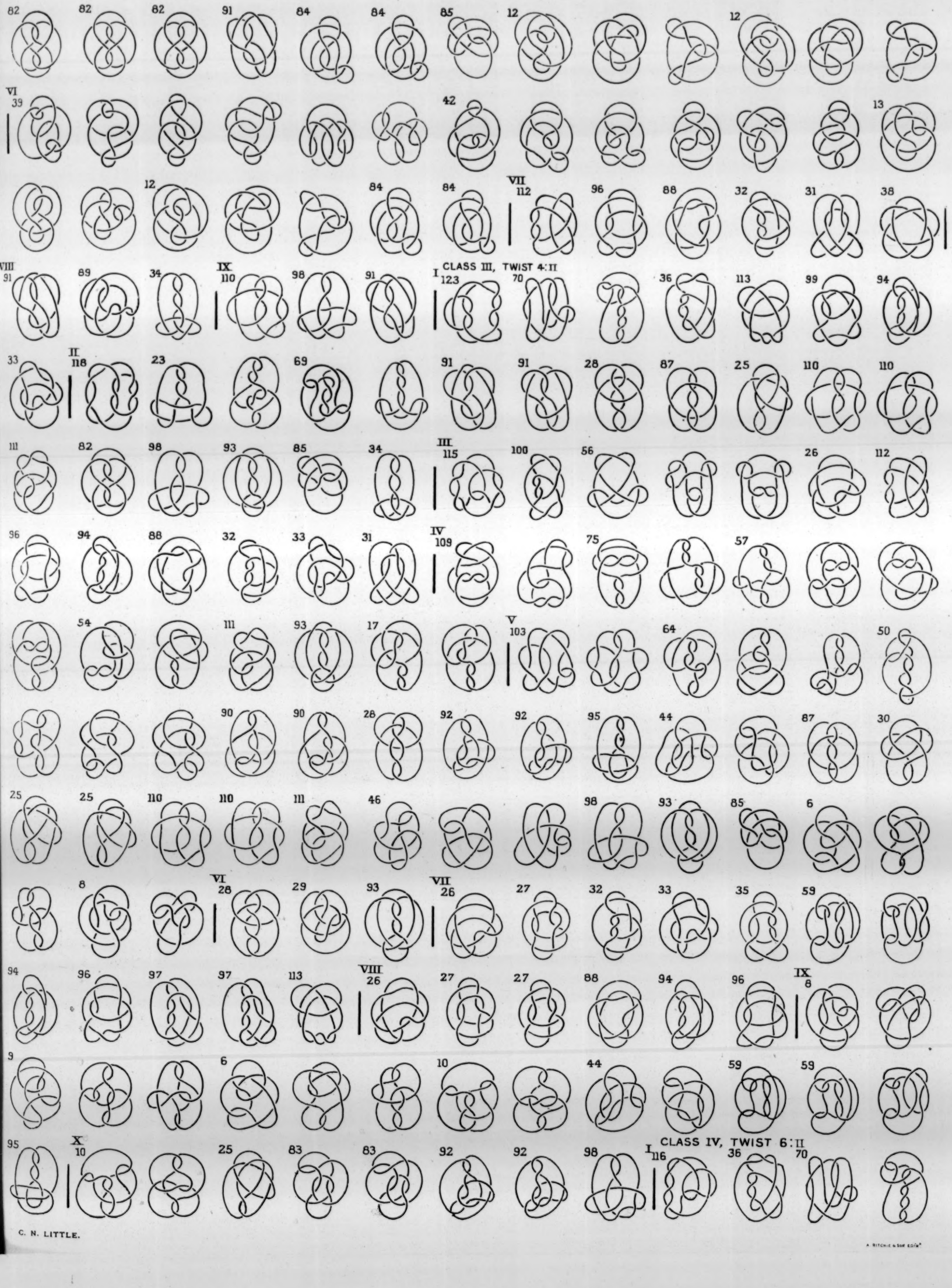
PROF. LITTLE: NON-ALTERNATE ± KNOTS.

PLATE I.



PROF. LITTLE: NON-ALTERNATE ± KNOTS.

PLATE II.



C. N. LITTLE.

A. RITCHIE & SON EDIN.

PROF. LITTLE: NON-ALTERNATE ± KNOTS.

PLATE III.

C. N. LITTLE.

A. Ritchie & Son, Edin.

XXXI.—*The Meteorology of Ben Nevis in Clear and in Foggy Weather.*

By J. Y. Buchanan, F.R.S. (With Eight Plates.)

(Read July 3, 1899.)

The publication by the Royal Society of Edinburgh in 1890 of the hourly observations made at the observatory on the summit of Ben Nevis along with corresponding observations at five different hours at Fort-William at the sea level was an important contribution to meteorological science. As I held at the time the Lectureship in Geography at the University of Cambridge it furnished a welcome supply of facts for the illustration of important points in physical geography. What struck me most at the time in the study of this work was the unique character of the observatory of Ben Nevis itself, without reference to its companion at the sea level, as a first-class observatory in a locality where the atmosphere is for the greater part of the year completely saturated with moisture. Being also situated on a true peak, and that the highest point in the British Islands, the observations may fairly be taken to represent the meteorology of the clouds, or at least of such clouds as are in contact with mountain slopes, and in any case they must afford rich material for the study of the physics of the atmosphere.

The weather on Ben Nevis falls naturally into three categories: the first includes the days when the mountain is continuously enveloped in fog or cloud, the second, those periods when the mountain is clear of clouds, and the third, periods during which frequent alternations of clear and foggy weather occur. It seemed to me that the conditions would be best understood by studying the continuously clear weather and the continuously foggy weather, each by itself, and that this would make it more easy to understand the conditions of mixed weather. When the meteorological observations on Ben Nevis are looked at as physical observations, and the object is to study the physics of the atmosphere, apart from all considerations of weather, then a separation such as that above indicated becomes an obvious preliminary. Had meteorology been first practised in the Tropics it would now be in a more advanced state than it is. The meteorology of Europe, like the tides on its coasts, is the most intricate and involved that can be found anywhere in the world, and it is the worst possible material on which the study could be commenced. Within the Tropics, and round the Poles, the conditions are simple and uninvolved, and the meteorological observations, at least those made within the Tropics, reflect this simplicity. Generally, in the Tropics there are two kinds of weather, indicated by the terms Dry season, Rainy season. In the Indian peninsula, for instance, during the months December, January, February, March, and April, hardly a drop of rain falls, there is little cloud, the air is very dry, and the temperature shows considerable daily range.

In the months from June to October it rains at least two days out of every three; the tension of aqueous vapour is high, and the atmosphere is generally nearly saturated with it; this is the weather which accompanies the strong south-west wind known as the Monsoon.

If we take the meteorological record of a year at any such place we find that the sorting out of different kinds of weather has been done by Nature, and the discussion has necessarily reference to the different kinds of weather, even although this may not always find expression. At the Poles, where the year consists of one long day and one long night, we cannot doubt that the meteorology must be characterised by much simplicity. In Europe, we have dry seasons and rainy seasons; but, whereas, in India the dry season occupies completely one-half of the year and the rainy season occupies completely the other half without any mutual interference, our wet and dry seasons alternate as many times in the course of a single day. Here it would seem to be indicated that we should do for ourselves what Nature does for us in the Tropics, and sort out the weather before discussing it.

The prime factor in determining the climate of a place is its Latitude, because this determines the altitude to which the sun rises from day to day and the length of time that it is above the horizon each day. The heating power of the sun on any horizontal portion of the earth's surface varies with the sine of its altitude at the moment. All the elements of climate and weather depend ultimately on this factor, and its variability produces a corresponding variability in the weather. It is only necessary to consult a table of sines to see where the greatest variability is likely to occur. Thus, at the equator the sine of the sun's meridian altitude varies between 0·924 and 1·000, at either of the Tropics, between 0·684, and 1·000 in lat. 45°, between 0·367 and 0·930, and in the latitude of Ben Nevis (56° 48′ N.) it varies between 0·169 and 0·835. At the equator the sun's heating power at noon only varies by $7\frac{1}{2}$ per cent. of its maximum amount, while at Ben Nevis the variation is 80 per cent.

But the latitude of a place determines not only the intensity of the sun's heat which it receives, it also determines the intensity of the cooling to which it is exposed by radiation into the upper regions of the atmosphere and into space. This goes on all day independently of the sun's heating, but it becomes more apparent after the sun has set, and produces the greater effect the longer is the duration of the night, and this is a function of the latitude. While the altitude which the sun attains measures the heat which it supplies, and the length of the night determines the amount which is lost, both the heat received and that lost at any particular place may be in excess or in default of what it is entitled to, owing to its latitude alone. This is due to secondary actions set up by the primary heating and cooling, whereby one place may receive, in addition to its own, a supply of heat or of cold to which it is not entitled, thereby altering its climate as well as that of the other place which has supplied the heat or cold.

The principal secondary agencies through which the sun works are the atmosphere, in its motion, both horizontal and vertical, and in its changes of volume; and the sub-

stance of water in its changes of state of aggregation. It cannot be doubted that, considering the Earth as a whole, the nett effect of these secondary agencies is *nil*. Yet their local effect is in many cases very great, as witness the difference in population between Great Britain and places in the same latitude in North America. The waters of the ocean are also a powerful secondary agency in the distribution of the sun's heat.

The $3\frac{1}{2}$ years' published observations on Ben Nevis, though furnishing excellent material for the illustration of lectures, were not sufficient for the thorough discussion of the climate of the mountain. There was no lack of foggy weather; but it is of very little use attempting to study a secondary effect without knowing something of the primary effect which is disturbed, and we must consider the fog as a disturbance of the normal solar climate. For this purpose it is important to have a sufficient number of examples of the clear weather of Ben Nevis; and now that ten more years have passed, thirteen whole years' observations are available, and these have been obligingly put at my disposal by Dr BUCHAN.

Many special features of the meteorology of Ben Nevis have been discussed in interesting papers by Mr OMOND, Mr RANKINE, Mr MOSSMAN, and by Dr BUCHAN. I have little doubt that these papers would have gained in interest if the clear and foggy weather had been distinguished. It has been my wish to do something towards this end myself, and at the beginning of last winter I set about sorting the thirteen years into periods of clear and of foggy weather, leaving out all days of mixed weather. The burden of this work has been borne by Mr ANDREW KING, who for a number of years ably assisted me in my chemical work. Without his aid the collection of books, with the hourly observations sorted into spells of clear and of foggy weather, would never have been brought together; and I desire to express my great obligation to him for this and for the extensive work of computation which has accompanied it.

Great as this labour was, it would have been several times greater had our meteorological observatories followed the fashion, which it is attempted to force on them from abroad, more especially from the continent of Europe, of using Celsius' scale for the measurement of the temperature of the air.*

The books, which already form almost a little library, will be deposited with the Scottish Meteorological Society, and will furnish the indispensable preliminary arrangement for permitting every feature of the meteorology of Ben Nevis to be discussed with reference to clear or foggy weather. I attach particular value to discussions by men like Mr OMOND, Mr RANKINE, Mr BRUCE, and others who have resided on the summit for a considerable time as observers. They have known the mountain in all its moods, and explanations will occur to them which would never be suggested to another by the mere contemplation of numbers.

The published observations were begun on Ben Nevis in 1884, but, to avoid a broken series, I have taken the years from 1st January 1885 to 31st December 1897.

* My views on this subject are more fully explained in a letter which appeared in *Nature* (1899), vol. lx. p. 364.

The principle on which the dates were selected was:—For foggy weather, to take spells of three or more whole days of continuous foggy weather, and *continuous foggy weather* is defined by twenty-four consecutive entries of fog in the log of each day. The supply of foggy days seemed to be so abundant that the minimum length of spell was able to be fixed at three whole days. When it became a question of selecting the spells of clear weather it was necessary to adopt the hour as unit, and twenty-four consecutive hours during which fog was not once entered in the log was adopted as the specification of the spell of clear weather of minimum duration. It was not possible to enforce the limitation that the twenty-four hours should all belong to the same day. It will be observed that a *clear* day only means twenty-four hours free of fog, and implies nothing with regard to the presence or absence of cloud overhead.

Summit and Base.—Although observations have been made at the base of the mountain since the observatory on the summit was established, and for the last ten years a first-class observatory has existed at Fort-William, in this paper no account is taken of the observations made at the base. After the meteorology of the summit has been thoroughly studied by itself, and that of the base by itself, there will be greater light for the study of the combined observations and more assurance of the validity of the conclusions arrived at.

Details of Method of Selection.—A list of dates selected from the years 1885–1897 was made. The principle of the selection was, in the case of *foggy* weather, to pick out every block of at least three days of continuous fog; in the case of *clear* weather, to pick out every spell of at least twenty-four hours continuous clear weather. By a spell of *clear* weather is meant one in which *no fog* was logged. The blocks of *foggy weather* vary in length from three days to eleven days; the spells of clear weather vary in length from twenty-four hours to two hundred and eighty-three hours.

This list of dates having been made, the *barometer* observations for every hour of the days involved were copied from the Ben Nevis sheets into a series of note-books: one note-book containing all the data belonging to a particular month for all the years. Thus, one note-book contains all the January blocks of foggy and spells of clear weather; another, the February, and so on—a series of twelve books being thus made up. In the case of the spells of clear weather the entries for the entire day in which the spell begins and for that on which it ends were copied out (besides those for the intermediate days), and afterwards the exact hours at which the spell begins and ends were marked. Thus, in the case of a spell of clear weather extending from nine o'clock on 15th August to two o'clock on 19th August, all the entries for 15th, 16th, 17th, 18th, and 19th August were made. The advantage of this arrangement is that it shows plainly the barometric conditions prevailing at epochs of transition from one kind of weather to another, and it has greater completeness. The same thing was done in the case of the *temperature*, *rainfall*, *tension of aqueous vapour*, *wind*, *cloud*, and *sunshine*; so that we have a series of eighty-four note-books, containing records of the meteorological elements mentioned above, for the selected list of dates. The dates of

clear and of *foggy* weather are clearly indicated, so that there is a valuable collection of data available for any future work on the conditions prevailing in the two kinds of weather specified.

Note-books were then prepared for the tabulation of the *sums* and *means* of the different meteorological elements in each month. These were ruled with the various years (1885–1897) as the side argument, and the twenty-four hours of the day as the top argument.

Take the barometric pressure during clear weather for illustration. Under each hour and opposite each year were recorded the *sum* of the barometric pressures, and the number of observations involved at that particular hour of that particular year. Thus, for each month we have thirteen longitudinal columns and forty-eight vertical columns: twenty-four of the vertical columns being the record of the sums in the individual years, and twenty-four being the record of the number of observations dealt with. On summing the vertical columns and dividing the sums of the pressures by the sums of the number of observations we find the mean pressures at each hour of each month for the whole period of thirteen years.

The same process was applied to the Barometric observations during foggy weather, and to the *temperature, rainfall, and tension of aqueous vapour* for both clear and foggy weather. In the case of the *wind* observations note-books were also prepared for the tabulation of the number of winds in each month. The top argument in this case was the sixteen points of the compass (N., N.N.E., N.E., E.N.E., etc.), and the side argument the twenty-four hours of the day. Under each point and opposite each hour was recorded the number of times that that particular direction of wind had been logged at the hour under consideration in the whole thirteen years. By summing the columns vertically the total number of times that each particular direction of wind had been logged in each month for the whole thirteen years was obtained without regard to the hour of the day.

When the totals were obtained they were reduced to eight points, viz. : N., N.E., E., S.E., S., S.W., W., N.W. This was done by distributing the N.N.E. winds equally among N. and N.E., the E.N.E. equally among N.E. and E., and so on.

A tabulation of the number of times the wind was observed to blow in a particular direction in a particular month does not, however, give a true idea of its relative importance, without also taking into consideration the number of observations dealt with altogether in that particular month. Thus, we find in June that in our clear weather and foggy weather we have westerly winds logged about an equal number of times. It must be remembered, however, that in June we are dealing with a very much larger number of observations of clear weather than of foggy weather; hence we find that the westerly winds are only 6 per cent. of the total dealt with in June in clear weather, whereas they are 20 per cent. of those in foggy weather. A table has, therefore, been made, showing for each month the percentage of each direction.

Although we have in the tables and curves all the data relating to clear and

foggy weather on Ben Nevis, arranged in convenient form for study, the study of them takes time. The object of this paper, however, is less to present a complete account of the meteorology of the mountain in the two different classes of weather, than to present the data in the form which appears the most suitable for fruitful study, leaving the actual study free and open to everyone, in view of the publication very shortly of the complete series of observations to the end of 1897. While, therefore, no complete discussion of the data is attempted in this paper, the chief facts of interest which proceed from them are noticed in the following pages.

First, as regards the selected dates: the principle of selection has been already stated. In Tables I. and II. we have the complete list of clear and of foggy spells. The clear spells are indicated in Table I. by the year and month, and the day and hour at which each spell begins and ends. The foggy spells, consisting only of whole days and a minimum of three consecutive days, are indicated simply by the days in the squares corresponding to the year and month. The observations of thirteen years, namely, 1885 to 1897 (inclusive), are dealt with in this paper.

The following short Table shows a summary of the quantity of material which was available on the principle of selection adopted.

Summary of Material Used.

Month.	Foggy Weather.		Clear Weather.	
	No. of Spells.	No. of Days.	No. of Spells.	No. of Hours.
January, . . .	22	86	13	606
February, . . .	12	54	26	1,158
March, ...	20	78	27	1,335
April, ...	16	59	30	1,728
May, ...	7	27	37	2,228
June, ...	12	42	46	2,895
July, ...	11	46	30	1,502
August, ...	17	58	19	971
September, . . .	16	79	31	1,296
October, . . .	17	64	21	951
November, . . .	20	88	19	983
December, . . .	15	66	14	697
Totals, ...	185	747	313	16,350

From it, it will be seen that 70 per cent. of the clear weather occurs in the months of April, May, and June, and generally there is much more prolonged clear weather in

the spring half of the year than in the autumnal half. There is also less continuous foggy weather in the spring half, there being a well-marked minimum in May. The months of August and especially September are rich in continuous foggy weather, and this accounts for the very bad reputation of the mountain among tourists. The month which figures in our lists with the maximum of foggy spells is November, the maximum of clear spells falls in June.

The distribution of the spells of clear and of foggy weather in the months of different years is worth attention. In January there were no spells of twenty-four hours clear weather in the six years, 1889 to 1894 (inclusive). In March, May, June, and July, there is one year without a spell of clear weather; in February, April, August, and September, there are two years; in October and November, three; and in December, five years without a spell of clear weather. With regard to the spells of foggy weather of three days and upwards, we find only one year where they were awanting in January. In March, April, September, November, and December, there are two years; in June and August, three years; in February, July, and October, four years; and in May, six years where they were awanting.

Adding up the total number of hours of clear weather and days of foggy weather considered, we find that they make 681 and 747 days respectively, or a total of 1428 days out of 4748 days contained in the thirteen years, or, roughly speaking, one-third.

TABLE *giving the Number of Minutes after Greenwich Mean Noon when it is Local Apparent Noon at Ben Nevis at Different Dates.*

Date.	Minutes.	Date.	Minutes.	Date.	Minutes.	Date.	Minutes.
Jan. 1	24	Mar. 30	24	Aug. 18	24	Nov. 2	3·7
,, 3	25	April 4	23	,, 24	23	,, 9	4
,, 5	26	,, 7	22	,, 27	22	,, 17	5
,, 7	27	,, 10	21	,, 30	21	,, 21	6
,, 9	28	,, 15	20	Sept. 1	20	,, 26	7
,, 12	29	,, 18	19	,, 4	19	,, 28	8
,, 15	30	,, 24	18	,, 7	18	,, 30	9
,, 20	31	,, 30	17	,, 9	17	Dec. 2	10
,, 24	32	May 15	16	,, 12	16	,, 4	11
,, 29	33	,, 25	17	,, 15	15	,, 7	12
Feb. 1	34	June 3	18	,, 17	14	,, 10	13
,, 11	34·5	,, 9	19	,, 20	13	,, 12	14
,, 19	34	,, 15	20	,, 22	12	,, 14	15
,, 27	33	,, 21	21	,, 25	11	,, 16	16
Mar. 2	32	,, 25	22	,, 30	10	,, 18	17
,, 7	31	,, 30	23	Oct. 3	9	,, 21	18
,, 10	30	July 2	24	,, 5	8	,, 23	19
,, 15	29	,, 12	25	,, 11	7	,, 24	20
,, 17	28	,, 18	26	,, 13	6	,, 26	21
,, 20	27	Aug. 1	26	,, 19	5	,, 28	22
,, 25	26	,, 12	25	,, 27	4	,, 30	23
,, 28	25						

Hence, out of the thirteen years we can make up four years in which the weather was either continuously clear or continuously foggy in the sense of our specification of these classes of weather.

Time used in the Observatory.—The observations are made at every hour by a clock showing Greenwich mean time. The mountain lies 5° of longitude west of Greenwich, so that the local mean time of Ben Nevis is twenty minutes earlier than mean time at Greenwich. With the varying values of the equation of time the local apparent time varies from 3·5 to 34·5 minutes earlier than Greenwich mean time. In the first of the accompanying tables (printed on the preceding page) is given, for a number of dates, the number of minutes after Greenwich mean noon when it is local apparent noon at Ben Nevis. It will be seen that at the beginning of November the sun crosses the meridian at only 3·5 minutes after noon by the clock; while at the beginning of February it does not do so till nearly thirty-five minutes after noon by the clock. Between 15th January and 10th March, 1 P.M. is nearer true local noon than 12 o'clock by the clock.

The second Table gives the dates of days of lengths differing by half-an-hour, with the number of days elapsed since the vernal equinox.

Both of these Tables are useful in interpreting the meaning of the results. The first naturally suggests the question whether at first-class observatories the hours of observa-

BEN NEVIS.—*Dates when the Length of the Day is an exact number of Half-hours.*

Days from vernal equinox.	Diff.	Date.	Length of Day.	Date.	Diff.	Days from vernal equinox.
93	23	June 21	17 h. 32 m.	21 June		93
70	9	May 29	17 hrs.	14 July	23	116
61	8	„ 20	16½	23 „	9	125
53	7	„ 12	16	31 „	8	133
46	8	„ 5	15½	8 Aug.	8	141
38	6	April 27	15	15 „	7	148
32	7	„ 21	14½	21 „	6	154
25	6	„ 14	14	28 „	7	161
19	6	„ 8	13½	4 Sept.	7	168
13	7	„ 2	13	10 „	6	174
6	6	Mar. 26	12½	16 „	6	180
365	6	„ 20	12	23 „	7	187
359	6	„ 14	11½	29 „	6	193
353	6	„ 8	11	5 Oct.	6	199
347	7	„ 2	10½	11 „	6	205
340	6	Feb. 23	10	18 „	7	212
334	6	„ 17	9½	24 „	6	218
328	7	„ 11	9	30 „	6	224
321	7	„ 4	8½	6 Nov.	7	231
314	7	Jan. 28	8	13 „	7	238
307	9	„ 21	7½	21 „	8	246
298	22	„ 12	7	30 „	9	255
276	21	Dec. 21	6 h. 28 m.	21 Dec.	21	276

tion should not be according to local apparent time, especially when, as at Ben Nevis, hourly eye readings are made. They would be equally available for popular purposes, whereas, the correction of popular time by as much as half-an-hour is a serious matter for scientific purposes. When self-recording instruments are used, it is as easy to take off the values for one time as for another. It is certain that it is apparent time and not any conventional time that rules meteorological phenomena.

Winds.—The data regarding the winds are contained in Tables III. to VIII. Tables III. and VI. contain the total number of observations of each wind to sixteen points, as logged in each month in clear and in foggy weather. Tables IV. and VII. contain the same data reduced to eight points of the compass, and Tables V. and VIII. are these figures reduced to percentages per month.

Only the direction of the wind is given in the Tables. In foggy weather the prevailing directions are N., N.W., W., and S.W., the greatest percentage being from the West. In clear weather the prevailing directions are N., S.E., and S., with a maximum at S.E., while calms occur very frequently. The cyclonic gales so common on our coasts, especially in winter, blow hardest between S.W. and N.W., and on these occasions the summit is usually enveloped in fog. On the other hand, in anti-cyclonic weather the air has hardly any sensible horizontal motion, and is generally clear and warm.

In the following short Table are given the general results of the classification of the winds according to percentage, and distributed over eight points of the compass :—

Weather.	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.	
Clear . .	17·45	6·01	11·63	18·85	16·74	8·94	6·17	3·54	10·67	100
Foggy . .	16·88	5·28	3·63	5·64	6·95	17·82	23·72	16·65	3·43	100

If the winds were equally distributed round the compass we should have 12·5 per cent. from each point. In clear weather the amounts are above the average for N., S.E. and S. ; in foggy weather, they are above the average for N., S.W., W., and N.W. ; the maximum falling on W., with 23·72 per cent., and the minimum on E., with 3·63 per cent. The winds in both clear and foggy weather require to be studied in connection with the other meteorological elements, but for this purpose the individual observations must be combined. It has not been possible to overtake this.

Rainfall.—The data with respect to rainfall are contained in Tables IX. to XI. Table IX. contains the total rain measured in each month of each year in continuous clear weather. It will be seen that in weather of this description rain is of very rare occurrence, so much so that the rain in clear weather is not further dealt with in tabular form. Table X. is the corresponding Table for foggy weather. In this kind of weather the rainfall is very large, and in Table XI. it is reduced to mean hourly

values for each month, with the maximum hourly rainfall during the thirteen years for each hour. Table XII. contains the diurnal variation of the monthly mean hourly values.

On studying these Tables we note the following facts:—

In continuous foggy weather there is always some precipitation measured, and it is usually very abundant. There is only one spell of foggy weather in which no precipitation was registered, and that was in January 1897. In continuous clear (not necessarily cloudless) weather it may be said that it never rains at all. The only exception which could be taken to this might be the cases of June and August. In June, in the 2895 selected hours, the total amount of rain measured was 1·243 ins., giving an average of one-hundredth of an inch in twenty-four hours. In August, in the 971 selected hours, 0·72 in. fell, giving an average of 0·018 in. per twenty-four hours. The rainless character of continuous clear weather is well shown in Table IX., in which the total amount of rain which fell in each month in each year during clear weather is given. There are only five occasions in the course of the thirteen years when more than one-tenth of an inch of rain fell during the clear weather of any one month, and the chief of these are 0·685 in. in August 1890, and 0·298 and 0·774 in. in June of 1887 and 1893 respectively. Two-thirds of the whole rain fell in these five months, the remaining third is distributed over forty months, and 111 months out of the total of 156 have none at all.

If we now turn to Table X., drawn in the same form for foggy weather, we see a very different state of things. As already pointed out, there is only one spell of foggy weather where no rain or precipitation has been measured. The smallest mean daily rainfall in foggy weather is 0·590 in. in April. The highest daily mean is 1·286 in. in September, and next to it comes February with 1·270 in., and October with 1·163 in. April, May, June, July, and August are all under the mean for the year, which is 0·998 in., or almost exactly one inch. When it is remembered that in foggy weather the air is always saturated, and besides that it has disseminated through it particles of water, which form the fog, on which the vapour can immediately and without resistance condense, and that in summer there is a much larger percentage of water vapour present in the atmosphere than is found in winter, it appears somewhat remarkable that the summer rainfall per foggy day should be so much lower than that of winter. When we turn to the maximum rainfall of any foggy day in each month during the thirteen years, we find these same months below the average again. The absolute maximum for the thirteen years falls on the 3rd October 1890, when 7·287 ins. fell in the twenty-four hours; the next highest value, 6·668 ins., falls on 6th February 1894, and the maximum rainfall on a foggy day is above five inches in December, January, and March. The greatest fall in one hour occurred between fifteen and sixteen hours on 14th January 1890, when 0·850 in. fell.

The following short table puts together concisely the results of the rainfall measurements in the two kinds of weather, and shows clearly the great contrast between them in this respect:—

Month.	Clear Weather.			Foggy Weather.					
	Number of Hours.	Total Rainfall in Month.	Mean Rainfall per 24 Hours.	Number of Days.	Total Rainfall in Month.	Mean Rainfall per Day.	Maximum Rainfall in one Day.	Date of Maximum.	
		Inches.	Inch.		Inches.	Inches.	Inches.	Day.	Year.
January, . .	606	0·004	0·00015	86	93·029	1·082	5·476	29	'92
February, . .	1,158	·033	·00069	54	68·587	1·270	6·668	6	'94
March, . . .	1,335	·003	·00005	78	82·602	1·059	5·210	11	'90
April, . . .	1,728	·110	·00153	59	34·782	0·590	2·530	22	'90
May, . . .	2,228	·392	·00422	27	20·555	0·761	2·880	15	'92
June, . . .	2,895	1·243	·01030	42	32·579	0·776	2·108	16	'97
July, . . .	1,502	0·169	·00270	46	41·452	0·901	2·337	13	'90
August, . .	971	·720	·01777	58	55·852	0·963	3·483	29	'92
September, . .	1,296	·046	·00085	79	101·646	1·2 6	4·930	31	'91
October, . .	951	·064	·00161	64	74·438	1·163	7·287	3	'90
November, . .	983	·076	·00185	88	91·660	1·042	4·294	28	'93
December, . .	697	·000	·00000	66	71·049	1·076	5·340	12	'85
Sums, . . .	16,350	2·860		747	768·231				
Mean, . . .			0·00389			0·998	4·379		

Barometric Pressure.—The particulars of barometric pressure are given in Tables XIII. to XIX. Tables XIII. and XIV. contain the actual maximum and minimum pressures observed at every hour during the thirteen years in clear and in foggy weather. Tables XV. and XVI. contain the monthly mean hourly pressures for clear and foggy weather. Tables XVII. and XVIII. contain the diurnal variations in the monthly mean hourly pressures in clear and in foggy weather; and Table XIX. gives the monthly mean hourly excess of the barometric pressure in clear weather over that in foggy weather.

The last of these tables shows the most striking difference between clear and foggy weather on Ben Nevis, namely, the large and continuous excess of pressure in clear weather over that in foggy weather. At every hour of the year, without a single exception, the mean monthly pressure is several tenths of an inch higher in clear than in foggy weather. The maximum excess recorded in the table is 0·602 in. at twenty-one hours in January, and the minimum is 0·296 in. at fourteen hours in February. The average excess for the whole year is 0·456 in., and the following are the mean excesses for each month:—

Jan.	Feb.	Mar.	April.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.	Mean.
0·570	0·320	0·501	0·353	0·372	0·390	0·414	0·433	0·519	0·550	0·537	0·513	0·456 inch
+ ·114	− ·126	+ ·045	− ·103	− ·084	− ·066	− ·042	− ·023	+ ·063	+ ·094	+ ·081	+ ·057	0·000

Roughly speaking, in spring and summer the excess is below the mean, excepting the month of March, and in autumn and winter it is above the mean. The drop from

the maximum to the minimum value in one month, from January to February, is very remarkable, but the recovery in March to a much higher value, and indeed to a second maximum, suggests that there are special reasons for this irregularity which may be accidental to the particular years under consideration. It is certainly difficult to imagine, though it is well worth while to try to find out, what conditions vary so much and so rapidly between January and April that the excess of pressure in question should show the variations which we observe in the table.

The curves of yearly march of pressure, Plate VIII., show a remarkable rise in February in foggy weather and a sharp fall in clear weather, so that the curves are brought much nearer each other. In both weathers there is a minimum in March and a maximum at the beginning of June. In foggy weather this is the principal maximum. In clear weather the principal maximum is in September.

Turning now to the distribution of pressure over the day in each month, Tables XV. to XVIII., Plates I. and II., we find considerable differences. In Plate I. we have the months of the winter half-year, and in Plate II. those of the summer half-year, both on a scale of 20 : 1. It is seen at once that the curves for foggy weather are smoother than those for clear weather, although sharp irregularities are not wanting, as, for instance, in the month of April. Looking at the clear weather curves, especially those of the winter half-year, it would be imagined that they were curves of a single day's observations, but they are the mean curves for the clear days of the month during thirteen years.

It will be observed that in all the clear weather curves there is a maximum in the evening between eighteen hours and midnight, this is particularly developed in winter. In these months, especially December, January, and February, the curves are most irregular. In summer they are a good deal smoothed out, and run down to the early morning minimum, which is strongly marked in nearly all the curves. The midsummer months, June and July, show curves which, without straining, could be smoothed into one having a single minimum between three and four A.M., and a single maximum about six P.M. In all the others a double period is perceptible, and in the winter months the serrated character of the curves is too remarkable to be accidental.

In foggy weather we have, as has been pointed out, greater regularity. The continual change of water from the liquid to the gaseous state and *vice versâ* has a regulating influence on the changes of pressure as it undoubtedly has on the changes of temperature. In comparing the curves for clear and for foggy weather, it has to be remembered that the mean pressures, the hourly deviations from which are shown in the curves, are always much lower than the corresponding ones for clear weather.

The curves for May and June very much resemble those for June and July in clear weather. They give at once a smooth curve with single minimum and single maximum, but these points fall rather later, the minimum at 4 to 5 A.M., and the maximum at 9 P.M. A second minimum at 6 P.M. appears in July, providing a second maximum at 2 P.M., and the curve preserves this form in August and September. In October the second

Summary.—Barometric Pressure—Inches.

		n.	Jan.	Feb.	March.	April.	May.	June.	July.	August.	Sept.	Oct.	Nov.	Dec.	Mean.
			Inches.	Inches.	Inches.	Inches.	Inches.	Inches.	Inches.	Inches.	Inches.	Inches.	Inches.	Inches.	Inches.
Clear Weather.	Absolute maximum, . . .	1	26·005	26·063	25·946	26·003	26·014	25·992	25·929	25·925	25·981	26·047	26·068	25·914	25·991
	Absolute minimum, . . .	2	24·791	24·686	24·324	25·072	25·115	25·179	25·217	25·231	24·979	24·957	25·003	24·798	24·946
	Range, ... (1 – 2)	3	1·214	1·377	1·622	0·931	0·899	0·813	0·712	0·694	1·002	1·090	1·065	1·116	1·045
	Mean, ...	4	25·618	25·532	25·472	25·536	25·659	25·657	25·628	25·660	25·712	25·670	25·663	25·536	25·612
Foggy Weather.	Absolute maximum, . . .	5	25·838	25·807	25·663	25·824	25·744	25·755	25·612	25·691	25·761	25·749	25·947	25·800	25·766
	Absolute minimum, . . .	6	24·011	23·869	23·943	24·188	24·830	24·757	24·686	24·360	24·260	24·371	24·085	23·742	24·259
	Range, ... (5 – 6)	7	1·827	1·938	1·720	1·636	0·914	0·998	0·926	1·331	1·501	1·378	1·862	2·058	1·507
	Mean, ...	8	25·048	25·212	24·971	25·183	25·287	25·267	25·214	25·227	25·193	25·120	25·126	25·023	25·156
	Difference . . . . (4 – 8)	9	0·570	0·320	0·501	0·353	0·372	0·390	0·414	0·433	0·519	0·550	0·537	0·513	0·456

Summary.—Barometric Pressure—Millimetres.

		n.	Jan.	Feb.	March.	April.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.	Mean.
			Mm.	Mm.	Mm.	Mm.	Mm.	Mm.	Mm.	Mm.	Mm.	Mm.	Mm.	Mm.	Mm.
Clear Weather.	Absolute maximum, . . .	1	660·66	661·93	659·13	660·40	660·66	660·15	658·62	658·62	659·89	661·67	662·18	658·12	660·17
	Absolute minimum, . . .	2	629·67	627·13	617·73	636·78	637·80	639·57	640·59	640·84	634·49	633·99	635·00	629·92	633·63
	Range, ... (1 – 2)	3	30·99	34·80	41·40	23·62	22·86	20·58	18·03	17·78	25·40	27·68	27·18	28·20	26·54
	Mean, ...	4	650·75	648·46	646·94	648·72	651·77	651·77	651·00	651·77	653·04	652·02	651·77	648·72	650·50
Foggy Weather.	Absolute maximum, . . .	5	656·34	655·58	651·77	655·83	653·80	654·31	650·50	652·53	654·31	654·05	659·13	655·32	654·46
	Absolute minimum, . . .	6	609·86	606·30	608·08	611·89	630·68	628·91	627·13	618·75	616·21	619·00	611·89	603·00	615·98
	Range, ... (5 – 6)	7	46·48	49·28	43·69	43·94	23·12	25·40	23·37	33·78	38·10	35·05	47·24	52·32	38·48
	Mean, ...	8	636·27	640·34	634·24	639·57	642·37	641·86	640·34	640·84	639·83	638·05	638·30	635·51	639·07
	Difference . . . . (4 – 8)	9	14·48	8·12	12·70	9·15	9·40	9·91	10·66	10·93	13·21	13·97	13·47	13·21	11·43

maximum and minimum have been nearly smoothed out, and the curve changes character with the approach of winter. The most striking feature of these curves is the sharp rise of pressure between midnight and 1 A.M., this is apparent in all the curves from November to April, with the single exception of December. The morning minimum is found about 6 A.M., with a forenoon maximum at 11 A.M., and these are followed by an afternoon minimum and an evening maximum more or less pronounced. The extraordinary serration of the curve after midday in April is remarkable, as it is the only one belonging to foggy weather which shows this feature.

A general summary of the movements of the barometer during the year is presented in the tables on the preceding page, in terms of the inch and the millimetre respectively.

Tension of Aqueous Vapour.—This element is dealt with in Tables XX. to XXVI. Tables XX. and XXI. contain the absolute maximum and minimum values observed at every hour in the thirteen years in clear and in foggy weather respectively. Tables XXII. and XXIII. contain the monthly mean hourly values of the aqueous tension; Tables XXIV. and XXV. contain the diurnal variation of the same element; and Table XXVI. contains the monthly mean hourly excess of vapour tension in clear weather over foggy weather. Excepting in the middle of summer this difference is always negative, the aqueous tension in the colder part of the year being always higher in foggy than in clear weather.

In foggy weather the vapour tension is that of saturation at the temperature of the air. As the water particles which form the fog permeate the whole of the air any addition of heat has the effect of changing into vapour a portion of the fog, and any loss of heat has the opposite effect, of condensing some of the vapour of the air on the water (or ice) of the fog, both being accompanied by a certain change in temperature. In the clear weather the atmosphere is generally in a state far removed from saturation, and the effect of addition or removal of heat is simply a rise or fall of temperature. In itself the air in clear weather has no means of increasing its supply of water vapour.

It follows from this that in the case of non-saturated air the percentage of aqueous vapour present in it forms a valuable means of identification. If it has no means of receiving or parting with water this percentage must remain the same. If the percentage has changed, then we conclude that the whole air has changed either by mixture or by replacement. Hence the percentage of aqueous vapour affords a ready means of detecting changes of air.

The barometric pressure is the measure of the sum of the tensions of the various gaseous constituents of the atmosphere. If this pressure is diminished all the constituents expand proportionately, and their tensions diminish also proportionately. If the pressure is increased they contract in the same proportion, and their tensions increase proportionately.

The tension of a given quantity of gas varies inversely with the volume which it occupies. From this it follows that, if, in a gaseous mixture, we know the total tension of the mixture, which is given by the barometric pressure, and the tension of any one

of the constituents, then the volume at barometric pressure of this constituent is to the total volume of the mixture as the observed tension of the constituent is to the total tension of the mixture or the barometric pressure. It should be noted that in the analysis of a gaseous mixture the volumes of the constituents are always given in terms of the same temperature and pressure.*

At all meteorological observatories the barometric pressure and the tension of aqueous vapour in the atmosphere are among the chief objects of observation, and they furnish at once the means of ascertaining the exact composition by volume of the air in so far as it consists of permanent gas and aqueous vapour.

In clear weather in winter on Ben Nevis the tension of aqueous vapour in the atmosphere is very low. The mean for January is 0·0892 in., with a mean diurnal range of 0·016 in. In July the mean value is 0·2362 in., with a mean diurnal range of 0·062 in. In January the mean height of the barometer is 25·618, so that the air contains 0·35 per cent. by volume of aqueous vapour, and the range during the day is from 0·316 to 0·379 per cent. In July the mean barometric pressure is 25·628 ins., and the mean vapour tension being 0·236 in., the air contains 0·92 per cent. by volume of aqueous vapour. This ranges during the day from 0·80 to 1·04 per cent. The difference between these figures is considerable, amounting to 24 per cent. of the larger amount, and it indicates a considerable change of air.

If we look at the tables or curves giving the monthly mean hourly values of the vapour tension and of the barometric pressure in clear weather in each month, and if we consider the range through which each of these elements varies in the mean monthly day, we find that these ranges vary in the case of vapour tension from 16 to 30 per cent. of the whole vapour tension, while the barometric pressure does not range through more than a quarter of one per cent.

In continuously clear weather, as we have specified it, the air at a height of 4000 feet has no means of changing its percentage of aqueous vapour except by mixture with air from greater heights, which usually reduces the percentage, or with air from lower levels, which usually increases the percentage. From the character of the vapour tension curves it appears that, on the whole, at night drier air descends from above, while during the day moister air rises from below. The changes in the composition of the air, which we recognise by the accurate analytical method which we have thus in our hands, afford us quantitative proof of the activity of changes of air in the atmosphere, which on other grounds we are forced to conclude must exist, in order that it may preserve unimpaired its life-supporting properties.

The accompanying Table gives a summary of the movements of vapour tension in the year, and combined with the atmospheric pressure the movements in percentage by volume of the amount of aqueous vapour in the atmosphere :—

* Thus, if H be the barometric pressure, and h the observed vapour tension, then the percentage by volume of aqueous vapour in the air is given by the equation $V = 100 \frac{h}{H}$.

Summary.—Tension of Aqueous Vapour in Inches, and Percentage of the same in the Air.

		n.	January.	February.	March.	April.	May.	June.	July.	August.	Sept.	October.	Nov.	Dec.	Mean.
			Inches.	Inches.	Inches.	Inches.	Inches.	Inches.	Inches.	Inches.	Inches.	Inches.	Inches.	Inches.	Inches.
Clear Weather.	Absolute maximum, . .	1	0·217	0·197	0·207	0·272	0·333	0·456	0·374	0·424	0·367	0·289	0·264	0·257	0·305
	Absolute minimum, . .	2	0·020	0·012	0·007	0·021	0·024	0·048	0·087	0·093	0·072	0·038	0·016	0·010	0·037
	Range, . . . (1 – 2)	3	0·197	0·185	0·200	0·251	0·309	0·408	0·287	0·331	0·295	0·251	0·248	0·247	0·268
	Mean, ...	4	0·0892	0·0790	0·0933	0·1230	0·1731	0·2218	0·2365	0·2290	0·1944	0·1395	0·1110	0·0848	0·1479
Foggy Weather.	Absolute maximum, . .	5	0·221	0·223	0·274	0·214	0·265	0·323	0·336	0·348	0·374	0·348	0·265	0·238	0·286
	Absolute minimum, . .	6	0·077	0·030	0·078	0·091	0·113	0·143	0·160	0·171	0·136	0·113	0·091	0·071	0·106
	Range, . . . (5 – 6)	7	0·144	0·193	0·196	0·123	0·152	0·180	0·176	0·177	0·238	0·235	0·174	0·167	0·180
	Mean, ...	8	0·1464	0·1519	0·1471	0·1555	0·1794	0·2074	0·2239	0·2447	0·2190	0·1975	0·1700	0·1478	0·1825
	Difference, . . (4 – 8)	9	– 0·0572	– 0·0729	– 0·0538	– 0·0325	– 0·0063	– 0·0144	+ 0·0126	– 0·0157	– 0·0246	– 0·0580	– 0·0590	– 0·0630	– 0·0346
Clear Weather.	Mean Barometric Pressure,	10	25·618	25·532	25·472	25·536	25·659	25·657	25·628	25·660	25·712	25·670	25·663	25·536	25·612
	Mean % of H_2O (100 × 4/10),	11	0·344	0·309	0·366	0·482	0·674	0·864	0·921	0·892	0·756	0·553	0·433	0·332	0·576
Foggy Weather.	Mean Barometric Pressure,	12	25·048	25·212	24·971	25·183	25·287	25·267	25·214	25·227	25·193	25·120	25·126	25·023	25·156
	Mean % of H_2O (100 × 8/12),	13	0·583	0·600	0·589	0·675	0·710	0·819	0·888	0·970	0·868	0·785	0·675	0·590	0·725
	Difference, . (11 – 13)	14	– 0·239	– 0·291	– 0·223	– 0·133	– 0·036	+ 0·045	+ 0·033	– 0·078	– 0·112	– 0·232	– 0·242	– 0·258	– 0·149

Summary.—Tension of Aqueous Vapour in Millimetres, and Percentage of the same in the Air.

		n.	January.	February.	March.	April.	May.	June.	July.	August.	Sept.	October.	Nov.	Dec.	Mean.
			Mm.	Mm.	Mm.	Mm.	Mm.	Mm.	Mm.	Mm.	Mm.	Mm.	Mm.	Mm.	Mm.
Clear Weather.	Absolute maximum, . .	1	5·51	5·00	5·26	6·91	8·46	11·58	9·50	10·77	9·32	7·34	6·71	6·53	7·74
	Absolute minimum, . .	2	0·51	0·31	0·18	0·53	0·61	1·22	2·21	2·36	1·83	0·97	0·41	0·25	0·95
	Range, . . . (1 – 2)	3	5·00	4·69	5·08	6·38	7·85	10·36	7·29	8·41	7·49	6·37	6·30	6·25	6·79
	Mean, ...	4	2 26	2·01	2·36	3·12	4·39	5·64	5·99	5·82	4·93	3·53	2·82	2·16	3·76
Foggy Weather.	Absolute maximum, . .	5	5·61	5·66	6·96	5·44	6·73	8·20	8·53	8·84	9·50	8·84	6·73	6·05	7·26
	Absolute minimum, . .	6	1·96	0·76	1·98	2·31	2·87	3·63	4·06	4·34	3·45	2·87	2·31	1·80	2·70
	Range, . . . (5 – 6)	7	3·65	4·90	4·98	3·13	3·86	4·57	4·47	4·50	6·05	5·97	4·42	4·25	4·56
	Mean, ...	8	3·71	3·86	3·73	3·94	4·55	5·26	5·69	6·22	5·56	5·03	4·32	3·76	4·65
	Difference, . . (4 – 8)	9	– 1·45	– 1·85	– 1·37	– 0·82	– 0·16	+ 0·38	+ 0·30	– 0·40	– 0·63	– 1·50	– 1·50	– 1·60	– 0·89
Clear Weather.	Mean Barometric Pressure,	10	650·75	648·46	646·94	648·72	651·77	651·77	651·00	651·77	653·04	652·02	651·77	648·72	650·50
	Mean % of H_2O (100 × 4/10),	11	0·344	0·309	0·366	0·482	0·674	0·864	0·921	0·892	0·756	0·553	0·433	0·332	0·576
Foggy Weather.	Mean Barometric Pressure	12	636·47	640·34	634·24	639·57	642·37	641·86	640 34	640·84	639·83	638·05	638·30	635·51	639·07
	Mean % of H_2O (100 × 8/12),	13	0·583	0·600	0·589	0·615	0·710	0 819	0·888	0·970	0·868	0·785	0·675	0·590	0·725
	Difference, . (11 – 13)	14	– 0·239	– 0·291	– 0·223	– 0·133	– 0·036	+ 0·045	+ 0·033	– 0·078	– 0·112	– 0·232	– 0·242	– 0·258	– 0·149

If we consider the variations of vapour tension with reference to the vapour tension itself, and compare the two kinds of weather in this respect, we find a marked difference. Looked at from this point of view, we find some analogy between the foggy weather on Ben Nevis and the rainy season in the Tropics, and between the clear weather and the dry season.

The following figures illustrate this :—

In India the months June, July, August, and September are the chief Monsoon months. The station Poona lies with respect to the west coast of the peninsula and to the Indian Ocean beyond, in a position not unlike that of Ben Nevis to the west coast of Scotland and to the Atlantic. Also, at Poona in the rainy season, and on Ben Nevis during foggy weather, the prevailing wind is from the westerly quarter. On the other hand, Poona is 60 miles inland, and only 2000 feet above the sea, whereas Ben Nevis is on the coast and 4400 feet above the sea. If we compare the vapour tensions in the months June to September at Poona, and in foggy weather on Ben Nevis, and the same elements in the opposite months, December, January, February, and March, the dry season at Poona with June to September on Ben Nevis in clear weather, we have at Poona* in the rainy season for the mean vapour tension 0·708 in., and the mean range 0·037 in., or 5·2 per cent. of the whole vapour tension. On Ben Nevis during foggy weather in the same months the mean vapour tension is 0·224 in., and the range 0·0153 in., or 6·8 per cent. of the whole vapour tension. In the months December to March at Poona the mean vapour tension is 0·465 in., and the range 0·087 in., or 18·7 per cent. of the whole. In the months June to September on Ben Nevis in clear weather the mean vapour tension is 0·220, and the mean range 0·049 in., or 22·2 per cent. of the whole. These figures show that the conditions in the two cases run in parallel lines.

The temperatures are given in Tables XXVII. to XXXIII., and in Plates IV., V., VI., VII., and VIII. Tables XXVII. and XXVIII. contain the absolute maximum and minimum temperatures at every hour during the thirteen years. Tables XXIX. and XXX. contain the monthly mean hourly temperatures in clear and in foggy weather. Tables XXXI. and XXXII. contain the corresponding diurnal variation of temperature, and Table XXXIII. contains the hourly difference between the temperature in clear and in foggy weather.

The mean temperature of the year is 3·57° F. higher in clear weather than in foggy weather. Amongst the monthly values this excess is greatest in June, when it reaches 10·11° F. In the first three months of the year the difference is in the opposite sense. The temperature is then higher in foggy than in clear weather, the excess being 2·92° F. in February. The mean temperature of the months October to March is almost the same for both kinds of weather, being 28·70° for clear and 28·51° for foggy weather. Amongst the hourly values the greatest excess of temperature in clear over that in foggy weather is 13·4° F. at fifteen hours in June, and the greatest difference in the

* *The Meteorology of the Bombay Presidency.* By Charles Chambers, F.R.S., Official Publication of the India Office, p. 148.

opposite sense is 4·3° F. at one hour in March. The range of mean hourly temperature is much greater in clear than in foggy weather in every month.

In this respect again the difference between clear weather and foggy weather on Ben Nevis is the same in kind as that between the dry season and the rainy season in the Tropics.

In foggy weather the mean daily range of temperature is very small in winter, and by no means large in summer. The maximum mean daily range is 2·5° F. in May, when the mean monthly temperature is 31·42° F. The maximum mean monthly temperature, 39·58° F., falls in August, and then the daily range is only 1·7° F. In clear weather the maximum mean monthly temperature, 46·46° F., falls in July, and the maximum mean daily range, 8·2° F., falls in the same month. In clear weather the effect of adding heat is to raise the temperature of the air and to increase its volume. In foggy weather a large portion of it is rendered latent by the evaporation of a portion of the water in a very fine state of subdivision disseminated through it. It may be doubted whether the direct heat of the sun has any part in producing the diurnal range of temperature during continuous foggy weather. At the upper surface of the fog-cloud we would expect the sun's rays to be largely reflected from the dazzlingly white surface which such a cloud always presents when viewed from above in bright sunshine, and that the remainder would be absorbed in evaporating the upper layer of fog, leaving nothing to be transmitted downwards. There is, as a rule, a resultant current of air upwards during the day, and when the cloud, as is often the case, does not spread to a great distance from the mountain side, the lower ground is enjoying sunshine and the upward circulation is very active. The fact that the month of May has the greatest amount of clear weather and also the greatest daily range of temperature in foggy weather seems to support this view. In winter the daily range of temperature is under 1° F.

Mean Hourly March of Temperature in each Month.—This is conveniently treated with respect to the hourly change of temperature during the day and without respect to the actual temperatures at these hours.

If we review the curves which have been drawn for each month we see that they fall into two distinct types. The one type is predominant in the winter months and the other in the summer months. For this purpose the winter months are October to March, and the summer months April to September. The principal feature which distinguishes the winter months from the summer months in clear weather is the occurrence in the former of a pronounced nocturnal heating effect. This shows itself particularly in the curves for November, December, January, and February ; in October and March the passage between the summer and winter types is apparent.

Confining our attention more especially to the four months about midwinter we find the abnormal feature referred to very strongly marked in November. The mean temperature of the twenty-four hours is 32·74°, and it occurs four times, namely, at midnight, at 1.40 A.M., 9 A.M., and 3.50 P.M., the curve showing two maxima at

1 A.M. and 12.30 P.M. respectively, and two minima at 7 A.M. and 6.30 P.M. respectively. The shape of the curve produces the impression that immediately the sun had set another sun rose. It will be observed that the diurnal heating and cooling occupies twelve hours and the nocturnal heating and cooling occupies the same length of time. The area contained between the curve and the base line, forming a tangent to the minimum part of the curve, is proportional to the heat added during the time corresponding to the interval between the initial and final ordinates, and it is given directly in a convenient unit, namely, in *Hour-degrees* (*F.*), which may be indicated by H.° F. Effecting the necessary measurements we find the diurnal heating represented by 16·75 H.°F. and the nocturnal heating by 8·70 H.°F. Thus the nocturnal heating is rather more than half of that produced by the sun during the day. It is, therefore, roughly equivalent to what would be supplied by the sun itself if it rose again after setting and shone for the same number of hours as during the day, eight hours; attaining, however, only one-half of its meridian altitude during the day.

This nocturnal effect persists in the month of December, and, apparently, more intensely relatively to the normal solar heating than in November. At the first glance at the curve it would not be thought so, for there is no prominent rise and fall of the curve during the night, as in November. The fall is then just before sunrise, but the minimum and with it the fall and rise between the diurnal and the nocturnal heating is almost obliterated, so that the curve, after rising in the forenoon to the normal maximum in the early afternoon, hardly falls at all, and merges into the nocturnal heating, forming a very flat curve, nearly parallel to the base line.

We have seen from measurements on the November curve where the two effects are sharply separated that the nocturnal heat supply is rather more than one-half of the diurnal supply. If we calculate from HALLEY'S * well-known formulae, the relative amounts of heat communicated by the sun in a day in the middle of November and in one at the middle of December in the latitude of Ben Nevis, we find that they are in the proportion 356 : 189, or almost exactly in the proportion of the diurnal to the nocturnal heating in November. If the nocturnal heating is the same in December as in November, and as the night is longer it has every chance of being greater, the heat due to the two sources would be equal, and the rise of temperature due to the one would counteract the fall of temperature due to the other, producing a day with an excessively small range of temperature. From the table the range is 1·4° F. The mean temperature in December is 28·45°, or 4·29° below that of November. This is a very small difference, especially when we find, from LAMBERT'S † fundamental calculations of the yearly march of heat under different latitudes, that the

* On the proportional Heat of the Sun in all Latitudes, with the method of collecting the same. By E. Halley, *Phil. Trans.*, 1693, vol. xvii. p. 878. Abridgment iii., p. 576.

† Johann Heinrich Lambert's *Pyrometrie oder vom Maasse des Feuers und der Wärme*, Berlin, 1779, p. 339 and fig. 35.

normal difference due to solar and terrestrial radiations alone would be about 17° F.

Nocturnal heating is observed in foggy weather in most of the months, and it takes the form of a sharp rise of temperature between midnight and 1 A.M., which then falls gradually to a minimum at or near the hour of sunrise; but there are exceptions to this, as in November, and particularly in January, when the temperature rises very uniformly from a minimum at midnight to a maximum at noon, and then falls again as uniformly to the midnight minimum again.

In dealing with the winter months at Ben Nevis, it must not be forgotten that the solar influence is very small. Lying in lat. 56°48′ N. the sun's meridian altitude at the winter solstice is only 9°35′, and the length of the day is under $6\frac{1}{2}$ hours. It is, therefore, chiefly at this season that we might expect terrestrial or geographical influences to produce their most apparent effect. With the march of the season the influence of the sun increases very rapidly, and it has a tendency to obliterate the effects of other agents, especially in the hours of the day when its heating power is increasing or diminishing most rapidly. In the summer months, when the diurnal range of temperature is considerable, it is only in the neighbourhood of the epochs of maximum and of minimum temperature that other influences can make themselves felt. At these times, and especially at the time of maximum temperature, the heating and cooling influences are for a time in a condition approaching equilibrium, during which the temperature remains nearly constant, and its curve runs sensibly parallel to the line of time-abscissæ. Here we might expect other influences to show themselves; and, in fact, if we inspect the curves, especially those relating to clear weather, we see that most of them show great irregularities in the neighbourhood of the extremes. Nearly all the clear weather curves have strongly-marked irregularities near the date of minimum temperature, and most of them, as February, March, August, and October, show similar irregularities near the date of maximum. In foggy weather the hour to hour irregularities of mean temperature are, as might be expected, much less striking, with, however, the exception of the month October, when we have a very remarkable oscillation of temperature from hour to hour during the whole morning. If we were to calculate the mean temperature of the morning from the odd hours, 1, 3, 5 . . . . 11, we should find it quite half a degree lower than if we used the even hours, 2, 4 . . . . 12. One reason for the greater uniformity of the temperature curves in foggy weather than in clear weather is that there are no effects of alternating cloud and sunshine. In clear weather we have these effects, and to some extent they must be held responsible for the irregularities apparent. This, however, applies only to the daylight hours, and we see that the irregularities in clear weather are by no means confined to these hours.

In dealing with the barometric pressure we have found something very similar to what we have just noticed in regard to temperature, namely, that the curves for foggy weather are much smoother and more uniform than those for clear weather, and the clear weather barometric curves present nearly as irregular and serrated an outline as

those of temperature. It naturally occurs to see if there is any connection between the two.

Taking clear weather and the winter months during which the curves, both of temperature and of pressure, show the greatest irregularity, and considering first the mean curves of temperature and pressure for the six winter months, October to March, on Plate VII., we see from 6 A.M. onwards a very considerable agreement in the irregularities of the two curves. The alternations of flatness and steepness in the two curves are very remarkable even during the hours when the sun's increasing influence is most felt. Even where a rise of the barometer is not accompanied by a rise of temperature we see a sharp rise or fall of the barometer, accompanied by an increased or diminished steepness of the temperature curve. The nocturnal rise of temperature between 6 P.M. and midnight, which is so marked a feature in the midwinter months, affects the mean of the whole six winter months, and the nocturnal maximum of pressure is still more pronounced in the six winter months. Turning now to the curves for the individual months, we find in October the parallelism in the irregularities strongly marked. In this month the sun's power is still considerable, and during the morning rise and afternoon fall of temperature the rise of barometer between 5 and 7 P.M. may be put in connection with the flattening of the temperature curve at the same time, when, with the disappearance of the sun, the rate of cooling ought to be the greatest. The nocturnal heating is characteristic of all the winter months, and is sensible even in April and May. A nocturnal maximum of pressure is common to all the months of the year, but it is not accompanied by a temperature maximum in the warmer months. On Ben Nevis, as in most places, the barometer is subject normally to more than one rise and fall during the twenty-four hours, and it is meant here only to point out parallelisms without expressing any opinion as to cause or effect. The parallelisms in the changes of curvature from hour to hour are more particularly apparent in the case of January. In February and March, and in a less degree in other months, in clear weather there is agreement in the irregularities in both curves in the early afternoon when the temperature is about its maximum.

In foggy weather the march of both temperature and pressure is more uniform than in clear weather. The most striking departure from uniformity occurs about midnight and the early morning hours. In the months of March, April, June, July, August, September, October, November, and December there is a sharp rise of temperature between midnight and 1 or 2 A.M. This is greatest in summer, in July the difference in the mean temperature at midnight and at 1 A.M. is 1·3° F. A similar feature is observed in the barometric pressure in the months November, January, February, March, and April. The maximum excess of the pressure at 1 A.M. over that at midnight is 0·034 in. in January. Hence, in foggy weather the most striking feature of parallelism, although common to some months, reaches its maximum values at midsummer in the case of temperature, and at midwinter in the case of pressure.

The Table on page 803 gives a summary for the year of the movements of tempera-

ture. The mean dew-point for each month is given for clear weather, and the excess over it of the mean monthly temperature in foggy weather is given. It is only in June and July that the mean temperature of the air in foggy weather is under the mean dew-point in clear weather. As the air is always completely saturated in foggy weather on Ben Nevis, the temperature of the air is also its dew-point.

Summary.

The paper deals only with observations made at the observatory on the summit of Ben Nevis. The position of the observatory is lat. 56° 48′ N., long. 5° 0′ W., and 4407 feet or 1343 metres above sea-level. Greenwich mean time is used at the observatory. The apparent time is always earlier than that shown by the clock, and the amount varies from three to thirty-five minutes. A table is given at page 785 to facilitate the conversion of G.M.T. into local apparent time. The length of the day varies from six and a half hours at mid-winter to seventeen and a half hours at mid-summer. A table is given at page 786, showing the dates when the length of the day is an exact number of half hours.

The observations are made every hour, and by eye. The period covered by this paper is from 1st January 1885 to 31st December 1897, or thirteen complete years.

The weather on Ben Nevis is characterised by great prevalence of fog or mist. The principal object of this paper is to select dates of continuous foggy weather and of continuous clear weather, and to discuss them separately. A spell of continuous foggy weather means three or more consecutive days during which fog has been logged at every hour. Clear weather is comparatively so rare that the minimum length of spell was taken at twenty-four hours, and the characteristic of the weather is that fog shall not be logged once during the spell. At page 784 a table gives a summary of the material used. It amounts in the aggregate to about one-third of the total available, without distinction of weather. Full details of the method of selection are given at page 782. The data for pressure, temperature, tension of aqueous vapour, rainfall, wind, cloud, and sunshine were sorted out, and fill eighty-four note-books. These will be deposited with the Scottish Meteorological Society.

The information contained in these note-books has been concentrated into the thirty-four Tables which form the main part of this paper, and they are illustrated by the eight Plates of curves at the end. The Tables and the Plates explain themselves. It is sufficient to call attention to one or two salient features.

It must be remembered that, for the purpose of this paper, *clear weather* means only absence of fog, and predicates nothing with regard to presence or absence of cloud overhead.

Rainfall.—In continuous clear weather it practically never rains on the mountain at all. In continuous foggy weather, on the other hand, the average daily rainfall is almost exactly 1 inch. The maximum amount of rain in any one day during the thirteen years fell on the 2nd October 1890, and it amounted to 7·287 in. The greatest rainfall in any one hour occurred between fifteen and sixteen hours on 14th January 1890, when 0·85 in. fell.

Barometric Pressure.—The difference between clear and foggy weather is shown very clearly. There is a large and continuous excess of pressure in clear weather over that in foggy weather. At every hour of the year, without a single exception, the mean monthly pressure is several tenths of an inch higher in clear weather than in foggy weather. The mean excess for the whole year is 0·456 in. (11·6 millim.). At page 791 the results of the barometric observations are summarised in two tables in terms of the inch and the millimetre respectively.

The *Tension of Aqueous Vapour* is an important element of the meteorology of this mountain. In foggy weather the vapour tension is that of saturation at the temperature of the air, and the variations are slight. In clear weather the variations are considerable, but then the air never reaches the point of saturation, consequently it never loses moisture by precipitation. At a height of 4000 feet it has also no opportunity of increasing its supply of moisture, consequently any change in the relative amount of moisture is due to change of air as a whole. Attention is directed to the fact that, in the barometric pressure and the tension of aqueous vapour, we have the data for an exact analysis by volume of the air, in so far as it consists of aqueous vapour and permanent gas. At page 794 the results of the observations of tension of aqueous vapour are summarised in inches and millimetres, and the percentage by volume of the aqueous vapour in the non-saturated air is included.

Temperature.—The mean temperature of the year is 3·57° Fahr. higher in clear weather than in foggy weather. Amongst the monthly values this excess is greatest in June, when it reaches 10·11° Fahr. In the first three months of the year the difference is in the opposite sense. The temperature is then higher in foggy weather than in clear weather, the excess being 2·92° Fahr. in February. The range of mean hourly temperature is much greater in clear than in foggy weather in every month.

The Ben Nevis observations show very clearly the nocturnal heating in the winter months, which has been observed before both on mountains and in balloons. This occurs in both clear and foggy weather, though it is more pronounced in the clear weather.

No distinction is made between one kind of fog and another, and they are not distinguished in the monthly sheets of the observatory. There are, however, several different kinds of fog, and these are clearly distinguished by the observers living on the mountain. There is the very wet fog, which is called *mist* in the log, and there is the comparatively dry fog, which is logged *fog*. Then both the fog and mist in winter seem to be much denser than in summer. These belong to the elements of meteorology

which cannot be expressed in numbers. They are as important as those which can be so expressed; and they can be brought into the discussion of the meteorology of the mountain with their due weight and importance only by men who have spent a considerable time there as observers. Whether wet or dry, the fog which characterises the climate of the mountain is nothing but *cloud* under another name. The lower surface of the clouds which form on the hills rising out of the Western Ocean is found generally at a height of about three thousand feet above the sea. On the west coast of Scotland the air is very damp, and the clouds abundant, consequently the observatory on the summit of Ben Nevis is usually situated in the heart of the common clouds of the country. It may, therefore, be claimed that it is in reality an observatory established in the clouds, and that the observations made in it furnish a record of the meteorology of the clouds. In this respect the observatory of Ben Nevis is unique.

[Summary Tables.

Summary.—Temperature and Dew-Point in Fahrenheit's Degrees.

		n.	Jan.	Feb.	March.	April.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.	Mean.
Clear Weather.			° Fah.	° Fah.	° Fah.	° Fah.	° Fah.	° Fah.	° Fah.	° Fah.	° Fah.	° Fah.	° Fah.	° Fah.	° Fah.
	Absolute maximum,	1	48·3	46·1	41·2	52·5	54·1	66·2	64·0	62·2	59·9	55·5	52·0	41·9	53·7
	Absolute minimum,	2	10·2	7·0	5·7	11·5	17·9	25·0	30·3	32·0	26·4	16·2	13·2	14·3	17·5
	Range, (1 – 2)	3	38·1	39·1	35·5	41·0	36·2	41·2	33·7	30·2	33·5	39·3	38·8	27·6	36·2
	Mean,	4	26·44	24·46	24·12	30·81	38·14	45·34	46·47	45·54	43·48	35·96	32·74	28·46	35·16
	Mean Dew-point,	5	15·20	12·40	16·20	22·75	31·05	37·29	38·95	38·12	33·95	25·77	20·15	14·00	25·49
Foggy Weather.															
	Absolute maximum,	6	37·6	37·3	40·0	37·3	41·8	47·0	48·1	49·0	51·0	49·0	41·8	39·0	43·2
	Absolute minimum,	7	12·9	10·5	13·1	16·7	21·1	26·3	29·0	30·6	25·1	21·0	16·4	11·2	19·5
	Range, (6 – 7)	8	24·7	26·8	26·9	20·6	20·7	20·7	19·1	18·4	25·9	28·0	25·4	27·8	23·7
	Mean,	9	26·61	27·37	26·62	28·05	31·43	35·24	37·24	39·59	36·53	33·71	30·12	26·62	31·59
	Difference, (4 – 9)	10	– 0·17	– 2·71	– 2·50	2·76	+ 6·71	+ 10·10	+ 9·23	+ 5·95	+ 6·95	+ 2·25	+ 2·62	+ 1·84	+ 3·59
	Difference, (5 – 9)	11	– 11·41	– 14·97	– 10·42	– 5·30	– 0·38	+ 2·05	+ 1·71	– 1·47	– 2·58	– 7·94	– 9·97	– 12·62	– 6·11

Summary.—Temperature and Dew-Point in Celsius' Degrees.

		n.	Jan.	Feb.	March.	April.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.	Mean.
Clear Weather.			° C.	° C.	° C.	° C.	° C.	° C.	° C.	° C.	° C.	° C.	° C.	° C.	° C.
	Absolute maximum,	1	9·06	7·83	5·11	11·39	12·28	19·00	17·78	16·78	15·50	13·06	11·11	5·50	12·05
	Absolute minimum,	2	– 12·11	– 13·89	– 14·61	– 11·39	– 7·83	– 3·89	– 0·94	0·00	– 3·11	– 8·78	– 10·44	– 9·83	– 8·05
	Range, (1 – 2)	3	21·17	21·72	19·72	22·78	20·11	22·89	18·72	16·78	18·61	21·84	21·55	15·33	20·10
	Mean,	4	– 3·08	– 4·20	– 4·38	– 0·67	3·41	7·41	8·04	7·53	6·38	2·20	0·96	– 1·97	1·76
	Mean Dew-point,	5	– 9·33	– 10·89	– 8·78	– 5·14	– 0·53	2·94	3·86	3·40	1·08	– 3·46	– 6·58	– 10·00	– 3·62
Foggy Weather.															
	Absolute maximum,	6	3·11	2·94	4·44	2·94	5·44	8·33	8·94	9·44	10·56	9·44	5·44	3·89	6·22
	Absolute minimum,	7	– 10·61	– 11·94	– 10·50	– 8·50	– 6·06	– 3·17	– 1·67	– 0·78	– 3·83	– 6·11	– 8·67	– 11·56	– 6·94
	Range, (6 – 7)	8	13·72	14·88	14·94	11·44	11·50	11·50	10·61	10·22	14·39	15·55	14·11	15·45	13·16
	Mean,	9	– 2·99	– 2·58	– 2·98	– 2·19	– 0·31	1·80	2·91	4·22	2·53	0·95	– 1·05	– 2·99	– 0·22
	Difference, (4 – 9)	10	– 0·09	– 1·62	– 1·40	+ 1·52	+ 3·72	+ 5·61	+ 5·13	+ 3·31	+ 3·85	+ 1·25	+ 2·01	+ 1·02	+ 1·98
	Difference, (5 – 9)	11	– 6·34	– 8·31	– 5·80	– 2·95	– 0·22	+ 1·14	+ 0·95	– 0·82	– 1·45	– 4·41	– 5·53	– 7·01	– 3·40

TABLES I. to XXXIV. and PLATES I. to VIII. are attached to this paper.

BEN NEVIS SUMMIT OBSERVATORY.

Latitude	56° 48′ N.
Longitude	5° 0′ W.
Height above sea	4407 feet.
Do.	1343 metres.
Time used	Greenwich mean time.
Period covered	1st January 1885 to 31st December 1897.

BEN NEVIS SUMMIT.

Month.	1885.				1886.				1887.				1888.					
	from		to		from		to		from		to		from		to		from	
	Day.	Hour.	Day.	Hour.	Day.	Hour.	Day.	Hour.	Day.	Hour.	Day.	Hour.	Day.	Hour.	Day.	Hour.	Day.	Hour.
January,	14	9	17	11	5	15	7	4	...	...	...	...	10	14	12	23	...	...
	19	15	21	17	19	22	21	2	14	17	15	16	13	6	16	15	...	...
	21	22	23	17	...	...	...	...	...	...	...	...	17	5	20	6	...	...
	...	...	...	...	...	...	...	...	...	...	..	...	27	24	28	22	...	...
February,	...	...	...	...	2	23	3	24	7	20	10	7	15	3	16	19	...	...
	15	22	17	10	4	4	5	17	11	19	13	6	24	18	25	21	...	...
	..	...	...	...	15	17	18	10	25	18	26	21	27	11	29	1	...	...
	...	...	...	...	...	...	...	...	27	9	28	19	...	...	...	...	...	...
March,	...	...	...	...	6	22	8	23	...	...	...	...	...	...	...	...	...	...
	12	20	14	12	9	2	10	15	4	13	7	8	17	22	21	10	...	...
	22	15	23	20	10	22	13	3	18	18	19	10	27	13	28	15	26	20
	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
	...	...	...	...	...	...	...	..	...	...	...	...	...	...	...	...	...	...
April,	...	...	...	...	11	14	12	15	7	6	8	22	...	...	...	...	...	...
	7	22	8	23	20	15	23	9	8	24	12	22	23	24	25	5	...	...
	12	18	17	23	26	7	27	10	13	1	14	20	...	...	...	...	...	...
	...	...	...	...	28	4	30	3	...	...	...	...	...	...	...	...	...	...
	...	...	...	...	30	21	...	...	29	12	...	...	...	...	...	...	...	...
May,	...	...	...	...	...	...	2	19	...	...	1	4	...	...	...	...	...	...
	...	...	...	...	6	15	8	2	15	6	17	6	9	12	12	9	12	8
	...	..	...	...	21	17	22	18	...	...	...	...	21	8	26	24	25	17
	..	...	...	...	22	23	23	24	...	...	...	...	...	...	...	...	...	...
	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
	...	...	...	...	31	10	...	...	27	9	...	...	..	...	...	...	...	...
June,	7	14	9	3	...	...	1	10	...	...	4	14	...	...	...	...	...	...
	10	2	11	5	2	16	5	5	...	...	...	...	4	16	6	14	4	1
	24	5	26	2	7	4	8	3	15	20	27	6	13	24	15	2	10	1
	26	4	27	3	17	22	20	22	29	6	30	5	15	14	27	8	16	10
	27	7	28	6	...	...	...	...	...	...	...	...	...	...	...	...	23	15
	...	...	...	...	29	19	...	...	...	...	...	...	30	6	..	...	25	8
July,	1	14	3	2	...	...	1	23	5	24	6	24	...	...	1	22	...	..
	20	1	22	4	...	...	...	...	17	10	18	14	11	22	12	24	...	..
	23	5	24	22	..	...	...	...	...	...	...	...	19	1	20	21	1	23
	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
	...	...	...	...	...	..	...	...	...	...	...	...	...	...	...	...	...	...
	26	2	...	...	...	...	..	..	...	...	...	...	...	...	...	...	...	...
August,	...	...	2	6	...	...	...	...	...	...	...	...	...	..	...	...	...	...
	4	22	6	7	3	10	4	19	3	2	6	6	15	9	19	2	7	23
	17	13	22	15	...	...	...	...	...	...	...	...	...	...	...	...	...	...
	22	22	24	10	...	...	...	...	...	...	...	..	...	...	...	...	...	...
	30	22	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
September,	...	..	...	...	...	...	...	...	...	..	..	...	...	...	...	...	...	...
	...	...	1	20	...	...	...	...	...	...	...	...	...	...	...	...	...	...
	...	...	...	...	2	18	5	5	17	10	13	15	8	4	9	16	...	...
	...	...	...	...	15	12	17	2	19	18	21	7	16	5	17	5	6	1
	...	...	...	...	...	...	...	...	21	20	23	13	18	23	24	4	...	...
	...	...	...	...	...	...	...	...	23	15	24	20	...	...	...	...	...	...
	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
	...	...	...	...	...	...	...	...	...	...	..	...	...	...	...	...	...	...
October,	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	2	18
	16	1	17	2	...	...	...	...	...	...	...	...	18	12	19	22	16	23
	21	1	22	23	...	...	...	...	...	...	...	...	20	13	23	4	27	21
	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
November,	9	14	11	23	...	...	...	...	9	6	10	10	...	...	...	...	...	...
	15	1	19	10	...	...	...	...	13	9	14	19	...	...	...	...	10	18
	21	1	23	7	...	...	...	...	22	16	23	18	...	...	...	...	...	...
	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
	...	...	...	...	...	...	...	...	...	...	...	...	...	...	..	...	...	...
December,	17	13	19	22	13	10	15	14	28	16	29	18	10	6	11	11	...	...
	...	...	...	...	26	12	27	14	...	...	...	...	29	9	30	13	...	...

TABLE I.

Complete List of Dates used in Spells of Continuous Clear Weather.

	1889.				1890.				1891.				1892.				1893.				
to	from		to		from		to		from		to		from		to		from		to		fro
Hour.	Day.	Hour.	Day.	Hour.	Day.	Hour.	Day.	Hour.	Day.	Hour.	Day.	Hour.	Day.	Hour.	Day.	Hour.	Day.	Hour.	Day.	Hour.	Day.
23	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
15	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
6	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
22	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
19	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
21	...	...	...	...	6	15	9	16	18	10	21	9	18	2	19	5	...	...	...	...	13
1	...	...	...	...	23	22	26	6	22	21	24	3	19	11	20	15	...	...	...	...	...
...	...	...	...	...	...	...	...	...	24	6	27	24	...	...	...	...	...	...	...	...	...
...	...	...	...	...	...	...	...	...	...	...	...	...	4	13	6	4	18	9	19	22	...
10	...	...	...	...	1	11	3	5	7	16	9	19	11	10	12	15	21	3	24	4	...
15	26	20	17	24	...	...	...	...	10	15	14	10	14	18	16	13	24	10	25	16	22
...	...	...	...	...	...	...	...	...	...	...	...	...	19	7	20	20	26	8	27	8	...
...	...	...	...	...	...	...	...	...	...	...	...	...	30	19	...	...	27	22	29	21	...
...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	4	15	...	...	...	...	...
5	...	...	...	...	2	7	4	16	7	13	10	7	8	5	12	2	3	21	6	4	18
...	...	...	...	...	11	7	12	21	16	19	20	18	14	4	15	18	8	1	10	13	20
...	...	...	...	...	27	11	28	22	13	1	24	3	28	16	29	16	21	4	25	20	27
...	...	...	...	...	30	14	...	...	24	10	26	3	...	...	...	...	...	...	...	...	...
...	...	...	...	...	...	...	2	16	...	...	...	...	...	...	...	...	...	...	...	...	...
9	12	8	14	3	3	14	4	19	9	9	12	18	3	15	7	3	9	1	10	17	16
24	25	17	28	4	8	10	9	21	...	...	...	...	9	11	12	20	14	4	16	8	21
...	...	...	...	...	22	22	26	5	...	...	...	...	...	...	...	...	...	...	...	...	...
...	...	...	...	...	26	15	28	13	...	...	...	...	...	...	...	...	...	...	...	...	...
...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	31
...	...	...	...	...	...	...	...	...	1	7	2	13	...	...	...	...	...	...	...	...	...
14	4	1	7	3	...	...	...	...	2	17	5	3	5	10	6	10	8	20	9	22	2
2	10	1	11	20	...	...	...	...	5	11	7	10	20	23	22	5	11	10	12	16	5
8	16	10	23	13	...	...	...	...	8	7	12	24	...	...	...	...	12	18	15	24	7
...	23	15	25	1	...	...	...	...	19	16	24	5	...	...	...	...	17	6	19	3	13
...	25	3	26	16	...	...	...	...	...	...	...	...	...	...	...	...	29	24	...	...	28
22	...	...	...	...	...	...	...	...	...	...	...	...	9	22	10	23	...	...	2	3	...
24	...	...	...	...	16	4	18	4	8	22	10	22	12	7	14	9	4	21	7	3	22
21	1	23	6	14	18	24	20	5	13	15	16	6	15	10	16	12	15	10	16	10	27
...	...	...	...	...	...	...	...	...	31	1	31	24	23	4	24	7	26	24	28	3	30
...	...	...	...	...	...	...	...	...	...	..	...	...	24	9	25	22	...	...	...	...	...
...	...	...	...	...	...	...	...	...	...	...	...	...	27	21	30	13	...	...	...	...	...
...	...	...	...	...	..	...	...	...	...	...	...	...	...	...	...	...	8	20	9	19	22
2	7	23	8	21	7	22	9	16	...	...	...	...	9	10	10	10	12	16	14	7	24
...	...	...	...	...	10	13	11	13	...	...	...	...	...	...	...	...	...	...	...	...	28
...	...	...	...	...	..	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
...	...	...	...	...	...	..	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	1
...	...	..	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	8
16	...	...	...	...	6	4	7	20	11	5	13	3	...	...	...	...	3	13	4	23	11
5	6	9	7	16	12	8	13	24	22	1	23	12	20	21	23	4	10	13	11	11	15
4	...	...	...	...	15	10	16	21	...	...	...	...	...	...	...	...	...	...	...	...	17
...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	23
...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	27
...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	29
...	2	14	3	21	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
22	16	22	17	24	7	23	8	21	28	8	29	19	24	15	26	11	...	...	...	...	2
4	27	21	28	24	18	19	19	21	30	7	31	21	...	...	...	...	...	...	...	...	10
...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	30
...	...	...	...	...	...	...	..	...	...	...	...	...	...	...	...	...	8	19	14	8	...
...	10	15	12	2	26	14	28	7	1	13	4	7	22	21	24	19	20	11	21	11	...
...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
11	...	...	...	...	...	...	...	...	16	14	17	14	23	22	28	13	...	...	...	...	1
13	...	...	...	...	...	...	...	...	20	21	22	23	...	...	...	...	...	...	...	...	...

[*To face page* 804.

	1893.				1894.				1895.				1896.				1897.			
to	from		to		from		to		from		to		from		to		from		to	
Hour.	Day.	Hour.	Day.	Hour.	Day.	Hour.	Day.	Hour.	Day.	Hour.	Day.	Hour.	Day.	Hour.	Day.	Hour.	Day.	Hour.	Day.	Hour.
...	...	...	...	...	...	...	...	...	...	...	...	...	4	1	6	6	...	...	...	...
...	...	...	...	...	...	...	...	...	7	22	9	2	22	9	23	8	...	...	...	...
...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
...	...	...	...	...	...	...	...	...	8	3	9	4	...	...	...	...	2	14	3	16
5	...	...	...	...	13	21	14	24	9	6	11	21	23	11	25	2	11	18	12	22
15	...	...	...	...	...	...	...	...	11	23	13	7	...	...	...	...	...	...	...	...
...	...	...	...	...	...	...	...	...	16	13	20	11	...	...	...	...	...	...	...	...
4	18	9	19	22	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
15	21	3	24	4	...	...	...	...	...	...	...	...	11	21	13	5	29	12	30	15
13	24	10	25	16	22	11	31	16	...	...	...	...	29	14	30	14	30	18	31	16
20	26	8	27	8	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
...	27	22	29	21	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
15	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
2	3	21	6	4	18	24	20	2	...	...	...	...	...	...	...	...	21	12	23	18
18	8	1	10	13	20	7	21	23	13	18	19	5	...	...	...	...	25	24	27	12
16	21	4	25	20	27	19	28	20	...	...	...	..	...	...	...	...	...	...	...	...
...	...	...	...	...	...	...	...	...	...	..	...	...	...	...	...	...	...	...	...	...
...	...	...	...	...	...	...	...	...	2	20	5	5	1	22	3	2	...	...	...	...
3	9	1	10	17	16	16	18	18	5	8	9	1	3	10	5	3	12	9	13	15
20	14	4	16	8	21	24	25	2	10	8	12	16	5	24	14	10	15	15	24	16
...	...	...	...	...	...	...	...	...	17	9	18	20	14	24	16	6	...	...	...	...
...	...	...	...	...	...	...	...	...	23	14	24	16	26	10	28	1	...	...	...	...
...	...	...	...	...	31	21	...	...	29	13	31	12	29	19	31	14	31	10	...	...
...	...	...	...	...	...	...	2	4	...	...	...	...	...	...	...	...	...	...	1	20
10	8	20	9	22	2	6	4	3	2	7	3	18	11	11	12	19	3	10	4	20
5	11	10	12	16	5	16	6	16	4	23	7	20	13	2	16	6	27	10	28	14
...	12	18	15	24	7	4	8	20	14	4	16	5	...	..	...	...	...	...	...	...
...	17	6	19	3	13	1	14	8	23	2	26	5	...	...	...	...	...	...	...	...
...	29	24	...	...	28	1	...	...	...	...	...	...	...	...	...	...	...	...	...	...
23	...	...	2	3	...	...	2	3	...	...	...	...	...	...	...	...	...	...	...	...
9	4	21	7	3	22	12	24	20	...	...	...	...	27	23	29	3	...	...	...	...
12	15	10	16	10	27	20	30	3	24	19	25	18	...	...	...	...	10	18	21	16
7	26	24	28	3	30	7	31	22	...	..	...	...	...	...	...	...	...	...	...	...
22	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
13	...	...	...	...	...	...	...	...	...	...	...	...	31	9	...	...	...	...	...	...
...	8	20	9	19	22	3	23	15	...	...	...	...	...	...	1	10	...	...	...	...
10	12	16	14	7	24	15	27	1	...	...	...	...	9	5	10	10	3	8	4	21
...	...	...	...	...	28	23	30	7	...	...	...	...	...	...	...	...	...	...	...	...
...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
...	...	...	...	...	1	15	2	18	...	...	...	...	...	...	...	...	...	...	...	...
...	...	...	...	...	8	23	10	20	...	...	...	...	...	...	...	...	...	...	...	...
...	3	13	4	23	11	20	13	17	8	23	9	24	...	...	...	...	9	1	10	16
4	10	13	11	11	15	16	16	21	22	19	24	7	...	...	...	...	10	18	11	16
...	...	...	...	...	17	6	19	14	...	...	...	...	...	...	...	...	...	...	...	...
...	...	...	...	...	23	2	24	4	...	...	...	...	...	...	...	...	...	...	...	...
...	...	...	...	...	27	12	28	11	...	...	...	...	...	...	...	...	...	...	...	...
...	...	...	...	...	29	18	...	...	28	1	...	...	...	...	...	...	...	...	...	...
...	...	...	...	...	...	...	2	2	...	...	1	19	...	...	...	...	4	4	5	6
11	...	...	...	...	2	4	5	22	16	12	17	23	12	16	14	8	20	5	26	23
...	...	...	...	...	10	20	11	20	...	...	...	...	14	22	17	5	...	...	...	...
...	...	...	...	...	30	4	31	6	31	16	...	...	...	...	...	...	...	...	...	...
...	8	19	14	8	...	...	...	...	...	...	1	24	1	4	2	7	...	...	...	...
19	20	11	21	11	...	...	...	...	24	16	26	13	3	5	4	17	2	1	6	8
...	...	...	...	...	...	...	...	...	...	..	...	...	24	13	25	12	...	...	...	...
...	...	...	...	...	...	...	...	...	...	...	...	...	25	19	27	20	...	...	...	...
...	...	...	...	...	...	...	...	...	...	..	...	...	29	15	...	...	...	...	...	...
13	...	...	...	...	1	17	5	5	...	...	...	...	...	...	1	19	18	12	20	15
...	...	...	...	...	...	...	...	...	...	...	...	...	19	9	20	9	21	4	24	1

TABLE II

Complete List of Days used in Spells of

Month.	1885.	1886.	1887.	1888.	1889.	1890.
January, . . .	6, 7, 8, 9, 10	1, 2, 3, 4 27, 28, 29, 30, 31	25, 26, 27, 28, 29	2, 3, 4, 5	3, 4, 5 15, 16, 17 28, 29, 30, 31	14, 15, 16 19, 20, 21 24, 25, 26
February, . . .		7, 8, 9 27, 28, (1)	19, 20, 21, 22, 23, 24	3, 4, 5, 6, 7, 8, 9		1, 2, 3
March, . . .		17, 18, 19	23, 24, 25	6, 7, 8, 9	5, 6, 7 22, 23, 24, 25	10, 11, 12, 13
April, ...		3, 4, 5, 6, 7, 8, 9	18, 19, 20, 21	27, 28, 29	19, 20, 21	22, 23, 24
May, ...			8, 9, 10	4, 5, 6, 7		
June, ...	18, 19, 20, 21	23, 24, 25, 26	6, 7, 8, 9	8, 9, 10		1, 2, 3, 4 23, 24, 25
July, ...	10, 11, 12. 15, 16, 17	11, 12, 13, 14, 15, 16	22, 23, 24, 25			13, 14, 15 28, 29, 30, 31
August, . . .		24, 25, 26		7, 8, 9 11, 12, 13 20, 21, 22 26, 27, 28, 29	16, 17, 18 26, 27, 28, 29	16, 17, 18 20, 21, 22
September, . . .	9, 10, 11, 12, 13, 14, 15, 16. 20, 21, 22	(29, 30, 31), 1		5, 6, 7	17, 18, 19 27, 28, 29	24, 25, 26, 27, 28, 29
October, . . .	1, 2, 3. 26, 27, 28			24, 25, 26, 27, 28	4, 5, 6, 7, 8 29, 30, 31	3, 4, 5, 6 9, 10, 11 14, 15, 16, 17
November, . . .	1, 2, 3, 4. 30, (1, 2, 3, 4)		16, 17, 18	6, 7, 8. 12, 13, 14. 21, 22, 23, 24		(31), 1, 2 12, 13, 14, 15
December, . . .		21, 22, 23	(30), 1, 2, 3	25, 26, 27	17, 18, 19	(30), 1, 2, 3

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TABLE II.

in Spells of Continuous Foggy Weather.

1890.	1891.	1892.	1893.	1894.	1895.	1896.	1897.
, 15, 16 , 20, 21 , 25, 26	18, 19, 20 23, 24, 25, 26, 27, 28	1, 2, 3 27, 28, 29, 30, 31	18, 19, 20, 21, 22, 23, 24, 25, 26 29, 30, 31	16, 17, 18, 19, 20, 21 24, 25, 26		16, 17, 18	6, 7, 8
, 2, 3		5, 6, 7	2, 3, 4, 5 7, 8, 9, 10, 11	6, 7, 8, 9, 10, 11 26, 27, 28		9, 10, 11	20, 21, 22, 23, 24, 25, 26, 27
11, 12, 13	1, 2, 3, 4, 5 23, 24, 25		4, 5, 6 9, 10, 11, 12	1, 2, 3 8, 9, 10, 11, 12, 13 18, 19, 20	23, 24, 25, 26 28, 29, 30	1, 2, 3 15, 16, 17, 18	10, 11, 12, 13 14, 15 17, 18, 19, 20 21, 22 24, 25, 26, 27
2, 23, 24	2, 3, 4, 5, 6	21, 22, 23, 24	(31), 1, 2 14, 15, 16, 17 28, 29, 30		21, 22, 23	4, 5, 6, 7, 8 16, 17, 18 26, 27, 28	15, 16, 17 28, 29, 30
	26, 27, 28, 29	13, 14, 15	23, 24, 25, 26, 27, 28	6, 7, 8			2, 3, 4, 5
, 2, 3, 4 3, 24, 25	13, 14, 15	25, 26, 27	22, 23, 24, 25			18, 19, 20, 21	14, 15, 16 20, 21, 22
3, 14, 15 29, 30, 31	24, 25, 26	5, 6, 7, 8	19, 20, 21, 22		7, 8, 9, 10, 11		3, 4, 5, 6, 7, 8, 9
6, 17, 18 0, 21, 22	7, 8, 9 24, 25, 26	13, 14, 15	22, 23, 24, 25, 26		27, 28, 29, 30 31, (1)	28, 29, 30	16, 17, 18 28, 29, 30
25, 26, 27, 28, 29	(31), 1, 2 15, 16, 17, 18	(31), 1, 2 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19 24, 25, 26, 27, 28, 29	13, 14, 15, 16, 17, 18, 19, 20		15, 16, 17	15, 16, 17, 18, 19	21, 22, 23, 24, 25, 26
, 4, 5, 6 , 10, 11 15, 16, 17		6, 7, 8 27, 28, 29, 30, 31	11, 12, 13, 14, 15, 16 19, 20, 21, 22 27, 28, 29		5, 6, 7 11, 12, 13	1, 2, 3	8, 9, 10, 11
31), 1, 2 , 13, 14, 15	9, 10, 11	27, 28, 29	1, 2, 3 27, 28, 29	4, 5, 6, 7, 8, 9, 10, 11, 12	5, 6, 7, 8, 9 27, 28, 29, 30, (1, 2, 3, 4)	10, 11, 12, 13, 14, 15 20, 21, 22, 23	11, 12, 13 17, 18, 19, 20, 21, 22, 23 26, 27, 28, 29, 30
0), 1, 2, 3	4, 5, 6 26, 27, 28, 29, 30, 31	18, 19, 20, 21	3, 4, 5, 6, 7, 8, 9 11, 12, 13, 14, 15, 16 20, 21, 22, 23, 24, 25	17, 18, 19 21, 22, 23 28, 29, 30, 31			9, 10, 11

BEN NEVIS SUMMIT. TABLE III.

Total Number of Times each Wind was Logged in each Month of the Thirteen Years during Continuance of Clear Weather.

Month.	N.	N.N.E.	N.E.	E.N.E.	E.	E.S.E.	S.E.	S.S.E.	S.	S.S.W.	S.W.	W.S.W.	W.	W.N.W.	N.W.	N.N.W.	Variable.	Calm.	Total.
January, . .	79	27	31	10	36	24	98	16	121	57	57	9	15	2	3	3	3	15	606
February, .	165	25	43	24	95	55	91	75	152	80	123	42	49	4	22	23	10	80	1158
March, . .	202	29	67	33	70	31	225	135	130	76	130	29	38	11	27	22	6	74	1335
April, . .	255	49	119	43	141	109	265	97	160	51	77	27	58	26	28	22	25	176	1728
May, . . .	318	25	82	43	236	190	321	126	236	42	92	29	106	28	40	38	46	230	2228
June, . . .	428	49	109	56	253	280	357	114	249	73	134	35	141	40	67	51	47	412	2895
July, . . .	311	27	46	24	123	116	202	95	110	37	65	16	58	26	27	24	17	178	1502
August, . .	236	26	35	14	54	43	51	38	124	34	33	11	42	24	42	27	11	126	971
September, .	211	15	15	7	34	36	151	85	242	76	107	43	100	20	47	29	2	76	1296
October, . .	166	18	30	6	63	26	137	55	142	37	69	15	69	8	9	6	11	84	951
November, .	107	27	72	22	126	41	132	93	129	27	33	16	35	3	15	8	5	92	983
December, .	81	12	25	12	36	23	67	66	128	43	68	40	36	24	13	4	1	18	697

BEN NEVIS SUMMIT. TABLE VI.

Total Number of Times each Wind was Logged in each Month of the Thirteen Years during Continuance of Foggy Weather.

Month.	N.	N.N.E.	N.E.	E.N.E.	E.	E.S.E.	S.E.	S.S.E.	S.	S.S.W.	S.W.	W.S.W.	W.	W.N.W.	N.W.	N.N.W.	Variable.	Calm.	Total.
January, .	240	48	122	25	39	57	60	48	138	99	330	162	240	112	239	53	41	11	2064
February, .	188	19	59	23	43	25	50	12	47	48	137	93	187	112	195	30	27	1	1296
March, . .	266	35	69	24	55	65	107	41	63	62	207	115	289	146	258	40	23	7	1872
April, . .	129	15	52	3	47	99	44	18	59	49	127	104	250	123	180	44	62	11	1416
May, . . .	154	14	37	8	12	24	7	9	21	17	42	34	88	55	88	15	15	8	648
June, . . .	196	21	60	22	37	9	33	18	54	23	110	55	146	57	92	28	22	25	1008
July, . . .	165	17	47	7	19	4	14	10	48	34	160	92	192	70	141	23	44	17	1104
August, . .	193	9	12	1	11	23	79	22	69	45	180	110	344	77	118	27	38	34	1392
September, .	286	25	41	14	16	1	22	22	47	53	247	176	469	140	212	54	54	17	1896
October, . .	298	36	86	8	13	7	37	20	51	58	236	104	232	90	163	51	38	8	1536
November, .	257	18	48	7	26	140	124	27	99	89	267	156	318	141	255	86	33	21	2112
December, .	256	51	76	27	16	10	67	28	74	96	170	91	245	89	194	37	40	17	1584

BEN NEVIS SUMMIT. TABLE IV.

Data of Table III., distributed over Eight Points of the Compass.

Month.	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm and Variable.	Total.
January, . . .	94	49	53	118	158	90	20	6	18	606
February, . . .	189	67	135	156	229	184	72	36	90	1158
March, ...	228	98	102	308	235	183	58	43	80	1335
April, ...	291	165	217	368	234	116	84	52	201	1728
May, ...	350	116	352	479	320	128	134	73	276	2228
June, ...	478	161	421	554	343	188	178	113	459	2895
July, ...	337	71	193	308	176	91	79	52	195	1502
August, ...	262	55	83	91	160	56	59	68	137	971
September, . . .	233	26	55	212	322	167	131	72	78	1296
October, ...	178	42	79	178	188	95	80	16	95	951
November, . . .	125	96	158	199	189	54	45	20	97	983
December, . . .	89	37	54	111	183	109	68	27	19	697
Total	2854	983	1902	3082	2737	1461	1008	578	1745	16350

BEN NEVIS SUMMIT. TABLE V.

Data of Table IV. reduced to Percentage.

Month.	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm and Variable.
January, ...	15·49	8·08	8·75	19·47	26·07	14·85	3·30	0·99	2·97
February, . . .	16·31	5·78	11·65	13·46	19·76	15·88	6·21	3·11	7·77
March, ...	17·08	7·34	7·64	23·08	17·56	13·71	4·34	3·22	6·00
April, ...	16·88	9·57	12·58	21·34	13·57	6·73	4·87	3·01	11·66
May, ...	15·71	5·21	15·80	21·50	14·37	5·74	6·01	3·27	12·39
June, ...	16·49	5·56	14·53	19·11	11·84	6·49	6·14	3·90	15·84
July, ...	22·46	4·73	12·86	20·53	11·72	6·06	5·26	3·46	13·00
August, ...	26·99	5·66	8·55	9·37	16·48	5·77	6·08	7·00	14·11
September, . . .	17·98	2·00	4·24	16·36	24·84	12·88	10·11	5·55	6·02
October, ...	18·72	4·41	8·31	18·72	19·77	9·99	8·41	1·68	9·99
November, . . .	12·71	9·76	16·07	20·24	19·22	5·49	4·58	2·04	9·86
December, . . .	12·77	5·31	7·75	15·93	26·26	15·64	9·76	3·87	2·72
Mean	17·45	6·01	11·63	18·85	16·74	8·94	6·17	3·54	10·67

BEN NEVIS SUMMIT.

TABLE VII.

Data of Table VI., distributed over Eight Points of the Compass.

Month.	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm and Variable.	Total.
January, . . .	290	159	80	112	212	460	377	322	52	2064
February, . . .	213	80	67	68	77	208	289	266	28	1296
March, ...	304	98	100	160	114	296	419	351	30	1872
April, ...	159	61	98	102	93	203	364	263	73	1416
May, ...	168	48	28	24	34	67	133	123	23	648
June, ...	221	81	53	46	75	149	202	134	47	1008
July, ...	185	59	24	21	70	223	273	188	61	1104
August, ...	211	17	23	101	103	257	438	170	72	1392
September, . . .	326	60	24	33	85	361	627	309	71	1896
October, ...	341	108	21	50	90	317	329	234	46	1536
November, . . .	309	61	99	208	157	389	467	368	54	2112
December, . . .	300	115	34	86	136	264	335	257	57	1584
Total	3027	947	651	1011	1246	3194	4253	2985	614	17928

BEN NEVIS SUMMIT.

TABLE VIII.

Data of Table VII. reduced to Percentage.

Month.	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm and Variable.
January, ...	14·05	7·70	3·87	5·43	10·27	22·29	18·26	15·60	2·52
February, . . .	16·44	6·17	5·17	5·25	5·94	16·05	22·30	20·53	2·16
March, ...	16·24	5·24	5·34	8·55	6·09	15·81	22·38	18·75	1·60
April, ...	11·23	4·31	6·23	7·20	6·51	14·34	25·71	18·57	5·16
May, ...	25·93	7·41	4·32	3·70	5·21	10·34	20·52	18·98	3·55
June, ...	21·92	8·04	5·26	4·56	7·44	14·78	20·04	13·29	4·66
July, ...	16·76	5·34	2·17	1·90	6·34	20·20	24·73	17·03	5·53
August, ...	15·16	1·22	1·65	7·26	7·40	18·46	31·47	12·21	5·17
September, . . .	17·19	3·16	1·26	1·74	4·48	19·04	33·07	16·30	3·74
October, ...	22·20	7·03	1·37	3·26	5·86	20·64	21·42	15·23	2·99
November, . . .	14·63	2·89	4·69	9·85	7·43	18·42	22·11	17·42	2·56
December, . . .	18·94	7·26	2·14	5·43	8·59	16·67	21·15	16·22	3·60
Mean	16·88	5·28	3·63	5·64	6·95	17·82	23·72	16·65	3·43

BEN NEVIS SUMMIT.

TABLE IX.

Total Monthly Rainfall in each Year during Continuance of Clear Weather.

Year.	January.		February.		March.		April.		May.		June.		July.		August.		September.		October.		November.		December		Year.		
	No. of Hrs.	Total Rain-fall.	No. of Hrs.	Total Rain-fall.	No. of Hrs.	Total Rain-fall.	No. of Hrs.	Total Rain-fall.	No. of Hrs.	Total Rain-fall.	No. of Hrs.	Total Rain-fall.	No. of Hrs.	Total Rain-fall.	No. of Hrs.	Total Rain-fall.	No. of Hrs.	Total Rain-fall	No. of Hrs.	Total Rain-fall.	No. of Hrs.	Total Rain-fall.	No. of Hrs.	Total Rain-fall.	No. of Hrs.	Total Rain-fall.	Rain-fall per 24 Hrs.
		Inch.		Inch.		Inch.		Inch.		Inch.		Inch.		Inch.		Inch.		Inch.		Inch.		Inch.		Inch.		Inch.	Inch.
1885, . .	170	0·001	37	0·004	71	0·000	152	0·000	0	0·000	160	0·031	131	0·000	367	0·000	47	0·000	73	0·000	219	0·000	58	0·000	1485	0·036	0·00058
1886, . .	67	·001	130	·000	142	·000	169	·011	160	·054	159	·002	53	·011	34	·001	99	·000	0	·000	0	000	80	·000	1093	·080	·00161
1887, . .	24	·000	159	·000	95	·000	180	·000	90	·003	497	·298	54	007	77	·032	140	·000	0	·000	91	·074	27	·000	1434	·414	·00693
1888, . .	238	·002	108	·000	112	·000	30	·000	207	·000	398	·014	72	·018	90	·000	188	·000	99	·064	0	·000	59	·000	1601	·098	·00147
1889, . .	0	·000	0	·000	29	·000	0	·000	104	·041	364	·002	112	·010	23	·000	32	·000	87	·000	36	·000	0	·000	787	·053	·00162
1890, . .	0	·000	131	·010	43	·000	133	·000	244	·032	0	·000	79	·123	68	·685	118	·000	50	·000	42	·002	0	·000	908	·852	·02252
1891, . .	0	·000	194	·015	144	·000	232	·037	82	·000	362	·000	137	·000	0	·000	83	·006	74	·000	67	·000	76	·000	1451	·058	·00096
1892, . .	0	·000	57	·000	152	·000	284	·036	167	·056	56	·000	235	·000	25	·000	56	·000	45	·000	47	·000	112	·000	1236	·092	·00179
1893, . .	0	·000	0	·000	216	·003	230	·006	94	·000	183	·774	160	·000	64	·002	58	·005	0	·000	159	·000	0	·000	1164	·790	·01629
1894, . .	0	·000	28	·000	222	·000	94	·000	126	·000	275	·022	153	·000	129	·000	258	·032	200	·000	0	·000	85	·000	1570	·054	·00083
1895, . .	29	·000	218	·004	0	·000	132	·020	316	·112	232	·085	24	·000	0	·000	154	·003	36	·000	79	·000	0	·000	1220	·224	·00440
1896, . .	78	·000	40	·000	58	·000	0	·000	389	·094	110	·000	29	·000	56	·000	0	·000	97	·000	139	·000	78	·000	1074	·094	·00210
1897, . .	0	·000	56	·000	51	·000	92	·000	249	·000	99	·015	263	·000	38	000	63	·000	190	·000	104	·000	122	·000	1327	·015	·00027
Total, . .	606	0·004	1158	0·033	1335	0·003	1728	0·110	2228	0·392	2895	1·243	1502	0·169	971	0·720	1296	0·046	951	0·064	983	0·076	697	0·000	16,350	2·860	
Rainfall per 24 Hours, .	0·00015		0·00069		0·00005		0·00153		0·00422		0·01030		0·00270		0·01777		0·00085		0·00161		0·00185		0·00000		0·00419		

BEN NEVIS SUMMIT.

TABLE X.

Total Monthly Rainfall in each Year during Continuance of Foggy Weather.

YEAR.	JANUARY.		FEBRUARY.		MARCH.		APRIL.		MAY.		JUNE.		JULY.		AUGUST.		SEPTEMBER.		OCTOBER.		NOVEMBER.		DECEMBER.		YEAR.		
	No. of Days.	Total Rainfall.	No. of Days.	Total Rainfall.	No. of Days.	Total Rainfall.	No. of Days.	Total Rainfall.	No. of Days.	Total Rainfall.	No. of Days.	Total Rainfall.	No. of Days.	Total Rainfall.	No. of Days.	Total Rainfall.	No. of Days.	Total Rainfall.	No. of Days.	Total Rainfall.	No. of Days.	Total Rainfall.	No. of Days.	Total Rainfall.	No. of Days.	Total Rainfall.	Mean Daily Rainfall.
		Inches.		Inches.		Inches.		Inches.		Inches.		Inches.		Inches.		Inches.		Inches.		Inches.		Inches.		Inches.		Inches.	Inches.
1885, . .	5	3·605	0	0·000	0	0·000	0	0·000	0	0·000	4	3·121	6	4·223	0	0·000	11	12·565	6	11·669	9	11·362	4	10·216	45	56·761	1·261
1886, . .	9	8·481	6	0·838	3	0·086	7	3·601	0	0·000	4	2·432	6	4·218	3	1·123	4	1·877	0	0·000	0	0·000	3	1·155	45	23·811	0·529
1887, . .	5	7·504	6	7·592	3	1·815	4	2·896	3	1·676	4	3·868	4	4·022	0	0·000	0	0·000	0	0·000	3	1·234	4	7·956	36	38·563	1·071
1888, . .	4	5·902	7	6·431	4	5·186	3	2·693	4	3·963	3	1·186	0	0·000	13	9·175	3	2·261	5	6·635	10	9·972	3	1·610	59	55·014	0·932
1889, . .	10	9·935	0	0·000	7	7·219	3	1·946	0	0·000	0	0·000	0	0·000	7	8·698	6	4·052	8	5·355	0	0·000	3	4·046	44	41·251	0·938
1890, . .	9	11·460	3	3·439	4	13·410	3	4·973	0	0·000	7	6·796	7	6·598	6	5·591	6	13·647	11	22·314	7	6·870	4	2·623	67	97·721	1·459
1891, . .	9	8·711	0	0·000	8	11·249	5	0·300	4	1·267	3	1·875	3	1·119	6	6·192	7	20·636	0	0·000	3	3·629	9	9·516	57	64·494	1·131
1892, . .	8	14·678	3	3·687	0	0·000	4	2·552	3	4·777	3	3·546	4	5·743	3	3·636	20	20·194	8	3·635	3	5·385	4	1·734	63	69·567	1·104
1893, . .	12	10·153	9	6·117	7	5·377	10	5·015	6	1·441	4	2·630	4	4·264	5	4·010	8	11·470	13	12·985	6	13·263	19	21·329	103	98·054	0·952
1894, . .	9	8·037	9	22·109	12	9·930	0	0·000	3	2·016	0	0·000	0	0·000	0	0·000	0	0·000	0	0·000	9	7·594	10	8·563	52	58·249	1·120
1895, . .	0	0·000	0	0·000	7	7·881	3	1·120	0	0·000	0	0·000	5	4·015	6	6·986	3	2·608	6	6·190	13	9·650	0	0·000	43	38·450	0·894
1896, . .	3	4·563	3	4·763	7	6·880	11	6·138	0	0·000	4	3·058	0	0·000	3	4·914	5	3·550	3	0·524	10	6·366	0	0·000	49	40·756	0·832
1897, . .	3	0·000	8	13·611	16	13·569	6	3·548	4	5·415	6	4·067	7	7·250	6	5·527	6	8·786	4	5·131	15	16·335	3	2·301	84	85·540	1·018
Total, . .	86	93·029	54	68·587	78	82·602	59	34·782	27	20·555	42	32·579	46	41·452	58	55·852	79	101·646	64	74·438	88	91·660	66	71·049	747	768·231	
Mean, . .	1·082		1·270		1·059		0·590		0·761		0·776		0·901		0·963		1·286		1·163		1·042		1·076		1·028		
Maximum in one day, .	5·476		6·668		5·210		2·530		2·880		2·108		2·337		3·483		4·930		7·287		4·292		5·340				
Date of maximum,	29/92		6/94		11/90		22/90		15/92		16/97		13/90		29/92		31/91		3/90		28/93		12/85				

TABLE XII.

Diurnal Variations of Monthly Mean Hourly Rainfall during Continuance of Foggy Weather.

Month.	January.	February.	March.	April.	May.	June.	July.	August.	September.	October.	November.	December.	Mean.
Daily Mean of Month.	Inch. 0·045	Inch. 0·053	Inch. 0·044	Inch. 0·025	Inch. 0·032	Inch. 0·034	Inch. 0·038	Inch. 0·040	Inch. 0·053	Inch. 0·048	Inch. 0·044	Inch. 0·045	Inch. 0·042
Difference of Hourly Mean from Daily Mean of Month.													
Hour.													
1	−0·005	−0·002	−0·002	+0·007	−0·001	−0·005	−0·004	+0·010	+0·004	+0·003	+0·008	−0·003	+0·001
2	− ·005	+ ·003	− ·002	+ ·004	− ·004	− ·003	− ·003	+ ·010	0	+ ·004	+ ·006	+ ·001	+ ·001
3	+ ·003	+ ·002	+ ·006	+ ·003	− ·005	0	− ·007	+ ·014	+ ·010	+ ·010	+ ·006	+ ·006	+ ·004
4	+ ·001	+ ·001	− ·001	+ ·001	+ ·007	− ·010	0	+ ·010	+ ·010	+ ·004	+ ·006	− ·003	+ ·002
5	+ ·001	+ ·003	+ ·002	− ·001	+ ·005	+ ·003	− ·007	+ ·002	+ ·006	+ ·018	− ·002	− ·009	+ ·002
6	− ·001	− ·010	− ·002	+ ·007	− ·011	− ·005	− ·001	− ·002	− ·001	+ ·007	+ ·003	− ·014	− ·003
7	− ·004	− ·014	+ ·006	+ ·003	− ·010	− ·004	+ ·004	− ·002	+ ·003	+ ·003	− ·002	− ·008	− ·002
8	+ ·001	− ·018	− ·002	− ·001	− ·010	− ·007	− ·007	− ·001	− ·002	0	+ ·008	− ·005	− ·004
9	+ ·004	− ·011	− ·002	− ·004	− ·009	0	− ·009	− ·004	+ ·006	+ ·004	+ ·001	− ·006	− ·003
10	+ ·015	+ ·004	− ·003	+ ·001	− ·012	+ ·012	− ·003	− ·003	+ ·007	+ ·008	− ·006	+ ·003	− ·002
11	+ ·001	+ ·008	0	0	− ·006	− ·007	− ·002	0	+ ·008	+ ·001	− ·004	+ ·010	0
12	+ ·005	− ·002	− ·004	+ ·007	− ·006	− ·001	− ·002	− ·002	− ·001	− ·004	+ ·002	+ ·014	0
13	− ·001	+ ·006	+ ·005	+ ·001	+ ·005	+ ·010	0	+ ·012	− ·007	+ ·011	0	+ ·015	+ ·004
14	− ·005	− ·004	0	− ·002	− ·005	+ ·009	− ·004	+ ·009	− ·012	− ·005	− ·007	+ ·016	− ·001
15	+ ·004	+ ·005	+ ·006	− ·006	− ·002	+ ·009	− ·003	+ ·008	− ·006	0	− ·009	+ ·003	+ ·001
16	− ·007	+ ·006	+ ·006	− ·006	− ·002	− ·001	0	− ·004	− ·009	− ·002	− ·003	− ·002	− ·002
17	+ ·009	+ ·006	+ ·011	− ·003	+ ·020	− ·002	+ ·017	− ·011	− ·012	− ·004	0	+ ·006	+ ·003
18	− ·003	− ·004	− ·002	+ ·003	+ ·004	+ ·007	+ ·014	− ·004	− ·012	− ·004	− ·008	− ·001	− ·001
19	+ ·002	− ·006	0	− ·003	+ ·013	+ ·003	+ ·002	− ·007	+ ·006	− ·004	− ·009	− ·008	− ·001
20	− ·008	− ·001	− ·007	− ·008	+ ·012	0	+ ·003	− ·002	− ·006	− ·007	− ·007	− ·008	− ·003
21	+ ·002	+ ·006	− ·007	− ·004	+ ·012	− ·001	− ·003	− ·014	+ ·009	− ·012	+ ·008	0	0
22	− ·004	− ·001	− ·004	0	− ·002	− ·003	+ ·002	− ·003	+ ·004	− ·011	0	0	− ·002
23	− ·008	+ ·011	+ ·001	− ·003	+ ·004	− ·001	− ·002	− ·006	0	− ·005	− ·001	− ·005	− ·001
24	− ·005	+ ·006	− ·003	− ·007	− ·006	− ·002	+ ·003	− ·004	+ ·002	− ·004	0	− ·009	− ·002

BEN NEVIS SUMMIT. TABLE X

Monthly Mean Rainfall and Actual Maximum Rainfall observed at every Hour

Hour.	January.		February.		March.		April.		May.		June.		July.	
	Mean.	Max.	Mean.	Max.	Mean.	Max.	Mean.	Max.	Mean.	Max.	Mean.	Max.	Mean.	Max.
	Inch.	Inch.	Inch.	Inch.	Inch.	Inch.	Inch.	Inch.	Inch.	Inch.	Inch.	Inch.	Inch.	Inch.
1	0·040	0·372	0·051	0·461	0·042	0·372	0·032	0·201	0·031	0·190	0·029	0·359	0·034	0·138
2	·040	·352	·056	·471	·042	·374	·029	·150	·028	·175	·031	·333	·035	·173
3	·048	·460	·055	·308	·050	·500	·028	·192	·027	·109	·034	·284	·031	·152
4	·046	·413	·054	·705	·043	·526	·026	·195	·039	·235	·024	·105	·038	·181
5	·046	·444	·056	·254	·046	·365	·024	·195	·037	·165	·037	·200	·031	·123
6	·044	·425	·043	·225	·042	·351	·032	·621	·021	·075	·029	·151	·037	·250
7	·041	·367	·039	·353	·050	·593	·028	·447	·022	·103	·030	·142	·042	·210
8	·046	·419	·035	·200	·042	·245	·024	·142	·022	·115	·027	·183	·031	·116
9	·049	·279	·042	·449	·042	·276	·021	·136	·023	·102	·034	·387	·029	·118
10	·060	·315	·057	·581	·041	·248	·026	·150	·020	·070	·046	·529	·035	·122
11	·046	·377	·061	·812	·044	·270	·025	·133	·026	·094	·027	·150	·036	·238
12	·050	·290	·051	·663	·040	·180	·032	·190	·026	·121	·033	·210	·036	·205
13	·044	·285	·059	·583	·049	·247	·026	·177	·037	·255	·034	·157	·038	·188
14	·040	·400	·049	·273	·044	·253	·023	·141	·027	·164	·044	·291	·034	·140
15	·049	·500	·058	·388	·050	·318	·019	·100	·030	·152	·043	·204	·035	·182
16	·038	·850	·059	·349	·050	·411	·019	·142	·030	·150	·033	·187	·038	·142
17	·054	·830	·059	·669	·055	·330	·022	·160	·052	·525	·032	·223	·055	·390
18	·042	·372	·049	·481	·042	·258	·028	·201	036	·210	·041	·321	·052	·328
19	·047	·382	·047	·341	·044	·303	·022	·130	·045	·298	·037	·209	·040	·190
20	·037	·219	·052	·416	·037	·165	·017	·184	·044	·300	·034	·192	·041	·338
21	·047	·417	·059	·720	·037	·222	·021	·117	·044	·193	·033	·180	·035	·197
22	·041	·200	·052	·329	·040	·430	·025	·134	·030	·230	·031	·180	·040	·310
23	·037	·245	·064	·395	·045	·262	·022	·142	·036	·194	·033	·180	·036	·160
24	·040	·281	·059	·395	·041	·333	·018	·148	·026	·180	·032	·342	·041	·253
Mean	0·045	0·396	0·053	0·451	0·044	0·326	0·025	0·189	0·032	0·184	0·034	0·238	0·038	0·202
No. of Days employed	86		54		78		59		27		42		46	

TABLE XI.

[*To face page* 810.

at every Hour in each Month of Thirteen Years during Continuance of Foggy Weather.

July.		August.		September.		October.		November.		December.		Mean.		No. of Hourly Observations.
Mean.	Max.	Mean.	Max.	Mean.	Max.	Mean.	Max.	Mean.	Max.	Mean.	Max.	Mean.	Max.	
Inch.	Inch.	Inch.	Inch.	Inch.	Inch.	Inch.	Inch.	Inch.	Inch.	Inch.	Inch.	Inch.	Inch.	
0·034	0·138	0·050	0·264	0·057	0·808	0·051	0·368	0·052	0·728	0·042	0·245	0·043	0·376	747
·035	·173	·050	·270	·053	·750	·052	·417	·050	·382	·046	·256	·043	·342	„
·031	·152	·054	·303	·063	·511	·058	·445	·050	·323	·051	·292	·046	·323	„
·038	·181	·050	·291	·063	·369	·052	·298	·050	·440	·042	·325	·044	·340	„
·031	·123	·042	·191	·059	·619	·066	·340	·042	·202	·036	·200	·044	·275	„
·037	·250	·038	·198	·052	·301	·055	·620	·047	·435	·031	·152	·039	·317	„
·042	·210	·038	·215	·056	·302	·051	·533	·042	·164	·037	·234	·040	·305	„
·031	·116	·039	·214	·051	·226	·048	·410	·052	·430	·040	·208	·038	·242	„
·029	·118	·036	·177	·059	·343	·052	·360	·045	·341	·039	·190	·039	·263	„
·035	·122	·037	·215	·060	·375	·056	·480	·038	·330	·048	·437	·044	·321	„
·036	·238	·040	·165	·061	·357	·049	·450	·040	·200	·055	·390	·042	·303	„
·036	·205	·038	·212	·052	·335	·044	·460	·046	·282	·059	·675	·042	·319	„
·038	·188	·052	·324	·046	·320	·059	·404	·044	·321	·060	·622	·046	·324	„
·034	·140	·049	·192	·041	·364	·043	·442	·037	·282	·061	·397	·041	·278	„
·035	·182	·048	·389	·047	·526	·048	·343	·035	·273	·048	·313	·043	·307	„
·038	·142	·036	·199	·044	·211	·046	·326	·041	·413	·043	·275	·040	·305	„
·055	·390	·029	·304	·041	·312	·044	·250	·044	·346	·051	·351	·045	·391	„
·052	·328	·036	·387	·041	·176	·044	·183	·036	·381	·044	·322	·041	·302	„
·040	·190	·033	·282	·059	·403	·044	·238	·035	·279	·037	·184	·041	·270	„
·041	·338	·038	·228	·047	·219	·041	·215	·037	·313	·037	·270	·039	·255	„
·035	·197	·026	·250	·062	·780	·036	·243	·052	·711	·045	·321	·041	·363	„
·040	·310	·037	·275	·057	·282	·037	·232	·044	·342	·045	·293	·040	·270	„
·036	·160	·034	·255	·053	·402	·043	·210	·043	·309	·040	·275	·040	·252	„
·041	·253	·036	·290	·055	·786	·044	·225	·044	·356	·036	·225	·039	·318	„
0·038	0·202	0·040	0·254	0·053	0·420	0·048	0·354	0·044	0·358	0·045	0·311	0·042	0·307	...
46		58		79		64		88		66		...		...

BEN NEVIS SUMMIT. TABLE XI

Actual Maximum and Minimum Barometric Pressure observed at every Hour in

Hour.	January.		February.		March.		April.		May.		June.		July.	
	Max.	Min.	Max.	Min.	Max.	Min.	Max.	Min.	Max.	Min.	Max.	Min.	Max.	Min.
	Inches.	Inches.	Inches.	Inches.	Inches.	Inches.	Inches.	Inches.	Inches.	Inches.	Inches.	Inches.	Inches.	Inche
1	25·997	24·867	26·053	24·779	25·943	24·444	25·935	25·101	26·000	25·172	25·976	25·247	25·907	25·255
2	·990	·909	·040	·686	·940	·447	·931	·093	25·994	·156	·970	·247	·901	·252
3	·987	·935	·020	·689	·929	·455	·931	·085	·990	·136	·966	·247	·901	·250
4	·998	·953	·010	·691	·926	·452	·937	·072	·991	·115	·968	·248	·897	·252
5	·980	·963	·006	·688	·930	·456	·934	·080	·993	·190	·969	·253	·897	·264
6	·976	·983	·012	·697	·931	·458	·944	·077	26·000	·196	·970	·216	·904	·276
7	·975	·999	·011	·697	·928	·466	·952	·081	·004	·216	·979	·232	·907	·280
8	·974	25·015	·009	·702	·933	·473	·959	·090	25·998	·212	·984	·232	·912	·284
9	·985	·047	·016	·695	·940	·465	·970	·090	26·003	·213	·985	·279	·915	·292
10	·992	·064	·030	·709	·942	·471	·980	·094	·008	·222	·988	·179	·920	·291
11	·995	·079	·033	·709	·943	·478	·987	·099	·011	·209	·992	·206	·921	·287
12	·999	·090	·027	·711	·944	·481	·993	·100	·013	·220	·988	·237	·920	·285
13	·994	·100	·029	·709	·946	·320	·997	·108	·012	·260	·988	·262	·927	·292
14	·997	·110	·022	·717	·937	·324	26·003	·094	·014	·248	·986	·260	·929	·294
15	26·003	24·798	·015	·719	936	·332	25·990	·090	·009	·235	·983	·306	·924	·295
16	·005	·874	·009	·726	·926	·341	·989	·084	·003	·223	·979	·268	·925	·296
17	·000	·897	·011	·733	·927	·344	·984	·088	25·993	·129	·972	·321	·909	·299
18	·001	·926	·022	·747	·933	·355	·983	·092	·993	·139	·963	·275	·910	·297
19	·001	·953	·028	·749	·939	·369	·980	·096	·990	·149	·960	·320	·912	·217
20	·004	·969	·032	·761	·940	·391	·979	·098	·994	·153	·965	·310	·908	·224
21	·005	·988	·032	·766	·940	·400	·982	·108	26·000	·159	·971	·229	·908	·231
22	·001	·991	·063	·777	·942	·411	·978	·104	·001	·166	·975	·238	·908	·239
23	·007	·791	·061	·784	·945	·419	·971	·105	·005	·170	·977	·239	·912	·239
24	·002	25·029	·052	·783	·943	·432	·951	·104	·000	·173	·978	·245	·912	·243
Mean	25·994	24·972	26·027	24·726	25·937	24·416	25·968	25·093	26·001	25·186	25·976	25·254	25·912	25·268
No. of Hourly Observations	606		1158		1335		1728		2228		2895		1502	

ABLE XIII.

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ry Hour in each Month of Thirteen Years during Continuance of Clear Weather.

July.		August.		September.		October.		November.		December.		Mean.		No. of Hourly Observations.
Max.	Min.	Max.	Min.	Max.	Min.	Max.	Min.	Max.	Min.	Max.	Min.	Max.	Min.	
nches.	Inches.	Inches.	Inches.	Inches.	Inches.	Inches.	Inches.	Inches.	Inches.	Inches.	Inches.	Inches.	Inches.	
5·907	25·255	25·904	25·355	25·975	25·391	26·026	25·166	26·047	25·023	25·901	24·885	25·972	25·057	689
·901	·252	·904	·334	·971	·376	·020	·155	·038	·020	·897	·882	·966	·046	691
·901	·250	·903	·231	·961	·362	·022	·147	·026	·016	·898	·878	·961	·036	680
·897	·252	·904	·233	·960	·348	·021	24·957	·020	·011	·895	·866	·961	·016	676
·897	·264	·899	·236	·957	·335	·020	·979	·007	·004	·890	·857	·957	·025	671
·904	·276	·907	·254	·958	·313	·012	25·007	·006	·004	·878	·850	·958	·028	664
·907	·280	·907	·267	·960	·304	·013	·031	·003	·003	·876	·851	·960	·036	661
·912	·284	·911	·279	·962	·295	·019	·064	·009	·093	·881	·837	·963	·048	658
·915	·292	·914	·286	·962	·281	·023	·088	·004	·092	·886	·845	·967	·056	664
·920	·291	·918	·299	·960	·258	·027	·110	25·983	·092	·887	·838	·970	·052	682
·921	·287	·921	·309	·957	·223	·035	·130	·993	·092	·887	·836	·973	·055	675
·920	·285	·921	·321	·952	·203	·032	·142	·990	·082	·884	·814	·972	·057	679
·927	·292	·923	·337	·953	·169	·027	·142	26·057	·071	·883	·816	·978	·049	681
·929	·294	·925	·346	·965	·150	·026	·140	·056	·064	·883	·798	·979	·045	690
·924	·295	·922	·354	·959	·125	·018	·141	·052	·059	·894	·915	·975	·031	697
·925	·296	·916	·358	·957	·089	·024	·144	·056	·057	·900	·925	·974	·032	698
·909	·299	·910	·367	·951	·060	·022	·141	·064	·056	·897	·918	·970	·030	683
·910	·297	·912	·379	·958	·016	·026	·153	·067	·057	·901	·921	·972	·030	690
·912	·217	·910	·385	·971	24·979	·032	·155	·068	·055	·909	·925	·975	·029	689
·908	·224	·917	·371	·979	25·389	·037	·165	·062	·055	·914	·919	·978	·067	687
·908	·231	·917	·368	·979	·375	·047	·174	·055	·054	·907	·916	·979	·064	680
·908	·239	·912	·362	·981	·355	·022	·172	·049	·050	·904	·903	·978	·064	688
·912	·239	·910	·358	·971	·338	·028	·161	·054	·042	·898	·906	·978	·046	686
·912	·243	·904	·361	·979	·306	·027	·154	·053	·032	·899	·902	·975	·064	685
25·912	25·268	25·912	25·323	25·964	25·252	26·025	25·117	26·034	25·049	25·894	24·875	25·970	25·044	...
1502		971		1296		951		983		697		...		...

BEN NEVIS SUMMIT. TABLE XI

Actual Maximum and Minimum Barometric Pressure observed at every Hour in e

Hour.	January.		February.		March.		April.		May.		June.		July.	
	Max.	Min.	Max.	Min.	Max.	Min.	Max.	Min.	Max.	Min.	Max.	Min.	Max.	Min.
	Inches.	Inches.	Inches.	Inches.	Inches.	Inches.	Inches.	Inches.	Inches.	Inches.	Inches.	Inches.	Inches.	Inches.
1	25·838	24·067	25·768	24·331	25·640	24·140	25·824	24·197	25·681	24·830	25·755	24·788	25·612	24·734
2	·838	·105	·768	·335	·631	·100	·812	·224	·685	·856	·734	·799	·597	·719
3	·839	·104	·741	·314	·625	·069	·804	·259	·684	·904	·719	·767	·582	·703
4	·833	·062	·741	·290	·609	23·985	·799	·276	·683	·905	·700	·757	·576	·686
5	·822	·034	·743	·275	·614	24·014	·789	·288	·686	·919	·690	·765	·572	·688
6	·817	·105	·743	·270	·631	23·997	·772	·284	·692	·934	·681	·762	·563	·708
7	·806	·091	·752	·341	·640	·982	·782	·303	·698	·940	·674	·772	·580	·719
8	·807	·071	·772	·444	·650	·977	·776	·311	·701	·933	·666	·774	·567	·736
9	·808	·011	·785	·511	·659	·975	·767	·323	·708	·935	·670	·780	·558	·753
10	·815	·146	·789	·476	·663	·966	·772	·340	·719	·922	·643	·788	·563	·792
11	·803	·217	·801	·444	·658	·963	·769	·327	·728	·931	·641	·799	·563	·821
12	·811	·239	·795	·435	·661	·952	·770	·356	·733	·922	·641	·809	·553	·838
13	·816	·255	·806	·397	·651	·945	·763	·376	·735	·916	·633	·822	·558	·867
14	·811	·227	·794	·357	·647	·943	·760	·387	·742	·922	·624	·844	·554	·868
15	·815	·191	·798	·329	·645	·949	·750	·387	·744	·925	·622	·880	·564	·837
16	·808	·177	·803	·290	·640	·954	·743	·408	·744	·922	·620	·901	·569	·823
17	·812	·150	·800	·229	·644	·962	·718	·401	·740	·935	·611	·934	·569	·805
18	·816	·107	·804	·173	·651	·971	·723	·394	·736	·935	·598	·907	·572	·747
19	·829	·057	·807	·100	·654	·981	·723	·393	·734	·947	·583	·881	·569	·762
20	·826	·018	·806	·041	·650	·997	·732	·369	·730	·928	·591	·867	·565	·751
21	·822	·028	·803	·023	·650	24·008	·738	·280	·728	·936	·609	·846	·565	·758
22	·793	·054	·789	23·976	·650	·045	·741	·203	·723	·883	·630	·823	·553	·756
23	·787	·070	·794	·960	·656	·022	·751	·200	·714	·875	·644	·816	·550	·747
24	·773	·096	·784	·869	·642	·033	·750	·188	·702	·865	·654	·792	·545	·740
Mean	25·811	24·112	25·783	24·259	25·644	23·997	25·764	24·311	25·715	24·913	25·651	24·820	25·567	24·765
No. of Days employed	86		54		78		59		27		42		46	

ABLE XIV.

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ery Hour in each Month of Thirteen Years during Continuance of Foggy Weather.

July.		August.		September.		October.		November.		December.		Mean.		No. of Hourly Observations.
Max.	Min.	Max.	Min.	Max.	Min.	Max.	Min.	Max.	Min.	Max.	Min.	Max.	Min.	
nches.	Inches.	Inches.	Inches.	Inches.	Inches.	Inches.	Inches.	Inches.	Inches.	Inches.	Inches.	Inches.	Inches.	
5·612	24·734	25·678	24·420	25·755	24·483	25·749	24·475	25·942	24·300	25·800	24·167	25·753	24·411	747
·597	·719	·667	·377	·752	·454	·731	·462	·937	·301	·740	·028	·741	·397	„
·582	·703	·664	·372	·750	·437	·724	·383	·938	·285	·718	23·866	·732	·372	„
·576	·686	·655	·360	·747	·427	·714	·379	·920	·251	·703	·742	·723	·343	„
·572	·688	·659	·380	·751	·395	·719	·371	·921	·217	·685	·749	·721	·341	„
·563	·708	·668	·400	·754	·380	·723	·421	·917	·240	·685	·789	·720	·357	„
·580	·719	·677	·448	·754	·365	·734	·407	·907	·244	·675	·913	·723	·377	„
·567	·736	·680	·526	·753	·338	·739	·417	·916	·278	·673	24·019	·725	·402	„
·558	·753	·683	·584	·753	·330	·739	·417	·915	·220	·662	23·935	·726	·398	„
·563	·792	·685	·638	·752	·318	·725	·425	·913	·220	·642	·888	·723	·410	„
·563	·821	·686	·686	·748	·304	·732	·434	·910	·212	·649	·921	·724	·422	„
·553	·838	·687	·717	·742	·286	·721	·436	·930	·176	·632	·941	·723	·426	„
·558	·867	·691	·754	·734	·271	·711	·445	·937	·146	·616	24·037	·721	·436	„
·554	·868	·685	·781	·726	·260	·699	·448	·937	·113	·622	·040	·717	·432	„
·564	·837	·678	·767	·712	·304	·702	·457	·930	·099	·617	·076	·715	·433	„
·569	·823	·673	·737	·701	·379	·703	·442	·947	·091	·628	·099	·715	·435	„
·569	·805	·663	·716	·701	·411	·708	·471	·942	·088	·641	·106	·716	·434	„
·572	·747	·666	·678	·689	·435	·713	·457	·942	·085	·649	·143	·713	·419	„
·569	·762	·672	·653	·700	·493	·711	·475	·939	·096	·659	·159	·715	·416	„
·565	·751	·678	·624	·715	·558	·701	·479	·944	·117	·670	·179	·717	·411	„
·565	·758	·686	·605	·729	·553	·709	·470	·950	·137	·673	·188	·722	·403	„
·553	·756	·687	·565	·739	·535	·711	·452	·938	·161	·674	·209	·719	·388	„
·550	·747	·690	·515	·752	·529	·704	·418	·931	·194	·668	·186	·720	·378	„
·545	·740	·684	·474	·761	·513	706	·423	·941	·213	·664	·213	·717	·368	„
25·567	24·765	25·677	24·574	25·736	24·407	25·718	24·436	25·931	24·187	25·668	24·025	25·722	24·400	...
46		58		79		64		88		66		...		...

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TABLE XV.

Monthly Mean Barometric Pressure at every Hour of the Day during Continuance of Clear Weather.

Hour.	January.	February.	March.	April.	May.	June.	July.	August.	September.	October.	November.	December.	Mean.	No. of Hourly Observations.
	Inches.	Inches.	Inches.	Inches.	Inches.	Inches.	Inches.	Inches.	Inches.	Inches.	Inches.	Inches.	Inches.	
1	25·579	25·555	25·482	25·542	25·659	25·651	25·615	25·660	25·711	25·673	25·663	25·544	25·611	689
2	·580	·525	·478	·537	·670	·645	·611	·665	·698	·668	·658	·537	·606	691
3	·610	·516	·477	·532	·647	·641	·605	·648	·691	·657	653	·533	·601	680
4	·609	·506	·471	·520	·647	·637	·608	·645	·693	·644	·642	·537	·597	676
5	·624	·507	·466	·517	·654	·638	·613	·642	·696	·646	·633	·531	·597	671
6	·635	·523	·466	·526	·658	·639	·615	·644	·716	·652	·629	·542	·604	664
7	·624	·510	·470	·528	·662	·643	·618	·649	·713	·670	·633	·541	·605	661
8	·631	·513	·481	·531	·669	·648	·618	·657	·718	·684	·655	·541	·612	658
9	·624	·520	·477	·534	·663	·658	·624	·656	·724	·683	·658	·527	·612	664
10	·630	·530	·477	·547	·663	·655	·627	·652	·722	·687	·658	·516	·614	682
11	·634	·537	·482	·548	·659	·658	·635	·662	·722	·688	·668	·548	·620	675
12	·635	·532	·475	·547	·669	·663	·633	·664	·724	·695	·661	·500	·617	679
13	·629	·536	·455	·548	·675	·665	·639	·659	·725	·694	·666	·533	·619	687
14	·640	·518	·458	·542	·664	·664	·638	·644	·718	·686	·662	·499	·611	690
15	·605	·519	·448	·539	·660	·668	·640	·667	·716	·672	·661	·540	·611	697
16	·600	·530	·470	·539	·650	·664	·638	·671	·711	·666	·658	·529	·611	698
17	·605	·536	·479	·538	·655	·664	·636	·669	·703	·667	·660	·536	·612	683
18	·634	·544	·461	·538	·651	·671	·639	·667	·702	·675	·662	·536	·815	690
19	·640	·537	·465	·535	·647	·663	·638	·666	706	·678	·681	·535	·616	689
20	·646	·568	·483	·538	·657	·667	·638	·672	·726	·672	·691	·553	·626	687
21	·649	·547	·482	·539	·661	·672	·641	·673	·719	·664	·692	·559	·625	680
22	·590	·568	·489	·538	·667	·669	·635	·676	·718	·656	·691	·552	·621	688
23	·590	·531	·451	·537	·664	·667	·635	·669	·714	·653	·688	·557	·613	686
24	·579	·550	·486	·535	·657	·661	·624	·666	·713	·658	·687	·548	·614	685
Mean	25·618	25·532	25·472	25·536	25·659	25·657	25·628	25·660	25·712	25·670	25·663	25·536	25·612	16,350
No. of Hourly Observations	606	1158	1335	1728	2228	2895	1502	971	1296	951	983	697	...	...

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TABLE XVI.

Monthly Mean Barometric Pressure at every Hour of the Day during Continuance of Foggy Weather.

Hour.	January.	February.	March.	April.	May.	June.	July.	August.	September.	October.	November.	December.	Mean.	No. of Hourly Observations.
	Inches.	Inches.	Inches.	Inches.	Inches.	Inches.	Inches.	Inches.	Inches.	Inches.	Inches.	Inches.	Inches.	
1	5·075	5·216	5·000	5·201	5·283	5·258	5·214	5·222	5·203	5·124	5·150	5·026	25·164	747
2	·066	·212	4·987	·193	·276	·252	·208	·215	·198	·116	·142	·021	·157	,,
3	·059	·208	·976	·187	·273	·249	·201	·211	·190	·109	·138	·014	·151	,,
4	·050	·205	·965	·181	·268	·246	·201	·206	·184	·103	·132	·006	·146	,,
5	·041	·208	·963	·180	·266	·248	·198	·206	·182	·103	·128	·001	·144	,,
6	·038	·203	·973	·183	·269	·249	·201	·207	·184	·106	·129	·001	·145	,,
7	·040	·209	·963	·185	·274	·252	·204	·211	·186	·108	·126	·001	·147	,,
8	·043	·213	·966	·188	·278	·257	·209	·216	·189	·114	·132	·005	·151	,,
9	·046	·217	·963	·191	·281	·262	·211	·220	·191	·116	·133	·014	·154	,,
10	·049	·216	·965	·196	·285	·261	·216	·226	·192	·120	·136	·017	·157	,,
11	·054	·225	·969	·195	·288	·265	·219	·228	·195	·122	·134	·022	·160	,,
12	·052	·222	·970	·199	·290	·270	·221	·231	·197	·123	·127	·004	·159	,,
13	·047	·222	·970	·189	·295	·272	·223	·235	·200	·123	·122	·016	·160	,,
14	·044	·222	·968	·196	·296	·274	·224	·238	·195	·123	·116	·015	·159	,,
15	·042	·220	·964	·181	·296	·275	·222	·238	·192	·123	·114	·021	·157	,,
16	·041	·215	·976	·188	·293	·279	·220	·239	·189	·124	·115	·026	·159	,,
17	·042	·217	·968	·170	·297	·281	·217	·237	·186	·125	·117	·030	·157	,,
18	·044	·217	·971	·184	·297	·281	·215	·236	·187	·128	·119	·035	·160	,,
19	·043	·216	·974	·168	·299	·281	·217	·239	·190	·129	·120	·042	·160	,,
20	·046	·209	·972	·183	·297	·281	·218	·241	·195	·132	·121	·048	·162	,,
21	·047	·206	·972	·163	·301	·283	·219	·243	·197	·131	·119	·050	·161	,,
22	·047	·202	·972	·176	·298	·282	·219	·240	·199	·131	·117	·052	·161	,,
23	·046	·199	·971	·155	·296	·281	·217	·239	·201	·127	·115	·048	·158	,,
24	·041	·191	·970	·168	·288	·277	·215	·232	·199	·123	·114	·044	·155	,,
Mean	25·048	25·212	24·971	25·183	25·287	25·267	25·214	25·227	25·193	25·120	25·126	25·023	25·156	
No. of Days employed	86	54	78	59	27	42	46	58	79	64	88	66		

BEN NEVIS SUMMIT.

TABLE XVII.

Diurnal Variation of Monthly Mean Hourly Pressure during Continuance of Clear Weather.

Month.	January.	February.	March.	April.	May.	June.	July.	August.	September.	October.	November.	December.	Mean.
Daily Mean of Month.	Inch. 25·618	Inch. 25·532	Inch. 25·472	Inch. 25·536	Inch. 25·659	Inch. 25·657	Inch. 25·628	Inch. 25·660	Inch. 25·712	Inch. 25·670	Inch. 25·663	Inch. 25·536	Inch. 25·612
Difference of Hourly Mean from Daily Mean of Month.													
Hour.													
1	−0·039	+0·023	+0·010	+0·006	0	−0·006	−0·013	0	−0·001	+0·003	0	+0·008	−0·001
2	− ·038	− ·007	+ ·006	+ ·001	+ ·011	− ·012	− ·017	+ ·005	− ·014	− ·002	− ·005	+ ·001	− ·006
3	− ·008	− ·016	+ ·005	− ·004	− ·012	− ·016	− ·023	− ·012	− ·021	− ·013	− ·010	− ·003	− ·011
4	− ·009	− ·026	− ·001	− ·016	− ·012	− ·020	− ·020	− ·015	− ·019	− ·026	− ·021	+ ·001	− ·015
5	+ ·006	− ·025	− ·006	− ·019	− ·005	− ·019	− ·015	− ·018	− ·016	− ·024	− ·030	− ·005	− ·015
6	+ ·017	− ·009	− ·006	− ·010	− ·001	− ·018	− ·013	− ·016	+ ·004	− ·018	− ·034	+ ·006	− ·008
7	+ ·006	− ·022	− ·002	− ·008	+ ·003	− ·014	− ·010	− ·011	+ ·001	0	− ·030	+ ·005	− ·007
8	+ ·013	− ·019	+ ·009	− ·005	+ ·010	− ·009	− ·010	− ·003	+ ·006	+ ·014	− ·008	+ ·005	0
9	+ ·006	− ·012	+ ·005	− ·002	+ ·004	+ ·001	− ·004	− ·004	+ ·012	+ ·013	− ·005	− ·009	0
10	+ ·012	− ·002	+ ·005	+ ·011	+ ·004	− ·002	− ·001	− ·008	+ ·010	+ ·017	− ·005	− ·020	+ ·002
11	+ ·016	+ ·005	+ ·010	+ ·012	0	+ ·001	+ ·007	+ ·002	+ ·010	+ ·018	+ ·005	+ ·012	+ ·008
12	+ ·017	0	+ ·003	+ ·011	+ ·010	+ ·006	+ ·005	+ ·004	+ ·012	+ ·025	− ·002	− ·036	+ ·005
13	+ ·011	+ ·004	− ·017	+ ·012	+ ·016	+ ·008	+ ·011	− ·001	+ ·013	+ ·024	+ ·003	− ·003	+ ·007
14	+ ·022	− ·014	− ·014	+ ·006	+ ·005	+ ·007	+ ·010	− ·016	+ ·006	+ ·016	− ·001	− ·037	− ·001
15	− ·013	− ·013	− ·024	+ ·003	+ ·001	+ ·011	+ ·012	+ ·007	+ ·004	+ ·002	− ·002	+ ·004	− ·001
16	− ·018	− ·002	− ·002	+ ·003	− ·009	+ ·007	+ ·010	+ ·011	− ·001	− ·004	− ·005	− ·007	− ·001
17	− ·013	+ ·004	+ ·007	+ ·002	− ·004	+ ·007	+ ·008	+ ·009	− ·009	− ·003	− ·003	0	0
18	+ ·016	+ ·012	− ·011	+ ·002	− ·008	+ ·014	+ ·011	+ ·007	− ·010	+ ·005	− ·001	0	+ ·003
19	+ ·022	+ ·005	− ·007	− ·001	− ·012	+ ·006	+ ·010	+ ·006	− ·006	+ ·008	+ ·018	− ·001	+ ·004
20	+ ·028	+ ·036	+ ·011	+ ·002	− ·002	+ ·010	+ ·010	+ ·012	+ ·014	+ ·002	+ ·028	+ ·017	+ ·014
21	+ ·031	+ ·015	+ ·010	+ ·003	+ ·002	+ ·015	+ ·013	+ ·013	+ ·007	− ·006	+ ·029	+ ·023	+ ·013
22	− ·028	+ ·036	+ ·017	+ ·002	+ ·008	+ ·012	+ ·007	+ ·016	+ ·006	− ·014	+ ·028	+ ·016	+ ·009
23	− ·028	− ·001	− ·021	+ ·001	+ ·005	+ ·010	+ ·007	+ ·009	+ ·002	− ·017	+ ·025	+ ·021	+ ·001
24	− ·039	+ ·018	+ ·014	− ·001	− ·002	+ ·004	− ·004	+ ·006	+ ·001	− ·012	+ ·024	+ ·012	+ ·002

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TABLE XVIII.

Diurnal Variation of Monthly Mean Hourly Barometric Pressure during Continuance of Foggy Weather.

Month.	January.	February.	March.	April.	May.	June.	July.	August.	September.	October.	November.	December.	Mean.
Daily Mean of Month.	Inch. 25·048	Inch. 25·212	Inch. 24·971	Inch. 25·183	Inch. 25·287	Inch. 25·267	Inch. 25·214	Inch. 25·227	Inch. 25·193	Inch. 25·120	Inch. 25·126	Inch. 25·023	Inch. 25·156
Difference of Hourly Mean from Daily Mean of Month.													
Hour.													
1	+0·027	+0·004	+0·029	+0·018	−0·004	−0·009	0	−0·005	+0·010	+0·004	+0·024	+0·003	+0·008
2	+ ·018	0	+ ·016	+ ·010	− ·010	− ·015	− ·006	− ·012	+ ·005	− ·004	+ ·016	− ·002	+ ·001
3	+ ·011	− ·004	+ ·005	+ ·004	− ·014	− ·018	− ·013	− ·016	− ·003	− ·011	+ ·012	− ·009	− ·005
4	+ ·002	− ·007	− ·006	− ·002	− ·019	− ·021	− ·013	− ·021	− ·009	− ·017	+ ·006	− ·017	− ·010
5	− ·007	− ·004	− ·008	− ·003	− ·021	− ·019	− ·016	− ·021	− ·011	− ·017	+ ·002	− ·022	− ·012
6	− ·010	− ·009	+ ·002	0	− ·018	− ·018	− ·013	− ·020	− ·009	− ·014	+ ·003	− ·022	− ·011
7	− ·008	− ·003	− ·008	+ ·002	− ·013	− ·015	− ·010	− ·016	− ·007	− ·012	0	− ·022	− ·009
8	− ·005	+ ·001	− ·005	+ ·005	− ·009	− ·010	− ·005	− ·011	− ·004	− ·006	+ ·006	− ·015	− ·005
9	− ·002	+ ·005	− ·008	+ ·008	− ·006	− ·005	− ·003	− ·007	− ·002	− ·004	+ ·007	− ·009	− ·002
10	+ ·001	+ ·004	− ·006	+ ·013	− ·002	− ·006	+ ·002	− ·001	− ·001	0	+ ·010	− ·006	+ ·001
11	+ ·006	+ ·013	− ·002	+ ·012	+ ·001	− ·002	+ ·005	+ ·001	+ ·002	+ ·002	+ ·008	− ·001	+ ·004
12	+ ·004	+ ·010	− ·001	+ ·016	+ ·003	+ ·003	+ ·007	+ ·004	+ ·004	+ ·003	+ ·001	− ·019	+ ·003
13	− ·001	+ ·010	− ·001	+ ·006	+ ·008	+ ·005	+ ·009	+ ·008	+ ·007	+ ·003	− ·004	− ·007	+ ·004
14	− ·004	+ ·010	− ·003	+ ·013	+ ·009	+ ·007	+ ·010	+ ·011	+ ·002	+ ·003	− ·010	− ·008	+ ·003
15	− ·006	+ ·008	− ·007	− ·002	+ ·009	+ ·008	+ ·008	+ ·011	− ·001	+ ·003	− ·012	− ·002	+ ·001
16	− ·007	+ ·003	− ·005	+ ·005	+ ·006	+ ·012	+ ·006	+ ·012	− ·004	+ ·004	− ·011	+ ·003	+ ·003
17	− ·006	+ ·005	− ·003	− ·013	+ ·010	+ ·014	+ ·003	+ ·010	− ·007	+ ·005	− ·009	+ ·007	+ ·001
18	− ·004	+ ·005	0	+ ·001	+ ·010	+ ·014	+ ·001	+ ·009	− ·006	+ ·008	− ·007	+ ·012	+ ·004
19	− ·005	+ ·004	+ ·003	− ·015	+ ·012	+ ·014	+ ·003	+ ·012	− ·003	+ ·009	− ·006	+ ·019	+ ·004
20	− ·002	− ·003	+ ·001	0	+ ·010	+ ·014	+ ·004	+ ·014	+ ·002	+ ·012	− ·005	+ ·025	+ ·004
21	− ·001	− ·006	+ ·001	− ·020	+ ·014	+ ·016	+ ·005	+ ·016	+ ·004	+ ·011	− ·007	+ ·027	+ ·005
22	− ·001	− ·010	+ ·001	− ·007	+ ·011	+ ·015	+ ·005	+ ·013	+ ·006	+ ·011	− ·009	+ ·029	+ ·005
23	− ·002	− ·013	0	− ·028	+ ·009	+ ·014	+ ·003	+ ·012	+ ·008	+ ·007	− ·011	+ ·025	+ ·002
24	− ·007	− ·021	− ·001	− ·015	+ ·001	+ ·010	+ ·001	+ ·005	+ ·006	+ ·003	− ·012	+ ·021	− ·001

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TABLE X

Actual Maximum and Minimum Tension of Aqueous Vapour observed at every Hou

Hour.	January.		February.		March.		April.		May.		June.		July.	
	Max.	Min.	Max.	Min.	Max.	Min.	Max.	Min.	Max.	Min.	Max.	Min.	Max.	Min.
	Inch.	Inch.	Inch.	Inch.	Inch.	Inch.	Inch.	Inch.	Inch.	Inch.	Inch.	Inch.	Inch.	Inch.
1	0·193	0·023	0·174	0·014	0·177	0·013	0·236	0·021	0·267	0·058	0·375	0·048	0·343	0·089
2	·198	·024	·169	·019	·177	·013	·247	·026	·258	·046	·373	·059	·344	·088
3	·196	·023	·193	·020	·172	·012	·247	·026	·254	·050	·377	·055	·338	·087
4	·189	·022	·189	·019	·181	·012	·248	·029	·250	·024	·357	·060	·343	·086
5	·188	·020	·188	·020	·191	·015	·246	·028	·272	·042	·357	·071	·328	·089
6	·205	·021	·193	·021	·167	·014	·240	·027	·285	·043	·351	·062	·343	·089
7	·200	·022	·181	·020	·168	·015	·218	·029	·302	·046	·358	·078	·349	·090
8	·212	·021	·177	·017	·197	·014	·245	·036	·306	·044	·362	·083	·331	·091
9	·199	·023	·149	·026	·172	·014	·250	·033	·325	·047	·366	·086	·339	·113
10	·214	·024	·171	·028	·191	·017	·243	·046	·333	·066	·366	·091	·340	·111
11	·217	·020	·166	·020	·191	·018	·249	·045	·329	·085	·382	·086	·360	·114
12	·214	·023	·179	·024	·201	·019	·261	·067	·329	·095	·396	·121	·355	·114
13	·213	·023	·155	·010	·197	·013	·264	·064	·306	·094	·386	·118	·339	·130
14	·193	·021	·158	·013	·196	·014	·268	·056	·298	·099	·386	·112	·333	·162
15	·195	·026	·156	·017	·205	·016	·271	·062	·288	·086	·393	·143	·344	·159
16	·186	·024	·159	·017	·207	·012	·247	·049	·286	·091	·389	·147	·370	·174
17	·184	·025	·146	·019	·180	·011	·250	·057	·286	·086	·393	·135	·374	·179
18	·184	·022	·149	·021	·179	·010	·257	·053	·302	·090	·412	·130	·373	·181
19	·196	·021	·160	·023	·187	·008	·272	·042	·285	·081	·421	·115	·371	·152
20	·186	·020	·174	·014	·189	·007	·263	·061	·285	·069	·456	·110	·371	·119
21	·214	·023	·197	·014	·184	·008	·260	·059	·269	·031	·410	·080	·369	·129
22	·211	·026	·191	·013	·177	·010	·252	·027	·269	·044	·403	·068	·370	·124
23	·204	·033	·174	·012	·176	·013	·247	·028	·256	·029	·399	·061	·379	·120
24	·199	·028	·174	·013	·175	·011	·251	·037	·265	·062	·379	·060	·374	·106
Mean	0·1996	0·0233	0·1718	0·0181	0·1849	0·0129	0·2513	0·0420	0·2877	0·0628	0·3853	0·0908	0·3533	0·120
No. of Hourly Observations	606		1158		1335		1728		2228		2895		1502	

ABLE XX.

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t every Hour in each Month of Thirteen Years during Continuance of Clear Weather.

July.		August.		September.		October.		November.		December.		Mean.		No. of Hourly Observations.
Max.	Min.	Max.	Min.	Max.	Min.	Max.	Min.	Max.	Min.	Max.	Min.	Max.	Min.	
Inch.	Inch.	Inch.	Inch.	Inch.	Inch.	Inch.	Inch.	Inch.	Inch.	Inch.	Inch.	Inch.	Inch.	
·343	0·089	0·301	0·093	0·355	0·086	0·288	0·049	0·262	0·019	0·252	0·018	0·2678	0·0443	689
·344	·088	·308	·101	·356	·078	·285	·066	·261	·018	·248	·021	·2687	·0466	691
·338	·087	·303	·096	·356	·075	·253	·057	·253	·017	·256	·018	·2665	·0447	680
·343	·086	·324	·100	·335	·084	·245	·052	·250	·030	·245	·017	·2630	·0446	676
·328	·089	·361	·114	·339	·074	·274	·050	·212	·033	·244	·014	·2667	·0475	671
·343	·089	·369	·119	·333	·072	·275	·055	·194	·021	·242	·012	·2664	·0463	664
·349	·090	·377	·120	·334	·074	·266	·057	·197	·016	·228	·012	·2648	·0483	661
·331	·091	·345	·102	·339	·075	·289	·071	·183	·018	·220	·011	·2672	·0486	658
·339	·113	·356	·101	·329	·081	·280	·038	·184	·017	·224	·012	·2644	·0493	664
·340	·111	·313	·097	·319	·082	·284	·050	·181	·020	·220	·014	·2646	·0538	682
·360	·114	·324	·121	·335	·078	·228	·063	·202	·020	·221	·015	·2670	·0571	675
·355	·114	·348	·127	·335	·078	·233	·079	·219	·026	·219	·016	·2741	·0658	679
·339	·130	·379	·128	·336	·077	·280	·065	·263	·022	·220	·014	·2782	·0632	681
·333	·162	·388	·140	·357	·078	·285	·078	·264	·030	·228	·012	·2795	·0679	690
·344	·159	·378	·113	·335	·076	·244	·085	·247	·016	·229	·010	·2738	·0674	697
·370	·174	·375	·167	·339	·075	·256	·077	·252	·017	·214	·010	·2733	·0717	698
·374	·179	·400	·158	·367	·077	·236	·073	·252	·033	·233	·013	·2751	·0722	683
·373	·181	·410	·116	·361	·081	·227	·076	·241	·034	·258	·014	·2794	·0690	690
·371	·152	·404	·124	·349	·090	·231	·082	·225	·038	·229	·017	·2775	·0661	689
·371	·119	·410	·106	·345	·083	·225	·081	·224	·027	·231	·024	·2799	·0601	687
·369	·129	·424	·098	·335	·082	·239	·071	·225	·022	·237	·023	·2803	·0533	680
·370	·124	·388	·100	·334	·081	·267	·073	·237	·025	·255	·023	·2795	·0512	688
·379	·120	·374	·101	·345	·078	·287	·081	·226	·023	·230	·025	·2748	·0503	686
·374	·106	·356	·102	·356	·081	·276	·061	·208	·025	·257	·023	·2725	·0508	685
0·3533	0·1207	0·3631	0·1143	0·3427	0·0790	0·2605	0·0663	0·2276	0·0236	0·2350	0·0162	0·2719	0·0558	...
1502		971		1296		951		983		697		...		...

BEN NEVIS SUMMIT. TABLE

Actual Maximum and Minimum Tension of Aqueous Vapour observed at every Hour

Hour.	January.		February.		March.		April.		May.		June.		July.	
	Max.	Min.	Max.	Min.	Max.	Min.	Max.	Min.	Max.	Min.	Max.	Min.	Max.	Min.
	Inch.	Inch.	Inch.	Inch.	Inch.	Inch.	Inch.	Inch.	Inch.	Inch.	Inch.	Inch.	Inch.	Inch.
1	0·219	0·083	0·220	0·030	0·211	0·078	0·208	0·093	0·247	0·117	0·280	0·156	0·327	0·162
2	·220	·078	·222	·030	·206	·079	·208	·093	·246	·113	·280	·154	·325	·167
3	·218	·077	·222	·030	·216	·080	·210	·093	·244	·114	·277	·153	·323	·174
4	·215	·079	·222	·031	·215	·082	·208	·094	·241	·117	·274	·154	·318	·179
5	·214	·083	·220	·040	·220	·082	·213	·093	·240	·118	·267	·154	·309	·173
6	·217	·084	·204	·073	·220	·085	205	·091	·241	·120	·275	·152	·311	·174
7	·218	·085	·204	·072	·214	·082	·209	·093	·241	·119	·279	·153	·308	·172
8	·218	·083	·207	·073	·211	·082	·205	·093	·245	·121	·280	·149	·307	·171
9	·219	·082	·214	·075	·213	·086	·199	·093	·247	·121	·293	·147	·319	·170
10	·221	·082	·218	·076	·218	·090	·209	·095	·255	·126	·297	·151	·321	·173
11	·222	·083	·211	·078	·220	·093	·211	·093	·256	·132	·306	·156	·321	·172
12	·222	·086	·214	·078	·222	·092	·214	·094	·264	·135	·297	·161	·325	·176
13	·220	·088	·218	·075	·227	·090	·213	·095	·261	·136	·295	·167	·329	·174
14	·220	·085	·218	·075	·230	·089	·213	·097	·265	·137	·299	·164	·329	·173
15	·219	·085	·212	·074	·238	·090	·210	·095	·265	·142	·301	·160	·333	·174
16	·221	·082	·214	·074	·247	·090	·211	·094	·265	·136	·311	·164	·335	·174
17	·220	·082	·212	·072	·238	·090	·212	·097	·261	·128	·321	·162	·335	·171
18	·218	·086	·212	·072	·236	·090	·212	·097	·261	·127	·323	·158	·336	·168
19	·217	·084	·210	·072	·222	·090	·208	·097	·257	·125	·317	·156	·336	·167
20	·218	·085	·208	·072	·219	·090	·208	·099	·252	·120	·306	·147	·334	·161
21	·217	·082	·205	·069	·218	·090	·209	·100	·250	·114	·294	·147	·323	·160
22	·214	·083	·212	·069	·213	·090	·209	·099	·255	·116	·279	·146	·323	·160
23	·218	·082	·218	·069	·217	·090	·204	·097	·250	·116	·288	·143	·321	·161
24	·214	·082	·223	·070	·274	·090	·204	·094	·251	·115	·288	·155	·324	·160
Mean	0·2183	0·0830	0·2142	0·0645	0·2235	0·0871	0·2088	0·0950	0·2525	0·1235	0·2928	0·1545	0·3238	0·169
No. of Days employed	86		54		78		59		27		42		46	

TABLE XXI.

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at every Hour in each Month in Thirteen Years during Continuance of Foggy Weather.

July.		August.		September.		October.		November.		December.		Mean.		No. of Hourly Observations.
Max.	Min.	Max.	Min.	Max.	Min.	Max.	Min.	Max.	Min.	Max.	Min.	Max.	Min.	
Inch.	Inch.	Inch.	Inch.	Inch.	Inch.	Inch.	Inch.	Inch.	Inch.	Inch.	Inch.	Inch.	Inch.	
0·327	0·162	0·311	0·171	0·362	0·144	0·325	0·123	0·239	0·116	0·233	0·076	0·2652	0·1124	747
·325	·167	·308	.174	·361	·141	·334	·124	·239	·117	·236	·078	·2654	·1123	„
·323	·174	·313	·177	·357	·136	·336	·123	·245	·116	·231	·079	·2660	·1127	„
·318	·179	·317	·179	·352	·140	·338	·124	·246	·115	·230	·078	·2647	·1143	„
·309	·173	·302	·180	·360	·142	·340	·120	·247	·117	·228	·074	·2633	·1147	„
·311	·174	·309	·181	·352	·139	·336	·118	·259	·114	·227	·079	·2630	·1175	„
·308	·172	·313	·182	·327	·143	·324	·120	·244	·114	·230	·078	·2593	·1178	„
·307	·171	·318	·193	·319	·143	·324	·120	·265	·116	·232	·078	·2609	·1185	„
·319	·170	·311	·190	·325	·146	·318	·125	·263	·119	·229	·079	·2625	·1194	„
·321	·173	·311	·195	·330	·149	·335	·126	·257	·117	·231	·082	·2669	·1218	„
·321	·172	·316	·194	·335	·153	·336	·127	·250	·116	·236	·083	·2683	·1233	„
·325	·176	·321	·201	·348	·153	·340	·126	·255	·118	·234	·083	·2713	·1253	„
·329	·174	·324	·192	·356	·156	·346	·125	·258	·118	·231	·081	·2732	·1248	„
·329	·173	·343	·201	·351	·157	·348	·128	·255	·113	·231	·081	·2752	·1250	„
·333	·174	·348	·201	·344	·152	·346	·127	·253	·110	·230	·080	·2749	·1242	„
·335	·174	·339	·193	·348	·151	·346	·123	·248	·113	·231	·079	·2763	·1228	„
·335	·171	·331	·194	·352	·148	·343	·122	·244	·098	·231	·075	·2750	·1199	„
·336	·168	·318	·190	·358	·149	·328	·122	·244	·098	·230	·074	·2730	·1193	„
·336	·167	·306	·185	·367	·142	·336	·123	·240	·103	·231	·071	·2706	·1179	„
·334	·161	·306	·183	·374	·147	·329	·122	·244	·100	·234	·072	·2693	·1165	„
·323	·160	·301	·183	·370	·148	·335	·118	·230	·091	·238	·071	·2658	·1144	„
·323	·160	·301	·182	·351	·152	·335	·118	·237	·096	·236	·072	·2638	·1153	„
·321	·161	·307	·179	·351	·152	·321	·116	·238	·096	·234	·074	·2639	·1146	„
·324	·160	·290	·179	·362	·152	·322	·113	·238	·096	·231	·073	·2684	·1141	„
0·3238	0·1694	0·3152	0·1866	0·3509	0·1473	0·3342	0·1222	0·2474	0·1095	0·2319	0·0771	0·2678	0·1183	
46		58		79		64		88		66		–		–

BEN NEVIS SUMMIT.

TABLE XIX.

Monthly Mean Hourly Excess of Barometric Pressure in Clear Weather over that in Foggy Weather.

Hour.	January.	February.	March.	April.	May.	June.	July.	August.	September.	October.	November.	December.	Mean.
	Inch.	Inch.	Inch.	Inch.	Inch.	Inch.	Inch.	Inch.	Inch.	Inch.	Inch.	Inch.	Inch.
1	0·504	0·339	0·482	0·341	0·376	0·393	0·401	0·438	0·508	0·549	0·513	0·518	0·447
2	·514	·313	·491	·344	·394	·393	·403	·450	·500	·552	·516	·516	·449
3	·551	·308	·501	·345	·374	·392	·404	·437	·501	·548	·515	·519	·450
4	·559	·301	·506	·339	·379	·391	·407	·439	·509	·541	·510	·531	·451
5	·583	·299	·503	·337	·388	·390	·415	·436	·514	·543	·505	·530	·453
6	·597	·320	·493	·343	·389	·390	·414	·437	·532	·546	·500	·541	·459
7	·584	·301	·507	·343	·388	·391	·414	·438	·527	·562	·507	·540	·458
8	·588	·300	·515	·343	·391	·391	·409	·441	·529	·570	·523	·536	·461
9	·578	·303	·514	·343	·382	·396	·413	·436	·533	·567	·525	·513	·458
10	·581	·314	·512	·351	·378	·394	·411	·426	·530	·567	·522	·499	·457
11	·580	·312	·513	·353	·371	·393	·416	·434	·527	·566	·534	·526	·460
12	·583	·310	·505	·348	·379	·393	·412	·433	·527	·572	·534	·496	·458
13	·582	·314	·485	·359	·380	·393	·416	·424	·525	·571	·544	·517	·459
14	·596	·296	·490	·346	·368	·390	·414	406	·523	·563	·546	484	·452
15	·563	·299	·484	·358	·364	·393	·418	·429	·524	·549	·547	·519	·454
16	·559	·315	·494	·351	·357	·385	·418	·432	·522	·542	·543	·503	·452
17	·563	·319	·511	·368	·358	·383	·419	·432	·517	·542	·543	·506	·455
18	·590	·327	·490	·354	·354	·390	·424	·431	·515	·547	·543	·501	·455
19	·597	·321	·491	·367	·348	·382	·421	·427	·516	·549	·561	·493	·456
20	·600	·357	·511	·355	·360	·386	·420	·431	·531	·540	·570	·505	·464
21	·602	·341	·510	·376	·360	·389	·421	·430	·522	·533	·573	·509	·464
22	·543	366	·517	·362	·369	·387	·416	·436	·519	·525	·574	·500	·460
23	·544	·334	·480	·382	·368	·386	·418	·430	·513	·526	·573	·509	·455
24	·538	·359	·516	·367	·369	·384	·409	·434	·514	·535	·573	·504	·459
Mean	0·570	0·320	0·501	0·353	0·372	0·390	0·414	0·433	0·519	0·550	0·537	0·513	0·456

BEN NEVIS SUMMIT.

TABLE XXII.

Monthly Mean Tension of Aqueous Vapour at every Hour of the Day during Continuance of Clear Weather.

Hour.	January.	February.	March.	April.	May.	June.	July.	August.	September.	October.	November.	December.	Mean.	No. of Hourly Observations.
	Inch.	Inch.	Inch.	Inch.	Inch.	Inch.	Inch.	Inch.	Inch.	Inch.	Inch.	Inch.	Inch.	
1	0·093	0·081	0·089	0·112	0·159	0·207	0·226	0·215	0·191	0·135	0·108	0·080	0·1413	689
2	·097	·076	·089	·113	·155	·204	·218	·219	·187	·134	·110	·081	·1402	691
3	·095	·078	·084	·113	·151	·203	·212	·216	·187	·129	·106	·081	·1379	680
4	·089	·076	·083	·111	·150	·199	·208	·212	·182	·130	·105	·081	·1355	676
5	·088	·075	·083	·113	·157	·199	·205	·208	·183	·132	·107	·081	·1359	671
6	·093	·076	·079	·108	·157	·199	·206	·208	·175	·128	·109	·081	·1349	664
7	·087	·071	·083	·111	·160	·198	·209	·212	·177	·126	·111	·079	·1353	661
8	·081	·071	·087	·116	·166	·206	·210	·210	·176	·127	·107	·078	·1363	658
9	·086	·074	·088	·116	·170	·211	·222	·220	·184	·127	·106	·084	·1407	664
10	·086	·081	·095	·123	·176	·221	·226	·219	·189	·137	·111	·084	·1457	682
11	·089	·084	·097	·127	·184	·229	·235	·226	·193	·142	·115	·089	·1508	675
12	·089	·081	·099	·134	·187	·235	·247	·240	·201	·149	·110	·091	·1552	679
13	·087	·080	·106	·133	·189	·241	·254	·251	·209	·156	·123	·094	·1603	687
14	·088	·078	·106	·134	·192	·242	·255	·254	·211	·156	·122	·090	·1607	690
15	·088	·080	·106	·136	·192	·245	·260	·253	·211	·154	·118	·091	·1612	697
16	·088	·082	·107	·136	·195	·247	·267	·253	·213	·150	·121	·086	·1621	698
17	·088	·079	·106	·135	·191	·245	·266	·251	·207	·147	·112	·088	·1596	683
18	·086	·076	·096	·134	·190	·243	·265	·248	·211	·144	·112	·083	·1573	690
19	·088	·082	·094	·133	·187	·237	·261	·244	·205	·142	·111	·083	·1556	689
20	·087	·078	·095	·130	·179	·233	·251	·237	·197	·144	·110	·086	·1522	687
21	·094	·083	·094	·125	·172	·225	·253	·233	·191	·144	·108	·091	·1511	680
22	·090	·086	·091	·123	·168	·221	·250	·230	·187	·139	·109	·084	·1482	688
23	·092	·084	·090	·121	·165	·219	·238	·222	·201	·141	·108	·083	·1470	686
24	·091	·084	·093	·115	·163	·214	·231	·216	·198	·136	·106	·086	·1444	685
Mean	0·0892	0·0790	0·0933	0·1230	0·1731	0·2218	0·2365	0·2290	0·1944	0·1395	0·1110	0·0848	0·1479	16,350
No. of Hourly Observations	606	1158	1335	1728	2228	2895	1502	971	1296	951	983	697	...	...

BEN NEVIS SUMMIT.

TABLE XXIII.

Monthly Mean Tension of Aqueous Vapour at every Hour of the Day during Continuance of Foggy Weather.

Hour.	January.	February.	March.	April.	May.	June.	July.	August.	September.	October.	November.	December.	Mean.	No. of Hourly Observations.
	Inch.	Inch.	Inch.	Inch.	Inch.	Inch.	Inch.	Inch.	Inch.	Inch.	Inch.	Inch.	Inch.	
1	0·133	0·149	0·146	0·154	0·175	0·208	0·228	0·246	0·212	0·197	0·170	0·149	0·1806	747
2	·144	·148	·147	·153	·175	·206	·227	·246	·214	·196	·170	·150	·1813	,,
3	·145	·148	·147	·153	·174	·203	·227	·244	·219	·196	·171	·150	·1814	,,
4	·146	·147	·145	·152	·174	·201	·226	·243	·218	·197	·171	·149	·1808	,,
5	·147	·148	·145	·151	·173	·202	·219	·239	·218	·197	·171	·148	·1798	,,
6	·148	·148	·144	·152	·173	·200	·223	·238	·217	·196	·171	·147	·1798	,,
7	·148	·148	·143	·152	·174	·200	·222	·239	·217	·196	·171	·147	·1798	,,
8	·149	·148	·144	·153	·175	·202	·218	·241	·218	·197	·171	·148	·1803	,,
9	·149	·150	·146	·156	·177	·206	·225	·243	·219	·198	·170	·147	·1822	,,
10	·149	·153	·148	·157	·181	·207	·222	·245	·221	·199	·171	·149	·1835	,,
11	·150	·153	·150	·159	·184	·209	·228	·248	·223	·201	·172	·150	·1856	,,
12	·150	·156	·151	·159	·186	·213	·228	·250	·224	·202	·172	·150	·1867	,,
13	·150	·157	·151	·162	·187	·215	·229	·252	·225	·202	·172	·145	·1873	,,
14	·150	·157	·152	·162	·190	·212	·231	·254	·225	·201	·171	·149	·1878	,,
15	·151	·156	·152	·161	·189	·215	·232	·253	·225	·199	·170	·149	·1877	,,
16	·148	·156	·149	·160	·187	·215	·231	·251	·224	·198	·169	·148	·1863	,,
17	·147	·153	·147	·158	·185	·214	·229	·241	·221	·197	·168	·147	·1839	,,
18	·147	·153	·147	·157	·183	·213	·221	·247	·219	·197	·169	·147	·1833	,,
19	·146	·153	·147	·156	·181	·211	·222	·244	·218	·196	·169	·146	·1824	,,
20	·147	·153	·146	·155	·179	·209	·221	·243	·218	·196	·169	·146	·1818	,,
21	·145	·153	·146	·154	·177	·206	·218	·243	·217	·195	·168	·146	·1807	,,
22	·146	·153	·147	·153	·176	·205	·216	·242	·216	·195	·168	·147	·1803	,,
23	·145	·153	·144	·152	·176	·204	·216	·240	·215	·196	·168	·146	·1796	,,
24	·133	·153	·146	·151	·175	·202	·215	·240	·214	·195	·168	·146	·1782	,,
Mean	0·1464	0·1519	0·1471	0·1555	0·1794	0·2074	0·2239	0·2447	0·2190	0·1975	0·1700	0·1478	0·1825	...
No. of Days employed	86	54	78	59	27	42	46	58	79	64	88	66	...	...

BEN NEVIS SUMMIT.

TABLE XXIV.

Diurnal Variation of Monthly Mean Hourly Tension of Aqueous Vapour during Continuance of Clear Weather.

Month.	January.	February.	March.	April.	May.	June.	July.	August.	September.	October.	November.	December.	Mean.
Daily Mean of Month.	Inch. 0·0892	Inch. 0·0790	Inch. 0·0933	Inch. 0·1230	Inch. 0·1731	Inch. 0·2218	Inch. 0·2365	Inch. 0·2290	Inch. 0·1944	Inch. 0·1395	Inch. 0·1110	Inch. 0·0848	Inch. 0·1479
Difference of Hourly Mean from Daily Mean of Month.													
Hour.													
1	+0·0038	+0·0020	−0·0043	−0·0110	−0·0141	−0·0148	−0·0105	−0·0140	−0·0034	−0·0045	−0·0030	−0·0048	−0·0063
2	+ ·0078	− ·0030	− ·0043	− ·0100	− ·0181	− ·0178	− ·0185	− ·0100	− ·0074	− ·0055	− ·0010	− ·0038	− ·0077
3	+ ·0058	− ·0010	− ·0093	− ·0100	− ·0221	− ·0188	− ·0245	− ·0130	− ·0074	− ·0105	− ·0050	− ·0038	− ·0100
4	− ·0002	− ·0030	− ·0103	− ·0120	− ·0231	− ·0228	− ·0285	− ·0170	− ·0124	− ·0095	− ·0060	− ·0038	− ·0124
5	− ·0012	− ·0040	− ·0103	− ·0100	− ·0161	− ·0228	− ·0315	− ·0210	− ·0114	− ·0075	− ·0040	− ·0038	− ·0120
6	+ ·0038	− ·0030	− ·0143	− ·0150	− ·0161	− ·0228	− ·0305	− ·0210	− ·0194	− ·0015	− ·0020	− ·0038	− ·0130
7	− ·0022	− ·0080	− ·0103	− ·0120	− ·0131	− ·0238	− ·0275	− ·0170	− ·0174	− ·0135	0	− ·0058	− ·0126
8	− ·0082	− ·0080	− ·0063	− ·0070	− ·0071	− ·0158	− ·0265	− ·0190	− ·0184	− ·0125	− ·0040	− ·0068	− ·0116
9	− ·0032	− ·0050	− ·0053	− ·0070	− ·0031	− ·0108	− ·0145	− ·0090	− ·0104	− ·0125	− ·0050	− ·0008	− ·0072
10	− ·0032	+ ·0020	+ ·0017	0	+ ·0029	− ·0008	− ·0105	− ·0100	− ·0054	− ·0025	0	− ·0008	− ·0022
11	− ·0002	+ ·0050	+ ·0037	+ ·0040	+ ·0109	+ ·0072	− ·0015	− ·0030	− ·0014	+ ·0025	+ ·0040	+ ·0042	+ ·0029
12	− ·0002	+ ·0020	+ ·0057	+ ·0110	+ ·0139	+ ·0132	+ ·0105	+ ·0110	+ ·0066	+ ·0095	− ·0010	+ ·0062	+ ·0073
13	− ·0022	+ ·0010	+ ·0127	+ ·0100	+ ·0159	+ ·0192	+ ·0175	+ ·0220	+ ·0146	+ ·0165	+ ·0120	+ ·0092	+ ·0124
14	− ·0012	− ·0010	+ ·0127	+ ·0110	+ ·0189	+ ·0202	+ ·0185	+ ·0250	+ ·0166	+ ·0165	+ ·0110	+ ·0052	+ ·0128
15	− ·0012	+ ·0010	+ ·0127	+ ·0130	+ ·0189	+ ·0232	+ ·0235	+ ·0240	+ ·0166	+ ·0145	+ ·0070	+ ·0062	+ ·0133
16	− ·0012	+ ·0030	+ ·0137	+ ·0130	+ ·0219	+ ·0252	+ ·0305	+ ·0240	+ ·0186	+ ·0105	+ ·0100	+ ·0012	+ ·0142
17	− ·0012	0	+ ·0127	+ ·0120	+ ·0179	+ ·0232	+ ·0295	+ ·0220	+ ·0126	+ ·0075	+ ·0010	+ ·0032	+ ·0117
18	− ·0032	− ·0030	+ ·0027	+ ·0110	+ ·0169	+ ·0212	+ ·0285	+ ·0190	+ ·0166	+ ·0045	+ ·0010	− ·0018	+ ·0094
19	− ·0012	+ ·0030	+ ·0007	+ ·0100	+ ·0139	+ ·0152	+ ·0245	+ ·0150	+ ·0106	+ ·0025	0	− ·0018	+ ·0077
20	− ·0022	− ·0010	+ ·0017	+ ·0070	+ ·0059	+ ·0112	+ ·0145	+ ·0080	+ ·0026	+ ·0045	− ·0010	+ ·0012	+ ·0043
21	+ ·0048	+ ·0040	+ ·0007	+ ·0020	− ·0011	+ ·0032	+ ·0165	+ ·0040	− ·0034	+ ·0045	− ·0030	+ ·0062	+ ·0032
22	+ ·0008	+ ·0070	− ·0023	0	− ·0051	− ·0008	+ ·0135	+ ·0010	− ·0074	− ·0005	− ·0020	− ·0008	+ ·0003
23	+ ·0028	+ ·0050	− ·0033	− ·0020	− ·0081	− ·0028	+ ·0015	− ·0070	+ ·0066	+ ·0015	− ·0030	− ·0018	− ·0009
24	+ ·0018	+ ·0050	− ·0003	+ ·0080	− ·0101	− ·0078	− ·0055	− ·0130	+ ·0036	− ·0035	− ·0050	+ ·0012	− ·0035

TABLE XXV.

Diurnal Variations of Monthly Mean Hourly Tension of Aqueous Vapour during Continuance of Foggy Weather.

Month.	January.	February.	March.	April.	May.	June.	July.	August.	September.	October.	November.	December.	Mean.
Daily Mean of Month.	Inch. 0·1464	Inch. 0·1519	Inch. 0·1471	Inch. 0·1555	Inch. 0·1794	Inch. 0·2074	Inch. 0·2239	Inch. 0·2447	Inch. 0·2190	Inch. 0·1975	Inch. 0·1700	Inch. 0·1478	Inch. 0·1825
Difference of Hourly Mean from Daily Mean of Month.													
Hour.													
1	−0·0134	−0·0029	−0·0011	−0·0015	−0·0044	+0·0006	+0·0041	+0·0013	−0·0070	−0·0005	0	+0·0012	−0·0019
2	− ·0024	− ·0039	− ·0001	− ·0025	− ·0044	− ·0014	+ ·0031	+ ·0013	− ·0050	− ·0015	0	+ ·0022	− ·0012
3	− ·0014	− ·0039	− ·0001	− ·0025	− ·0054	− ·0044	+ ·0031	− ·0007	0	− ·0015	+ ·0010	+ ·0022	− ·0011
4	− ·0004	− ·0049	− ·0021	− ·0035	− ·0054	− ·0064	+ ·0021	− ·0017	− ·0010	− ·0005	+ ·0010	+ ·0012	− ·0017
5	+ ·0006	− ·0039	− ·0021	− ·0045	− ·0064	− ·0054	− ·0049	− ·0057	− ·0010	− ·0005	+ ·0010	+ ·0002	− ·0027
6	+ ·0016	− ·0039	− ·0031	− ·0035	− ·0064	− ·0074	− ·0009	− ·0067	− ·0020	− ·0015	+ ·0010	− ·0008	− ·0027
7	+ ·0016	− ·0039	− ·0041	− ·0035	− ·0054	− ·0074	− ·0019	− ·0057	− ·0020	− ·0015	+ ·0010	− ·0008	− ·0027
8	+ ·0026	− ·0039	− ·0031	− ·0025	− ·0044	− ·0054	− ·0059	− ·0037	− ·0010	− ·0005	+ ·0010	+ ·0002	− ·0022
9	+ ·0026	− ·0019	− ·0011	+ ·0005	− ·0024	− ·0014	+ ·0011	− ·0017	0	+ ·0005	0	− ·0008	− ·0003
10	+ ·0026	+ ·0011	+ ·0009	+ ·0015	+ ·0016	− ·0004	− ·0019	+ ·0003	+ ·0020	+ ·0015	+ ·0010	+ ·0012	+ ·0010
11	+ ·0036	+ ·0011	+ ·0029	+ ·0035	+ ·0046	+ ·0016	+ ·0041	+ ·0033	+ ·0040	+ ·0035	+ ·0020	+ ·0022	+ ·0031
12	+ ·0036	+ ·0041	+ ·0039	+ ·0035	+ ·0066	+ ·0056	+ ·0041	+ ·0053	+ ·0050	+ ·0045	+ ·0020	+ ·0022	+ ·0042
13	+ ·0036	+ ·0051	+ ·0039	+ ·0065	+ ·0076	+ ·0076	+ ·0051	+ ·0073	+ ·0060	+ ·0045	+ ·0020	− ·0028	+ ·0048
14	+ ·0036	+ ·0051	+ ·0049	+ ·0065	+ ·0106	+ ·0046	+ ·0071	+ ·0093	+ ·0060	+ ·0035	+ ·0010	+ ·0012	+ ·0053
15	+ ·0046	+ ·0041	+ ·0049	+ ·0055	+ ·0096	+ ·0076	+ ·0081	+ ·0083	+ ·0060	+ ·0015	0	+ ·0012	+ ·0052
16	+ ·0016	+ ·0041	+ ·0019	+ ·0045	+ ·0076	+ ·0076	+ ·0071	+ ·0063	+ ·0050	+ ·0005	− ·0010	+ ·0002	+ ·0038
17	+ ·0006	+ ·0011	− ·0001	+ ·0025	+ ·0056	+ ·0066	+ ·0051	− ·0037	+ ·0020	− ·0005	− ·0020	− ·0008	+ ·0014
18	+ ·0006	+ ·0011	− ·0001	+ ·0015	+ ·0036	+ ·0056	− ·0029	+ ·0063	0	− ·0005	− ·0010	− ·0008	+ ·0008
19	− ·0004	+ ·0011	− ·0001	+ ·0005	+ ·0016	+ ·0036	− ·0019	− ·0007	− ·0010	− ·0015	− ·0010	− ·0018	− ·0001
20	+ ·0006	+ ·0011	− ·0011	− ·0005	− ·0004	+ ·0016	− ·0029	− ·0017	− ·0010	− ·0015	− ·0010	− ·0018	− ·0007
21	− ·0014	+ ·0011	− ·0011	− ·0015	− ·0024	− ·0014	− ·0059	− ·0017	− ·0020	− ·0025	− ·0020	− ·0018	− ·0018
22	− ·0004	+ ·0011	− ·0001	− ·0025	− ·0034	− ·0024	− ·0079	− ·0027	− ·0030	− ·0025	− ·0020	− ·0008	− ·0022
23	− ·0014	+ ·0011	− ·0031	− ·0035	− ·0034	− ·0034	− ·0079	− ·0047	− ·0040	− ·0015	− ·0020	− ·0018	− ·0029
24	− ·0134	+ ·0011	− ·0011	− ·0045	− ·0044	− ·0054	− ·0089	− ·0047	− ·0050	− ·0025	− ·0020	− ·0018	− ·0043

BEN NEVIS SUMMIT.

TABLE XXVI.

Monthly Mean Hourly Excess of Tension of Aqueous Vapour in Clear Weather over that in Foggy Weather.

Hour.	January.	February.	March.	April.	May.	June.	July.	August.	September.	October.	November.	December.	Mean.
	Inch.	Inch.	Inch.	Inch.	Inch.	Inch.	Inch.	Inch.	Inch.	Inch.	Inch.	Inch.	Inch.
1	−0·040	−0·068	−0·057	−0·042	−0·016	−0·001	−0·002	−0·031	−0·021	−0·062	−0·062	−0·069	−0·0393
2	− ·047	− ·072	− ·058	− ·040	− ·020	− ·002	− ·009	− ·027	− ·027	− ·062	− ·060	− ·069	− ·0411
3	− ·050	− ·070	− ·063	− ·040	− ·023	0	− ·015	− ·028	− ·032	− ·067	− ·065	− ·069	− ·0435
4	− ·057	− ·071	− ·062	− ·041	− ·024	− ·002	− ·018	− ·031	− ·036	− ·067	− ·066	− ·068	− ·0453
5	− ·059	− ·073	− ·062	− ·038	− ·016	− ·002	− ·014	− ·031	− ·035	− ·065	− ·064	− ·069	− ·0439
6	− ·055	− ·072	− ·065	− ·044	− ·016	− ·001	− ·017	− ·030	− ·042	− ·068	− ·062	− ·066	− ·0449
7	− ·058	− ·077	− ·060	− ·041	− ·014	− ·002	− ·013	− ·027	− ·040	− ·070	− ·060	− ·068	− ·0445
8	− ·068	− ·077	− ·057	− ·037	− ·009	+ ·004	− ·008	− ·031	− ·042	− ·070	− ·064	− ·070	− ·0440
9	− ·063	− ·076	− ·058	− ·040	− ·007	+ ·005	− ·003	− ·023	− ·035	− ·071	− ·064	− ·063	− ·0415
10	− ·063	− ·072	− ·053	− ·034	− ·005	+ ·014	+ ·004	− ·026	− ·032	− ·062	− ·060	− ·065	− ·0378
11	− ·061	− ·069	− ·053	− ·032	0	+ ·020	+ ·007	− ·022	− ·030	− ·059	− ·057	− ·061	− ·0348
12	− ·061	− ·075	− ·052	− ·025	+ ·001	+ ·022	+ ·019	− ·010	− ·023	− ·053	− ·062	− ·059	− ·0315
13	− ·063	− ·077	− ·045	− ·029	+ ·002	+ ·026	+ ·025	− ·001	− ·016	− ·046	− ·049	− ·051	− ·0270
14	− ·062	− ·079	− ·046	− ·028	+ ·002	+ ·030	+ ·024	0	− ·014	− ·045	− ·049	− ·059	− ·0271
15	− ·063	− ·076	− ·046	− ·025	+ ·003	+ ·030	+ ·028	0	− ·014	− ·045	− ·052	− ·058	− ·0265
16	− ·060	− ·074	− ·042	− ·024	+ ·008	+ ·032	+ ·036	+ ·002	− ·011	− ·048	− ·048	− ·062	− ·0242
17	− ·059	− ·074	− ·041	− ·023	+ ·006	+ ·031	+ ·037	+ ·010	− ·014	− ·050	− ·056	− ·059	− ·0243
18	− ·061	− ·077	− ·051	− ·023	+ ·007	+ ·030	+ ·044	+ ·001	− ·008	− ·053	− ·057	− ·064	− ·0260
19	− ·058	− ·071	− ·053	− ·023	+ ·006	+ ·026	+ ·039	0	− ·013	− ·054	− ·058	− ·063	− ·0268
20	− ·060	− ·075	− ·051	− ·025	0	+ ·024	+ ·030	− ·006	− ·021	− ·052	− ·059	− ·060	− ·0296
21	− ·051	− ·070	− ·052	− ·029	− ·005	+ ·019	+ ·035	− ·010	− ·026	− ·051	− ·060	− ·055	− ·0296
22	− ·056	− ·067	− ·056	− ·030	− ·008	+ ·016	+ ·034	− ·012	− ·029	− ·056	− ·059	− ·063	− ·0321
23	− ·053	− ·069	− ·054	− ·031	− ·011	+ ·015	+ ·022	− ·018	− ·014	− ·057	− ·060	− ·063	− ·0326
24	− ·042	− ·069	− ·053	− ·036	− ·012	+ ·012	+ ·016	− ·024	− ·016	− ·059	− ·062	− ·060	− ·0338
Mean	−0·0572	−0·0729	−0·0538	−0·0325	−0·0063	+0·0144	+0·0126	−0·0157	−0·0246	−0·0580	−0·0590	−0·0630	−0·0346

BEN NEVIS SUMMIT. TABLE XX

Actual Maximum and Minimum Temperature observed at every Hour in eac

Hour.	JANUARY.		FEBRUARY.		MARCH.		APRIL.		MAY.		JUNE.		JULY.	
	Max.	Min.	Max.	Min.	Max.	Min.	Max.	Min.	Max.	Min.	Max.	Min.	Max.	Mi
	° Fah.	° Fah.	° Fah.	° Fah.	° Fah.	° Fah.	° Fah.	° Fah.	° Fah.	° Fah.	° Fah.	° Fah.	° Fah.	° F
1	47·1	10·3	42·9	9·0	38·4	6·0	45·1	13·0	49·0	18·2	55·9	26·0	56·0	31
2	46·0	10·2	43·1	8·5	37·0	5·9	46·1	12·5	49·4	18·1	55·1	25·9	55·5	31
3	46·0	10·2	46·1	7·3	37·0	6·3	46·0	13·7	49·3	17·9	55·3	25·2	56·0	31
4	46·7	11·4	46·0	7·8	36·9	7·0	46·3	11·8	49·2	18·0	56·2	25·0	55·0	31
5	45·2	12·1	45·2	7·9	35·9	6·3	46·9	11·5	50·4	18·9	57·1	25·0	55·3	30
6	43·7	11·8	44·9	7·5	37·5	7·8	48·2	12·0	49·6	20·6	60·6	26·0	56·1	31
7	43·7	11·5	45·0	7·0	38·6	7·9	48·2	13·0	48·0	21·2	60·9	26·8	56·8	32
8	43·3	12·2	44·5	7·4	40·0	8·0	48·3	12·8	48·4	22·1	61·4	26·9	57·5	32
9	44·0	12·2	44·0	7·8	40·6	8·3	50·0	14·9	48·7	19·7	62·3	28·0	58·1	33
10	44·0	12·5	43·3	7·7	41·2	9·0	50·9	14·5	49·4	21·0	63·0	27·9	59·0	34
11	44·2	13·0	42·2	9·4	39·9	9·0	51·8	16·3	52·0	21·2	62·9	29·0	60·5	36
12	45·5	13·0	43·7	9·9	40·2	8·8	52·5	18·3	53·6	21·5	63·7	29·6	61·7	34
13	45·4	12·8	43·7	11·0	39·8	9·6	52·0	18·1	54·1	22·2	65·3	28·0	63·0	34
14	45·3	13·0	43·0	11·7	40·4	10·3	50·4	17·8	54·0	21·9	64·9	27·9	64·0	40
15	45·3	12·4	41·6	11·9	41·2	9·3	50·7	18·0	53·9	20·9	66·2	34·0	63·3	39
16	44·8	12·3	42·8	11·3	40·9	8·2	50·4	17·0	53·3	21·3	65·9	28·8	62·4	39
17	44·2	11·4	42·0	9·8	40·0	8·0	49·8	16·7	52·3	21·0	66·2	30·0	62·0	38
18	44·6	11·7	41·2	9·5	38·5	7·0	49·0	17·5	52·3	20·8	65·0	29·7	61·2	37
19	45·7	11·2	41·2	8·5	38·1	6·4	48·1	15·5	51·2	20·2	63·5	28·8	60·6	35
20	45·6	12·0	43·1	10·0	38·1	5·7	48·1	15·3	50·0	19·4	60·6	27·8	59·1	35
21	46·8	11·1	42·5	10·0	37·7	5·9	48·6	15·7	48·9	19·5	60·1	27·0	57·8	34
22	47·9	12·0	42·3	10·0	38·0	8·0	48·1	14·5	48·3	19·4	58·1	26·3	57·0	32
23	47·4	12·0	42·0	9·8	38·0	5·6	46·9	14·5	48·1	18·7	59·9	26·0	56·6	31
24	48·3	10·3	41·8	9·6	38·0	6·2	46·0	14·0	47·2	18·5	56·2	26·0	56·0	30
Mean	45·47	11·78	43·25	9·18	38·83	7·52	48·68	14·95	50·44	20·09	61·10	27·57	58·77	34
No. of Hourly Observations	606		1158		1335		1728		2228		2895		1502	

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ur in each Month for Thirteen Years during Continuance of Clear Weather.

July.		August.		September.		October.		November.		December.		Mean.		No. of Hourly Observations.
Max.	Min.	Max.	Min.	Max.	Min.	Max.	Min.	Max.	Min.	Max.	Min.	Max.	Min.	
° Fah.	° Fah.	° Fah.	° Fah.	° Fah.	° Fah.	° Fah.	° Fah.	° Fah.	° Fah.	° Fah.	° Fah.	° Fah.	° Fah.	
56·0	31·0	53·0	33·9	52·2	27·0	47·1	18·0	45·9	15·1	40·5	16·7	47·76	18·68	689
55·5	31·5	53·0	33·2	52·7	26·4	47·9	17·3	45·9	15·0	40·1	16·3	47·65	18·40	691
56·0	31·1	52·4	33·4	51·8	27·5	47·6	17·0	46·1	14·9	40·9	16·1	47·88	18·38	680
55·0	31·1	51·6	33·0	51·6	28·2	47·4	17·3	45·5	15·1	41·9	16·1	47·86	18·48	676
55·3	30·8	52·4	32·0	51·8	29·1	48·0	17·5	45·1	15·1	41·1	15·5	47·87	18·39	671
56·1	31·4	53·0	32·1	51·3	26·7	47·6	16·4	45·1	14·3	41·6	14·4	48·27	18·42	664
56·8	32·1	53·6	33·8	51·9	29·7	48·3	17·4	46·0	13·2	40·9	16·1	48·49	19·14	661
57·5	32·4	55·7	33·1	53·2	29·1	50·1	17·1	46·8	14·5	41·1	16·7	49·19	19·36	658
58·1	33·2	55·5	35·4	55·0	30·0	52·2	18·8	47·8	15·0	41·7	16·1	49·99	19·95	664
59·0	34·9	56·3	32·0	55·5	32·4	53·1	18·9	49·6	17·2	41·1	16·1	50·53	20·34	682
60·5	36·0	58·2	33·5	57·9	33·0	53·7	19·8	50·4	17·0	40·8	16·5	51·21	21·14	675
61·7	34·8	58·4	34·8	57·5	29·7	54·6	20·1	51·0	18·6	40·8	16·2	51·93	21·28	679
63·0	34·9	59·5	35·1	59·0	29·9	55·5	20·5	52·0	17·2	41·2	16·0	52·54	21·28	681
64·0	40·0	60·6	36·0	59·9	30·0	55·5	20·5	52·0	17·0	41·3	15·3	52·65	21·78	690
63·3	39·4	62·2	36·0	59·3	31·1	55·0	19·6	51·9	16·3	41·4	15·3	52·67	22·02	697
62·4	39·9	60·1	37·0	57·7	31·0	54 7	18·2	50·0	15·2	40·2	16·1	51·93	21·36	698
62·0	38·5	59·0	36·1	57·8	30·5	52·6	17·8	47·8	15·3	41·8	16·1	51·29	20·93	683
61·2	37·9	56·7	36·1	54·9	29·5	51·0	17·5	47·3	16·1	42·9	16·4	50·38	20·81	690
60·6	35·0	56·2	36·0	53·2	29·0	49·8	17·2	47·0	16·1	40·8	16·0	49·62	19·99	689
59·1	35·0	55·5	35·1	52·8	28·8	48·5	16·2	46·8	15 5	41·7	17·0	49·16	19·82	687
57·8	34·1	54·8	35·0	52·9	27·0	48·6	16·5	46·4	15·9	41·5	16·9	48·88	19·55	680
57·0	32·1	54·0	34·7	51·9	26·7	48·4	16·4	46·6	16·0	41·5	16·8	48·51	19·41	688
56·6	31·0	53·0	34·4	53·1	26·4	49·0	17·0	46·2	16·3	40·9	15·5	48·43	18·93	686
56·0	30·3	52·4	34·1	51·7	26·6	48·8	17·2	46·1	15·1	41·9	14·3	47·87	18·52	685
58·77	34·10	55·71	34·42	54·44	28·97	50·63	17·93	47·72	15·71	41·23	16·02	49·69	19·85	...
1502		971		1296		951		983		697		...		...

BEN NEVIS SUMMIT. TABLE XXV

Actual Maximum and Minimum Temperature observed at every Hour in each

Hour.	January.		February.		March.		April.		May.		June.		July.	
	Max.	Min.	Max.	Min.	Max.	Min.	Max.	Min.	Max.	Min.	Max.	Min.	Max.	Min.
	° Fah.	° Fah.	° Fah.	° Fah.	° Fah.	° Fah.	° Fah.	° Fah.	° Fah.	° Fah.	° Fah.	° Fah.	° Fah.	° Fah.
1	36·9	14·3	37·0	12·3	37·6	13·1	35·3	16·8	40·0	21·9	43·3	28·5	47·3	29·4
2	37·0	13·2	37·2	12·2	37·9	13·4	35·6	16·8	39·9	21·1	43·3	28·1	47·2	30·0
3	36·8	12·9	37·2	11·9	36·6	13·5	35·0	16·8	39·7	21·3	43·0	28·0	47·0	31·0
4	36·4	13·3	37·2	11·8	36·4	14·0	35·6	17·2	39·4	21·8	42·7	28·3	46·6	31·8
5	36·3	14·3	37·0	12·0	37·0	14·2	36·1	16·9	29·3	22·0	42·0	28·2	45·9	30·9
6	36·6	15·0	34·9	11·7	37·0	14·8	35·2	16·3	39·4	22·4	42·8	27·8	46·0	31·0
7	36·7	14·9	35·1	11·6	36·3	14·2	35·3	16·7	39·4	22·3	43·2	28·0	45·8	30·8
8	36·8	14·3	35·4	11·9	35·6	14·1	36·1	16·7	39·8	22·7	43·3	27·4	45·7	30·6
9	36·9	14·1	36·3	12·2	36·1	15·2	35·9	16·9	40·0	22·7	44·6	27·0	46·7	30·4
10	37·2	14·2	36·8	12·5	36·7	16·0	36·1	17·4	40·8	23·5	44·8	27·7	46·8	30·9
11	37·2	14·4	35·7	13·1	37·0	16·8	35·9	16·9	40·9	24·5	45·6	28·5	46·8	30·8
12	37·6	15·2	36·3	13·0	37·2	16·5	36·2	17·2	41·7	25·0	44·8	29·2	47·2	31·4
13	37·0	15·5	36·7	12·4	37·8	16·1	36·1	17·5	41·4	25·2	44·6	30·0	47·5	31·1
14	37·0	14·8	36·8	12·2	38·1	15·9	36·6	17·8	41·8	25·3	45·0	29·6	47·5	30·9
15	36·9	14·9	36·0	12·1	39·1	16·1	35·8	17·4	41·8	26·1	45·1	29·1	47·8	31·0
16	37·1	14·2	36·2	12·1	40·0	16·0	35·9	17·2	41·8	25·1	46·0	29·6	48·0	31·0
17	37·0	14·2	36·0	11·6	39·0	16·1	36·0	17·7	41·4	23·9	46·8	29·4	48·0	30·6
18	36·7	15·0	36·0	11·5	38·8	16·1	36·4	17·8	41·4	23·6	47·0	28·8	48·1	30·1
19	36·6	14·5	35·8	11·3	37·2	16·0	37·0	17·8	41·0	23·3	46·5	28·5	48·1	30·0
20	36·7	14·7	35·5	11·3	36·9	16·2	36·9	18·2	40·5	22·5	45·6	27·0	47·9	29·2
21	36·6	14·0	35·2	10·6	36·8	16·0	37·2	18·7	40·3	21·3	44·5	27·0	47·0	29·1
22	36·2	14·3	36·0	10·5	36·1	16·2	37·3	18·2	40·8	21·6	43·2	26·8	47·0	29·0
23	36·7	14·1	36·7	10·5	36·6	16·2	35·1	17·7	40·3	21·6	44·0	26·3	46·8	29·2
24	36·3	14·1	37·3	10·8	37·4	16·0	35·3	17·2	40·4	21·5	44·0	28·3	45·7	29·1
Mean	36·80	14·35	36·26	11·80	37·30	15·36	36·00	17·33	40·55	23·01	44·40	28·21	47·02	30·39
No. of Days employed	86		54		78		59		27		42		46	

BLE XXVIII.

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Hour in each Month of Thirteen Years during Continuance of Foggy Weather.

July.		August.		September.		October.		November.		December.		Mean.		No. of Hourly Observations.
Max.	Min.	Max.	Min.	Max.	Min.	Max.	Min.	Max.	Min.	Max.	Min.	Max.	Min.	
° Fah.	° Fah.	° Fah.	° Fah.	° Fah.	° Fah.	° Fah.	° Fah.	° Fah.	° Fah.	° Fah.	° Fah.	° Fah.	° Fah.	
47·3	29·4	46·0	30·6	50·1	26·5	47·2	23·0	39·2	21·7	38·5	12·5	41·53	20·88	747
47·2	30·0	45·8	31·0	50·0	26·0	47·9	23·1	39·2	21·3	38·8	13·2	41·65	20·78	„
47·0	31·0	46·2	31·5	49·7	25·1	48·1	23·0	39·8	20·2	38·3	13·3	41·45	20·71	„
46·6	31·8	46·5	31·8	49·3	25·8	48·2	23·1	39·9	19·7	38·1	13·2	41·36	20·98	„
45·9	30·9	45·2	32·0	49·9	26·1	48·4	22·5	40·0	19·0	37·9	12·2	41·25	20·86	„
46·0	31·0	45·9	32·0	49·3	25·6	48·1	22·0	41·2	19·0	37·8	13·3	41·18	20·91	„
45·8	30·8	46·2	32·5	47·3	26·3	47·1	22·4	41·0	19·0	38·1	13·1	40·96	20·98	„
45·7	30·6	46·6	33·7	46·7	26·3	47·1	22·5	41·8	19·0	38·4	13·2	41·11	21·03	„
46·7	30·4	46·0	33·3	47·2	26·8	46·6	23·4	41·6	19·0	38·0	13·3	41·33	21·19	„
46·8	30·9	46·0	33·9	47·6	27·4	48·0	23·5	41·0	19·7	38·2	14·1	41·67	21·73	„
46·8	30·8	46·4	33·8	48·0	28·0	48·1	23·6	40·3	20·1	38·8	14·3	41·73	22·07	„
47·2	31·4	46·8	34·7	49·0	28·0	48·4	23·5	40·8	19·3	38·6	14·3	42·05	22·28	„
47·5	31·1	47·1	33·5	49·6	28·4	48·9	23·4	41·1	20·0	38·2	13·9	42·17	22·25	„
47·5	30·9	48·6	34·7	49·2	28·6	49·0	23·9	40·8	20·1	38·3	13·7	42·39	22·29	„
47·8	31·0	49·0	34·7	48·9	27·8	48·9	23·7	40·6	19·9	38·1	13·5	42·33	22·19	„
48·0	31·0	48·3	33·6	49·1	27·6	48·9	23·0	40·1	19·0	38·2	13·3	42·47	21·81	„
48·0	30·6	47·7	33·8	49·5	27·2	48·6	22·9	39·7	18·0	38·2	12·3	42·33	21·48	„
48·1	30·1	46·6	33·3	50·0	27·4	47·4	22·8	39·7	18·1	38·1	12·0	42·18	21·38	„
48·1	30·0	45·6	32·6	50·9	26·2	48·1	23·0	39·3	19·0	38·3	11·2	42·03	21·12	„
47·9	29·2	45·6	32·3	51·0	27·0	47·5	22·8	38·2	18·5	38·6	11·3	41·74	20·92	„
47·0	29·1	45·1	32·3	50·7	27·2	46·5	22·0	38·0	16·4	39·0	11·2	41·41	20·48	„
47·0	29·0	45·1	32·1	49·8	27·8	46·2	22·0	38·9	17·5	38·8	11·3	41·28	20·61	„
46·8	29·2	45·7	31·7	49·8	27·8	46·8	21·6	37·8	17·6	38·6	12·1	41·24	20·53	„
45·7	29·1	44·2	31·8	50·1	27·8	46·9	21·0	39·0	17·6	38·3	11·8	41·24	20·58	„
47·02	30·39	46·34	32·80	49·28	27·03	47·79	22·82	40·00	19·11	38·34	12·82	41·67	21·25	...
46		58		79		64		88		66		...		...

BEN NEVIS SUMMIT.

TABLE XXIX.

Monthly Mean Temperature of the Air at every Hour of the Day during Continuance of Clear Weather.

Hour.	January.	February.	March.	April.	May.	June.	July.	August.	September.	October.	November.	December.	Mean.	No. of Hourly Observations.
	° Fah.	° Fah.	° Fah.	° Fah.	° Fah.	° Fah.	° Fah.	° Fah.	° Fah.	° Fah.	° Fah.	° Fah.	° Fah.	
1	25·44	24·20	22·44	29·21	35·93	42·59	43·29	43·60	41·16	34·48	32·37	28·40	33·59	689
2	25·46	23·91	23·13	29·10	35·49	42·19	43·04	43·40	40·99	34·80	32·37	28·43	33·52	691
3	26·28	23·21	22·52	29·02	35·66	41·97	42·96	43·29	40·89	34·54	32·70	28·51	33·46	680
4	26·44	22·89	22·92	28·93	35·75	41·91	42·53	42·89	40·67	34·32	32·65	28·54	33·37	676
5	27·25	22·91	22·62	28·94	36·25	42·26	43·16	42·75	40·53	34·08	32·45	28·25	33·45	671
6	27·40	23·30	22·97	29·21	37·01	42·89	43·82	43·19	40·52	34·46	32·01	28·37	33·76	664
7	26·88	23·01	23·54	29·83	37·57	43·58	44·51	43·45	41·11	34·88	31·44	28·69	34·04	661
8	27·27	23·34	24·43	30·55	38·35	44·37	45·74	44·62	42·24	35·67	32·46	28·31	34·78	658
9	26·61	24·14	24·81	31·32	38·50	45·28	46·67	45·57	43·44	36·79	32·69	27·78	35·30	664
10	27·54	25·11	25·28	31·65	39·21	45·88	47·35	45·75	44·52	37·60	33·23	27·94	35·92	682
11	27·71	25·48	25·83	31·97	40·05	46·94	48·32	46·99	45·91	38·22	33·79	28·23	36·62	675
12	27·38	25·64	25·86	32·53	40·59	48·09	49·03	48·16	46·52	38·36	35·15	28·21	37·13	679
13	27·60	25·72	25·83	32·75	40·99	48·76	49·90	48·58	46·78	38·68	34·70	28·66	37·41	687
14	27·61	26·01	26·13	32·92	41·04	49·08	50·37	48·73	47·18	38·87	34·36	28·76	37·59	690
15	26·69	25·42	25·79	33·05	40·84	49·49	50·69	48·69	47·19	38·25	34·12	29·21	37·45	697
16	26·19	25·24	26·08	32·96	40·71	48·95	50·57	48·80	46·57	37·01	32·40	28·54	37·00	698
17	25·72	24·76	25·06	32·49	40·09	48·38	49·97	48·11	46·40	36·16	32·01	28·23	36·45	683
18	25·86	24·26	24·18	31·82	39·31	47·77	49·10	47·59	44·84	35·88	31·67	28·49	35·90	690
19	25·99	24·44	23·80	30·92	38·53	46·97	48·09	46·74	44·02	35·49	31·41	28·43	35·40	689
20	26·18	25·04	23·81	30·53	37·51	45·57	47·01	45·67	43·32	35·19	32·18	28·94	35·08	687
21	26·08	25·00	23·73	30·22	36·97	44·62	46·13	44·97	42·84	34·88	32·18	28·67	34·69	680
22	25·15	25·02	22·97	30·07	36·62	43·98	45·11	44·12	42·35	34·90	32·23	28·64	34·26	688
23	25·06	24·63	22·73	30·00	36·37	43·43	44·39	43·81	41·69	34·48	32·39	28·40	33·95	686
24	24·86	24·28	22·52	29·51	36·00	43·14	43·47	43·52	41·81	35·00	32·70	28·44	33·77	685
Mean	26·44	24·46	24·12	30·81	38·14	45·34	46·47	45·54	43·48	35·96	32·74	28·46	35·16	16350
No. of Hourly Observations.	606	1158	1335	1728	2228	2895	1502	971	1296	951	983	697	...	...

BEN NEVIS SUMMIT.

TABLE XXX.

Monthly Mean Temperature of the Air at every Hour of the Day during Continuance of Foggy Weather.

Hour.	January.	February.	March.	April.	May.	June.	July.	August.	September.	October.	November.	December.	Mean.	No. of Hourly Observations.
	° Fah.	° Fah.	° Fah.	° Fah.	° Fah.	° Fah.	° Fah.	° Fah.	° Fah.	° Fah.	° Fah.	° Fah.	° Fah.	
1	26·22	27·09	26·50	27·80	30·82	35·20	37·60	39·63	36·66	33·47	30·00	26·71	31·48	747
2	26·19	26·91	26·53	27·73	30·81	35·01	37·47	39·66	36·58	33·87	30·19	26·83	31·48	"
3	26·26	26·90	26·50	27·65	30·72	34·70	37·49	39·45	36·42	33·37	30·29	26·87	31·38	"
4	26·39	26·89	26·26	27·47	30·60	34·46	37·32	39·35	36·32	33·90	30·21	26·79	31·33	"
5	26·52	26·92	26·26	27·38	30·57	34·28	37·07	39·00	36·27	33·42	30·24	26·64	31·21	"
6	26·66	26·71	26·12	27·42	30·44	34·27	36·96	38·80	36·25	33·84	30·22	26·51	31·18	"
7	26·78	26·67	26·01	27·58	30·54	34·36	36·98	38·96	36·21	33·36	30·24	26·53	31·18	"
8	26·90	26·75	26·19	27·73	30·88	34·61	37·01	39·13	36·31	33·95	30·27	26·62	31·36	"
9	26·91	27·04	26·41	28·11	31·13	34·99	37·22	39·40	36·45	33·64	30·19	26·59	31·51	"
10	26·94	27·41	26·78	28·38	31·74	35·18	37·40	39·67	36·73	34·33	30·32	26·79	31·80	"
11	27·03	27·71	27·11	28·59	32·05	35·54	37·57	39·94	36·90	34·08	30·38	27·01	31·99	"
12	27·07	28·03	27·38	28·64	32·35	35·95	37·56	40·17	37·04	34·68	30·44	27·03	32·19	"
13	27·04	28·04	27·39	29·01	32·52	36·19	37·76	40·31	37·18	34·19	30·41	26·97	32·25	"
14	27·02	28·12	27·50	29·00	32·90	36·28	37·95	40·53	37·15	34·09	30·32	26·89	32·31	"
15	26·94	28·04	27·50	29·05	32·83	36·14	38·06	40·46	37·20	33·92	30·13	26·78	32·25	"
16	26·78	27·94	27·02	28·77	32·52	36·17	37·94	40·30	37·06	33·76	30·02	26·63	32·08	"
17	26·65	27·54	26·67	28·51	32·18	36·05	37·82	40·07	36·78	33·59	29·92	26·53	31·86	"
18	26·56	27·47	26·58	28·28	31·99	35·86	37·31	39·71	36·51	33·54	29·93	26·47	31·68	"
19	26·52	27·42	26·51	28·05	31·59	35·69	37·02	39·56	36·33	33·41	29·96	26·35	31·53	"
20	26·42	27·55	26·46	27·92	31·35	35·41	36·80	39·45	36·31	33·41	30·00	26·30	31·45	"
21	26·35	27·42	26·39	27·80	31·04	35·02	36·50	39·24	36·20	33·30	29·90	26·33	31·29	"
22	26·27	27·42	26·37	27·63	31·00	34·92	36·29	39·28	36·08	33·32	29·69	26·34	31·22	"
23	26·21	27·51	26·33	27·40	30·93	34·79	36·26	39·15	35·99	33·34	29·77	26·19	31·16	"
24	26·15	27·46	26·21	27·27	30·80	34·63	36·35	38·95	35·85	33·22	29·86	26·24	31·08	"
Mean	26·61	27·37	26·62	28·05	31·43	35·24	37·24	39·59	36·53	33·71	30·12	26·62	31·59	...
No. of Days employed	86	54	78	59	27	42	46	58	79	64	88	66	...	...

BEN NEVIS SUMMIT.

TABLE XXXI.

Diurnal Variation of Monthly Mean Hourly Temperature during Continuance of Clear Weather.

Month.	January.	February.	March.	April.	May.	June.	July.	August.	September.	October.	November.	December.	Mean.
Daily Mean of Month.	° Fah. 26·44	° Fah. 24·46	° Fah. 24·12	° Fah. 30·81	° Fah. 38·14	° Fah. 45·34	° Fah. 46·47	° Fah. 45·54	° Fah. 43·48	° Fah. 35·96	° Fah. 32·74	° Fah. 28·46	° Fah. 35·16
	Difference of Hourly Mean from Daily Mean of Month.												
Hour.													
1	−1·00	−0·26	−1·68	−1·60	−2·21	−2·75	−3·18	−1·94	−2·32	−1·48	−0·37	−0·06	−1·57
2	−0·98	−0·55	−0·99	−1·71	−2·65	−3·15	−3·43	−2·14	−2·49	−1·16	−0·37	−0·03	−1·64
3	−0·16	−1·25	−1·60	−1·79	−2·48	−3·37	−3·51	−2·25	−2·59	−1·42	−0·04	+0·05	−1·70
4	0	−1·57	−1·20	−1·88	−2·39	−3·43	−3·94	−2·65	−2·81	−1·64	−0·09	+0·08	−1·49
5	+0·81	−1·55	−1·50	−1·87	−1·89	−3·08	−3·31	−2·79	−2·95	−1·88	−0·29	−0·21	−1·71
6	+0·96	−1·16	−1·15	−1·60	−1·13	−2·45	−2·55	−2·35	−2·96	−1·50	−0·73	−0·09	−1·40
7	+0·44	−1·45	−0·58	−0·98	−0·57	−1·76	−1·96	−2·09	−2·37	−1·08	−1·30	+0·23	−1·12
8	+0·83	−1·12	+0·31	−0·26	+0·21	−0·97	−0·73	−0·92	−1·24	−0·29	−0·28	−0·15	−0·38
9	+0·17	−0·32	+0·69	+0·51	+0·36	−0·06	+0·20	+0·03	−0·04	+0·83	−0·05	−0·68	+0·14
10	+1·10	+0·65	+1·16	+0·84	+1·07	+0·54	+0·88	+0·21	+1·04	+1·64	+0·49	−0·52	+0·76
11	+1·27	+1·02	+1·71	+1·16	+1·91	+1·60	+1·85	+1·45	+2·43	+2·26	+1·05	−0·23	+1·46
12	+0·94	+1·18	+1·74	+1·72	+2·45	+2·75	+2·56	+2·62	+3·04	+2·42	+2·41	−0·25	+1·97
13	+1·16	+1·26	+1·71	+1·94	+2·85	+3·42	+2·43	+3·04	+3·30	+2·72	+1·96	+0·20	+2·25
14	+1·17	+1·55	+2·01	+2·09	+2·90	+3·74	+3·90	+3·19	+3·70	+2·91	+1·62	+0·30	+2·43
15	+0·25	+0·96	+1·67	+2·24	+2·70	+4·15	+4·22	+3·15	+3·71	+2·29	+1·38	+0·75	+2·29
16	−0·25	+0·78	+1·96	+2·15	+2·54	+3·61	+4·00	+3·26	+3·09	+1·05	−0·34	+0·08	+1·84
17	−0·72	+0·30	+0·94	+1·68	+1·95	+3·04	+3·50	+2·57	+2·92	+0·20	−0·73	−0·23	+1·29
18	−0·58	−0·20	+0·06	+1·01	+1·17	+2·43	+2·63	+2·05	+1·36	−0·08	−1·07	+0·03	+0·74
19	−0·45	−0·02	−0·32	+0·11	+0·39	+1·63	+1·62	+1·20	+0·54	−0·47	−1·33	−0·03	+0·24
20	−0·26	+0·58	−0·31	−0·28	−0·63	+0·23	+0·54	+0·13	−0·16	−0·77	−0·56	+0·48	−0·08
21	−0·36	+0·54	−0·39	−0·59	−1·17	−0·72	−0·34	−0·57	−0·64	−1·08	−0·56	+0·21	−0·47
22	−1·29	+0·56	−1·15	−0·74	−1·52	−1·36	−1·36	−1·42	−1·13	−1·06	−0·51	+0·18	−0·90
23	−1·38	+0·17	−1·39	−0·81	−1·77	−1·91	−2·08	−1·73	−1·79	−1·48	−0·35	−0·06	−1·21
24	−1·58	−0·16	−1·60	−1·30	−2·14	−2·20	−3·00	−2·02	−1·67	−0·96	−0·04	−0·02	−1·39

BEN NEVIS SUMMIT.

TABLE XXXII.

Diurnal Variation of Monthly Mean Hourly Temperature during Continuance of Foggy Weather.

Month.	January.	February.	March.	April.	May.	June.	July.	August.	September.	October.	November.	December.	Year.
Daily Mean of Month.	° Fah. 26·61	° Fah. 27·37	° Fah. 26·62	° Fah. 28·05	° Fah. 31·43	° Fah. 35·24	° Fah. 37·24	° Fah. 39·59	° Fah. 36·53	° Fah. 33·71	° Fah. 30·12	° Fah. 26·62	° Fah. 31·59
Difference of Hourly Mean from Daily Mean of Month.													
1	−0·39	−0·28	−0·12	−0·25	−0·61	−0·04	+0·36	+0·04	+0·13	−0·24	−0·12	+0·09	−0·11
2	−0·42	−0·46	−0·09	−0·32	−0·62	−0·23	+0·23	+0·07	+0·05	+0·16	+0·07	+0·21	−0·11
3	−0·35	−0·47	−0·12	−0·40	−0·71	−0·54	+0·25	−0·14	−0·11	−0·34	+0·17	+0·25	−0·21
4	−0·22	−0·48	−0·36	−0·58	−0·83	−0·78	+0·08	−0·24	−0·21	+0·19	+0·09	+0·17	−0·26
5	−0·09	−0·45	−0·36	−0·67	−0·86	−0·96	−0·17	−0·59	−0·26	−0·29	+0·12	+0·02	−0·38
6	+0·05	−0·66	−0·50	−0·63	−0·99	−0·97	−0·28	−0·79	−0·28	+0·13	+0·10	−0·11	−0·41
7	+0·17	−0·70	−0·61	−0·47	−0·89	−0·88	−0·26	−0·63	−0·32	−0·35	+0·12	−0·09	−0·41
8	+0·29	−0·62	−0·43	−0·32	−0·55	−0·63	−0·23	−0·46	−0·22	+0·24	+0·15	0	−0·23
9	+0·30	−0·33	−0·21	+0·06	−0·30	−0·25	−0·02	−0·19	−0·08	−0·07	+0·07	−0·03	−0·08
10	+0·33	+0·04	+0·16	+0·33	+0·31	−0·06	+0·16	+0·08	+0·20	+0·62	+0·20	+0·17	+0·21
11	+0·42	+0·34	+0·49	+0·54	+0·62	+0·30	+0·33	+0·35	+0·37	+0·37	+0·26	+0·39	+0·40
12	+0·46	+0·66	+0·76	+0·59	+0·92	+0·71	+0·32	+0·58	+0·51	+0·97	+0·32	+0·41	+0·60
13	+0·43	+0·67	+0·77	+0·96	+1·09	+0·95	+0·52	+0·72	+0·65	+0·48	+0·29	+0·35	+0·66
14	+0·41	+0·75	+0·88	+0·95	+1·47	+1·04	+0·71	+0·94	+0·62	+0·38	+0·20	+0·27	+0·72
15	+0·33	+0·67	+0·88	+1·00	+1·40	+0·90	+0·82	+0·87	+0·67	+0·21	+0·01	+0·16	+0·66
16	+0·17	+0·57	+0·40	+0·72	+1·09	+0·93	+0·70	+0·71	+0·53	+0·05	−0·10	+0·01	+0·49
17	+0·04	+0·17	+0·05	+0·46	+0·75	+0·81	+0·58	+0·48	+0·25	−0·12	−0·20	−0·09	+0·27
18	−0·05	+0·10	−0·04	+0·23	+0·56	+0·62	+0·07	+0·12	−0·02	−0·17	−0·19	−0·15	+0·09
19	−0·09	+0·05	−0·11	0	+0·16	+0·45	−0·22	−0·03	−0·20	−0·30	−0·16	−0·27	−0·06
20	−0·19	+0·18	−0·16	−0·13	−0·08	+0·17	−0·44	−0·14	−0·22	−0·30	−0·12	−0·32	−0·14
21	−0·26	+0·05	−0·23	−0·25	−0·39	−0·22	−0·74	−0·35	−0·33	−0·41	−0·22	−0·29	−0·30
22	−0·34	+0·05	−0·25	−0·42	−0·43	−0·32	−0·95	−0·31	−0·45	−0·39	−0·43	−0·28	−0·37
23	−0·40	+0·14	−0·29	−0·65	−0·50	−0·45	−0·98	−0·44	−0·54	−0·37	−0·35	−0·43	−0·43
24	−0·46	+0·09	−0·41	−0·78	−0·63	−0·61	−0·89	−0·64	−0·68	−0·49	−0·26	−0·38	−0·51

BEN NEVIS SUMMIT.

TABLE XXXIII.

Monthly Mean Hourly Excess of Temperature of the Air in Clear Weather over that in Foggy Weather.

Hour.	January.	February.	March.	April.	May.	June.	July.	August.	September.	October.	November.	December.	Mean.
	° Fah.	° Fah.	° Fah.	° Fah.	° Fah.	° Fah.	° Fah.	° Fah.	° Fah.	° Fah.	° Fah.	° Fah.	° Fah.
1	−0·78	−2·89	−4·06	+1·41	+5·11	+7·39	+5·69	+3·97	+4·50	+1·01	+2·37	+1·69	+2·11
2	−0·73	−3·00	−3·40	+1·37	+4·68	+7·18	+5·57	+3·74	+4·41	+0·97	+2·18	+1·60	+2·04
3	+0·02	−3·69	−3·98	+1·37	+4·94	+7·27	+5·47	+3·84	+4·47	+1·17	+2·41	+1·64	+2·08
4	+0·05	−4·00	−3·34	+1·46	+5·15	+7·45	+5·21	+3·54	+4·35	+0·42	+2·44	+1·75	+2·04
5	+0·73	−4·01	−3·64	+1·56	+5·68	+7·98	+6·09	+3·75	+4·46	+0·66	+2·21	+1·61	+2·24
6	+0·74	−3·41	−3·15	+1·79	+6·57	+8·62	+6·86	+4·39	+4·27	+0·62	+1·78	+1·86	+2·58
7	+0·10	−3·66	−2·47	+2·25	+7·03	+9·22	+7·53	+4·49	+4·90	+1·52	+1·20	+2·16	+2·86
8	+0·37	−3·41	−1·76	+2·82	+7·47	+9·76	+8·73	+5·49	+5·93	+1·72	+2·19	+1·69	+3·42
9	−0·30	−2·90	−1·60	+3·21	+7·37	+10·29	+9·45	+6·17	+6·99	+3·15	+2·50	+1·19	+3·79
10	+0·60	−2·30	−1·50	+3·27	+7·47	+10·70	+9·95	+6·08	+7·79	+3·27	+2·91	+1·15	+4·12
11	+0·68	−2·23	−1·28	+3·38	+8·00	+11·40	+10·75	+7·05	+9·01	+4·14	+3·41	+1·22	+4·63
12	+0·31	−2·37	−1·52	+3·89	+8·24	+12·14	+11·47	+7·99	+9·48	+3·68	+4·71	+1·18	+4·94
13	+0·56	−2·32	−1·56	+3·74	+8·47	+12·57	+12·14	+8·27	+9·60	+4·49	+4·29	+1·69	+5·16
14	+0·59	−2·11	−1·37	+3·92	+8·14	+12·80	+12·42	+8·20	+10·03	+4·78	+4·04	+1·87	+5·28
15	−0·25	−2·62	−1·71	+4·00	+8·01	+13·35	+12·63	+8·23	+9·99	+4·33	+3·99	+2·43	+5·20
16	−0·59	−2·70	−0·94	+4·19	+8·19	+12·78	+12·63	+8·50	+9·51	+3·25	+2·38	+1·91	+4·92
17	−0·83	−2·78	−1·61	+3·98	+7·91	+12·33	+12·15	+8·04	+9·62	+2·57	+2·08	+1·73	+4·59
18	−0·70	−3·21	−2·40	+3·54	+7·32	+11·91	+11·79	+7·88	+8·53	+2·34	+1·74	+2·02	+4·22
19	−0·53	−2·98	−2·71	+2·87	+6·94	+11·28	+11·07	+7·18	+7·69	+2·08	+1·45	+2·08	+3·87
20	−0·22	−2·51	−2·65	+2·61	+6·16	+10·16	+10·21	+6·22	+7·01	+1·78	+2·18	+2·64	+3·63
21	−0·27	−2·42	−2·66	+2·42	+5·93	+9·60	+9·63	+5·73	+6·64	+1·58	+2·28	+2·34	+3·40
22	−1·12	−2·40	−3·40	+2·44	+5·62	+9·06	+8·82	+4·84	+6·27	+1·58	+2·54	+2·30	+3·04
23	−1·15	−2·88	−3·60	+2·60	+5·44	+8·64	+8·13	+4·66	+5·70	+1·14	+2·62	+2·21	+2·79
24	−1·29	−3·18	−3·69	+2·24	+5·20	+8·51	+7·12	+4·57	+5·96	+1·78	+2·84	+2·20	+2·69
Mean	−0·17	−2·71	−2·50	+2·76	+6·71	+10·10	+9·23	+5·95	+6·95	+2·25	+2·62	+1·84	+3·57

BEN NEVIS SUMMIT.

TABLE XXXIV.

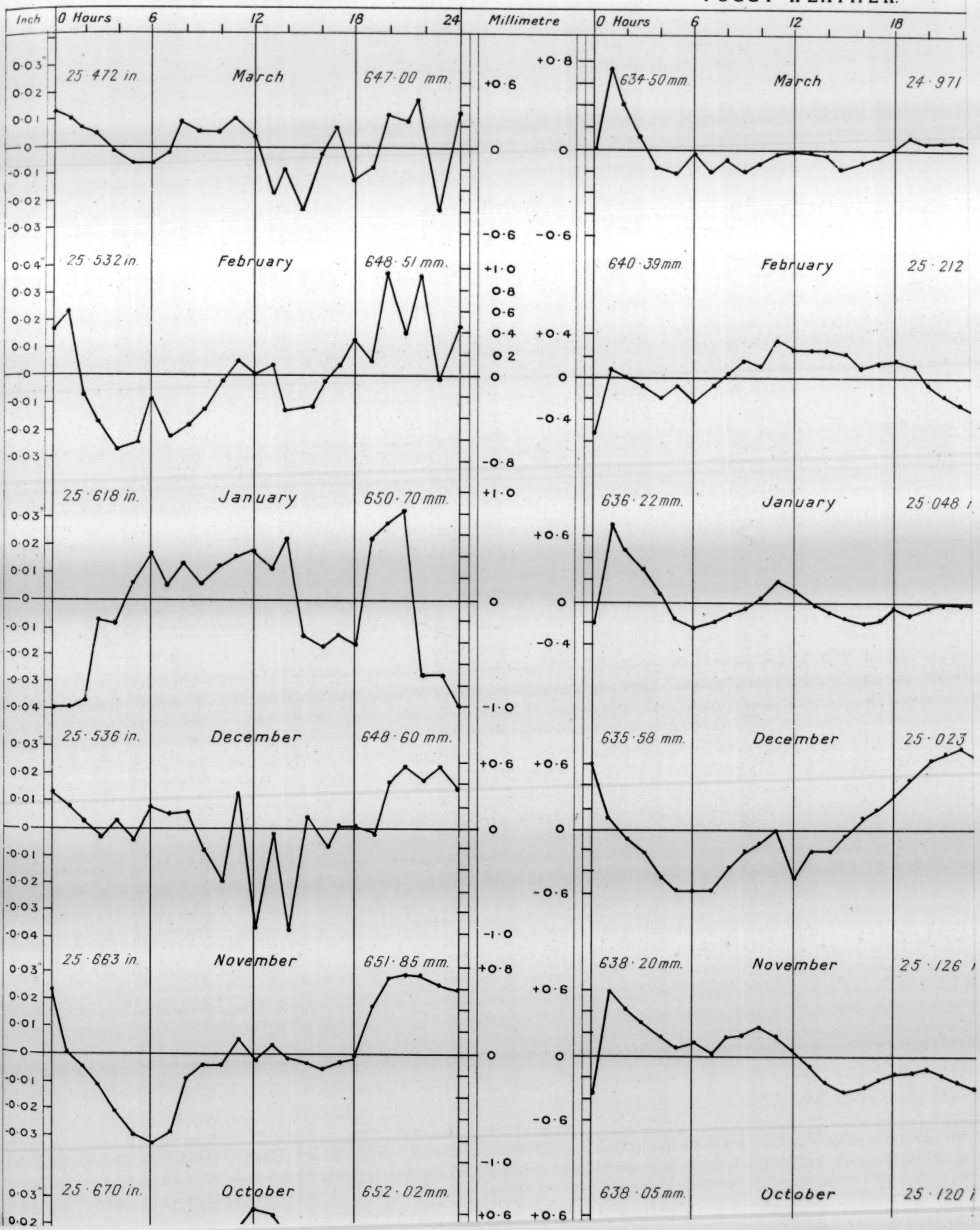
Diurnal Variation of Mean Hourly Temperature and Barometric Pressure in Summer (April to September), in Winter (October to March), and in the Year.

Season.	Clear Weather.						Foggy Weather.					
	Temperature.			Barometric Pressure.			Temperature.			Barometric Pressure.		
	Summer.	Winter.	Year.	Summer.	Winter.	Year.	Summer.	Winter.	Year.	Summer.	Winter.	Year.
Daily Mean of Season.	° Fah. 41·63	° Fah. 28·70	° Fah. 35·16	Inches. 25·642	Inches. 25·582	Inches. 25·612	° Fah. 34·68	° Fah. 28·51	° Fah. 31·59	Inches. 25·229	Inches. 25·083	Inches. 25·156
Difference of Hourly Mean from Daily Mean of Season.												
Hour.												
1	−2·33	−0·64	−1·58	−0·002	+0·001	−0·001	−0·06	−0·17	−0·11	+0·001	+0·016	+0·008
2	−2·60	−0·68	−1·64	− ·004	− ·009	− ·006	−0·13	−0·11	−0·11	− ·005	+ ·008	+ ·001
3	−2·67	−0·74	−1·71	− ·015	+ ·009	− ·011	−0·30	−0·13	−0·21	− ·010	+ ·001	− ·005
4	−2·83	−0·74	−1·81	− ·017	− ·014	− ·015	−0·43	−0·09	−0·26	− ·015	− ·006	− ·010
5	−2·65	−0·77	−1·70	− ·015	− ·014	− ·015	−0·56	−0·19	−0·37	− ·016	− ·009	− ·012
6	−2·17	−0·61	−1·39	− ·009	− ·007	− ·008	−0·65	−0·17	−0·41	− ·013	− ·008	− ·011
7	−1·62	−0·62	−1·12	− ·006	− ·007	− ·007	−0·58	−0·24	−0·41	− ·010	− ·008	− ·009
8	−0·65	−0·12	−0·40	− ·002	+ ·002	0	−0·41	−0·06	−0·23	− ·006	− ·004	− ·005
9	+0·17	+0·11	+0·14	+ ·001	0	0	−0·13	−0·06	−0·09	− ·003	− ·001	− ·002
10	+0·76	+0·75	+0·75	+ ·006	+ ·001	+ ·002	+0·17	+0·24	+0·21	0	+ ·001	+ ·001
11	+1·73	+1·22	+1·54	+ ·005	+ ·011	+ ·008	+0·42	+0·36	+0·40	+ ·003	+ ·005	+ ·004
12	+2·52	+1·42	+1·97	+ ·008	+ ·001	+ ·005	+0·59	+0·59	+0·59	+ ·006	0	+ ·003
13	+2·83	+1·50	+2·22	+ ·010	+ ·004	+ ·007	+0·82	+0·49	+0·66	+ ·007	0	+ ·004
14	+3·25	+1·59	+2·43	+ ·003	− ·005	− ·001	+0·96	+0·48	+0·72	+ ·008	− ·002	+ ·003
15	+3·36	+1·23	+2·30	+ ·007	− ·008	− ·001	+0·94	+0·36	+0·65	+ ·005	− ·002	+ ·001
16	+3·12	+0·55	+1·83	+ ·004	− ·006	− ·001	+0·79	+0·18	+0·49	+ ·006	0	+ ·003
17	+2·61	−0·04	+1·29	+ ·002	− ·001	0	+0·56	−0·02	+0·27	+ ·002	0	+ ·001
18	+1·78	−0·31	+0·74	+ ·003	+ ·003	+ ·003	+0·27	−0·09	+0·09	+ ·004	+ ·003	+ ·004
19	+0·92	−0·44	+0·23	+ ·001	+ ·007	+ ·004	+0·04	−0·14	−0·05	+ ·003	+ ·004	+ ·004
20	−0·09	−0·14	−0·09	+ ·008	+ ·020	+ ·014	−0·16	−0·16	−0·16	+ ·007	+ ·005	+ ·004
21	−0·67	−0·11	−0·48	+ ·009	+ ·017	+ ·013	−0·39	−0·24	−0·31	+ ·005	+ ·004	+ ·005
22	−1·26	−0·54	−0·90	+ ·009	+ ·009	+ ·009	−0·48	−0·27	−0·37	+ ·007	+ ·004	+ ·005
23	−1·68	−0·75	−1·21	+ ·006	− ·004	+ ·001	−0·58	−0·29	−0·43	+ ·003	+ ·001	+ ·002
24	−2·06	−0·73	−1·39	+ ·001	+ ·003	+ ·002	−0·71	−0·31	−0·51	+ ·001	− ·003	− ·001

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PLATE I. BEN NEVIS. BAROMETRIC PRESSURE. (SCALE 20 : 1) DIURNAL V

CLEAR WEATHER. FOGGY WEATHER.



:VIS.

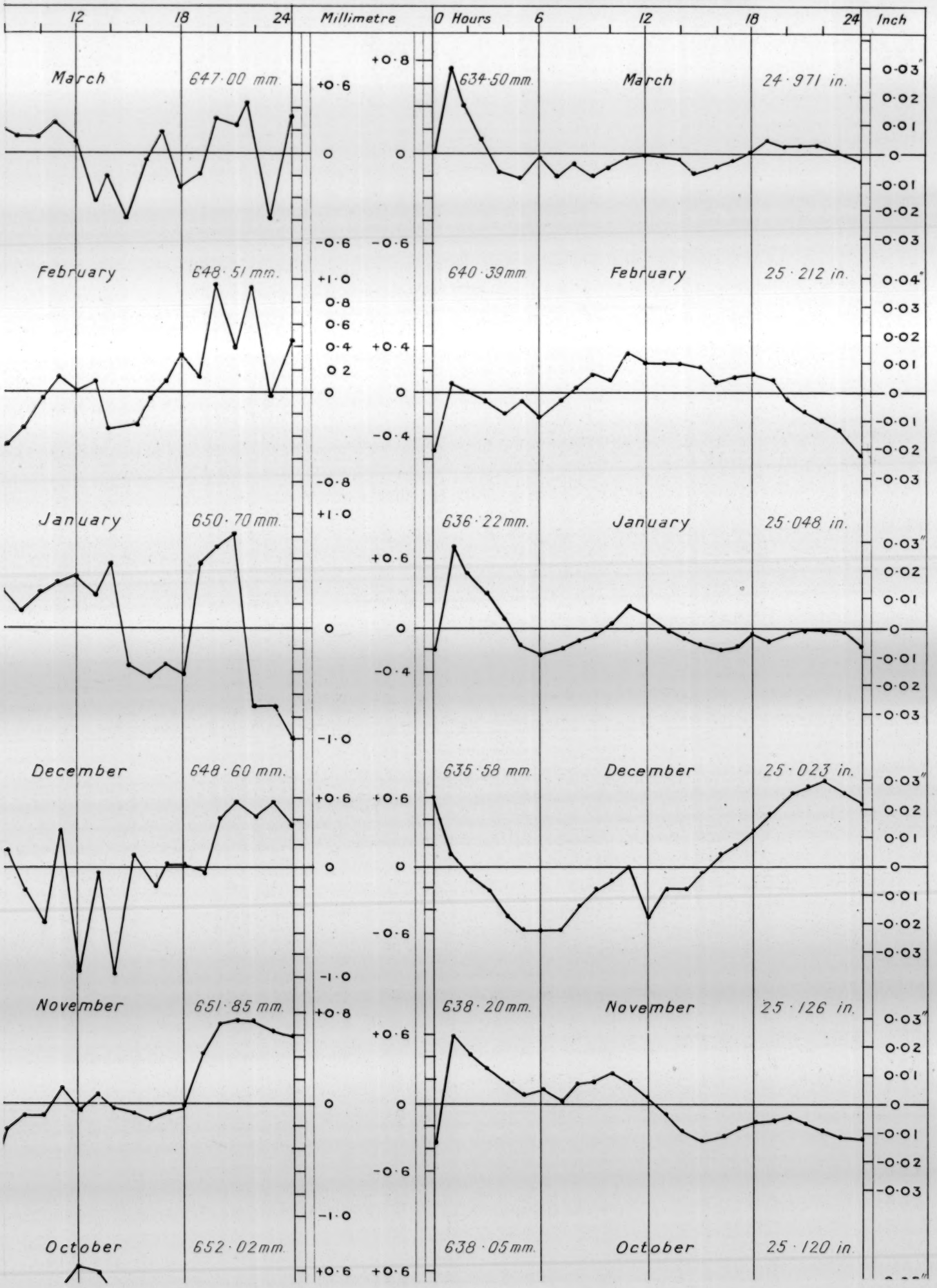
BAROMETRIC PRESSURE.

(SCALE 20 : 1)

DIURNAL VARIATION.

EAR WEATHER.

FOGGY WEATHER.



25·532 in. February 648·51 mm.

25·618 in. January 650·70 mm.

25·536 in. December 648·60 mm.

25·663 in. November 651·85 mm.

25·670 in. October 652·02 mm.

640·39 mm. February 25·21

636·22 mm. January 25·04

635·58 mm. December 25·02

638·20 mm. November 25·12

638·05 mm. October 25·12

Inch 0 Hours 6 12 18 24 Millimetre 0 Hours 6 12 18

February 648·51 mm. 640·39 mm. February 25·212 in.

January 650·70 mm. 636·22 mm. January 25·048 in.

December 648·60 mm. 635·58 mm. December 25·023 in.

November 651·85 mm. 638·20 mm. November 25·126 in.

October 652·02 mm. 638·05 mm. October 25·120 in.

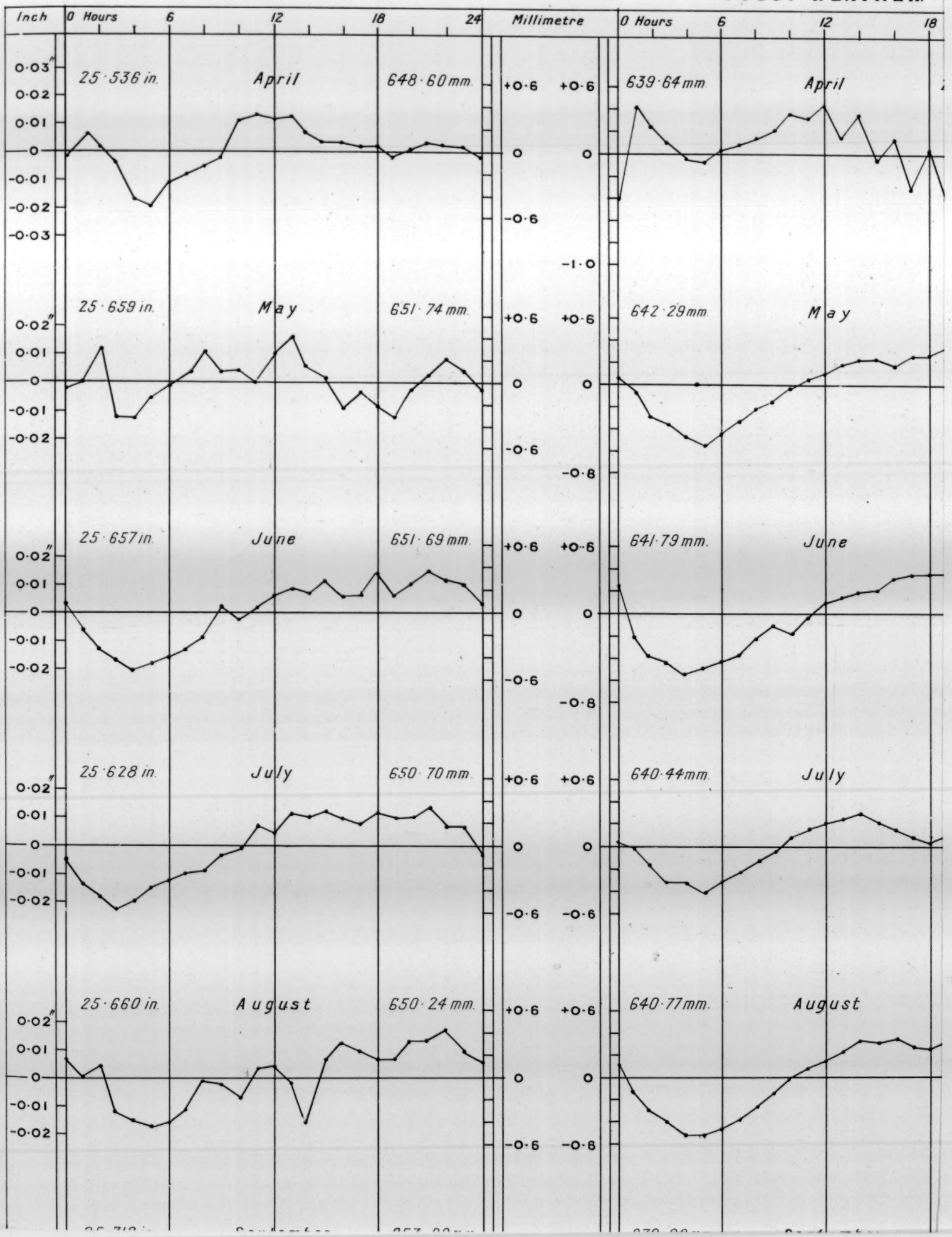
12 18 24 Millimetre 0 Hours 6 12 18 24 Inch

A. RITCHIE & SON EDIN.R

Trans. Roy. Soc. Edin.

PLATE II. BEN NEVIS. BAROMETRIC PRESSURE. (SCALE 20 : 1) DIURNA

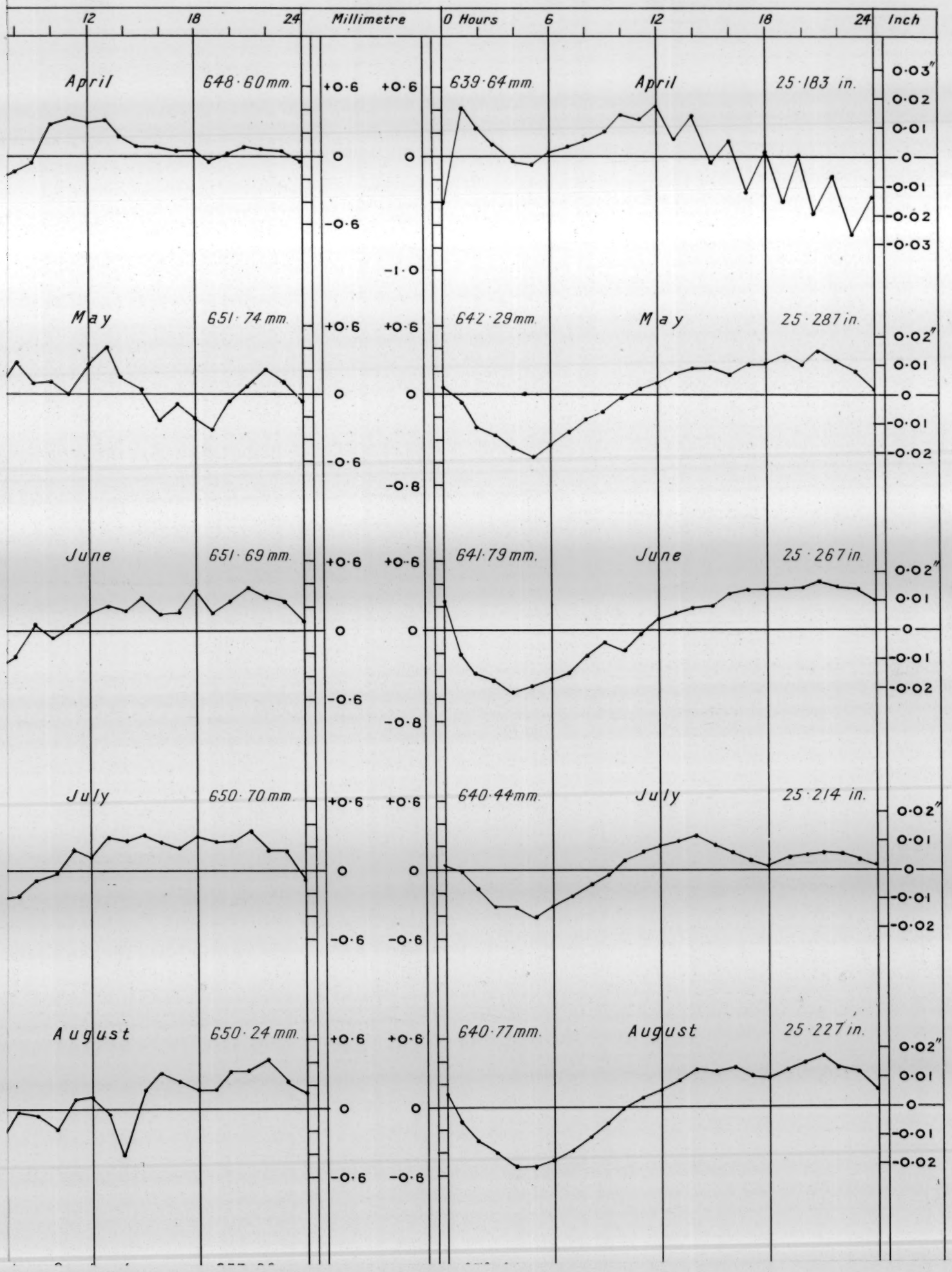
CLEAR WEATHER. FOGGY WEATHER.

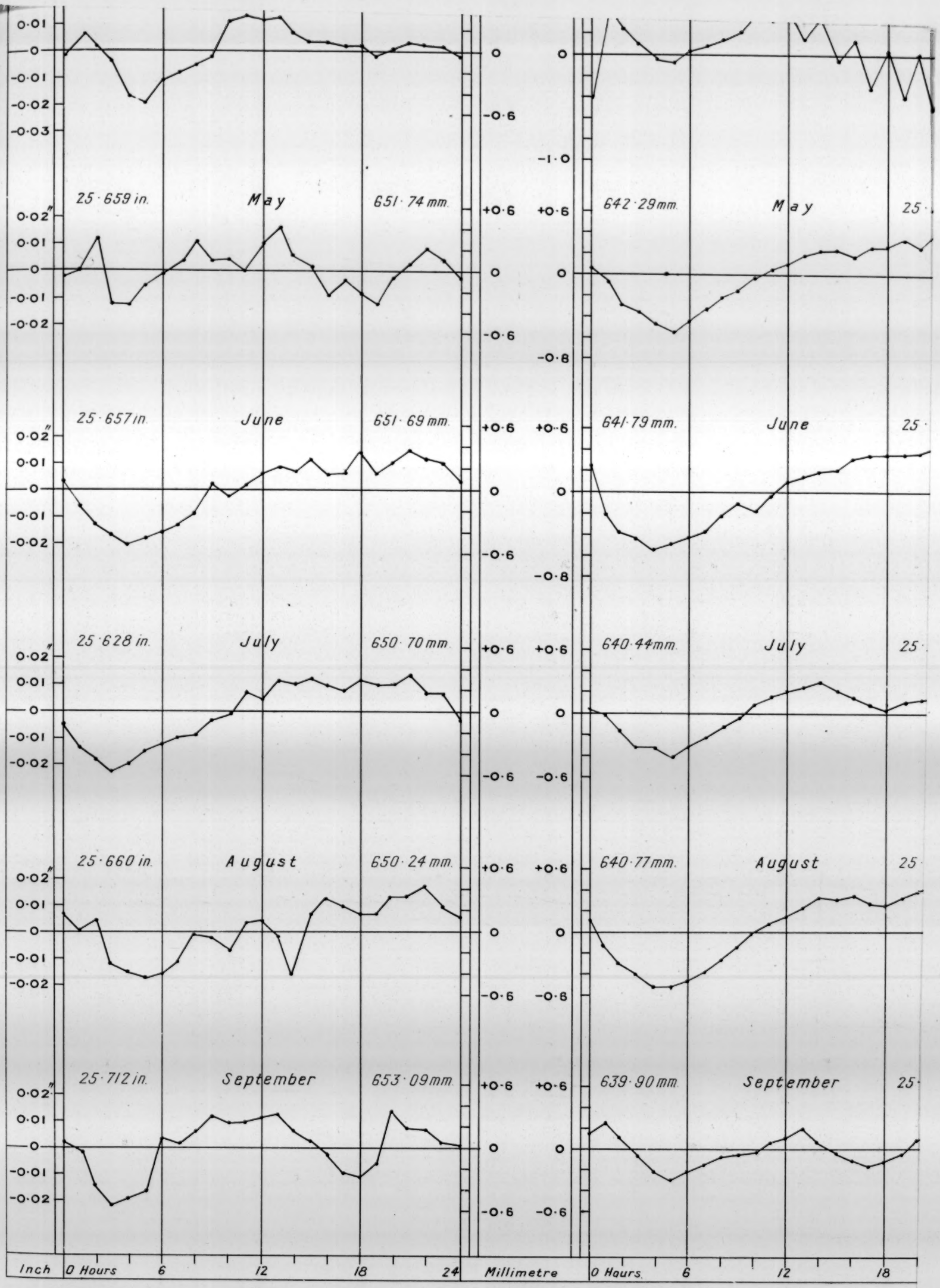


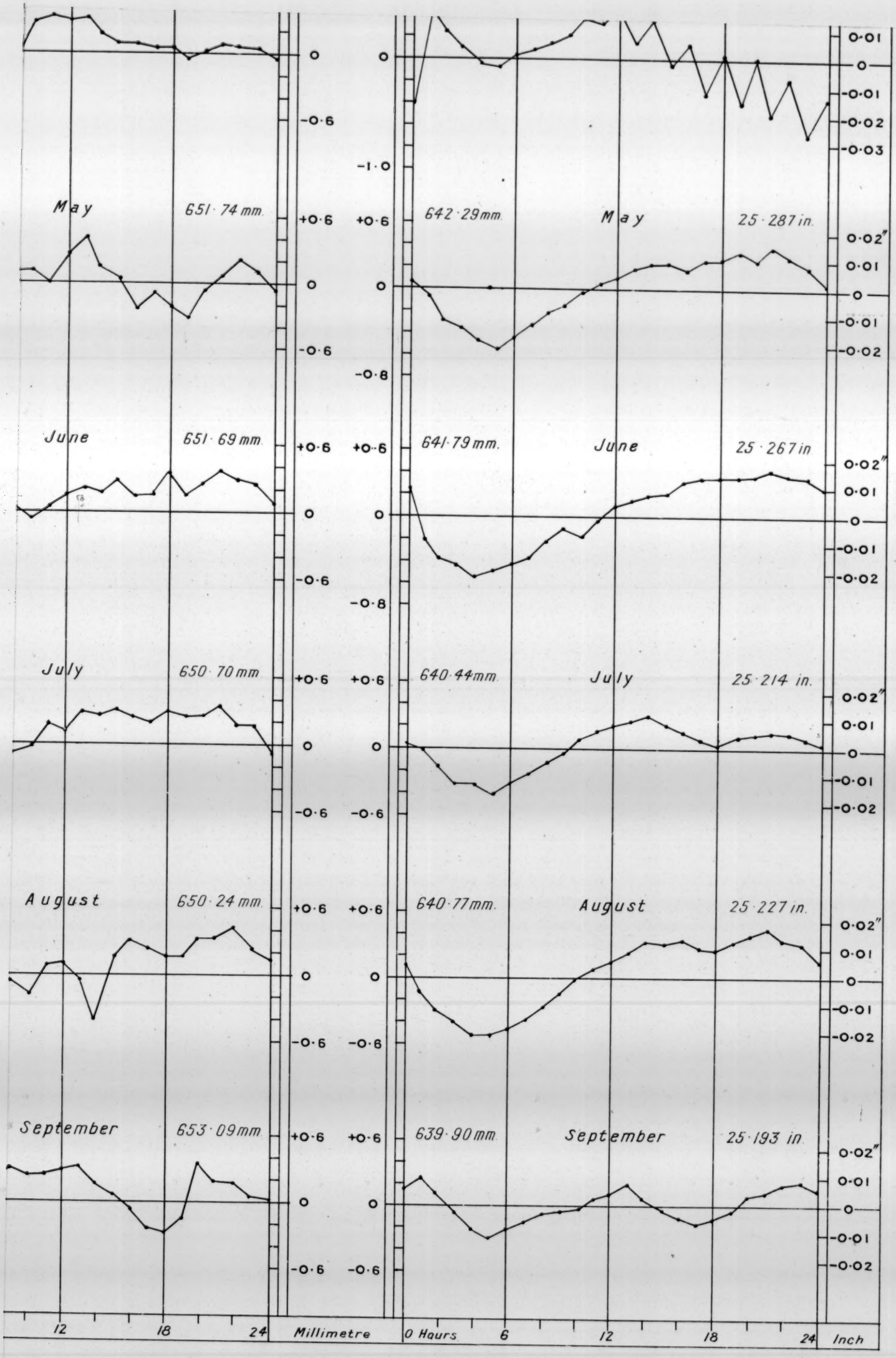
ɛVIS. BAROMETRIC PRESSURE. DIURNAL VARIATION.

(SCALE 20 : 1)

EAR WEATHER. FOGGY WEATHER.



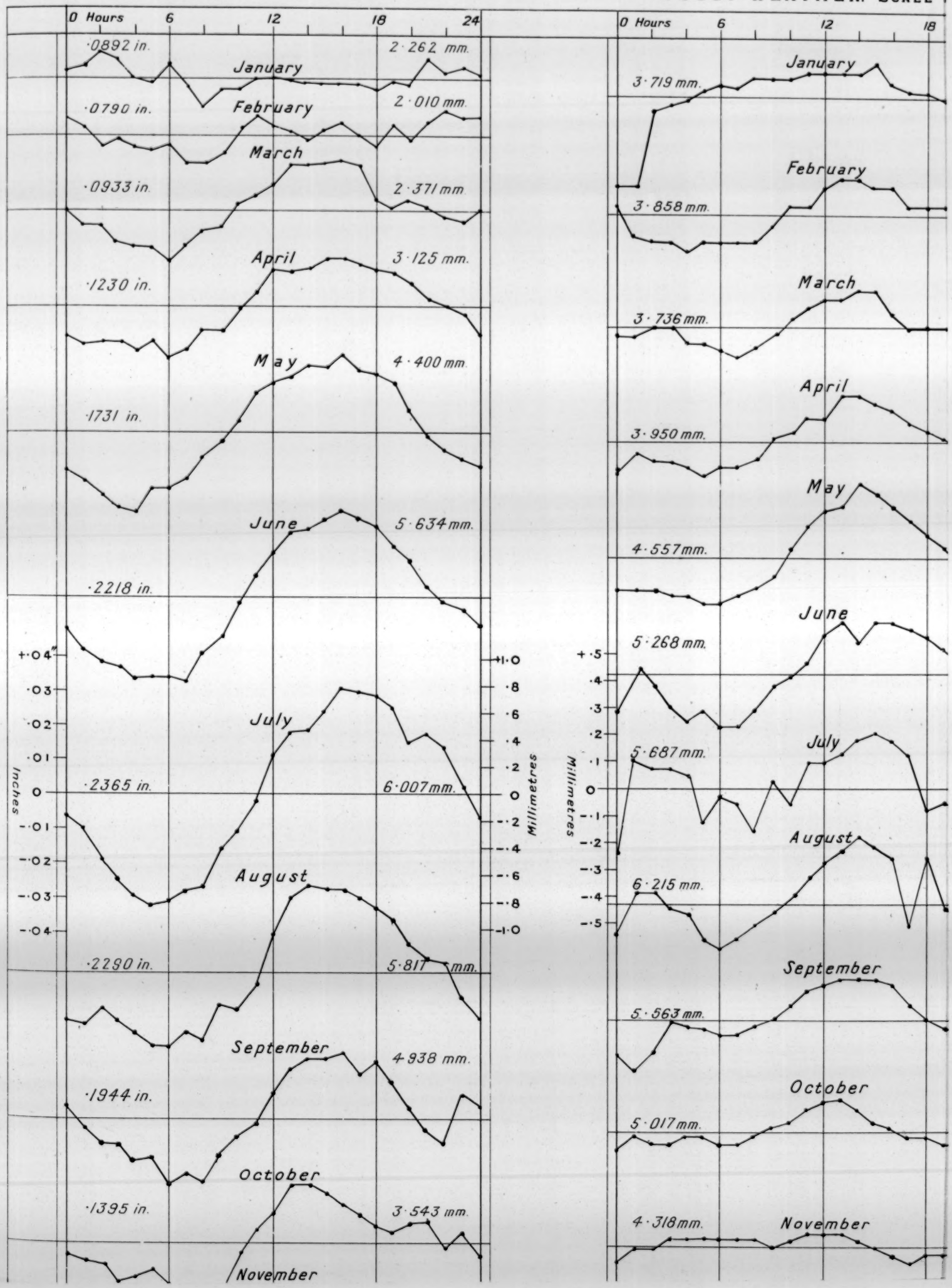

25·659 in.
May
651·74 mm.
642·29 mm.
May
25·657 in.
June
651·69 mm.
641·79 mm.
June
25·628 in.
July
650·70 mm.
640·44 mm.
July
25·660 in.
August
650·24 mm.
640·77 mm.
August
25·712 in.
September
653·09 mm.
639·90 mm.
September
Inch
0 Hours
6
12
18
24
Millimetre
0 Hours
6
12
18


May
651·74 mm.
642·29 mm.
May
25·287 in.
June
651·69 mm.
641·79 mm.
June
25·267 in.
July
650·70 mm.
640·44 mm.
July
25·214 in.
August
650·24 mm.
640·77 mm.
August
25·227 in.
September
653·09 mm.
639·90 mm.
September
25·193 in.
0
-0·6
+0·6
-1·0
-0·8
0·01
-0·01
-0·02
-0·03
0·02"
12
18
24
Millimetre
0 Hours
6
12
18
24
Inch

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PLATE III. BEN NEVIS. VAPOUR TENSION. DIURNA

CLEAR WEATHER. (SCALE 25 : 1) (FOGGY WEATHER. SCALE



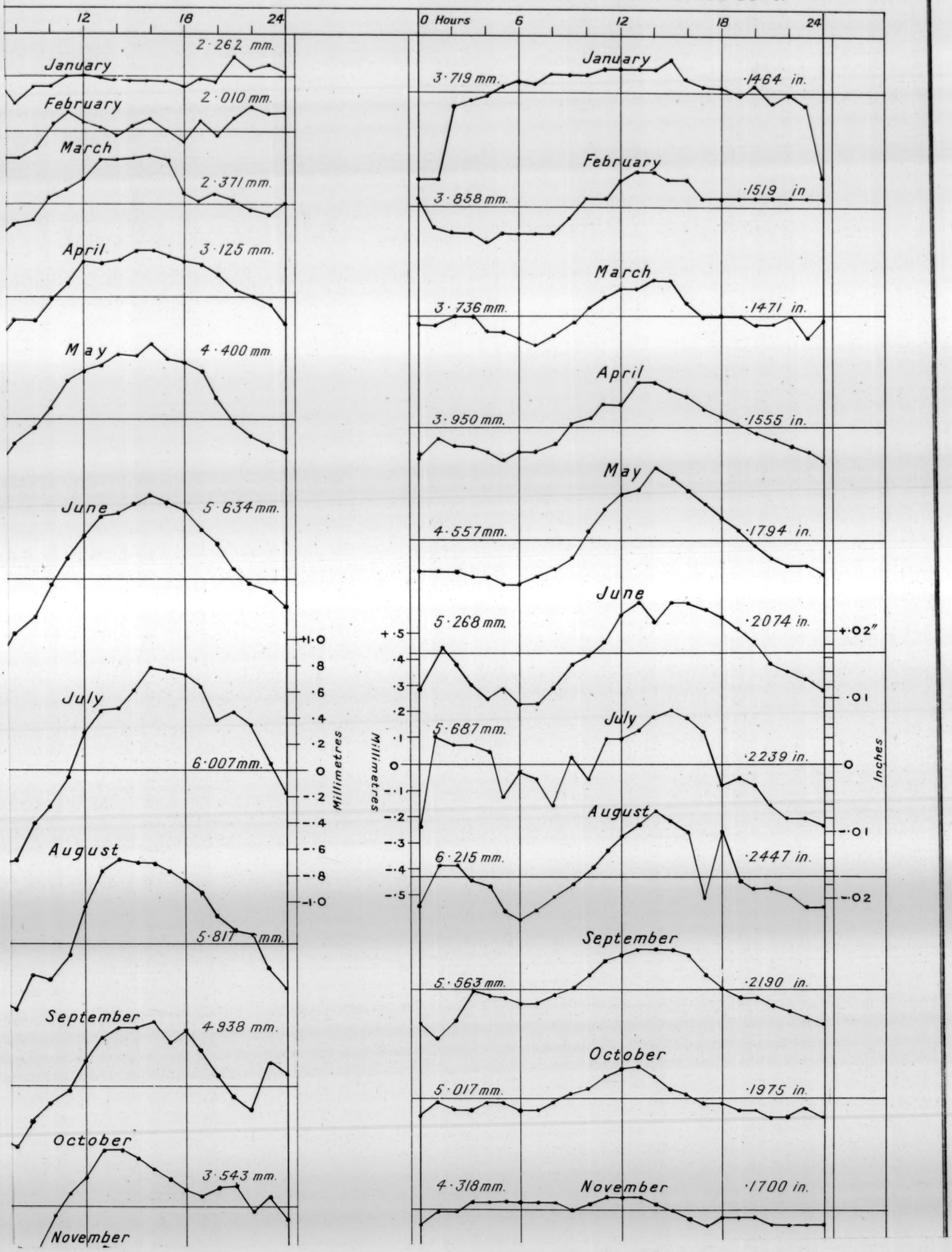
EVIS.

VAPOUR TENSION.

DIURNAL VARIATION.

WEATHER. (SCALE 25 : 1)

(FOGGY WEATHER. SCALE 50 : 1)



·0190 in. February

March
·0933 in. 2·371 mm.

April
·1230 in. 3·125 mm.

May
·1731 in. 4·400 mm.

June
·2218 in. 5·634 mm.

July
·2365 in. 6·007 mm.

August
·2290 in. 5·817 mm.

September
·1944 in. 4·938 mm.

October
·1395 in. 3·543 mm.

November
·1100 in. 2·819 mm.

December
·0848 in. 2·154 mm.

+·04 ·03 ·02 ·01 0 -·01 -·02 -·03 -·04 Inches

+1·0 ·8 ·6 ·4 ·2 0 -·2 -·4 -·6 -·8 -1·0 Millimetres

0 Hours 6 12 18 24

3·858 mm. February ·15

3·736 mm. March ·14

3·950 mm. April ·15

4·557 mm. May ·17

5·268 mm. June ·2

5·687 mm. July ·22

6·215 mm. August ·24

5·563 mm. September ·21

5·017 mm. October ·19

4·318 mm. November ·17

3·754 mm. December ·14

+·5 ·4 ·3 ·2 ·1 0 -·1 -·2 -·3 -·4 -·5 Millimetres

0 Hours 6 12 18

February
March
2·371 mm.
April
3·125 mm.
May
4·400 mm.
June
5·634 mm.
July
6·007 mm.
August
5·817 mm.
September
4·938 mm.
October
3·543 mm.
November
2·819 mm.
December
2·154 mm.
+1·0 ·8 ·6 ·4 ·2 0 -·2 -·4 -·6 -·8 -1·0
Millimetres
6 12 18 24

February
3·858 mm.
·1519 in
March
3·736 mm.
·1471 in.
April
3·950 mm.
·1555 in.
May
4·557 mm.
·1794 in.
June
5·268 mm.
·2074 in.
July
5·687 mm.
·2239 in.
August
6·215 mm.
·2447 in.
September
5·563 mm.
·2190 in.
October
5·017 mm.
·1975 in.
November
4·318 mm.
·1700 in.
December
3·754 mm.
·1478 m.
+·5 ·4 ·3 ·2 ·1 0 -·1 -·2 -·3 -·4 -·5
Millimetres
+·02" ·01 0 -·01 -·02
Inches
0 Hours 6 12 18 24

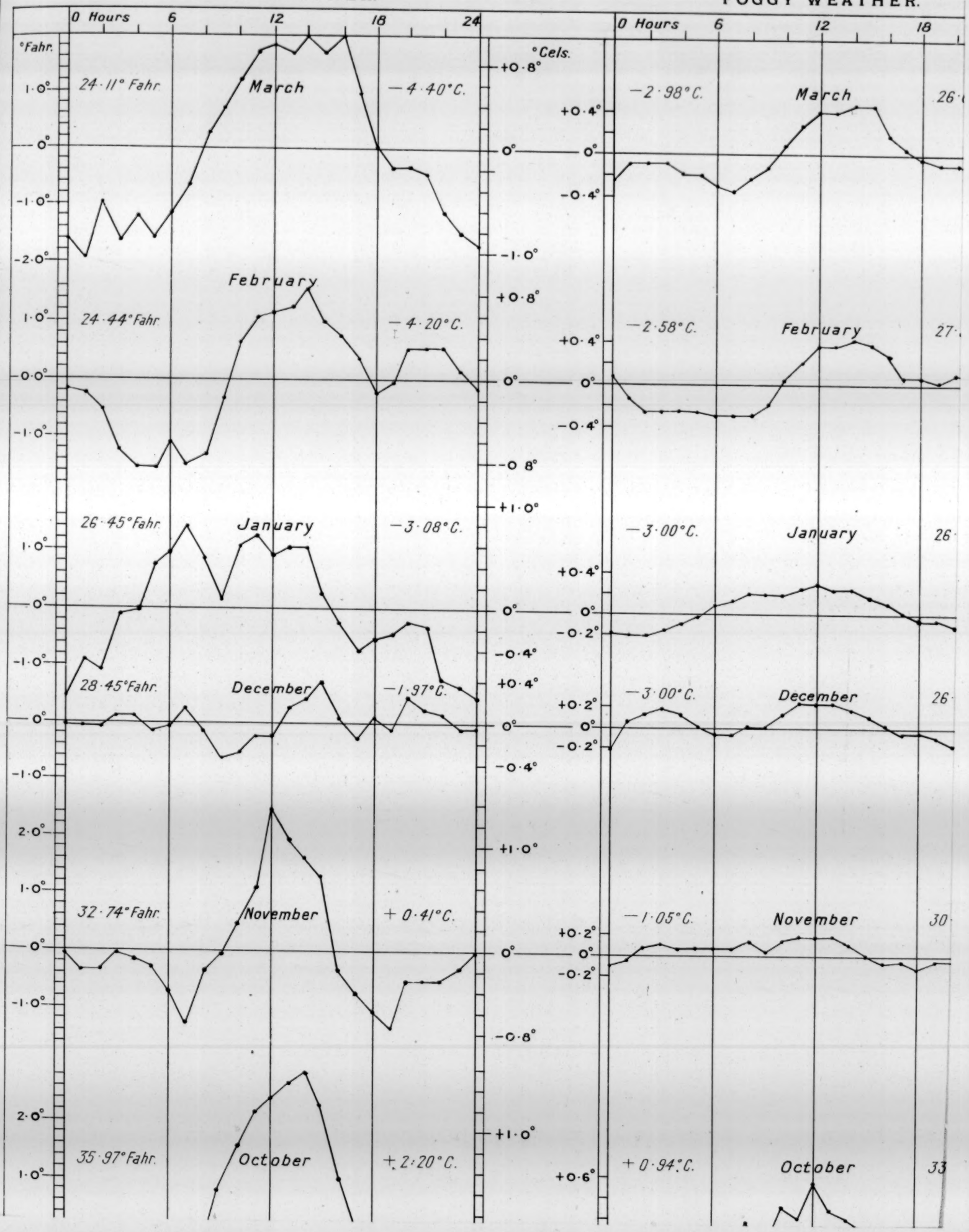
A. H. TOBIE & SON, EDIN.

Trans. Roy. Soc. Edin.

PLATE IV. BEN NEVIS. TEMPERATURE OF THE AIR. DIURNAL

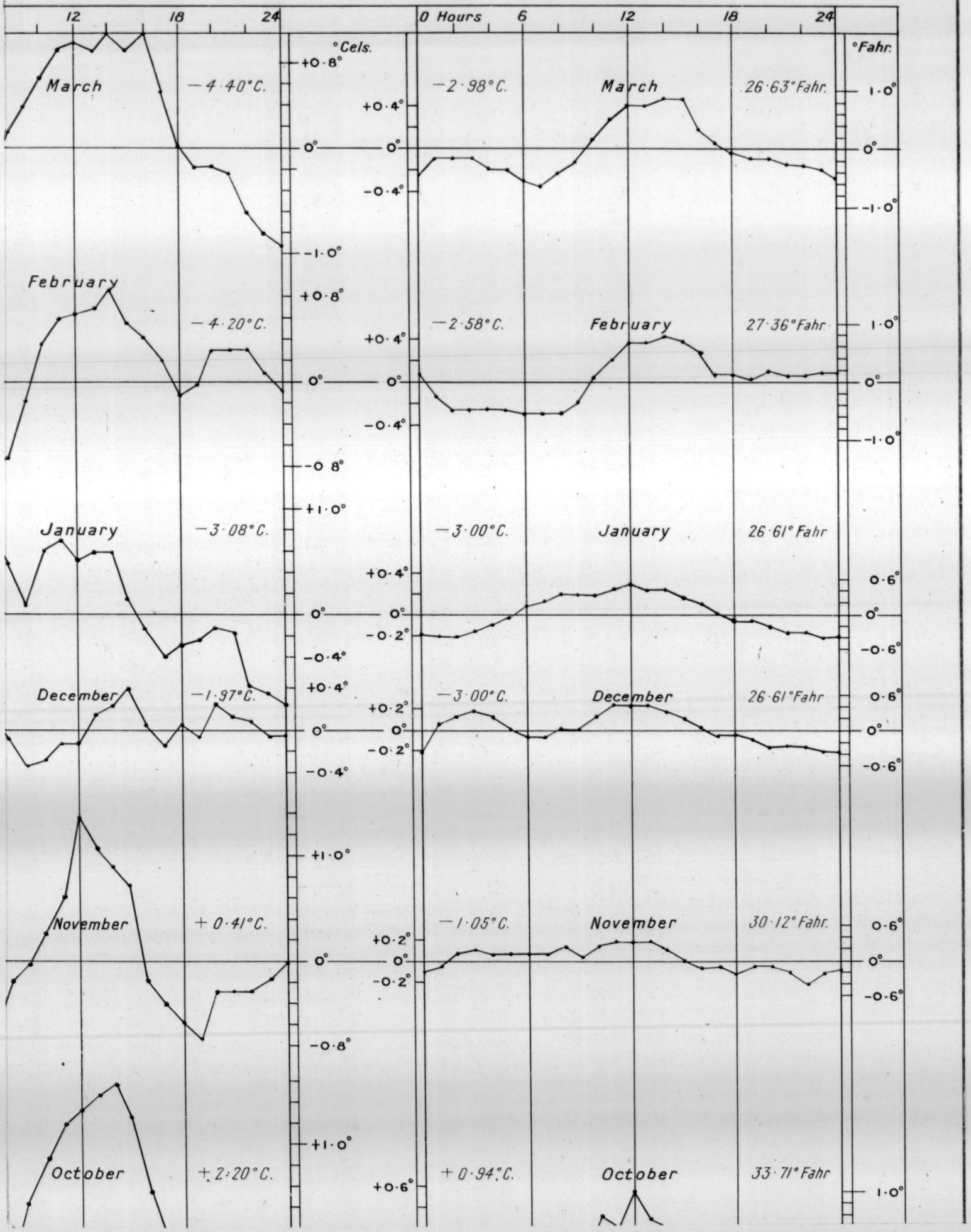
CLEAR WEATHER.

FOGGY WEATHER.



VIS. TEMPERATURE OF THE AIR. DIURNAL VARIATION.

EAR WEATHER. FOGGY WEATHER.



March 24·11° Fahr. −4·40°C.

February 24·44° Fahr. −4·20°C.

January 26·45° Fahr. −3·08°C.

December 28·45° Fahr. −1·97°C.

November 32·74° Fahr. +0·41°C.

October 35·97° Fahr. +2·20°C.

March −2·98°C. 26·63°F

February −2·58°C. 27·36°F

January −3·00°C. 26·61°F

December −3·00°C. 26·61°F

November −1·05°C. 30·12°F

October +0·94°C. 33·71°F

0 Hours 6 12 18 24

March — 4·40°C.

February — 4·20°C.

January — 3·08°C.

December — 1·97°C.

November + 0·41°C.

October + 2·20°C.

March — 2·98°C. 26·63°Fahr.

February — 2·58°C. 27·36°Fahr.

January — 3·00°C. 26·61°Fahr

December — 3·00°C. 26·61°Fahr

November — 1·05°C. 30·12°Fahr.

October + 0·94°C. 33·71°Fahr

0 Hours 6 12 18 24

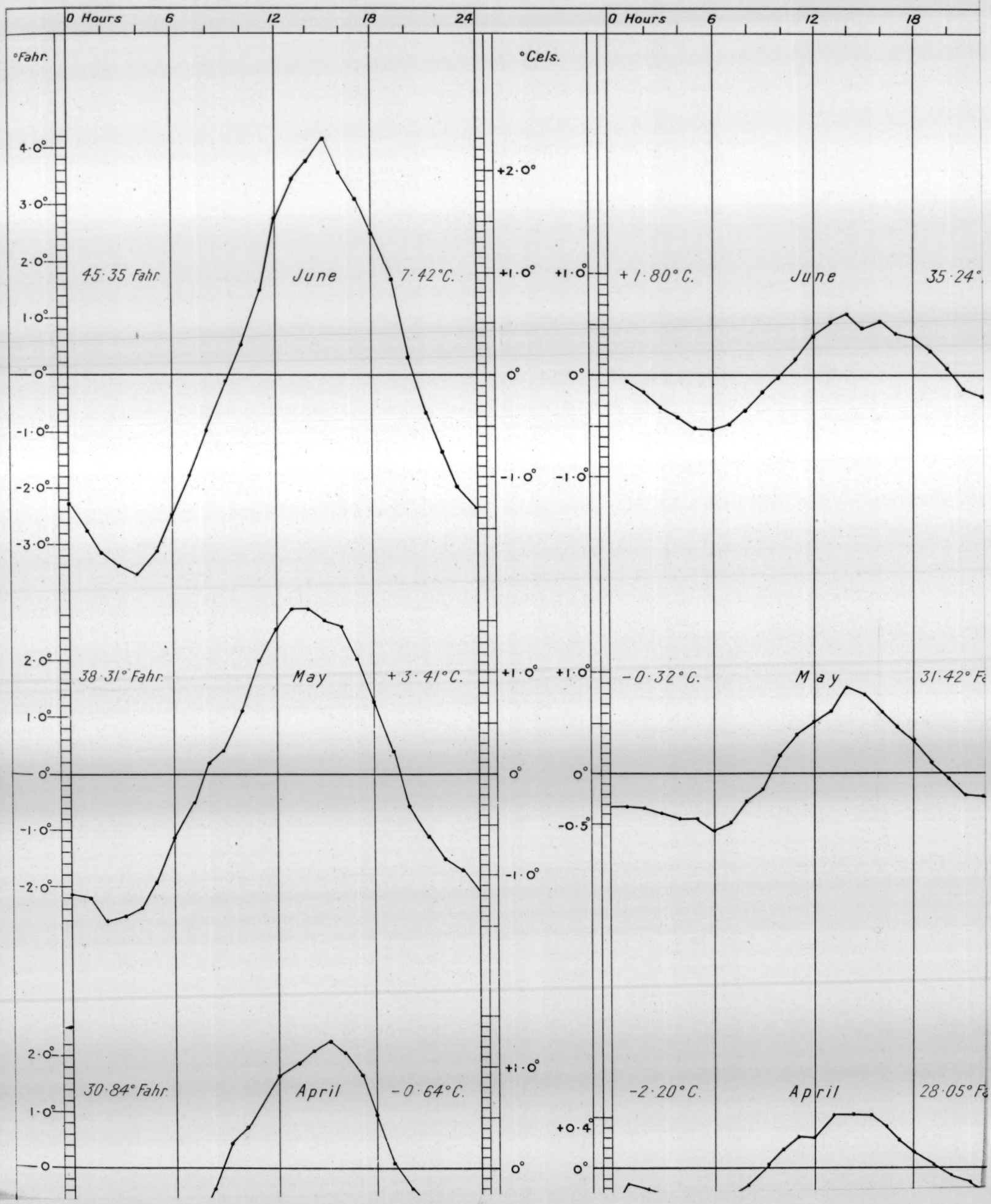
12 18 24

A. RITCHIE & SON EDIN.

PLATE V. BEN NEVIS. TEMPERATURE OF THE AIR. DIURNAL V

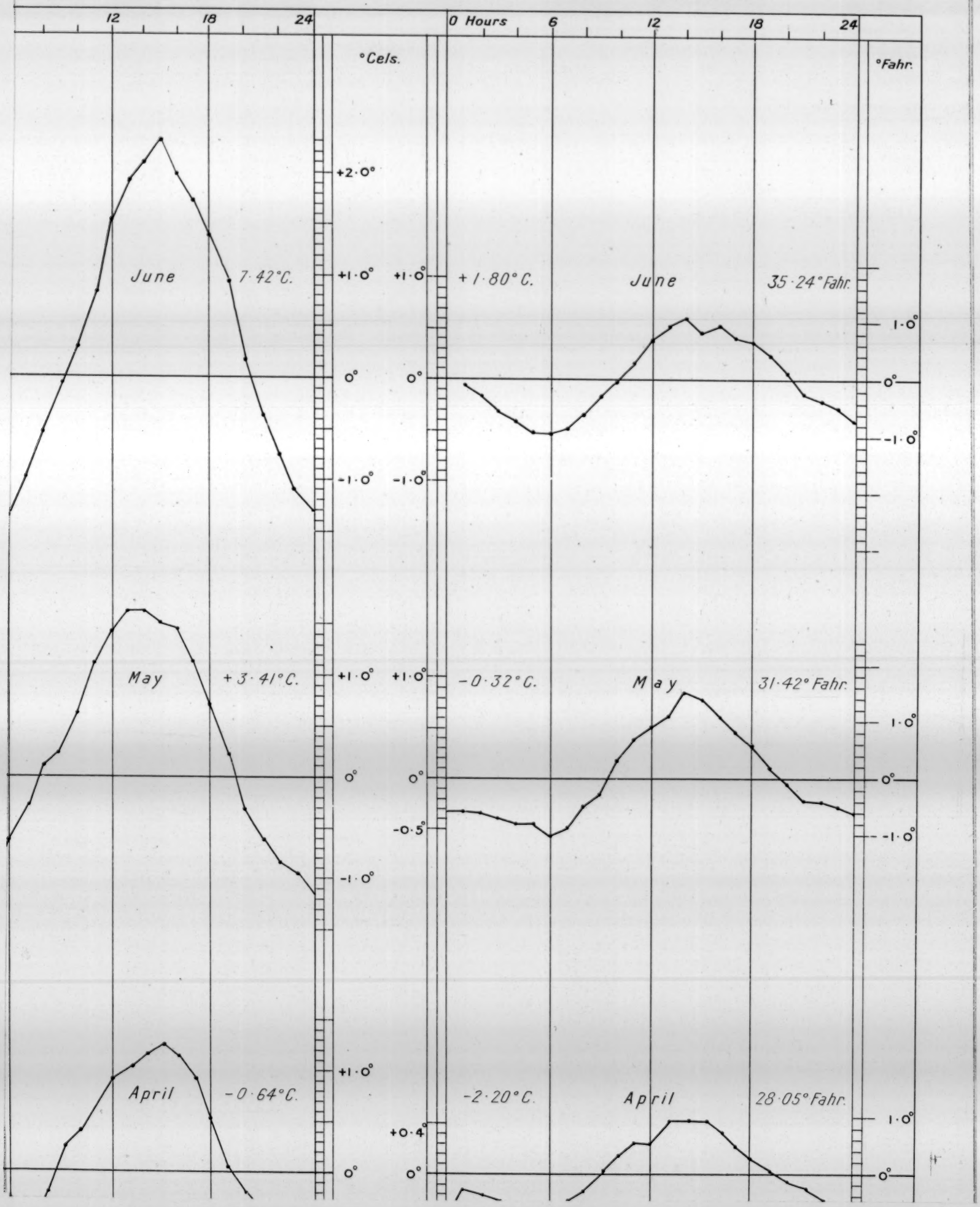
CLEAR WEATHER.

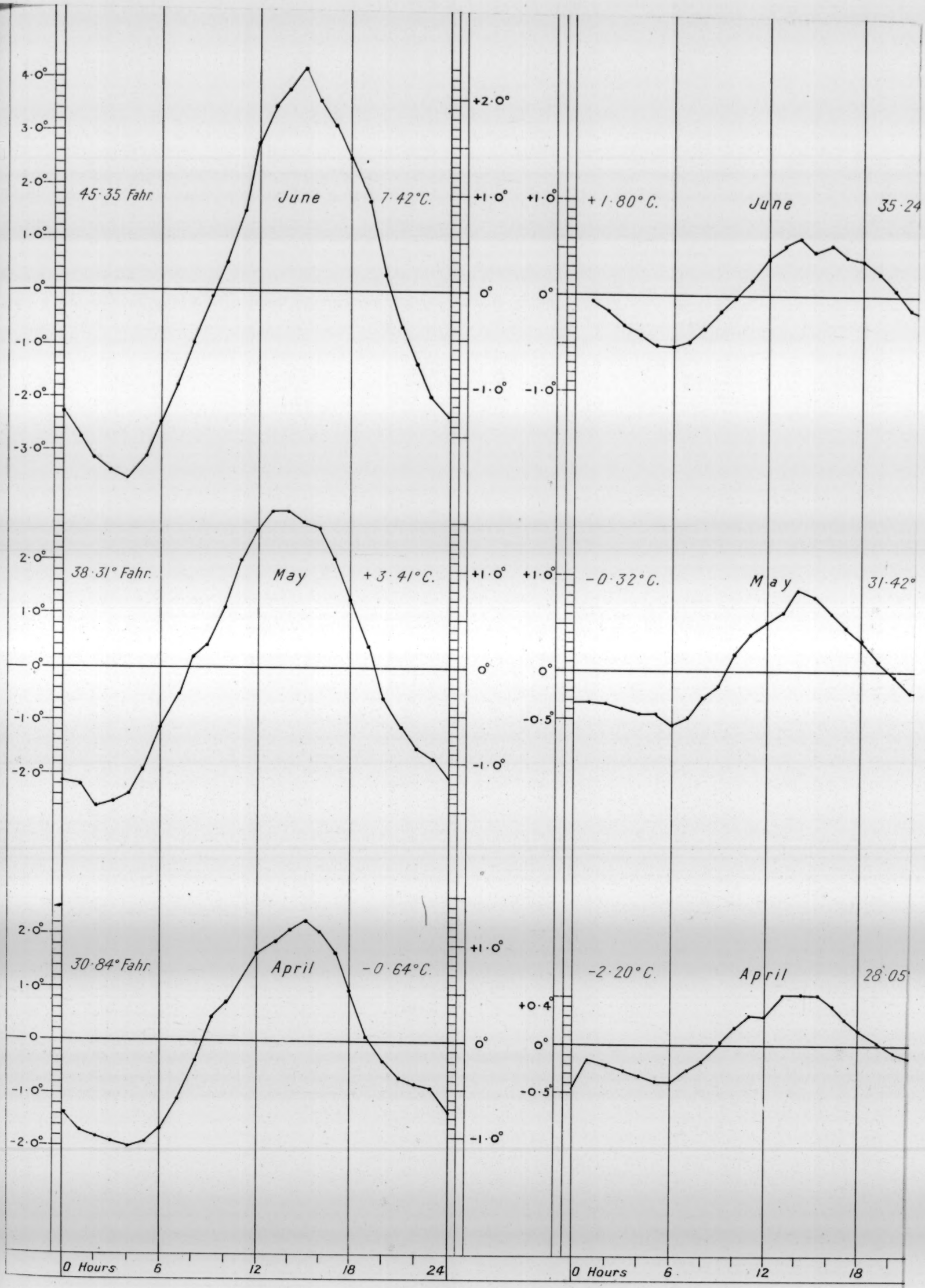
FOGGY WEATHER.



NEVIS. TEMPERATURE OF THE AIR. DIURNAL VARIATION.

CLEAR WEATHER. FOGGY WEATHER.




4·0°
3·0°
2·0°
1·0°
0°
-1·0°
-2·0°
-3·0°
45·35 Fahr.
June
7·42°C.
+2·0°
+1·0°
0°
-1·0°
+1·0°
+1·80°C.
June
35·24
0°
-1·0°
2·0°
1·0°
0°
-1·0°
-2·0°
38·31° Fahr.
May
+3·41°C.
+1·0°
0°
-1·0°
+1·0°
-0·32°C.
May
31·42°
0°
-0·5°
2·0°
1·0°
0
-1·0°
-2·0°
30·84° Fahr.
April
-0·64°C.
+1·0°
0°
-1·0°
-2·20°C.
April
28·05°
+0·4°
0°
-0·5°
0 Hours
6
12
18
24
0 Hours
6
12
18

June 7·42°C.

+2·0° +1·0° 0° -1·0°

June +1·80°C. June 35·24°Fahr.

+1·0° 0° -1·0° 1·0° 0° -1·0°

May +3·41°C.

+1·0° 0° -1·0°

May -0·32°C. May 31·42°Fahr.

+1·0° 0° -0·5° 1·0° 0° -1·0°

April -0·64°C.

+1·0° 0° -1·0°

April -2·20°C. April 28·05°Fahr.

+0·4° 0° -0·5° 1·0° 0° -1·0°

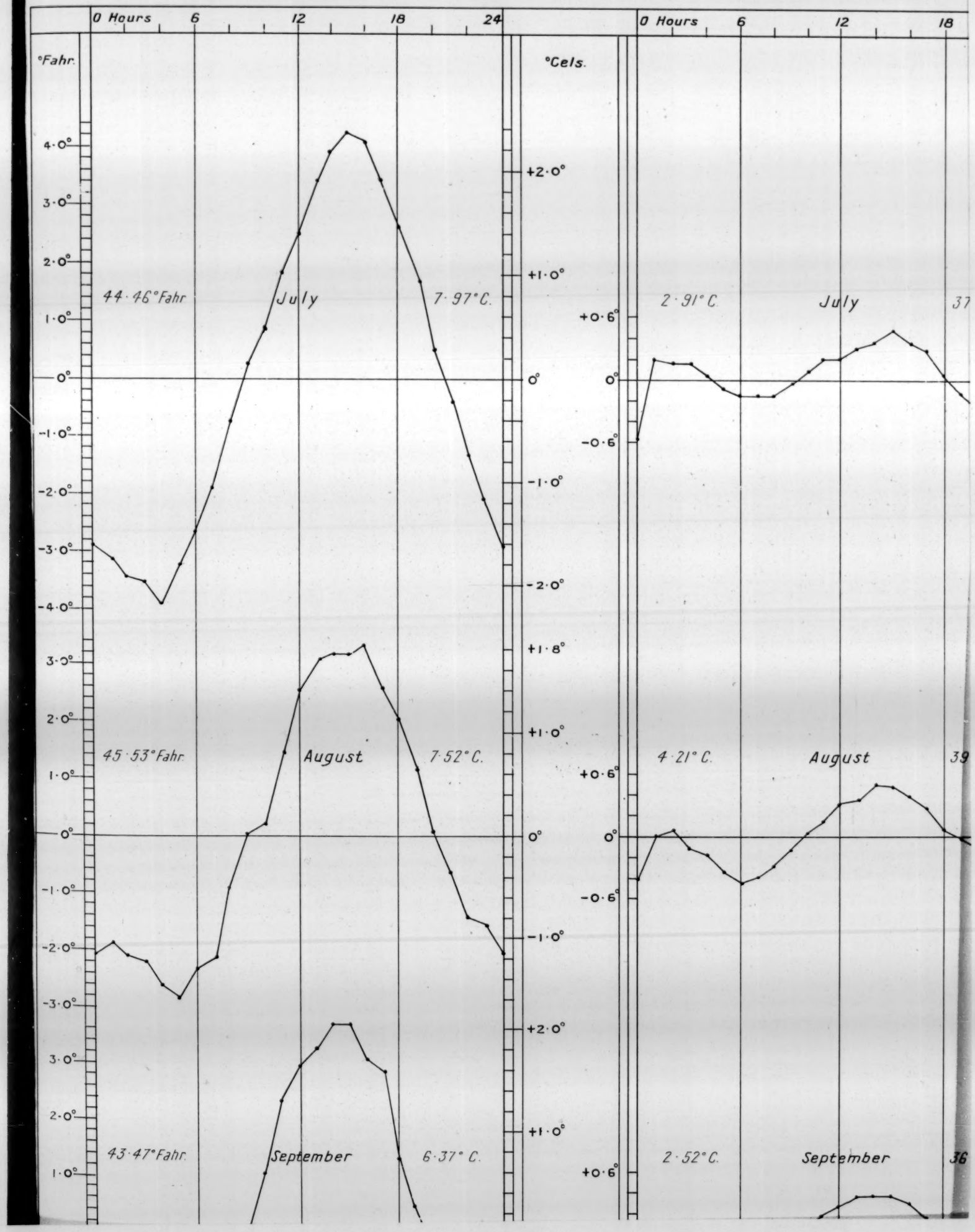
12 18 24 0 Hours 6 12 18 24

A REICHERT & SON EDIN.

PLATE VI. BEN NEVIS. TEMPERATURE OF THE AIR. DIURNA

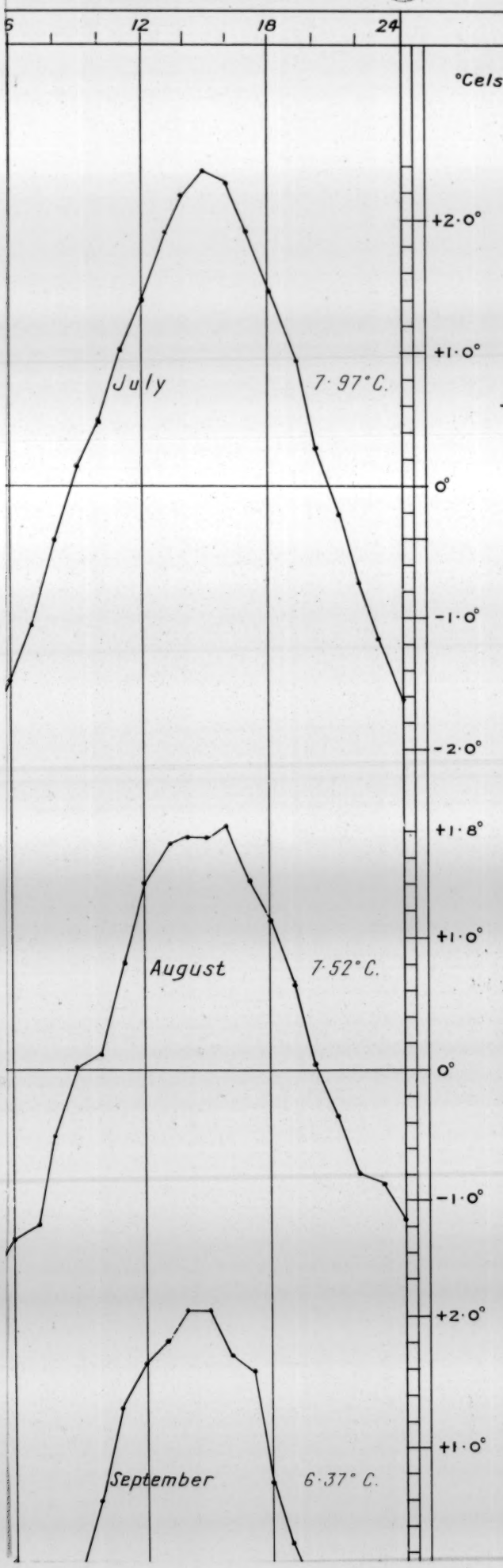
CLEAR WEATHER.

FOGGY WEATHER.

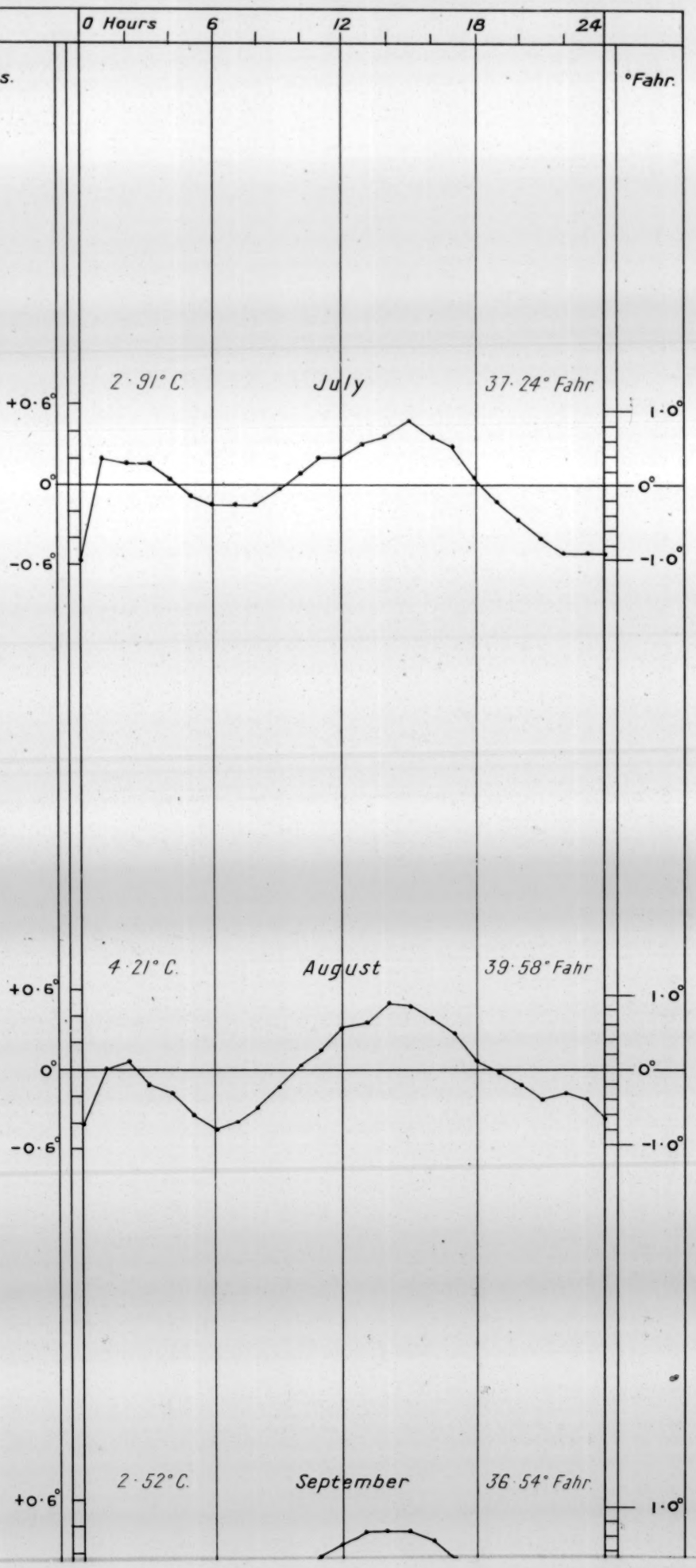


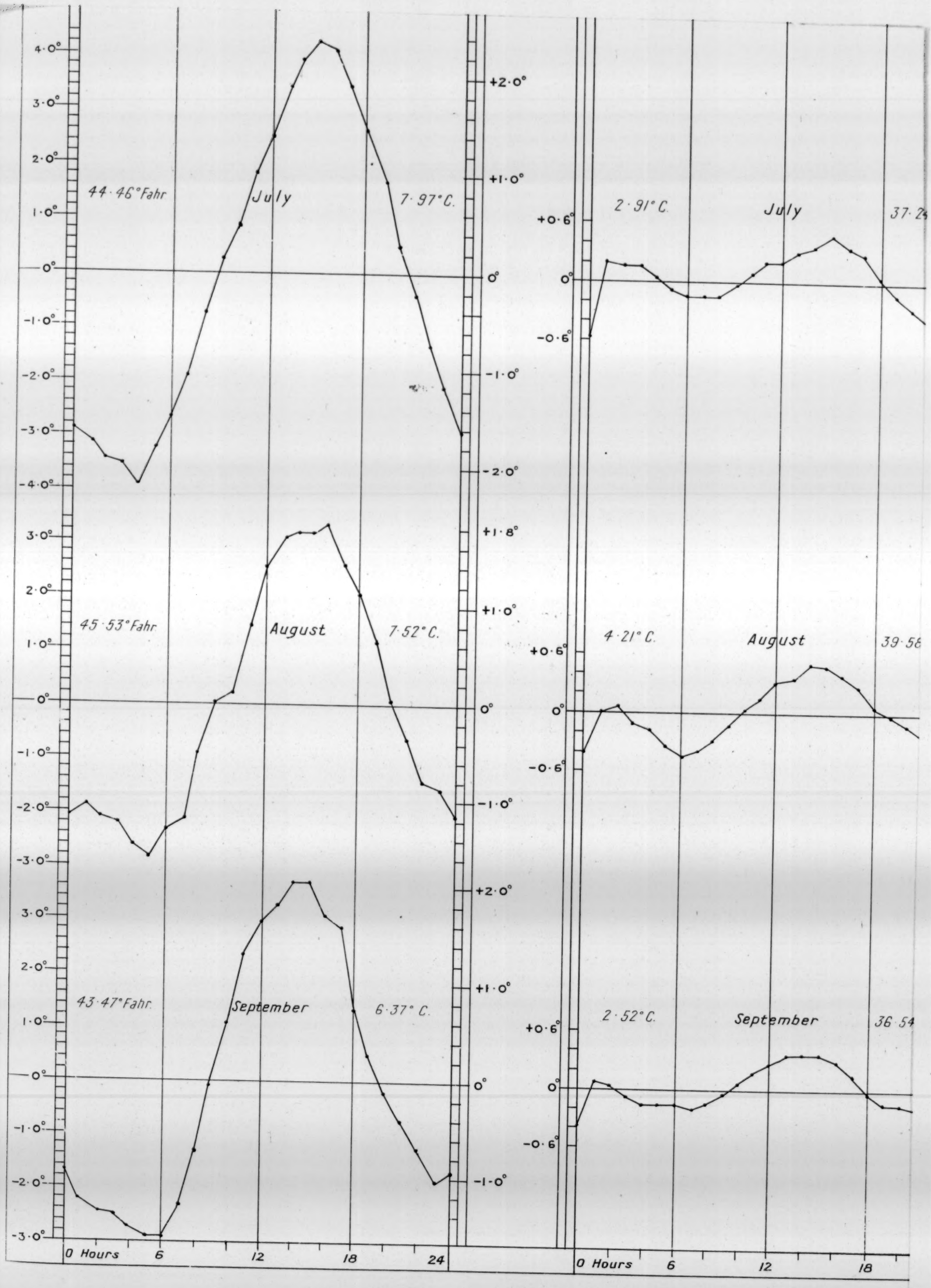
NEVIS. TEMPERATURE OF THE AIR. DIURNAL VARIATION.

CLEAR WEATHER.



FOGGY WEATHER.




4·0°
3·0°
2·0°
1·0°
0°
-1·0°
-2·0°
-3·0°
-4·0°
44·46°Fahr.
July
7·97°C.
+2·0°
+1·0°
0°
-1·0°
-2·0°
3·0°
2·0°
1·0°
0°
-1·0°
-2·0°
-3·0°
45·53°Fahr.
August
7·52°C.
+1·8°
+1·0°
0°
-1·0°
-2·0°
3·0°
2·0°
1·0°
0°
-1·0°
-2·0°
-3·0°
43·47°Fahr.
September
6·37°C.
+2·0°
+1·0°
0°
-1·0°
0 Hours
6
12
18
24
+0·6°
0°
-0·6°
2·91°C.
July
37·2
+0·6°
0°
-0·6°
4·21°C.
August
39·58
+0·6°
0°
-0·6°
2·52°C.
September
36·54
0 Hours
6
12
18

July 7·97°C.

August 7·52°C.

September 6·37°C.

+2·0° +1·0° 0° -1·0° -2·0° +1·8° +1·0° 0° -1·0° +2·0° +1·0° 0° -1·0°

12 18 24

July 2·91°C. 37·24° Fahr.

August 4·21°C. 39·58° Fahr.

September 2·52°C. 36·54° Fahr.

+0·6° 0° -0·6° 1·0° 0° -1·0°

0 Hours 6 12 18 24

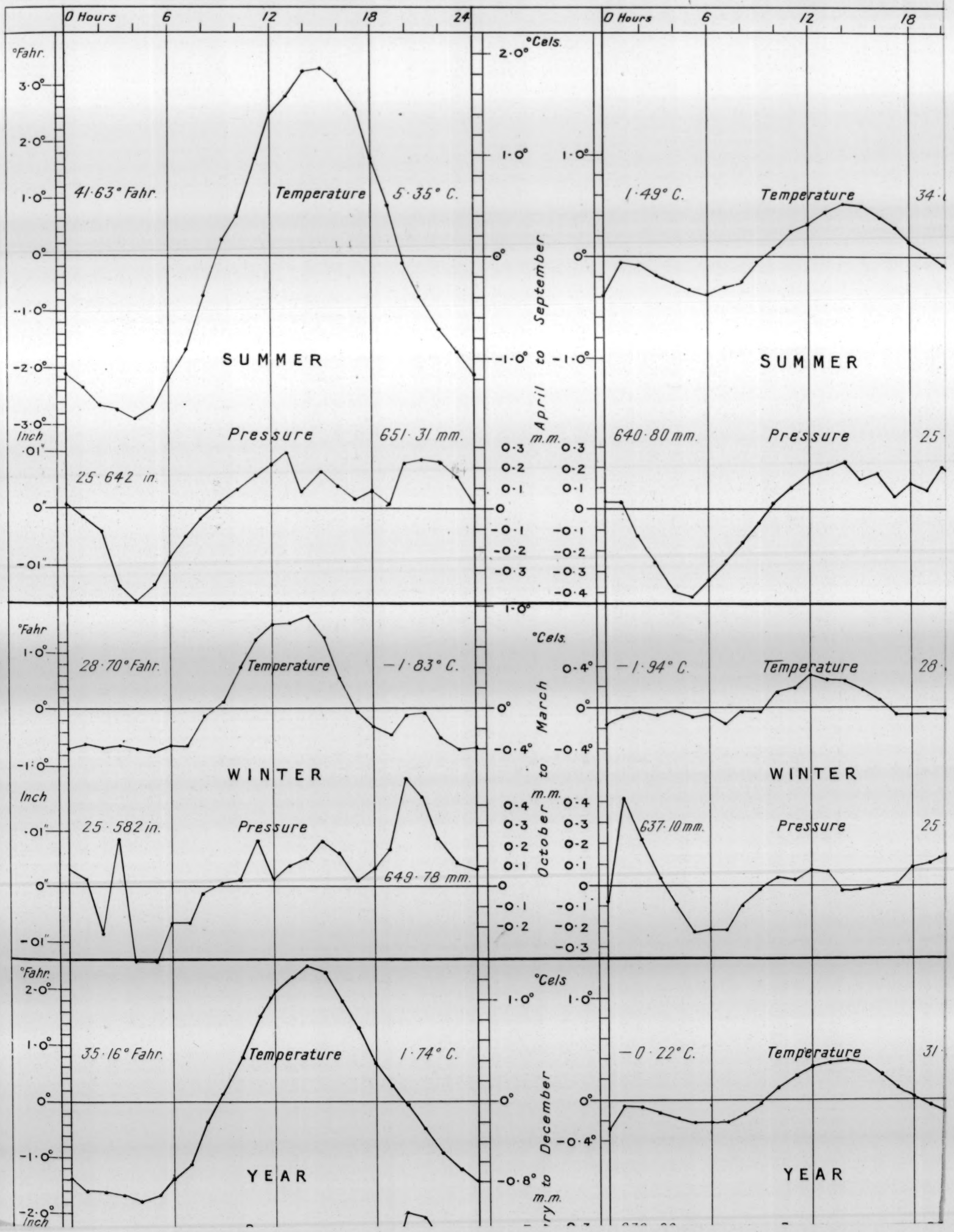
A. RITCHIE & SON EDIN.

Trans. Roy. Soc. Edin.

PLATE VII. BEN NEVIS. TEMPERATURE AND PRESSURE IN SEASONS. DIURNA

(SCALE OF PRESSURE 40 : 1)

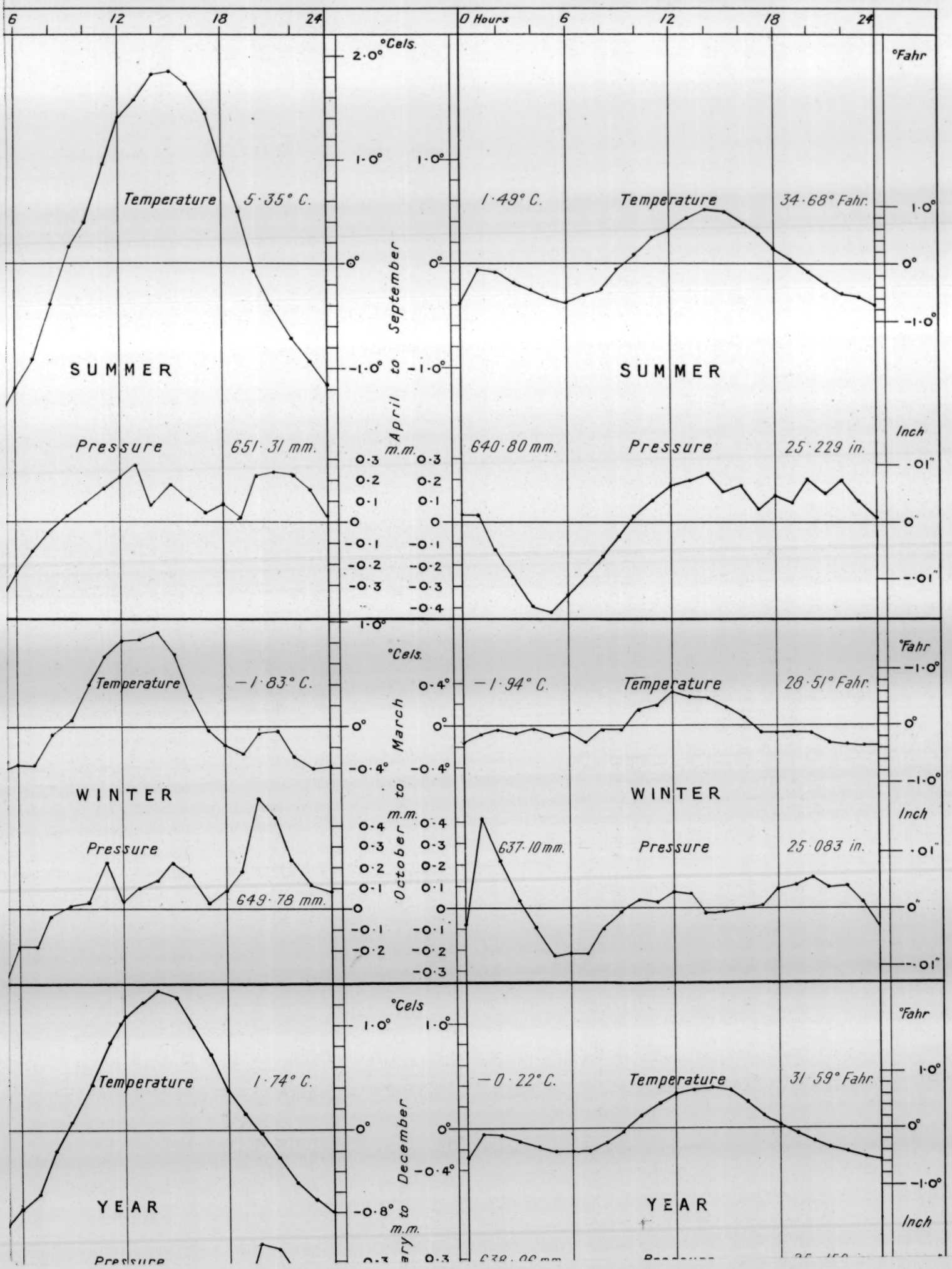
CLEAR WEATHER. FOGGY WEATHER.

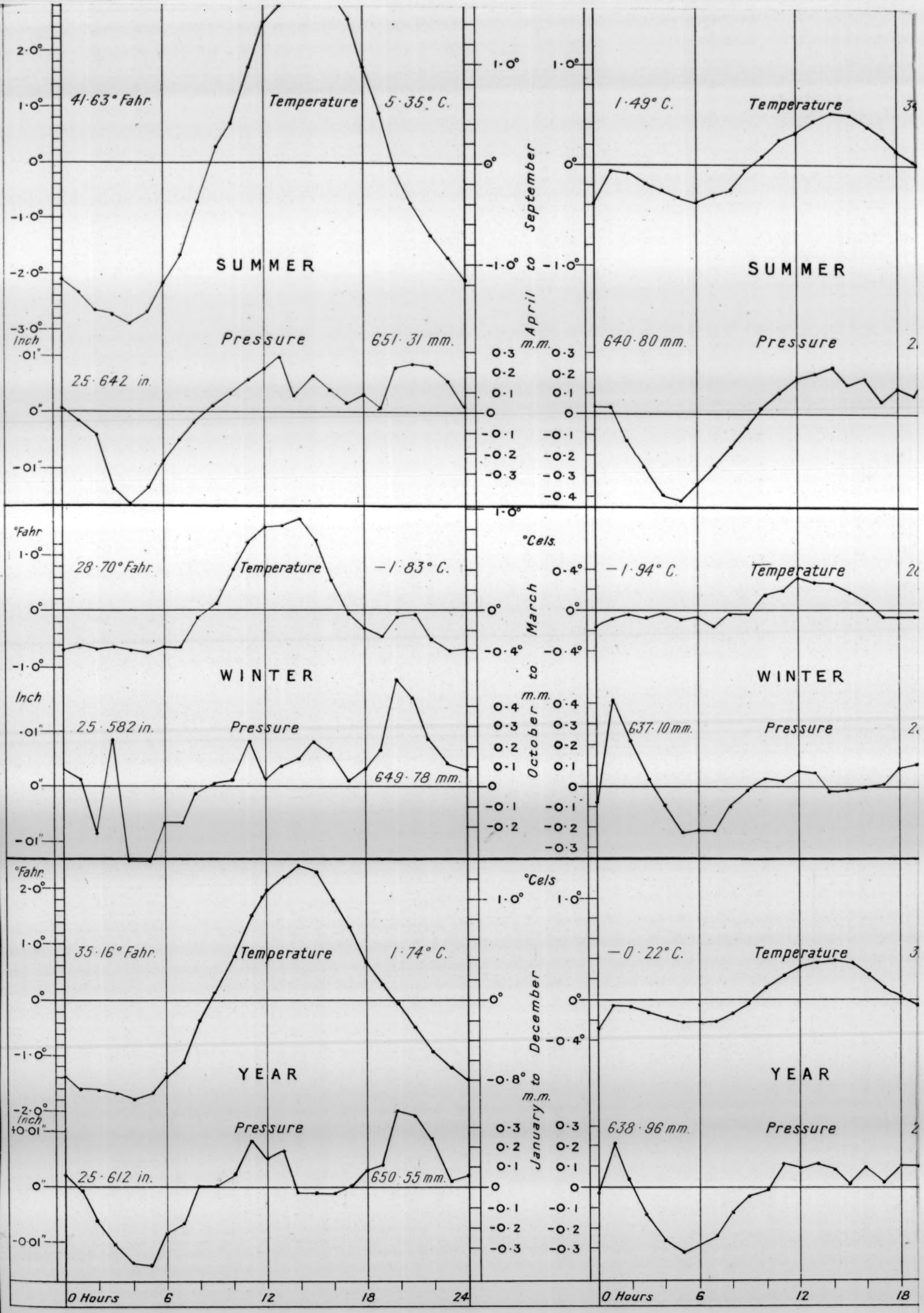


NEVIS. TEMPERATURE AND PRESSURE IN SEASONS. DIURNAL VARIATION.

(SCALE OF PRESSURE 40 : 1)

CLEAR WEATHER. FOGGY WEATHER.




41·63° Fahr.
Temperature
5·35° C.
SUMMER
Pressure
651·31 mm.
25·642 in.
1·49° C.
Temperature
SUMMER
640·80 mm.
Pressure
April to September
m.m.
°Fahr.
28·70° Fahr.
Temperature
−1·83° C.
WINTER
Inch
25·582 in.
Pressure
649·78 mm.
−1·94° C.
Temperature
WINTER
637·10 mm.
Pressure
°Cels.
March to October
m.m.
°Fahr.
35·16° Fahr.
Temperature
1·74° C.
YEAR
Inch
Pressure
25·612 in.
650·55 mm.
−0·22° C.
Temperature
YEAR
638·96 mm.
Pressure
°Cels
December to January
m.m.
0 Hours
6
12
18
24

SUMMER

Temperature 5·35° C.

Pressure 651·31 mm.

SUMMER

1·49° C. Temperature 34·68° Fahr.

640·80 mm. Pressure 25·229 in.

April to September

WINTER

Temperature −1·83° C.

Pressure 649·78 mm.

WINTER

−1·94° C. Temperature 28·51° Fahr.

637·10 mm. Pressure 25·083 in.

October to March

YEAR

Temperature 1·74° C.

Pressure 650·55 mm.

YEAR

−0·22° C. Temperature 31·59° Fahr.

638·96 mm. Pressure 25·156 in

January to December

°Cels. m.m. °Fahr Inch

0 Hours 6 12 18 24

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PLATE VIII. BEN NEVIS. TEMPERATURE PRESSURE AND VAPOUR TENSION. YEAR

(SCALE OF PRESSURE AND VAPOUR TENSION 5:1)

CLEAR WEATHER. FOGGY WEATHER.

D J F M A My J Jy A S O N D D J F M A My J Jy A S

°Fahr. °Cels.

+10·0° +6·0° +5·0° +5·0°

+8·0° 35·16° Fahr. Temperature 1·76° C. +4·0° +4·0° −0·28°C. Temperature 31

+6·0° +3·0° +3·0°

+4·0° +2·0° +2·0°

+2·0° +1·0° +1·0°

0° 0° 0°

-2·0° -1·0° -1·0°

-4·0° -2·0° -2·0°

-6·0° -3·0° -3·0°

-8·0° -4·0°

-10·0° -5·0°

-6·0°

Inch Millimetre

25·612 in. Pressure 650·55 mm. 638·95 mm. Pressure 25

+0·1" +3·0 +3·0

+2·0 +2·0

+1·0 +1·0

0 0 0

-0·1" -1·0 -1·0

-2·0 -2·0

-3·0 -3·0

-4·0 -4·0

0·1479 in. Vapour Tension 3·755 mm. 4·630 mm. Vapour Tension 0

+0·1" +3·0 +3·0

+2·0 +2·0

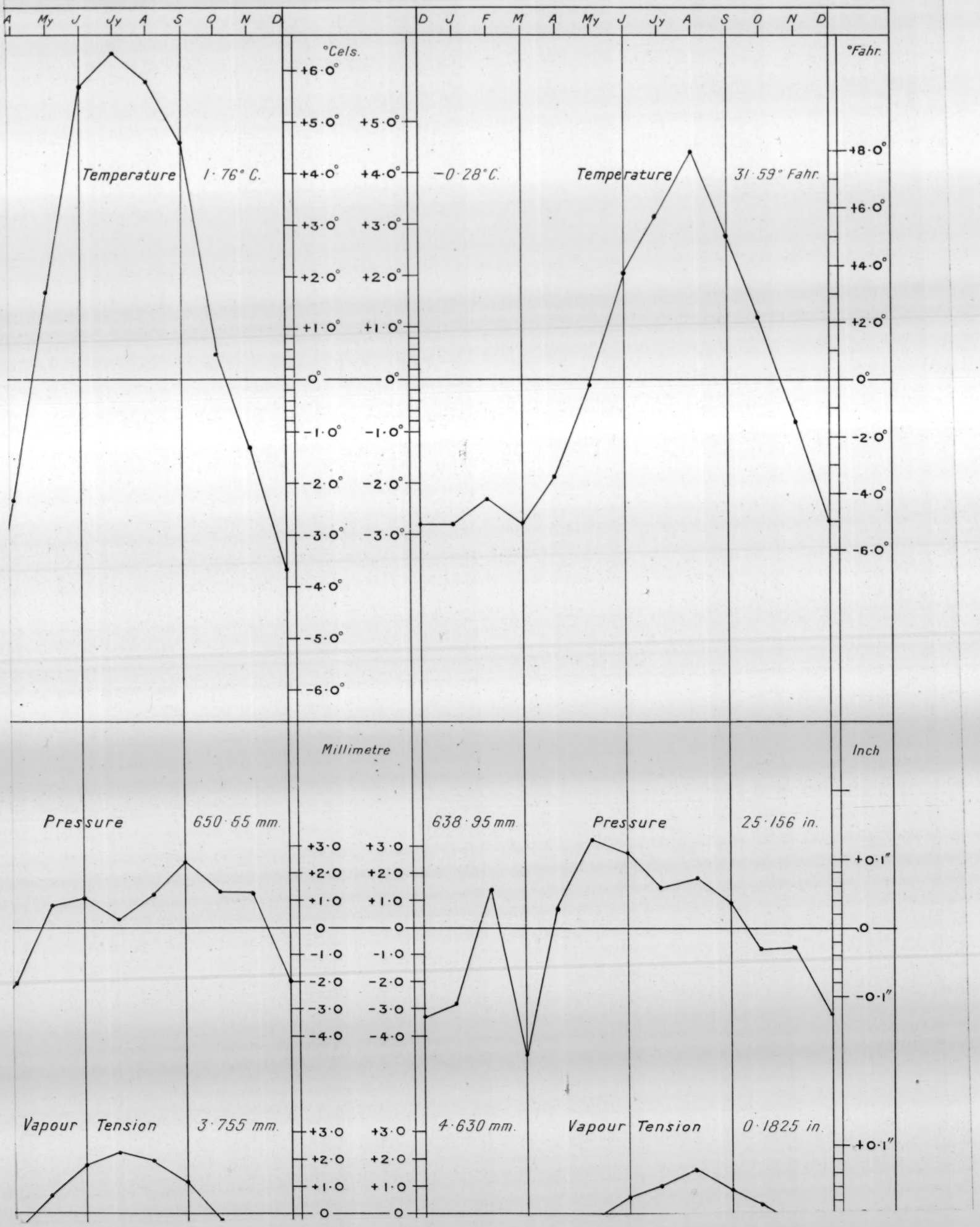
+1·0 +1·0

0 0 0

EVIS. TEMPERATURE PRESSURE AND VAPOUR TENSION. YEARLY VARIATION.

(SCALE OF PRESSURE AND VAPOUR TENSION 5:1)

AR WEATHER. FOGGY WEATHER.



35·16° Fahr. Temperature 1·76° C. −0·28° C. Temperature 3

+10·0° +8·0° +6·0° +4·0° +2·0° 0° −2·0° −4·0° −6·0° −8·0° −10·0°

+5·0° +4·0° +3·0° +2·0° +1·0° 0° −1·0° −2·0° −3·0° −4·0° −5·0° −6·0°

Inch Millimetre

25·612 in. Pressure 650·55 mm. 638·95 mm. Pressure 2

+0·1" 0 −0·1"

+3·0 +2·0 +1·0 0 −1·0 −2·0 −3·0 −4·0

0·1479 in. Vapour Tension 3·755 mm. 4·630 mm. Vapour Tension 0

+0·1" 0 −0·1"

+3·0 +2·0 +1·0 0 −1·0 −2·0 −3·0

D J F M A My J Jy A S O N D

D J F M A My J Jy A S

Temperature 1·76° C. −0·28° C. Temperature 31·59° Fahr.

Millimetre Inch

Pressure 650·55 mm 638·95 mm Pressure 25·156 in.

Vapour Tension 3·755 mm 4·630 mm Vapour Tension 0·1825 in.

A M J Jy A S O N D D J F M A My J Jy A S O N D

A. RITCHIE & SON, EDINT.

XXXII.—*Report on Fossil Fishes collected by the Geological Survey of Scotland in the Silurian Rocks of the South of Scotland.*

By Ramsay H. Traquair, M.D., LL.D., F.R.S., Keeper of the Natural History Collections in the Museum of Science and Art, Edinburgh. (With Five Plates.)

(Read July 4, 1898.)

Introduction.

In the autumn of last year (1897), Sir A. Geikie, F.R.S., Director-General of the Geological Survey, kindly placed in my hands for description an important collection of fossil fish-remains from the Silurian rocks of the Lesmahagow district, which had been made by Messrs Macconochie and Tait, collectors to the Survey. I accordingly prepared a brief report on these fishes, which was included by the Director-General in his *Summary of Progress* for that year.

The collection was, however, considerably increased by additional work on the part of Mr Tait in the spring of the present year (1898). Many better and more complete specimens were procured, and I was able to define one new genus and species, which had previously been represented only by undeterminable fragments. The results gained by the examination of the entire collection I now propose, with Sir Archibald Geikie's sanction, to lay before this Society in detailed form.

As might be expected, this collection is of the greatest importance both from a geological and from a zoological standpoint. Though I understand that Mr James Young of Lesmahagow had indeed found a fish in one of the beds, the "*Ceratiocaris* Band," before the collectors of the Geological Survey came on the scene, the only previous record, in any scientific publication, of the occurrence of such remains in the Scottish Silurian rocks is to be found in a paper (xii.) by the late Dr Hunter-Selkirk of Braidwood, in which he mentions the finding of "a few small scales, something like those of Acanthodian fishes," in the rocks of Logan Water. Of special geological interest is also the fact, that while fishes of the family Cœlolepidæ, represented by detached scales in the Upper Silurian rocks of England and Russia, occur in these Lesmahagow beds, no trace has as yet been found in them of the Pteraspidæ, the Cephalaspidæ, or of the Selachian spines and teeth which have been yielded by rocks of similar horizon elsewhere. Moreover, all the species and all the genera but one are new to science, and some of these throw unexpected light upon forms concerning which next to nothing was previously known. Others, too, are of appearance so strange that their precise place in the system has yet to be determined.

The fishes occur in special bands in the horizons designated by the Geological Survey

as "Ludlow" and "Downtonian," the latter forming the uppermost part of the Silurian system.

The localities are—

Ludlow—"*Ceratiocaris*" and "*Pterygotus*" bands in Logan Water.

Downtonian—Seggholm, Birkenhead Burn, Dippal Burn, Monk's Burn; in the Lesmahagow district. Lyneslie Burn in the Pentland Hills.

The fish-bands consist of hard, grey, flaggy shale, and in all the localities save Seggholm the actual substance of the fossils seems preserved. At the last-mentioned locality, however, the shale is somewhat decayed, and is in consequence soft, inclines to a brownish-yellow coloration, and the scales and plates of the contained fishes are only seen in impression.

It will serve our purpose best to begin with the description of the fishes themselves and of the facts concerning them, and thereafter to enter into the consideration of the bearing of these new facts on certain previously obscure questions in palæozoic ichthyology.

Part I.—Descriptive.

Order HETEROSTRACI, Lankester.

Family Cœlolepidæ, Pander.

Head and anterior part of body flattened, trapezoidal, broader behind than in front; the sides forming posteriorly a right and left angular flap-like projection, the contour of which, sharply marked off from the tail behind, is continuous or nearly so with the lateral margin of the head. Tail narrow, provided with a deeply bifurcate and strongly heterocercal caudal fin. The flap- or lappet-like projections probably represent the pectoral members; but there is no trace of ventrals, or of dorsal or anal fins. Dermal covering, consisting either of minute shagreen-like scales having an internal pulp cavity, and usually a hole in the base, or of minute pointed conical spines, without any basal plate, hollow internally and widely open below.

There is no trace of teeth or jaws or of any internal skeleton except in one instance, the unique specimen of *Thelodus Pagei* of the Lower Old Red Sandstone of Forfarshire, which shows certain markings, probably caused by a branchial apparatus (xxxvi. p. 599).

This family was instituted by Pander (xxi. p. 64) for the genera *Cœlolepis*, Pander; *Thelodus*, Agassiz; *Nostolepis*, Pander; and *Pachylepis*, Pander. Of these *Nostolepis* and *Pachylepis* are synonyms of *Thelodus* (Rohon, xxviii. p. 31), while *Cœlolepis*, in my opinion, falls under the same category (xxxvi. p. 601). These genera were founded on scattered scales, and nothing further has hitherto been known as to the creatures to which these scales belonged. The Cœlolepid scales have been considered to be "*incertæ sedis*" (Smith Woodward, xxxviii. p. 157), or to have belonged to sharks (M'Coy, xvii. p. 15; xviii. p. 576; Rohon, xxviii. p. 15; Reis, xxvi. p. 211), or to Acanthodians (Zittel, xliii. p. 30). Those who have referred the Cœlolepidæ to sharks have also been inclined to correlate with them the spines known as *Onchus*, which in England and Russia often occur in the same beds with the scales in question.

Genus THELODUS, Agassiz, 1839.

Syn. Cœlolepis, Thelolepis, Pachylepis and *Nostolepis*, Pander.

Generic Characters.—Form of the body as in the definition of the family. Scales consisting of a base and of a crown separated by a constriction or neck. Crown of scale round, oval, quadrangular, or sometimes acutely pointed behind, smooth or sculptured; base usually with an opening of greater or lesser size (sometimes absent) leading into the central pulp cavity.

Range in time.—Although *Thelodus* is a characteristically Silurian genus, its range in time extends to the Upper Devonian, as Prof. ROHON has described and figured scales of a species (*Th. Tulensis*) from strata of the latter age in Russia. I have also shown that the "*Cephalopterus*" *Pagei* of Powrie from the Forfarshire Lower Old Red belongs to the same genus (xxxvi.).

Remarks.—Although the name *Thelodus*, given to the detached scales in the Ludlow Bone Bed by AGASSIZ, implies that these little bodies are *teeth*, the laws of priority forbid its alteration into *Thelolepis*, as proposed by PANDER and adopted by ROHON.

For a more detailed account of the history of the genus I must refer the reader to my paper on *Thelodus Pagei* (xxxvi.).

Thelodus Scoticus, Traquair.

Plate I. figs. 1–10.

1898. *Thelodus Scoticus*, Traquair, in Director-General's *Summary of Progress* for 1897, p. 72.

Specific Characters.—Scales in front having the crown rounded, somewhat convex, smooth, the edges distinctly crenulated, the crenulation passing down the neck as a sort of fluting; base well developed with a conspicuous basal opening. Scales behind having the crown acutely pointed posteriorly, and sculptured with several longitudinal ridges and furrows.

Note.—The anterior scales conform in general shape to the ordinary type of *Thelodus* scales, but the crenulations are fewer than in any which I have seen figured. The posterior scales resemble those of *Thelodus* (*Cœlolepis*) *Schmidti*, Pander, in having their crowns pointed behind and longitudinally sculptured, but judging from PANDER's and ROHON's figures, the sculpture in these scales appears to consist of incised lines on a nearly flat surface, while in our present species the surface is convex and the ridges are sharp and elevated. It is interesting to reflect that were *Thelodus Scoticus* only known by detached scales, the two different forms of those bodies would certainly figure as distinct species.

Description.—Fig. 1 on Plate I. represents a specimen from the "*Ceratiocaris* Band" of the Ludlow horizon in Logan Water, which, though it shows the upper lobe of the caudal fin, is very deficient anteriorly; this is the largest specimen known, as its total length when entire would probably not be under $8\frac{1}{2}$ inches. Fig. 2 is taken from a

smaller specimen from the same horizon and locality, in which both lobes of the caudal fin are seen, but the general form of the body is somewhat obscure. The length of this specimen is $5\frac{1}{4}$ inches.

In fig. 3 we have a small example from the Downtonian beds of Seggholm, which gives a very good view of the general shape of the entire creature, the pectoral fin-flaps and the deeply-cleft heterocercal tail being very conspicuous. Fig. 4 is a still smaller specimen from Monk's Burn, in which the tail is also very well exhibited, but the pectoral flaps are rather drawn in.

The length of the head and body to the posterior origin of the pectoral fin-flaps is contained on an average rather less than three times in the total, while the caudal fin, with its two lobes, occupies also rather less than the terminal third of the fish.

In some specimens, as in fig. 3, the apices of the pectoral fins seem rounded, but in most, as in fig. 4, they are angulated, and this latter condition I take to represent the normal and unaltered contour. As to the caudal fin, both lobes are pointed, and the upper one is longer than the lower.

Owing to the state of preservation of the specimens, there are few opportunities of making out the exact form of the scales, an obscurely granulated surface being very often all that is shown by the dermal covering. Careful examination, however, of a considerable number of specimens reveals not only the shape of the scales, but the fact that the scales on the head are of a different form from those further back. I was fortunate enough to find both forms of scales together in an iron-stone nodule from Logan Water, imbedded in a softish material, considered by Mr B. N. Peach to be coprolitic in its nature, and with a little trouble I succeeded in isolating the scales represented in Pl. I. figs. 6–9. On comparing these with the scales and impressions of scales which are now and then to be seen in the entire fishes themselves, there can be no doubt as to the identity of the species.

The anterior or head-scales (figs. 5–7) have the regular *Thelodus* form, with well-developed base and conspicuous basal opening (fig. 6); a smooth, slightly convex crown (fig. 5), separated from the base by a constricted neck, as seen in fig. 7. It will be seen that the edges of the upper surface show eight or nine shallow crenulations or crimpings, which are continued down the neck as a sort of "fluting," very much as in the scales of *Th. Pagei.*

Fig. 8 represents one of the posterior scales seen from above, and fig. 9 a similar one seen from the side. They can be absolutely identified with those which are seen beautifully preserved *in situ* in some parts of the fish represented in fig. 1, and whose arrangement is represented in fig. 10.

These scales are very small, the examples figured measuring only $\frac{1}{50}$ inch in long diameter. The crown is rounded in front, pointed behind, where it projects backwards over the basal part,—convex above, and longitudinally ribbed in such a manner as to remind one of some small umbelliferous seed. The usual mode of ribbing is this:—Each side of the upper surface is constituted by a marginal ridge, inside which and at a some-

what higher level we have a second pair of ridges running parallel to the first, and also meeting behind in a point, while, lastly, between them is a longitudinal median depression. The base is shallow, but I have not got a good view of its under surface.

Observations.—The specimens from the Downtonian beds of Birkenhead Burn, Seggholm, etc., though well preserved as to external form, do not show the configuration of the scales in so distinct a manner as those from the Ludlow horizon in Logan Water; it is therefore not beyond the bounds of possibility that they may ultimately turn out to belong to another though closely allied species. Meanwhile, to avoid premature multiplication of names, I associate them with the Logan Water examples which constitute the types of *Scoticus*.

Position and Localities.—In the *Pterygotus* Band and in the *Ceratiocaris* Band in Logan Water; also in the Downtonian beds at Seggholm, Birkenhead Burn, and Monk's Water.

I may mention that I have lately, while examining some specimens of the well-known "Bone Bed" from Ludlow, found one or two scales which resemble the posterior ones of this species to a very close degree.

Thelodus planus, Traquair.

Plate II. figs. 1–3.

1898. *Thelodus planus*, Traq., in Director-General's *Summary of Progress* for 1897, p. 74.

Specific Characters.—Upper surface of anterior scales round or somewhat oval, slightly convex, smooth; scales behind becoming narrow, elongated, and pointed posteriorly, but without any strongly marked sculpture.

Description.—Fig. 1, Plate II., represents the only specimen of this form which has as yet been obtained. It measures seven and a half inches in length, but as the extremity of the tail is imperfect, it must originally have been at least two inches longer. The contour of the fish is pretty clear, although the edges are somewhat ragged; the lower lobe of the caudal fin is gone, while only the beginning of the upper one is preserved.

The scales are very small, and are represented magnified eight diameters in figs. 2 and 3 of the same plate. Unfortunately no view can be had of any of them, either from below or from the side—all are either broken through or show the upper surface only. In those of the head (fig. 2) this upper surface is round or oval, smooth, ganoid, slightly convex, and with an occasional tendency to crenulation round the edge, while in the posterior scales (fig. 3) the exposed surface is more elongated, is pointed posteriorly, and in some cases even a trace of a ridge on each side close to the margin may with some care be made out. As shown in the figure, these scales seem to be somewhat irregular in size.

This species is distinguished from *Th. Scoticus* by the smoothness of the posterior scales, the species to which it is most allied being *Th. glaber* (Pander), from the Upper Silurian of Oesel.

Position and Locality.—From the *Ceratiocaris* Band in Logan Water.

Genus LANARKIA, Traquair, 1898.

Contour of head, body, tail, and fins as in *Thelodus*. Dermal armature consisting of small, sharp, conical spines, hollow within and widely open below, without basal plate.

The generic name is taken from the County of Lanark, in which the fish-bearing Silurian beds are situated.

Lanarkia horrida, Traquair.

Plate III. figs. 1–6.

1898. *Lanarkia horrida*, Traq., in Director-General's *Summary of Progress* for 1897, p. 73.

Specific Characters.—Dermal spines mostly of one size, large for the size of the fish, and with expanded, somewhat trumpet-mouth shaped base.

Description.—This is a small species, the length of which seldom exceeds two inches, while the proportionally large size of its spines gives it a very decidedly prickly appearance.

Fig. 1, Plate III., represents a specimen from Birkenhead Burn, in which the form of the head and pectoral region is more than usually undistorted, but the caudal fin is absent. This deficiency is, however, supplied in fig. 2, in which it will, however, be observed that the parts in front are pressed somewhat awry. Both figures are enlarged by one-half.

The spines are nearly equal in size all over the fish, save on the pectoral and caudal fins, where they are smaller. On the pectoral fins there is also some appearance of smaller spines intermixed with the larger ones. These appendages (figs. 3–6) have a widely open trumpet-mouth-like base which passes, usually in an oblique manner (fig. 3), up into a conical pointed shaft, which is externally finely striated longitudinally (fig. 4). Fig. 5 represents a natural cast or impression of the interior of the base of one of those spines with the shaft broken off, and in fig. 3 a portion of the wall of the spine has flaked away, showing the mould of the cavity within. Fig. 6 represents a group consisting of three moulds of the interior and two impressions of the exterior of similar spines.

Position and Localities.—Downtonian Beds at Birkenhead Burn and Seggholm.

Lanarkia spinosa, Traquair.

Plate III. figs. 7–12; Plate IV. figs. 1–2.

1898. *Lanarkia spinosa*, Traq., in Director-General's *Summary of Progress* for 1897, p. 73.

Specific Character.—Skin closely covered with minute pointed spines, among which are scattered, in varying degrees of closeness, spines of a larger size, which are also conical pointed and striated externally.

Description.—This is a larger species than the former, some of the examples in the collection attaining a length of six inches. In general form it is identical with *L. horrida*, but the manner in which that form may be altered by distortion is well seen in the examples figured in Pl. III. figs. 7 and 8, and Pl. IV. fig. 12. My idea of the undistorted form is given in the accompanying restored outline.

The small spines are extremely minute, the larger ones (Pl. III. figs. 10–12) are very similar to those of the preceding species, though not quite so trumpet-shaped; they

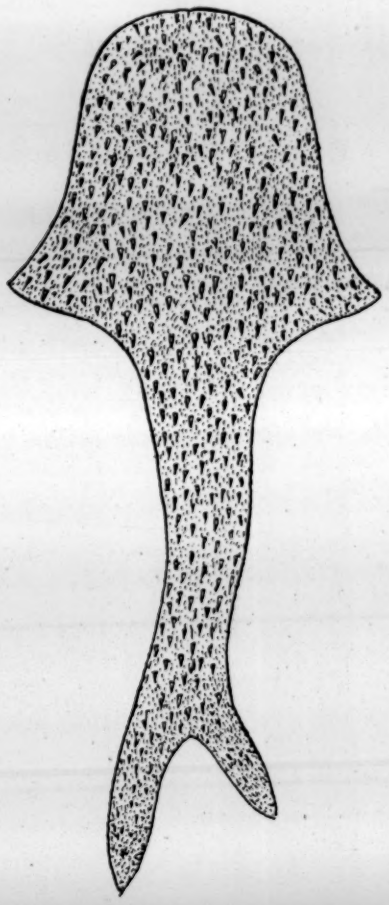


Fig. 1.—Restored outline of *Lanarkia spinosa* in the position in which it occurs as a fossil, namely, vertically compressed in front, but the tail twisted round so as to appear in profile.

are also hollow internally and minutely striated externally. The manner in which they are interspersed among the smaller ones is shown in fig. 9 of this plate and in fig. 2 of Plate IV.

Position and Localities.—From the Downtonian Beds of Seggholm, Birkenhead Burn, and Monk's Burn.

Lanarkia spinulosa, Traquair.

Plate IV. figs. 3–5.

1898. *Lanarkia spinulosa*, Traq., in Director-General's *Summary of Progress* for 1897, p. 73.

Specific Character.—All the dermal spines minute, and without admixture of larger ones.

Description.—As yet very few examples of this species have occurred, and of these not one is entire. The specimen represented in Pl. IV. fig. 3 shows a large portion of the fish, but is imperfect both in front and behind: from what remains we may judge that its original length was seven or eight inches. The whole surface is closely covered with minute, sharp, conical, striated spines, as seen magnified in fig. 4, and still further enlarged in fig. 5.

Badly preserved specimens of *Thelodus Scoticus* and *Lanarkia spinulosa* are sometimes difficult to distinguish from each other.

Position and Localities.—Downtonian Beds at Seggholm and Birkenhead Burn.

ORDER OSTEOSTRACI.

Family ATELEASPIDÆ.

Genus ATELEASPIS, n. g., Traquair.

Imperfectly known. General form of body apparently as in the Cœlolepidæ, but the dermal covering in front consists of small polygonal plates, while behind the pectoral fin-flaps it takes the form of flat rhombic sculptured scales. Orbits apparently on the top of the head as in *Cephalaspis*.

Ateleaspis tessellata, n. sp., Traquair.

Plate IV. figs. 6–12.

In my preliminary report (p. 74) I referred to "one or two fragments showing quadrangular osseous scales belonging to some other fish," no doubt undescribed. On account of the very fragmentary nature of those specimens I abstained from giving them a name, as well as from entering any further into their description.

Since that report was written, Mr TAIT was successful in obtaining a specimen at Seggholm, which I think I am justified in identifying with these fragments, and which, imperfect as it is, throws a certain amount of light on what is apparently a very remarkable fish.

Description.—The most perfect specimen is represented of the natural size in Pl. IV. fig. 6, and shows the anterior portion, including the pectoral fin-flaps, in a tolerable state of completeness; the tail is, however, obliquely cut through on the right side. We have here in front a contour essentially similar to that in *Thelodus* or *Lanarkia*, though the pectoral flaps appear to be a little more rounded. When examined by a lens, the surface appears covered with small polygonal tesseræ, which are not all of the same size or of the same number of angles, and these tesseræ were evidently covered with minute, closely-set, rounded tubercles. As there is scarcely any of the original bony matter left in the fossil, a study of the counterpart is necessary towards coming to a conclusion as to the condition of the surface. In the same Plate, fig. 9, we have a very little bit of the counterpart magnified five times, showing distinctly the impression of the tesseræ and

of their tubercular ornament. Fig. 10 again represents a "squeeze" in modelling wax taken from a similar piece of the counterpart, the tubercles appearing here in relief. The tesseræ become extremely small and at last indistinguishable on the parts corresponding to the fin-flaps in the two preceding genera; and it is also to be noted that tubercles of a larger size are to be seen in impression along the lateral and anterior margins of the head.

About the middle of the head there are to be seen, on the counterpart, two small crescentic markings, right and left, separated from each other by a space of one-third of an inch, and situated half an inch back from the front, the convexity of each being outwards and the concavity inwards. These markings, indicated in the accompanying outline, certainly do suggest the *outer margins* of a pair of orbits placed as in *Cephalaspis*, and on no other supposition can I explain their presence, although there is no trace of any *inner* margin to either, nor of any orbital space or opening, the impressions of the

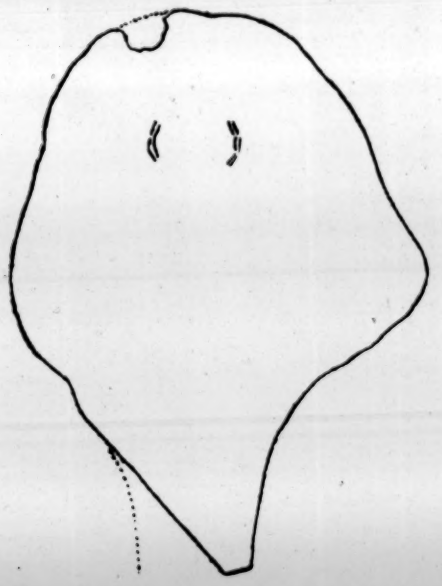


FIG. 2.—Outline sketch of the counterpart of the specimen of *Ateleaspis tessellata*, to show the position of the crescentic marking alluded to in the text.

tuberculated tesseræ being continued all over the intervening surface. This may, however, be due to that vertical pressure which has reduced to absolute flatness a contour which was no doubt originally more or less elevated or vaulted in the middle; it must, however, also be noted that there is no trace here of the ant-orbital fossæ or of the post-orbital valley which are prominent markings on the shield of *Cephalaspis*.

On the tail the dermal covering assumes the form of rhombic scales arranged in transverse rows, and the change from the polygonal tesseræ to this condition is seen to occur already in front of the posterior margins of the pectoral fin-flaps. In Pl. IV. fig 11 portions of three rows of these scales are represented, magnified three diameters, the drawing being made from a "squeeze" in modelling wax taken from the counterpart. As here exhibited, the sculpture of the surface consists of comparatively coarse, wavy, and tortuous ridges, tubercles and furrows, which pass across the scale from before backwards.

Fig. 7 represents a fragment apparently of the caudal part of a larger fish near its

origin. Part of the original osseous matter of the scales still adheres to the matrix, so that their sculpture (Pl. IV. fig. 12, magnified three diameters) is not properly seen; however, as far as one can judge, it is similar to that of the scales represented in fig. 11, though the ridges seem finer and more numerous.

Fig. 8 of the same plate is another fragment, which is evidently the upper lobe of a forked caudal fin, shaped like that of *Thelodus* or of *Lanarkia*. The scales on the body-prolongation are mostly rhombic, though sometimes nearly square in form, and show traces of an ornamentation resembling that in the two former specimens. On the part corresponding to the fin-membrane the scales become very small, and assume a more or less linear arrangement.

Microscopic Structure.—In a vertical section of scales from Birkenhead Burn which are evidently referable to this species, I distinguish three layers :—an *upper*, consisting of sections of the superficial tubercles and ridges; a *middle*, or cancellated layer; and a *lower*, or laminated one. The ridges and tubercles which make up the outer layer are usually solid in section, though sometimes hollow at the base, and are permeated, but not very closely so, by branching tubules, which pass in a radiating manner from the base to the periphery. In most cases, and especially towards the base of the tubercle or ridge, these tubules are seen to be provided with irregular dilatations which also give off other branches, and which cannot in fact be removed from the category of *lacunæ* or "bone-cells," however much they may differ from the orthodox lacunæ in their frequently irregular shape. In a few instances the tubules seem towards the base of the tubercle to be merely very coarse, without the presence of distinctly differentiated lacunæ, but that is rare. There is no surface layer of ganoine. The middle layer consists of a cancellated or spongy tissue in which distinct structure is hard to discern; the lower layer seems to be composed of thin laminæ superimposed upon each other, with obscure, elongated specks between, which I have no doubt are also lacunæ. I hope to figure these interesting details on a future occasion.

Observations.—In general form *Ateleaspis* resembles the Cœlolepidæ, but on the head the shagreen-bodies have coalesced into small polygonal plates,—behind, into flat rhombic scales; the thickness of these plates and scales being added to by ossification in a deeper layer of the skin. The tessellated aspect of the head-covering reminds us both of *Cephalaspis* and *Psammosteus*, but the position of the crescentic markings, which apparently indicate the outer margins of the orbits, shows a greater affinity with the former genus, which is confirmed by the microscopic examination of the scale, which discloses presence of undoubted bone-lacunæ with branching processes. There are, however, neither lateral cornua, nor post-orbital valley, nor pre-orbital fossæ; but as we seem to have here an approximation to the formation of a shield like that of *Cephalaspis*, I propose for the genus the name of *Ateleaspis*, or "imperfect shield" (ἀτελης and ἀσπις). Further consideration of the zoological bearings of this remarkable form I reserve for the second part of this paper.*

* The paragraphs above designated as "*Microscopic Structure*" and "*Observations*" have been rewritten since the

Position and Localities.—All the specimens figured are from the Downtonian Beds of Seggholm, though there is also in the collection a fragment from the beds of similar age at Birkenhead Burn. Mr TAIT has also recently collected a few detached scales from the Downtonian Beds of the Pentland Hills. It is unfortunate that a fish of such interest and importance should be so exceedingly rare, and the specimens as yet obtained so comparatively fragmentary.

Order ANASPIDA.

The two remarkable genera to be now described are so unlike any other fishes hitherto known that I feel under the necessity of erecting a new order for their reception. And as one of these forms, *Birkenia*, presents some features reminding us of *Cephalaspis*, we may as well, at least provisionally, place the order also in the sub-class Ostracodermi. Nevertheless, the structure of the substance forming dermal scales of *Birkenia* shows neither the bone-lacunæ of the Osteostraci nor the dentine tubules of the Heterostraci, but so far as I have been able to examine them microscopically, nothing is seen but a homogeneous, or slightly fibrillated mass, though this may possibly be the result of faulty preservation.

Family BIRKENIIDÆ.

Small fishes, fusiform in shape, with deeply bifurcate heterocercal caudal fin, but no paired limbs. Dermal hard parts in the form of scutes, which are in one form nearly entirely absent. No cranial shield; orbits, jaws, and internal skeleton unknown.

Genus BIRKENIA, Traquair, 1898.

Generic Characters.—Fusiform; body covered with several longitudinal rows of narrow scutes, arranged in lines running obliquely from above downwards and forwards; head bluntly rounded, also covered with narrow scutes; an oblique row of small round openings on the side just at the posterior boundary of the head; no orbit seen, and no evidence of cranial bones, jaws or shoulder-girdle; no paired fins; caudal palæoniscoid in shape, completely heterocercal, deeply bilobed and rayed; a small rounded dorsal situated far back, near the caudal.

Birkenia elegans, Traquair.

Plate V. figs. 1–4.

1898. *Birkenia elegans*, Traquair, in Director-General's *Summary of Progress* for 1897, p. 73.

Specific Characters.—Scutes finely tuberculated; five rows on the sides of the body, of which the upper two do not pass beyond the dorsal fin; a row of six median scutes between the anal region and the origin of the lower lobe of the caudal fin, each

paper was presented to the Society, and also since the publication of my notes in the Geological Survey's *Memoir on the Silurian Rocks of Scotland.*

furnished with an aculeate spine, first one being directed forwards, the remaining five backwards.

Description.— The specimen represented in Pl. V. fig. 1, enlarged by one-half, is three and a half inches in length, and is the largest complete one in the collection, though fragments occur indicating a somewhat larger size. This example does not show the contour of the head properly, but that deficiency is supplied in fig. 2, in which the rounded blunt form of that part is seen in its entirety. Fig. 3 represents another small specimen, the contour of which is shortened up, a phenomenon which, as well as the converse condition of lengthening out, is frequently observed in palæozoic fishes which had no ossified vertebral column to keep them in shape during fossilisation.

The configuration of the fish and the arrangement of its dermal scutes may best be understood by a reference to the accompanying restored outline, the contour of which is based on that of the specimen represented in Pl. V. fig. 1, the details being, however, completed by an examination of numerous other examples.

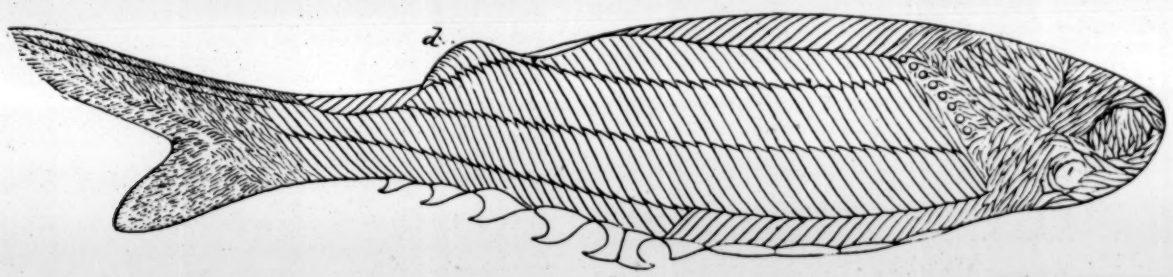


FIG. 3.—Restored outline of *Birkenia elegans*, Traq., one-half larger than natural size; *d*, dorsal fin.

From the elegantly fusiform shape of the body and the completely heterocercal deeply cleft inequilobate caudal fin, we might at first sight fancy that we had before us a member of the Palæoniscid family, with the rows of scales running the wrong way! The resemblance is, however, entirely superficial.

The head is bluntly rounded, and covered with small scutes, spindle-shaped in outline, and very peculiarly arranged. On the top of the head they are disposed in four areas separated by two cross lines, one longitudinal-median and the other transverse—the scutes of each area having their long axes parallel with each other, but directed at acute angles to those of its fellow of the opposite side, and also of the area immediately in front. Of course, in the figure only the two areas, anterior and posterior, of the right side can be seen. Then in front, just behind the rounded snout, the little scutes swirl round a circular space nearly where we would expect the orbit of a palæoniscid fish to be, but the area of this space is occupied by scutes like those of the rest of the head, but arranged with their long axes vertical. Below and behind this space is another rounded marking, around which the adjacent scutes also pass in a swirling manner, but what organ this can represent it is meanwhile impossible to guess. It is on the wrong aspect of the head for an orbit,—it cannot be a mouth, because it is paired, having a fellow on the opposite side. It is not even certain that

it is an opening at all. Immediately behind this the rest of the head is covered by an area of little scutes, which have their long axes directed downwards and backwards.

Now a narrow band-like prolongation of this last-mentioned area passes obliquely upwards and backwards along the posterior margin of the head as far as the first row of body-scutes. In this band we see, in every specimen, eight small round openings or their impressions. Are these *branchial* openings? That is indeed the only interpretation which I am able to put upon them.

In no specimen can any trace of mouth, of jaws, or of teeth be found, nor is there any certain evidence of eye-orbits.

Passing now to the body, we find that it is covered on each side with five longitudinal rows of scutes, which are much larger than those on the head, and of a narrow, elongated oblong shape, pointed at each end.

Commencing above, the *first* row extends from the back of the head to within a short distance of the dorsal fin. It consists of a succession of narrow parallel plates, whose long axes are directed from above downwards and backwards, that is, in a direction contrary to that of most of the scutes of which the other rows consist. This row is in contact in the middle line of the back with the corresponding series of the other side. The *second* row proceeds below the first as far as the dorsal fin, in fact passes into that appendage; the direction of its parallel scutes is downwards and forwards. So it is with the scutes of the *third* row until they pass the dorsal fin, when, from that point to the termination of the series on the tail pedicle, their direction is suddenly altered, and their long axes point downwards and backwards. The *fourth* series, which extends right back from the head to the tail pedicle, has its scutes directed downwards and forwards from beginning to end without interruption. The *fifth* row, which runs along the ventral margin from the head also to the tail pedicle, consists of scutes which in the front half of the fish point downwards and backwards, but exactly in the middle of the ventral curve the direction is suddenly reversed and the scutes come to have their long axes directed downwards and forwards until the band ends at the caudal fin.

In the middle line of the back, and placed exactly between the termination of the first row of body-scutes and the dorsal fin, is a single narrow oblong azygous plate.

Then on the opposite aspect of the body we have on the front half of the ventral curve five marginal scutes, of which the first four are narrow and spurless, while the fifth rises into a backwardly directed sharp conical elevation. So far as I can judge, these scutes seem to be placed on one side of the middle line, leaving us therefore to suppose that they are paired structures, but this I cannot prove, as in no specimen are their fellows of the opposite side to be seen, and analogy with *Lasanius* would lead us on the other hand to suppose that they formed one continuous series with the remaining five marginal scutes behind them, which undoubtedly are placed in the middle line, and are remarkable for the prominent thorn-like spine borne by each. The first one of these is apparently formed of two closely fused together, the two halves—anterior and pos-

terior—being separated by a vertical line, and each bearing a thorn, that of the front half being directed forwards, that of the hinder part backwards. The succeeding four scutes which lie between this double one and the tail pedicle gradually diminish in size, but each of them rises into a prominent backwardly curved and pointed thorn, of a rather formidable appearance it must be owned.

The *dorsal fin* forms a small rounded projection from the outline of the back, and is situated far behind, so as to be just in front of the tail pedicle. Its "rays" are simply continuous with the scutes of the body, in fact they belong to the second lateral row, but tend to become broken up distally.

The *caudal fin* is palæoniscoid in shape, being deeply cleft into two unequal lobes, of which the upper is the longer and contains the prolongation of the body axis, from the *under* aspect of which the fin-membrane exclusively arises. The body-prolongation is covered by innumerable minute rhombic or spindle-shaped scales, which result from the breaking up of the third, fourth and fifth series of lateral body-scutes. Along the dorsal margin we have a special band of tiny narrow oblique scales, which would remind us of the ridge-scales or "fulcra" of the upper lobe of a palæoniscoid tail, were it not that the band here apparently consists of two rows of scales one above the other. The fin-membrane is also covered with narrow scales which tend to be arranged linearly so as to give the fin a very decidedly rayed appearance. All the scales of the tail and of the caudal fin show the same minutely tuberculate ornamentation which occurs on the scutes of the body.

Position and Localities.—Extremely rare in the Ludlow horizon, one specimen only having been found by Mr TAIT in the "*Ceratiocaris* Band" at Shanks Castle, Logan Water. It is, however, by far the most common of the fishes which occur in the overlying Downtonian rocks, and has been obtained in that horizon in the following localities in Lanarkshire—Slot Burn, Seggholm; Birkenhead Burn; Dippal Burn; Monk's Burn. Detached scales have also occurred in the Downtonian of the Pentland Hills, at Lyneslie Burn, near its junction with the Lyne Water.

Genus LASANIUS, Traquair.

Generic Characters.—Elongated-fusiform in shape, with a deeply cleft heterocercal caudal fin. A median row of ventral scutes, each bearing a recurved thorn, runs along the ventral margin from behind the head to the origin of the caudal fin. Immediately behind the head is a series of eight slender parallel bony rods on each side, and directed downwards and forwards, each of which at its dorsal extremity sends a process inwards to the middle line of the back, there meeting its fellow of the opposite side. In front of the anterior one of these rods, and parallel with it, is a chain of short slender ossicles. No other hard parts are visible, but the form of the body is often more or less indicated by a delicate carbonaceous film.

[At first, judging from the position of the median aculeated scutes in *Birkenia*,

I thought it probable that they also occupied a ventral position in *Lasanius*; afterwards, however, it seemed to me more natural to suppose that such a long row of median thorny plates should run along the back instead of along the belly. The matter was, however, settled by a specimen of a new species of the same genus obtained by Mr TAIT since the beginning of the year 1899, and, in consequence, the description of *Lasanius* has been rewritten since this paper was presented to the Society.]

Lasanius problematicus, Traquair.

Plate V. figs. 5–11.

1898. *Lasanius problematicus*, Traq., in Director-General's *Summary of Progress* for 1897, p. 73.

Specific Characters.—Eighteen median scutes along the ventral line, aculei slender.

Description.—In ordinary specimens only two things are to be seen—the long row of median scutes and the arrangement of parallel rods in front—and though these always maintain the same relative position, it was impossible so long as no other parts were observed to decide as to which was the dorsal and which the ventral aspect of the fish. That the line of scutes is, however, ventral is proved by the occurrence of a specimen of the closely-allied species *L. armatus*, in which the arrangement of the rays of the heterocercal tail-fin can be clearly detected.

Pl. V. fig. 5 represents a typical specimen showing the position of the oblique rods lying above, and passing beyond the anterior extremity of the row of ventral scutes; the same relations are also shown in the restored outline, fig. 4, in the text.

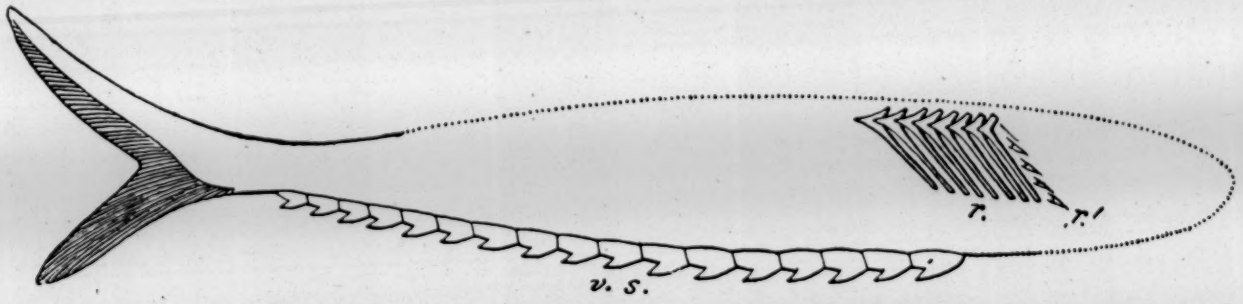


FIG. 4.—*Lasanius problematicus*, Traq., restored outline, enlarged.
v.s., ventral scutes; *r.*, post-cephalic rods; *r'.*, chain of ossicles.

These rods (see also figs. 7 and 8) are eight in number, and consist each of two parts or limbs, which meet above at an angle which increases in acuteness as we pass backwards along the series, and, where the two parts join, there is a sharp posteriorly directed process. The lower limb, slender and tapering, is directed obliquely downwards and forwards on the side of the fish,—the upper one is on the other hand short and directed inwards (fig. 7) to meet its fellow of the opposite side in the middle line of the back. In front of the foremost rod there runs parallel with it a row of five or six small ossicles, the lower extremity of each of which rides over the upper extremity of the one below, and each of them (fig. 9) is also furnished with a small backwardly directed thorn-like projection.

The median scutes are eighteen in number; they are oblong in form when seen from above or below, and the anterior extremity of each overlaps the posterior margin of the one in front, a mode of imbrication with which we are unacquainted as regards the scales of other fishes. Each of these plates is hollowed on its attached surface, elevated on its free aspect, and there it is provided (figs. 10 and 11) with a sharp backwardly directed thorn or aculeus. I have not observed any evidence of external sculpture in those scutes.

In a considerable number of specimens a delicate carbonaceous film indicates in parts the remains of the soft tissues and gives a clue as to the external form of the fish. It is thus clear that the head extended a little way beyond the oblique rods, and that it was, as in *Birkenia*, bluntly rounded in front. In the same way the caudal fin is seen to commence just at the posterior termination of the ventral row of scutes, and to be heterocercal, deeply divided into two slender lobes, of which the upper one is considerably the longer. This is well shown in fig. 8, but it is in the species next to be described that the actual fin rays have been observed.

As regards size, specimens have occurred so small as to have the row of ventral scutes only three-quarters of an inch in length, whereas in large examples this may extend over a length of two inches and a quarter.

Position and Localities.—Only in the Downtonian horizon, in which it occurs in the same localities with *Birkenia*, *Lanarkia*, etc.; Birkenhead Burn; Slot Burn, Seggholm; Dippal Burn; Monk's Burn; Smithy Burn, Hagshaw Hill. In the Pentland Hills detached scutes have been found by Mr TAIT at Lyneslie Burn, along with similar scattered remains of *Birkenia* and *Ateleaspis*.

Lasanius armatus, sp. nov., Traquair.

Plate V. figs. 12, 13.

Specific Character.—Aculei of ventral scutes, thick and stout.

Description.—Two specimens only have occurred, both of which are very small, the more perfect one being only $1\frac{1}{8}$ inch in length, including head and caudal fin, while the other would probably have given the same measurement had it not been cut off by the edge of the stone before the termination of the ventral scutes.

The first mentioned specimen is represented in Pl. V. fig. 12, magnified three diameters. It will be seen that the shape of the body is pretty well shadowed out by a dark film, the form of the head being slightly distorted. I attach no significance to the two round spots without film seen near the front of the head. The oblique rods are seen in their proper position; below them commences the ventral row of scutes, and these are seen to have their thorns disproportionally large when compared with those of *L. problematicus*. The shape of the individual thorns is better shown in the second specimen, from which fig. 13 is taken, and which represents one of these plates with its thorn seen from the side, and magnified four diameters. This difference in the ventral

aculei is not a matter of age, for the smallest specimens of *L. problematicus* have slender thorns, just as in adult examples.

The condition of the caudal fin is here of prime interest, for on wetting the specimen and examining it with a lens, the arrangement of its rays can distinctly be seen. And so, if we are to judge from the analogy of other heterocercal fish-tails, the arrangement here seen at once shows that the azygous row of scutes is on the ventral side of the body. In other words, we see the body-prolongation distinctly passing into the longer lobe of the fin, which is on the opposed side to the scutes, the rays of this lobe being short, while those of the smaller lobe are long.

Position and Locality.—Downtonian Beds at Slot Burn, Seggholm.

Part II.—Results.

The Cœlolepidæ.

The general form of the Cœlolepidæ has now been ascertained. They are shark-like fishes of comparatively small size, the largest example known being only fourteen or fifteen inches in total length. The head, with the anterior part of the body, is depressed, the pectoral fins are lappet-like, there is a strongly heterocercal caudal, but no other fins. The dermal covering is seen in its most primitive form in *Lanarkia*, where it consists of small hollow-pointed spines, open below, and without basal plate. It appears in a more specialised form in *Thelodus*, where we have small shagreen-like scales constricted below the crown, with a base more or less developed, in which there is usually an opening into a central pulp cavity. Where their microscopic structure has been examined, these scales are found to consist of simple dentine, with radiating tubules, with no Haversian canals; the crown is also covered with a layer of ganoine. No traces have been seen of jaws, teeth, eyes, branchial openings, or internal skeleton. The last-mentioned part of the organism must have been entirely cartilaginous.

In the absence of hard circumorbital plates, of teeth, and of opercula, it is not surprising that the position of the eyes, of the mouth, or of the branchial openings should not be ascertainable, though in the Devonian *Thelodus Pagei* (xxxvi. p. 599) the position of the branchiæ themselves seems to be indicated by certain transverse markings in the broad and depressed anterior part of the fish.

Of course the notion that the Selachian spines known as *Onchus*, or the teeth, which have been named *Monopleurodus* and *Anchistrodus*, had anything to do with the Cœlolepidæ, is now entirely disposed of. Nevertheless, looking at these fishes as a group by themselves, and taking into special consideration the nature of their dermal covering, we should have no hesitation in assigning to them a position among the Selachii, and, as Selachians I classed them in my preliminary notice of the Lesmahagow fishes.

In that notice I also applied to them the term "primitive." If that is so, then the

lappet-like appearance of the flaps, which I have interpreted as pectoral fins, would give a new and important corroboration to the lateral fold theory of the paired limbs, and would present us with a "ptychopterygium" still more archæic than the pectorals of *Cladoselache*, as described and interpreted by BASHFORD DEAN and others.

Whether the Cœlolepidæ are admissible to a place among the Selachii or not, the nature of their dermal hard parts points directly to an Elasmobranch affinity and Elasmobranch derivation. But their "primitive" nature now seems to me, on reconsideration of the subject, extremely doubtful. In fact, the depressed configuration of the anterior part of the fish, the absence of teeth, and of ventral, dorsal, and anal fins seem to me to be rather marks of a very considerable specialisation than of archæic simplicity. In this way the lappet-like fin-flap may well represent a degenerate form of pectoral fin instead of an original ptychopterygeal form of that member.

And as the Cœlolepidæ seem also to be so inseparably linked to a series of organisms whose typical representatives can hardly be looked upon as Selachii, hardly even as Elasmobranchs, I prefer to consider them as having definitely split off from the last named sub-class, from which they doubtless originally came.

THE DREPANASPIDÆ.

We may now take up the consideration of the Drepanaspidæ, a family the sole representative of which is the singular genus *Drepanaspis* of Schlüter, from the Lower Devonian slates of Gmünden, in Western Germany.

Drepanaspis Gmündenensis has hitherto been scarcely known to science. It was named, but very imperfectly described, also without figures, in 1887 by Prof. SCHLÜTER of Bonn (xxx.), who seemed to consider it as allied to *Cephalaspis*. In Mr SMITH WOODWARD'S *Catalogue*, Part II. p. 311, it is only mentioned by name and placed along with a number of other imperfectly known forms (*Aspidichthys*, *Anomalichthys*, &c.), which he considered as "perhaps for the most part" referable to the Coccosteidæ. In 1896 I noticed the fish before this Society,* and expressed the opinion that its affinities lay rather with the Pteraspidæ, a view which I am now prepared to defend and confirm, as well as to point out that on the other hand *Drepanaspis* is likewise related to the Cœlolepidæ.

I have now, by the help of Mr B. STÜRTZ of Bonn, got together, in the Edinburgh Museum of Science and Art, an important series of specimens of *Drepanaspis* from Gmünden, and I have also to thank my friend, Prof. O. JAEKEL, for procuring for me some beautiful casts of examples in the Natural History Museum at Berlin, and also in the collection of the Geological Survey in that city. I hope presently to use this material for a more exhaustive description of this remarkable form; meanwhile I shall indicate the principal points in its construction, as far as ascertainable, by the aid of the accompanying restored sketch of its dorsal aspect.

* *Nature*, vol. liv., 1896, p. 263.

The remains of *Drepanaspis Gmündenensis* occur in a pyritised condition in the dark purple roofing-slate (Hunsrückschiefer) of Gmünden, a mode of preservation which, though yielding beautiful fossils to laborious and careful preparation, forbids their affording any results to microscopic examination. It was a fish of considerable size, and large examples must have attained a length of over two feet.

From the accompanying sketch it will be seen that, as in *Thelodus*, the fish is divided into two parts—an anterior, broad and depressed, corresponding to the head and body, and a posterior, or tail, terminating in a heterocercal caudal fin. The anterior

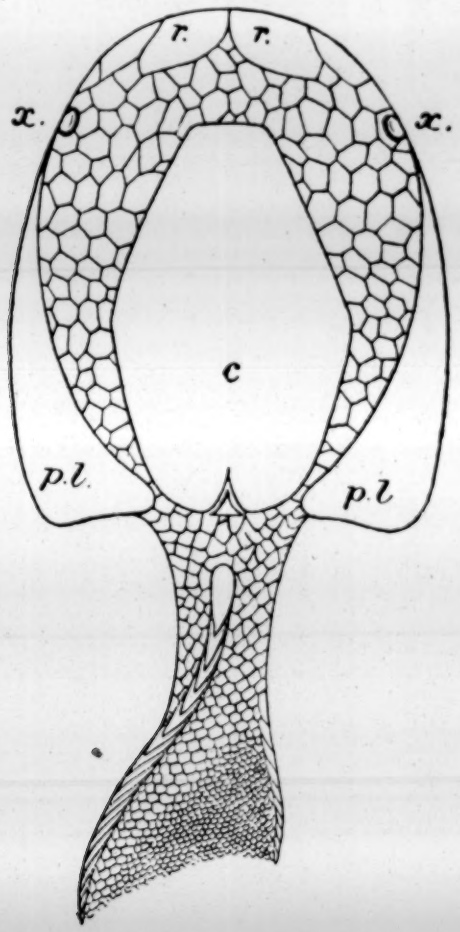


FIG. 5.—*Drepanaspis Gmündenensis*, Schlüter; restored outline of the dorsal aspect, the surface ornament omitted, and the tail twisted round so as to show the caudal fin in profile; *c.*, central plate; *p.l.*, postero-lateral plates; *r.*, rostral plates; *x.*, orbits?

part forms a broad oblong carapace, rounded in front, abruptly truncated behind, where there is on each side a prominent though rounded angle. From the middle third of the transverse posterior margin the tail arises. The carapace consists of numerous bony plates, large and small. In the centre there is a large *median dorsal* plate (*c*) of a somewhat ovate-hexagonal contour, the anterior margin being short and somewhat straight; the posterior is more rounded, and is also acutely notched in the middle. The two postero-lateral angles are formed each by a large, narrow, triangular falciform plate (*p.l.*), which narrows to a very acute point more than half-way to the front of the carapace. The rest of the space is covered by a multitude of small polygonal plates, mostly hexagonal in contour, but the anterior margin itself is generally formed by a

few plates of a larger size, which we may term *rostral* (*r.*). Lastly, on each lateral margin, a little way in front of the termination of the postero-lateral plate, may be seen a small rounded depression (*x.*) which may represent an orbit, though in one case I see on its floor a portion of bone sculptured like the rest of the plate, but this may be due to some accidental displacement. This depression or cavity is of such constant occurrence that it must mean something.

On the ventral surface there is likewise a large oblong central or median ventral plate, but its posterior notch is larger, and its direction is continued forward for a little distance by a slight elevation or fold of the surface—hence, when detached, the plate can always be readily distinguished from the corresponding one on the back. The postero-lateral angles and margins are formed by the same plates (*p.l.*), which we saw on the dorsal surface, but a much narrower area of each is exposed. There are also a few broad plates at the anterior margin, the rest of the surface being filled in by small polygonal ones, exactly as on the dorsal aspect of the carapace.

All the plates of the carapace, both above and below, are ornamented by tolerably closely-set stellate tubercles. There is no trace of jaws or of teeth, but as it is impossible to conceive of the absence of a mouth, we must conclude that it was placed exactly at the anterior margin.

The tail is rather shorter than the carapace, and is covered with tuberculated quadrangular scales, which, becoming finer and smaller, are continued on the caudal fin, which is heterocercal but scarcely bilobate. But in addition to those lateral scales, we have, running along both upper and lower margins of the tail and caudal fin, a series of stout elongated imbricating "fulcral scales," those of the dorsal series being the longer. No trace of any other fins is to be seen, nor do we find any remains of internal skeleton.

After this brief description of the leading points in the structure of *Drepanaspis*, it is impossible even in the absence of evidence as to the microscopic structure of its hard parts, to avoid the conclusion that it is related to the Cœlolepidæ—in fact, that it forms an onward stage in the evolution of a common series to which the last-named family belongs.

We cannot fail to recognise the general resemblance in form—the broad and depressed anterior portion, rounded in front and truncated behind; the want of jaws and teeth; the slender tail with heterocercal caudal fin. But whereas the minute shagreen-scales of *Thelodus* have in *Ateleaspis* coalesced into polygonal tesseræ in front and rhombic scales behind, in *Drepanaspis* we have, added to the rhombic scales of the tail, well-developed fulcra, and as to the anterior part, we find that it is now covered by a regular carapace, into which enter not merely a multitude of small polygonal plates, but also several large ones, among which the great median dorsal and ventral plates and the falciform plates at the postero-lateral angles are conspicuous. Then, if the postero-lateral lappet-like projections of the body of *Thelodus* and *Lanarkia* represent lateral fin-flaps, as I believe them to do, we find these parts in *Drepanaspis*, though still preserving the same general contour, rendered utterly functionless as fins by being enclosed

in unyielding bony plates. In fact, the shark-like Cœlolepidæ seem to have specialised themselves into a form, which in former days would certainly have been called a veritable "Placoderm."

But if *Drepanaspis* points backwards to the Cœlolepidæ, it also points forwards to the Pteraspidæ, but the consideration of this question I shall defer till we come to treat of the last-named group itself. But before doing so we must look into the question of the affinities of the Psammosteidæ, a family upon whose hitherto somewhat doubtful position the structure of *Drepanaspis* throws an unexpected light.

The Psammosteidæ.

The plates known as *Psammosteus* (Agassiz), which occur usually in a very fragmentary condition in the Devonian (Old Red Sandstone) rocks of Great Britain and Russia, have long been a puzzle to palæontologists. By Agassiz, to whom only small fragments were known, *Psammosteus* was classed as a "Cœlacanth" (ii. p. 61); by Trautschold, plates apparently belonging to the same genus were interpreted as swimming paddles of *Coccosteus* (xxxvii., Pl. VI., Pl. VII., fig. 2); but the most prevalent opinion at present is that these remains are elasmobranch in their nature, and belonged to some extinct group of "armoured sharks."

The remains of *Psammosteus* consist in the first place of large oblong plates, gently hollowed in boat-like fashion, obtusely pointed at one extremity (anterior), and truncated or obtusely notched at the other. Internally these plates are smooth, externally they are covered with minute closely-set tubercles, which are brilliantly ganoid and have beautifully crimped edges. In many instances these tubercles are arranged in polygonal areas, which in worn specimens are often removed, leaving shallow polygonal depressions behind, so as to give the surface of the plate something of a honeycombed appearance.

The inner layer of these plates is formed by a dense laminated substance perforated by vessels; the middle one is thicker, and shows a close network of vascular canals, the intermediate substance displaying numerous minute tubules, so that, as Agassiz already remarked, it appears more related to dentine than to bone. The outer layer consists of the tubercles themselves, which show a radiating arrangement of dentine tubules precisely similar to those figured by Rohon in the scales of *Thelodus* (xxviii. p. 33), while the external resemblance of these tubercles to certain *Thelodus* scales, especially to those of *Thelodus Pagei* from the Forfarshire Old Red (xxxvi. fig. 2), is obvious enough.

It seems, therefore, pretty clear that, as I have already remarked in my *Extinct Vertebrata of the Moray Firth Area* (xxxiv. p. 262), the stellate tubercles of *Psammosteus* are shagreen-granules which have coalesced, and have also become united to a plate formed in a deeper layer of the skin. Here an analogy seems to be afforded by the condition of the dermal covering in *Ateleaspis*, on the anterior part of which we have also reason to believe that minute scales have run together into polygonal

plates, which, small as they are, remind us strongly of the polygonal areas so often seen on the surface of the plates of *Psammosteus*.

It need scarcely be added that the large median dorsal and ventral plates of *Drepanaspis* and the oblong boat-like shields of *Psammosteus* mutually remind us of each other, and though the former never show any polygonal areas, the smaller plates with which they are surrounded are eminently suggestive of these areas as they occur in *Psammosteus paradoxus*, Agassiz, and *Ps. Taylori*, Traq.

There are, however, two other forms of plates often found associated with the large oblong ones above referred to, the structure and external sculpture of which lead us to refer them also to *Psammosteus*. Of these we have first certain flattened and somewhat falciform pieces, pointed and often worn at one extremity, and having towards that extremity the characteristic *Psammosteus*-ornament, which, however, covers more of the surface on one side than on the other. A fragment of one of these bodies was figured by PANDER (xxii. Pl. VII. fig. 22) as an "Ichthyodorulite." From their want of bilateral symmetry, they must have occupied a lateral position, and they were indeed, as already remarked, figured and described by TRAUTSCHOLD as the paddles of a species of *Coccosteus*. Relying on the microscopic structure of these plates or "ichthyodorulites," which, as described by PANDER, consists of true dentine without any bone lacunæ, I referred them in 1890 (xxxiii. p. 134) to the category of "Selachian Appendages," noting also the certain amount of resemblance which they bear to the peculiar carboniferous species known as *Oracanthus*. A year afterwards they were noticed by SMITH WOODWARD in the second volume of his *Catalogue* (xxxviii. p. 126), and referred by him to AGASSIZ'S *Psammosteus meandrinus*; and again, in 1895, in his *Problem of the Primæval Sharks*, he compares those bodies to *Oracanthus*, adding, as regards their position on the fish, that "they may have been arranged along the lower margin of the body, as in certain Acanthodian sharks (*e.g.*, *Climatius*), or the animal may have had only a single pair of these spines at the back of the head, as described by Dr TRAQUAIR in *Oracanthus*." The second supposition is more feasible than the first, and I rather think that the "ichthyodorulites" in question were backwardly directed developments of plates corresponding to the postero-laterals of *Drepanaspis*.

The third form of *Psammosteus*-plate, of which PANDER (xxii. Pl. VII. fig. 16) figured an example as possibly a caudal scale or spine of *Asterolepis*, is comparatively small in size, bilaterally symmetrical, oblong, bluntly pointed at one extremity, and sculptured externally with the usual shagreen-like tuberculation.

Mr SMITH WOODWARD remarks (xxxvii. p. 39), concerning these bodies, that they are shaped much like the rostrum of *Pteraspis*, but in my mind there is not the slightest doubt that they are ridge-scales of the tail, similar to those which are to be found *in situ* in *Drepanaspis*.

It is now pretty clear that *Psammosteus* is closely allied to *Drepanaspis*,—so closely that it may be a question as to whether there is any need for family distinction. I think, however, that it is better for the present to keep them in separate families until

the microscopic structure of the hard parts of *Drepanaspis* can be investigated and more also is known regarding the configuration of *Psammosteus* and the arrangement of its plates.

But as regards *Oracanthus*, I must now abandon all idea of its being related to *Psammosteus*, retaining it indeed as a veritable Selachian. Certainly it is so, if the carboniferous fish which I named and described as *Oracanthus armigerus* (xxxii. p. 86) has anything at all to do with the genus in which I placed it. The position of the spines of *O. armigerus* is at the back of the head, one on each side, like the cornua of *Cephalaspis*, but the dentition is cochliodont, and the creature is evidently closely allied to *Menaspis armata*, Ewald, from the German Kupferschiefer.

Meanwhile the problem of the "armoured sharks" is, I think, solved, by our ceasing to consider them, properly speaking, as sharks at all, and transferring them to the Heterostraci. This is in accordance with views already expressed by Dr O. M. Reis of Munich, as we shall presently see.

The Pteraspidæ.

The Pteraspidæ, the only family which by common consent has hitherto been included in the Heterostraci of Lankester (Aspidorhini, Rohon), have, like the Psammosteidæ, no bone lacunæ in the substance of their dermal plates, but the middle layer, instead of consisting altogether of a dense reticulation of Haversian canals, forms in its lower part at least a stratum of polygonal or prismatic cavities, which is the cancellated layer. The outer layer consists of dentine, or kosmine, with fine arborescent tufts of minute tubules. The external sculpture consists of fine concentric and sub-parallel ridges and grooves, but the ridges, as Prof. Lankester observes, "are usually crenated at the margins, and give the notion in some species of a linear series of minute tubercles fused together." Further on he also observes:—"Indeed each of the sections of the ridges recalls very strongly the structure of a tooth or of a dermal defence of a placoid fish" (xiv. pp. 11–12).

In fact the microscopic appearance of the ridges when seen cut in transverse section at once recalls the structure of the tubercles of *Psammosteus*, or of the crowns of the scales of *Thelodus*, and I have no doubt that we have the explanation of their origin in the idea of the fusion of Cœlolepid dermal tubercles or shagreen-bodies in linear order. That these ridges originated by the fusion of "placoid" scales of some sort at least was strongly advocated by Rohon in the first part of his monograph on the *Upper Silurian Fishes of Oesel* (xxvii. p. 75), where he says:—

"Ich muss Prof. Ray Lankester beistimmen, wenn er in dem Bau der äusseren oder Leistenschicht des *Pteraspis*-Schildes die Structur der Placoidschuppen erblickt und diese Schicht aus demselben Grunde auf die Placoidschuppen zurückführt. Wird an dieser vollkommen richtigen Anschauung festgehalten, so ist die Existenz der Knochenzellen wie ganz treffend Ray Lankester hervorhebt, für die Wirbelthiernatur

des *Pteraspis* vollständig überflüssig, dass aber die leistenartigen Erhabenheiten der Schildoberfläche vom *Pteraspis* aus der Verschmelzung zahlreicher Placoidschuppen hervorgegangen sind davon kann man sich ohne Rücksicht auf den histiologischen Bau auch bei macroscopischer Betrachtung überzeugen."

However, in the second part of his researches on the *Silurian Fishes of Oesel* (xxviii. p. 105), the same author proposes an alternative and opposite view of the case—namely, that the ridges of the Pteraspidæ might have been the most primitive condition of the dermal skeleton of the Vertebrata, out of which, by differentiation, the dermal denticles (placoid scales) of the Selachii, as well as their modifications in the Ganoids, Teleostei, &c., have arisen. Again, to quote his words :—

"Indessen könnte ebensogut die gegentheilige Ansicht gelten, d. h. die Streifchen und Plättchen der Pteraspiden könnten den ursprünglichen Zustand des Hautskelets der Vertebraten bilden, so dass also aus den Streifen und Plättchen der Pteraspiden durch Differenzirung die Hautzähnchen (Placoidschuppen) der Selachier gleichwie die entsprechenden Modificationen bei deren Descendenten (Ganoiden, Teleostiern, Amphibien, &c.), entstanden wären. Demgemäss würden die länglichen Streifen, Leistchen oder Plättchen der Pteraspiden als die auf der niedersten Entwickelungstufe befindlichen Hartgebilde an der Körperoberfläche bei den Vertebraten darbieten."

It seems to me that, on this occasion at least, second thoughts have not proved the best, for to suppose that the ridges of so specialised a structure as a dermal plate of *Pteraspis* are likely to be more archæic in character than the simple shagreen-bodies of the Cœlolepidæ formed round a simple papilla, seems to me to be indeed rather like putting the cart before the horse. But Prof. Rohon does not seem to insist very strongly on his new theory, as may be seen from the remarks which immediately follow his enunciation of it. It is, moreover, interesting to see that in comparing the microscopic structure of the ridges of *Pteraspis* with that of placoid scales, he makes in this paper special reference to the Cœlolepidæ :—"da die Streifen und Plättchen der Pteraspiden die gleiche microscopische Structur wie die den recenten Placoidschuppen gegenüber als einfachere und ältere Hautzähnchen erkannten Cœlolepiden aufweisen" (*ib.*, p. 106).

Much remains still to be learned about the Pteraspidæ, but the configuration and structure of the carapace which covers the head and anterior part of the body in the type genus *Pteraspis* is pretty well known through the researches of Huxley, Lankester, Alth, and others.

The head and anterior part of the body of *Pteraspis* is enclosed in a carapace, the dorsal part of which shows evidence of having been originally composed of at least seven distinct plates, though in these fossils, as we usually find them, the pieces are not actually separate. Text-figure 6 shows the arrangement of the parts of the dorsal surface of the carapace, while the accompanying text-figure 7 is a restoration of the entire fish slightly altered from the figure given by Smith Woodward in the second volume of his *Catalogue*, p. 161.

The greater part of the carapace, as seen from above, is composed of the oblong *median dorsal* plate (D), which is slightly bilobate in front, while posteriorly it presents a narrow deep notch, into which fits the *dorsal spine* (S). In front of the median plate is the anteriorly-pointed *rostral* one (R); while fitting into the angle between these two, on each side, is the triangular *orbital* plate (O), each of which shows a small opening, just on the edge of the carapace, and supposed to be for the eye. Behind this is the *cornual* plate (C), one on each side of the median dorsal, and forming a right and left prominent postero-lateral angle. Each of these plates, also visible on the ventral aspect of the fish, forms the sharp lateral edge of the carapace behind the orbital region, and is perforated near its hinder angle by a pretty large oblique opening (B), usually supposed to be *branchial* in its function. Lastly it is to be mentioned that on the inner aspect of the carapace, just between the median dorsal and rostral plates is a small but very distinct round median pit, which was probably supported by a minute eighth or *pineal* plate. (*See* LANKESTER, xiv. p. 28; ALTH, iii. p. 43.)

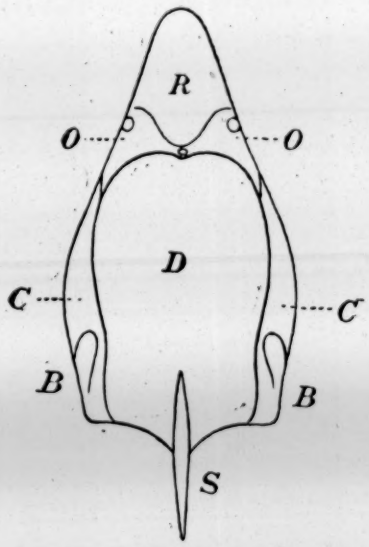


FIG. 6.

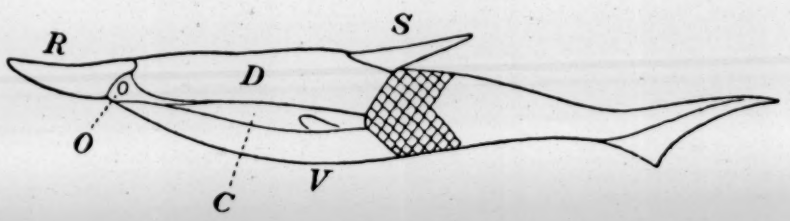


FIG 7.

Fig. 6, Diagram of the dorsal surface of the carapace of *Pteraspis rostrata*, from specimens in the British Museum. Fig. 7, Restored outline of the same fish seen from the side, slightly altered from a figure by Mr A. SMITH WOODWARD. (D) Median dorsal plate; (S) Spine; (C) Cornual plate; (O) Orbital plate; (R) Rostrum; (V) Ventral plate; (B) Branchial opening. The posterior caudal scales are here omitted.

On the ventral surface (text-figure 7) is a large oblong *median ventral* plate (V), once described by LANKESTER as a distinct genus (*Scaphaspis*). The mouth must have been placed between the anterior margin of this plate and the posterior-ventral aspect of the rostrum.

It is impossible to compare this carapace with that of *Drepanaspis* (text-figure 5) without being struck by the general resemblance in the arrangement of the parts in both. In each there is on the dorsal surface a great oblong median plate notched behind, although there is no dorsal spine in *Drepanaspis*. The cornual plates of *Pteraspis* are represented by the postero-laterals of *Drepanaspis*, though in the latter genus no branchial opening is observable, at least in the position in which it

occurs in the former. The position of the orbits in *Pteraspis* on the edges of the anterior part of the carapace is relatively identical with that of the rounded depressions which I take to represent orbits in *Drepanaspis*. But there is no distinct specialised rostral plate in *Drepanaspis*, and the mass of small polygonal ones which surround the median plate have altogether disappeared in the Pteraspidæ.

Then again on the under surface of *Drepanaspis* there is a great median ventral plate, comparable to that of *Pteraspis*, though it is again surrounded by small ones like the dorsal one above.

So far as the arrangement of the plates of the *carapace* is concerned, *Pteraspis* then suggests to us a specialised form of *Drepanaspis*.

The *tail* of *Pteraspis* is only known by three fragments, two of which were figured by LANKESTER (xiv., Pl. V., figs. 1, 3, 5, 8). One of these, from Cradley, Herefordshire (*tab. cit.*, figs. 3 and 8), and now in the British Museum, is the basis of the restored squamous portion of the tail in LANKESTER and SMITH WOODWARD'S reconstructions, and shows that this part was covered by somewhat imbricating rhombic scales. The second specimen, from the Powrie collection, and now in the Edinburgh Museum of Science and Art, is from the Bridge of Allan, where it occurred along with carapaces of *Pteraspis Mitchelli*, Powrie. To this specimen LANKESTER did not devote much space in his monograph, merely remarking (*op. cit.*, p. 33), under *Pteraspis Mitchelli*, that "a few rhomboidal scales also obtained by Mr POWRIE from this locality (Pl. V., fig. 1) probably belong to this or another species."

This patch of scales is, however, worthy of careful examination. As may be seen in Prof. LANKESTER'S figure, it is a narrow band two inches in length by three-eighths in breadth, starting apparently from a larger mass, of which the remains are, however, no more than barely indicated. It is to be noted that the scales, quadrangular in form, become smaller towards what was apparently the distal extremity of the patch, while on one aspect there is something to be seen which Prof. LANKESTER had apparently overlooked—namely, some undoubted remains or traces of a fin-membrane covered by minute scales. I have therefore no doubt that this specimen is the remains of the upper lobe of a heterocercal Pteraspidian tail, and belonging in all probability to *Pt. Mitchelli*.

The third example is from the Lower Devonian of the Rhine country, and has been briefly alluded to by Prof. SCHLÜTER (xxx. p. 125) under the name of *Scaphaspis Bonnensis*. The specimen is in the Geological Museum of the University of Bonn, where I have on two occasions had the opportunity of looking at it for a few moments, but I did not find that it showed anything more than the fact already known—namely, that the Pteraspidian tail was provided with a covering of small scales.

The tail, then, of *Pteraspis*, so far as we know it, is also in accordance with that of *Drepanaspis*, and the conclusion seems to me to be certainly warranted—namely, that the Pteraspidæ are related to the Drepanaspidæ as more highly specialised forms of one common series.

No doubt this conclusion would be invalidated if it were proved that any Pteraspidian possessed distinct Crossopterygian-like paired fins, like those indicated by Claypole in his "attempted restoration" of *Palæaspis Americana* (v. p. 560). But this is far from being the case. Judging from Prof. CLAYPOLE's sketches it does seem to me that the resemblance of the objects in question to Crossopterygian fins is rather superficial, and the author himself admits that "no specimen (as aforesaid) shows this organ in position." Prof. JAEKEL of Berlin (xiii. p. 467) has expressed the opinion that the supposed fins are isolated portions of the dermal armature, and probably *scales*.* A different view, but equally decisive against these objects being *fins*, has been put forward by Dr BASHFORD DEAN of New York (viii. p. 71, footnote) in the following words :—

"The presence of paired fins in Palæaspis, as determined by CLAYPOLE, has not been confirmed. The present writer, to whom the type specimens were kindly shown by their describer, must regard these structures as elasmobranchian (Chimæroid?) spines, in crushed condition, accidentally associated with the head region of the fossil."

So with due apologies to Prof. CLAYPOLE, we cannot accept the occurrence of distinct paired fins in the Pteraspidæ as an ascertained fact.

THE HETEROSTRACI.

If the Pteraspidæ are related to the Drepanaspidæ, then it follows, by the same line of reasoning, that they are also related to the Psammosteidæ, and finally to the Cœlolepidæ. The conception of the Heterostraci is, therefore, widened by the addition of three families, showing almost every gradation from the shark-like *Thelodus*, with its shagreen-covered skin and lappet-like pectoral fin-folds, to *Pteraspis*, which, with its box-like carapace, composed of a limited number of sculptured plates, its scaly tail, presents us with as good a type of the so-called "Placoderm" or "Panzerfisch" as any palæozoic creature which has ever been brought under that designation.

What characters are now to be considered as common to the members of this order?

They all have this in common, that the microscopic structure of the dermal hard parts (unknown, however, in Drepanaspidæ) is either that of dentine, or at least of a substance partaking more of the nature of dentine than of bone.† In none has any internal skeleton been found, or any distinct jaws or teeth. The eyes, where their position has been observed, are situated on the outer edge on each side of the anterior part of the carapace. Those whose external form is sufficiently known have all a strongly heterocercal caudal, but no other median fins.

* "Nach Alledem glaube ich mit voller Sicherheit annehmen zu müssen, dass die für Flossen gehaltenen Skeletstücke nichts anderes als isolirte Hautpanzertheile und zwar Schuppen des Fisches sind."

† SCHMIDT (xxxi. p. 16–17) described and figured the occurrence of lacunae in the ridges of the *Pteraspis*-shield, but this was denied by LANKESTER, and finally disproved by ROHON (xxvii. p. 74–75). As regards *Psammosteus* the last named author maintains the occurrence, in the lowest layer of the shield of simple spindle-shaped bone cells which, however, "weisen fast gar keine Primitivröhrchen auf" (xxviii. p. 70–71). These I have yet not seen, but Prof. ROHON says they are only visible in especially well preserved examples.

Specialisation from the most primitive form (*Lanarkia*) to the most specialised (Pteraspidæ) has been accompanied by:—

1. Fusion of the spinelets (*Lanarkia*) or shagreen grains (*Thelodus*) into plates, scutes, and rhombic scales, supported by hard matter, developed in a deeper layer of the skin.

2. Alterations in the pectoral fin-flaps, which becoming covered up by the postero-lateral plates in *Drepanaspis*, are finally no longer recognisable in the Pteraspidæ.

Now, if it be admitted that the Cœlolepidæ are of Elasmobranch origin—they have hitherto, from the structure of their shagreen-bodies, been looked upon as actual sharks —then it follows, if my views are correct, that the entire group of Heterostraci owes its origin to an Elasmobranch source. This idea has been already foreshadowed, at least as regards the Pteraspidæ and Psammosteidæ, by Dr O. M. Reis of Munich, in his remark—"dass Pteraspiden und Psammosteiden sehr nahe mit einander verwandt eine einheitliche Degenerationsgruppe der Elasmobranchier bilden, für welche ich den Namen Psammacanthiden vorschlage" (xxv. p. 64). Again, in another and later publication, he says—"Pteraspiden und Psammosteiden gehören zusammen auf Grund der microscopischen Structur, welche zwar placoid ist, aber die Eigenheit zeigt, dass das Dentin auf die ausserste Schicht beschränkt wird; es ist dies aber eine Structur-differenzirung zu gross plattiger und massiver Stachel- und Hautplattenentfaltung deren Beginn auch bei den Holocephalenzähnen zeigt" (xxvi. pp. 213-214). However, he does not seem to have suspected any special affinity between his Psammacanthiden (Heterostraci) and the Cœlolepidæ, for, on a previous page of the same paper (p. 211), he adheres to the old view as to the correlation of *Thelodus* and *Onchus*,—"mit mehreren andern Forschern halte auch ich es für sehr wahrscheinlich, dem die zusammenvorkommenden *Thelodus*-Schuppen und *Onchus*-Flossenstachel einer und derselben Haifischgattung angehören."

It follows now that, if the views which I have here supported as to the derivation of the Heterostraci be accepted, two other theories which have been propounded regarding their affinities must fall to the ground.

The first is that originated by the late Prof. E. D. Cope, who, placing the Pteraspids, Cephalaspids, and afterwards also the Asterolepids,* in one sub-class of "Ostracodermi," associated this sub-class with the Marsipobranchii or Cyclostomes (Lampreys and Hags) in one class of Agnatha, apart altogether with the Pisces or Fishes. This idea, founded on the apparent absence of lower jaw and shoulder-girdle in the Ostracodermi, has been accepted by Smith Woodward in his obituary notice of Cope (xl. p. 379), as well as in his recently published *Manual of Vertebrate Palæontology* (xli. p. 1). It also appears in Dr Bashford Dean's *Fishes, Living and Fossil*, where, in his table of Classification of Fishes, he boldly adopts, in place of "Agnatha," the term "Marsipobranchii," previously used only for the Lampreys and Hagfishes themselves. In the table of

* The Asterolepids or "Antiarcha" were, however, first added to the Ostracodermi by Mr Smith Woodward (xxxviii. p. 17).

distribution of fishes in geological time, however, which is given on the next page of the same work, though still placing the Pteraspids, Cephalaspids, and Pterichthyids under the Marsipobranchii, he qualifies the position by the judicious use of points of interrogation. A few pages further on (p. 66), speaking of the Ostracodermi, including the Pteraspidæ, he indicates his opinion that they "are in no way closely connected with the ancient shark types."

For this doctrine of an affinity between the Marsipobranchii and any of the groups reckoned to the "Ostracodermi," I myself never could perceive any real justification, and I consequently found myself quite in agreement with the opinions expressed by Prof. LANKESTER in the short paper (xvi.) which he wrote on the opposite side of the question. Prof. LANKESTER lays principal stress on the entire absence of any proof that the Ostracodermi were monorhinal like the Lampreys and Hags. I would go further and ask, Where is the evidence that they were really "agnathous"? These fossils never show any trace of endo-skeleton at all—it must have been entirely cartilaginous—so that it would be just as reasonable to affirm that they had no chondrocranium. Again, it is by no means so certain that the term "Agnatha," if taken to indicate a primitive deficiency of the mandible or mandibular arch, can properly be applied even to the Cyclostomes, for though the distinguished embryologist F. M. BALFOUR believed that their ancestors never had lower jaws, and that they themselves are the "remnants of a primitive and prægnathostomatous group,"* the opinions of those who consider these creatures to be degenerates from originally gnathostomatous forms cannot be entirely overlooked (see HOWES, x.). HUXLEY (xi.), and following him, HOWES (*op. cit.*) have even maintained that, in the cartilaginous framework of the Marsipobranch head, elements are present which represent parts at least of the mandibular arch in the true Gnathostomata,† though the latter author, from other developmental reasons, emphasises the enormity of the gap which lies between the Marsipobranchii and the other and higher Vertebrata.

It is not, however, necessary to enter further into the agnathous question, if the palæontological facts described in this report indicate that the Pteraspidæ, Drepanaspidæ, and Psammosteidæ, are derivable from the Cœlolepidæ, and that the latter are of Elasmobranch origin.

For the same reason we need not enter into any very detailed discussion of the second theory to which I have referred—namely, that of Prof. W. PATTEN, who seems to be endeavouring to revive, on a scientific basis, what we have long considered to be

* *Comparative Embryology*, London, 1881, vol. ii. p. 69.

† HUXLEY says regarding *Petromyzon* (*op. cit.*, p. 427):—"The posterior lateral cartilages are directly connected with that end of the suborbital arch, which answers to the articular end of the suspensorium in the frog, and in their position exaggerate the peculiar arrangement of the tadpole's meckelian cartilage. That they are parts of the mandibular arch I believe to be certain, but in the absence of any knowledge of their mode of development, I leave the question as to their exact homology open." This view is supported by HOWES, who also finds a representative of the mandible in the prepalatine cartilage of *Myxine*, remarking,—"In its relationships to the superficial branches of the trigeminal nerve, this 'prepalatine' closely corresponds with that which HUXLEY claimed as MECKEL'S cartilage in the Lamprey, and with that I hold it to be homologous notwithstanding PARKER'S view to the contrary" (*op. cit.*, p. 133).

obsolete guesses as to a true zoological (we should now call it a *genetic*) affinity between the ancient plated fishes of the Silurian and Devonian epochs and Arthropoda. It will be remembered that nearly ten years ago Prof. PATTEN compared the sutures and other markings on the head of a Trilobite with those on the cranial shields of *Pterichthys* and *Bothriolepis*, though unsuccessfully, as he had unfortunately taken his figures from old restorations, in which sutures and sensory grooves were confounded. He now (xxii.) finds a strange similarity between the microscopic characters of certain chitinous trabecular structures "underlying the external chitinous covering of the body" of *Limulus* and those of the plates of the carapace of *Pteraspis*. These trabeculæ contain in their centres, or cores, minute cavities, which he compares to bone-lacunæ; and each of these sends off a delicate tubule or canaliculus vertically to the surface. "When the chitinous network forms a rather thin layer, as in the eye region and elsewhere on the thoracic shield, the innermost trabeculæ unite to form a nearly continuous layer, perforated by pores, that lead into the irregular sinuses above them." This he compares with the basal layer of *Pteraspis*, while the layer above, "crossed in various directions by the chitinous trabeculæ containing the lacunæ," he divides into two, of which the lower corresponds to the cancellated, the upper to the reticulated layer in the same fish-plates. Lastly, the "thick outer cuticula" he compares to the outer layer of *Pteraspis*, the most superficial and colourless stratum representing the ganoine, while he evidently looks upon the deeper part which is permeated by "innumerable canaliculæ (pore canals of authors)" as the dentine or kosmine layer. He concludes the descriptive part of his paper by stating that "the only animals known to show such an exoskeleton as Limulus are some of the remarkable fishes known as Cephalaspidæ," under which designation he includes also the Pteraspidæ.

I have not myself examined the dermal structure of *Limulus* microscopically, but I have read Prof. PATTEN's paper very carefully, and must own that I fail to see, more especially in his figures, such a correspondence between the structure described in *Limulus* and the structure of the plates of *Pteraspis* as would warrant us in supposing that the two forms were "genetically related." *Pteraspis* has no bone-lacunæ, but here Prof. PATTEN calls in the aid of *Cephalaspis*, in which, however, the lacunæ are quite different from the appearances figured as such in *Limulus*. Nor do I see anything in Prof. PATTEN's figures having the slightest resemblance to the dentine or kosmine layer of *Pteraspis*. Prof. PATTEN says—"As to its peculiar surface ornamentation, the shield of Pteraspis is exceptional among the Cephalaspidæ, and need not at present be considered." But it is here where the gist of the matter comes in. The crenulated ridges of *Pteraspis* obviously consist of shagreen-bodies, like those of *Thelodus*, but run together in lines, and the derivation of the family from an Elasmobranch source is indicated in a way, which superficial resemblances to other groups cannot in any way touch.

To sum up, then, I must consider the Heterostraci to be of Elasmobranch deriva-

tion, and would include under them the following families, in their order of specialisation:—*Cœlolepidæ*, *Psammosteidæ*, *Drepanaspidæ*, and *Pteraspidæ*.

Heterostraci and Osteostraci.

Although the classification of the Orders Heterostraci (Pteraspids), Osteostraci (Cephalaspids), and Antiarcha (Asterolepids), together in one great division or class of "Ostracodermi," as adopted by Cope and Smith Woodward, is, in the present state of knowledge, very convenient, and has for that reason been used by myself, it has not, however, gained universal assent. Reis protests against the union of the Pteraspids and Cephalaspids as "unbegründet" and "unheilvoll," while Prof. Lankester, who thirty years ago treated them in his classical monograph as "sections" of one group,* has emphasised another view of the matter in the short paper in *Natural Science* to which I have already referred (xv. p. 46), and where he says—"There is absolutely no reason for regarding *Cephalaspis* as allied to *Pteraspis* beyond that the two genera occur in the same rocks, and still less for concluding that either has any connection with *Pterichthys*."

I must nevertheless hold that the configuration and structure of the remarkable, though imperfectly known genus *Ateleaspis* does seem to indicate that there is, after all, an actual connection between the two groups. So far as the external form of *Ateleaspis*, shown in the specimen represented in Pl. IV. fig. 6, is concerned, the resemblance to *Thelodus* is so striking, that the idea of a genetic connection between them is well-nigh unavoidable, and in truth I placed it at first immediately after the Cœlolepidæ in the order Heterostraci, even although certain misgivings were aroused in my mind by the crescentic markings (see text fig. 2) which seemed to point to the presence of orbits on the top of the head as in *Cephalaspis*. But when I succeeded in obtaining microscopic sections of the scales, and saw that the indications of a *Cephalaspis*-like position of the orbits were correlated with the presence of true bone lacunæ in the dermal hard parts, then I felt compelled to transfer *Ateleaspis* to the Osteostraci, though with a tolerably strong conviction that we have here an annectent form—in fact, a "missing link." Here, however, it must form the type of a very distinct family, characterised especially by the want of the pre- and post-orbital openings or markings of the Trematapidæ and Cephalaspidæ, the acute lateral cornua of the latter being also absent. We do not know if the shield was flexible; at all events its composition, externally at least, out of a multitude of polygonal tuberculated plates gives it a resemblance to that of *Cephalaspis* as well as of *Psammosteus*. For though the apparent tessellation of the shield of *Cephalaspis* may be "deceptive," inasmuch as the tesseræ are not actually separate from each other, but take part in the formation of

* It must, however, not be forgotten that Prof. Lankester in the concluding page of that monograph (xiv. p. 62), even then stated that "The Heterostraci are associated at present with the Osteostraci because they are found in the same beds, because they have, like *Cephalaspis*, a large head-shield, and because there is nothing else with which to associate them."

one continuous structure in the forms with which we are acquainted, we are, I think, justified in supposing that they were originally distinct pieces.

If we now turn to the lappet-like expansions behind the head in the Cœlolepidæ and Ateleaspidæ, which I have interpreted as pectoral fins, we shall see that they bear a strong resemblance to the flap-like structures in *Cephalaspis*, which are organically continuous with the head shield, and placed immediately internal to the cornua and behind the head. It will be remembered that these were originally described by LANKESTER as pectoral fins (xiv. p. 41), though SMITH WOODWARD designates them with a query as opercula (xxxvi. p. 176). If the homology of these parts is accepted, it follows that if they represent pectoral fins in *Thelodus*, as such they must also be looked upon in *Cephalaspis*, and consequently LANKESTER was right in his original interpretation.

I am therefore meanwhile of opinion, that the association of Heterostraci and Osteostraci in one great subclass of Ostracodermi is not a mere delusion founded on the occurrence together, geologically, of their fossil remains, and on the presence in both of a cephalic shield, but is supported by the facts brought forward in this paper. The position of the Asterolepidæ does not come within the scope of the present observations.

But unless the Ostracodermi are to be maintained, though it might be only as a "lumber room," we have no place in the system for the two remarkable genera of fishes, with the consideration of which I may now conclude this report.

THE ANASPIDA.

I have placed the genera *Birkenia* and *Lasanius* together in one family, because both possess a fusiform body with bluntly rounded head, a bilobate heterocercal tail, and a median row of aculeated scutes on the ventral margin. In neither do we find any jaws, teeth, paired fins, shoulder-girdle, or ossified internal skeleton.

Birkenia has the head and body completely covered with scutes. *Lasanius*, though nearly naked, has a row of median ventral aculeated plates resembling those of *Birkenia*, and the only other dermal hard parts which it possesses—the parallel rods behind the head—are directed, for the greater part of their extent, downwards and forwards like the elongated lateral scutes of *Birkenia*.

So I am inclined to look upon *Lasanius* as standing much in the same relation to *Birkenia* as the nearly naked *Phanerosteon* does to the other genera of Palæoniscidæ, whose bodies are covered with osseous scales. *Lasanius* seems to have lost its scutes, and is consequently a more specialised form.

As to *Birkenia* itself, its heterocercal tail, its small posteriorly situated dorsal fin, and its narrow tuberculated body-scutes, do remind us strangely of *Cephalaspis*, in which genus it must also be noted that the lateral scutes, if not directed actually downwards and forwards like those of the former fish, are at least vertical, and do not pass obliquely backwards like the bands of scales on the sides of the ordinary "Ganoids."

The under surface of the head of *Cephalaspis* is as yet imperfectly known, and no

branchial openings have as yet been seen or described in that genus. But if we consider the row of eight openings, apparently branchial, which in *Birkenia* are arranged in an oblique line on the side, at the junction of head and body (see text figure 3), we are struck by the very interesting fact that ROHON has already figured two rows, right and left, of similar openings on the ventral surface of the Osteostracan genus *Trematapis* (xxvii. p. 70 and xxix. p. 8), though the number of openings here is only six on each side. Now, in *Trematapis* the head is broad and depressed, while in *Birkenia* the creature, as it is always found lying on its side, must have been more laterally compressed, and consequently the position of the branchial openings must have been in the two genera relatively as in the skate and in the shark.

For these reasons I have often felt inclined to refer the Birkeniidæ to the Osteostraci, but there are two serious difficulties in the way of this idea. The first is the absence of the cranial buckler with orbits on the top, which is so prominent a feature in all known Osteostraci. The second difficulty is the utter absence of any microscopic proof, as the apparent substance of the scutes of *Birkenia*, as preserved in the schists of such localities as Birkenhead Burn, shows in sections under the microscope absolutely no structure whatever, except a very faint fibrillation or striation.

But whatever may be the position which increased knowledge may afterwards assign to *Birkenia* and *Lasanius*, for the present they are best placed in an Order by themselves, for which, I repeat, no place can be found in the system unless we admit it to the "Ostracodermi."

CONCLUSION.

The fossil fishes from the Silurian rocks of the South of Scotland, described in the preceding pages, constitute eight species, which are all new to science. They may be arranged in five genera, four of which are also new; the remaining one, *Thelodus*, having been named by AGASSIZ in 1831 from detached scales occurring in the Ludlow Bone Bed. The following classification of the species has been adopted :—

Sub-class—OSTRACODERMI.

Order—HETEROSTRACI.

Family—*Cœlolepidæ*.

1. *Thelodus Scoticus*, Traq.,—Ludlow and Downtonian horizons.
2. „ *planus*, Traq.,—Ludlow.
3. *Lanarkia horrida*, Traq.,—Downtonian.
4. „ *spinosa*, Traq.,—Downtonian.
5. „ *spinulosa*, Traq.,—Downtonian.

Order—OSTEOSTRACI.

Family—*Ateleaspidæ.*

6. *Ateleaspis tessellata*, Traq.,—Downtonian.

Order—ANASPIDA.

Family—*Birkeniidæ.*

7. *Birkenia elegans*, Traq.,—Ludlow and Downtonian.
8. *Lasanius problematicus*, Traq.,—Downtonian.

And whether the views which I have expressed regarding the phylogeny and classification of these and allied forms be adopted or not by subsequent writers, it cannot, I think, be denied that these recent discoveries by the Geological Survey have opened out a new vista in the field of palæozoic ichthyology.

I must conclude by thanking the Director-General of the Geological Survey for his kindness in submitting this very important collection to me for description, and the officers of his staff for the friendly courtesy and readiness with which they afforded me every facility for its examination.

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EXPLANATION OF THE PLATES.

Plate I.

Thelodus Scoticus, Traq.

Fig. 1. A specimen from the "*Ceratiocaris* Band," Logan Water. Natural size. The head is imperfect, but the sculpture and arrangement of the scales is exceedingly well shown in places.

Fig. 2. Another specimen from the same horizon and locality. Natural size. The contour in front is somewhat obscured by distortion.

Fig. 3. A specimen from the Downtonian horizon at Seggholm, showing the shape of the body unusually well. Natural size.

Fig. 4. Another specimen from a similar horizon at Monk's Burn.

Fig. 5. Upper surface of an isolated head-scale from Logan Water, magnified twenty diameters.

Fig. 6. The same scale, from below.

Fig. 7. The same scale, from the side.

Fig. 8. Upper surface of a scale from behind the head, also from Logan Water, magnified twenty diameters.

Fig. 9. A similar scale seen from the side, magnified twenty diameters.

Fig. 10. Arrangement of the posterior scales, from the specimen represented in fig. 1. Same enlargement as in figs. 8 and 9.

Plate II.

Thelodus planus, Traq.

Fig. 1. Entire specimen, natural size, from the "*Ceratiocaris* Band," Logan Water.

Fig. 2. Scales from the front; outer surface, magnified eight diameters.

Fig. 3. Scales from the caudal region, outer surface; same enlargement.

Plate III.

Lanarkia horrida, Traq.

Fig. 1. Specimen, showing the head region well, but imperfect posteriorly. Downtonian, Birkenhead Burn. Enlarged by one half.

Fig. 2. Another specimen from the same horizon and locality, showing a nearly perfect caudal extremity. Also enlarged one half.

Figs. 3-4. Isolated dermal spines, magnified twelve diameters.

Fig. 5. Natural cast of the interior of a spine, seen from above, apex broken off; same enlargement.

Fig. 6. A group of similar spines; same enlargement.

Lanarkia spinosa, Traq.

Fig. 7. Entire specimen, somewhat distorted on right side; natural size. Downtonian, Seggholm.

Fig. 8. Another entire specimen from the same horizon and locality; natural size.

Fig. 9. Portion of integument from a specimen from Birkenhead Burn, magnified, showing the larger dermal spines intermixed with those of minute size; magnified nine diameters.

Figs. 10-12. Three of the larger dermal spines also from a Birkenhead Burn specimen, magnified twelve diameters.

Plate IV.

Lanarkia spinosa, Traq.

Fig. 1. A small specimen from Seggholm, in which the larger spines are very strongly marked. Enlarged one half.

Fig. 2. Portion of integument of the same specimen, magnified four diameters.

Lanarkia spinulosa, Traq.

Fig. 3. Imperfect specimen from the Downtonian horizon, Birkenhead Burn; natural size.

Fig. 4. Spiny integument of the same specimen, magnified six diameters.

Fig. 5. Isolated dermal spine from another specimen, magnified twenty diameters.

Ateleaspis tessellata, Traq.

Fig. 6. Specimen from the Downtonian horizon, Seggholm, fairly complete in front, but obliquely cut off behind.

Fig. 7. Portion of the body of another specimen; natural size.

Fig. 8. Upper lobe of the caudal fin of another specimen; natural size.

Fig. 9. Tesseræ of the surface of the head, seen in impression on the counterpart of the specimen represented in fig. 6, and magnified five diameters.

Fig. 10. A squeeze in modelling wax taken from a similar part, also magnified five times.

Fig. 11. Scales from behind the head of the specimen shown in fig. 6 as seen in a squeeze in modelling wax taken from the counterpart, magnified three diameters.

Fig. 12. Scales from specimen shown in fig. 7, magnified two diameters.

All these specimens of *Ateleaspis* are from the Downtonian Beds of Seggholm.

Plate V.

Birkenia elegans, Traq.

Fig. 1. Entire specimen from Birkenhead Burn (Downtonian), enlarged one half.

Fig. 2. Another specimen from the same locality, having the body shortened up, enlarged one half.

Fig. 3. Head of another specimen from the same locality, enlarged one half.

Fig. 4. Impression of a detached scute of the median ventral series, magnified four diameters. From Lyneslie Burn, Pentland Hills.

Lasanius problematicus, Traq.

Fig. 5. A specimen from Birkenhead Burn, showing the fish as it usually occurs—the ventral scutes and the gridiron-like arrangement of rods being in position with regard to each other. Enlarged by one half.

Fig. 6. A specimen from Seggholm, in which the form of the head as well as of the caudal fin is shown by a carbonaceous film. The lower lobe of the caudal is slightly distorted; the body is bent nearly double. Magnified two diameters.

Fig. 7. Gridiron-like arrangement of parallel rods of both sides, vertically compressed and spread out, and showing how those of opposite sides meet together in the dorsal middle line. The left set of rods is crossed by the anterior part of the series of ventral scutes (*v*). From Birkenhead Burn, and magnified two diameters.

Fig. 8. Left series of parallel rods in a specimen from Seggholm; enlarged by one half. The peculiar chain of ossicles or rodlets is well seen in front of the anterior rod.

Fig. 9. The three uppermost rodlets in a specimen from Seggholm; magnified two diameters.

Fig. 10. Outline of a usual form of the ventral scutes, seen from the side. Birkenhead Burn; magnified four diameters.

Fig. 11. Another form, in which the elevation is less. Seggholm; magnified four diameters.

Lasanius armatus, Traq.

Fig. 12. The more perfect of the two specimens known, showing in a carbonaceous film the blunt form of the head, and posteriorly the heterocercal caudal fin with its rays distinctly preserved. Seggholm; magnified three diameters.

Fig. 13. Outlines of two of the ventral scutes of the other specimen, magnified four diameters.

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Dr R. H. TRAQUAIR ON SILURIAN FISHES, PLATE I.

Fig. 1.

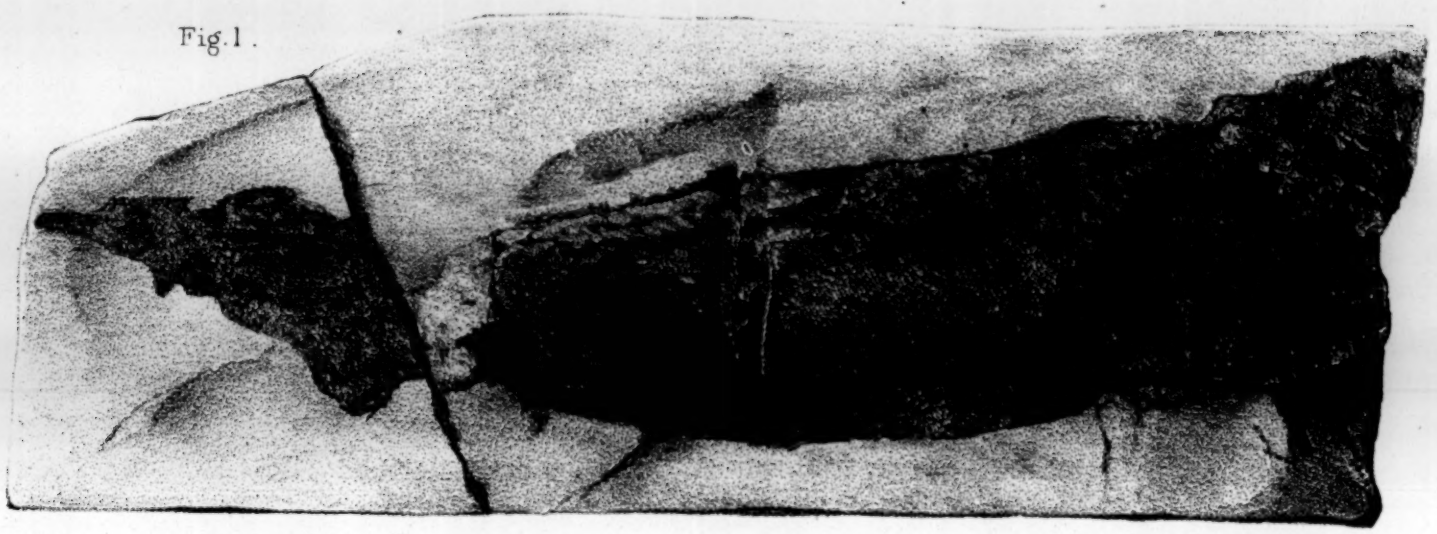




Fig. 3.

Fig. 10.  20/1

Fig. 8.  20/1 Fig. 9.  20/1

Fig. 7.  20/1

Fig. 5.  20/1

Fig. 6.  20/1



Fig. 4.

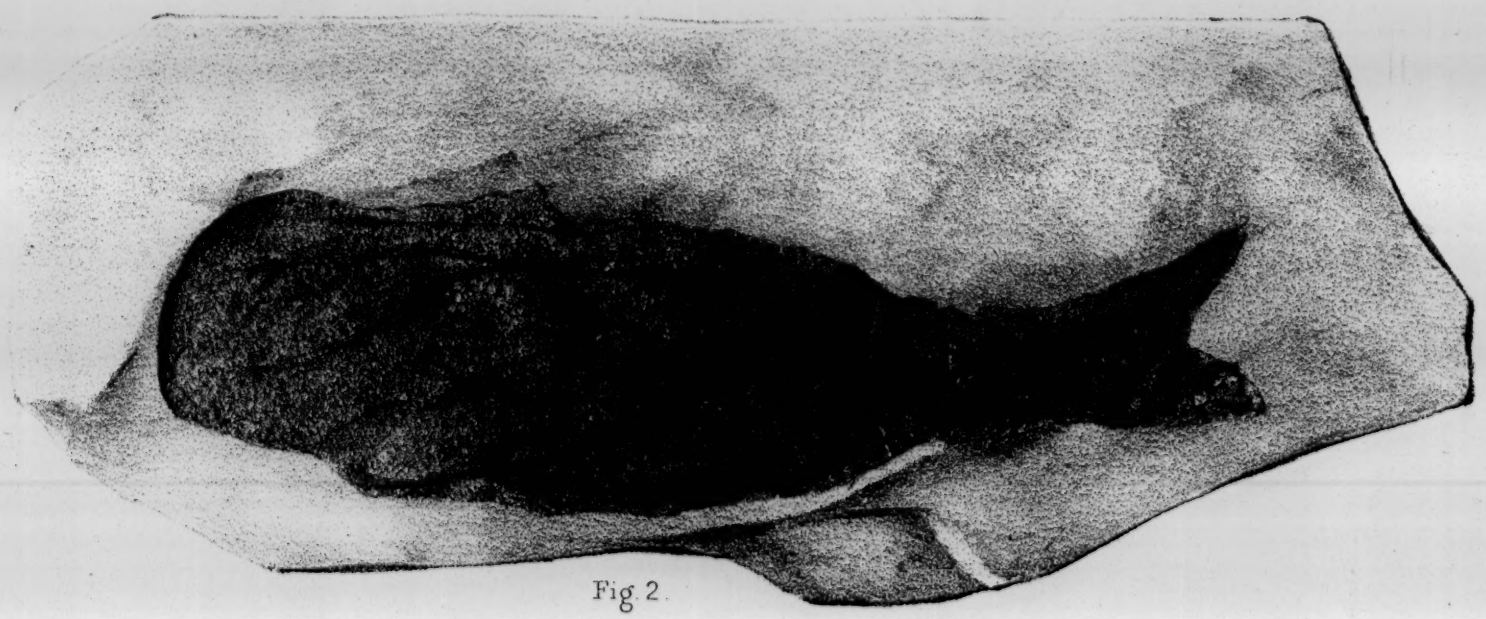


Fig. 2.

Figs 1-4 J Green.
Figs 5-10 R H Traquair.

J Green imp. Edin.
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Dr R.H. TRAQUAIR ON SILURIAN FISHES, PLATE II.

Fig. 2. 8/1

Fig. 3. 8/1

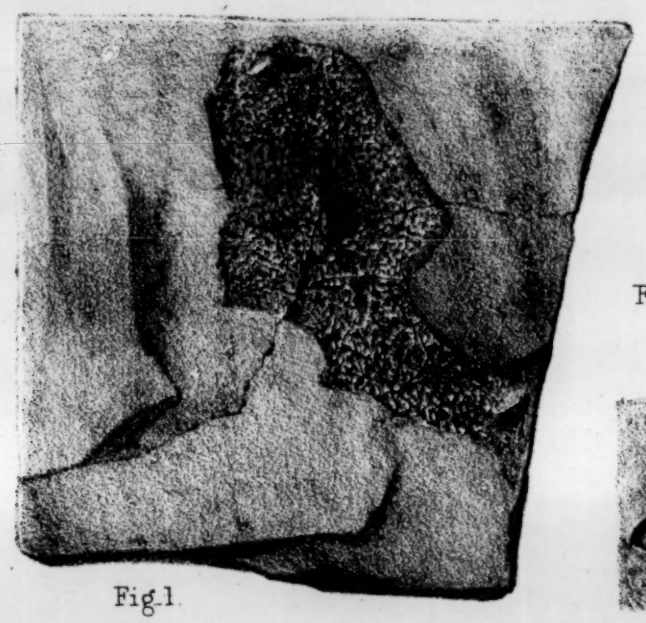
Fig 1.



Fig 1 J Green del
Fig 2-3. R.H Traquair del.

J. Green in lap.
Mintern Bros. imp. London.

Dʳ R. H. TRAQUAIR ON SILURIAN FISHES, PLATE III.

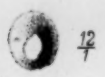


1½

Fig. 1.

Fig. 2.

1½



12/1

Fig. 5.



Fig. 3. 12/1



Fig. 4. 9/1

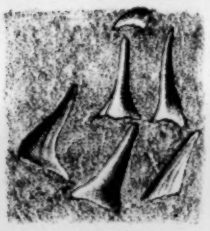


Fig. 6.

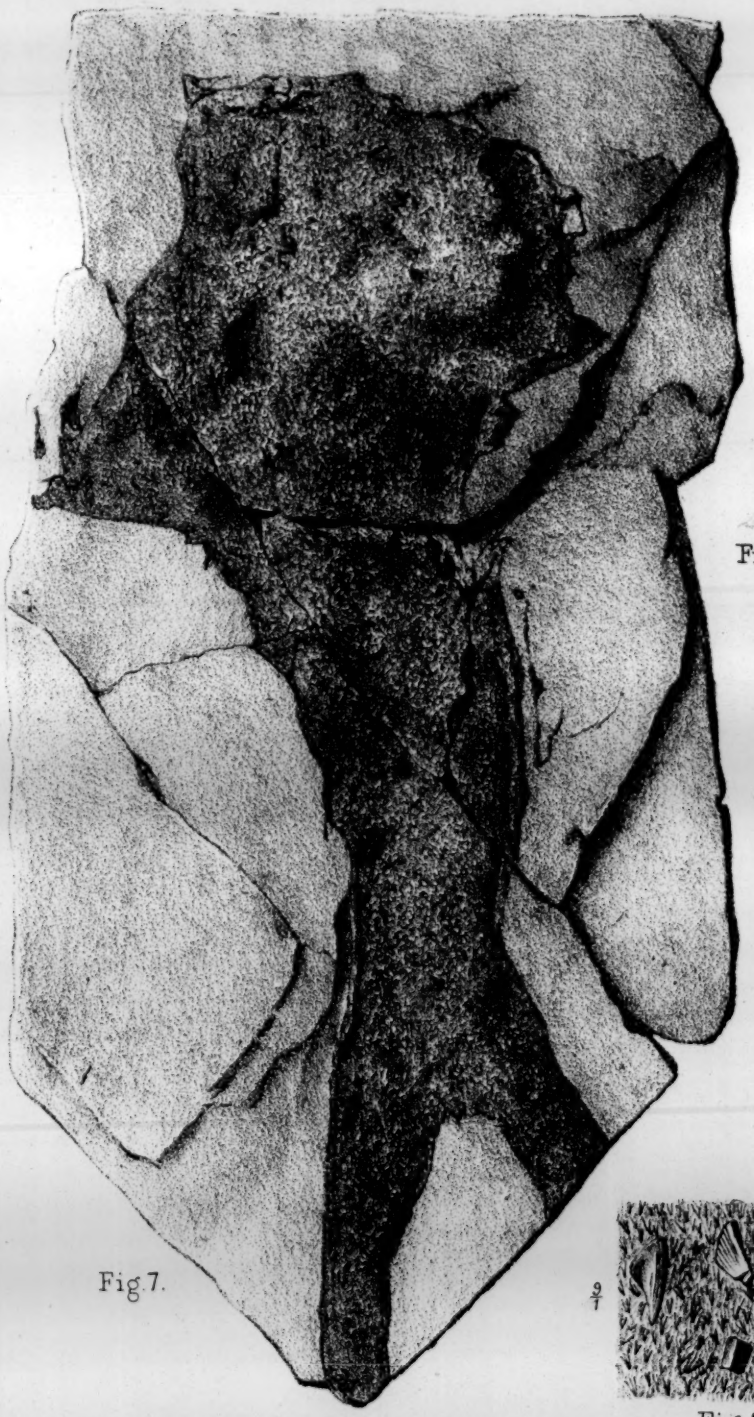


Fig. 7.

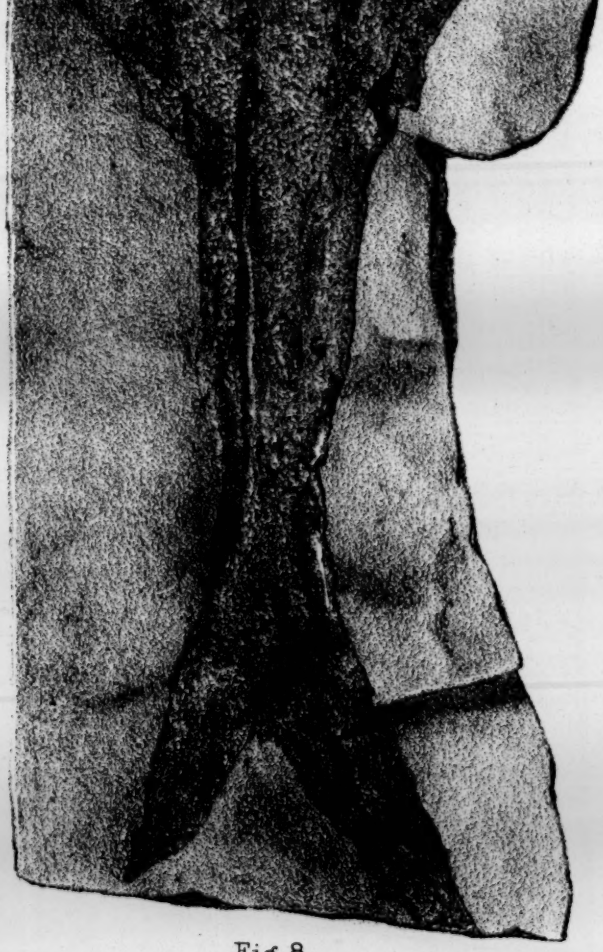


Fig. 8.



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Fig. 12.



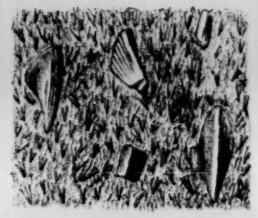
12/1

Fig. 11.



12/1

Fig. 10.



9/1

Fig. 9.

Figs 1, 2, 7, 8, J. Green del.
Figs 3-6, 9-12 R. H. Traquair del.

J. Green in lap
Mintern Bros. imp. London.

D.r R.H. TRAQUAIR ON SILURIAN FISHES, PLATE IV.

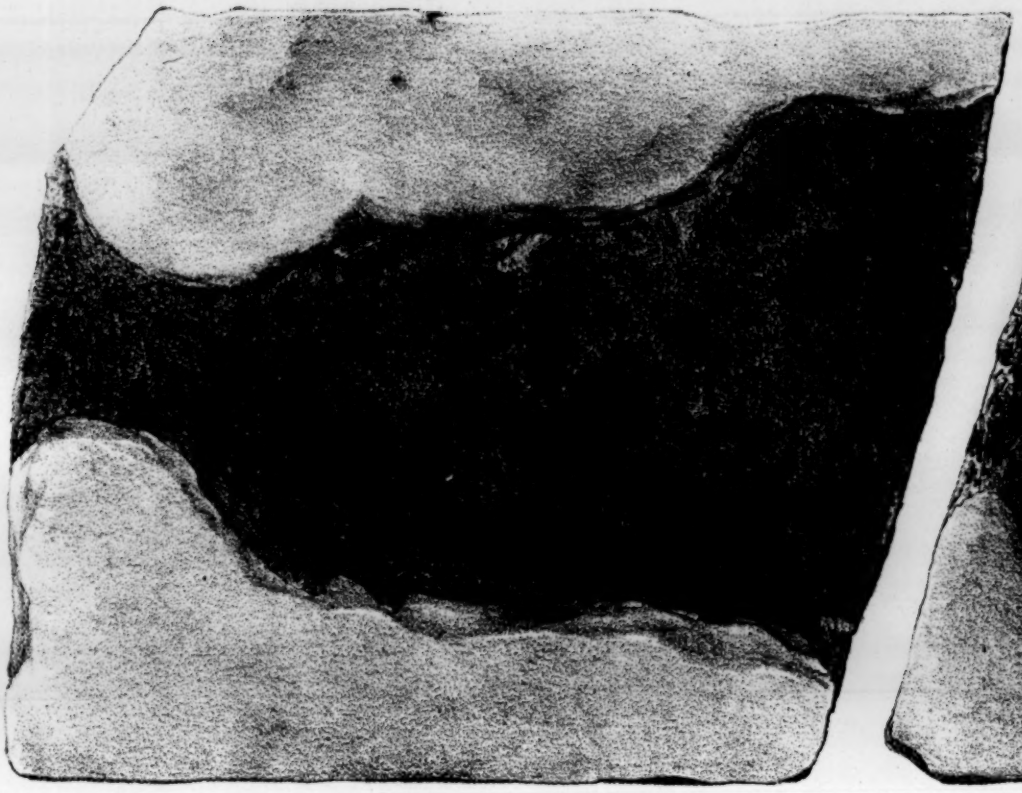


Fig. 3.



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Fig. 1.

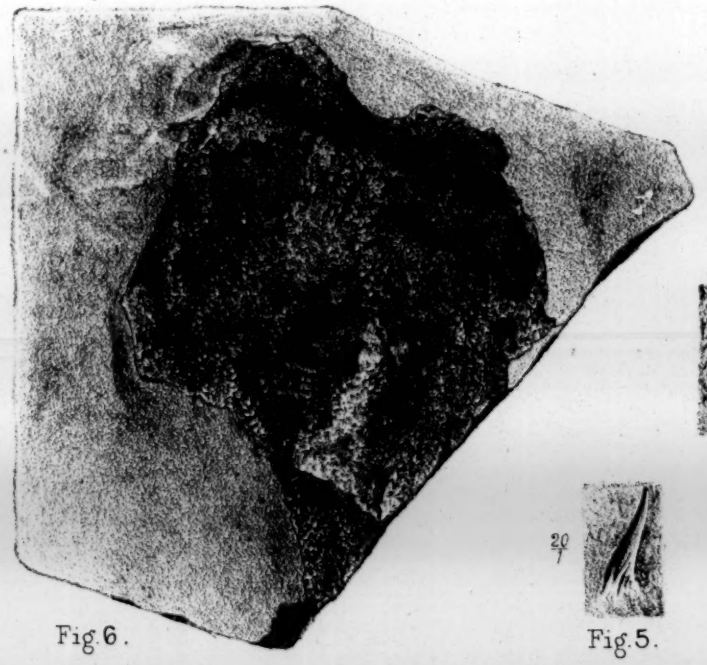


Fig. 6.



$\frac{4}{1}$

Fig. 2.

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Fig. 5.

$\frac{6}{1}$

Fig. 4.

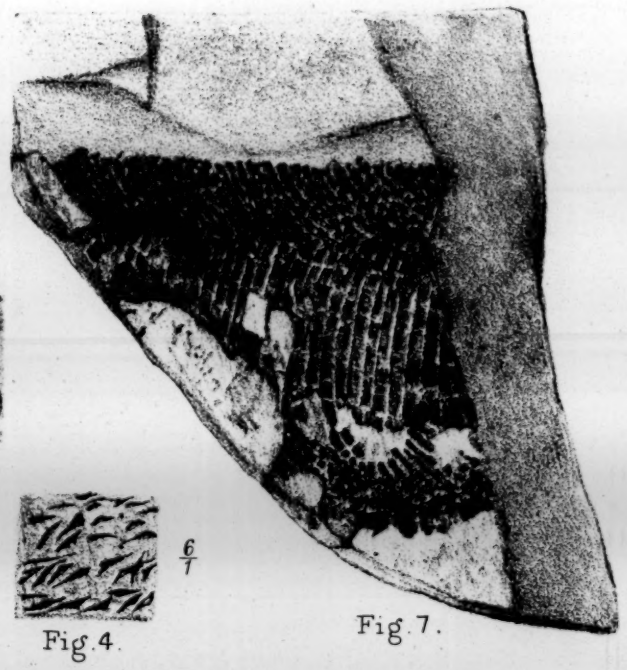


Fig. 7.



Fig. 9. $\frac{5}{1}$

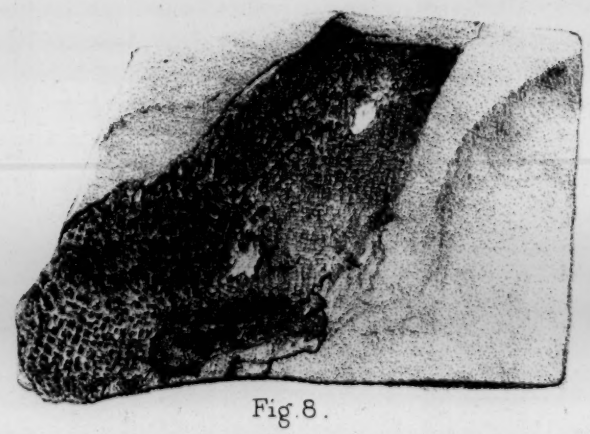


Fig. 8.



Fig. 11. $\frac{3}{1}$

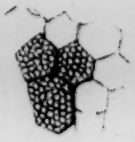


Fig. 10. $\frac{5}{1}$



Fig. 12. $\frac{2}{1}$

Figs. 1 3 6 7 8 J. Green del.
Figs 2 4 5 9 12. R H Traquair del.

J. Green. in lap.
Mintern Bros. imp. London.

DR R.H. TRAQUAIR ON SILURIAN FISHES, PLATE V.



Fig.1.

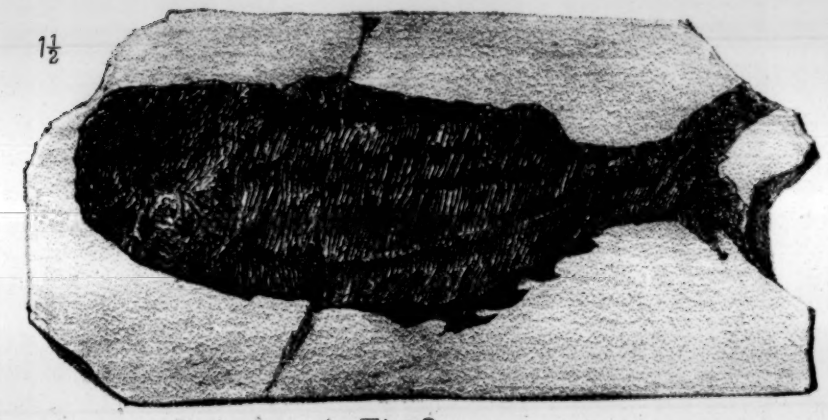


Fig.2.

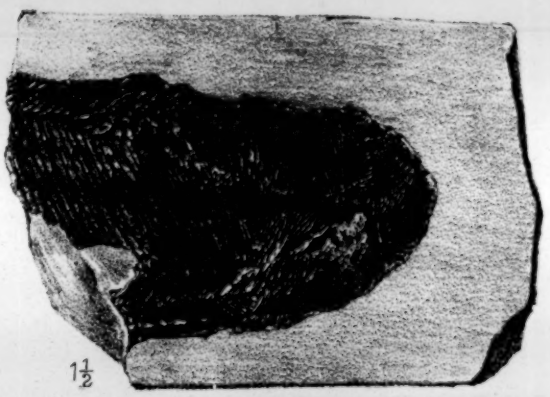


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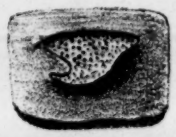


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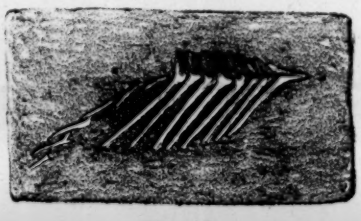


Fig.8.

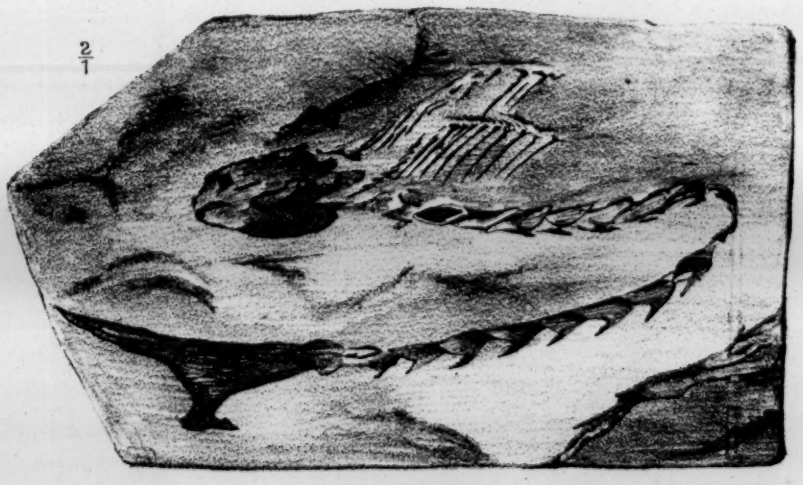


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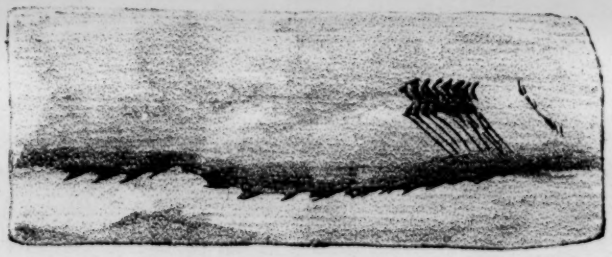


Fig.5.

Fig.9.

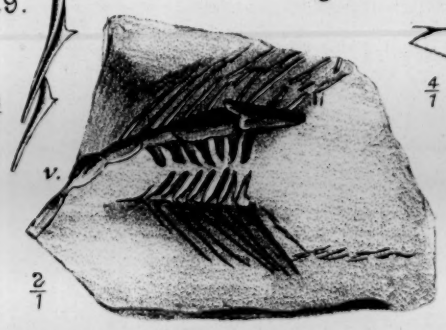


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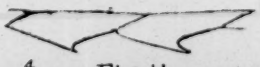


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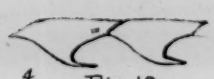


Fig.10.

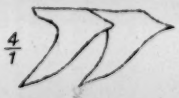


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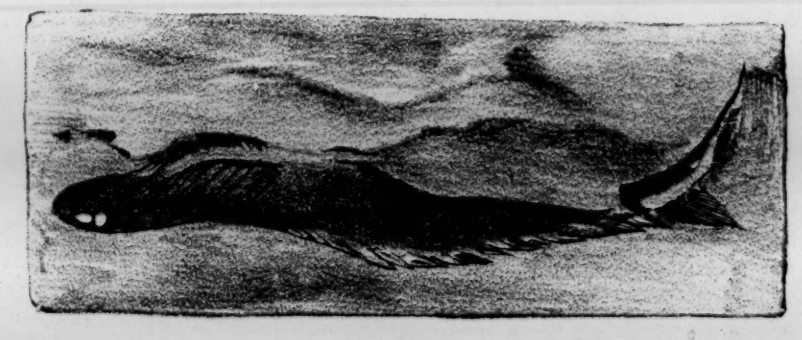


Fig.12. 4/1

Figs. 1-3, J. Green del.
Figs. 4-13, R.H. Traquair del.

J. Green in lap.
Mintern Bros. imp. London.

XXXIII.—*The Trap Dykes of the Orkneys.*—By John S. Flett, M.A., B.Sc.
(With Three Plates.)

(Read June 19, 1899.)

Previous Accounts of the Trap Dykes of the Orkneys.

The existence of a series of trap dykes cutting the Old Red Sandstone strata of the Orkneys has been noted by most of the geologists who have examined the islands. In his description of the Mainland or Pomona, Professor Jameson states (I., p. 233, vol i.) that "at Yesknaby is the only basaltic rock which I observed in the whole island. It forms veins which traverse the common argillaceous sandstone. The crystals of hornblende, which are contained in it, are larger than usual in such rocks, being more than an inch long and half an inch broad. I sometimes observed small cavities filled with bitumen." Sir Archibald Geikie, in his account of the Old Red Sandstone of Orkney, remarks (II., p. 408): "Here and there a few basalt dykes—far outlying portions, no doubt, of the great Tertiary series of the West of Scotland—cut through the flagstones with a prevalent direction towards west or north-west." Messrs Peach and Horne, in their paper on *The Old Red Sandstone of Orkney* (III., p. 14), describe them in the following terms: "Several dykes of basalt were observed among the islands. They are most numerous and conspicuous on the west coast of the Mainland from Breckness to Skaill, but as they have been so often described, it is unnecessary to refer to them in detail. They have the same lithological characters, and behave in exactly the same manner as the dykes in other parts of Scotland, which have been regarded as the product of volcanic energy in Miocene times. A noticeable feature about the Orcadian representatives is, that they are usually divided up the centre of the dyke by a line of vesicles. This is not an uncommon feature elsewhere." In the chapter on the Geology of Orkney in Tudor's *The Orkneys and Shetland*, by the same authors, the dykes are referred to in similar terms (IV., p. 191). A somewhat more minute examination of these dykes was made by Professor M. Foster Heddle. He notes the presence of augite, olivine, and hornblende in certain dykes near Skaill (V., p. 118), and gives a map of the dyke which cuts the west end of the granite outcrop at Inganess (V., pl. viii.). He figures also a crystal of augite of simple form which he found in a dyke on Scabra Head, Rousay (V., p. 128).

The present account is based on investigations made in the field in the summer vacations of 1897 and 1898. The microscopic and chemical work was done in the geological laboratory of the University of Edinburgh. The result has been to show

that these dykes belong to the bostonites, camptonites, monchiquites, and alnöites, and that we have in Orkney a very interesting and remarkable series of this peculiar group of rocks.

I. The Geological Features of the Trap Dykes.

Their Number and Distribution.—The number of dykes from which specimens have been collected and examined during this investigation cannot be less than a hundred, and this might be considerably increased were we to regard as separate dykes what are really the branches and offshoots of one. Where two dykes are seen close together and running in parallel courses, it is not always possible to say definitely whether they are distinct, or would unite to form one, could they be traced for some distance. And there can be no doubt whatever that this is merely a fraction of the total number in the district. Far the most satisfactory exposures are those to be obtained by following a bare and rocky shore which runs in a direction at right angles to their trend, such as, for example, the west coast of the Mainland. Where the beach is low and covered with gravel, a dyke may easily escape notice, or be quite concealed, even though the occurrence of trap in loose blocks may indicate its presence. And, in the inland districts, it can only be a very small minority which meets the eye of the geologist. They are to be found in quarries, in burns, on the shores of inland lochs, and very occasionally they can be traced in a roadside or on the bare slopes of a hill. But in the cultivated fields, or the low moors covered by peat or boulder clay, any number of dykes might be so effectively concealed as to yield no evidence of their existence.

The rocks of which they consist are, as a rule, too soft and too readily decomposed to enable them to form outstanding wall-like outcrops. Such, however, do occasionally occur, as at Netherton (near Stromness), Crowness (near Kirkwall), and in the north of Birsay. They are usually denuded to the level of the surrounding flags, or lie even in trenches formed by their rapid decay. It is very rare to find that any noticeable feature of the scenery can be attributed to their presence. On the lofty cliffs of the west coast they sometimes, by reason of their perfect jointing, give rise to a narrow "geo" with parallel sides. On Rowe Head, near Skaill, the flagstone between two parallel dykes has been tunnelled out by the sea, forming a natural arch, the Hole of Rowe, the sides of which are lined in part by the trap dykes. Occasionally a dyke is so resistant as to form the face of an exposed cliff, as, for example, at Skaill in Rousay. In the inland streams their presence is commonly marked by small waterfalls, but at Corrigal, in Harray, the burn flows in a deep, narrow trough, out of which a trap dyke has been washed. The burn has evidently been turned out of its preglacial valley by a dam of boulder clay, and has followed the outcrop of the trap dyke in cutting its new course.

Sufficient is known, however, to prove that their distribution is far from uniform, and that from some districts they are entirely, or almost entirely, absent; while in others they are so numerous that several may occur in a distance of a few hundred

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yards. Without doubt it is in the West Mainland that they principally occur. In the splendid coast section which stretches from Breckness, near Stromness, to the Brough of Birsay, there is not a mile in which one cannot be found, and in several places there are three, four, or six to a mile. On the east coast of the West Mainland also, in Evie, Rendall, and Firth, although here the shores are low and gravelly, dykes are known to be present in considerable numbers. From the East Mainland, on the other hand, specimens of only two have been collected. The town of Kirkwall seems to stand on the boundary-line between the region of dykes and that from which dykes are almost absent. Five dykes were observed immediately to the west of the town, while none was seen for ten miles to the east of it.

Of the North Isles, Rousay is the only one which has yielded trap dykes; along the western and southern shore they are very frequent, and bear a great resemblance to those of the West Mainland. From the other islands they are not yet recorded, and although I have examined the whole shore of the principal islands, I failed to obtain any, so that they at least cannot be numerous. Of the South Isles I cannot speak with the same certainty, but in South Ronaldshay several are known which present interesting features of their own. While there cannot be any doubt that in the future many additions will be made to the numbers recorded, yet it is probable that the main facts of their distribution are already established.

Moreover, it is frequently to be observed that they tend to occur in groups, the members of which are at no great distance from one another, and show a close resemblance in petrological character. One such group, for example, includes those of South Ronaldshay. The dykes of Rousay, Birsay, and Evie form a well-marked class, closely similar to one another. Those also which are exposed in the coast section just north of the Black Craig on the west coast of the Mainland have certain common features, which at the same time are fairly distinctive. In the little bay at Binniaro, Firth, eight dykes occur, which in some points are quite unlike any other Orkney dykes. Instances might be multiplied, but sufficient has been adduced to show that a tendency exists for similar dykes to be grouped within a short distance of one another. It seems quite certain that this is not due to the branching and subdivision of one main dyke, though that is admittedly of common occurrence. In that case the distance which separates the branch dykes is never great, and they are inconstant, frequently uniting within a short distance only to diverge again, so that their true nature is quite obvious. It seems rather that the dykes of one group were all of the same, or about the same age, while those of other groups were injected at a different period. The common magma from which they all proceeded was one which from time to time underwent considerable change, and the fact that each group has common petrographical characters which differ from those of other groups would indicate that each series was the product of a separate phase in the cycle of change through which the magma passed. As will be seen later, there is evidence of another kind that certain of these dykes are of later origin than others.

This rule, however, is not without exceptions. Near the Bridge of Waithe two dykes occur—one a bostonite, the most acid of the whole series; the other a biotite-monchiquite or alnöite, one of the most basic. At Rennibuster, on the road from Kirkwall to Stromness, camptonite dykes, rich in felspar, are associated with monchiquites, from which felspar is absent. In these cases, however, the trend is different, and the association is merely fortuitous.

Breadth, Jointing, Contact Metamorphism, etc.—The breadth of the dykes may be stated to be on an average 3 feet to 3 feet 6 inches. Dykes under a foot are rare, except as lateral veins proceeding from a larger mass. On the other hand, one of 7 feet may be considered large. The dyke of Borwick, near Yeskenaby, is 9 feet; at Avalshay, Rousay, there is one of 10 feet, and the broadest of all is one near the Spoord, on the west coast of Birsay, which measures 12 feet from side to side. As usual in dykes, a series of transverse joints divides the mass into horizontal angular prisms, and the jointing is commonly finer and closer at the sides than in the centre. This is very well seen where the side of the dyke is exposed on the face of a sea cliff. In the thicker dykes the jointing is apt to be rude. Very often they weather in a spheroidal manner, exactly resembling the diabases. This, as a whole, is most characteristic of the rather basic camptonites, but it is far from universal. The general presence of fine-grained, chilled edges proves that the flagstones at the time of injection were comparatively cold. Some of the dykes, as will be later described, are remarkably coarsely porphyritic; but this is confined to the centre, the edges being fine grained, and, under the microscope, showing traces of residual glassy material, even though they never have a vitreous appearance to the naked eye. As remarked by Peach and Horne (III., p. 14), the dykes are often vesicular, and the vesicles, which are filled with white calcite, tend to be central in position. The flags adjacent to the dyke show usually a contact alteration, which rarely extends to more than a few inches, and amounts only to induration with the development of a closer texture and a splintery fracture. They frequently break down under the hammer into small sharp-edged cubical fragments. Jameson (*antea*) noted that where the flags contain much organic matter, bitumen may appear in the vesicles.

Regularity of the Trend, Branching, etc.—In most cases the exposures are very limited, and it is always to be observed that a dyke follows one of the two series of usually very perfect joints by which the flags are cut. These must have been antecedent to the dykes, and furnished lines of weakness along which the flagstones opened out. Wherever a dyke can be followed for more than a few yards, its course is found to be not absolutely straight. It passes frequently from one joint to another by means of a cross joint, then resumes its previous direction. This feature is so common that instances need hardly be cited, but it is well seen in a dyke at the Brough of Birsay, which runs parallel to the shore line, and can be traced in the rocky beach for about 400 yards, during which space it maintains its general direction with frequent tem-

porary deviations. Instances of curvature are rare; one was observed by Professor HEDDLE on the west coast, south of Skaill (at Borwick). "From a W. by N. it suddenly curves round to a W.S.W., and as suddenly regains its original direction, exhibiting thus a bold sigmoid feature" (V., p. 119). On Rowe Head, Skaill, there is a dyke well exposed on the bare cliff top for about 200 yards. About 100 yards back from the face of the cliff it forks into two branches—one of which maintains the original trend of the dyke—a little S. of W.; the other curves away and takes a nearly S.W. course. These are the only instances of curvature which have come under my notice.

It is very common, on the other hand, to find a dyke branching into two or more which run in parallel courses a few feet apart, and may unite again in a short space to form a single dyke. A fine example of this is to be seen below Widewall, South Ronaldshay. A large dyke, seven feet broad, is exposed in the beach at the mouth of the mill stream. It runs nearly parallel to the bank which overlooks the shore. A few yards further south it is found to be much diminished in breadth, while lateral dykes accompany it on each side. For about 400 yards it can be followed, and its general direction is well maintained, though it frequently passes laterally for a few feet, then resumes its former course. Sometimes there are three, at other times four or even more parallel dykes, but it is to be noted that the sum of their respective breadths is always equal to that of the original dyke where first seen; and under the microscope they are all very similar in character. At the Oyce, Finstown, a dyke penetrates a bed of contorted slickensided flagstones, in which the jointing is disturbed and irregular, and in consequence it breaks up into a plexus of ramifying veins. The flag between them is markedly indurated. On the west coast of Birsay, north of Skaill Bay, there are several dykes, which, traced along their courses, are found to die out with a wedge-shaped termination. In a parallel joint a few feet away a new dyke appears, beginning in the same manner as the other ends, and increasing step by step as it diminishes. No connecting vein can be seen. There is no doubt that in these cases the dyke has passed laterally along a bedding plane, and indeed this is a common feature of the exposures seen in vertical cliffs.

In the course of my examination of the Orkney dykes, two points have impressed me as particularly clear. One is that in every case they availed themselves of the highly perfect joints by which the flags were intersected. It is very rare to find any brecciation, and, except where the dykes branch, fragments of flagstone are never found included in the intrusive rock. The other is that in no case has a dyke been seen to pass laterally into a sheet or sill. This may be due to the ease with which the sedimentary masses opened along the joint planes, or possibly also to the pressure exerted by a great thickness of superjacent rock, which has been swept away by denudation since the time of injection.

The Trend of the Dykes.—Observations were made by means of a pocket compass of the direction in which every dyke was running so far as exposed, or, in case of branching or deviation, of the average course as near as could be judged; and although it is not

pretended that high accuracy was attained, sufficient care was taken to establish the following conclusion.

Of the camptonites, which form almost 90 per cent. of the whole, the average trend is E. 25° N., and rarely do they differ from this by more than 10°. Their regularity in this respect will be best seen from the tables appended. It has not proved possible to break them up into groups, each with a characteristic direction. Their extreme range is from E. 5° N. to E. 35° N., and two dykes within a hundred yards of one another may differ by as much as 10°. If we neglect one or two instances in which the outcrop is too limited to justify positive observations, or where in all probability it is a cross vein and not the main dyke that is exposed to view, their uniformity in this respect is so marked as to indicate with certainty that they are very closely allied in age and origin. The single dyke of bostonite is distinguished by having the most easterly trend of the whole series (east and west). On the other hand, the most basic dykes, the biotite-monchiquites and alnöites, have a very distinct northerly trend—the dykes at Quanterness N. 20° E., at the burn of Ireland N. 5° W., at Naversdale N. 20° W., the average being almost a direct north and south. Finally, the monchiquites proper seem to be connected with the camptonite series, with which they agree closely in direction, having perhaps a slightly more northerly direction.

Sequence of the Different Types.—The main road from Kirkwall to Stromness, four miles from the former town, crosses the mouths of two little burns which are flowing into the Bay of Firth. The streams are about 100 yards apart, and the road runs here along the sea shore; a little cottage stands just above the beach between the streams, and nearer the east one. In the west burn, just beside the bridge which carries the road across it, a large trap dyke is seen 6 feet 6 inches broad, which stands up boldly and forms a little waterfall. It is running east 20° north, or almost parallel with the Stromness road, and is a notable dyke, as the centre is coarsely porphyritic, with large hornblende crystals up to an inch in length; the edges are fine grained. It disappears in the grassy bank of the stream, but if we pass eastward, cross the road, and examine the sea beach below it, we find, a little further east, that two dykes appear in the beach with the same course as the burn dyke, and respectively 4 feet and 2 feet 6 inches in breadth. In character they exactly resemble the larger dyke in the burn, except that they are not so coarsely porphyritic, and their united breadths are equal to that of the burn dyke. It is clear that the burn dyke has swung northwards along a cross joint, hidden by soil and road, and after crossing the road has resumed its previous direction, but now as two dykes running a few feet apart. These two shore dykes can be traced in the gravelly beach till they disappear beneath the cottage. Here they must still further split up, for in the east burn, after passing the cottage, we see only a thin dyke with the characteristic trend, but not over a foot in breadth. The other branches are covered by gravel or boulder clay, and are not to be found. We have here, in fact, a case exactly similar to that already described from Widewall, South Ronaldshay, a single, rather large dyke, with the usual direction, which repeatedly branches into

parallel dykes of smaller breadth, and deviates occasionally from its straight course by passing for a short distance along cross joints. The microscope shows that they are all very similar, and belong to a peculiar group of the camptonites, with abundant felspar.

Now, in the beach, below high-water mark, and just north of the cottage which stands between the burns, three other dykes are to be seen, 3 feet, 1 foot 6 inches, and 2 feet wide, running nearly straight in parallel courses, but in a direction almost at right angles to that of the camptonite dyke described (N. 20° E.). They are 6 feet and 30 feet apart, but they are to be regarded as really one dyke, and belong to a very basic type, the biotite-monchiquites or alnöites, and contain no felspar. They are exposed for 40 or 50 yards, and, as we trace them southwards towards the little cottage, the east dyke thins, and finally dies out in a small vein; the middle dyke twists a good deal in its course, and approximates gradually to the west dyke, which is the most constant of the three. The west and mid dykes are finally covered by the gravel of the shore just behind the cottage. In the manner in which the east dyke dies out in a thin vein, while the other dykes twist from side to side and send veinlets into the chinks of the flags between them, which behind the house are filled with little ramifying jets of trap, we have something so different from the usual straightforward behaviour of the trap dykes as to prove that some peculiar obstruction lay in their course. This must have been the camptonite dyke which runs beneath the cottage, and which would necessarily bind the flagstones together and render irregular the series of cracks into which the later dykes were injected. The actual junction is covered by gravel and by the cottage, but there can be no doubt that we have here dykes of two different series, running each with the characteristic trend of its group, and crossing one another, and that the more basic were later in origin.

From no other locality has evidence been obtained which would elucidate the question of the succession in time of the different types. From the way in which the monchiquites are interspersed among camptonite dykes, agreeing with them in trend and in other characters, it seems, on the whole, most probable that they are of closely allied origin and of very similar date. Like the alnöites, the bostonites have a distinctive course, which would point to their having had a separate period of injection, but nothing is to hand to show whether earlier or later than the others.

The Age of the Dykes.—It has been generally assumed that the trap dykes of the Orkneys are outlying members of the great Tertiary series so well developed in many parts of Scotland. That they are not basaltic does not disprove this, as, in the opinion of Brögger (VI., p. 26) and others, the camptonites may be products of a basaltic magma. On the other hand, there is no conclusive evidence to establish this opinion. They cut the highest rocks of the Orcadian Old Red Sandstone in the district, the red sandstones of the John o' Groats series, near Newark Bay in Deerness. They are not yet proved to be later than the Upper Old Red Sandstone of Hoy; they are not confined to the Orkneys, as, in a collection of rock sections from Shetland, kindly lent

me by Mr Horne, of H.M. Geological Survey of Scotland, there is one of a much decomposed camptonite dyke from Brenista Bay. Their great freshness, in many cases, would point to a comparatively recent origin, for the igneous rocks of Old Red age are in every case much more decomposed (VII., p. 412). But, till we know more of the distribution of these rocks in the north-east of Scotland, it will not be possible to arrive at any definite conclusion as to the geological period in which they were formed.

That they are later than certain of the faults by which the flagstones are intersected has been shown by observations in two different localities. In the cliffs on the east coast of Holm there is a narrow inlet, the "Long Geo," where the sea has eaten its way inwards along the line of a fault. The "geo" runs east and west, and crossing it obliquely, there is a small trap dyke, which is not brecciated or broken by the fault, but, on the other hand, can be seen to twist out of its true course, and to split into two branches, where it cuts the crushed fault rock. On the west shore of the Wart Holm of Copinshay there is a beautiful monchiquite dyke, one side of which has been exposed by the sea, and forms the surface of the cliff for a short distance. It differs from all the other dykes in being not vertical, but inclined at an angle of 60° (to the vertical plane). Along the shore there runs a fault, as is shown by a crushed and slickensided belt of flagstones, and this has proved a line of least resistance, which has been taken advantage of by the dyke. Although all the flags are much disturbed and twisted, the dyke shows no deformation whatever, and is undoubtedly of later origin than the fault.

II. Petrographical Characters.

1. *The Bostonites.*

The only dyke of bostonite known to occur within the area was found on Onston Ness, a small promontory on the Loch of Stennis, just east of the Bridge of Waithe. Its width is a little over two feet, and its colour is greenish-grey, and much paler than is usual in the camptonites in the centre, but mixed with darker streaks; while at the edges, for about two inches, the rock is finer grained and darker in colour. In the hand specimen it shows neither porphyritic nor fluxion structure, and contains occasional vesicles which are filled with calcite.

Sections were made of the edges and of both the lighter and darker varieties of the centre. In the centre the rock is porphyritic, having large phenocrysts of anorthoclase felspar simply twinned on the Carlsbad plan, and with a honeycombed appearance indicating the former existence of glass enclosures (Pl. I. fig. 2). These lie in a groundmass, rather coarse grained, of felspars mostly simply twinned, and almost isodiametric, but without crystalline form. Among them are scattered long irregular sections of plagioclase crystals. This groundmass is holocrystalline, and fluxion structure is not apparent. Calcite and limonite in irregular patches are scattered about the section. There is no trace of quartz or of undecomposed ferro-magnesian minerals. But pale

green chlorite, mixed with calcite and limonite, is rather abundant, and fills up the interspaces between the felspars of the groundmass.

Specimens from nearer the margin contain felspar phenocrysts (partly of plagioclase) in a groundmass of long lath-shaped felspars, for the most part plagioclase, though simply twinned sections are frequent. The structure is trachytic and markedly fluidal (Pl. I. fig. 1), the felspars of the groundmass having their long axes parallel and winding in streams around the phenocrysts. Chlorite, with limonite and calcite, are here more abundant, but the nature of the original component from which derived is not determinable.

At the extreme margin the rock is finer grained, long narrow crystals, both orthoclase and plagioclase, most of them with forked irregular terminations lying in a groundmass still in some measure glassy, but filled with minute pointed felspar microliths arranged in no definite order.

The difference between this rock and even the most felspathic of the camptonite dykes is so marked as entirely to justify its recognition as the type of a distinct group. The predominance of felspar and abundance of orthoclase, the paucity of ferromagnesian ingredients, the pale colour and low specific gravity (2·653), all support this view. In some of the sections the resemblance to the bostonites is very close, but in others the abundance of plagioclase and of chlorite show that the rock is of a more basic character than any bostonite with which I have been able to compare it. Needless to say, the sections show no quartz. The presence of glassy material in the selvage is also a feature not characteristic of bostonite dykes.

	1.	2.	3.	4.	5.
SiO_2	52·00	62·28	63·25	67·16	56·50
TiO_2	·98	...	...	...	0·85
Al_2O_3	18·06	19·17	22·12	14·53	18·14
Fe_2O_3	2·18	3·39	...	4·17	3·12
FeO	5·14	...	...	...	2·86
MnO	·25	...	...	...	...
MgO	2·84	tr.	...	0·41	1·22
CaO	4·59	1·44	0·56	1·26	3·38
Na_2O	3·78	5·37	6·29	5·55	5·28
K_2O	4·68	5·93	5·92	6·10	1·60
H_2O	1·84	2·33	0·93	1·10	1·26
P_2O_5	...	...	...	...	...
CO_2	3·59	...	...	...	5·11
	99·93	99·91	99·07	100·28	99·32

1. Onston Ness, Orkney.
2. Lake Champlain, U.S.A. (IX. p. 20).
3. M'Gill College, Montreal (cited from XIX. p. 211).
4. Chateaugay Lake, Clinton Co., N.Y., U.S.A. (cited from XIX. p. 211).
5. Maena, Gran, Christiania (VIII. p. 207).

Chemical analysis was made of a specimen from about six inches from the margin so as to obtain as nearly as possible the average composition. It is given below No. 1, and may be compared with the analyses of similar rocks appended.

It will be seen that with Nos. 2, 3, and 4, which are typical bostonites, the affinities of the Onston dyke are by no means close. No. 5 is regarded by Professor BRÖGGER as the representative of a distinct group which he names the maenites or lime-bostonites. The characteristics he insists on are the large percentage of lime and the preponderance of soda over potash. In the latter respect the Onston dyke is far more near the typical analyses 2, 3, and 4, as it shows a slight excess of potash. Otherwise the two analyses agree pretty well; but in the Orkney dyke the high percentage of iron, mostly in the ferrous condition, and of magnesia and water, and the lower percentage of alumina, indicate a greater abundance of a chloritic mineral.

The Camptonite Dykes.

The dykes included in this series may be porphyritic or non-porphyritic, and vary in coarseness of grain, having often fine chilled edges and a coarser centre. They consist of olivine, augite, brown hornblende, and plagioclase felspar; the commonest accessory minerals being magnetite, ilmenite, pyrite, apatite, biotite, and orthoclase. Olivine is the commonest porphyritic ingredient; augite and brown hornblende occur less frequently. The groundmass contains hornblende or augite, or most frequently both of these, with plagioclase felspar, and sometimes orthoclase. The structure is typically panidiomorphic, all the minerals having well-developed crystalline form. The fresh rocks are black or very dark green, rarely showing fluxion structure, and usually with lighter spots, due to the presence of calcite and "ocelli" of felspar. When weathered they are green or rusty brown, and the tendency to spheroidal decomposition is very marked.

The groundmass of the porphyritic dykes resembles in most respects, as regards composition and structure, the usual type of non-porphyritic camptonite; and the variety exhibited by the porphyritic minerals gives greater interest to the study of the dykes containing them. I shall select for detailed description the most important dykes with porphyritic structure before considering the much commoner rocks in which this structure is absent.

The Rennibuster Dykes.—The dyke already mentioned as crossing the west burn at Rennibuster, four miles from Kirkwall, is a remarkably fresh and interesting camptonite. It is 6 feet 6 inches wide, and at the edges finer grained for a few inches, but showing in a compact groundmass crystals of olivine, augite, and hornblende, much altered. Towards the centre it becomes coarser grained, and contains large phenocrysts of hornblende, augite, and olivine. The hornblendes measure in some cases one inch in length, and nearly half an inch in breadth, and are occasionally arranged in a parallel manner, so as to indicate fluxion structure. The augite and olivine are not over a quarter inch in diameter. The weathered material is dark green, the fresh rock is black.

Prismatic jointing is well developed, and the flags for a few inches from the margin are slightly baked and hardened.

Sections from the centre rarely contain the large phenocrysts of hornblende so striking in the hand specimen, as their perfect cleavage renders them exceedingly apt to be shattered in grinding. They do not appear to be idiomorphic, their outlines being rounded and irregular. They are not markedly zonal, but have sometimes a slightly darker border. Twinning is common, and the usual hornblende cleavage very perfect. The only enclosures observed were calcite, apatite, and magnetite. Extinction and pleochroism are the same as in the smaller crystals. That olivine was an original ingredient is proved by the presence of pseudomorphs with the outlines of olivine in specimens the augite of which is quite undecomposed. The serpentine is often covered with a meshwork of magnetite, and mixed with more or less calcite. The augite phenocrysts are the most numerous (Pl. I., fig. 3). They are of rather a pale greenish variety, and perfectly idiomorphic, showing in the prism zone the faces 110, 100, 010, and terminated apparently by 111 and $10\bar{1}$. The prismatic cleavage is highly perfect, and there are in addition numerous irregular cracks. The enclosures are apatite, magnetite, olivine (altered into serpentine), calcite, and occasionally a minute patch of hornblende. The crystals are sometimes twinned on 100. An extinction angle up to 43° was measured. Between crossed nicols there is neither zonal nor hour-glass structure. Decomposition begins at the edges and in the strongly marked cracks, and proceeds gradually over the mineral, so that in early stages there is a perfectly fresh centre, while an outer zone, ·3 mm. or more in depth, is passing into deep green chlorite. The increase of volume due to hydration produces a series of fissures perpendicular to the surface of the crystal which make this border very evident. Its inner limits are usually so sharp as to suggest that it is due to an outer layer of slightly different composition, and more liable to alteration than the central mass; and, indeed, where least altered, this border appears to have a more brownish colour. In other sections the mineral is entirely altered into a pale chlorite mixed with finely divided calcite, and covered with a fine dust of magnetite, with anatase or rutile. Even in these pseudomorphs an outer zone is clearly distinguishable from the central part. Magnetite is abundant in large rounded masses, and in small sharply angular crystals disseminated through the other minerals. Around the larger crystals there is usually a number of smaller grains, radiately disposed, and mixed with biotite. Pyrite is rather common, and the presence of ilmenite is indicated by occasional traces of alteration into leucoxene. Biotite in small scales of deep brown colour and very intense pleochroism is frequently adherent to the iron oxides, and is found in small quantity scattered through the other minerals. There are numerous long delicate pointed crystals of apatite.

The groundmass consists of augite, hornblende, and felspar. The augite is similar in general character to the porphyritic crystals, but of smaller size, and not quite so perfectly idiomorphic; it is only in small amount, and is commonly decomposed, while the hornblende remains quite fresh. The hornblende forms almost one-half of the

groundmass in long, narrow prisms of a rich brown colour. In cross-section it shows the usual faces (110, 010), the prism faces being best developed. In longitudinal section it is sometimes ten times as long as broad, and the ends are irregular or forked. Twinning is common (on 100), and the only enclosures are apatite and magnetite. The pleochroism is—$\mathfrak{c}$, dark brown; $\mathfrak{b}$, pale brown; $\mathfrak{a}$, clear yellow—absorption; $\mathfrak{c}>\mathfrak{b}>>\mathfrak{a}$. From the measurement of twinned crystals the angle $\mathfrak{c}:c$ is about 10°. In all the sections the hornblende is quite undecomposed. The felspar lies between the other ingredients, and being evidently the last to crystallise, shows less perfect crystalline form. Much of it is plagioclase, with both albite and pericline twinning, and often zonal. The long, narrow crystals lie scattered in every direction; around them is often a considerable amount of simply twinned orthoclase, never with crystalline outlines, and commonest in the areas richest in felspar. The plagioclase it surrounds shows sharp idiomorphism towards the orthoclase, which has been the last mineral to develop. According to the extinction of the twin lamellæ, the plagioclase is principally a variety between andesine and labradorite. Both minerals are quite fresh; they enclose grains of magnetite and hornblende and apatite in long needle-like crystals.

To prove the presence of orthoclase, several grammes of the rock were crushed and sifted. The powder selected had an average diameter of $\frac{1}{80}$ inch. This was cast into a Thoulet's solution of specific gravity 2·62. The grains which floated were removed and introduced into a Sollas's diffusion column. The indicators used were pure adularia and cordierite of specific gravity 2·59. None of the particles floated so high as the former, and the great majority had a specific gravity of between 2·58 and 2·59. The felspar is hence an anorthoclase. When removed, washed, and examined under the microscope, they had all the characters of orthoclase felspar. The enclosures were apatite (sp. gr., 3·16 to 3·22) and possibly needles of hornblende.

In structure the rock is panidiomorphic, all the constituents, except orthoclase, showing sharp crystalline outlines; it is holocrystalline, no trace of glassy base being present. The arrangement of the groundmass is mostly perfectly irregular. Around the olivine phenocrysts the later hornblendes are often grouped with their axes parallel to the crystal faces, a rough approach to ocellar structure; and in places there are pale spots rich in felspar, which may be 3 or 4 mm. in diameter. The long plagioclases converge towards the centre. Between them is orthoclase and a little hornblende in long radiating prisms; a cavity filled with calcite occupies the centre, and into it the sharp points of the surrounding crystals project, showing that it is really of miarolitic origin. (See Pl. I. fig. 3.)

The dykes in the shore and in the east burn are neither so fresh nor so coarse-grained as the centre of the large dyke described, but they are practically identical with it in composition and in structure. As we approach their edges, all the dykes become finer-grained, but retain the panidiomorphic structure. There is no trace of a glassy selvage. Olivine and augite are the phenocrysts; hornblende does not seem to have formed in the first generation; hornblende and plagioclase form the groundmass; there is little

evidence of augite. The ferro-magnesian minerals are much altered, the products being chlorite, calcite, and much anatase. An occasional cluster of epidote and a few grains of sphene are to be found. Magnetite in skeleton networks, ilmenite weathering into leucoxene, and pyrite in large cubes are abundant in some of the slides.

The order of crystallisation appears to have been as follows:—Pyrite, magnetite, and ilmenite; apatite; olivine; augite; biotite and hornblende; and in the groundmass, augite; hornblende; plagioclase; orthoclase.

The Dyke from Scabra Head, Rousay.—The presence of large phenocrysts, such as occur in the Rennibuster dyke, is not exactly a common feature of the Orkney camptonites; it occurs in all in about a dozen. These are usually 5 feet or more in breadth, but several of the broadest dykes show no porphyritic crystals. On the very summit of Scabra Head, Rousay, there is one which resembles in many respects that of Rennibuster. It is 6 feet broad, and of several dykes in this region it is the broadest, and the only one with porphyritic structure. The edges are fine-grained, but in the central part the dyke is not homogeneous, fine-grained bands alternating with others which are porphyritic. This is perhaps due to enclosures of blocks of sandstone, a few of which are to be seen slightly indurated, and in contact with them the dyke shows a chilled edge. Some of the veins which run in the joints of the flags have a compact edge and a porphyritic centre, though not over 3 inches in total breadth.

The minerals of the first generation are pyrite in large crystals, magnetite, and apatite; olivine in small quantity and entirely altered into serpentine; biotite in scattered scales; augite and hornblende (Pl. I. fig. 4). The augite is perfectly idiomorphic, and this dyke is probably the source of the crystal figured by Heddle, showing the faces (110, 100, 010, 111, $10\bar{1}$), as I saw similar crystals in the weathered material on the surface. It encloses olivine, apatite, magnetite, and pyrite. Most of it has a distinctly violet tinge and a slight pleochroism, otherwise it resembles that of Rennibuster. In some of the crystals scattered flakes of hornblende occur, and they have all the same extinction, though few and far apart, being evidently in parallel growth with the augite which surrounds them. There is in many cases a narrow marginal zone of paler colour and slightly different extinction from the central area of the crystal. The hornblende crystals are smaller, but probably more numerous than at Rennibuster, and are more frequently preserved in the sections. Some are almost without cleavage. It is almost quite idiomorphic, in crystals not very elongated. Zonal structure is very common; mostly the centre is of a darker colour than the margin, but there may be several zones of alternately lighter and darker shade. The various zones have parallel outlines; it encloses apatite, magnetite, pyrite, and sometimes partly envelops an augite. Pleochroism and absorption resemble those described for Rennibuster, but the colours are more reddish and the absorption more intense, especially in the interior, where b and c are sometimes dark greenish-brown, a pale brown.

The groundmass is similar in structure to that of the Rennibuster dyke as regards panidiomorphism and irregular arrangement of the ingredients; but the white felspathic

spots are few; the grain is finer (the hornblendes averaging ·03 mm. in breadth); and augite, which is not very abundant in the former rock, is practically absent here, even where the material is fresh enough to ensure its identification if present. There is apparently no orthoclase. This rock is a typical camptonite in the sense in which the term is used by some authors.

The Dyke at Stromness.—Just outside Stromness harbour, on the isthmus which connects the Holms with the Mainland at low water, there is a porphyritic camptonite dyke which differs from these two in some important respects. The phenocrysts are olivine (serpentinised) and augite, very similar to that of Rennibuster. It is of a pale violet colour, sometimes rather greenish, and has a narrow marginal zone of a slightly darker tint. Flakes of hornblende in parallel growth are scattered through the augite. There are no hornblende phenocrysts. Magnetite occurs in rather large areas, fringed with biotite scales (Pl. I. fig. 5).

The groundmass contains augite and hornblende in about equal amount, in crystals of good crystalline form, and not more than four or five times as long as broad. On the surface of the olivine little hornblendes are planted in ocellar fashion. The augite and hornblende are often in parallel growth. Augite forms the centre, hornblende the periphery, or each may form one side of a crystal, and give its outlines to that part it constitutes. Prisms, also, are common, of which the ends are hornblende, while the centre is augite. Together they form one-half of the groundmass. The rest is felspar in long, narrow prisms, often aggregated to form radiate white spots with a central area of calcite.

North Galton.—Another porphyritic dyke of great interest occurs at North Galton, Sandwick (Pl. I. fig. 6). The phenocrysts are olivine (entirely altered into serpentine, magnetite, and calcite) and augite of a brownish-violet tint, like the titaniferous augites of many basalts, and with a marked dispersion of the axes. As usual, it is perfectly idiomorphic, and encloses flakes of brown hornblende in parallel growth. At the margins the groundmass is that typical of the camptonites, consisting of hornblende and augite in about equal proportions, the augite brownish-violet, the hornblende reddish-brown. Both are frequently twinned, and parallel growths are exceedingly common. Here also the augite is internal, central, or lateral; the hornblende external, peripheral, and sometimes terminal. The augite was evidently the first of the two minerals to finish crystallising. Plagioclase felspar forms the other half of the groundmass in long, narrow prisms, while felspathic spots of radiating crystals are numerous.

In the centre of the dyke we have quite a different structure (Pl. I. fig. 6). The groundmass is much coarser; it contains felspar, partly in long, narrow polysynthetic crystals, partly in large, broader, simply twinned crystals. It is unusually abundant in this rock, and the only other mineral of the second generation is brown hornblende. Both minerals are partly idiomorphic, the hornblende much less so than usual. Around the augite phenocrysts lath-shaped plagioclases are arranged parallel to the crystal faces, and the interstices between them are filled up by hornblende, which is entirely

moulded upon felspar. The structure is intersertal. In other parts of the section the hornblende has more continuity, and is less penetrated by felspar, but it is everywhere less idiomorphic than usual, and obviously later than felspar, around which it is very commonly deposited. This is a reversal of the normal sequence in these dykes, and indicates a transition to the diabases and proterobases. There is no second generation of augite in the slides. It is noteworthy that the usual deposit of hornblende on the surface of the phenocrysts does not appear, being replaced by plagioclase felspar. The sequence of crystallisation must have been magnetite, ilmenite, apatite, olivine, biotite, augite, with a little hornblende in parallel growth; then plagioclase, and finally plagioclase and hornblende.

The South Ronaldshay Dykes.—In South Ronaldshay occurs a group of dykes, in many ways the most remarkable of any of the camptonite dykes of the Orkneys. These include that already described as running in the shore at Widewall, and breaking up by repeated branching into parallel dykes. The two others are in Hoxa, a few hundred yards to the west of the landing-place. These dykes are all comparatively broad (5 feet to 7 feet), and carry large crystals of hornblende, augite, and olivine; they are more basic than the general type of the camptonites; some of them, in fact, contain so little felspar and so much glassy base as to pass gradually into monchiquites. I have retained them, however, in this class, as they form a natural group, closely connected, and their general facies is that of camptonites with little felspar.

They all contain olivine in phenocrysts, usually decomposed, but in sections taken from the east dyke of Hoxa, one foot from the edge, it occurs in crystals almost quite fresh, and showing serpentine only along the borders and cracks. Augite and hornblende are present in crystals up to one-half inch in length in the west dyke on Hoxa; in the east dyke and at Widewall they are smaller, but numerous. The augite phenocrysts resemble those described in the Stromness and Rennibuster dykes in most respects. Like them, they contain flakes of brown hornblende in parallel growth, and have commonly a margin of a darker brown. Zonal structure is very marked, and some of the crystals contain a nucleus of a bright green colour, which seems to have suffered from corrosion, being only partly idiomorphic. This augite is slightly dichroic in shades of yellow, green, and darker green; it has a smaller extinction angle than the brownish augite which surrounds it. In one section the nucleus extinguished at 31°, the periphery at 44°. The central augite has also a greater axial angle, and apparently a weaker dispersion. The hornblende is in large, short, stout crystals, with evident traces of corrosion, and distinctly zonal. A darker margin surrounds a paler centre, but there may be five or six zones of varying shades of brown. As a rule, these are strictly parallel to one another. In pleochroism and other characters they resemble the hornblende in the dyke on Scabra Head (Pl. II. fig. 3).

The majority of the phenocrysts are of a more complicated structure, and consist of several zones of different composition succeeding one another in a definite order. In the centre of the crystal we may have a rounded remnant of the green augite above

described, which is apparently one of the earliest minerals to crystallise out, as it occurs only enclosed in brown augite or in hornblende. It encloses glass cavities, with an immobile bubble, magnetite in small octahedra, and apatite. Olivine accompanies it, apparently formed at the same period.

The innermost and oldest hornblende is of a deep brown or dark greenish-brown colour, with a pleochroism $\mathfrak{c}$, deep greenish-brown; $\mathfrak{b}$, dark brown; $\mathfrak{a}$, pale brown. The absorption for all the rays is more intense than usual. This is in part due to the great amount of magnetite in very minute grains which are scattered throughout the mineral, though not quite equally distributed. In some places they make it almost opaque. They lie parallel to the prism faces, and as they run in streaks following the cleavage, they give the sections a laminated appearance resembling diallage (Pl. II. figs. 2 and 6). Higher magnification shows, however, that this is not cleavage, as in the thinnest sections they are not sharply defined lines, but broadish bands along which magnetite in rounded grains has been deposited, while in the centre of the bands a cleavage crack may occasionally be found. The enclosures found in this form of hornblende are apatite, magnetite, occasional glass cavities, and bright green augite and olivine. It has usually the form of a highly corroded remnant of very irregular outlines, into which tongue-shaped intrusions have penetrated from every side (Pl. II. figs. 4 and 6). It may indeed resemble a spongy mass of hornblende, the interstices of which have been filled up by later products of crystallisation. This hornblende is found only in the rocks which contain large porphyritic crystals, and in them only as a central nucleus. Still such remnants may be of considerable size, measuring over $\frac{1}{4}$ inch in diameter.

Around this is commonly found a thin zone of brown hornblende, clear and less green in colour. It is usually free from enclosures, except scattered grains of magnetite. Its external outlines follow those of the mass it surrounds, and it is never more than ·01 mm. in breadth (Pl. II. fig. 4).

The third zone in these complex crystals covers the two described, or, when they are absent, forms the nucleus itself. It has always a dark appearance, owing to the abundant grains of magnetite it encloses. These are of larger size than in the central mass, where they are mere dust. It consists of an intergrowth of pale violet augite, with dark hornblende, usually in about equal proportions, though often the hornblende predominates. In polarised light the clearer colours of the augite, owing to its smaller absorption, distinguish it at once, and all the augite granules extinguish in the same position, while the scattered patches of hornblende are also in optical continuity, and are moreover in parallel growth with the central hornblende on which they were deposited. The structure is similar to that of the granophyres, where the graphic quartz and felspar of the groundmass are in optical continuity with the phenocrysts. Whether this graphic intergrowth does not enclose a certain number of glass cavities is not easy to establish with certainty (Pl. II. figs. 1–6).

The external zone varies in character. It may be brown hornblende, comparatively pure, and sometimes showing zones of varying tint. It may be brown augite of the

usual variety. In either case the result is the completion of the crystalline form, which is that of the external mineral, and is often perfectly sharp and well preserved. But more usually this zone is composed of both minerals in parallel growth, each forming part of the external rim, and lending its form to the part it constitutes. The augite often forms the lateral faces, the hornblende the terminal ones (Pl. II. fig. 2). But their relative position and amount follow no fixed rule. At one end there may be augite, at another hornblende, or hornblende may be the earlier and form the inner part, while a mere film of augite surrounds it, thickest at the corners over the prism faces and thinnest on the pinakoids, and giving the whole the octagonal transverse section of pyroxene. Between crossed nicols it is easily seen that the marginal augite is in optical continuity with the scattered patches in the graphic augite-hornblende intergrowth in the zone within; and that it is in parallel growth with the hornblende is shown by the disposition of the cleavages, and the simultaneous extinction of both minerals in certain sections, those, namely, which are parallel to the ortho-diagonal. The pinakoidal faces of both minerals are parallel to one another. The relation of their terminal planes could not be made out, owing to the fact that where these crystalline faces of one mineral are perfect, those of the other are deficient. The crystals are frequently twinned, and the twin plane passes straight through both minerals (see Pl. II. figs. 1, 2, 3, 4, 5, and description, p. 904).

This is the commonest, and at the same time the most highly developed and complex type of structure, but many sections are found in which all these zones are not present. The centre may be an augite-hornblende intergrowth; the periphery ordinary hornblende or augite; or on a nucleus of greenish-brown laminated hornblende, a border of ordinary hornblende may have formed. There is, in fact, a very great variety of combinations. But the order of succession is invariably that given, and is never reversed, although certain of the zones may be wanting. The dark brown hornblende full of minute magnetite dust is never any but the innermost zone, and never occurs without another zone of whatever sort surrounding it. Neither does the graphic intergrowth of hornblende and augite form the external surface in any case. The absence of certain elements may perhaps be explained as due to the section having missed that part of the crystal which they constitute, and this is most probable when the central zones are apparently absent. It is obvious that these structures are the result of a complex series of periods of alternate crystal growth and destruction by corrosion. Whether the augite-hornblende intergrowth is a direct formation or a resorption product it is not easy to establish. It has rarely external crystalline outlines, and it is filled with magnetite sometimes to such an extent as to be almost opaque, and in these respects it recalls many instances of the complete or partial destruction of hornblende and substitution of augite and magnetite. But this does not explain the graphic structure, and it seems more probable that this zone has originated by direct crystallisation from the magma. Its external outlines may be those characteristic of augite or of hornblende, but more usually are rounded or irregular.

In the west dyke of Hoxa these complex phenocrysts are largest and most numerous.

The groundmass consists of brown hornblende, brownish-violet augite, plagioclase felspar, and glassy base. The hornblende preponderates, and is commonly in parallel growth with the augite, and both have sharp crystalline form. The felspar in irregular masses fills up the interspaces. The glassy base is not abundant, is turbid with decomposition products, and filled with colourless, ramifying microliths, which appear to be felspar. Its presence indicates that the rock is a transition to the monchiquites, where such a material is a constant constituent. Rounded pale ocelli, consisting of felspar prisms mixed with a little hornblende and much glassy matter, are scattered through the slides (Pl. II., fig. 4).

The other dykes contain the same minerals, though the phenocrysts are smaller, and the groundmass richer in felspar. Towards the edges of the dykes there is a tendency to the development of much glassy material, which, with the absence of felspar, makes the sections very similar to those of some monchiquites.

In all the South Ronaldshay dykes, but particularly in the west dyke of Hoxa, sections of a colourless isotropic mineral, clear, transparent, without regular cleavage or trace of crystalline form, are to be observed. It rarely shows any double refraction, and agrees in all its characters with analcite. The manner of its occurrence is pretty varied. It is abundant in the pale ocelli, mixed with felspar and pale, turbid, decomposed glass. There it is usually central, but appears in the ocellus itself, and not in the miarolitic cavity, which is occupied by calcite. It is surrounded by the glassy base, and merges into it gradually on every side. It is apparently formed by the decomposition and hydration of areas of a glass, which, as Pirsson has shown (XI.), has a close resemblance in chemical composition to analcite. In scattered spots in the groundmass it is also seen, and is here of similar origin. The tendency of the still liquid magma to aggregate into globules after the crystallisation of the hornblende and augite of the groundmass which has given rise to ocelli, has produced also smaller rounded patches of glass, mixed with a little felspar at the edges, which readily change to analcite. Within the larger phenocrysts, and especially the honeycombed, corroded, nuclei of hornblende, analcite is abundant, occupying sac-shaped cavities. In many cases, around it are a few felspathic needles, lying in a little turbid glass with hornblende in parallel growth with the enclosing crystal, and it may be a little magnetite. These are, in fact, glass cavities decomposed and reconstituted into analcite. At the periphery, the hornblende and magnetite have crystallised out with a little felspar, leaving in the centre an area of glass. In other cases, the cavity is apparently entirely filled with analcite, or analcite mixed with calcite, which may surround it or be enclosed in it, or each may occupy one side. Similarly, a rounded patch of analcite may lie in the groundmass, and may exactly resemble a corroded phenocryst. Out of a large number of sections the best example of this is photographed in fig. 3, Pl. II. In view of the suggestion made by Lindgren (XII.), and adopted by Cross (XIII.) and Pirsson (XI.), that analcite may be a primary porphyritic ingredient in rocks allied to these, I have given special attention to these

crystals. I find no evidence of their primary origin. In that case the period of their crystallisation would have been utterly indeterminate. It must have been one of the earliest, one of the intermediate, and probably also the very last mineral to crystallise. It has never traces of idiomorphism, or of that expansive crystalline force of which Cross finds evidence (XIII. p. 686). Its constant association with calcite is practically decisive as regards its origin in this particular rock.

Another suggestion offers itself, viz., that the apparent phenocrysts may have resulted from the decomposition of some such mineral as nepheline. In fact, out of over thirty sections, two show each one crystal partly enclosed in porphyritic hornblende, and corroded where surrounded by the groundmass. The only mineral these resemble is nepheline, but in view of the chemical composition of the rock it is difficult to see how it could have been produced. It may be recalled, also, that the very early phenocrysts of augite resemble aegirine augite very closely. What this mineral is I have been unable to decide with certainty.

General Features of the other Camptonite Dykes.

Of the greater number of the camptonite dykes scattered so abundantly over the West Mainland and Rousay, little remains to be said. They resemble in most respects the groundmass of those already described. Most of them show in section phenocrysts of olivine, less commonly of augite, and of hornblende only very seldom. The groundmass is panidiomorphic and holocrystalline, consisting of augite and hornblende in very variable relative proportions, and usually intergrown, the hornblende surrounding the augite or forming the terminations of the crystals. Felspar is never absent, and may form quite one-half of the rock. It is mostly plagioclase, which stands as a whole between andesine and labradorite, though orthoclase would seem to be present in small amount in many dykes. It surrounds the plagioclase, or forms a sort of interstitial substance without crystalline form. Often the long plagioclase felspars are grouped in radiating fan-like aggregates, spreading out from a common axis. Except in rocks which approach the monchiquites, traces of a glassy base are absent. The felspars are usually filled with hair-like apatites. They are often greenish, and may be in part actinolite, as suggested by Brögger (VIII. p. 50).

Most of the rocks contain pale rounded felspathic ocelli. These consist of radiating plagioclase crystals, mixed with long narrow hornblendes. Even when augite occurs in the groundmass elsewhere, it is absent from the ocelli. In the Hoxa dykes the felspars are separated by much glassy material. Orthoclase may be comparatively abundant, though not obvious in other parts of the rock, and is found chiefly in the interior. In the centre there is always an irregular mass of calcite, filling what appears to have been a miarolitic cavity. These ocelli consist, in fact, of the last minerals to crystallise, and are due to the accumulation of globules of the still liquid magma at a late period in consolidation. Crystallisation has proceeded from margin to centre, leaving a cavity to

be subsequently filled by matter infiltrated from the surrounding rocks, which are highly calcareous.

The large dykes tend to be coarse grained in the centre, where all the ingredients have a short broad form. At the margin, on the other hand, both felspars and ferromagnesian minerals become very long and narrow, and the rock is very fine grained. The presence of glassy material in the chilled edge of the dykes is very unusual in the camptonites. Occasionally steam cavities of rounded form are to be noted. They are filled in with chlorite and calcite.

All states of preservation are to be found. Some dykes are so fresh that all the minerals except olivine are practically unchanged. Others, again, have decomposed into a rusty-brown earth, enclosing rounded nodules of less altered material, from which microscopic sections can hardly be prepared. The augite is far more susceptible to weathering than the hornblende, while the biotite, which is probably never entirely absent, though never abundant, is so resistant as to be particularly conspicuous in the more advanced stages of decay. Olivine passes into serpentine and magnetite, and later into calcite and limonite. The augite yields a deep green mineral, which is often chlorite, dusted over with innumerable grains of anatase. The resulting pseudomorph is fibrous, with the fibres parallel to the long axis of the section, and a straight, or nearly straight extinction. The polarisation colours are not uncommonly too high for chlorite, and I suspect that uralite is a very frequent product, especially of the pyroxene of the groundmass, but chloritisation is the usual change in the phenocrysts. Hornblende and biotite pass into chlorite with abundant anatase and occasional rutile. Kaolin and calcite result from the decay of the felspars, and in the more altered rocks rich yellow epidote is often to be seen. That the iron ores are usually titaniferous is evident from the abundance of leucoxene. Calcite is everywhere present, and increases greatly in amount as decomposition advances.

A peculiar secondary amphibole is found in one or two dykes near Outshore Point in Birsay. The sections show primary hornblende of a deep brown colour in large irregular phenocrysts, with numerous cracks, now filled with calcite and chlorite. It resembles that of the dykes previously described (*e.g.*, Scabra Head), and its extinction angle is about 15°. Though much cracked and infiltrated with secondary products, it is mostly quite fresh, but on the outer surface, and lining the fissures, there are occasional spots of a deep blue hornblende. This has a very pronounced cleavage, and may indeed be said to be fibrous. The two minerals are in parallel growth. In one section, which must be nearly parallel to the clinopinakoid, the brown hornblende extinguishes at an obliquity of 15°, the other at 7° on the opposite side of the traces of the prismatic cleavage, and the axis of elasticity most nearly parallel to the prism axis is that of greatest elasticity. The pleochroism is intense.

$\mathfrak{c}$ = pale yellow-green, $\mathfrak{b}$ = pale bluish-green, $\mathfrak{a}$ = deep blue; absorption,—$\mathfrak{a} > \mathfrak{b} > \mathfrak{c}$.

This is, in fact, a secondary mineral of the Riebeckite group, very similar to that which was thoroughly investigated by Cross (X.) in a rock which has much in

common with the camptonites. Here, however, it does not surround augite, nor does it form those frayed-out fringes which he describes.

Classification of the Camptonites.—Rosenbusch (XV., vol. ii. p. 540) has proposed to subdivide this series of rocks, according to the porphyritic minerals they contain, into hornblende-camptonites with olivine, augite, and hornblende; biotite-camptonites with olivine, augite, and biotite; biotite-hornblende-camptonites with olivine, augite, biotite, and hornblende; but, if applied to the Orkney dykes, this would result in a very artificial grouping. Many of them contain no phenocrysts, except occasional olivine. Others contain only olivine and augite. Biotite is very wide-spread, and perhaps universal, but in such small amount, and always so inconspicuous, as to be best regarded as an accessory. In the groundmass both hornblende and augite usually occur, but in a few slides, which are perfectly fresh, only hornblende is to be seen. The same rock may contain both minerals in the groundmass at the edges, while only hornblende is present at the centre (*e.g.*, Galton).

In the literature of this group many descriptions are given of rocks which contain only hornblende in the groundmass, mixed with green chlorite, which is regarded as secondary after hornblende. I very much doubt whether this is always the case. The augite decomposes so much more readily than the amphibole, that it may be quite decomposed while the other is fresh, and, owing to the frequent parallel growths, the outlines may be those of hornblende, and the crystal appear to be a hornblende, which at one part has weathered into chlorite. In many such cases sections of fresher material from the same dyke have shown the presence of augite, and, after a careful study, the conviction has been forced upon me that it is only very rarely it can be concluded that augite is entirely absent from the groundmass. To subdivide the rocks into camptonites with hornblende and plagioclase in the groundmass, and augite-camptonites which contain augite in addition, would lead us in some dykes to give different specific names to the central and marginal parts of the same mass. These two minerals, in fact, are so closely related, both as phenocrysts and as constituents of the groundmass, that in the Orkney dykes such a subdivision would be highly unnatural.

What might be called a pure augite-camptonite, were the term not already employed in a different signification, is shown in a section from a loose block of trap I picked up on the beach a little to the south of Burness, Firth. The original dyke must be concealed by the gravel of the shore. The section shows phenocrysts only of olivine altered into serpentine, and consists of a very pale violet augite, quite idiomorphic, in short stout prisms, which are often in radiating groups and perfectly fresh. Magnetite, ilmenite, and scales of biotite are scattered through the slide. There is no hornblende. Plagioclase felspar in well-formed crystals occupies the spaces between the augites, and forms occasional ocelli, with calcite in the centre. The structure is that of the camptonites, but their most characteristic mineral, brown hornblende, is quite absent. The general appearance of the slide will be seen from the photograph (Pl. III. fig. 1).

In the Bay of Binniaro, a little south of the locality just mentioned, occurs a group

of dykes, which in some points are quite different from any others in the Orkneys. The most interesting is that just below the house of Binniaro. It is porphyritic, and the only Orkney camptonite to show phenocrysts of felspar. These are numerous, well shaped, free from inclusions, and belong to a labradorite of average composition. They show albite and pericline twinning, and are tabular on the brachypinakoid. The other crystals of the first generation are olivine, altered into serpentine, and augite, of a pale greenish variety, in idiomorphic crystals, aggregated into nodular groups and decomposing into chlorite. It has an extinction angle of 43°. The groundmass consists of felspar (labradorite of a rather acid kind) and augite largely chloritised, and is filled with skeleton growths of magnetite. The felspar is ·05 to ·01 mm. in section, and up to ·5 mm. in length. The crystals lie scattered irregularly through the slide; their form is sharp and perfect. Between them lies the augite in grains of irregular form, mostly altered into chlorite, and evidently later than the felspar in crystallisation. The structure hence is intersertal, and the rock approaches closely to the diabases. The association of diabase with camptonite is too well known to call for any remark. The usual sequence of crystallisation in the groundmass is, in this instance, reversed (see photograph No. 2, Pl. III.).

The dykes associated with this one are intermediate in character between it and the normal camptonites. The nearest one is a fairly typical camptonite, rather rich in augite, but with brown hornblende in smaller quantity, and panidiomorphic in structure. Their chilled edges have more resemblance to diabase, being fine grained, full of magnetite skeletons, and consist of small lath-shaped idiomorphic felspar, with chloritised augite between.

Chemical Analyses of the Camptonites.

The dykes selected for analysis were :—

1. The dyke at Rennibuster.
2. A dyke from the Wart of Skaill, Sandwick; very typical of the West Mainland dykes, with phenocrysts of decomposed olivine and a groundmass of augite, hornblende, and plagioclase.
3. The west dyke at Hoxa.
4. The dyke approaching diabase at Binniaro Firth.

With them a series of typical selected analyses is given for comparison.

5. Camptonite, Maena, Gran (Brögger, VI., p. 26).
6. Camptonite, Egge (Brögger, VI., p. 26).
7. Camptonite, Campton Falls (cited from XIX., p. 235).
8. Camptonite, Montreal (cited from XIX., p. 235).

It is to be noted that in 1 and 2 the alumina carries a considerable amount of titanic acid, which was not estimated. Another feature of the analyses is the high

percentage of potash in 1 and 2, where it preponderates over soda. This is partly due to the composition of the hornblende (see analyses, p. 892), but indicates also the presence of a small amount of orthoclase. The high percentage of iron oxides, and the high specific gravity, are due to abundant magnetite, pyrite, and ilmenite, as well as to the hornblende and augite. No. 3 shows a transition to the monchiquites in the lower silica and alumina, the higher magnesia, and the relative amounts of the alkalies; but in the titanic acid, and the large amount of iron oxides, it belongs to the camptonites. This is owing to the abundance of hornblende, magnetite, and ilmenite. In No. 4 the alumina is rather low, the iron oxides rather high, a feature of the camptonites, but otherwise it approaches very closely to many olivine diabases. The alkalies are in less quantity than in the camptonites, particularly the potash, and in this respect, and also in their relative proportion, this rock is of a more normal type. To this is to be ascribed the absence of the peculiar brown hornblende.

ANALYSES OF CAMPTONITES.

	1.	2.	3.	4.	5.	6.	7.	8.
SiO_2	41·99	42·13	39·13	45·96	40·60	42·05	41·94	40·95
TiO_2	...	...	4·02	...	4·20	5·60	4·15	3·39
Al_2O_3	17·58	16·31	11·38	12·68	12·55	12·30	15·36	16·45
FeO	8·33	7·93	8·13	7·94	9·52	9·52	9·89	...
Fe_2O_3	6·17	6·43	7·33	7·63	5·47	3·81	3·27	13·47
MnO	0·29	·50	·42	·61	...	...	0·25	0·33
MgO	8·03	7·37	8·64	8·25	8·96	4·83	5·01	6·10
CaO	8·53	9·62	11·77	8·36	10·80	11·55	9·47	10·53
K_2O	2·81	2·48	1·93	·98	1·19	1·11	0·19	1·28
Na_2O	2·12	2·27	2·47	1·88	2·54	2·18	5·15	4·00
CO_2	1·80	2·12	2·41	2·43	2·68	2·68	2·47	...
P_2O_5	...	...	...	...	...	...	...	0·29
H_2O	2·99	3·16	2·87	4·10	2·28	2·88	3·29	3·84
	100·64	100·32	100·50	100·78	100·79	98·51	100·44	100·63
Sp. Gr.	3·01	2·98	3·07	2·96	...	...	...	2·927

The Monchiquite Dykes.

Compared with the camptonites, the monchiquites are few in number, but they are of general distribution, and occur in all the principal areas, with the exception of the west coast of the Mainland.

A typical monchiquite forms a small dyke in the shore of the Peerie Sea, Kirkwall, below the mouth of a spring which issues from the fields of Grainbank. It is a fine-grained dark green rock, with few spots of calcite. Under the microscope (Pl. III. fig. 3) it shows olivine in sharply idiomorphic crystals, altered into serpentine; these contain

magnetite in fine grains and little brown octrahedra of chromite or perofskite. Augite phenocrysts are few and small, but it forms most of the groundmass in small crystals five or six times as long as broad, and averaging ·02 mm. in breadth. It is brownish-violet, very feebly dichroic, with rather perfect idiomorphism, occasional twinning, and frequent hour-glass structure. It lies scattered irregularly through the field, and grouped on the faces of the olivines. Hornblende is practically absent, but a deep brown biotite is present in small amount. It is hexagonal in transverse section, with sometimes a darker rim and a pale centre, far more dichroic than hornblende (the a ray being nearly colourless), and usually adherent to the iron ores and the olivine. In convergent light it is very nearly uniaxial. The rock is very fresh, but there is a little calcite and chlorite. Between the minerals of the groundmass there is a very small quantity of a transparent, colourless, almost isotropic groundmass, slightly turbid, and showing the weakest irregular double refraction. It is moulded on the other minerals, and has no definite outlines; the other ingredients are apatite and magnetite. Here and there occur small ocelli, with calcite in the centre, surrounded by a little felspar and a prism or two of hornblende.

The more northerly of two dykes at the house of Skaill, Rousay, on the shores of Eynhallow Sound, is almost exactly similar to this rock.

The Dykes of Tingwall (*Rendall*), *of Quoynamuckle* (*Rendall*), *and of Kongie Geo, Saviskail* (*Rousay*).—They contain similar olivine and titaniferous augite, often in radiating groups of six or seven. Biotite is absent, but hornblende is rather more abundant, especially in the little ocelli, which may contain a small amount of felspar. The rocks are much decomposed, and the numerous small vesicles are filled with calcite and chlorite. In most of the slides there is a considerable quantity of a very turbid brownish glass, which is filled with little needles of augite and skeleton growths of magnetite; it contains also microliths of hornblende, and possibly a little felspar. Even where clearest and least altered, it has in polarised light the appearance of a devitrified base, granular and speckled with decomposition products.

On the shores of Kirkwall Bay, near the Skerry of Quanterness, I picked up in 1896 two blocks of trap, which in section turn out to be very interesting rocks. They are coarsely porphyritic, and the phenocrysts resemble in every respect those described for the camptonites of Hoxa. Olivine is absent from the sections (Pl. II. figs. 5 and 6), and the rock is thus a *hornblende fourchite*. The phenocrysts are augite and hornblende, which show all the varieties and intergrowths described in the west dyke of Hoxa (see photos, figs. 5 and 6, Pl. II.). Both are very fresh, the augite showing only incipient change into chlorite. The groundmass contains little augite, and consists almost entirely of hornblende similar to that of the camptonites, embedded in a clear glass. This glass has a tendency to collect in certain parts of the slides, forming paler ocelli; it is colourless or pale greenish, filled with innumerable microliths, which are often hornblende in beautiful dendritic growths very like those of the pitchstones of Arran. These consist of spicules so thin as to have only a faint brownish or greenish colour;

and without doubt many of the microliths enclosed in the felspars of the camptonites, and often taken for apatite, are of similar nature. The glass is in places perfectly structureless and isotropic, elsewhere is devitrified and turbid, and in polarised light shows a granular appearance. Very occasionally it encloses ill-defined radiating skeletons of felspar. Apatite, magnetite, and ilmenite are the accessory ingredients.

The dyke already mentioned as rising along a fault fissure on the west side of the Wart Holm of Copinshay is a monchiquite, remarkable for the large hornblende phenocrysts it includes. These may be as large as $2\frac{1}{2}$ inches by $1\frac{1}{4}$ inches. Owing to their perfect cleavage they are difficult to obtain in section, but they seem to be corroded, often zonal, and almost free from enclosures. Decomposed olivine and idiomorphic violet-brown augite, with sometimes a bright green centre, are the other phenocrysts. The groundmass is fine grained, and resembles that of the Grainbank dyke, consisting of titaniferous augite and rather plentiful biotite. Hornblende is confined to the rather numerous ocelli, where it is mingled with felspar in fine radiating fibres. A partly devitrified glassy base fills up the interspaces between the other minerals, and not unfrequently shows in polarised light the presence of a small quantity of ill-developed felspar.

The Nature of the Groundmass of the Monchiquites.—This is a subject to which attention has recently been directed by Professor Pirsson (XI., p. 680), who, in a very able paper, has advanced the view that it is in reality primary analcite, which he considers an essential mineral of rocks of this class. It had previously been regarded as a glass, which, owing to its richness in alkalies and alumina, had a very marked tendency to decompose into zeolites and analcite. Lindgren had previously (XII., p. 51) called attention to the presence of this mineral in apparent phenocrysts in the analcite basalts of the Highwood Mountains, Montana, and also in a second generation in the groundmass. He suggests that the mineral could have formed from igneous magmas in presence of water, and crystallising under sufficient pressure to retain it. Still more recently Cross (XIII., p. 684) has described certain "analcite basalts," in which the analcite, though the last product of crystallisation, in many ways resembles the phenocrysts of porphyritic rocks. He, too, considers it a primary mineral.

The grounds on which Pirsson bases his opinion are, that it is in itself improbable that rocks so basic in composition, solidifying under great pressure, should produce any glassy material, while under similar circumstances the acid dykes of the same districts are always crystalline. That it is not impossible, however, is proved by the frequent tachylite selvages of basalt dykes, and the association of variolite with diabase. Comparison of the limburgites with the basalts will show that it is not merely the percentage of silica a rock contains which determines the abundance or frequency of glass in the groundmass. The chemical analysis of the glass in a monchiquite from Brazil has, as Pirsson shows, a striking resemblance to analcite, and he considers it has been made with great care and skill on excellent material. We must remember that the ground-

mass is decomposing into zeolites, and (XVI., p. 452) that the solution from which the material was precipitated had been diluted to a specific gravity of 2·3, which is less than that of most natural glasses, so that the material must have in any case contained a disproportionate amount of these secondary minerals. A single analysis is, moreover, a slender basis on which to build so important a theory. Two analyses of the material from LINDGREN'S rocks, which also are admittedly rich in analcite, are adduced to support the hypothesis, but they seem to differ too much from one another to be suitable for the deduction of a mineral formula. Finally, in PIRSSON'S view, the ready gelatinisation with acids, and the weak and irregular double refraction, point to this substance being really analcite; but these points have long been known and not considered incompatible with the properties of a natural glass such that it readily decomposes with formation of zeolites (HUNTER and ROSENBUSCH, XVI., p. 452).

As a result of my examination of the Orkney trap dykes, I have been led to form the opinion that this material is really a glass, which, owing to its composition, very readily decomposes, with formation of zeolites and particularly of analcime. Where analcime can be identified, as in the west dyke of Hoxa, it is quite different in appearance from the glassy base, being clear and transparent, while the latter is turbid and full of microlites. In some rocks, as in the fourchite, this glass is perfectly clear, fresh, and isotropic, though filled with felspar microlites, which are grouped in radiate bundles very similar to those of the minettes. It may be colourless or pale brown. In decomposition it becomes turbid, granular, semi-opaque. In polarised light it is then filled with specks and fibres of undeterminable nature, which give it the appearance of a devitrified material rather than of analcite. Finally, it becomes a granular mosaic of calcite and analcite, mixed with various fibrous zeolites.

Moreover, its occurrence is that which is distinctive of a glassy base. In certain of the camptonites, as, for example, those of Hoxa, an isotropic matter is abundant at the edges of the dykes. Towards the centre felspar increases in abundance, and the base diminishes, but does not disappear. The sections of the edges have all the characters of hornblende monchiquites. Yet if the base is here analcite, it cannot be analcite which in the centre has been left after the crystallisation of the felspar. On the other hand, the passage from monchiquites to alnöites will be shown to be a very gradual one. There are forms of the latter rocks which at the centre show melilite, at the edges only a glassy base; other dykes contain a granular glass throughout. The transition is so perfectly gradual, that no line can be drawn to separate the "analcite rocks" on the one side from the felspathic and melilite-bearing rocks on the other.

Should ever this hypothesis be established, it must rest on a broad basis of chemical facts. The attempt which I made to separate out the groundmass of the Orkney dykes failed to yield material of suitable purity. It is difficult to see how it could be obtained, as the undoubtedly secondary analcite of the amygdules should certainly be first removed. The rock which ROSENBUSCH and HUNTER investigated (XVI.) is far more suitable for this purpose than any other I have seen; but it should not be forgotten that this is

an extreme member of the series, and, in its richness in alkalies, stands in striking contrast to the rocks which have later been associated with it (Analysis 5, below).

CHEMICAL ANALYSES OF MONCHIQUITE DYKES.

	1.	2.	3.	4.	5.
SiO_2	42·51	42·46	43·74	43·50	46·48
TiO_2	...	2·47	2·80	2·10	0·99
Al_2O_3	12·85	12·04	14·82	18·06	16·16
Fe_2O_3	2·67	2·19	2·40	7·52	6·17
FeO	7·52	5·34	7·52	7·64	6·09
MnO	0·83	0·16	...	...	...
MgO	12·00	12·40	6·98	3·47	4·02
CaO	11·83	12·14	10·81	13·39	7·35
Na_2O	2·75	1·21	3·08	2·00	5·85
K_2O	2·15	2·68	2·90	1·30	3·08
CO_2	3·46	·55	1·50	1·22	0·45
P_2O_5	...	·84	0·64	...	...
H_2O	2·96	4·03	2·94	...	4·27
	100·53	99·51	100·23	100·20	100·91
Spec. Grav.	2·905	2·94	2·914	3·051	2·736

1. Grainbank, Kirkwall.
2. Willow Creek, Castle Mountain (Pirsson, XIV., p. 115).
3. Monchiquite, Rio de Ouro. Rio de Janeiro (XVI., p. 464).
4. Hornblende Monchiquite, Magnet Cove, Arkansas. Analyst, W. A. Noyes (XXI., p. 295).
5. Monchiquite, Santa Cruz Bahn, Rio de Janeiro (XVI., p. 464).

The monchiquite dyke of the Wart Holm, Copinshay, carrying the large phenocrysts of hornblende, presented a specially favourable opportunity to obtain this mineral pure for analysis. About 20 grammes were broken out of various crystals, hand picked, crushed, and thrown into borotungstate of cadmium. The principal precipitation took place on dilution to a specific gravity of 3·15, and, on microscopic examination, proved to be pure hornblende, perfectly fresh, and free from enclosures. The material had previously been purified by passing over a strong electro-magnet to remove the magnetite. The analysis is given under No. 1.

As this rock carries a little felspar in the groundmass and in the ocelli, it approaches closely to camptonite, and the composition is probably the same as that of the hornblende in the mass of the dykes. From these pure material could not be obtained, owing to the frequent intergrowths with augite and admixture of its decomposition products. The average given in the third column shows that this is a typical basaltic hornblende.

The Alnöites and Biotite-Monchiquites.

To this group belongs a small number of dykes, so far as at present known confined to the district between Kirkwall and Stromness. They have all a very northerly

trend, and seem to have a marked tendency to branching. There can be no doubt that they are, both in geological origin and in petrographical characters, a closely related series.

	1.	2.	3.
SiO_2	40·65	40·79	39·88
TiO_2	4·52	...	4·86
Al_2O_3	17·12	17·36	14·83
Fe_2O_3	4·26	3·83	12·60
FeO	5·53	15·04	
MnO	0·34	0·30	...
CaO	12·88	10·83	12·68
MgO	9·96	6·97	12·27
Na_2O	1·74	4·17	3·39
K_2O	2·80		
H_2O	·36	0·71	...
	100·16	100·00	100·51
Spec. Grav.	3·153	...	...

1. Hornblende from hornblende-monchiquite, Copinshay, Orkney.
2. Analyses of hornblende from camptonite. Dixville North, N.H., U.S.A. (cited from XIX., p. 234).
3. Average of ten analyses of basaltic hornblende. Brögger (VI., p. 29), after Schneider's analyses.

One of the most interesting occurs in a small headland about 400 yards north of the farmhouse of Rennibuster, four miles from Kirkwall. It is 5 feet broad, and in the hand specimen is a dark green, almost black rock, with fine glancing scales of biotite and vesicles filled with calcite and pale-coloured zeolites. The weathered material is dark rusty brown.

Under the microscope, olivine proves to be abundant in phenocrysts, rather uniformly of small size. It is entirely decomposed into pale green, sometimes almost colourless, serpentine accompanied by magnetite, or into calcite and limonite. It encloses magnetite, and perofskite in small dark brown octahedra. Augite in large phenocrysts is comparatively scarce, and usually idiomorphic, with short, broad, longitudinal, and octagonal transverse sections. The centre is almost quite colourless, the margins purplish-brown. It is twinned, often repeatedly, on 100, and has the usual cleavage. It shows no decomposition. The enclosures are apatite, magnetite, and perofskite.

The most striking porphyritic mineral is biotite, in large irregular plates. Its pleochroism is intense, $\mathfrak{a}$ = pale yellow, $\mathfrak{b}$ and $\mathfrak{c}$ = reddish-brown, absorption $-$ $\mathfrak{c} = \mathfrak{b} > \mathfrak{a}$. A very narrow zone of intenser brown surrounds a large corroded nucleus, and may occasionally show a tendency to idiomorphism by the presence of angular corners recalling hexagonal outlines. It has a greater absorption than the central biotite for all

rays, but especially for b and c, which are dark brown. The optic axial angle is very small. Twinning on TSCHERMAK's law was not made out, and the extinction may have an obliquity of 5°. The biotite encloses olivine, augite, magnetite, perofskite, apatite. In weathering it passes into a deep green chlorite, but this change is confined to the margin of certain sections (see Pl. III. fig. 4).

Augite is very abundant in small crystals of the second generation. The prisms are elongated, and average ·015 mm. in breadth by ·1 mm. in length. They are very idiomorphic, and in transverse section have usually six sides, the clinopinakoid not being developed. They are irregularly scattered, and often in radiating groups of six or seven. Their colour is purplish-brown, resembling the outer zone of the phenocrysts. Owing to their small size, they are almost without cleavage, but it is usually to be traced in the longitudinal sections. Their extinction angle ranges up to 43°. They are not dichroic, and very commonly show hour-glass structure and twinning similar to that of the phenocrysts. These little augites are enclosed in the periphery of the biotite plates, but are never seen in the centre, and very frequently the edges of the biotite are moulded on them. In that case the dark brown rim is absent. In the groundmass biotite in small, sometimes sharply-formed crystals is mingled with the augite, and adherent to its margins or surrounding magnetite. On the surfaces of the olivines biotite and augite are usually implanted.

Melilite in some of the slides and in some parts of them is quite common, in others it is not recognisable owing to decomposition. It is best found fresh on the edges of the biotite. Here it has rather perfect form, squarish rectangular or lath shaped. It is colourless, not dichroic, with a cleavage parallel to the length of the section very well defined, and another less frequently seen at right angles to this. It contains glass inclusions, but regular peg-structure is not seen. The extinction is straight, and the polarisation colours are pale grey. Transverse sections are not recognisable. Parallel growths between biotite and melilite, as figured by BERWERTH (XVII., pl. 10), were not found. Care must be taken to distinguish it from the abundant apatite, which approaches it in size, but the lower refractive index, the more perfect cleavage, the sharply rectangular outlines, and the frequent decomposition, assist in identifying the melilite. The apatite penetrates all the ingredients, but the melilite is moulded on the others, except biotite occasionally. It is optically negative.

In some parts of the slides a clear isotropic groundmass resembling glassy matter is frequently observed, but usually it is turbid and granular, and in polarised light breaks up into a mosaic of angular grains of calcite with fibrous zeolites, and possibly analcite. This material shows mostly no cleavage or trace of regular structure, but at times it encloses little rectangular prisms which are apparently melilite mostly decomposed into zeolites, which are fibrous, and extinguish parallel to the prism axis, while weakly polarising grains of still fresh material may be found in the centre. Whether there has been originally a glassy residuum or a final crystallisation of nepheline, as ADAMS suggests (XVIII., p. 278), cannot now be made out.

If this rock be compared with the alnöite of St Anne, Montreal, it will be found that the chief differences are that it is less coarsely crystalline and much more decomposed. The olivine of the Canadian rock is mostly fresh, there is more porphyritic augite; the melilite is in larger grains, and far better preserved; otherwise the two rocks present a close parallel.

The sequence of crystallisation appears to have been perofskite, apatite, magnetite; olivine, first augite (later corroded), biotite, and before its completion, second augite, melilite, and then the brown rim of biotite; lastly, completion of augite of groundmass, and the bulk of the melilite, with perhaps, finally nepheline or a glassy base.

Towards the edge the dyke becomes finer grained, but the specimens are too decomposed for minute study. Numerous phenocrysts of olivine entirely altered into serpentine and calcite, with many porphyritic augites still fresh, are scattered in a fine grained dark groundmass. The augites are sharply idiomorphic, with a pale or colourless corroded centre and a purplish margin, and in polarised light have the remarkable zonal and hour-glass structure of those of the Montreal rock. The centre and periphery may differ 12° in extinction, and the marked dispersion of the bisectrices observed by ADAMS is evident here also. Biotite at the extreme edge of the dyke is present only in small scales, and increases in size and abundance as we approach the centre. It is not commonly idiomorphic. Melilite is not found at the margin of the dyke, but there is much of a granular turbid semi-opaque groundmass which was probably originally glassy. We remark here a curtailment of both the first and second generation of crystals. A few grains of melanite with broad dark corroded borders were noted, and here and there pale ocelli.

In the shore at the mouth of the burns of Rennibuster, as already described, three dykes are seen traversing the course of the camptonites. They are narrow, and run parallel to one another, and in all probability are subdivisions of one dyke; indeed, connecting veins were observed in the flagstones between. They are all alnöites, but vary somewhat in microscopic appearance. Phenocrysts of olivine occur in all, and those of augite are also numerous. They resemble those described in the previous case, and some contain a honeycombed centre which gives the impression that it has been full of glass cavities. The corroded interior has sometimes a diallage lamination parallel to the face 100. Biotite, in large irregular plates, is abundant in sections of the west dyke. It is not surrounded by a darker border, and is often attached to the surface of the porphyritic augite and olivine, and encloses the augites of the groundmass. It is not idiomorphic, and its period of crystallisation appears to have been deferred till the formation of the groundmass. Melilite is to be found in the groundmass often with typical rectangular sections, but weathering at the edges to fibrous zeolites, which lie parallel to the axis of the section and extinguish straight. In the centre of these, grains of fresh material may still be found. Elsewhere the groundmass consists of secondary products, chiefly calcite, zeolites, and analcite in irregular grains. The cleavage of the melilite is evident, but there is no peg-structure. In the

mid dyke and east dyke the ingredients are the same, but the biotite is less abundant, and in smaller crystals which are deep brown, and adhere to olivine, augite, and magnetite. The east dyke is much decomposed, and yielded no evidence of melilite. The mid dyke is fresher, and in the hand specimen has a finely nodular appearance. The little dark brown biotites are often idiomorphic, and distinct traces of melilite were observed in crystals irregular in shape or approaching rectangular, moulded on the augite, which plays the chief part in the groundmass, and undergoing the same decomposition as in the other slides.

In Naversdale, Orphir, a dyke is seen cutting the more northerly burn in the valley about half-a-mile above the sheepfold. It is 3 feet broad, and in the hand specimen the mica is very evident in little glittering scales, but there is no marked parallel structure; there is much calcite in rounded spots.

Under the microscope it is very similar to the east dyke at Rennibuster point (Pl. III. fig. 5). It contains many olivines, weathered to green and yellow serpentine and magnetite, and surrounded by a fringe of biotite and little augites. Large augites with similar appearance and properties to those in the Rennibuster dyke accompany the olivine. The biotite occurs also in large, irregular plates, with similar enclosures, but here without a darker rim; and the little scales in the groundmass are absent. The groundmass consists of little idiomorphic augite crystals scattered through what appears under low powers a turbid, granular material, but on higher magnification resolves itself into melilite in irregularly-shaped intersertal masses, which fill up the interstices between the other minerals. Over fairly large areas the melilite has the same cleavage and extinction, though penetrated by many augites. The structure is exactly that of the ophitic dolerites, where augite encloses idiomorphic plagioclase. Peg-structure is here very perfect, and perpendicular to it a rather ill-defined cleavage. The mineral is never absolutely fresh, but always granular, turbid, brownish in ordinary light; it is not dichroic. In polarised light decomposition is seen to have followed the direction of the pegs and of the cleavage to a less extent, with the production of a fine fibrous secondary product, which at first forms a rectangular network over the mineral, and, as the change proceeds, extends gradually over the meshes. The fibres are mostly parallel, and although the polarisation colours are too high for melilite, the aggregate has a straight extinction. Many fibres, however, have an indefinite orientation, so that, when the section is at its darkest, extinction is never absolute, but bright flakes are to be seen. These are mostly at the edges and in the cleavage cracks. In other sections the fibres have an entirely irregular or radiate arrangement; these may be pseudomorphs after melilite in transverse section. Recognisable calcite does not appear till comparatively late in the process. Analcite, calcite, and other minerals of secondary origin, which are undeterminable, occupy other parts of the sections. An isotropic mineral of irregular corroded form, full of dark inclusions, and resembling nosean, is present in very small quantity. Apatite and perofskite are both abundant.

The sequence of crystallisation appears to have been apatite, perofskite, magnetite;

olivine, augite, biotite; 2nd augite, melilite. In all probability there was no glassy residuum.

A rock, which appears to be a much decomposed alnöite, forms a small dyke which crosses the burn of Ireland, Stenness, where the two tributary burns unite. This dyke forks into two branches. It has all the features of the alnöites, but in the groundmass melilite is not recognisable, although in all probability originally present.

It has been already remarked that at the edges of the dykes the alnöites tend to develop a glassy groundmass, biotite being present in small quantity, and melilite absent. Sections taken from the thin veins which connect the dykes at Rennibuster burn have a totally different appearance from those of the dykes themselves. Large phenocrysts of olivine (altered into serpentine) and of augite lie in a dark brown turbid groundmass, evidently at one time glassy, but now decomposed and devitrified. No biotite and no melilite appears. In the groundmass are microliths of augite and minute brown grains, which may be biotite. The glass has rounded paler spots, which recall the ocelli of the camptonites, and the groundmass is distinctly fluidal.

On Rennibuster Point, a short distance to the west of the alnöite dyke first described, occurs another, a little over a foot broad, and with the same northerly trend. In section it is the freshest of the monchiquites and alnöites of the Orkneys, and the numerous phenocrysts of olivine show only slight decomposition into serpentine, and are sometimes absolutely unaltered. There are many phenocrysts of augite which exactly resemble those of the alnöites. A deep brown biotite, in small, fairly idiomorphic crystals, is rather plentiful. It encloses only apatite, magnetite, perofskite, and may surround porphyritic olivine or augite. Only very rarely do the augites of the groundmass encroach on it. It has a narrow, deep brown border. In the groundmass we have many small augites embedded in a brown granular glass. A careful search has revealed a few sections which resemble melilite. It is perfectly idiomorphic and sharply rectangular. Apatite and perofskite are frequent accessories. There is one large brown corroded crystal of melanite, with traces of twelve-sided outlines and a broad black border. This is a glassy alnöite, which very closely approaches in microscopic character certain of the monchiquites.

In the table on next page an analysis is given of the alnöite of Naversdale, the only one I have so far been able to overtake. It is accompanied by the principal analyses of similar rocks elsewhere.

Melilite-Monchiquite.—This name I propose for a rock which occurs in a small dyke at Long Geo, Holm, and has been already mentioned as crossing a small fault (p. 872). In consequence of the irregularity in its course, its true direction cannot with certainty be established. It is in every respect a monchiquite, except that the groundmass here consists wholly or in part of melilite in plates of irregular shape.

It contains phenocrysts of olivine, decomposed into calcite, pale serpentine, and magnetite, and enclosing brown octahedra of perofskite. The olivines are rounded, and

on their surface are implanted grains of augite, and biotite in small scales. Augite occurs in few small phenocrysts, purplish-brown, idiomorphic, not markedly zonal, and resembling those of the monchiquites rather than of the alnöites. The groundmass consists of augite, biotite, and melilite. The augites are in small prisms 5 to 10 times as long as broad, brownish-purple, idiomorphic, sometimes twinned in 100, frequently showing hour-glass structure, and with a high obliquity of extinction (44°). They are

	1.	2.	3.
SiO_2	35·54	35·91	24·19
TiO_2	2·03	0·23	tr.
Al_2O_3	11·72	11·51	12·00
Fe_2O_3	5·86	2·35	6·45
FeO	5·99	5·38	9·32
MnO	0·32	...	...
MgO	13·56	17·54	14·07
CaO	15·83	13·57	17·37
Na_2O	1·91	1·75	1·99
K_2O	2·24	2·87	3·06
H_2O	1·67	9·40	5·16
P_2O_5	...	...	3·96
CO_2	4·30	...	2·77
	100·97	100·51	101·16
Spec. Grav.	3·052	3·02	3·15

1. Alnöite, Naversdale, Orkney.
2. Alnöite, Ste. Anne de Bellevue, Montreal (anal. Le Rossignol, XVIII., p. 271).
3. Alnöite, Alnö (XIX., p. 235).

The accordance between the first and second is exceedingly close. The titanic acid was estimated by the method described by Hillebrand (XX., p. 45), and must be contained in the perofskite, titaniferous magnetite, and augite of the groundmass. The Canadian rock carries magnesia partly as carbonate.

not dichroic, and almost perfectly undecomposed. They lie irregularly dispersed, or groups in radiating rosettes. Biotite occurs only in small scales of irregular form, dark brown colour, and intense dichroism. It is not, however, like that of the alnöites, being less clear yellow, with a not colourless, but distinctly yellow-brown. It resembles the later biotite in these rocks, and is almost perfectly uniaxial. Around the olivines and magnetites, it often forms a narrow border, and in the groundmass it is mingled with augite and melilite. Except its less idiomorphism, there is nothing to distinguish it from the biotite in such a monchiquite as that of Grainbank. The melilite was the last mineral to crystallise, and is destitute of crystalline form, its irregular masses enclosing all the other ingredients. It is paler and fresher than in the alnöite of Naversdale, almost colourless, with very perfect peg-structure (see photo, Pl. III. fig. 6), though the cleavage is often indistinct, no dichroism, and gives polarisation colours of dark grey, about the same as apatite, or slightly lower. Its optical sign is negative. Its decom-

position is very similar to that already described in the former rock, fibrous zeolites, with straight extinction parallel to the peg-structure, forming first in a net-work, and spreading afterwards through the mineral. In the freshest parts of the slides it fills up all the spaces between the other ingredients of the groundmass, and leaves no room for any later mineral or for a glassy base. But where the rock is more decomposed, calcite, with chlorite, brilliantly polarising zeolites, and an isotropic mineral (analcite ?), in a mosaic of irregular grains and fibres, form the interstitial material. These have, no doubt, resulted from the final disintegration of the melilite and the adjacent minerals.

The rock contains a few vesicles filled with calcite, chlorite, and zeolites. In the hand specimen it is fine grained, non-porphyritic, and in places much decomposed. It closely resembles the monchiquites, but is distinctly paler in colour, being rather greyish green, while they are dark green.

A chemical analysis is given below (No. 1), and beside it certain others for comparison.

Chemical Analyses of Melilite Rocks, etc.

	1.	2.	3.	4.	5.
SiO_2	33·87	42·51	35·54	33·89	35·84
TiO_2	2·12	...	2·03	0·64	8·85
Al_2O_3	15·25	12·04	11·72	9·93	10·48
Fe_2O_3	2·37	2·67	5·86	15·63	7·25
FeO	5·15	7·52	5·99	...	6·62
MnO	·32	·83	·32	...	...
MgO	12·52	12·00	13·56	16·14	12·95
CaO	14·43	11·83	15·83	15·19	10·90
Na_2O	1·41	2·75	1·91	2·86	3·53
K_2O	1·02	2·15	2·24	...	1·51
H_2O	2·47	2·96	1·67	2·90	...
P_2O_5	·99	...	...	1·41	...
CO_2	8·64	3·46	4·30	1·41	2·84Cr_2O_3
	100·36	100·53	100·97	100·00	100·77
Sp.G.	3·033	2·905	3·052	3·015	3·051

1. Melilite monchiquite, Long Geo, Holm, Orkney.
2. Monchiquite, Grainbank, Kirkwall, Orkney.
3. Alnöite, Naversdale, Orphir, Orkney.
4. Melilite basalt, Hochbohl bei Owen.
5. Nepheline melilite basalt, Riedöschingen, Hegau, Baden.
(Nos. 4 and 5 cited from Rosenbusch (XIX., p. 360).)

A comparison with 4 and 5 will show that the melilite monchiquite from Orkney has a sufficiently close similarity to other well-known rocks of the melilite group. It is in a state of more advanced decomposition, as witness the high percentage of CO_2, which is partly combined with magnesia, as it was noticed that only a small amount of CO_2 was

evolved on treatment with cold dilute HCl, but on warming, the evolution of gas was renewed and rapid. To what extent the abundant secondary products have resulted from the decomposition of the dyke rock itself, or have infiltrated from the surrounding highly calcareous flagstones, it is impossible accurately to determine, but, from the investigations of MERRILL (XXII., p. 214), we may assume that the effect has been to increase the apparent percentage of alumina and to diminish that of silica and the alkalies. This would make the accordance still more complete.

An examination of analyses 1, 2, and 3 shows how closely connected are these three rocks which we have selected as types of different groups, and leads to the assumption that they must have proceeded from the same igneous magma. In the monchiquite we have a relatively high percentage of silica, and the alkalies, while lime, magnesia, alumina, and iron, are slightly low. The others have the extremely low silica percentage characteristic of the melilite rocks. In the alnöite potash predominates, in the melilite monchiquite soda. The alnöite is richer in magnesia, lime, and iron oxides, the melilite monchiquite in alumina. As decomposition has followed a similar course in these three rocks, more reliance can be placed on a comparison between them, and their chemical composition, taking into consideration the analyses of their constituents (see Table below), may reasonably be expected to throw some light on their diversity of character.

CHEMICAL ANALYSES OF CONSTITUENTS OF MONCHIQUITES AND ALNÖITES.

	1.	2.	3.	4.	5.
SiO_2	53·43	38·56	44·55	36·42	44·76
TiO_2	...	...	2·85	3·99	...
Al_2O_3	20·86	...	7·86	17·92	7·90
Fe_2O_3	2·61	1·36	3·81	2·83	5·16
FeO	...	12·65	4·53	7·04	1·39
MnO	...	0·11	0·38	...	...
MgO	0·29	44·37	12·71	20·52	8·60
CaO	1·14	...	20·84	...	27·47
Na_2O	11·63	...	1·29	2·60	2·65
K_2O	2·51	...	0·49	6·54	0·33
H_2O	7·06	2·91	...	2·50	1·42
CO_2	...	...	...	...	...
	99·53	99·96	99·31	100·36	99·68

1. Glassy base of monchiquite Sta. Cruz Bahn (HUNTER and ROSENBUSCH, XVI., p. 454).
2. Olivine from alnöite, Ste. Anne de Bellevue, Montreal, Canada (anal. Prof. Harrington (ADAMS, XVIII., p. 273).)
3. Augite from monchiquite, Rio de Ouro, Sierra de Tingua, Rio Janeiro (HUNTER and ROSENBUSCH, XVI., p. 462).
4. Biotite from monchiquite, Norberig, Oberbergen (cited from ROSENBUSCH, XIX., p. 234).
5. Melilite from melilite basalt, Hochbohl. (cited from ROSENBUSCH, XIX., p. 360).

In all three, crystallisation began in the same manner. Perofskite, magnetite, apatite, olivine, augite, were the first products. In the alnöite the augite was accompanied by much biotite, conditioned, no doubt, by the excess of magnesia and of potash (anal. 4), and these two went on crystallising till the still liquid residuum was rich in lime and comparatively poor in alumina and magnesia (anal. 5), crystallising finally as melilite. In the melilite monchiquite the conditions were less favourable for the production of biotite, and for some time augite alone was formed, till a similar condition supervened, and crystallisation was completed by the formation of melilite. But, in the monchiquite, owing to the higher percentage of silica and lower percentage of lime and magnesia the continued crystallisation of augite was giving rise to a residuum comparatively acid, containing the alkalies in fair amount and in nearly equal proportions, and practically devoid of lime, iron, and magnesia (*cf.* anal. 1). To its peculiar chemical composition we must ascribe its final solidification as a glass. As in the closely-related rocks 1 and 3 an anhydrous crystalline silicate was finally formed, there does not seem to be any reason to believe that in this one, had crystallisation taken place, the water would have been retained, and the hydrous silicate analcite have been the result.

Conclusion.

It will be seen that in the Orkneys we have one of the most abundant and interesting series of the camptonite-monchiquite-alnöite rocks which is anywhere known to exist. They fall naturally into two groups, the "leucocrate" rocks of Brögger, of which the sole representative is one dyke of bostonite, and the "melanocrate," to which the others belong. Between them there is no connecting link; and the theory which he expounds of their origin by complementary differentiation from the same magma, is, in view of their constant association, the most natural that has been suggested.

The "melanocrate" series must be regarded as a unity. The series of gradations by which they merge into one another is so complete that they cannot be separated. They have proceeded from one magma, which, by a progressive differentiation, was becoming more and more basic as time elapsed. Taking the Orkney dykes as a whole, the petrographical characters agree with the facts regarding their geological occurrence, in establishing that they have been emitted, no doubt in successive periods, from the same volcanic focus.

But when we pass to consider what the nature of that parent magma has been, it must be insisted that there are no data, within the Orkneys, which will enable us to decide. In chemical composition they correspond exactly to no plutonic magma, and seem to be in every case products of differentiation. They are typical dyke rocks, and occur only as dykes or thin intrusive sheets. It may be possible, when their distribution in the north-east of Scotland is better known, to trace them to some parent mass; till then we can argue only from general facts. From what is known of this series at the present day, it is difficult to point to any magma from which we could say with

certainty they had not been derived. Granites, augite-syenites, elaeolite-syenites, gabbros and diabases, and theralites, have all been shown to have occasionally given origin to dykes of one or other of the rocks of this series. In the present instance only one fact seems to point to a solution of the problem. In Orkney the camptonites are the most numerous, and form a central type from which the others diverge. Of these there is only one dyke, though it is connected with the others by intermediate steps, which corresponds chemically and structurally to the products of undifferentiated magmas. That is a diabase, and we may provisionally regard it as most probable that the original magma was, as BRÖGGER has shown for the camptonites of Gran, an olivine-gabbro-diabase magma. If so, it may well be, as Sir ARCHIBALD GEIKIE suggests, that they are outlying members of the great Tertiary series of basaltic dykes of the west of Scotland.

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List of the Trap Dykes of the Orkneys from which Material was obtained and examined.

BOSTONITES.

Locality.	Width.	Bearings.	Notes.
Onston Ness, Stenness,	2 ft. 6 in.	E. and W.	see description, p. 872, and figs. 1, 2, Pl. I.

CAMPTONITES.

Rennibuster (4 miles west of Kirkwall) :—			
West burn,	6 ft. 6 in.	E. 20 N.	see description, p. 874, and fig. 3, Pl. I.
East burn,	1 ft. 3 in.	E. 20 N.	
2 dykes in shore,	4 ft. and 2 ft. 6 in.	E. 20 N.	
Walliwall quarry, Kirkwall,	(?)	E. 15 N.	non-porphyritic, much weathered.
Crowness, Kirkwall,	2 ft.	E. 25 N.	...
Peerie Sea, Kirkwall, below A. Borwick's house,	...	...	...
Corner of F.C. Manse, Finstown,	2 ft.	E. 30 N.	...
Mouth of Oyce, Finstown,	...	E. 30 N.	much branched, p. 869.
Scarvataing,	9 in.	E. 35 N.	...

Locality.	Width.	Bearings.	Notes.
Binniaro Bay:—			
Below boat, 2 dykes,	1 ft. 6 in., 2 ft.	E. 20 N.	see p. 886, and fig. 2, Pl. III.
200 yards further N.,	6 in.	E. 20 N.	
Between boathouse and farm,	2 ft.	E. 25 N.	
Near corner of farmyard,	15 in.	E. 30 N.	
400 yards south of boathouse, 2 dykes,	2 ft. 6 in.	E. 20 N.	
North of Burness, Firth,	1 ft.	E. 40 N.	...
Corrigal Burn, Harray, 2 dykes,	about 3 ft.	E. 15 N.	see p. 866.
Quarry, near Lochside, Stenness,	...	...	...
South end of Kirbuster Loch, Orphir,	...	...	...
400 yards south of Tingwall, Rendall,	4 ft.	E. 20 N.	...
300 yards further south,	6 in.	E. 20 N.	...
Break, Rendall,	15 in.	E. 30 N.	...
Laith, Sandwick, shore of Harray Loch,	2 ft.	E. 30 N.	...
Below Ironsides' farm, shore of Harray Loch,	3 ft. 6 in.	E. 35 N.	...
Below Howe, Stromness,	2 ft.	E. 20 N.	...
Cumminess, Bay of Ireland,	18 in.	E. 20 N.	...
Stromness, point below Garson,	4 ft.	E. 30 N.	see p. 878.
,, Stacks of Netherton,	6 in.	E. 35 N.	...
Billiacroo, Breckness,	2 ft.	E. 30 N.	...
Burn of Sella, 2 dykes,	...	...	...
Neban Head, 4 dykes,	...	...	...
North Galton, 2 dykes,	...	...	see p. 878.
Inganess,	4 ft.	N.E.	cuts granite.
Borwick,	9 ft.	E. 10 N.	curving, see p. 869.
Ramna Geo,	...	E. 25 N.	...
100 yards south of Wart, Skaill,	4 ft.	E. 25 N.	forks, see p. 869.
Wart, Skaill,	...	...	...
Crooadee Burn, Birsay, 2 dykes,	each 2 ft.	E. 10 N.	...
,, ,, in quarry,	2 ft.	E. 20 N.	...
Boardhouse Loch, Birsay, 100 yards south of burn,	4 ft.	E. 30 N.	...
Near Quoyloo Church, Sandwick,	8 ft.	E. 25 N.	...
Howana Geo,	3 ft. 6 in.	E. 30 N.	...
30 yards further north,	3 ft.	E. 30 N.	...
Axin Geo, Birsay,	3 ft.	E. 25 N.	...
Just south of a little streamlet a little further north, 2 dykes,	...	E. 20 N.	...
Spoord, Birsay,	12 ft.	E. 25 N.	...
Stack of Spoord,	3 ft. 6 in.	E. 20 N.	...
Grassy banks south of Outshore Point,	8 ft.	E. 25 N.	see p. 884.
Oyce, Marwick,	2 ft.	twisting	...
Backaquoy Point, Birsay,	4 ft.	E. 20 N.	...
Banks Head, Birsay,	4 ft.	E. 30 N.	...
Costa Head, Evie,	3 ft.	E. 30 N.	...
Sole Geo, Evie,	2 ft.	E. 30 N.	...
At a landing-place a little further east,	2 ft.	E. 20 N.	...
At the Crane further east,	3 ft.	E. 20 N.	...
Dyke on top of Burgar Hill exposed in a road,	...	...	...
Veira Lodge, Rousay,	3 ft.	E. 30 N.	...
Westness, Rousay,	4 ft. 6 in.	E. 20 N.	...
Skaill, Rousay, between kirk and house	2 ft.	E. 20 N.	...
Fishing Geo, Rousay,	3 ft. 6 in.	E. 20 N.	...
South side of Scabra Head, Rousay,	15 in.	E. 30 N.	...
20 yards further north, seen *en face*,	...	E. 30 N.	...
Scabra Head, Rousay,	6 ft.	E. 30 N.	see p. 877.
Telegraph Hut near Hullion, Rousay,	6 ft.	E. 20 N.	...
Varmaday, Rousay, west dyke,	2 ft.	E. 30 N.	...
,, ,, 2 east dykes,	5 ft.	E. 30 N.	...
	2 ft.	E. 20 N.	...
Taing of Tratland, Rousay,	3 ft.	E. 20 N.	...
300 yards west of Trumland Pier, Rousay,	6 ft.	E. 20 N.	...
Just east of Trumland Pier, Rousay,	3 to 4 ft.	E. 20 N.	...

Locality.	Width.	Bearings.	Notes.
400 yards east of pier, Rousay,	4 ft.	E. 20 N.	...
Avalshay Point, Rousay,	10 ft.	E. 20 N.	...
Sourin Mill, Rousay,	2 ft.	E. 30 N.	...
Geo of Dykend, Saviskail, Rousay,	3 ft.	E. 20 N.	...
Behind farmhouse, Saviskail,	2 ft.	E. 20 N.	...
Hoxa, South Ronaldshay, west,	5 ft.	E. 20 N.	see p. 879. see Pl. II. figs. 1, 2, 4.
,, ,, east,	4 ft. 9 in.	E. 25 N.	see p. 879. see Pl. II. fig. 2.
Widewall, ,, 4 or more dykes,	7 ft.	E. 30 N.	see p. 869.
East of Mermaid's Rocks, Deerness,	2 ft. 6 in.	(?)	...
MONCHIQUITES.			
Grainbank, Kirkwall,	2 ft. 6 in.	about E.N.E.	see p. 887. see Pl. III. fig. 3.
Skaill, Rousay,	2 ft.	E. 20 N.	see p. 888.
Quoynamuckle, Rendall,	4 ft.	E. 25 N.	see p. 888.
Tingwall, Rendall,	2 ft.	E. 30 N.	see p. 888.
Kongie Geo, Saviskail, Rousay,	4 ft. 6 in.	E. 20 N.	see p. 888.
Wartholm, Copinshay,	3 ft. 6 in.	E. 40 N.	see p. 889.
ALNÖITES.			
Rennibuster Point, east,	5 ft.	N. 20 E.	see p. 892.
,, west,	2 ft.	N. 35 E. (?) twisting	see p. 896.
Rennibuster Burn, east,	2 ft.	N. 20 E.	see p. 894.
,, mid,	1 ft. 6 in.	N. 20 E.	see p. 894.
,, west,	3 ft.	N. 20 E.	see p. 894.
Knowes, Burn of Ireland, Stenness,	branches, 5 ft., 3 ft., 2 ft.	N. 10 W.	see p. 896.
Naversdale, Orphir,	...	N. 20 W.	see p. 895.
MELILITE-MONCHIQUITE.			
Long Geo, Goltick, Holm,	2 ft.	N. 10 E. twisting	see p. 896. see Pl. III. fig. 6.

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EXPLANATION OF THE PLATES.

Plate I.

Fig. 1. Photomicrograph of the Bostonite of Onston Ness, Stennis, Orkney—magnified about 20 diameters. The elongated sections of felspar, which is both orthoclase and plagioclase, are arranged in a fluidal manner. The black spots are magnetite and ilmenite weathering to leucoxene. Chlorite and other secondary products in small quantity occupy the interspaces between the felspars. From near the margin of the dyke.

Fig. 2. The Bostonite of Onston Ness, Stennis, Orkney, photomicrograph between crossed nicols—magnified about 30 diameters. The section is taken from the centre of the dyke and shows a large Carlsbad twinned crystal of anorthoclase, filled with decomposed glass cavities, and surrounded by a groundmass not markedly fluidal, of simply twinned felspars.

Fig. 3. Camptonite, Rennibuster Burn, Orkney—magnified about 20 diameters. Large idiomorphic phenocrysts of a pale augite, surrounded by a decomposed zone, lie in a groundmass of idiomorphic hornblende prisms and clear plagioclase. On the right margin a segment of an ocellus rich in felspar, radiately disposed, with, at the upper margin, an area of calcite occupying a miarolitic cavity.

Fig. 4. Camptonite, Scabra Head, Rousay, Orkney—magnified 20 diameters. A more fine grained rock than the preceding, but with a similar groundmass. The phenocrysts are augite, idiomorphic and pale in colour, and dark hornblende slightly corroded and showing zones of different shade.

Fig. 5. Camptonite, Stromness, Orkney—magnified 26 diameters. At the bottom two phenocrysts of a violet augite. The groundmass consists of hornblende and augite, intergrown and idiomorphic, surrounded by clear plagioclase. In the upper part two felspathic ocelli.

Fig. 6. Camptonite, North Galton, Sandwick, Orkney. From the centre of the dyke—magnified 35 diameters. Large augite phenocrysts are seen in the margins below and above. On their surfaces lie lath-shaped felspars which, in the centre of the photograph, are enclosed by allotriomorphic brown hornblende, in an ophitic fashion.

Plate II.

Fig. 1. Camptonite, West Dyke of Hoxa, South Ronaldshay, Orkney—magnified 20 diameters. The phenocrysts are augite, idiomorphic, almost free from enclosures, and pale in colour, and hornblende (almost in the centre of the figure), with a dark brown laminated centre and a clear peripheral zone. In the upper part are three crystals, the centre of which consists of an augite-hornblende intergrowth, dark with grains of magnetite, and showing only rude traces of crystalline boundaries, surrounded by hornblende which may consist of two zones, the inner paler in colour than the outer. Just below the centre of the figure is a similar crystal, the upper margin of which is formed by a thin rim of hornblende, while the remainder of the margin is a paler augite. Details of the groundmass are not shown, but it is rich in hornblende and augite, and contains comparatively little felspar.

Fig. 2. Camptonite, East Dyke of Hoxa, South Ronaldshay, Orkney—magnified 20 diameters. The nature of the groundmass is better seen than in the previous figure. Some of the clear spots are analcite in irregular grains. Pale augite phenocrysts are few. The elongated section of hornblende shows an internal dark laminated nucleus and a clearer peripheral zone. On the right is a phenocryst with graphic augite-hornblende centre, dark with magnetite, and two external zones of hornblende. In the centre and on the left are two similar crystals, but in the central one the sides are augite, the ends hornblende; the other is completely surrounded by a narrow border of augite.

Fig. 3. Camptonite, West Dyke of Hoxa, South Ronaldshay, Orkney—magnified 20 diameters. In the centre an "analcite phenocryst." The clear space on the left is due to the disappearance of a hornblende crystal. Above the centre a clear augite, to the right a zonal somewhat irregular hornblende. Below and to the left several graphic intergrowths with hornblende borders.

Fig. 4. Camptonite, West Dyke of Hoxa, South Ronaldshay, Orkney—magnified 20 diameters. On the upper margin, part of one of the pale ocelli, with long, prismatic, hornblende crystals, and clear felspars, embedded in a brownish, granular, somewhat decomposed glass. Below the centre a large phenocryst of dark brown laminated hornblende intensely corroded, surrounded by a thin, clear hornblende zone, and around this a narrow augite-hornblende intergrowth. The external border is partly augite, partly hornblende. To the left of this a phenocryst with a centre of graphic augite-hornblende intergrowth, dark with magnetite, and a border of hornblende on the upper end, of augite elsewhere.

Fig. 5. Fourchite, section from a loose block of trap found on the shore of Kirkwall Bay near Quanterness Skerry—magnified 40 diameters. In the centre a large phenocryst, which consists of a graphic intergrowth of augite and hornblende, with comparatively little magnetite, and mixed with a considerable quantity of a clear transparent glass. The upper margin is augite, the lower hornblende. The crystal is repeatedly twinned and the twin planes pass transversely through the whole structure. The groundmass contains magnetite and hornblende embedded in a clear transparent glass which is full of dendritic skeleton crystals of hornblende.

Fig. 6. Section of the same rock as No. 5—magnified 20 diameters. Showing a large, much corroded phenocryst of dark laminated hornblende, with enclosures of fine magnetite dust in lines parallel to the faces of the prism (110). It is surrounded by a thin margin of clear hornblende.

Plate III.

Fig. 1. Augite Camptonite, section from a loose block of trap found on the beach near Burness, Firth, Orkney—magnified 20 diameters. Pale violet augite in small rounded crystals, which are often perfectly idiomorphic, surrounded by plagioclase felspar, mixed with magnetite.

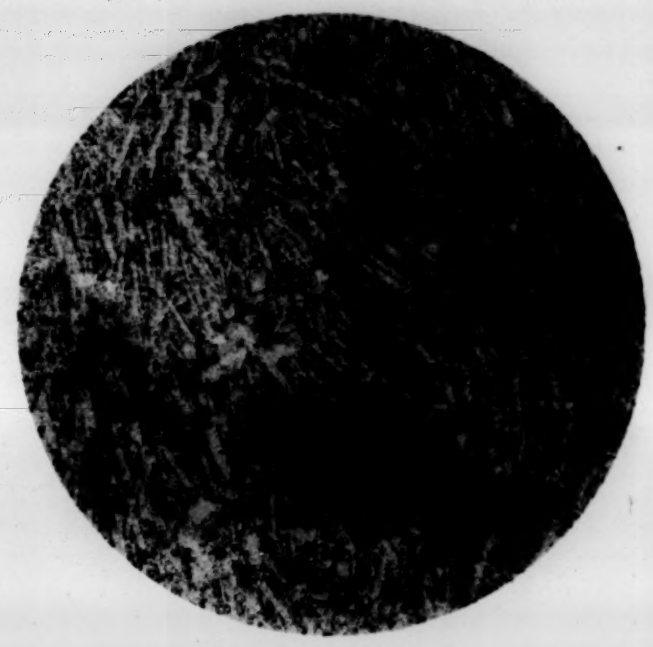
Fig. 2. Diabase, Binniaro, Firth, Orkney—magnified 20 diameters. Phenocrysts of olivine altered into serpentine (above) and of augite in a glomero-porphyritic aggregate (below). The groundmass consists of idiomorphic lath-shaped felspar, which is included in chloritised augite in an ophitic fashion.

Fig. 3. Monchiquite, Grainbank, Kirkwall, Orkney—magnified 20 diameters. Phenocrysts of olivine, altered into serpentine, and of augite, lying in a groundmass of augite, biotite, and magnetite, with a small quantity of a granular glassy base.

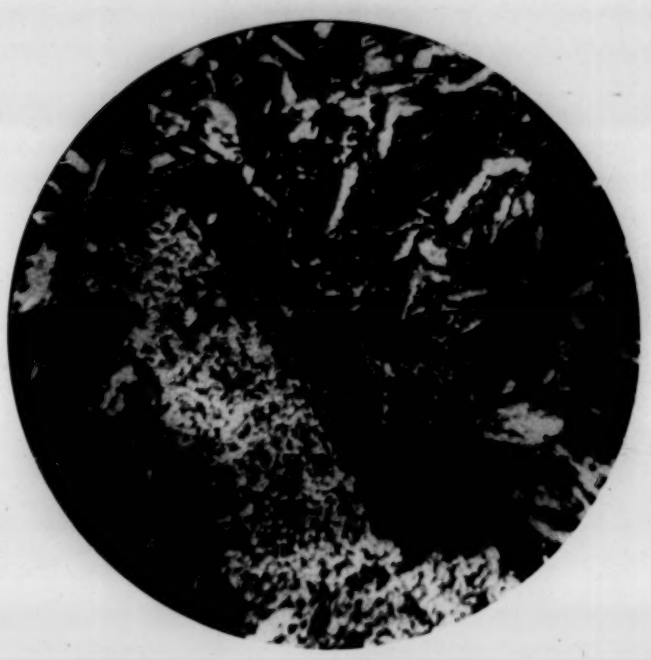
Fig. 4. Alnöite, Rennibuster Point (east dyke), near Kirkwall, Orkney—magnified 20 diameters. In the upper part of the figure a phenocryst of augite with a pale centre and dark violet-brown margin. Biotite is seen in large irregular plates below, and on the right cut perpendicular to the cleavage; on the left a plate of biotite cut parallel to the cleavage. The biotite encloses magnetite, apatite, and at its margins the small augites of the groundmass. A pseudomorph of calcite after olivine, enclosing perofskite on the left below. Details of the groundmass are not shown.

Fig. 5. Alnöite, Naversdale, Orphir, Orkney—magnified 20 diameters. The left of the figure shows one or two large plates of biotite cut parallel to the cleavage, and enclosing apatite, magnetite, augite, etc. On the right a large plate of biotite embedded in a groundmass of little augite prisms, biotite, magnetite, and melilite.

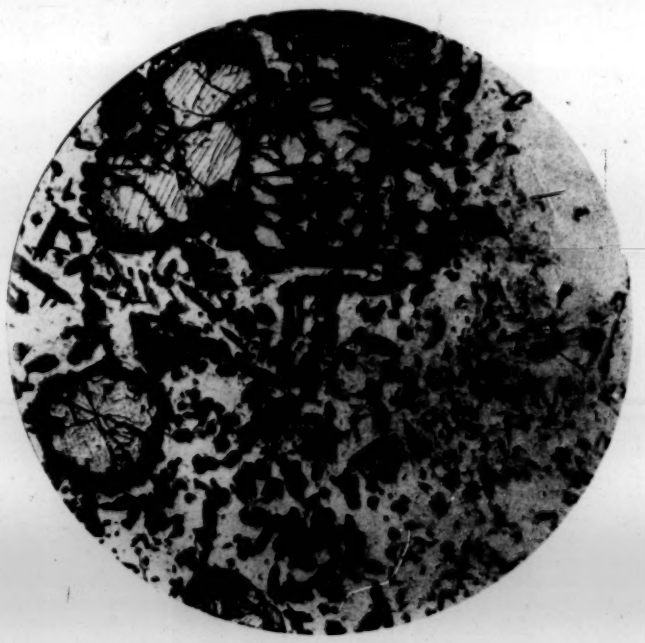
Fig. 6. Melilite monchiquite, Long Geo, Holm, Orkney—magnified 200 diameters. Augite in prismatic crystals and magnetite are embedded in clear melilite, which at the upper margin is decomposed, but on the left, a little above the centre, shows the peg structure. In the centre and on the right the pegs are not exactly in focus, but are indicated by the parallel striæ which cross the mineral.



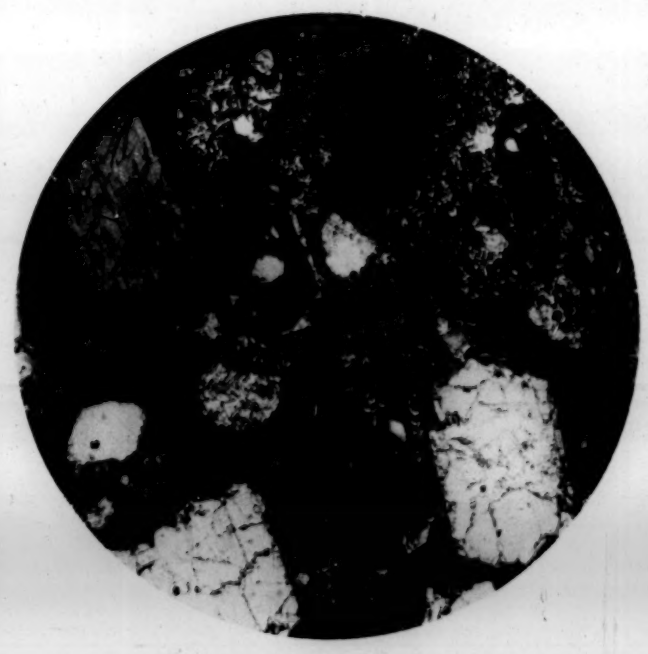
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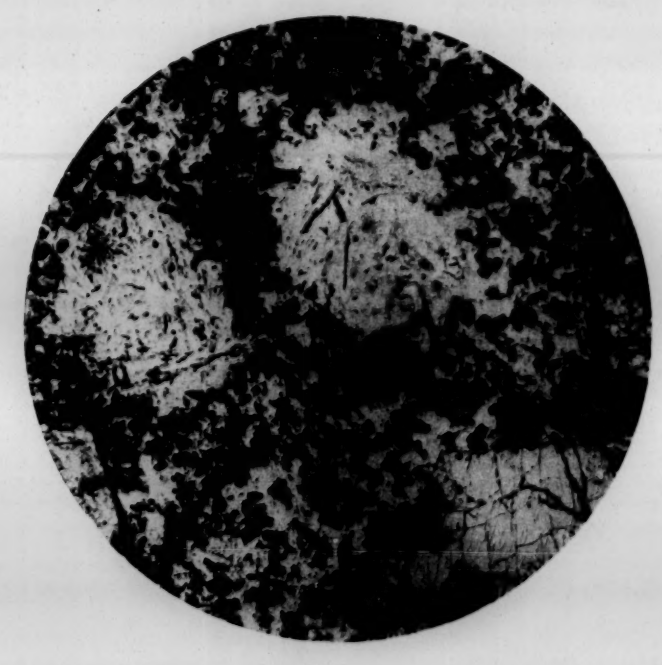
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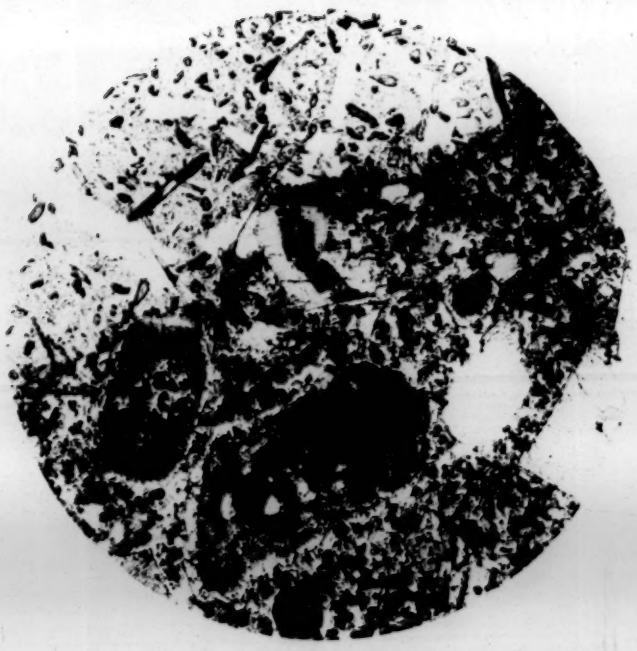
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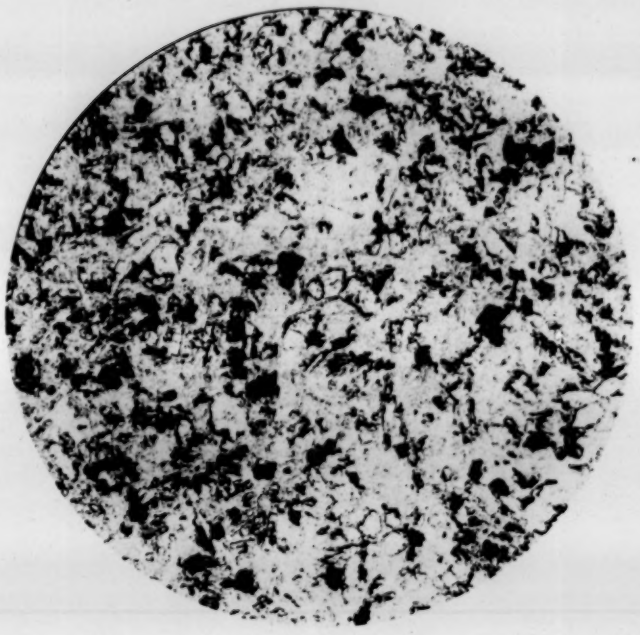
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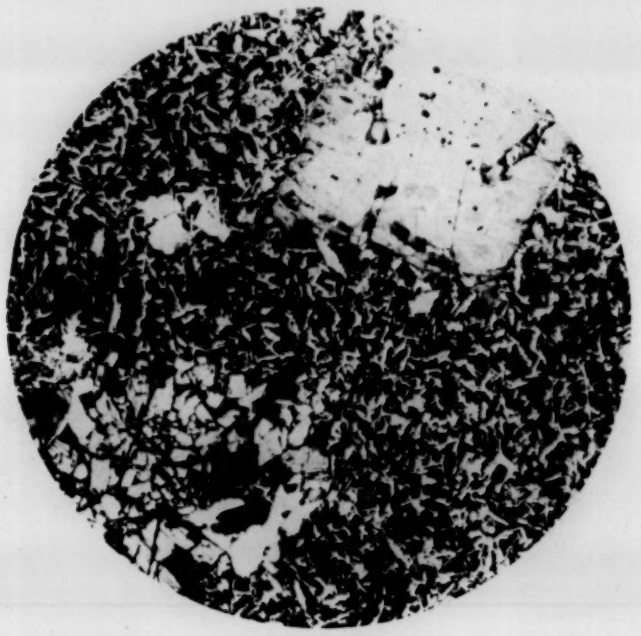
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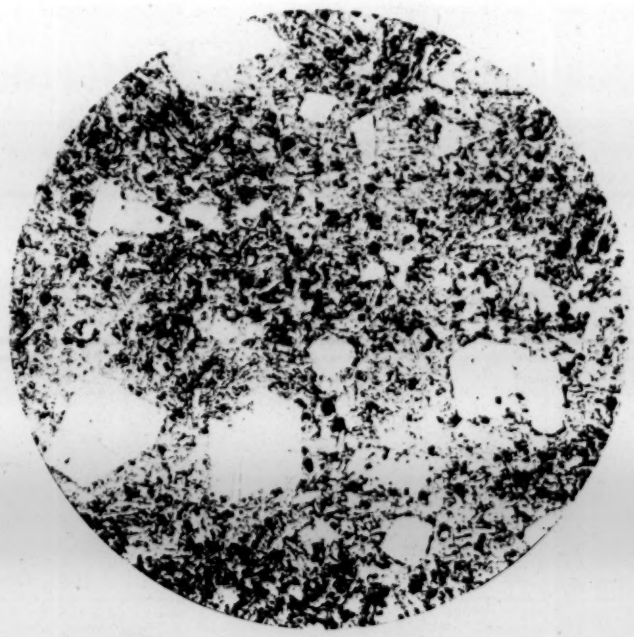
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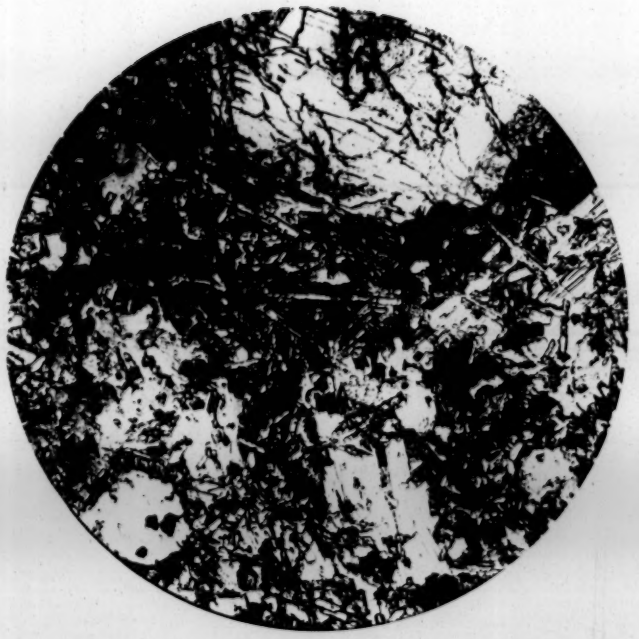
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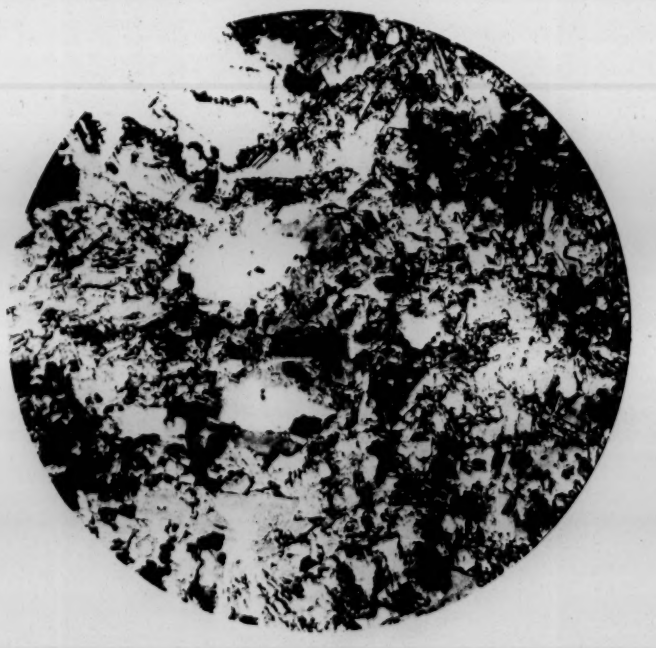
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XXXIV.—*On the Structure and Affinities of a Lepidodendroid Stem from the Calciferous Sandstone of Dalmeny, Scotland, possibly identical with Lepidophloios Harcourtii* (Witham).

By A. C. Seward, M.A., F.R.S., University Lecturer in Botany, and A. W. Hill, B.A., University Demonstrator in Botany, Cambridge.

(With Plates I.–IV.)

(Read June 19, 1899.)

Introductory.*

The unusually fine stem which forms the subject of this paper was found by Mr T. Kerr, of Edinburgh, in rocks of Calciferous sandstone age exposed in a railway cutting at Dalmeny, Linlithgowshire. Mr James Bennie, of the Geological Survey of Scotland, called the attention of Mr Kidston to the fossil, and the latter very generously placed part of the specimen in the hands of one of us for investigation.

The photograph reproduced in Pl. I. fig. 1, for which we are indebted to the kindness of Mr J. J. H. Teall, was taken from a polished slab of the stem in the Museum of Practical Geology, Jermyn Street. A similar specimen is in the Geological Museum, Edinburgh, and the remaining portions of the fossil are in Mr Kidston's possession and in the Botanical Laboratory Collection, Cambridge. A preliminary note on the structure and affinities of this Carboniferous plant was contributed to the Cambridge Philosophical Society in November 1898,† but no detailed account of the fossil has been published. A reference to the state of preservation of the tissues has also been made by one of us in a text-book published last year.‡

The specimen consists of a fairly thick piece of a stem, 33 cm. in diameter, and a polished transverse section of the block presents the appearance shown in Pl. I. fig. 1. The peripheral portion consists of a band of unequal thickness of partially disorganised bark; internal to this there is a light-coloured matrix of volcanic ash filling up the hollow trunk, and the detached central cylinder of wood and pith occupies an eccentric position close to the thicker part of the shell of bark. The marked contrast between the dark silicified plant tissues and the buff-coloured volcanic ash renders the specimen one of the most striking examples of a Palæozoic tree that has so far been discovered. The position of the larger angular fragments in the ash and of the central cylinder, point

* The numbers in brackets after the authors' names in the footnotes refer to the year of publication of the works given in the bibliography on page 928; *e.g.*, (99) means that the paper was published in 1899.

† Seward and Hill (99).

‡ Seward (98), figs. 14, A, B, and 15, p. 83.

to the stem having been preserved in the position in which it is represented in fig. 1. The tree may have been killed by the effects of neighbouring volcanic eruptions; and after the decay and removal of the inner and more delicate cortical tissues, the hollow shell was filled with volcanic ash. Silica was the petrifying medium, and the preservation of the tissues is unusually perfect: in the central cylinder there are numerous spherulitic patches scattered through the matrix; it would seem that silicification was first completed round definite isolated centres, and this was followed by a secondary crystallisation in the matrix, which partially obliterated some of the more delicate structural features.

Several of the photographs show the concentric bands of the circular areas of crystallisation—as in Pl. I. fig. 5 and Pl. II. figs. 13–15.*

The shell of the bark consists in the main of secondary tissue, from 7 to 8 cm. in thickness, for which we shall use Von Mohl's term—Phelloderm; externally this is broken up into irregular patches, as seen in the lower part of fig. 1, by numerous longitudinal fissures, some of which were probably formed during the life of the tree. Decay, previous to the introduction of the silicifying solution, removed the greater part of the cortical tissues, but it will be shown that the lost tissue no doubt consisted of thin-walled parenchyma. The central cylinder has a diameter of 6·5 cm.; the centre is occupied by concentric bands of silica, seen as a white spot in fig. 1; surrounding this is a band of unequal breadth of pith-tissue (fig. 1) about 11 mm. in breadth, succeeded by a ring, 3 mm. in width, of primary wood x′; beyond this is a broad zone of secondary wood x'', 2·2 cm. in diameter (*vide* also fig. 33, Pl. IV.). External to the secondary wood some of the tissues have fortunately been clearly preserved.

A. Description of the Tissues.

i. *Pith.*†—The band of pith (fig. 1, Pl. I.) has an irregular inner margin, where it is succeeded in the centre by concentric layers of silica; probably the pith was not quite solid in the old part of the living tree. The cells of this region are parenchymatous in form, and vary considerably in size; those in contact with the innermost tracheids of the primary wood have a more regular arrangement and a more uniform size than those towards the interior (Pl. I. fig. 4, Pl. IV. fig. 25). As we pass towards the centre of the pith, the cells become more irregular in shape, and many of them have the form of long filaments, which show a tendency to grow in a radial and more or less vertical direction, increasing in length by apical growth, and dividing by transverse walls. The general appearance of the parenchyma of the pith, especially the more internal portions, suggests an actively-growing stout hyphal tissue, comparable to the hyphæ in the

* *Vide* Seward, *loc. cit.*

† The term "pith" is used for the central tissue of the stele as a matter of convenience, but it would, perhaps, be better to adopt the expression "central conjunctive" [Flot (93)]. Bertrand has discussed the morphological nature of this region in his paper on *L. Harcourtii* [Bertrand (91)].

medullary region of *Fucus* and other genera of large brown seaweeds, which arise as outgrowths of short elements and insinuate themselves between neighbouring cells. This form of tissue is also well known in the middle cortical region of various Lepidodendroid plants, and a similar parenchyma occurs in the cortex of some recent Lycopodiaceæ.* The photograph shown in Pl. I. fig. 4 affords distinct proof of active cell-division taking place in both the smaller external cells and in the larger filamentous cell-rows.† This phenomenon of growth in the pith is referred to in a later part of the paper. It remains to call attention to the existence of numerous elements next the primary xylem, which may be spoken of as short or isodiametric tracheids with fine scalariform and reticulate bands of thickening; these are shown in Pl. IV. fig. 25. A solitary short tracheid was observed at the inner edge of the pith-tissue, similar to those of the peripheral region. These short tracheids are of the same character as the familiar "barred-cells" of WILLIAMSON, scattered through the axial region of the central cylinder of *Lepidodendron vasculare* (Binney).‡

ii. *Primary xylem.*—The limits of the annular band of primary xylem—*x'*—are readily recognised in Pl. I. fig. 1. This ring of wood consists of scalariform tracheids—about 20 to 30 tracheids in breadth; the narrowest tracheids form the outer limits of the band, and the diameter of the elements increases towards the inner edge of the xylem (Pl. IV. fig. 24). The outer edge of the xylem ring or corona has an undulating outline, prominent tapering teeth, which would appear as ridges in surface-view, and shorter, broader projections alternating with bays or depressions as shown in fig. 24. The two diagrams in fig. 34, Pl. IV., illustrate the general appearance of the outline of the primary wood. The character of the xylem surface appears to agree with that of *Lepidophloios Harcourtii*, as described by BERTRAND.§ The narrowest and most external tracheids, which constitute the protoxylem, have their thickening bands rather farther apart than in the wider elements, and the scalariform type passes in some cases into a spiral form of xylem element. There does not appear to be that well-marked distinction between definite spirally thickened protoxylem elements and the larger scalariform tracheids of the primary xylem, which we find in most recent Vascular Cryptogams. In some ferns, however, the protoxylem tracheids are not clearly differentiated from the rest of the xylem. The broad scalariform pits of the tracheids in both the primary and

* BOWER (93), p. 346.

† The growth and division of pith-cells has been noticed in old coniferous stems. *Cf.* STRASBURGER (88), p. 88.

‡ This specific name has been adopted, following the example of Solms-Laubach, in preference to that of *Lepidodendron selaginoides* used by CARRUTHERS, WILLIAMSON, and others. In 1862 BINNEY described the internal structure of certain stems, which he named *Sigillaria vascularis* and *Lepidodendron vasculare*; these were afterwards shown to be identical. The structure of the same species was also described by CARRUTHERS [CARRUTHERS (69)], and identified by him with *L. selaginoides* of Sternberg, a determination which WILLIAMSON, who extended CARRUTHERS' account, doubtfully accepted. STERNBERG'S species is, however, included by KIDSTON as a synonym of *L. Sternbergii*, Brongn. [KIDSTON (86), p. 151]. The simplest plan would seem to be to adopt BINNEY'S name, as we are quite certain as to the characters of the stem to which he gave the specific name of *vascularis*. KIDSTON has included *Sigillaria vascularis*, Binney, as a synonym of *Lepidodendron Harcourtii*, Witham (*loc cit.*, p. 169), but the difference in the anatomical characters is too well marked to render their identity possible.

§ BERTRAND (91).

secondary wood are clearly bordered (Pl. IV. figs. 25, 26 and 29), and the position of the pit-closing membrane is occupied by numerous delicate threads extending from one thickening band to the next; the appearance of these in surface-view is seen in figs. 25 and 26. In tangential longitudinal section the walls of the tracheids have a double contour, and each pit-closing membrane appears to consist of a double series of irregular fine threads, which have probably been formed by the contraction and tearing of the thin membrane which originally connected the scalariform bands. Precisely similar threads occur in *Lepidodendron vasculare* (Binney),* *Lepidophloios Wunschianus* (Will.),† *L. Harcourtii* (Witham),‡ *Lepidodendron mundum* (Will.),§ *L. brevifolium* (Will.),‖ *L. squamosum* (Göpp.),¶ and *Stigmaria ficoides*, Brongn.**

Count Solms-Laubach, in describing the same structure in a Culm species of Lepidodendron, speaks of the two sets of delicate threads, which appear as two lines in the drawing of Pl. IV. fig. 29, as probably corresponding to the "grenzhäutchen" of the tracheid wall, the middle lamella of which has disappeared. The same limiting membrane is shown in Pl. IV. fig. 26, and the irregular darker substance in the middle of each thickening band represents the shrunken internal portion of the wall. This histological feature is probably the result of post-mortem changes, and cannot be regarded as of any importance as a specific character.††

In contact with the narrower tracheids, already referred to, in the outer portions of the primary xylem, there are numerous short and more delicate elements characterised by fine spiral and reticulate thickening bands; some of these are shown in fig. 24, Pl. IV., but it is more convenient to deal with this feature in describing the origin and structure of the leaf-traces. At two points in the ring of primary xylem, indicated by · · in the diagram, fig. 33, Pl. IV., there is an indentation occupied by thin-walled parenchyma and a few fairly large isodiametric tracheids.

iii. *Secondary xylem.*—The broad band of secondary wood consists of radially disposed scalariform tracheids of smaller diameter than the innermost elements of the primary wood. At some points the smaller internal tracheids which compose the inner ends of the rows of secondary xylem are seen in contact with the elongated narrow elements of the projecting teeth of the corona, but for the most part the rows of secondary tracheids are separated from the primary tracheids by a zone of varying breadth consisting of elements of a different type. These intervening cells ‡‡ are short or isodiametric in form, and have spiral or reticulate bands on their walls; they are, in fact, short and delicate tracheids (Pl. II. fig. 13 and Pl. IV. fig. 24). When secondary

* Hovelacque (92), p. 42, fig. 6. Solms-Laubach (92), pl. ii. fig. 6.

† Williamson (80), pl. xiv. fig. 4. ‡ *Ibid.* (93), p. 20. § *Ibid.* (89), pl. v. fig. 14 A.

‖ *E.g.*, specimen 511 (Williamson Coll., British Museum).

¶ Solms-Laubach (92), p. 76. ** *Ibid.*, p. 76.

†† In a specimen of Williamson's *Lepidodendron brevifolium*, some of the tracheids show the pit-closing membrane intact, while in others it has the form of fine threads.

‡‡ "Fibres primitives" of Bertrand. It is probable that these elements described by Bertrand as cells with smooth walls possessed originally reticulate thickening bands.

thickening began there were some at least of these isodiametric elements present on the surface of the corona, and additional elements of this type were formed during the process of secondary growth; but this question must be considered in more detail in connection with the leaf-traces.*

The rows of secondary tracheids are separated at intervals by bands of one, two or several rows of radially elongated elements characterised by spiral or partially reticulate thickening bands. In the stem before us there appears to be an almost complete absence of any simple unthickened parenchymatous elements in the bands of medullary-ray tissue. The nature of the medullary-ray elements is shown in Pl. I. fig. 6,† which represents the appearance of a broad "ray" as seen in a transverse section of the secondary wood, also in Pl. IV. fig. 24; in figs. 7, 30 and 32, the medullary-ray tissue is seen in longitudinal sections of the wood. In a tangential section of the secondary xylem, the medullary rays are seen to vary considerably in size, consisting of a single row of a few cells deep, or of bands of considerable length and varying breadth (Pl. IV. fig. 30); a leaf-trace may be met with in the broadest rays, as seen in transverse section, on its way through the secondary wood (Pl. I. fig. 7 *l t*, and Pl. IV. fig. 30 *l t*). In the broad medullary ray shown in the slightly oblique tangential section of the wood in Pl. I. fig. 7, it is possible by careful focussing to examine the walls of the tissue, and in every case the elements appear to possess the characteristic thickening bands. In radial section the medullary-ray elements are clearly seen in Pl. IV. fig. 32; this section shows the radially elongated form and other characters of the tissue. The simple nature of the pits in the secondary tracheids which abut on medullary-ray elements is demonstrated in Pl. IV. fig. 29.

There is the same absence of any concentric lines of growth in the secondary tracheids as in Palæozoic stems generally. Here and there, however, the uniformity in the size of the constituents of the wood is interrupted by the occurrence of groups of smaller elements suddenly abutting on the walls of the larger tracheids. This is well shown in Pl. II. fig. 11. Some of the tracheids immediately internal to the patch of smaller elements are seen to have their cavities occupied by thin-walled parenchymatous cells *t*; a similar instance of tüllen in the tracheids has also been observed at one place in a longitudinal section of the wood. This local irregularity in the size of the tracheids may, no doubt, be regarded as the expression of some interference with the normal progress of secondary growth.

iv. *Leaf-trace bundles.*—The prominent ridges of the corona, as seen in Pl. IV. fig. 24, gradually detach themselves from the primary wood and constitute the leaf-traces in such manner as BERTRAND and other writers have described in various Lepidodendroid plants. For a short distance a leaf-trace pursues an almost vertical course approxi-

* WILLIAMSON and SCOTT [(94), p. 926], in their account of the secondary thickening in *Sphenophyllum plurifoliatum*, speak of part of the secondary xylem as separated from the primary xylem by a layer of thin-walled tissue.

† *Cf.* the medullary rays of *Stigmaria ficoides*, *Lepidodendron vasculare* (specimens 362 and 1609, Williamson Coll.), also *L. brevifolium* (No. 489), etc.

mately parallel to the surface of the primary wood. After becoming free it soon bends outwards almost at a right angle, and passes in a horizontal direction through a broad medullary ray until it reaches the limits of the secondary wood; at this point it again ascends, bending up abruptly and appearing in transverse section just beyond the outermost secondary tracheids (Pl. II. fig. 15 *l t* and Pl. III. 16 and 19).* The projecting ridges of smaller tracheids at the periphery of the primary xylem form a reticulum on the corona as seen in surface-view, and, as BERTRAND has shown in *L. Harcourtii*, the leaf-traces appear to be given off from the lower angle of the meshes. A comparison of the Dalmeny stem with the drawings in BERTRAND'S monograph leads us to consider the mode of attachment of the leaf-traces to the stele identical in the two cases. Each leaf-trace consists of a few primary elongated tracheids, accompanied by numerous thinner walled and shorter spiral and reticulately thickened elements; these latter elements are shown in fig. 24 in contact with the central projecting leaf-trace group. When seen in a tangential section of the secondary wood, the thicker and larger tracheids of the trace are recognised as a distinct group (Pl. I. fig. 7 and Pl. IV. fig. 30) surrounded by the smaller and more delicate elements which exhibit a more or less regular radiating arrangement. As seen beyond the limits of the secondary wood, a leaf-trace presents a characteristic diploxyloid structure, which on examination reveals certain points of particular interest. There is the well-defined group of long tracheids with the narrowest elements towards the exterior, but not on the edge of the primary elements, and now the radial disposition of the smaller elements, less distinctly shown in the tangential sections previously referred to (Pl. I. fig. 7, Pl. IV. fig. 30), is very striking (Pl. II. fig. 15, Pl. III. figs. 16 and 17). The smallest and somewhat crushed primary elements occupy a position near the outer edge of the primary xylem, the leaf-trace bundle may therefore be described as mesarch (Pl. III. fig. 19 *p x*). We have in each leaf-trace a group of primary tracheids partially enclosed by radially disposed rows of secondary tracheids, the latter being characterised by their smaller size and somewhat more delicate structure. The secondary xylem of each leaf-trace has usually the form of a fan, attached to the sides and outer edge of the primary xylem, but the fan-shaped secondary tissue may extend over almost the whole surface, as seen in Pl. III. fig. 16. The secondary thickening of the leaf-trace bundles is effected to some extent during their passage through the secondary wood of the stem, and further additions are made to the trace after it has passed beyond the limits of the wood of the stem. This latter point is illustrated by figs. 16 and 19, Pl. III. *m*, where the external secondary tracheids of the leaf-bundle pass gradually into thin-walled short meristematic cells. In Pl. III. fig. 21 a leaf-trace is seen in radial longitudinal section; the limit of the secondary wood of the stem is slightly to the right of *c*; at *a* we have some of the longer and thicker primary tracheids, and at *b* a broad band of the shorter and thinner secondary tracheids. A portion of the same section is more highly magnified in Pl. IV. fig. 28; *a* and *b* mark the position of the primary and secondary tracheids respectively. A tangential view of the secondary elements (fig. 21 *b*) is

* *Cf.* MARKFELDT (85), fig. 3.

afforded by the photograph in Pl. III. fig. 23; in the upper part of the section the more internal secondary elements are shown, with their reticulate, spiral or scalariform bands, and below we have the more delicate recently formed peripheral tracheids. This section shows very clearly the meristematic tissue which has been pointed out in fig. 19; the cells have extremely thin walls and square ends; they are slightly longer than broad, as seen in tangential section (Pl. III. fig. 23 *m*, *m*). In Pl. III. fig. 17 the transverse section has just missed a leaf-trace, but the accompanying thin-walled meristem is clearly shown, with the cells disposed in a fan-shaped manner.

The presence of undoubted secondary xylem in the leaf-trace bundles is a fact of considerable importance from the point of view of the affinities of the plant, and as proving that the distinction insisted upon by some writers between the Sigillariæ of the section Leiodermaria and Lepidodendroid leaf-bundles cannot be maintained.*

Owing to the absence of the greater portion of the cortex in the Dalmeny stem, the course of the leaf-traces cannot be followed through the cortex, but the partially disorganised tissues of two traces have been met with in the outer cortical tissues. Before passing to the consideration of the outer cortex, the tissues which immediately succeed the secondary xylem, and the question of secondary growth must be dealt with.

v. *Tissues in contact with the xylem; and secondary growth of the central cylinder.*—The first point to notice in connection with the growth of the secondary wood is the juxtaposition of small tracheids and the most external tracheids of normal size, as in the local patch shown in fig. 11, Pl. II. Instead of finding the large tracheids at the limits of the secondary wood succeeded by thinner-walled elements of the same breadth, which gradually pass into obvious cambium cells, we have a sudden transition from the wide elements (*t'*, Pl. III. fig. 19) to the much narrower elements *t' t''*. It is physically impossible that the latter should ever have been able to alter their form so as to fit on to the older and larger tracheids. It would seem that the constant association of these dissimilar tracheids at the periphery of the wood points to the existence of unfavourable conditions of growth; had the stem continued to grow, the period during which such conditions occurred would be shown by the occurrence of a ring of smaller tracheids; an extension round the whole circumference of such inequalities as are found here and there in the secondary wood (*e.g.*, Pl. II. fig. 11). It may not be unreasonable to suggest that this irregularity in the growth of the wood was due to the harmful effects of neighbouring volcanic activity. Prof. WILLIAMSON has expressed the opinion that the large Arran trees ("*Lepidodendron Wunschianum*") perished probably "in consequence of the mephitic vapours which filled the atmosphere"; the forests grew in a district "as volcanic as Auvergne." †

* In a tangential longitudinal section through the secondary wood of a specimen of *Lepidodendron vasculare* (No. 495) in the Williamson Collection, an outgoing leaf-trace seen in a wide medullary ray shows a tendency of some of the elements to arrange themselves in a fan-shaped manner like that of the secondary tracheids in Pl. I. fig. 7 and Pl. IV. fig. 30.

† WILLIAMSON (96), p. 175.

The occurrence of the narrower tracheids is in all probability merely the expression of the result of harmful conditions, and not an indication that the secondary thickening had reached its maximum extent. It might be suggested that the Lepidodendroid stems possessed a limited power of secondary growth, and that the stem before us represents the final stage of wood development; we find, however, that such continuous bands of smaller tracheids occasionally occur in the secondary wood of Lepidodendron* and Stigmaria,† and are succeeded by larger tracheids of the normal size. A section of one of the Arran stems in the Binney Collection shows an almost continuous zone of small tracheids succeeded externally by larger elements near the periphery of the secondary wood. The elements in the region t'' of fig. 19, Pl. III. correspond more closely in size with the large xylem tracheids than do the small elements in contact with the large tracheid *a*. In forming the small tracheids the segments cut off from the meristem cells must have divided radially; if the radial walls were not formed, the meristem segments would approximate more closely in breadth to the tracheids of normal size. But the mechanics of secondary growth in Lepidodendroid stems has still to be satisfactorily explained.

Passing beyond the small tracheids (t', figs. 16 and 19), we find a few rows of thinner-walled elongated elements, which are succeeded further out by several rows of delicate parenchymatous cells (*cm*, Pl. II. fig. 10, Pl. III. figs. 16 and 19). The cells composing this band of thin-walled tissue are rectangular, slightly elongated in a tangential direction, and somewhat flattened, as seen in a radial longitudinal section (Pl. II. fig. 10, *cm*). The thinnest and most recently formed cell-walls in this tissue occupy a radial position. In a few places the band *cm* may be traced continuously through the more elongated elements t'' to the narrow tracheids, but owing to the delicate nature of the parenchymatous cells the continuity of the tissue is frequently interrupted by tearing.

In Pl. II. fig. 10 we have an oblique radial section of the narrow tracheids and more external tissue; beyond the thicker tracheids t', corresponding to t' in the transverse sections (figs. 16, 17 and 19), we have shorter and thinner elements t'' (also in the transverse sections, figs. 16, 17, 19, and the radial longitudinal section, fig. 5), which we regard as partially developed tracheids, with transverse or slightly oblique walls, and beyond these the band *cm*, composed of flattened cells in a state of active division. At *mr*, in fig. 10, a strand of flatter cells is seen passing across the tracheids: this represents the outward extension of a medullary ray (*vide* Pl. I. fig. 5). This broad band of rectangular cells we regard as the meristematic zone, which is responsible for the growth in thickness of the xylem cylinder. From the examination of the Dalmeny stem and other species of Lepidodendroid plants, we are led to the conclusion that, as a general rule, in Lepidodendron and Lepidophloios there was no well-defined cambium in

* Williamson (72), pl. xliii. fig. 21.

† *Ibid.* (81), pl. liii. fig. 21. Section No. 1922 H. (Williamson Coll.) shows three complete rings of smaller tracheids in the outer part of the secondary wood. *Vide* also Williamson (87), pl. iv. fig. 19.

the sense of the typical cambium of recent Gymnosperms and Dicotyledons, or of such Palæozoic plants as Calamites, Sphenophyllum, Lyginodendron, and other genera; but in its place a broader band of meristematic elements, more similar in appearance to the zone of meristem which is formed in the pericycle of certain Monocotyledons or to the meristem zone of Isoetes and other recent plants.

The leaf-trace bundles in their course through the meristematic band, *cm*, are enclosed by an arc of the cambial tissue, *m*, which furnishes additions to the rows of secondary tracheids, and internal to each trace we see some flattened rows of the meristem zone of the stem (Pl. III. figs. 16 and 19). This relation of the leaf-bundles to the cambial tissue, and the arching round them of a special leaf-trace meristem band, may be compared with the behaviour of the petiole stele in *Lygniodendron Oldhamium* (Binney) as figured by Williamson and Scott.*

Beyond the meristem zone *cm* there are preserved portions of a partially disorganised tissue consisting in part of large sacs and smaller, more or less crushed cells (*sc*); this zone is directly continuous internally with the outermost cells of the band *cm*, as shown in Pl. II. fig. 9. This tissue, which may be termed the secretory zone, contains numerous dark and irregular patches which most probably represent the remains of secreted substances. The large circular sacs (*sc*, figs. 9, 15, and 16), which present practically the same appearance in longitudinal as in transverse section, were no doubt secretory in function. The areas bounded by the dark lines shown in fig. 9, *sc*, Pl. II., and fig. 16, *sc*, Pl. III., may enclose some small delicate cells; it is probable that these elements constitute glandular tissues, which gave rise to latex or other products of secretion. External to this secretory zone the tissues have disappeared, and the wide gap which intervenes between this region and the outer cortex is occupied by volcanic ash. The large sac-like spaces in this zone often present the appearance of having been tangentially stretched (figs. 9 and 16).

No tissue has been found which can be described as typical phlœm, but to this point reference is made in a later section of the paper. The so-called secretory zone occupies the position of phlœm tissue, and may represent it physiologically. The secretory zone may probably be regarded as the outermost tissue of the stelar region.†

vi. *Outer cortex, chiefly phelloderm.*—We now come to the shell of bark already mentioned; a piece of this cortical region is shown natural size in Pl. I. fig. 2. The innermost portion consists of imperfectly preserved large parenchymatous cells of polygonal form (*a* to *b*, fig. 2); the walls of the cells have been partially destroyed, but the angles are more perfectly preserved, and small intercellular spaces occur between the individual elements. At *a* in fig. 2 the remains of a leaf-trace occur, the primary tracheids are seen in slightly oblique transverse view, and the more delicate secondary elements are crushed and displaced. As we follow the parenchymatous tissue outwards,

* Williamson and Scott (95), p. 712, pl. xxii. fig. 6.

† This view is supported by an examination of some unusually perfect specimens of *Lepidophloios fuliginosus* in the Binney Collection. [Since this was written the sections referred to in the Binney Collection, Cambridge, have been figured and described. *Vide* Seward (99).]

t is found to gradually pass over, in the region *b* of fig. 2, into regular radially disposed rows of secondary tissue consisting of narrower and more elongated cells. This secondary tissue composes the whole of the outermost cortex from *b* to the surface of the stem. The appearance of the tissue is illustrated by fig. 18, Pl. III.; the cells are of fairly uniform size, but in some rows the elements have been tangentially stretched; intercellular spaces are comparatively large and abundant. The same tissue is seen in radial section in fig. 22, and in tangential view in fig. 20, Pl. III. The regular arrangement and square ends of these elongated cells, as shown in fig. 22, is a characteristic feature of the secondary cortical tissue of Lepidodendreæ and Sigillariæ.* The vacuolated cell-contents seen in the tangential section, fig. 20, bear a misleading resemblance to small cells; but the cellular appearance is simply due to the manner of occurrence of the cell-contents, and may be compared with the vacuolated tannin and other substances met with in unhealthy tissues of recent plants. The most interesting feature of the phelloderm is the occurrence of seven or eight fairly regular bands of secretory cell-rows, as seen at *c* in fig. 2, and at *d*, *e*, *f*, and in other parts of the phelloderm. There is a row of distinct secretory groups or canals just internal to the innermost cells of the phelloderm (*b*, fig. 2); each of these secretory patches contains dark indistinct traces of secreted products enclosed by smaller cells lining the cavity.

The row of secretory cells at *c* consists mainly of oval and circular areas, which may be either empty or occupied by portions of thin-walled cells and products of secretion, as shown in fig. 18, Pl. III. At *d* we have a similar row of secretory patches, but in these there is rather more of the secreted material; similarly at *e* and *f* (fig. 2) there are fairly regular bands of such tissue. In these more external rows the disorganisation of the secretory elements is less advanced, and their structure is more clearly seen (Pl. II. fig. 12). The cells forming each secretory strand are much lighter in colour than the neighbouring brown cells which make up the bulk of the phelloderm. It would seem that as the phelloderm was developed from a phellogen, at fairly regular intervals groups of the phelloderm cells underwent further division, and constituted definite strands of secretory cells. In longitudinal section a secretory strand presents the appearance shown in Pl. I. fig. 3; the cells have undergone various transverse and longitudinal divisions, and some of them have become disorganised, and have given rise to dark brown secretory products. A single secretory patch of cells is shown in Pl. IV. fig. 31; this is taken from the region *f* of the phelloderm (fig. 2), and shows a central group of cells deeply coloured by the resinous or other secretion. The method of development of these secretory strands appears to have been lysigenous. The superficial resemblance of the phelloderm with its rows of canals to the wood of resinous Conifers is fairly striking; a figure given by Tschirch † of the resin-ducts in the wood of *Copaifera Langdorfii* also agrees closely with our fig. 18, Pl. III.

* *Cf.* Williamson (72), pl. xxix. fig. 42, pl. xxxi. fig. 55; (72), pl. xliii. fig. 17; (78), pl. xxv. fig. 100. Also Renault (75), pl. x. fig. 11; Renault and Grand'Eury (75), pls. iii. and v.; Brongniart (39), pl. xxvi. figs. 2 and 3; and Renault and Roche (97), pl. vi. fig. 2.

† Tschirch (89), p. 217, fig. 216.

The phelloderm tissue is continued to the exterior of the fissured surface shown in the lower part of the photograph in Pl. I. fig. 1. In the upper part of the stem this secondary tissue was succeeded by the phellogen layer, and this again by the parenchyma of the leaf-cushions; probably this outermost tissue had been thrown off from the older portion of the stem during the life of the tree.*

Some secretory strands in a state of development are met with also in the neighbourhood of *g* in fig. 2, and beyond this region the phelloderm cells are less clearly preserved, and are characterised by walls of considerable thickness, the lumen being in some cases almost obliterated, and what remains of it is filled with a black substance. It is often difficult in dealing with petrified tissues to distinguish thick-walled tissue from tissue of which the cell-walls have been thickened by mineral deposition or by swelling of the membrane during mineralisation. In the present instance the cell-walls were probably fairly thick, but the thickened mass shown in the cell of Pl. IV. fig. 27 would seem to be the result of swelling of the wall, and this explanation is rendered probable by the fact that the cells of neighbouring rows are more or less flattened by pressure. The outermost portion of the phelloderm is characterised by many of the cells being tangentially elongated, and the occurrence of newly-formed radial walls in some of these elements points to the stretching being the result of growth, and not of mechanical or secondary origin. In a tangential section of the outermost phelloderm there are seen narrow bands of an opaque black substance forming an irregular anastomosing system; these dark lines extending through the imperfectly preserved tissue probably represent resinous or some other mineralised secretion.

There is another point of some interest brought out by the examination of the phelloderm connected with the leaf-traces. Reference has been made to the occurrence of two partially disorganised leaf-traces in the inner part of the outer cortex (region *a*, fig. 2, Pl. I.). In the peripheral part of the outer bark no indication of any leaf-trace xylem elements has been found, but in a radial section of the phelloderm one leaf-trace was met with in the region of the phelloderm indicated by *d* in fig. 2, Pl. I., passing outwards in an approximately horizontal direction; the xylem portion is crushed and imperfect, but sufficiently clear to enable one to easily recognise the tracheids (Pl. II. fig. 14 *tr*). The xylem is accompanied by a large patch of parenchyma composed of cells of varying shape, some of the elements having a hyphal or filamentous form. This mass of parenchyma is in marked contrast to the regular radial rows of phelloderm through which it is passing; there can be no doubt that we have here a portion of the middle cortex which accompanied the leaf-trace as the so-called parichnos (*pr*) of BERTRAND and other writers. The parichnos tissue affords an indication of the character of the lost middle cortex of which it formed a part. In tangential sections of the more external portion of the phelloderm, the mass of fibrous cells of which it consists is

* In a longitudinal section of Lepidophloios in the Williamson Collection (No. 1963) there appears to be an indication of an abscis layer of tissue in course of formation at the base of a leaf, suggesting that the leaf-fall was accompanied by the usual phenomenon familiar in the case of recent trees.

occasionally interrupted by the occurrence of an elongated elliptical group of parenchyma, with the cells slightly extended in a radial direction. One of these groups is shown in Pl. I. fig. 8; the cells vary considerably in size and shape, the central region consisting of smaller elements; the group shown in the photograph is about 3 mm. in length, and there is a second similar parenchymatous mass slightly above and to one side of the group shown in the figure. These parenchymatous masses agree in their structural features with the parenchyma accompanying the leaf-trace shown in Pl. II. fig. 14 *pr*, and it is probable that the more external groups like that of fig. 8 are the arms of the parichnos which have been shown by WILLIAMSON * and others to bifurcate in the more external region of the cortex in Lepidodendroid stems. In younger stems the two arms of a parichnos would appear on the same level as seen in a tangential section of the outermost cortex, and between them would be found the leaf-trace bundle; in the present instance we are dealing with an old trunk from which the leaves had fallen, and the parichnos arms alone persist, the vascular tissue having been gradually crushed and obliterated. It is well known that in the lower portions of large casts of Sigillarian stems the two large elliptical parichnos scars are often seen to lose their regular paired arrangement; one may occur at a higher level than the others, or they may appear as separate areas with no definite order.†

B. AFFINITIES OF THE DALMENY PLANT.

In the foregoing description no reference has been made to the relationship of the stem to other Lepidodendreæ. The size of the stem and its anatomical characteristics at once suggest a comparison with the famous Arran species described by WILLIAMSON and others. We have no doubt as to the specific identity of the Dalmeny and Arran trees; an examination of the series of sections of the Arran plant in the Williamson Collection (British Museum) reveals an identity in structural features. The Dalmeny specimen is, however, much more perfectly preserved, and enables us to considerably extend the diagnosis of the species.

The fossil trees of Arran were discovered by Mr WÜNSCH ‡ in volcanic beds of Calciferous sandstone age at Laggan Bay, and briefly described by him in 1867. He refers to the large stems as apparently Sigillarian, containing five or six "internal piths"; the smaller branches he identifies with Sigillaria, Halonia, and Lepidodendron. CARRUTHERS § afterwards visited the locality and gave a short account of the fossil stems; he speaks of having traced the radiating branches of a Stigmaria to a large trunk of Sigillaria, and describes the internal "piths" of WÜNSCH as young stems which had grown from spores carried into the hollow trunk of a large tree. CARRUTHERS ‖ proposed the name *Lepidodendron Wünschianum* in 1869 for a species from Arran in

* WILLIAMSON (93), pls. iii. and iv.

† RENAULT (96), p. 770. *Vide* also RENAULT and ROCHE (97), p. 17, where this point is referred to in the description of a silicified Syringodendron stem.

‡ WÜNSCH (67), p. 97. *Vide* also LYELL (78), p. 547, and SEWARD (98), p. 89.

§ CARRUTHERS (69), p. 178. ‖ *Ibid.* (69), p. 6.

his possession, but no diagnosis of the plant is given. In 1871 BINNEY * described three new Lepidostrobi from the Arran volcanic beds, and to one of these, a heterosporous form, he gives the name *Lepidostrobus Wünschianus.* In 1880 WILLIAMSON published an account of the Arran Lepidodendron,† in which he described and figured small twigs with a solid "vascular axis" (stele), and larger branches and stems with a parenchymatous axial region enclosed by a ring of wood. He regarded the smaller specimens as representing different stages in the growth of the large trunks, and discussed the evidence in favour of a gradual increase in diameter of the axial tissues, and the conversion of a solid vascular stele to a larger stele with a central parenchyma. In 1883 the same author‡ gave an account of an Halonial branch from Arran which with good reason he referred to the plant previously described. Ten years later WILLIAMSON made a brief reference to BINNEY's Arran Lepidostrobi, and in the second part of his Index,§ published in that year, he speaks of the Laggan Bay plant as *Lepidodendron Wünschianum* (Will.), having hitherto refrained from the use of a specific name in dealing with the "Arran Lepidodendron." Finally in 1895 WILLIAMSON ‖ gave a summary of his work on this species, and again discussed the question of growth and modification of the stele during the development of the stem.

SOLMS-LAUBACH ¶ has dealt at some length with WILLIAMSON's views on the growth of the Arran stems, and points out the physical impossibilities involved in some of the suggested explanations of the conversion of small solid steles into large steles with a wide central parenchyma. We must regard the various forms described by WILLIAMSON as representing, in part at least, branches of different orders, but all belonging to one specific type.

In a footnote to a paper in which some account is given of the Arran species, KIDSTON ** adds that the plant should be referred to the genus Lepidophloios and not to Lepidodendron, if WILLIAMSON was correct in connecting the Halonial branches which he described in 1883 with the large Laggan Bay trunks.††

It would necessitate an unreasonable extension of the paper to discuss in detail and to illustrate by drawings the numerous points of interest as regards the comparative anatomy of the Lepidodendreæ which are suggested by an examination of the Dalmeny specimen. There are certain considerations, however, which must be briefly touched on, as they have a direct bearing on the question of the systematic position of the plant. To deal first with the Arran plant, which should be designated *Lepidophloios Wünschianus.*

The large stems represented by various sections in the British Museum ‡‡ and in the Binney Collection, Cambridge, and by a magnificent specimen in the Owens College Museum, agree closely in size as well as in structure with the Dalmeny fossil. There

* BINNEY (71), p. 56, pl. xi. † WILLIAMSON (80). ‡ *Ibid.* (83), p. 466.
§ *Ibid.* (93), p. 15. ‖ *Ibid.* (95), p. 43.
¶ SOLMS-LAUBACH (91), p. 229. *Vide* also BERTRAND (91), p. 40. ** KIDSTON (97), p. 40.
†† For the evidence in favour of the connection between Halonial branches and Lepidophloios, *vide* KIDSTON (93), pp. 539 *et seq.* ‡‡ *E.g.*, sections Nos. 451, 452, etc.

are the same hyphal cells in the centre of the stele, the same spirally thickened elements in the medullary rays,* and an identity of structure in the primary and secondary wood. Isodiametric tracheids also occur internal to the primary xylem cylinder, as in the Dalmeny stem.† Unfortunately, the large Arran stems do not afford any positive evidence as to the structure of the leaf-trace after it becomes free from the secondary wood, the tissue in the innermost cortical region having been almost completely destroyed. In one section a leaf-trace with both primary and secondary elements was found close to the edge of the wood, but the fan-shaped group of secondary elements was directed inwards and not outwards as in the Dalmeny stem; as the trace is isolated and not enclosed in tissue, it has no doubt been twisted, and indeed it may not improbably belong to a Stigmaria and not to the stem with which it is associated.

The phelloderm of the Arran trunks is identical with that which we have described, there are the same rows of secretory strands as in our stem,‡ and in tangential sections of the phelloderm elliptical groups of cells are met with identical with that of Pl. I. fig. 8, which we regard as the parichnos.§ Passing to the smaller specimens, we may first mention the specimen represented in pl. xiv. fig. 5 of WILLIAMSON'S tenth Memoir.‖ One important fact to notice in connection with this example is the very small amount of secondary xylem in a stem about 12 cm. in diameter. Here, again, there is the closest agreement with the Dalmeny stem as regards the structure of the wood and cortical tissues; the large sac-like cells shown in our Pl. II. figs. 9 and 15, and Pl. III. fig. 16, are clearly seen in WILLIAMSON'S figured section. The middle portion of the cortex in WILLIAMSON'S specimen presents the characteristic structure which is shown in the parichnos, or portion of the inner cortex accompanying the leaf-trace, represented in our Pl. I. fig. 8. The small twigs described by WILLIAMSON show fairly well-preserved leaf-cushions, and the form of these, as seen in longitudinal section, suggests the Lepidophloios rather than the true Lepidodendron type. In the smaller axes, 1·2 cm. in diameter, there is no trace of secondary xylem, but each leaf-trace is accompanied by numerous short and spirally thickened tracheids, which in a transverse section of a leaf are seen to be arranged round the strand of elongated scalariform tracheids; their position and appearance recall the transfusion tissue of many recent plants.¶ These short tracheids are well shown also in longitudinal sections of the leaf-trace on the flanks of the tracheal strand.** The solid xylem strand occupying the axis of the small twig may indicate that these are slender shoots of an Halonial branch; their stele agrees

* The thin-walled spirally thickened elements accompanying the leaf-traces in their course through the secondary wood are clearly shown also in *Lepidodendron brevifolium*, Will. (*cf.* especially No. 499), and in Stigmaria, *L. vasculare*, etc.

† *Cf.* section 448 A.

‡ These secretory groups of cells are exceedingly well shown in some sections of the phelloderm of the largest Arran stem in the Binney Collection, Cambridge, as well as in British Museum sections. Similar cell-groups have been observed in sections of *L. vasculare*; *cf.* also HOVELACQUE (92), p. 58, fig. 17.

§ *E.g.*, sections 443, 447, 448, 449 in the Williamson Coll.; *cf.* also sections of Sigillaria (662 and 665).

‖ WILLIAMSON (80).

¶ *E.g.*, sections Nos. 431, 432, etc. (Williamson Coll.).

** *Cf.* section 1955, and several others in the British Museum.

closely with the small vascular strands given off from the stele of the larger Halonia described by WILLIAMSON in 1883. In the small twigs the two arms of the parichnos, composed of large thin-walled cells, are found in close proximity to the leaf-trace bundles, as seen in the fleshy leaf-bases.* In the older stems from which the leaves have fallen, and in which a considerable thickness of phelloderm has been developed, the leaf-trace becomes disorganised on its way through the outer cortex, and the parichnos strands lose to some extent their regularity of arrangement; their cells, too, appear to be somewhat compressed by the pressure of the mass of phelloderm, through which the parichnos is passing. The cells in the parichnos figured in Pl. I. fig. 8 are rather smaller and more compact than those in the parichnos arm of a leaf-base.

The Williamson Collection includes one specimen of a piece of rock from Arran (No. 62) which shows imperfect surface markings in the form of lozenge-shaped depressions, which no doubt represent the leaf-bases of a Lepidodendroid plant, but the preservation is far from satisfactory, and we have no proof of the connection between this cast and the petrified stems.

There is a specimen of *Lepidophloios Wünschianus* in the fossil-plant gallery of the British Museum, from the Calciferous sandstone of Craigleith (No. 52,625), which represents a stem with two steles 1 cm. in diameter; the stem has been cut across transversely just below the point at which the two branches of the bifurcating axis became free. It is interesting as showing the absence of secondary wood, although the section measures 19 by 14 cm.; this is one of the characters shared by *L. Wünschianus* and the stems known as *L. Harcourtii*.

We may next consider the question as to the possible identity of the Arran and Dalmeny stems with plants described under other specific names. There are many points of resemblance between these stems and the diploxyloid axes from Burntisland, named by WILLIAMSON *Lepidodendron brevifolium*,† but a comparison of the structural features does not lead us to regard the latter form as specifically identical with the Arran and Dalmeny species. There are various characteristics shared by WILLIAMSON's species, *Lepidophloios fuliginosus* (Will.), and *L. Wünschianus* (Will.), but the former species is no doubt distinct; the nature of its secondary xylem and other features prove it to be a well-defined specific type.

The most interesting comparison‡ is with the species long known as *Lepidodendron Harcourtii* (Witham). The plant bearing this name was first described and figured by WITHAM,§ and afterwards by LINDLEY and HUTTON in their Fossil Flora ‖; subsequently more accurate figures were given by BRONGNIART ¶ in 1837 and 1839.

* *Cf.* sections 428, 443, etc.

† WILLIAMSON (72), p. 310. In the index (Pt. II.), published in 1893, WILLIAMSON speaks of this species as Lepidophloios (p. 10).

‡ For the opinions expressed as to the possible identity of the Dalmeny plant and *L. Harcourtii*, one of us must be held responsible (A. C. S.).

§ WITHAM (32) and (33), p. 51, Pls. xii. and xiii. ‖ LINDLEY and HUTTON (33), vol. ii. pls. xcviii. and xcix.

¶ BRONGNIART (37), pls. xx. and xxi.; (39), pls. xxx. and xxxi.

The specimen on which the species was founded was presented by the Rev. C. G. V. Vernon Harcourt to Mr Phillips of York, and the latter forwarded it in 1832 to Mr WITHAM, who named the fossil *Lepidodendron Harcourtii*. It was found, as we are told by the authors of the Fossil Flora, in the roofstone of a bed of coal worked at Hesley Heath, near Rothbury in Northumberland. The seam is considered by LINDLEY and HUTTON to occupy a position "deep in the Mountain Limestone series." * WITHAM published several rather crude figures of the internal structure, and compared the anatomy of the stem with that of *Lycopodium clavatum*; he agreed with BRONGNIART'S conclusion, based on external characters, that the Lepidodendra should be regarded as Lycopodiaceous plants. WITHAM'S figures and descriptive text are repeated in his important work of 1833. Other drawings are given in the "Fossil Flora," but these are in many respects far from accurate. ADOLPHE BRONGNIART † afterwards published a few additional drawings in his *Histoire des végétaux fossiles* and in his classic paper on *Sigillaria elegans*. Under the name *Lepidodendron Harcourtii*, BINNEY ‡ described some exceedingly fine specimens from the Coal-measures near Dudley, but these belong most probably to the type since named by WILLIAMSON § *L. fuliginosum*.

WILLIAMSON in his latest paper, published in 1895,‖ calls attention to the probability that, like *Lepidophloios Wünschianus*, *L. Harcourtii* developed secondary xylem at a comparatively late period of its growth. BERTRAND ¶ has also expressed similar views as to the late appearance of the secondary wood in the latter species.

An exceedingly detailed account of *Lepidophloios Harcourtii* was published in 1891 by Prof. BERTRAND ** of Lille; this author draws attention to the close similarity between this species and the plant figured by CORDA †† in 1845 as *Lomatofloyos crassicule*.

The chief characteristic features usually associated with *L. Harcourtii* are (i) the absence or late appearance of secondary wood; (ii) the nature of the leaf-trace bundles, which consist of a strand of xylem tracheids accompanied by a group of "bast fibres"; (iii) the presence of numerous prominently projecting teeth from the surface of the corona; (iv) the usual absence of the middle cortical tissues. In 1887 WILLIAMSON ‡‡ contributed a note to the Royal Society, in which he proposed the name *Lepidodendron fuliginosum* for certain specimens incorrectly described as *L. Harcourtii*; he mentioned the "duplex structure" of the leaf-trace bundle as a special distinguishing feature of WITHAM'S specific type. SOLMS-LAUBACH §§ especially compares *L. Harcourtii* and WILLIAMSON'S *L. fuliginosum*, suggesting for the latter the specific name of *Williamsoni* ‖‖; he refers to the presence of the bast-fibres in *L. Harcourtii* as one of

* LINDLEY and HUTTON, *loc. cit.*, p. 45.
† BRONGNIART (37), pls. xx. and xxi.
‡ BINNEY (72), p. 77.
§ WILLIAMSON (87), *vide* also (93), p. 3.
‖ *Ibid.* (95), p. 47.
¶ BERTRAND (91), p. 150.
** *Ibid.* (91).
†† CORDA (45), pl. iii.
‡‡ WILLIAMSON (87), p. 7.
§§ SOLMS-LAUBACH (91), p. 226.
‖‖ The change of name suggested by Count SOLMS seems inadvisable, as the older name given to the species by WILLIAMSON has been generally recognised.

the distinguishing features between the two species. There are other differences which need not be considered here, as they are well known and apparently constant. The "bast-fibres" of the leaf-trace are referred to by BERTRAND * as consisting in part of possibly laticiferous tubes, and that they are not bast fibres but brown secretory cells is, we believe, the true explanation. Precisely similar groups of indistinctly preserved cells accompany the leaf-traces of *L. fuliginosum*, and these two are composed of a few partially-disorganised secretory elements. A comparative examination of the species *Lepidophloios Harcourtii*, *L. fuliginosus*, and *L. Wünschianus* shows that in each of these plants there is a fairly broad zone of secretory tissue a short distance beyond the xylem, as shown in several of the photographs of the Dalmeny stem. As the xylem tracheids of a leaf-trace pass into this zone they appear to carry in front of them a patch of the secretory tissue. A leaf-trace bundle when seen immediately external to the primary xylem of the stem has no strand of the so-called fibres accompanying it; but immediately beyond the secretory zone the second strand is present, and it consists, we believe, not of true fibres, but of somewhat crushed or partially-disorganised secretory cells.† This behaviour of the trace is very clearly demonstrated in some sections of *L. fuliginosus*, figured by BINNEY ‡ under the name of *L. Harcourtii*, and now in the Woodwardian Museum, also in several sections of *L. Wünschianus* § and *L. Harcourtii* in the British Museum. A more detailed account will be given of the Binney sections in another place. ‖

Another point of resemblance between *L. Harcourtii* and the Dalmeny plant is the occurrence of strands of secretory cells in the outer cortical region. In the type-specimen of WITHAM's species there are a few large canals near the periphery of the partially decorticated specimen; and in the large stem from Airdrie ¶ we find large canals just internal to the phelloderm precisely as in the Dalmeny stem, as well as other canals in the phelloderm tissue. In the Dalmeny plant the secretory areas immediately internal to the phelloderm (region *b* Pl. I. fig. 2) are rather larger than those in the phelloderm; in *L. Harcourtii* there is the same slight difference between the more internal and external canals.** BERTRAND †† has figured a secretory area in the cortex of a specimen of *L. Harcourtii* received from M. HOVELACQUE.

The short tracheids which occur in the inner margin of the primary wood of the Dalmeny stem (Pl. IV. fig. 25) are clearly represented in *L. Harcourtii*.‡‡ The margin of the corona in *L. Harcourtii* and in the Dalmeny stem is, we believe, identical, and the leaf-traces apparently become detached from the main mass of xylem in the same manner (figs. 24 and 34). The mesarch structure of the leaf-traces is another character shared by *L. Harcourtii* §§ and the Scotch plant; the same feature is well shown in

* BERTRAND (91), p. 119.

† *Cf.* specimen 448A, 456A, etc.

‡ BINNEY (72), pls. xiii. and xv. *Cf.* also specimens 1922A, 384, etc., in the Williamson Collection.

§ *E.g.*, sections Nos. 465, 448A, 428.

‖ [Since this was written a description of Binney's specimens has been published in the *Proceedings of the Cambridge Philosophical Society*, vol. x. p. 137; *vide* SEWARD (99).]

¶ Figured by WILLIAMSON (93), pl. i. fig. 3.

** *Cf.* specimens Nos. 1593, 380, 381.

†† BERTRAND (91), pl. v. fig. 26.

‡‡ Specimen No. 381.

§§ *Ibid.* (91).

some exceedingly well-preserved sections of *L. fuliginosus*, originally described by BINNEY as *Lepidodendron Harcourtii* and as *Halonia.**

There are no anatomical characteristics shown in the type-specimen of *L. Harcourtii* nor in the larger stem from Airdrie,† which cannot be matched, more or less exactly, in the Dalmeny and Arran species. This resemblance in structure agrees also with the geological horizon from which the type-specimen of WITHAM was obtained.

We have no precise information as to the geological age of the larger stem of *L. Harcourtii* described by WILLIAMSON from Airdrie; the matrix in which the specimen is embedded bears a close resemblance to that of the Dalmeny tree. There are, however, other examples‡ of what appears to be *L. Harcourtii* from the lower Coal-measures of Lancashire and Yorkshire, but of the identity of these we do not wish to speak with certainty at present. To be consistent with our belief of the specific identity of WITHAM'S type and the larger Airdrie stem with the Dalmeny stem, the latter should be spoken of as *L. Harcourtii* (Witham).

The conspicuous gaps accompanying the leaf-trace bundles, as seen in a transverse section of the outer cortical regions of the stems of *L. Harcourtii* hitherto figured, are not present in the large stem which we have described. The size of the parichnos, part of which is shown in Pl. II. fig. 14 *pr*, agrees with the large dimensions of this organ in the smaller specimens, and the absence of the gaps in the phelloderm-band in the Dalmeny stem is in all probability due to the considerable development of phelloderm, which compressed, and to a large extent obliterated, the parichnos tissue of the leaf-traces which were no longer connected with leaves, as the older stems were leafless. The slightly more prominent and obvious tooth-like projections of the corona in the smaller stems may be accounted for by the presence of the broad zone of secondary wood, but the long and narrow prominences shown in Pl. IV. figs. 24 and 34 are indeed very similar to those of the smaller examples of *L. Harcourtii*.

WILLIAMSON'S nineteenth Memoir, the last of the splendid series which he communicated as sole author to the Royal Society, concludes with the paragraph: "We have at least arrived at one result from the investigations recorded in the preceding pages; we now know that *L. Harcourtii* was not a distinct form, teaching an independent and special philosophy. So far as its history is now known, it is substantially identical with that of the other known Lepidodendra, teaching the same truths, and leading to the same inferences. We only want, in my opinion, to discover it in its arborescent condition to place it side by side with *L. Wünschianum* as a true typical exogenous Lycopod, and with which it has several external close relationships."§

In the Dalmeny stem, and in the much less perfect stems from Arran, we have most probably this "arborescent" form which WILLIAMSON refers to.

* BINNEY (72). [*Vide* also SEWARD (99).]

† Section No. 380 (received by Prof. Williamson from the late Prof. Balfour of Edinburgh).

‡ Specimens No. 1596A, 380B, etc., in the Williamson Collection [*vide* WILLIAMSON (93), pl. i.].

§ WILLIAMSON (93), p. 29.

In considering the affinities of the Dalmeny stem reference must be made to a specimen recently described by RENAULT and ROCHE * from the Culm of Esnost under the name *Syringodendron esnostense.* The petrified block shows several spirally disposed elliptical areas on the weathered surface, which represents the ends of parenchymatous cell-groups, called by the authors *organes aerifères.* Internally there is a segment of a band of secondary wood 20 mm. in diameter, succeeded by portions of an annular zone of primary wood and a large pith. Beyond the secondary wood, and separated from it by a space filled with siliceous material and plant débris, there is a broad band of secondary cortex described as consisting of concentric zones of "suber." In a tangential section of the secondary wood some broad medullary rays are represented containing diploxyloid † vascular bundles passing horizontally outwards, practically identical with our Pl. I. fig. 7. One of the vascular strands is also shown in transverse section immediately beyond the secondary wood of the stem, agreeing closely with the leaf-trace bundles of the Dalmeny stem (*e.g.*, Pl. III. figs. 16 and 19). These bundles are described as passing out to branches, and the true leaf-traces are considered to have been obliterated as the stem had most probably lost its leaves; one small and indistinct bundle is shown in a tangential section of the wood passing through one of the broad rays traversed by a "branch-bundle," which RENAULT and ROCHE speak of as a foliar vascular strand. It would seem, however, much more probable that the diploxyloid xylem strands are the true leaf-traces.

There is, in fact, the closest resemblance between the French and Scotch stems, and if not specifically identical, the two must be very nearly allied. The photograph of the "concentric bands" of secondary cortex suggests a comparison of the fairly regular lines, which break the regularity of the radially disposed phelloderm cells, with the rows of secretory strands in the Dalmeny plant. The elliptical groups of cells visible externally as slightly projecting areas in the French stem agree in structure, except in their somewhat larger dimensions, with the parichnos strands met with in tangential sections of the Arran and Dalmeny stems.

The old name Syringodendron, usually regarded as standing for a partially decorticated and old Sigillarian trunk, is used by RENAULT and ROCHE as an autonomous genus on the grounds that the parichnos strands occur singly and not in pairs, and that no evidence has been found suggesting the formation of the single groups by the fusion of two originally separate arms. These reasons seem hardly sufficient for raising Syringodendron to generic rank. In *Syringodendron esnostense* we have in all probability an old stem—very closely allied to the Dalmeny species—which was most probably either a Sigillaria, a Lepidodendron, or a Lepidophloios. To discuss this question would involve us in an examination of the anatomical and other characteristics which distinguish these three genera. The structure of Lepidodendron and Lepidophloios is

* RENAULT and ROCHE (97).

† The term "diploxyle" is used by RENAULT to describe vascular tissue consisting of primary (generally centripetal) xylem and secondary centrifugal xylem; he includes under this head the leaf-traces of Cycads, which are now termed Mesarch by many writers; they are not truly diploxyloid in that they consist of primary xylem only.

practically identical, and there are at least some Sigillarian stems * in which we cannot discover any structural features distinct from those of the Lepidodendreæ. Reasons have already been given for regarding the Dalmeny stem as a Lepidophloios, but whether the younger stems and branches of the French stem should be recognised as belonging to Lepidodendron, Lepidophloios, or Sigillaria must remain an open question. This uncertainty as to the respective anatomical characteristics of these three genera is perhaps a reason in favour of retaining such a generic term as Syringodendron.

Conclusion.

The more important anatomical features of the Dalmeny species may be briefly summarised as follows :—

i. The diploxyloid and mesarch leaf-trace bundles.
ii. The numerous canals or secretory strands of parenchyma in the phelloderm.†
iii. The presence of a fairly regular ring of rather larger secretory strands immediately internal to the secondary cortical tissue (phelloderm).
iv. The presence of short tracheids on the inner edge of the primary xylem, and the numerous and more delicate short tracheal elements external to the corona.
v. The delicate and lax nature of the broad inner cortex.
vi. The parenchymatous pith, which consists in part of elongated cell-filaments, especially in the larger stems; in smaller branches the pith is solid and composed of more regular cells.

The character of the parichnos, the nature of the secondary xylem, the presence of a band of secretory tissue in the innermost cortex and other features are not peculiar to this type of the Lepidodendreæ. The characters enumerated in the above list must not be regarded as being of specific rank, some at least are certainly shared by allied forms. We are much in need of an exhaustive comparative account of the Lepidodendroid species from which to select features common to the various types and others which may serve as specific characters. The apparent absence of typical phlœm has been briefly referred to in the above description; this is probably a character shared by other species of Lepidophloios and Lepidodendron; it would seem that we have as yet no satisfactory evidence in these genera of the existence of tissue possessing the ordinary anatomical characters of phlœm, and probably the functions associated with the phlœm of

* *E.g.*, a specimen described by Carruthers at the Edinburgh British Association Meeting in 1892; but the account was not published in the Annual Report. [Since the above was written a Sigillarian stem has been described by Prof. Bertrand which throws fresh light on the anatomy of this genus. Vide *Annals of Botany*, 1899.]

† We have observed similar secretory strands in the phelloderm of *L. vasculare* as well as in other species already referred to. [*Cf.* also a figure by Hovelacque (92), in which strands appear to be indicated.]

recent plants must in the Palæozoic types have been carried out by tissues of another kind and of a more primitive character. This question will be dealt with in a forthcoming account of some exceedingly well-preserved examples of *Lepidophloios fuliginosus* (Will.).* It is a remarkable fact that there is no band of tissue which can be spoken of as secondary phlœm at all comparable in extent to the secondary xylem. The absence of any well-defined initial meristem layer or typical cambium is another point of general interest. The meristem zone in the Dalmeny stem is separated from the newest tracheids by a comparatively broad band of cells, the elements of which appear to assume more gradually the form of xylem tracheids than is the case with most plants which possess the power of secondary growth; we should speak of a cambial zone rather than a cambium layer as constituting the generative region from which the wood was supplied with new elements.

The thick band of secondary cortex, described as phelloderm, is a character common to the Lepidodendra and Sigillariæ generally. The meristem-zone, which arises just internal to the leaf-cushions, at an early stage in the growth of these plants gives rise to little or no tissue on the outside, but on the other hand produces internally a considerable thickness of radially disposed elongated elements, and some of these cells undergo further divisions to form strands of secretory tissue; from analogy with other plants one may speak of the internal secondary cortex as phelloderm, a tissue with a fairly well-developed aërating system of intercellular spaces. The cork of recent plants, in the sense of an impermeable tissue developed on the outside of the phellogen, is barely represented. It may be noted that Worsdell has described a thick band of phelloderm in the stems of *Cycas media*, R. Br.,† and *Macrozamia*.‡

When we consider the small diameter of the woody central cylinder, as shown in Pl. I. fig. 1, in comparison with the size of the stem as a whole, the usefulness of the wood as a mechanical support to enable the stem to resist lateral strains would seem to have been extremely small. May we not regard the thick phelloderm of the fossil stem as a tissue of which one of the functions was mechanical; a strengthening band at the periphery of the stem corresponding to the hypodermal stereome or to the peripheral region of secondary wood in recent plants?§

The exact significance of secretory products in recent plants is by no means well understood, and seeing that we cannot determine the nature of the secretions in fossil tissues, we are not in a position to do more than hazard a guess as to the function of the glandular tissue. The abundance of secretory canals in so many of the Palæozoic genera is an interesting fact, and one which, with further data derived from physiological anatomical considerations, may help us to learn something as to the conditions under which these plants grew.

In the outermost stelar region of Lepidodendra the large secretory sacs form a striking feature, and the close connection between strands of this secretory zone and the outgoing leaf-trace reminds one of the large secretory strands which are met with in the

* [Seward (99).] † Worsdell (98), p. 443. ‡ *Ibid.* (96), p. 615. § *Cf.* Haberlandt (96), p. 163.

cortex of *Lycopodium inundatum* (Linn.), and other species, and associated with the foliar bundles of some Cycads.* It is possible that the secretory zone of Lepidophloios had some share in the conduction of plastic organic substances, while the secretory tissue of the phelloderm fulfilled a different purpose.

The abundance of short and comparatively thin-walled tracheids is a question which invites speculation as to the possibility of connecting certain anatomical characters with the habit of growth of these Palæozoic Lepidodendreæ. Some at least of these elements suggest a comparison with the transfusion tissue † of recent plants, to which reference has already been made, and at the same time there is the possibility that in these short tracheal cells we may have representatives of what have been called "Speicher-tracheiden."‡ There are, indeed, indications in the anatomical features of the Dalmeny Lepidophloios that the conditions of growth were such as to require the development of characters usually associated with a xerophytic habit. The existence of striking xerophytic peculiarities in recent plants which inhabit seashore regions, as well as those of arid regions, teaches the need of extreme caution in attempting to deduce from anatomical data conclusions as to the environment of Palæozoic species. On the whole, we are disposed to see in the presence of xerophytic characteristics an indication that some at least of the Carboniferous plants grew in seashore swamps under conditions which necessitated the development of halophytic peculiarities.

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* *Cf.* Worsdell (96).

† *Vide* Worsdell (97), Zimmermann (80), etc. Short tracheids, similar to those of Lepidophloios, accompanying the petiolar bundles have recently been described by Scott in *Medullosa anglica*. An excellent example of transfusion tissue in a fossil conifer has been figured by Solms-Laubach in Ullmannia [Solms-Laubach (84), pl. iii.].

‡ Haberlandt (96), p. 354. [In a new genus recently described under the name *Megaloxylon*, the abundance of large short tracheids, which probably served as reservoirs of water, is a characteristic feature. [*Vide* Seward (99).]

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EXPLANATION OF PLATES.

[The photographs reproduced in Pl. I. fig. 6, and Pl. III. figs. 20 and 22, were kindly supplied by Mr Kidston; all the other micro-photographs have been taken by Mr W. Tams, Cambridge.]

PLATE I.

Fig. 1. Transverse section of the stem. *p*, pith; *x'*, primary wood; *x*, secondary wood. *Circ.* $\frac{1}{4}$ nat. size.

Fig. 2. Portion of the outermost cortex, mainly composed of phelloderm, which extends from *b* to the surface of the stem. *a*, position of a leaf-trace; *b g*, regions referred to in the text. Nat. size.

Fig. 3. Longitudinal section of a secretory strand in the phelloderm. × 100.

Fig. 4. Longitudinal section of the pith, and portions of two xylem tracheids. × 100.

Fig. 5. Longitudinal section through the edge of the secondary wood. *t'*, narrow tracheids; *t''*, thinner and shorter elements representing partially developed tracheids; *mr*, medullary ray. × 80.

Fig. 6. Part of a medullary ray as seen in a transverse section of the secondary xylem. × 190.

Fig. 7. Tangential section of the secondary wood, showing a broad medullary ray and a leaf-trace, *lt*. × 90.

Fig. 8. Tangential section of the phelloderm showing a mass of parenchyma (parichnos) cut through transversely. × 80.

PLATE II.

Fig. 9. Transverse section of the secretory zone. *Sc. cm*, outer portion of the meristematic region. × 100.

Fig. 10. Radial longitudinal section, slightly oblique, of the outermost portion of the secondary xylem and neighbouring tissue. *t'* and *t''*, new and partially developed tracheids; *cm*, meristematic tissue. × 80.

Fig. 11. Transverse section of the secondary wood, showing a group of smaller tracheids. *t*, tüllen in the cavities of two tracheids. × 120.

Fig. 12. Transverse section of the phelloderm, with a row of secretory cell-groups. × 80.

Fig. 13. Radial longitudinal section through the junction of the primary and secondary xylem (*x'* and *x''*). × 90.

Fig. 14. Radial longitudinal section through the phelloderm, passing through a leaf-trace with partially disorganised tracheids *tr* and the parichnos *pr*. × 60.

Fig. 15. Transverse section through the stelar region and the outer edge of the secondary wood. *t*, leaf-traces; *sc*, secretory zone. × 50.

Plate III.

Fig. 16. Transverse section through the secretory zone *sc*, the meristem-zone *cm*, and the edge of the wood ; *t'* and *t''*, tracheids ; *m*, meristem cells surrounding the secondary tissue of a leaf-trace. × 90.

Fig. 17. From the same region as fig. 16, passing through a fan-shaped group of meristem cells. × 90.

Fig. 18. Transverse section of the phelloderm showing some older secretory canals. × 80.

Fig. 19. Transverse section of the outer edge of the secondary xylem, *t' t''*, the meristem region, *cm*, and a leaf-trace with its cambium arc, *m*. × 100.

Fig. 20. Tangential section of phelloderm cells with vacuolated cell-contents. × 80.

Fig. 21. Oblique radial longitudinal section of a leaf-trace, showing the primary and secondary xylem, *a*, *b* and *c*, region close to the secondary xylem. *sc*, secretory zone. × 100.

Fig. 22. Radial longitudinal section of phelloderm cells. × 50.

Fig. 23. Tangential section of a portion of a leaf-trace. *m*, meristematic cells. × 120.

Plate IV.

Fig. 24. Transverse section showing the corona (*x'*) and secondary xylem (*x''*). × 50.

Fig. 25. Radial longitudinal section of one of the large innermost tracheids of the primary xylem, with some small isodiametric tracheids in the peripheral region of the pith. × 90.

Fig. 26. Bordered pits with fine threads in the wall of a tracheid. × 240.

Fig. 27. A cell from the phelloderm (outer portion) with thickened wall. × 100.

Fig. 28. Portion of fig. 21, Pl. III., more highly magnified.

Fig. 29. Tangential longitudinal section showing the double bordered pits in the walls of adjacent tracheids, and the single bordered pits between the tracheids and medullary-ray cells. × 240.

Fig. 30. Tangential longitudinal section of the secondary xylem showing a leaf-trace in a medullary ray. × 50.

Fig. 31. A secretory group of cells in the phelloderm. × 90.

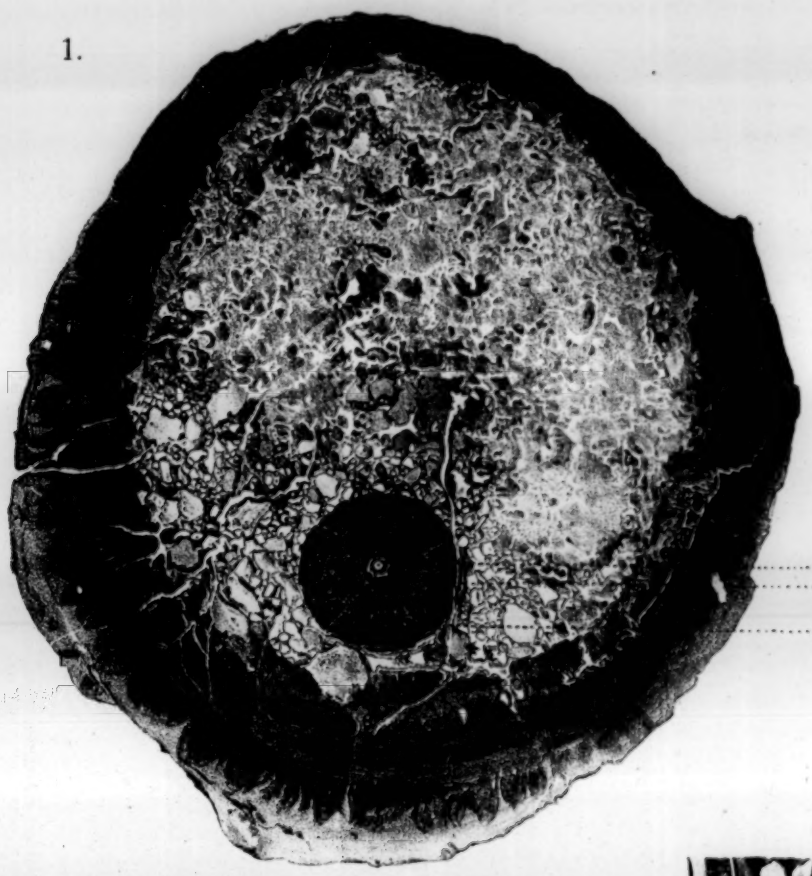
Fig. 32. Radial longitudinal section showing a medullary ray traversing the secondary xylem. × 50.

Fig. 33. Diagram of the annular primary xylem *x'*, with remains of pith. The dots . . mark the position of two indentations in the ring of wood.

Fig. 34. Diagrams illustrating the external contour of the corona.

Seward & Hill: Structure and Affinities of a Lepidodendroid Stem from the Calciferous Sandstone of Dalmeny.— Plate I.

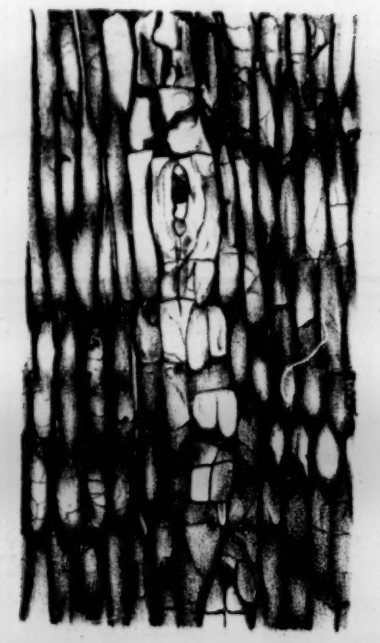
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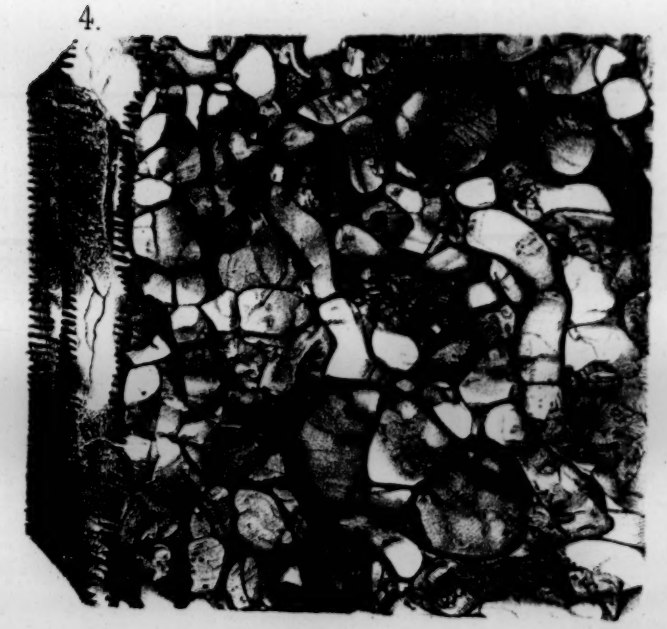
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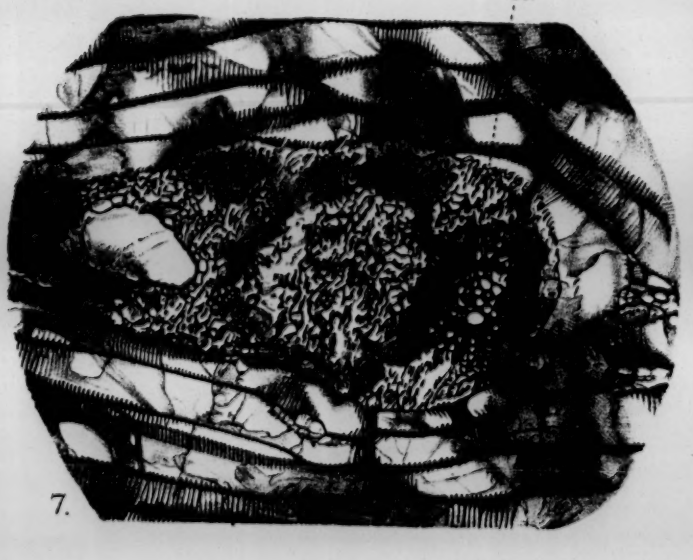
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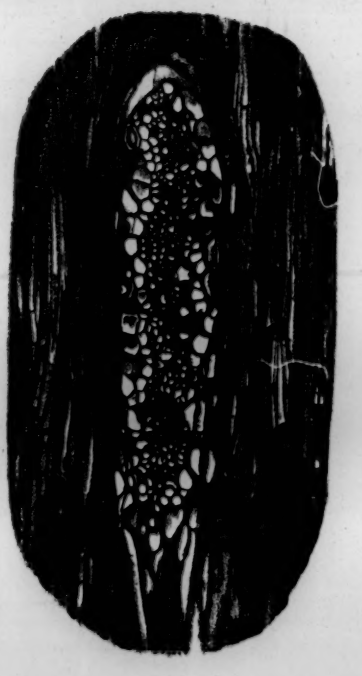
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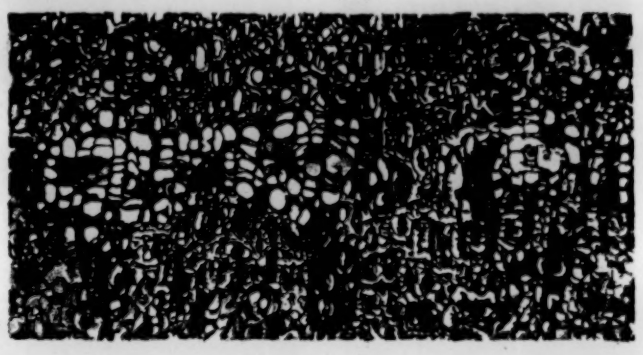
Seward & Hill: Structure and Affinities of a Lepidodendroid Stem from the Calciferous Sandstone of Dalmeny.— Plate II.



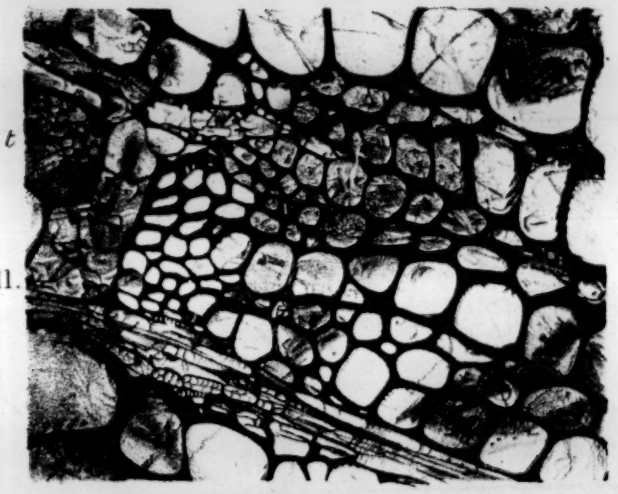
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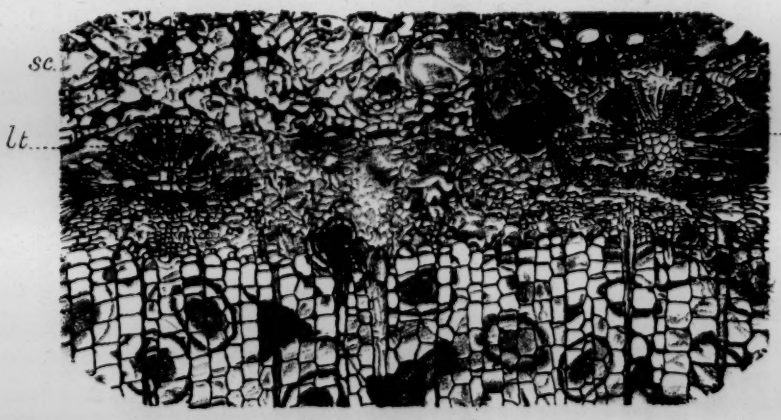


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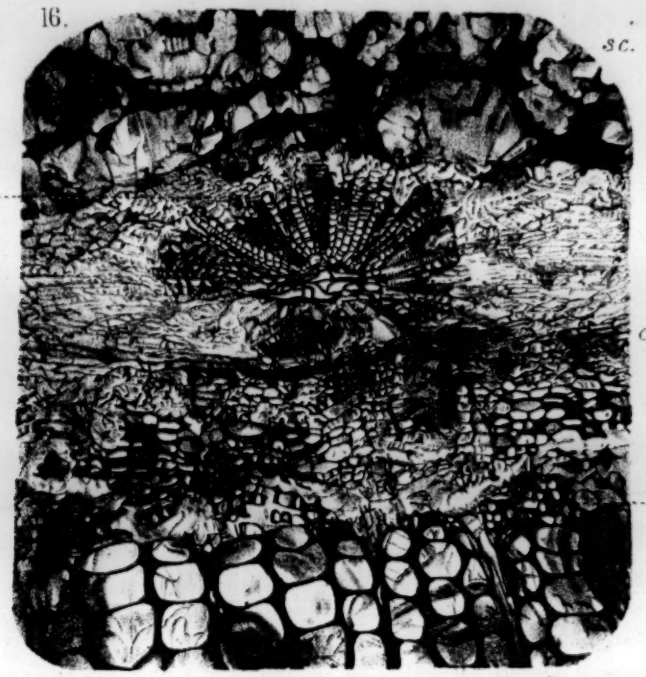
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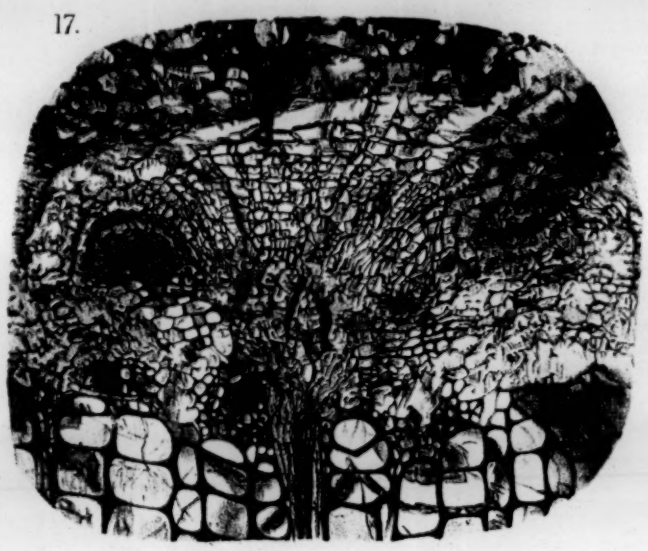
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SEWARD & HILL: STRUCTURE AND AFFINITIES OF A LEPIDODENDROID STEM FROM THE CALCIFEROUS SANDSTONE OF DALMENY.— PLATE III.

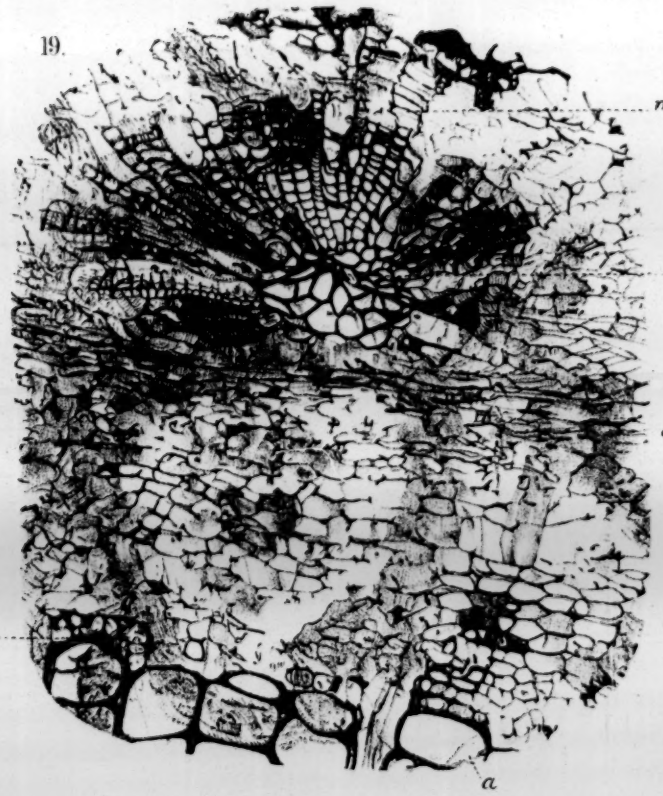
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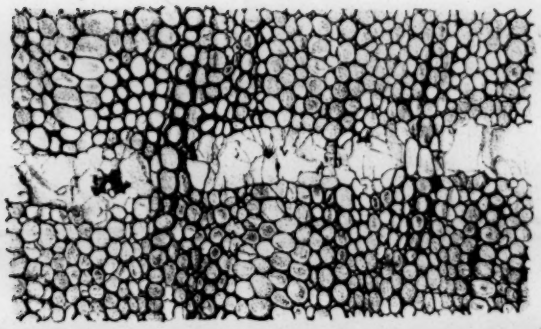


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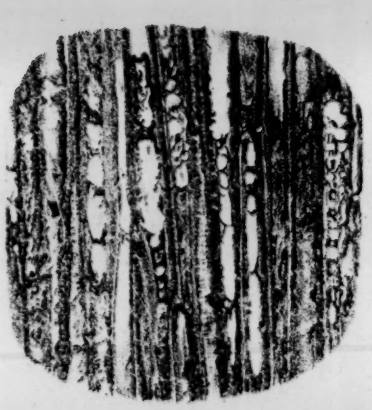
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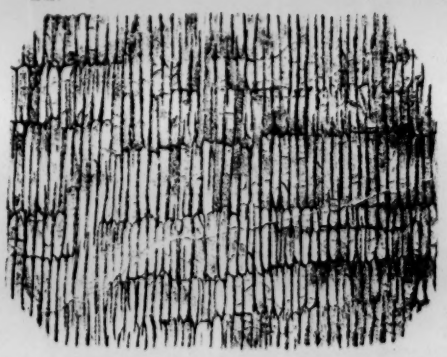
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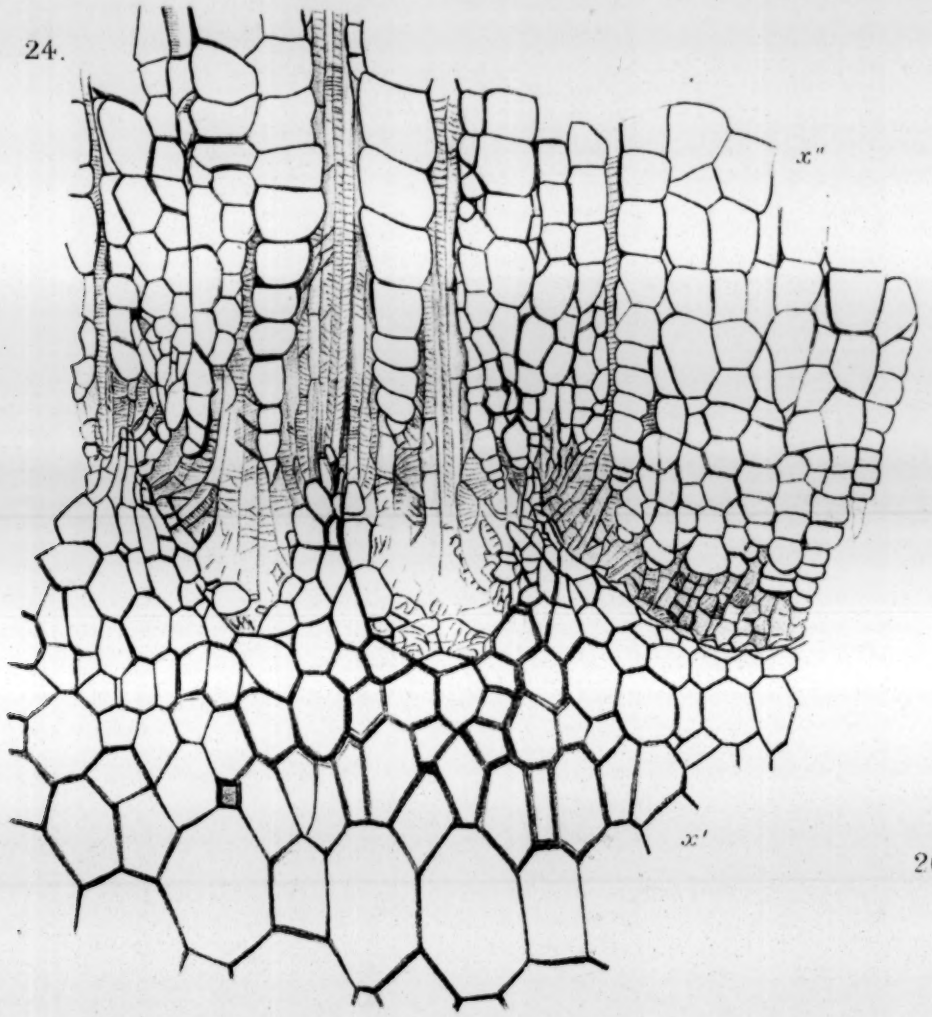
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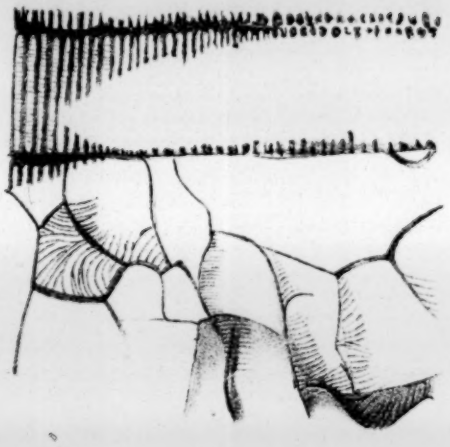
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Seward & Hill: Structure and Affinities of a Lepidodendroid Stem from the Calciferous Sandstone of Dalmeny — Plate IV.

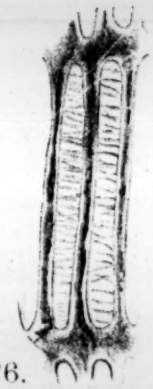
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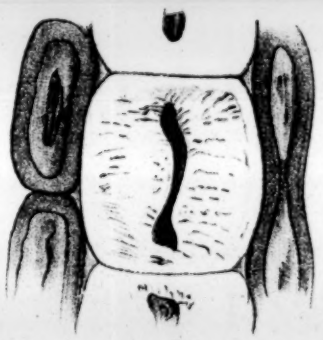
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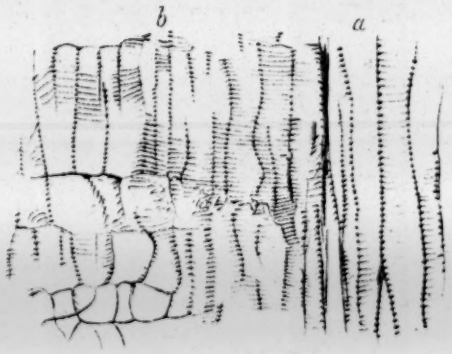


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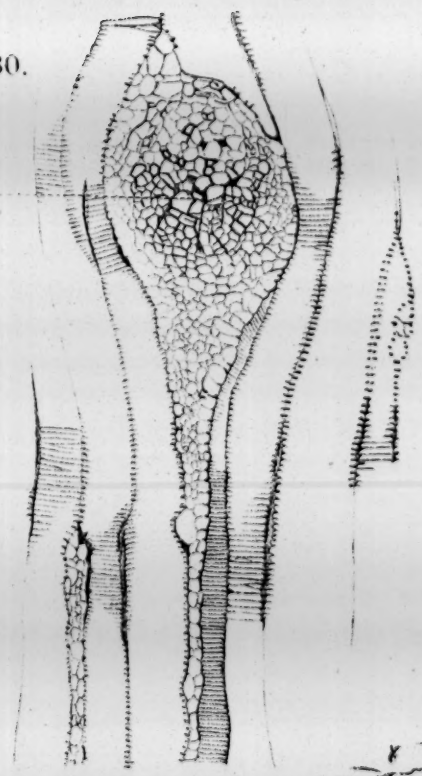
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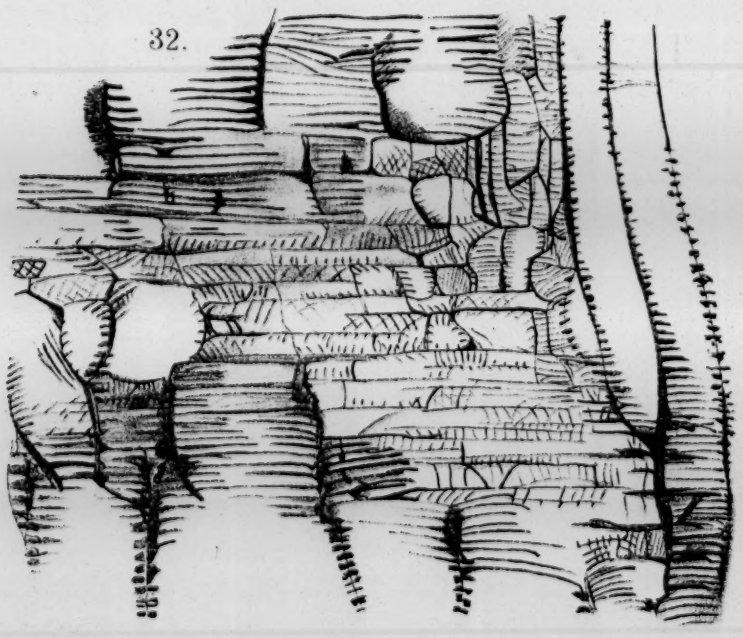
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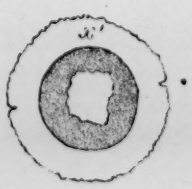


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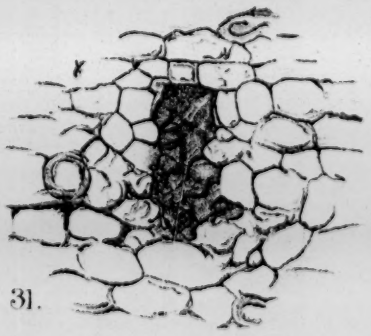


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APPENDIX.

TRANSACTIONS

OF THE

ROYAL SOCIETY OF EDINBURGH.

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ROYAL SOCIETY OF EDINBURGH.

LIST OF MEMBERS.

COUNCIL,

ALPHABETICAL LIST OF ORDINARY FELLOWS,

AND LIST OF HONORARY FELLOWS,

At November 1899.

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OF

THE ROYAL SOCIETY OF EDINBURGH.

NOVEMBER 1899.

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OF

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CORRECTED TO NOVEMBER 1899.

N.B.—*Those marked * are Annual Contributors.*

B. prefixed to a name indicates that the Fellow has received a Makdougall-Brisbane Medal.
K. ,, ,, ,, Keith Medal.
N. ,, ,, Neill Medal.
V. J. ,, ,, ,, the Gunning Victoria Jubilee Prize.
C. ,, ,, contributed one or more Communications to the Society's TRANSACTIONS or PROCEEDINGS.

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1871		Agnew, Sir Stair, K.C.B., M.A., Registrar-General for Scotland, 22 Buckingham Terrace
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1872		Barclay, George, M.A., 17 Coates Crescent
1883	C.	* Barclay, G. W. W., M.A., 91 Union Street, Aberdeen
1887		Barlow, W. H., Memb. Inst. C.E., F.R.S., High Combe, Old Charlton, Kent
1882	C.	Barnes, Henry, M.D., LL.D., 6 Portland Square, Carlisle
1893		Barnes, R. S. Fancourt, M.D., M.R.C.P.L., Consulting Physician to the British Lying-in Hospital, Woldhurstlea, Crawley, Sussex
1874		Barrett, William F., F.R.S., M.R.I.A., Professor of Physics, Royal College of Science, Dublin
1889		Barry, T. D. Collis, Staff Surgeon, M.R.C.S., F.L.S., Chemical Analyser to the Government of Bombay, and Prof. of Chemistry and Medical Jurisprudence to the Grant Medical College, and of Chemistry, Elphinstone College, Malabar Hill, Bombay
1887		* Bartholomew, J. G., F.R.G.S., The Geographical Institute, Dalkeith Road
1895	C.	Barton, Edwin H., D.Sc., A.M.I.E.E., Memb. Phys. Soc. of London, Senior Lecturer in Physics, University College, Nottingham
1888		* Beare, Thomas Hudson, B.Sc., Memb. Inst. C.E., Professor of Engineering and Mechanical Technology in University College, Gower Street, London
1897	C.	* Beattie, John Carruthers, D.Sc., Professor of Physics, South African College, Cape Town
1892		Beck, J. H. Meining, M.D., M.R.C.P.E., Rondebosch, Cape Town
1893	B. C.	* Becker, Ludwig, Ph.D., Regius Professor of Astronomy in the University of Glasgow, The Observatory, Glasgow
1882	C.	Beddard, Frank E., M.A. Oxon., F.R.S., Prosector to the Zoological Society of London, Zoological Society's Gardens, Regent's Park, London
1887		* Begg, Ferdinand Faithful, M.P. for the St Rollox Division of Glasgow, 13 Earl's Court Square, London, S.W.
1886		* Bell, A. Beatson, Advocate, 2 Eglinton Crescent
1874		Bell, Joseph, M.D., F.R.C.S.E., 2 Melville Crescent
1887		* Bernard, J. Mackay, B.Sc., 25 Chester Street
1875		Bernstein, Ludwik, M.D., Lismore, New South Wales
1893	C.	* Berry, George A., M.D., C.M., F.R.C.S., 31 Drumsheugh Gardens
1897	C.	* Berry, Richard J., M.D., F.R.C.S.E., 4 Howard Place
1881		* Berry, Walter, of Glenstriven, K.D., Danish Consul-General, 11 Atholl Crescent

Date of Election.		
1880	C.	* Birch, De Burgh, M.D., Professor of Physiology, Yorkshire College, Victoria University, 16 De Grey Terrace, Leeds
1884		* Black, John S., M.A., LL.D., 3 Down Street, Piccadilly, London
1850		Blackburn, Hugh, M.A., LL.D., Emeritus Professor of Mathematics in the University of Glasgow, Roshven, Ardgour
1897		* Blaikie, Walter Biggar, 6 Belgrave Crescent
1898		* Blyth, Benjamin Hall, M.A., M. Inst. C.E., 17 Palmerston Place
1878	C.	* Blyth, James, M.A., Professor of Natural Philosophy in Anderson's College, Glasgow
1894		* Bolton, Herbert, Curator of the Bristol Museum, Queen's Road, Bristol
1884		Bond, Francis T., B.A., M.D., M.R.C.S., Gloucester
1872	C.	Bottomley, J. Thomson, M.A., D.Sc., F.R.S., F.C.S., Lecturer on Natural Philosophy in the University of Glasgow, 13 University Gardens, Glasgow
1869	C.	Bow, Robert Henry, C.E., 7 South Gray Street
1886		* Bower, Frederick O., M.A., D.Sc., F.R.S., F.L.S., Regius Professor of Botany in the University of Glasgow, 45 Kerrsland Terrace, Hillhead, Glasgow
1884	C.	Bowman, Frederick Hungerford, D.Sc., F.C.S. (Lond. and Berl.), F.I.C., Assoc. Inst. C.E., Assoc. Inst. M.E., M.I.E.E., &c., Mayfield, Knutsford, Cheshire
1871		Boyd, Sir Thomas J., 41 Moray Place
1873		Boyd, William, M.A., 56 Palmerston Place
1886		* Bramwell, Byrom, M.D., F.R.C.P.E., 23 Drumsheugh Gardens
1895		* Bright, Charles, Assoc. M. Inst. C.E., M.I.E.E., F.R.A.S., F.G.S., 53 West Cromwell Road, London
1886		Brittle, John Richard, Memb. Inst. C.E., Farad Villa, Vanbrugh Hill, Blackheath, Kent
1877		Broadrick, George, Memb. Inst. C.E., Broughton House, Broughton Road, Ipswich
1893		Brock, G. Sandison, M.D., C.M., 47 Piazza Barberini, Rome, Italy
1892		* Brock, W. J., M.B., D.Sc., 5 Manor Place
1887		* Brown, A. B., C.E., 19 Douglas Crescent
1864	K. B. C.	Brown, Alex. Crum, M.D., D.Sc., F.R.C.P.E., LL.D., F.R.S. (SECRETARY), Professor of Chemistry in the University of Edinburgh, 8 Belgrave Crescent
1898		* Brown, David, F.C.S., F.I.C., Willowbrae House, Midlothian
1883	C.	* Brown, J. Graham, M.D., C.M., F.R.C.P.E., 3 Chester Street
1885	C.	Brown, J. Macdonald, M.B., F.R.C.S.E., 5 Lymington Road, West Hampstead, London, N.W.
1883	C.	* Bruce, Alexander, M.A., M.D., F.R.C.P.E., 13 Alva Street
1867		Bryce, A. Hamilton, LL.D., D.C.L., 12 Royal Circus
1898	C.	* Bryce, T. H., M.A., M.B. (Edin.), 2 Buckingham Terrace, Glasgow
1888		* Bryson, William A., Electrical Engineer, Engineer's Department, Town Hall, Leith
1869	B. C. V. J.	Buchan, Alexander, M.A., LL.D., F.R.S., Secretary to the Scottish Meteorological Society (CURATOR OF LIBRARY AND MUSEUM), 42 Heriot Row
1870	K. C.	Buchanan, John Young, M.A., F.R.S., 10 Moray Place, Edinburgh
1882		* Buchanan, T. R., M.A., M.P. for East Aberdeenshire, 10 Moray Place, Edinburgh, and 12 South Street, Park Lane, London, W.
1887	C.	* Buist, J. B., M.D., F.R.C.P.E., 1 Clifton Terrace
1894	C.	* Burgess, James, C.I.E., LL.D., M.R.A.S., M. Soc. Asiatique de Paris, *H.*A.R.I.B.A., 22 Seton Place
1887		* Burnet, John James, Architect, 18 University Avenue, Hillhead, Glasgow
1888		* Burns, Rev. T., F.S.A. Scot., Minister of Lady Glenorchy's Parish Church, Croston Lodge, Chalmers Crescent

Date of Election.		
1883		* Butcher, S. H., M.A., LL.D., Litt.D. Dub., Professor of Greek in the University of Edinburgh, 27 Palmerston Place 90
1896		* Butters, J. W., M.A., B.Sc., Secretary to the Edin. Mathematical Society, Rector of Ardrossan Academy
1887	C.	* Cadell, Henry Moubray, of Grange, B.Sc., Bo'ness
1897		* Caird, Robert, LL.D., Shipbuilder, Greenock
1879		* Calderwood, John, F.I.C., Belmont Works, Battersea, and Gowanlea, Spencer Park, Wandsworth, London, S.W.
1893	C.	Calderwood, W. L., Inspector of Salmon Fisheries of Scotland, 7 East Castle Road, Merchiston 95
1894		* Cameron, James Angus, M.D., Medical Officer of Health, Firhall, Nairn
1878		Campbell, John Archibald, M.D., Garland's Asylum, Carlisle
1898		* Campbell, Richard Vary, M.A., LL.B., Advocate, 37 Moray Place
1899	C.	* Carlier, Edmund W. W., M.D., B.Sc., Professor of Physiology in Mason College, Birmingham
1898		* Carter, Wm. Allan, M. Inst. C.E., 32 Great King Street 100
1898		Carus-Wilson, Cecil, F.R.G.S., F.G.S., Royal Societies Club, St James Street, London
1882		* Cay, W. Dyce, Memb. Inst. C.E., 107A Princes Street
1890		Charles, John J., M.A., M.D., C.M., Prof. of Anatomy and Physiology, Queen's College, Cork
1899		* Chatham, James, Actuary, Inverleith Park House
1874		Chiene, John, M.D., F.R.C.S.E., Professor of Surgery in the University of Edinburgh, President of the Royal College of Surgeons, 26 Charlotte Square 105
1875	C.	Christie, John, 19 Buckingham Terrace
1872		Christie, Thomas B., M.D., F.R.C.P.E., Royal India Asylum, Ealing, London
1880	K. C.	* Chrystal, George, M.A., LL.D., Professor of Mathematics in the University of Edinburgh (VICE-PRESIDENT), 5 Belgrave Crescent
1891		* Clark, John B., M.A., Mathematical and Physical Master in Heriot's Hospital School, 110 Craiglea Drive
1886		* Clark, Sir Thomas, Bart., 11 Melville Crescent 110
1875		Clouston, T. S., M.D., F.R.C.P.E., Tipperlinn House, Morningside
1892		* Coates, Henry, Pitcullen House, Perth
1887		* Cockburn, John, F.R.A.S., The Abbey, North Berwick
1888	C.	Collie, John Norman, Ph.D., F.R.S., F.C.S., Professor of Chemistry to the Pharmaceutical Society of Great Britain, 17 Bloomsbury Square, London
1886		Connan, Daniel M., M.A., Education Department, Cape of Good Hope 115
1872		Constable, Archibald, LL.D., 11 Thistle Street
1894		Cook, John, M.A., Principal of the Central College, Bangalore, India
1891		* Cooper, Charles A., 41 Drumsheugh Gardens
1890	C.	* Copeland, Ralph, Ph.D., F.R.A.S., Astronomer-Royal for Scotland, and Professor of Practical Astronomy in the University of Edinburgh (VICE-PRESIDENT), Royal Observatory, Blackford Hill, Edinburgh
1875		Craig, William, M.D., F.R.C.S.E., Lecturer on Materia Medica to the College of Surgeons, 71 Bruntsfield Place 120
1898		* Crawford, Francis Chalmers, 19 Royal Terrace
1887		* Crawford, William Caldwell, Lockharton Gardens, Slateford, Edinburgh
1870		Crichton-Browne, Sir Jas., M.D., LL.D., F.R.S., Lord Chancellor's Visitor and Vice-President of the Royal Institution of Great Britain, 61 Carlisle Place Mansions, Victoria Street, and Royal Courts of Justice, Strand, London

Date of Election.		
1886		* Croom, John Halliday, M.D., F.R.C.P.E., 25 Charlotte Square
1898		* Cullen, Alexander, F.S.A. Scot., Millburn House, by Hamilton
1878		* Cunningham, Daniel John, M.D., D.C.L., F.R.S., F.Z.S., Professor of Anatomy in Trinity College, Dublin, 43 Fitzwilliam Place, Dublin
1871		Cunynghame, R. J. Blair, M.D., 18 Rothesay Place
1898		* Currie, James, junior, M.A., Cantab., Larkfield, Golden Acre
1885		* Daniell, Alfred, M.A., LL.B., D.Sc., Advocate, 3 Great King Street
1897		* Davidson, Hugh, of Braedale, Lanark
1884		Davy, R., F.R.C.S., Surgeon to Westminster Hospital, Burstone House, Bow, North Devon
1894		* Denny, Archibald, Braehead, Dumbarton
1895		* Deuchar, David, F.I.A., F.F.A., Actuary, 12 Hope Terrace
1869	C.	Dewar, James, M.A., LL.D., F.R.S., F.C.S., Jacksonian Professor of Natural and Experimental Philosophy in the University of Cambridge, and Fullerian Professor of Chemistry at the Royal Institution of Great Britain, London
1884		* Dickson, Charles Scott, Q.C., Advocate, Solicitor-General for Scotland, 4 Heriot Row
1888	C.	* Dickson, H. N., B.Sc., 2 St Margaret's Road, Oxford
1876	C.	* Dickson, J. D. Hamilton, M.A., Fellow and Tutor, St Peter's College, Cambridge
1885	C.	Dixon, James Main, M.A., Professor of English Literature in the Washington University of St Louis, United States
1897		* Dobbie, James Bell, F.Z.S., 2 Hailes Street
1881	C.	* Dobbin, Leonard, Ph.D., Assistant to the Professor of Chemistry in the University of Edinburgh, 7 Cobden Road
1867	C.	Donaldson, J., M.A., LL.D., Principal of the University of St Andrews, St Andrews
1896		* Donaldson, William, M.A., Viewpark House, Bruntsfield Links
1882	C.	* Dott, D. B., Memb. Pharm. Soc., 29 Spring Gardens
1892		Doyle, Patrick, C.E., M.R.I.A., F.G.S., Editor of *Indian Engineering*, Calcutta
1866		Douglas, David, 22 Drummond Place
1876		* Duncan, James, 9 Mincing Lane, London
1889		* Duncan, James Dalrymple, F.S.A. Lond. and Scot., Meiklewood, Stirling
1878		* Duncanson, J. J. Kirk, M.D., F.R.C.P.E., 22 Drumsheugh Gardens
1859		Duns, Rev. Professor, D.D. (Vice-President), New College, Edinburgh, 5 Greenhill Place
1892	C.	Dunstan, M. J. R., B.A., F.C.S., Director of Technical Education in Agriculture, Newcastle Circus, The Park, Nottingham
1888		* Durham, James, F.G.S., Wingate Place, Newport, Fife
1899		* Duthie, George, M.A., Mathematical Master in the Edinburgh Academy, 6 Royal Crescent
1893		Edington, Alexander, M.B., C.M., Colonial Bacteriologist, Graham's Town, South Africa
1885		Elgar, Francis, Memb. Inst. C.E., LL.D., F.R.S., 18 York Ter., Regent's Park, London
1875		Elliot, Daniel G., Curator of Department of Zoology, Field Columbian Museum, Chicago, U.S.
1897	C.	* Erskine-Murray, James Robert, D.Sc., c/o Wireless Telegraph Co., 28 Mark Lane London, E.C.
1855		Etheridge, Robert, F.R.S., 14 Carlyle Square, Chelsea, London
1884		* Evans, William, F.F.A., 38 Morningside Park
1863	C.	Everett, J. D., M.A., D.C.L., F.R.S., Emeritus Professor of Natural Philosophy, Queen's College, Belfast, 11 Leopold Road, Ealing, London
1879	N. C.	* Ewart, James Cossar, M.D., F.R.C.S.E., F.R.S., F.L.S., Professor of Natural History, University of Edinburgh

Date of Election.		
1878	C.	* Ewing, James Alfred, M.A., B.Sc., Memb. Inst. C.E., F.R.S., Professor of Mechanism and Applied Mechanics in the University of Cambridge, Langdale Lodge, Cambridge
1875		Fairley, Thomas, Lecturer on Chemistry, 8 Newton Grove, Leeds
1888	C.	* Fawsitt, Charles A., 9 Foremount Terrace, Dowanhill, Glasgow
1859		Fayrer, Sir Joseph, Bart., K.C.S.I., M.D., F.R.C.P.L., F.R.C.S. L. and E., LL.D., F.R.S., Honorary Physician to the Queen, 16 Devonshire St., Portland Pl., London, W.
1883	C.	* Felkin, Robert W., M.D., F.R.G.S., Fellow of the Anthropological Society of Berlin 6 Crouch Hall Road, Crouch End, London 165
1899		* Fergus, Andrew Freeland, M.D., 22 Blythswood Square, Glasgow
1888		* Ferguson, John, M.A., LL.D., Professor of Chemistry in the University of Glasgow
1868	C.	Ferguson, Robert M., Ph.D., LL.D., 5 Douglas Gardens
1886		Field, C. Leopold, F.C.S., Upper Marsh, Lambeth, London
1898		* Findlay, John, M.A. Oxon., 3 Rothesay Terrace 170
1899		* Finlay, David W., B.A., M.D., F.R.C.P., D.P.H., Professor of Medicine in the University of Aberdeen, 2 Queen's Terrace, Aberdeen
1852		Fleming, Andrew, M.D., Deputy Surgeon-General, 8 Napier Road
1880		* Flint, Robert, D.D., Corresponding Member of the Institute of France, Corresponding Member of the Royal Academy of Sciences of Palermo, Professor of Divinity in the University of Edinburgh, Johnstone Lodge, 54 Craigmillar Park
1872	C.	Forbes, Professor George, M.A., Memb. Inst. C.E., Memb. Inst. E.E., F.R.S., F.R.A.S., 34 Great George Street, Westminster
1892		* Ford, John Simpson, F.C.S., 4 Nile Grove 175
1859		Forlong, Major-Gen. J. G., Assoc. C.E., F.R.G.S., R.A.S., &c., 11 Douglas Crescent
1828		Foster, John, Liverpool
1858		Fraser, A. Campbell, M.A., LL.D., D.C.L., Emeritus Professor of Logic and Metaphysics in the University of Edinburgh, Gorton House, Hawthornden
1896		* Fraser, John, M.B., F.R.C.P.E., one of H.M. Commissioners in Lunacy for Scotland, 19 Strathearn Road
1892		* Fraser, Patrick Neill, Rockville, Murrayfield 180
1867	K. B. C.	Fraser, Thomas R., M.D., LL.D., F.R.S., Professor of Materia Medica in the University of Edinburgh, President of the Royal College of Physicians, 13 Drumsheugh Gardens
1891		* Fullarton, J. H., M.A., D.Sc., Clyde Mussel Beds, Greenock
1891		* Fulton, T. Wemyss, M.D., Scientific Superintendent, Scottish Fishery Board, 417 Great Western Road, Aberdeen
1892		* Fyfe, Peter, Chief Sanitary Inspector, Glasgow
1888	C.	* Galt, Alexander, D.Sc., F.C.S., Physical Laboratory, The University, Glasgow 185
1899		Gatehouse, T. E., Assoc. Memb. Inst. C.E., M. Inst. M.E., M. Inst. E.E., 60 Gresham Road, Brixton, London
1894		Gatty, Charles Henry, M.A., LL.D., F.L.S., Felbridge Place, East Grinstead
1867		Gayner, Charles, M.D., Oxford
1889		* Geddes, George H., Mining Engineer, 8 Douglas Crescent
1880	C.	* Geddes, Patrick, Professor of Botany in University College, Dundee, and Lecturer on Zoology, Ramsay Garden, University Hall, Edinburgh 190
1861	B. C.	Geikie, Sir Archibald, LL.D., D.Sc. Dub., F.R.S., F.G.S., Corresponding Member of the Institute of France, Corresponding Member of the Royal Academy of Berlin, Director of the Geological Surveys of Great Britain, and Head of the Geological Museum, 28 Jermyn Street, London

Date of Election.		
1871	B. C.	Geikie, James, LL.D., D.C.L., F.R.S., F.G.S., Professor of Geology in the University of Edinburgh, Kilmorie, Colinton Road
1881		* Gibson, George Alexander, D.Sc., M.D., F.R.C.P.E., 17 Alva Street
1890		* Gibson, George A., M.A., Professor of Mathematics in the Glasgow and West of Scotland Technical College, 183 Renfrew Street, Glasgow
1877	C.	* Gibson, John, Ph.D., Prof. of Chemistry in the Heriot-Watt College, 20 George Square 195
1892		Gifford, Herbert James, Assoc. M. Inst. C.E., Longnor Hall, Leebotwood, Salop
1897	C.	* Gillespie, A. Lockhart, M.D., F.R.C.P. Ed., 23 Walker Street
1887		* Gilmour, William, 9 Inverleith Row
1880		* Gilruth, George Ritchie, Surgeon, 48 Northumberland Street
1898		* Glaister, John, M.D., F.F.P.S. Glasgow, D.P.H. Camb., Professor of Forensic Medicine in the University of Glasgow, 18 Woodside Place, W., Glasgow 200
1899		* Goodwin, Thomas S., F.C.S., Professor of Chemistry, Veterinary College, Glasgow
1897		Gordon-Munn, John Gordon, M.D., The Hall, Bushey, Herts
1850		Gosset, Major-General W. D., R.E., 70 Edith Road, West Kensington, London
1891		* Graham, Richard D., 11 Strathearn Road 205
1898	C.	* Gray, Albert A., M.D., 16 Berkeley Terrace, Glasgow
1883		* Gray, Andrew, M.A., LL.D., F.R.S., Regius Professor of Natural Philosophy in the University of Glasgow
1880	C.	Gray, Thomas, B.Sc., Professor of Physics, Rose Polytechnic Institute, Terre Haute, Indiana, U.S.
1886		* Greenfield, W. S., M.D., F.R.C.P.E., Professor of General Pathology in the University of Edinburgh, 7 Heriot Row
1897		Greenlees, Thomas Duncan, M.B. Edin., The Residency, Grahamstown, South Africa 210
1886	C.	* Griffiths, Arthur Bower, Ph.D., Lecturer at the National Dental Hospital and College, London, 12 Knowle Road, Brixton, London
1899		* Guest, Edward Graham, M.A., B.Sc., 5 Church Hill
1883		Gunning, His Excellency Robert Halliday, Grand Dignitary of the Order of the Rose of Brazil, M.A., M.D., LL.D., 12 Addison Crescent, Kensington
1888	C.	Guppy, Henry Brougham, M.B.
1867		Hallen, James H. B., C.I.E., F.R.C.S.E., Veterinary Lieut.-Colonel in H.M. Indian Army, Retired, Pebworth Fields, under Stratford-on-Avon 215
1899		Hamilton, Allan M'Lane, M.D., 44 East Twenty-ninth Street, New York
1881	C.	* Hamilton, D. J., M.B., F.R.C.S.E., Professor of Pathological Anatomy in the University of Aberdeen, 41 Queen's Road, Aberdeen
1876	C.	Hannay, J. Ballantyne, Cove Castle, Loch Long
1896		* Harris, David, Fellow of the Statistical Society, West Grange, Grange Loan, Edinburgh
1896	C.	* Harris, David Fraser, B.Sc. (Lond.), M.D., F.S.A. Scot., Lecturer on Physiology in the University of St Andrews 220
1888		* Hart, D. Berry, M.D., F.R.C.P.E., 29 Charlotte Square
1869		Hartley, Sir Charles A., K.C.M.G., Memb. Inst. C.E., 26 Pall Mall, London
1877	C.	Hartley, W. N., F.R.S., F.I.C., Prof. of Chemistry, Royal College of Science for Ireland, Dublin
1881		* Harvie-Brown, J. A., of Quarter, Dunipace House, Larbert, Stirlingshire
1880	C.	* Haycraft, J. Berry, M.D., D.Sc., Professor of Physiology in the University College of South Wales and Monmouthshire, Cardiff 225
1892	C.	* Heath, Thomas, B.A., Assistant Astronomer, Royal Observatory, Edinburgh

Date of Election.		
1862		Hector, Sir J., K.C.M.G., M.D., F.R.S., Director of the Geological Survey, Colonial Laboratory, Meteorological and Weather Departments, and of the New Zealand Institute, Wellington, New Zealand
1893		Hehir, Patrick, M.D., F.R.C.S.E., M.R.C.S.L., L.R.C.P.E., Surgeon-Captain, Indian Medical Service, Principal Medical Officer, H.H. the Nizam's Army, Hyderabad, Deccan, India
1890	C.	Helme, T. A., M.D., 258 Oxford Road, Manchester
1884		* Henderson, John, Meadowside Works, Partick, Glasgow 230
1890	C.	* Hepburn, David, M.D., Lecturer on Regional Anatomy in the University of Edinburgh
1896	C.	* Herbertson, Andrew J., Ph.D., Lecturer on Commercial Geography in the Heriot-Watt College, The Loan, Colinton
1881	N. C.	* Herdman, W. A., D.Sc., F.R.S., F.L.S., Prof. of Natural History in University College, Liverpool
1894		Hill, Alfred, M.D., M.R.C.S., F.I.C., Medical Officer of Health, The Council House, Birmingham
1859		Hills, John, Major-General, C.B., Bombay Engineers, United Service Club, London, and Love's Grove, Aberystwith, Wales 235
1879		Hislop, John, LL.D., formerly Secretary to the Department of Education, Forth Street, Dunedin, New Zealand
1885		Hodgkinson, W. R., Ph.D., F.I.C., F.C.S., Prof. of Chemistry and Physics at the Royal Military Acad. and Royal Artillery Coll., Woolwich, 18 Glenluce Road, Blackheath, Kent
1881	N. C.	* Horne, John, F.G.S., Geological Survey of Scotland, Sheriff-Court Buildings, Edinburgh
1896		Horne, J. Fletcher, M.D., F.R.C.S.E., The Poplars, Barnsley
1897		Houston, Alex. Cruikshanks, M.B., C.M., D.Sc., 19 Culworth St., Regent's Park, London 240
1893		Howden, Robert, M.A., M.B., C.M., Professor of Anatomy in the University of Durham, —address: 24 Burdon Terrace, Newcastle-on-Tyne
1899		Howie, W. Lamond, F.C.S., Hanover Lodge, West Hill, Harrow
1883	C.	* Hoyle, William Evans, M.A., M.R.C.S., 25 Brunswick Road, Withington, Manchester
1886		Hunt, Rev. H. G. Bonavia, Mus. D. Dub., Mus. B. Oxon., F.L.S., La Belle Sauvage, London
1872		Hunter, Colonel Charles, of Pläs Cöch, Llanfairpwll, Anglesea, and Junior United Service Club, London 245
1887	C.	* Hunter, James, F.R.C.S.E., F.R.A.S., Rosetta, Liberton, Midlothian
1887	C.	* Hunter, William, M.D., M.R.C.P. L. and E., M.R.C.S., 54 Harley Street, London
1882	C.	* Inglis, J. W., Memb. Inst. C.E., Kenwood, Liberton, Midlothian
1898		* Irons, James Campbell, M.A., S.S.C., 10 Royal Terrace
1886	N. C.	* Irvine, Robert, F.C.S., Royston, Granton, Edinburgh 250
1899		Isherwood, Thomas, M.A., LL.D., D.C.L., 1 Cambridge Road, Southport
1875		Jack, William, M.A., LL.D., Professor of Mathematics in the University of Glasgow
1894		Jackson, Sir John, 10 Holland Park, London
1889	C.	* James, Alexander, M.D., F.R.C.P.E., 10 Melville Crescent
1882		* Jamieson, A., Memb. Inst. C.E., Professor of Engineering in The Glasgow and West of Scotland Technical College, Glasgow 255
1860	C.	Jamieson, George Auldjo, Actuary, 24 St Andrew Square
1880		Japp, A. H., LL.D., The Limes, Elmstead, near Colchester
1869		Johnston, J. Wilson, M.D., Surgeon Lt.-Col., Benmore, 30 Bidston Road, Oxton, Cheshire
1895		Johnston, Surgeon-Major Henry Halcro, R.A.M.S., D.Sc., M.D., F.L.S., Orphir House, Kirkwall, Orkney
1874		Jones, Francis, Lecturer on Chemistry, Beaufort House, Alexandra Park, Manchester 260

Date of Election.		
1888		Jones, John Alfred, Memb. Inst. C.E., Fellow of the University of Madras (F.V.M.), Sanitary Engineer to the Government of Madras, Fort St George, Madras
1896		* Jong, E. F. de, L.R.C.P.E., L.R.C.S.E., F.R.C.V.S.
1847	K. C. V. J.	Kelvin, The Right Hon. Lord, G.C.V., LL.D., D.C.L., F.R.S. (PRESIDENT), Grand Officer of the Legion of Honour of France, Member of the Prussian Order *Pour le Mérite*, Foreign Associate of the Institute of France, and Emeritus Professor of Natural Philosophy in the University of Glasgow, Netherhall, Largs
1892		* Kerr, Rev. John, M.A., Manse, Dirleton
1891		Kerr, Joshua Law, M.D., Biddenden Hall, Cranbrook, Kent 265
1886	N. C.	* Kidston, Robert, F.G.S., 12 Clarendon Place, Stirling
1877		* King, Sir James, of Campsie, Bart., LL.D., 115 Wellington Street, Glasgow
1880		* King, W. F., Lonend, Russell Place, Trinity
1883		* Kinnear, The Right Hon. Lord, one of the Senators of the College of Justice, 2 Moray Place
1878		* Kintore, The Right Hon. the Earl of, M.A. Cantab., Keith Hall, Inglismaldie Castle, Laurencekirk 270
1880	K. C.	* Knott, C. G., D.Sc., Lecturer on Applied Mathematics in the University of Edinburgh (late Prof. of Physics, Imperial University, Japan), 42 Upper Gray Street, Edinburgh
1896	C.	* Kuenen, J. P., Ph.D. (Leiden), Prof. of Natural Philosophy in University College, Dundee
1886		* Laing, Rev. George P., 17 Buckingham Terrace
1878	C.	* Lang, P. R. Scott, M.A., B.Sc., Professor of Mathematics in the University of St Andrews
1885	C.	* Laurie, A. P., B.A., B.Sc., Woodside, Baldwin's Hill, Loughton 275
1894	C.	* Laurie, Malcolm, B.A., D.Sc., F.L.S., Prof. of Zoology, St Mungo's College, Glasgow
1870		Laurie, Simon S., M.A., LL.D., Professor of Education in the University of Edinburgh, 22 George Square
1872		Lee, Alexander H., C.E., 58 Manor Place
1863		Leslie, Hon. G. Waldegrave, Leslie House, Leslie
1874	K. C.	Letts, E. A., Ph.D., F.I.C., F.C.S., Professor of Chemistry, Queen's College, Belfast 280
1899		Lewis, Joseph Slater, M. Inst. C.E., M. Inst. E.E., M. Inst. M.E., Norwood, Ellesmere Park, Eccles
1889		* Lindsay, Rev. James, D.D., B.Sc., F.G.S., Corresponding Member of the Royal Academy of Sciences, Letters and Arts, of Padua, Minister of St Andrew's Parish, Springhill Terrace, Kilmarnock
1870	B. C.	Lister, The Right Hon. Lord, M.D., F.R.C.S.L., F.R.C.S.E., LL.D., D.C.L., P.R.S., Foreign Associate of the Institute of France, Emeritus Professor of Clinical Surgery, King's College, Surgeon Extraordinary to the Queen, 12 Park Crescent, Portland Place, London
1897	C.	Lloyd, Richard John, M.A., D.Lit., 49A Grove Street, Liverpool
1898		* Lothian, Alexander Veitch, M.A., B.Sc., Baxter Demonstrator in Geology, University of Glasgow, 11 Holborn Terrace, Kelvinside, Glasgow 285
1884		* Low, George M., Actuary, 15 Chester Street
1888		* Lowe, D. F., M.A., LL.D., Headmaster of Heriot's Hospital School, Lauriston
1849		Lowe, W. H., M.D., F.R.C.P.E., Woodcote, Inner Park, Wimbledon
1894		* Mabbott, Walter John, M.A., Rector of County High School, Duns, Berwickshire
1855		Macadam, Stevenson, Ph.D., Lecturer on Chemistry, Surgeons' Hall, Edinburgh, 11 East Brighton Crescent, Portobello 290
1888		* Macadam, W. Ivison, F.I.C., F.C.S., Lecturer on Chemistry, Slioch, Lady Road, Newington, Edinburgh

Date of Election.		
1887		M'Aldowie, Alexander M., M.D., 6 Brook Street, Stoke-on-Trent
1891		Macallan, John, F.I.C., 3 Charlemont Terrace, Clontarf, Dublin
1888	C.	M'Arthur, John, F.C.S., 196 Trinity Road, Wandsworth Common, London
1885		* M'Bride, Charles, M.D., Wigtown
1883		* M'Bride, P., M.D., F.R.C.P.E., 16 Chester Street
1867		M'Candlish, John M., W.S., 27 Drumsheugh Gardens
1899		* M'Cubbin, James, B.A., Rector of the Burgh Academy, Kilsyth
1894		* Macdonald, James, Secretary of the Highland and Agricultural Society of Scotland, 9 Lauriston Gardens
1897	C.	* Macdonald, James A., M.A., B.Sc., 22 Summerside Place, Leith
1886		* Macdonald, The Rt. Hon. J. H. A., C.B., Q.C., LL.D., F.R.S., M.I.E.E., Lord Justice-Clerk, and Lord President of the Second Division of the Court of Session, 15 Abercromby Place
1886		* Macdonald, William J., M.A., Comiston Drive
1888	C.	* M'Fadyean, John, M.B., B.Sc., Professor of Pathology and Dean of the Royal Veterinary College, Camden Town, London
1878	C.	Macfarlane, Alexander, M.A., D.Sc., LL.D., Lecturer in Physics in Lehigh University, Pennsylvania
1885	C.	* Macfarlane, J. M., D.Sc., Professor of Biology in the University of Pennsylvania, Lansdowne, Delaware Co., Pennsylvania
1897		* M'Gillivray, Angus, C.M., M.D., South Tay Street, Dundee
1878		* M'Gowan, George, F.I.C., Ph.D., 3 Mount Avenue, Ealing, Middlesex
1886		* MacGregor, Rev. James, D.D., 3 Eton Terrace
1880	C.	MacGregor, J. G., M.A., D.Sc., Prof. of Physics in Dalhousie Coll., Halifax, Nova Scotia
1869	N.C.	M'Intosh, William Carmichael, M.D., LL.D., F.R.S., F.L.S., Professor of Natural History in the University of St Andrews, 2 Abbotsford Crescent, St Andrews
1895	C.	* Macintyre, John, M.D., 179 Bath Street, Glasgow
1882		* Mackay, John Sturgeon, M.A., LL.D., Mathematical Master in the Edinburgh Academy, 69 Northumberland Street
1873	B. C.	M'Kendrick, John G., M.D., F.R.C.P.E., LL.D., F.R.S. (Vice-President), Professor of Physiology in the University of Glasgow
1840		Mackenzie, John, 6 Manor Place
1894		* Mackenzie, Robert, M.D., Napier Villa, 2 Morningside Road
1898		Mackenzie, W. Cossar, D.Sc., Principal of the College of Agriculture, Gheezeh, Egypt
1843	C.	Maclagan, Sir Douglas, M.D., F.R.C.P.E., LL.D., (Honorary Vice-President), Emeritus Professor of Medical Jurisprudence in the University of Edinburgh, 28 Heriot Row
1894		* Maclagan, Philip R. D., F.F.A. (Treasurer), St Catherine's, Liberton
1869		Maclagan, R. C., M.D., F.R.C.P.E., 5 Coates Crescent
1864		M'Lagan, Peter, of Pumpherston
1869	C.	M'Laren, The Hon. Lord, LL.D. Edin. and Glasg., F.R.A.S., one of the Senators of the College of Justice, 46 Moray Place
1899		Maclean, Ewan John, M.D., M.R.C.P., London, 51 Linden Gardens, Bayswater, London
1888	C.	* Maclean, Magnus, M.A., D.Sc., Prof. of Electrical Engineering in the Glasgow and West of Scotland Technical College, 51 Kerrsland Terrace, Hillhead, Glasgow
1876		* Macleod, Rev. Norman, D.D., Westwood, Inverness
1896		* M'Lintock, James, M.D., B.Sc., 5 Atholl Crescent
1872		Macmillan, Rev. Hugh, D.D., LL.D., 70 Union Street, Greenock
1876		* Macmillan, John, M.A., D.Sc., M.B., C.M., F.R.C.P.E., 27 Warrender Park Road

Date of Election.		
1893		* M'Murtrie, The Rev. John, M.A., D.D., 5 Inverleith Place
1884		* Macpherson, Rev. J. Gordon, M.A., D.Sc., Mathematical Examiner in the University of St Andrews, Ruthven Manse, Meigle
1888		Mactear, James, F.C.S., 28 Victoria Street, Westminster, London 330
1890		* M'Vail, John C., M.D., 2 Strathallan Terrace, Dowanhill, Glasgow
1898		Mahalanobis, S. C., B.Sc., Demonstrator of Physiology, University College, Cardiff
1880	C.	Marsden, R. Sydney, M.B., C.M., D.Sc., F.I.C., F.C.S., 64 Park Road, South, and Town Hall, Birkenhead
1882	C.	Marshall, D. H., M.A., Professor of Physics in Queen's University and College, Kingston, Ontario, Canada
1888	C.	* Marshall, Hugh, D.Sc., Assistant to the Professor of Chemistry in the University of Edinburgh, 131 Warrender Park Road 335
1892		* Martin, Francis John, W.S., 17 Rothesay Place
1864		Marwick, Sir James David, LL.D., Town-Clerk, Glasgow
1866		Masson, David, LL.D., Litt. D. Dub., Emeritus-Professor of Rhetoric and English Literature in the Univ. of Edin., H.M. Historiographer for Scotland, Gowanlea, Juniper Green
1885	C.	* Masson, Orme, D.Sc., Professor of Chemistry in the University of Melbourne
1898	C.	* Masterman, Arthur Thomas, M.A., D.Sc., Lecturer on Zoology in the New Medical School, Edinburgh, 1 Mortonhall Road, Blackford Hill 340
1890		* Matheson, The Rev. George, M.A., B.D., D.D., Minister of St Bernard's, Edinburgh, 19 St Bernard's Crescent
1888		* Methven, C. W., Memb. Inst. C.E., Engineer-in-Chief to the Natal Harbour Board, Equitable Buildings, Durban, Natal
1885	B. C.	* Mill, Hugh Robert, D.Sc., LL.D., Librarian, Royal Geographical Society, 1 Saville Row, and 22 Gloucester Place, Portman Square, London
1886		* Milne, William, M.A., B.Sc., 57 Springbank Terrace, Aberdeen
1899		* Milroy, T. H., M.D., B.Sc., 57 Inverleith Row 345
1866		Mitchell, Sir Arthur, K.C.B., M.A., M.D., LL.D. (Vice-President), 34 Drummond Place
1889	C.	Mitchell, A. Crichton, D.Sc., Professor of Pure and Applied Mathematics, and Principal of the Maharajah's College, Trivandrum, Travancore, India
1897		* Mitchell, George Arthur, M.A., 2 Lilybank Gardens, Glasgow
1871		Moncrieff, Rev. Canon William Scott, of Fossaway, Easington Rectory, Castle Eden, County Durham
1890	C.	Mond, R. L., M.A. Cantab., F.C.S., The Poplars, 20 Avenue Road, Regent's Park, London 350
1887	C.	Moos, N. A. F., L.C.E., B.Sc., Professor of Physics, Elphinstone College, and Director of the Government Observatory, Colaba, Bombay
1896		* Morgan, Alexander, M.A., D.Sc., Pres. of the Edin. Mathematical Society, 6 Cluny Terrace
1892		Morrison, J. T., M.A., B.Sc., Professor of Physics and Chemistry, Victoria College, Stellenbosch, Cape Colony
1892	C.	* Mossman, Robert C., 10 Blacket Place
1874	K. C.	Muir, Thomas, M.A., LL.D., Superintendent-General of Education for Cape Colony, Education Office, and The Hall, Mowbray, Cape Town 355
1888		* Muirhead, George, Commissioner to His Grace the Duke of Richmond and Gordon, K.G., Speybank, Fochabers
1887		Mukhopâdhyay, Âsûtosh, M.A., LL.D., F.R.A.S., M.R.I.A., Professor of Mathematics at the Indian Association for the Cultivation of Science, 77 Russa Road North, Bhowanipore, Calcutta

Date of Election.		
1870		Munn, David, M.A., 12 Danube Street
1894		* Munro, J. M. M., M.I.E.E., 136 Bothwell Street, Glasgow
1889		* Munro, Rev. Robert, M.A., B.D., F.S.A. Scot., Free Church Manse, Old Kilpatrick 360
1891	C.	* Munro, Robert, M.A., M.D., Hon. Memb. R.I.A., Hon. Memb. Royal Soc. of Antiquaries of Ireland, Secretary of the Society of Antiquaries of Scotland, 48 Manor Place
1896		* Murray, Alfred A., M.A., LL.B., 20 Warriston Crescent
1892	C.	* Murray, George Robert Milne, F.R.S., F.L.S., Keeper of the Botanical Department, British Museum (Natural Hist.), Cromwell Road, London
1857		Murray, John Ivor, M.D., F.R.C.S.E., M.R.C.P.E., 24 Huntriss Row, Scarborough
1877	B. N.C.	* Murray, Sir John, K.C.B., LL.D., Ph.D., D.Sc., F.R.S., Member of the Prussian Order *Pour le Mérite* (SECRETARY), (Society's Representative on George Heriot's Trust), Director of the Challenger Expedition Publications. Office, 45 Frederick St. House, Challenger Lodge, Wardie, and United Service Club 365
1888		* Murray, R. Milne, M.A., M.B., F.R.C.P.E., 11 Chester Street
1887		Muter, John, M.A., F.C.S., South London Central Public Laboratory, 325 Kennington Road, London
1888		Napier, A. D. Leith, M.D., C.M., M.R.C.P.L., General Hospital, Adelaide, S. Australia
1895		* Napier, James, M.A., Drums, Old Kilpatrick
1877		* Napier, John, C. Audley Mansions, Grosvenor Square, London 370
1897		Nash, Alfred George, B.Sc., Greenvale House, Mile Gully P.O., Jamaica, W.I.
1887		* Nasmyth, T. Goodall, M.D., C.M., D.Sc., Cupar-Fife
1898		Newman, George, M.D., D.P.H. Cambridge, 2 Woburn Square, London
1884		* Nicholson, J. Shield, M.A., D.Sc., Professor of Political Economy in the University of Edinburgh, 3 Belford Park
1880	C.	* Nicol, W. W. J., M.A., D.Sc., 15 Blacket Place 375
1878		Norris, Richard, M.D., M.R.C.S. Eng., 3 Walsall Road, Birchfield, Birmingham
1888		* Ogilvie, F. Grant, M.A., B.Sc., Principal of the Heriot-Watt College
1888		* Oliphant, James, M.A., 11 Ramsay Gardens
1886	C.	Oliver, James, M.D., F.L.S., Physician to the London Hospital for Women, 18 Gordon Square, London
1895		Oliver, Thomas, M.D., F.R.C.P., Professor of Physiology in the University of Durham, 7 Ellison Place, Newcastle-upon-Tyne 380
1884	K. C.	* Omond, R. Traill, Hon. Superintendent of Ben Nevis Observatory, Fort-William, 1 Newbattle Terrace, Edinburgh
1877		Panton, George A., 73 Westfield Road, Edgbaston, Birmingham
1892		Parker, Thomas, Memb. Inst. C.E., Manor House, Tettenhall, Wolverhampton
1886	C.	* Paton, D. Noël, M.D., B.Sc., F.R.C.P.E., 22 Lyndoch Place
1889		* Patrick, David, M.A., LL.D., c/o W. & R. Chambers, 339 High Street 385
1892		* Paulin, David, Actuary, 6 Forres Street
1881	N.C.	* Peach, B. N., F.R.S., F.G.S., Acting Palæontologist of the Geological Survey of Scotland, 86 Findhorn Place
1889		* Peck, William, F.R.A.S., Town's Astronomer, City Observatory, Calton Hill, Edinburgh
1863		Peddie, Alexander, M.D., F.R.C.P.E., 15 Rutland Street
1887	B.C.	* Peddie, Wm., D.Sc., Assistant to the Professor of Natural Philosophy, Edinburgh University, 14 Ramsay Gardens 390
1886	C.	* Peebles, D. Bruce, Tay House, Bonnington, Edinburgh
1893		Perkin, Arthur George, 8 Montpellier Terrace, Hyde Park, Leeds

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1889		* Philip, R. W., M.A., M.D., F.R.C.P.E., 45 Charlotte Square
1883		Phillips, Charles D. F., M.D., LL.D., 10 Henrietta St., Cavendish Sq., London, W.
1877	C.	Pole, William, Hon. Memb. Inst. C.E., Mus. Doc., F.R.S., Athenæum Club, London
1886		* Pollock, Charles Frederick, M.D., F.R.C.S.E., 1 Buckingham Terrace, Hillhead, Glasgow
1874		Powell, Baden Henry Baden-, C.I.E., M.R.A.S., Ferlys Lodge, 29 Bunbury Road, Oxford
1852		Powell, Eyre B., C.S.I., M.A., 28 Park Road, Haverstock Hill, Hampstead, London
1888		Prain, David, Surgeon-Major, Indian Medical Service, and Superintendent, Royal Botanic Gardens, Shibpur, Calcutta
1892		* Pressland, Arthur, M.A., Camb., Edinburgh Academy
1875	C.	Prevost, E. W., Ph.D., Elton, Newnham, Gloucester
1885		Pullar, J. F., Rosebank, Perth
1880		* Pullar, Sir Robert, Tayside, Perth
1898		* Purves, John Archibald, D.Sc., 53 York Place
1897		* Rainy, Harry, M.B., C.M., F.R.C.P. Ed., 25 George Square
1899		* Ramage, Alexander G., 9 Derby Street, Trinity
1884		Ramsay, E. Peirson, M.R.I.A., F.L.S., C.M.Z.S., F.R.G.S., F.G.S., Fellow of the Imperial and Royal Zoological and Botanical Society of Vienna, Curator of Australian Museum, Sydney, N.S.W.
1891		* Rankine, John, M.A., LL.D., Advocate, Professor of the Law of Scotland in the University of Edinburgh, 23 Ainslie Place
1885	C.	* Rattray, John, M.A., B.Sc., Dunkeld
1883	C.	* Readman, J. B., D.Sc., F.C.S., 20 Moray Place
1889		Redwood, Boverton, F.I.C., F.C.S., Assoc. Inst. C.E., Glenwathen, Ballard's Lane, Finchley, Middlesex
1875		Richardson, Ralph, W.S., 10 Magdala Place
1872		Ricarde-Seaver, Major F. Ignacio, Athenæum Club, Pall Mall, London
1883		* Ritchie, R. Peel, M.D., F.R.C.P.E., President of the Scottish Microscopical Society, 1 Melville Crescent
1898		Roberts, Alexander Wm., D.Sc., F.R.A.S., Mathematical Lecturer, Lovedale, S. Africa
1880		Roberts, D. Lloyd, M.D., F.R.C.P.L., 23 St John Street, Manchester
1872		Robertson, D. M. C. L. Argyll, M.D., F.R.C.S.E., LL.D., Surgeon Oculist to the Queen for Scotland, 18 Charlotte Square
1886		* Robertson, The Right Hon. Lord, Q.C., LL.D., Lord of Appeal in Ordinary, 19 Drumsheugh Gardens
1896		* Robertson, Robert, M.A., 27 Hartington Place, Viewforth
1896	C.	* Robertson, W. G. Aitchison, D.Sc., M.D., F.R.C.P.E., 26 Minto Street
1877	C.	* Robinson, George Carr, F.I.C., F.C.S., Lecturer on Chemistry in the College of Chemistry, Royal Institution, Hull
1881		* Rogerson, John Johnston, B.A., LL.B., LL.D., 3 Abbotsford Park
1881		Rosebery, The Right Hon. the Earl of, K.G., K.T., LL.D., D.C.L., F.R.S., Dalmeny Park, Edinburgh
1880		Rowland, L. L., M.A., M.D., President of the Oregon State Medical Society, and Professor of Physiology and Microscopy in Williamette University, Salem, Oregon
1880		* Russell, Sir Jas. Alex., M.A., B.Sc., M.B., F.R.C.P.E., LL.D., Woodville, Canaan Lane
1897		* Sanderson, William, Talbot House, Ferry Road
1864		Sandford, The Right Rev. Bishop D. F., LL.D., Boldon Rectory, Newcastle-on-Tyne

Date of Election.		
1895		Savage, Thomas, M.D., F.R.C.S. England, M.R.C.P. London, Professor of Gynæcology, Mason College, Birmingham, The Ards, Knowle, Warwickshire
1891		Sawyer, Sir James, Knt., M.D., F.R.C.P., J.P., Consulting Physician to the Queen's Hospital, Haseley Hall, Warwick
1885	C.	Scott, Alexander, M.A., D.Sc., F.R.S., The Davy-Faraday Research Laboratory of the Royal Institution, London 430
1880		Scott, J. H., M.B., C.M., M.R.C.S., Prof. of Anatomy in the Univ. of Otago, New Zealand
1888		* Scott, John, C.B., Memb. Inst. C.E., Halkshill, Largs
1875		Scott, M., Memb. Inst. C.E., care of A. Grahame, Esq., 30 Gt. George St., Westminster
1872	C.	Seton, George, M.A., Advocate, Ayton House, Abernethy, Perthshire
1897		* Shepherd, John William, Carrickarden, Bearsden, Glasgow 435
1894		* Shield, Wm., M.Inst.C.E., Executive Engineer, National Harbour of Refuge, Peterhead
1872		Sibbald, Sir John, M.D., Commissioner in Lunacy (retired), 18 Great King Street
1870		Sime, James, M.A., Craigmount House, 52 Dick Place
1871		Simpson, A. R., M.D., Vice-President of the Royal College of Physicians, Professor of Midwifery in the University of Edinburgh, 52 Queen Street
1876		* Skinner, William, W.S., 35 George Square 440
1868		Smith, Adam Gillies, C.A., Agsacre, North Berwick
1891	C.	* Smith, Alex., B.Sc., Ph.D., Professor of General Chemistry, University of Chicago, Ills., United States
1882	C.	Smith, C. Michie, B.Sc., F.R.A.S., Professor of Physical Science, Christian College, and Officiating Government Astronomer, Madras Presidency, The Observatory, Kodaikánal, Palani Hills, South India
1885		* Smith, George, F.C.S., Polmont Station
1871	C.	Smith, John, M.D., F.R.C.S.E., LL.D., 11 Wemyss Place 445
1880		Smith, William Robert, M.D., D.Sc., Barrister-at-Law, Professor of Forensic Medicine in King's College, 74 Great Russell Street, Bloomsbury Square, London
1846	K. B. C.	Smyth, Piazzi, LL.D., Ex-Astronomer-Royal for Scotland, and Emeritus Professor of Astronomy in the University of Edinburgh, Clova, Ripon
1899		Snell, Ernest Hugh, M.D., B.Sc., D.P.H., Camb., Coventry
1880		Sollas, W. J., M.A., D.Sc., LL.D., F.R.S., late Fellow of St John's College, Cambridge, and Professor of Geology and Palæontology in the University of Oxford
1889	C.	Somerville, Wm., M.A., D.Sc., D.Oec., Prof. of Agriculture in the Univ. of Cambridge 450
1882		* Sorley, James, F.I.A., C.A., 32 Onslow Square, London
1896		* Spence, Frank, M.A., B.Sc., 25 Craiglea Drive
1874	C.	Sprague, T. B., M.A., LL.D., Actuary, 29 Buckingham Terrace
1891		* Stanfield, Richard, Professor of Mechanics and Engineering in the Heriot-Watt College
1886	C.	* Stevenson, Charles A., B.Sc., Memb. Inst. C.E., 28 Douglas Crescent 455
1884		* Stevenson, David Alan, B.Sc., Memb. Inst. C.E., 45 Melville Street
1877		* Stevenson, James, F.R.G.S., Largs
1888		* Stevenson, Rev. John, LL.D., Minister of Glamis, Forfarshire
1868		Stevenson, John J., 4 Porchester Gardens, London
1888	C.	* Stewart, Charles Hunter, D.Sc., M.B., C.M., Professor of Public Health in the University of Edinburgh, 9 Learmonth Gardens 460
1868		Stewart, Major-General J. H. M. Shaw, late R.E., Assoc. Inst. C.E., F.R.G.S., 7 Inverness Terrace, London, W

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1866	C.	Stewart, Sir Thomas Grainger, M.D. Edin. and Dub., F.R.C.P.E., LL.D., Professor of the Practice of Physic in the University of Edinburgh, 19 Charlotte Square
1873		Stewart, Walter, 3 Queensferry Gardens
1877		* Stirling, William, D.Sc., M.D., Brackenbury Professor of Physiology and Histology in Owens College and Victoria University, Manchester
1889	C.	* Stockman, Ralph, M.D., F.R.C.P.E., Professor of Materia Medica and Therapeutics in the University of Glasgow 465
1896		* Sutherland, John Francis, M.D., Dep. Com. in Lunacy for Scotland, 4 MerchistonBank Avenue
1885	C.	* Symington, Johnson, M.D., F.R.C.S.E., Prof. of Anatomy in Queen's College, Belfast
1872		Tait, The Very Reverend Andrew, D.D., LL.D., M.R.I.A., Dean of St. Mary's Cathedral, Tuam, Deanery, Moylough, Ballinasloe, County Galway, Ireland
1861	K. C. V. J.	Tait, P. Guthrie, M.A., D.Sc., Professor of Natural Philosophy in the University of Edinburgh (GENERAL SECRETARY), 38 George Square
1898		* Tait, William Archer, B.Sc., M. Inst. C.E., 38 George Square 470
1895		Talmage, James Edward, D.Sc., Ph.D., F.R.M.S., F.G.S., Professor of Geology, Univ. of Utah, Salt Lake City, Utah
1890	C.	Tanakadate, Aikitu, Prof. of Nat. Phil. in the Imperial University of Japan, Tokyo, Japan
1870		Tatlock, Robert R., F.C.S., City Analyst's Office, 156 Bath Street, Glasgow
1899		* Taylor, James, M.A., Mathematical Master in the Edinburgh Academy, 3 Melgund Terrace
1892		* Taylor, W. A., M.A. (Camb.), 3 East Mayfield 475
1872		Teape, Rev. Charles R., M.A., Ph.D., Rector of St Andrews Episcopal Church, 15 Findhorn Pl.
1892		Thackwell, J. B., M.B., C.M., Ravenswood Hospital, Ravenswood, Queensland
1885	C.	* Thompson, D'Arcy W., C.B., B.A., F.L.S., Prof. of Natural Hist. in Univ. College, Dundee
1884		* Thoms, George Hunter, of Aberlemno, Advocate, Sheriff of the Counties of Orkney and Zetland, 13 Charlotte Square
1870		Thomson, Rev. Andrew, D.D., 63 Northumberland Street 480
1887		* Thomson, Andw., M.A., D.Sc., Mathematical Master in the Perth Academy, Ardenlea, Pitcullen, Perth
1896		* Thomson, George Ritchie, M.B., C.M., 306 Bath Street, Glasgow
1887	C.	* Thomson, J. Arthur, M.A., Regius Prof. of Natural History in the Univ. of Aberdeen
1880		Thomson, John Millar, LL.D., F.R.S., Prof. of Chem. in King's College, Lond., 85 Addison Road, London
1899		* Thomson, The Right Hon. Mitchell, Lord Provost of Edinburgh, 6 Charlotte Square 485
1899		* Thomson, R. Tatlock, 156 Bath Street, Glasgow
1870		Thomson, Spencer C., Actuary, 10 Eglinton Crescent
1882		Thomson, Wm., M.A., B.Sc., Registrar, University of the Cape of Good Hope, University Chambers, Cape Town
1876		Thomson, William, Royal Institution, Manchester
1874	N. C.	Traquair, R. H., M.D., LL.D., F.R.S., F.G.S., Keeper of the Natural History Collections in the Museum of Science and Art, Edinburgh, 8 Dean Park Crescent 490
1874		Tuke, Sir J. Batty, M.D., D.Sc., F.R.C.P.E., 20 Charlotte Square
1888		* Turnbull, Andrew H., Actuary, The Elms, Whitehouse Loan
1861	N. C.	Turner, Sir William, M.B., F.R.C.S.E., LL.D., D.C.L., D.Sc. Dub., F.R.S., (VICE-PRESIDENT), Professor of Anatomy in the University of Edinburgh, 6 Eton Terrace
1895		* Turton, Albert H., M.E., A.I.M. and M., F.G.S., F.C.S., F.R.G.S.
1898		* Tweedie, Charles, M.A., B.Sc., Lecturer on Mathematics in the University of Edinburgh, 15 Dalrymple Crescent, Grange 495

Date of Election.		
1877		* Underhill, Charles E., B.A., M.B., F.R.C.P.E., F.R.C.S.E., 8 Coates Crescent
1889		Underhill, T. Edgar, M.D., F.R.C.S.E., Dunedin, Barnt Green, Worcestershire
1891		Vernon, Henry Hannotte, M.D., Shipbrook, Cambridge Road, Southport, Lancashire
1875		Vincent, Charles Wilson, F.I.C., F.C.S., M.R.I., Librarian of the Reform Club, Pall Mall, London, 38 Queen's Road, South Hornsey, Middlesex
1888		Walker, James, Memb. Inst. C.E., Engineer's Office, Tyne Improvement Commission, Newcastle-on-Tyne 500
1891	B. C.	* Walker, James, D.Sc., Ph.D., Professor of Chemistry in University College, Dundee, 19 Springfield, Dundee
1873	C.	Walker, Robert, M.A., University, Aberdeen
1886	C.	* Wallace, R., F.L.S., Prof. of Agriculture and Rural Economy in the Univ. of Edin.
1898		Wallace, W., M.A., Principal, Southern Higher Grade School, Leeds
1891		* Walmsley, R. Mullineux, D.Sc., Prin. of the Northampton Inst., Clerkenwell, London 505
1866		Watson, Patrick Heron, M.D., F.R.C.S.E., LL.D., 16 Charlotte Square
1862	C.	Watson, Rev. Robert Boog, B.A., LL.D., F.L.S., Past President of the Conchological Society, 11 Strathearn Place
1896		* Webster, John Clarence, B.A., M.D., F.R.C.P.E., Lecturer on Gynecology, M'Gill University, Montreal, 287 Mountain Street, Montreal, Canada
1882		* Wenley, James A., 5 Drumsheugh Gardens
1896		Wenley, R. M., M.A., D.Sc., D.Phil., Prof. of Philosophy in the Univ. of Michigan, U.S. 510
1896	C.	White, Philip J., M.B., Prof. of Zoology in University College, Bangor, North Wales
1890		White, Sir William Henry, K.C.B., Memb. Inst. C.E., LL.D., F.R.S., Assistant Controller of the Navy, and Director of Naval Construction, The Admiralty, London
1881		Whitehead, Walter, F.R.C.S.E., Professor of Clinical Surgery, Owens College and Victoria University, 499 Oxford Road, Manchester
1894		Whymper, Edward, F.R.G.S., 29 Ludgate Hill, London
1879		* Will, John Charles Ogilvie, M.D., 379 Union Street, Aberdeen 515
1868		Williams, W., Principal and Professor of Veterinary Medicine and Surgery, New Veterinary College, Leith Walk
1897		* Williams, W. Owen, F.R.C.V.S., Johnville, Portobello
1879		* Wilson, Andrew, Ph.D., F.L.S., Lecturer on Zoology and Comparative Anatomy, 110 Gilmore Place
1895		Wilson-Barker, David, F.R.G.S., F.R. Met. Soc., Captain-Superintendent Thames Nautical Training College, H.M.S. "Worcester," Greenhithe, Kent
1882		Wilson, George, M.A., M.D., 7 Avon Place, Warwick 520
1891		* Wilson, John Hardie, D.Sc., University of St Andrews (39 South Street, St Andrews)
1886	C.	* Woodhead, German Sims, M.D., F.R.C.P.E., Prof. of Pathology in the Univ. of Cambridge
1884		Woods, G. A., M.R.C.S., Tregarthyn House, 31 Brunswick Road, Brighton
1890		* Wright, Johnstone Christie, Oakdene, Woking, Surrey
1896		* Wright, Robert Patrick, Professor of Agriculture, West of Scotland Technical College, Glasgow, Laventille, Crow Road, Partick, Glasgow 525
1882		* Young, Frank W., F.C.S., Lecturer on Natural Science, High School, Dundee, Woodmuir Park, West Newport, Fife
1892	C.	Young, George, Ph.D., Firth College, Sheffield
1896	C.	* Young, James Buchanan, M.B., D.Sc., Dalveen, Braeside, Liberton
1882		* Young, Thomas Graham, Westfield, West Calder 529

LIST OF HONORARY FELLOWS

AT NOVEMBER 1899.

His Royal Highness The PRINCE OF WALES.

FOREIGNERS (LIMITED TO THIRTY-SIX BY LAW X.).

Elected.		
1897	Alexander Agassiz,	*Cambridge (Mass.).*
1897	E.-H. Amagat,	*Paris.*
1889	Marcellin Pierre Eugène Berthelot,	*Paris.*
1895	Ludwig Boltzmann,	*Vienna.*
1897	Stanislao Cannizzaro,	*Rome.*
1883	Luigi Cremona,	*Rome.*
1877	Carl Gegenbaur,	*Heidelberg.*
1888	Ernst Haeckel,	*Jena.*
1883	Julius Hann,	*Graz.*
1884	Charles Hermite,	*Paris.*
1879	Jules Janssen,	*Paris.*
1864	Albert von Kölliker,	*Würzburg.*
1864	Rudolph Leuckart,	*Leipzig.*
1897	Gabriel Lippmann,	*Paris.*
1895	Éleuthère-Élie-Nicolas Mascart,	*Paris.*
1888	Demetrius Ivanovich Mendeléef,	*St Petersburg.*
1895	Carl Menger,	*Vienna.*
1864	Theodore Mommsen,	*Berlin.*
1897	Fridtjof Nansen,	*Christiania.*
1881	Simon Newcomb,	*Washington.*
1895	Max von Pettenkofer,	*Munich.*
1895	Jules Henri Poincaré,	*Paris.*
1889	Georg Hermann Quincke,	*Heidelberg.*
1886	Alphonse Renard,	*Ghent.*
1897	Ferdinand von Richthofen,	*Berlin.*
1897	Henry A. Rowland,	*Baltimore.*
1897	Giovanni V. Schiaparelli,	*Milan.*
1878	Otto Wilhelm Struve,	*St Petersburg.*
1886	Tobias Robert Thalén,	*Upsala.*
1874	Otto Torell,	*Lund.*
1868	Rudolph Virchow,	*Berlin.*
1897	Ferdinand Zirkel,	*Leipzig.*

Total, 32.

BRITISH SUBJECTS (LIMITED TO TWENTY BY LAW X.).

Elected.		
1889	Sir Robert Stawell Ball, Kt., LL.D., F.R.S., M.R.I.A., Lowndean, Professor of Astronomy in the University of Cambridge,	*Cambridge.*
1892	Colonel Alexander Ross Clarke, C.B., R.E., F.R.S.,	*Redhill, Surrey.*
1897	George Howard Darwin, M.A., LL.D., F.R.S., Plumian Professor of Astronomy in the University of Cambridge,	*Cambridge.*
1892	David Gill, LL.D., F.R.S., Her Majesty's Astronomer at the Cape of Good Hope,	*Cape of Good Hope.*
1895	Albert C. L. G. Günther, Ph.D., F.R.S.,	*London.*
1883	Sir Joseph Dalton Hooker, K.C.S.I., M.D., LL.D., D.C.L., F.R.S., Corresp. Mem. Inst. of France,	*London.*
1884	William Huggins, LL.D., D.C.L., F.R.S., Corresp. Mem. Inst. of France,	*London.*
1892	Sir James Paget, Bart., LL.D., D.C.L., F.R.S., Corresp. Mem. Inst. of France,	*London.*
1886	The Lord Rayleigh, D.C.L., LL.D., D.Sc. Dub., Sec. R.S., Corresp. Mem. Inst. of France,	*London.*
1881	The Rev. George Salmon, D.D., LL.D., D.C.L., F.R.S., Corresp. Mem. Inst. of France,	*Dublin.*
1884	Sir J. S. Burdon Sanderson, Bart., M.D., LL.D., D.Sc. Dub., F.R.S.,	*Oxford.*
1864	Sir George Gabriel Stokes, Bart., M.P., LL.D., D.C.L., F.R.S., Corresp. Mem. Inst. of France,	*Cambridge.*
1892	The Right Rev. W. Stubbs, D.D., LL.D., Bishop of Oxford,	*Oxford.*
1895	Sir Charles Todd, K.C.M.G., F.R.S., Government Astronomer, South Australia,	*Adelaide.*
1883	Alexander William Williamson, LL.D., F.R.S., Corresp. Mem. Inst. of France,	*London.*

Total, 15.

ORDINARY FELLOWS ELECTED

During Session 1896–97.

Arranged according to the Date of their Election.

6th December 1896.

Robert Caird. James Bell Dobbie, F.Z.S. Wm. Sanderson.

1st February 1897.

James Robert Erskine-Murray. A. Lockhart Gillespie, M.D., F.R.C.P.E.

1st March 1897.

Walter Biggar Blaikie. Richard J. Berry, M.D., F.R.C.S.E.
John Carruthers Beattie, D.Sc. Thomas Duncan Greenlees, M.B.
Harry Rainy, M.B., C.M., F.R.C.P.E. Alex. Cruikshank Houston, M.B., C.M., D.Sc.
W. Owen Williams, F.R.C.V.S.

3rd May 1897.

John William Shepherd. Alfred George Nash, B.Sc.

7th June 1897.

James A. Macdonald, M.A., B.Sc. George Arthur Mitchell, M.A.
Hugh Davidson, of Braedale. John Gordon Gordon-Munn, M.D.
Angus M'Gillivray, M.D., C.M. Richard John Lloyd, M.A., D.Lit.

HONORARY FELLOWS ELECTED

During Session 1896–97.

FOREIGN.

Alexander Agassiz, Director of the Museum of Comparative Zoology, Harvard College, Cambridge, U.S.

E.-H. Amagat, Correspondent of the Institute of France, Répétiteur at the École Polytechnique, Paris.

Stanislao Cannizzaro, Professor of Chemistry in the University of Rome.

Gabriel Lippmann, Member of the Institute of France and Professor of Physics in the University of Paris.

Fridtjof Nansen, Professor of Zoology in the University of Christiania.

Baron Ferdinand von Richthofen, Corresponding Member of the Institute of France, Berlin.

Henry A. Rowland, Professor of Physics in the Johns Hopkins University, Baltimore, U.S.

Giovanni V. Schiaparelli, Director of the Royal Brera Observatory, Milan.

Ferdinand Zirkel, Professor of Mineralogy in the University of Leipzig.

BRITISH.

The Very Rev. John Caird, D.D., LL.D., Principal of the University of Glasgow.

George Howard Darwin, M.A., LL.D., F.R.S., Plumian Professor of Astronomy in the University of Cambridge.

Sir William Flower, K.C.B., LL.D., D.C.L., F.R.S., Director of the Natural History Department, British Museum.

FELLOWS DECEASED, RESIGNED, OR CANCELLED

DURING SESSION 1896–97.

ORDINARY FELLOWS DECEASED.

EDMUND CHISHOLM BATTEN, of Aigas, M.A.
Professor MATTHEW CHARTERIS, M.A.
Professor HENRY DRUMMOND.
GEORGE ELDER, Knock Castle.
Admiral Sir ALEXANDER MILNE, Bart., G.C.B.
Very Rev. Dean MONTGOMERY, M.A., D.D.
JAMES GREIG SMITH, M.A., M.B., C.M.
Rev. JOHN WILSON, M.A.
THOMAS BRUMBY JOHNSTON, Geographer to the Queen.

RESIGNED.

WILLIAM JOLLY.

CANCELLED.

H. A. WEBSTER.

HONORARY FELLOWS DECEASED.

SESSION 1896–97.

FOREIGN.

ÉMILE DUBOIS-RAYMOND.
JOHANNES IAPETUS SMITH STEENSTRUP.

BRITISH.

JAMES JOSEPH SYLVESTER, LL.D., F.R.S.

ORDINARY FELLOWS ELECTED

During Session 1897–98,

Arranged according to the Date of their Election.

6th December 1897.

James Currie, Jun., M.A. John Archibald Purves, B.Sc. Charles Tweedie, M.A., B.Sc.

17th January 1898.

Alexander Cullen, F.S.A. (Scot.). W. Wallace, M.A. Prof. G. Adami, M.A., M.D.

7th February 1898.

Cecil Carus-Wilson, F.R.G.S.
James Campbell Irons, S.S.C.
Arthur Thomas Masterman, B.A., D.Sc.
Benjamin Hall Blyth, M.A., M.Inst. C.E.
Richard Vary Campbell, M.A., LL.B.
The Hon. John Abercromby.

7th March 1898.

Prof. John Glaister, M.D.
Francis Chalmers Crawford.
George Newman, M.D.
David Brown, F.C.S.
W. Allan Carter, M.Inst. C.E.
Alexander Veitch Lothian, M.A., B.Sc.
S. C. Mahalanobis, B.Sc.

4th April 1898.

W. Cossar MacKenzie, D.Sc. T. H. Bryce, M.A., M.B.

2nd May 1898.

William Archer Tait, B.Sc., M.Inst. C.E. Albert A. Gray, M.D.
Alexander W. Roberts, F.R.A.S.

4th July 1898.

John Findlay, M.A.

FELLOWS DECEASED

During Session 1897–98.

ORDINARY FELLOWS DECEASED.

James Edward Tierney Aitchison, C.I.E., M.D.
Sir James Bain.
Frederick W. Barry, M.D., D.Sc.
Professor Henry Calderwood.
Professor M. Forster Heddle.
Charles Hayes Higgins, M.D.
Henry Marshall, M.D.
Henry Newcombe, F.R.C.S.E.
The Right Hon. Lord Playfair, G.C.B.
The Hon. Bouverie F. Primrose, C.B.
James Syme.
Robert Wilson, M.Inst. C.E.

HONORARY FELLOW DECEASED.

Session 1897–98.

BRITISH.

The Very Rev. John Caird, D.D., LL.D.

ORDINARY FELLOWS ELECTED

During Session 1898–99,

Arranged according to the Date of their Election.

5th December 1898.

James R. Appleyard.
James Chatham.
T. E. Gatehouse, Assoc. Memb. Inst. C.E.
Ewen John Maclean, M.D.
Prof. E. W. W. Carlier, M.D.
T. H. Milroy, M.D.

9th January 1899.

Thomas Isherwood, M.A., LL.D.

6th February 1899.

Allan M'Lane Hamilton, M.D.
Prof. David W. Finlay, M.D.
Joseph Slater Lewis, Assoc. Memb. Inst. C.E.
George Duthie, M.A.
James M'Cubbin, B.A.

6th March 1899.

Prof. Thomas S. Goodwin.
R. Tatlock Thomson.
Edward Graham Guest, M.A., B.Sc.

3rd April 1899.

James Taylor, M.A.

1st May 1899.

Andrew Freeland Fergus.
The Right Hon. Mitchell Thomson, Lord Provost of Edinburgh.
W. Lamond Howie.

5th June 1899.

Alexander G. Ramage.

3rd July 1899.

Ernest Hugh Snell, M.D.

FELLOWS DECEASED OR RESIGNED

During Session 1898–99.

ORDINARY FELLOWS DECEASED.

Emeritus Professor George J. Allman.
James Lambert Bailey.
Professor D. Campbell Black.
Emeritus Professor W. Garden Blaikie.
David Chalmers.
Robert Cox, of Gorgie, M.P.
John Duncan, M.D., LL.D.
James Simpson Fleming.
Sir John Fowler, Bart., K.C.M.G.
James Graham, LL.D.
George Fosbery Lyster, M. Inst. C.E.
W. Bowman Macleod.
John J. Moir, M.D.
Professor William Rutherford, M.D.
Sir John Struthers, M.D.
Joseph Tillie, M.D.
R. H. B. Wickham, M.D.
George Williamson, F.S.A. Scot.

RESIGNED.

Most Hon. The Marquis of Lothian.
A. E. Scougall.
D. S. Sinclair.

HONORARY FELLOWS DECEASED.

Session 1898–99.

BRITISH.

Sir J. William Dawson, C.M.G.
Sir William Flower, K.C.B.
Edward Frankland, D.C.L., F.R.S.

FOREIGN.

Robert William Bunsen.
Gustav Heinrich Wiedemann.

LAWS

OF THE

ROYAL SOCIETY OF EDINBURGH.

AS REVISED 5TH MARCH 1900.

LAWS.

[By the Charter of the Society (printed in the *Transactions*, Vol. VI. p. 5), the Laws cannot be altered, except at a Meeting held one month after that at which the Motion for alteration shall have been proposed.]

I.

THE ROYAL SOCIETY OF EDINBURGH shall consist of Ordinary and Honorary Fellows. Title.

II.

Every Ordinary Fellow, within three months after his election, shall pay Two Guineas as the fee of admission, and Three Guineas as his contribution for the Session in which he has been elected; and annually at the commencement of every Session, Three Guineas into the hands of the Treasurer. This annual contribution shall continue for ten years after his admission, and it shall be limited to Two Guineas for fifteen years thereafter.* Fellows may compound for these contributions on such terms as the Council may from time to time fix. The fees of Ordinary Fellows residing in Scotland.

III.

All Fellows who shall have paid Twenty-five years' annual contribution shall be exempted from further payment. Payment to cease after 25 years.

IV.

The fees of admission of an Ordinary Non-Resident Fellow shall be £26, 5s., payable on his admission; and in case of any Non-Resident Fellow coming to reside at any time in Scotland, he shall, during each year of his residence, pay the usual annual contribution of £3, 3s., payable by each Resident Fellow; but after payment of such annual contribution for eight years, he shall be exempt Fees of Non-Resident Ordinary Fellows.

* A modification of this rule, in certain cases, was agreed to at a Meeting of the Society held on the 3rd January 1831.

At the Meeting of the Society, on the 5th January 1857, when the reduction of the Contributions from £3, 3s. to £2, 2s., from the 11th to the 25th year of membership, was adopted, it was resolved that the existing Members shall share in this reduction, so far as regards their future annual Contributions.

Case of Fellows becoming Non-Resident.

from any further payment. In the case of any Resident Fellow ceasing to reside in Scotland, and wishing to continue a Fellow of the Society, it shall be in the power of the Council to determine on what terms, in the circumstances of each case, the privilege of remaining a Fellow of the Society shall be continued to such Fellow while out of Scotland.

V.

Defaulters.

Members failing to pay their contributions for three successive years (due application having been made to them by the Treasurer) shall be reported to the Council, and, if they see fit, shall be declared from that period to be no longer Fellows, and the legal means for recovering such arrears shall be employed.

VI.

Privileges of Ordinary Fellows.

None but Ordinary Fellows shall bear any office in the Society, or vote in the choice of Fellows or Office-Bearers, or interfere in the patrimonial interests of the Society.

VII.

Numbers Unlimited.

The number of Ordinary Fellows shall be unlimited.

VIII.

Fellows entitled to Transactions.

The Ordinary Fellows, upon producing an order from the TREASURER, shall be entitled to receive from the Publisher, gratis, the Parts of the Society's Transactions which shall be published subsequent to their admission.

IX.

Mode of Recommending Ordinary Fellows.

Candidates for admission as Ordinary Fellows shall make an application in writing, and shall produce along with it a certificate of recommendation to the purport below,* signed by at least *four* Ordinary Fellows, two of whom shall certify their recommendation from personal knowledge. This recommendation shall be delivered to the Secretary, and by him laid before the Council, and shall afterwards be printed in the circulars for three Ordinary Meetings of the Society, previous to the day of election, and shall lie upon the table during that time.

* "A. B., a gentleman well versed in Science (*or Polite Literature, as the case may be*), being "to our knowledge desirous of becoming a Fellow of the Royal Society of Edinburgh, we hereby "recommend him as deserving of that honour, and as likely to prove a useful and valuable Member."

X.

Honorary Fellows shall not be subject to any contribution. This class shall consist of persons eminently distinguished for science or literature. Its number shall not exceed Fifty-six, of whom Twenty may be British subjects, and Thirty-six may be subjects of foreign states.

Honorary Fellows, British and Foreign.

XI.

Personages of Royal Blood may be elected Honorary Fellows, without regard to the limitation of numbers specified in Law X.

Royal Personages.

XII.

Honorary Fellows may be proposed by the Council, or by a recommendation (in the form given below*) subscribed by three Ordinary Fellows; and in case the Council shall decline to bring this recommendation before the Society, it shall be competent for the proposers to bring the same before a General Meeting. The election shall be by ballot, after the proposal has been communicated *viva voce* from the Chair at one meeting, and printed in the circulars for two ordinary meetings of the Society, previous to the day of election.

Recommendation of Honorary Fellows.

Mode of Election.

XIII.

The election of Ordinary Fellows shall only take place at the first Ordinary Meeting of each month during the Session. The election shall be by ballot, and shall be determined by a majority of at least two-thirds of the votes, provided Twenty-four Fellows be present and vote.

Election of Ordinary Fellows.

XIV.

The Ordinary Meetings shall be held on the first and third Mondays of each month from November to March, and from May to July, inclusive; with the exception that when there are five Mondays in January, the Meetings for that month shall be held on its second and fourth Mondays. Regular Minutes shall be kept of the proceedings, and the Secretaries shall do the duty alternately, or according to such agreement as they may find it convenient to make.

Ordinary Meetings.

* We hereby recommend_____
for the distinction of being made an Honorary Fellow of this Society, declaring that each of us from our own knowledge of his services to (*Literature or Science, as the case may be*) believe him to be worthy of that honour.

(To be signed by three Ordinary Fellows.)

_____

_____

_____

To the President and Council of the Royal Society of Edinburgh.

XV.

The Transactions.

The Society shall from time to time publish its Transactions and Proceedings. For this purpose the Council shall select and arrange the papers which they shall deem it expedient to publish in the *Transactions* of the Society, and shall superintend the printing of the same.

The Council shall have power to regulate the private business of the Society. At any Meeting of the Council the Chairman shall have a casting as well as a deliberative vote.

XVI.

How Published.

The Transactions shall be published in parts or *Fasciculi* at the close of each Session, and the expense shall be defrayed by the Society.

XVII.

The Council.

That there shall be formed a Council, consisting—First, of such gentlemen as may have filled the office of President; and Secondly, of the following to be annually elected, viz.:—a President, Six Vice-Presidents (two at least of whom shall be resident), Twelve Ordinary Fellows as Councillors, a General Secretary, Two Secretaries to the Ordinary Meetings, a Treasurer, and a Curator of the Museum and Library.

XVIII.

Retiring Councillors.

Four Councillors shall go out annually, to be taken according to the order in which they stand on the list of the Council.

XIX.

Election of Office-Bearers.

An Extraordinary Meeting for the election of Office Bearers shall be held annually on the fourth Monday of October, or on such other lawful day in October as the Council may fix, and each Session of the Society shall be held to begin at the date of the said Extraordinary Meeting.

XX.

Special Meetings; how called.

Special Meetings of the Society may be called by the Secretary, by direction of the Council; or on a requisition signed by six or more Ordinary Fellows. Notice of not less than two days must be given of such Meetings.

XXI.

Treasurer's Duties.

The Treasurer shall receive and disburse the money belonging to the Society, granting the necessary receipts, and collecting the money when due.

He shall keep regular accounts of all the cash received and expended, which shall be made up and balanced annually; and at the Extraordinary Meeting in October, he shall present the accounts for the preceding year, duly audited.

At this Meeting, the Treasurer shall also lay before the Council a list of all arrears due above two years, and the Council shall thereupon give such directions as they may deem necessary for recovery thereof.

XXII.

Auditor.

At the Extraordinary Meeting in October, a professional accountant shall be chosen to audit the Treasurer's accounts for that year, and to give the necessary discharge of his intromissions.

XXIII.

General Secretary's Duties.

The General Secretary shall keep Minutes of the Extraordinary Meetings of the Society, and of the Meetings of the Council, in two distinct books. He shall, under the direction of the Council, conduct the correspondence of the Society, and superintend its publications. For these purposes he shall, when necessary, employ a clerk, to be paid by the Society.

XXIV.

Secretaries to Ordinary Meetings.

The Secretaries to the Ordinary Meetings shall keep a regular Minute-book, in which a full account of the proceedings of these Meetings shall be entered; they shall specify all the Donations received, and furnish a list of them, and of the Donors' names, to the Curator of the Library and Museum; they shall likewise furnish the Treasurer with notes of all admissions of Ordinary Fellows. They shall assist the General Secretary in superintending the publications, and in his absence shall take his duty.

XXV.

Curator of Museum and Library.

The Curator of the Museum and Library shall have the custody and charge of all the Books, Manuscripts, objects of Natural History, Scientific Productions, and other articles of a similar description belonging to the Society; he shall take an account of these when received, and keep a regular catalogue of the whole, which shall lie in the Hall, for the inspection of the Fellows.

XXVI.

Use of Museum and Library.

All Articles of the above description shall be open to the inspection of the Fellows at the Hall of the Society, at such times and under such regulations as the Council from time to time shall appoint.

XXVII.

Register Book.

A Register shall be kept, in which the names of the Fellows shall be enrolled at their admission, with the date.

THE KEITH, MAKDOUGALL-BRISBANE, NEILL, AND GUNNING VICTORIA JUBILEE PRIZES.

The above Prizes will be awarded by the Council in the following manner :—

I. KEITH PRIZE.

The KEITH PRIZE, consisting of a Gold Medal and from £40 to £50 in Money, will be awarded in the Session 1899–1900 for the "best communication on a scientific subject, communicated, in the first instance, to the Royal Society during the Sessions 1897–98 and 1898–99." Preference will be given to a paper containing a discovery.

II. MAKDOUGALL-BRISBANE PRIZE.

This Prize is to be awarded biennially by the Council of the Royal Society of Edinburgh to such person, for such purposes, for such objects, and in such manner as shall appear to them the most conducive to the promotion of the interests of science; with the *proviso* that the Council shall not be compelled to award the Prize unless there shall be some individual engaged in scientific pursuit, or some paper written on a scientific subject, or some discovery in science made during the biennial period, of sufficient merit or importance in the opinion of the Council to be entitled to the Prize.

1. The Prize, consisting of a Gold Medal and a sum of Money, will be awarded at the commencement of the Session 1900–1901, for an Essay or Paper having reference to any branch of scientific inquiry, whether Material or Mental.

2. Competing Essays to be addressed to the Secretary of the Society, and transmitted not later than 8th July 1900.

3. The Competition is open to all men of science.

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4. The Essays may be either anonymous or otherwise. In the former case, they must be distinguished by mottoes, with corresponding sealed billets, superscribed with the same motto, and containing the name of the Author.

5. The Council impose no restriction as to the length of the Essays, which may be, at the discretion of the Council, read at the Ordinary Meetings of the Society. They wish also to leave the property and free disposal of the manuscripts to the Authors; a copy, however, being deposited in the Archives of the Society, unless the paper shall be published in the Transactions.

6. In awarding the Prize, the Council will also take into consideration any scientific papers presented to the Society during the Sessions 1898–99, 1899–1900, whether they may have been given in with a view to the prize or not.

III. NEILL PRIZE.

The Council of the Royal Society of Edinburgh having received the bequest of the late Dr PATRICK NEILL of the sum of £500, for the purpose of "the interest thereof being applied in furnishing a Medal or other reward every second or third year to any distinguished Scottish Naturalist, according as such Medal or reward shall be voted by the Council of the said Society," hereby intimate,

1. The NEILL PRIZE, consisting of a Gold Medal and a sum of Money, will be awarded during the Session 1900–1901.

2. The Prize will be given for a Paper of distinguished merit, on a subject of Natural History, by a Scottish Naturalist, which shall have been presented to the Society during the three years preceding the 8th July 1900,—or failing presentation of a paper sufficiently meritorious, it will be awarded for a work or publication by some distinguished Scottish Naturalist, on some branch of Natural History, bearing date within five years of the time of award.

IV. GUNNING VICTORIA JUBILEE PRIZE.

This Prize, founded in the year 1887 by Dr R. H. GUNNING, is to be awarded triennially by the Council of the Royal Society of Edinburgh, in recognition of original work in Physics, Chemistry, or Pure or Applied Mathematics.

Evidence of such work may be afforded either by a Paper presented to the Society, or by a Paper on one of the above subjects, or some discovery in them elsewhere communicated or made, which the Council may consider to be deserving of the Prize.

The Prize consists of a sum of money, and is open to men of science resident in or connected with Scotland. The first award was made in the year 1887.

In accordance with the wish of the Donor, the Council of the Society may on fit occasions award the Prize for work of a definite kind to be undertaken during the three succeeding years by a scientific man of recognised ability.

AWARDS OF THE KEITH, MAKDOUGALL-BRISBANE, NEILL, AND GUNNING VICTORIA JUBILEE PRIZES, FROM 1827 TO 1898.

I. KEITH PRIZE.

1st Biennial Period, 1827–29.—Dr Brewster, for his papers "on his Discovery of Two New Immiscible Fluids in the Cavities of certain Minerals," published in the Transactions of the Society.

2nd Biennial Period, 1829–31.—Dr Brewster, for his paper "on a New Analysis of Solar Light," published in the Transactions of the Society.

3rd Biennial Period, 1831–33.—Thomas Graham, Esq., for his paper "on the Law of the Diffusion of Gases," published in the Transactions of the Society.

4th Biennial Period, 1833–35.—Professor J. D. Forbes, for his paper "on the Refraction and Polarization of Heat," published in the Transactions of the Society.

5th Biennial Period, 1835–37.—John Scott Russell, Esq., for his Researches "on Hydrodynamics," published in the Transactions of the Society.

6th Biennial Period, 1837–39.—Mr John Shaw, for his experiments "on the Development and Growth of the Salmon," published in the Transactions of the Society.

7th Biennial Period, 1839–41.—Not awarded.

8th Biennial Period, 1841–43.—Professor James David Forbes, for his papers "on Glaciers," published in the Proceedings of the Society.

9th Biennial Period, 1843–45.—Not awarded.

10th Biennial Period, 1845–47.—General Sir Thomas Brisbane, Bart., for the Makerstoun Observations on Magnetic Phenomena, made at his expense, and published in the Transactions of the Society.

11th Biennial Period, 1847–49.—Not awarded.

12th Biennial Period, 1849–51.—Professor Kelland, for his papers "on General Differentiation, including his more recent communication on a process of the Differential Calculus, and its application to the solution of certain Differential Equations," published in the Transactions of the Society.

13th Biennial Period, 1851–53.—W. J. Macquorn Rankine, Esq., for his series of papers "on the Mechanical Action of Heat," published in the Transactions of the Society.

14th Biennial Period, 1853–55.—Dr Thomas Anderson, for his papers "on the Crystalline Constituents of Opium, and on the Products of the Destructive Distillation of Animal Substances," published in the Transactions of the Society.

15th Biennial Period, 1855–57.—Professor Boole, for his Memoir "on the Application of the Theory of Probabilities to Questions of the Combination of Testimonies and Judgments," published in the Transactions of the Society.

16TH BIENNIAL PERIOD, 1857–59.—Not awarded.

17TH BIENNIAL PERIOD, 1859–61.—JOHN ALLAN BROUN, Esq., F.R.S., Director of the Trevandrum Observatory, for his papers "on the Horizontal Force of the Earth's Magnetism, on the Correction of the Bifilar Magnetometer, and on Terrestrial Magnetism generally," published in the Transactions of the Society.

18TH BIENNIAL PERIOD, 1861–63.—Professor WILLIAM THOMSON, of the University of Glasgow, for his Communication "on some Kinematical and Dynamical Theorems."

19TH BIENNIAL PERIOD, 1863–65.—Principal FORBES, St Andrews, for his "Experimental Inquiry into the Laws of Conduction of Heat in Iron Bars," published in the Transactions of the Society.

20TH BIENNIAL PERIOD, 1865–67.—Professor C. PIAZZI SMYTH, for his paper "on Recent Measures at the Great Pyramid," published in the Transactions of the Society.

21ST BIENNIAL PERIOD, 1867–69.—Professor P. G. TAIT, for his paper "on the Rotation of a Rigid Body about a Fixed Point," published in the Transactions of the Society.

22ND BIENNIAL PERIOD, 1869–71.—Professor CLERK MAXWELL, for his paper "on Figures, Frames, and Diagrams of Forces," published in the Transactions of the Society.

23RD BIENNIAL PERIOD, 1871–73.—Professor P. G. TAIT, for his paper entitled "First Approximation to a Thermo-electric Diagram," published in the Transactions of the Society.

24TH BIENNIAL PERIOD, 1873–75.—Professor CRUM BROWN, for his Researches "on the Sense of Rotation, and on the Anatomical Relations of the Semicircular Canals of the Internal Ear."

25TH BIENNIAL PERIOD, 1875–77.—Professor M. FORSTER HEDDLE, for his papers "on the Rhombohedral Carbonates," and "on the Felspars of Scotland," published in the Transactions of the Society.

26TH BIENNIAL PERIOD, 1877–79.—Professor H. C. FLEEMING JENKIN, for his paper "on the Application of Graphic Methods to the Determination of the Efficiency of Machinery," published in the Transactions of the Society; Part II. having appeared in the volume for 1877–78.

27TH BIENNIAL PERIOD, 1879–81.—Professor GEORGE CHRYSTAL, for his paper "on the Differential Telephone," published in the Transactions of the Society.

28TH BIENNIAL PERIOD, 1881–83.—THOMAS MUIR, Esq., LL.D., for his "Researches into the Theory of Determinants and Continued Fractions," published in the Proceedings of the Society.

29TH BIENNIAL PERIOD, 1883–85.—JOHN AITKEN, Esq., for his paper "on the Formation of Small Clear Spaces in Dusty Air," and for previous papers on Atmospheric Phenomena, published in the Transactions of the Society.

30TH BIENNIAL PERIOD, 1885–87.—JOHN YOUNG BUCHANAN, Esq., for a series of communications, extending over several years, on subjects connected with Ocean Circulation, Compressibility of Glass, &c.; two of which, viz., "On Ice and Brines," and "On the Distribution of Temperature in the Antarctic Ocean," have been published in the Proceedings of the Society.

31ST BIENNIAL PERIOD, 1887–89.—Professor E. A. LETTS, for his Papers on the Organic Compounds of Phosphorus, published in the Transactions of the Society.

32ND BIENNIAL PERIOD, 1889–91.—R. T. OMOND, Esq., for his Contributions to Meteorological Science, many of which are contained in Vol. XXXIV. of the Society's Transactions.

33RD BIENNIAL PERIOD, 1891–93.—Professor THOMAS R. FRASER, F.R.S., for his Papers on *Strophanthus hispidus*, Strophanthin, and Strophanthidin, read to the Society in February and June 1889 and in December 1891, and printed in Vols. XXXV., XXXVI., and XXXVII. of the Society's Transactions.

34TH BIENNIAL PERIOD, 1893–95.—Dr CARGILL G. KNOTT, for his papers on the Strains produced by Magnetism in Iron and in Nickel, which have appeared in the Transactions and Proceedings of the Society.

35TH BIENNIAL PERIOD, 1895–97.—Dr THOMAS MUIR, for his continued Communications on Determinants and Allied Questions.

II. MAKDOUGALL-BRISBANE PRIZE.

1ST BIENNIAL PERIOD, 1859.—Sir RODERICK IMPEY MURCHISON, on account of his Contributions to the Geology of Scotland.

2ND BIENNIAL PERIOD, 1860–62.—WILLIAM SELLER, M.D., F.R.C.P.E., for his "Memoir of the Life and Writings of Dr Robert Whytt," published in the Transactions of the Society.

3RD BIENNIAL PERIOD, 1862–64.—JOHN DENIS MACDONALD, Esq., R.N., F.R.S., Surgeon of H.M.S. "Icarus," for his paper "on the Representative Relationships of the Fixed and Free Tunicata, regarded as Two Sub-classes of equivalent value; with some General Remarks on their Morphology," published in the Transactions of the Society.

4TH BIENNIAL PERIOD, 1864–66.—Not awarded.

5TH BIENNIAL PERIOD, 1866–68.—Dr ALEXANDER CRUM BROWN and Dr THOMAS RICHARD FRASER, for their conjoint paper "on the Connection between Chemical Constitution and Physiological Action," published in the Transactions of the Society.

6TH BIENNIAL PERIOD, 1868–70.—Not awarded.

7TH BIENNIAL PERIOD, 1870–72.—GEORGE JAMES ALLMAN, M.D., F.R.S., Emeritus Professor of Natural History, for his paper "on the Homological Relations of the Cœlenterata," published in the Transactions, which forms a leading chapter of his Monograph of Gymnoblastic or Tubularian Hydroids—since published.

8TH BIENNIAL PERIOD, 1872–74.—Professor LISTER, for his paper "on the Germ Theory of Putrefaction and the Fermentive Changes," communicated to the Society, 7th April 1873.

9TH BIENNIAL PERIOD, 1874–76.—ALEXANDER BUCHAN, A.M., for his paper "on the Diurnal Oscillation of the Barometer," published in the Transactions of the Society.

10TH BIENNIAL PERIOD, 1876–78.—Professor ARCHIBALD GEIKIE, for his paper "on the Old Red Sandstone of Western Europe," published in the Transactions of the Society.

11TH BIENNIAL PERIOD, 1878–80.—Professor PIAZZI SMYTH, Astronomer-Royal for Scotland, for his paper "on the Solar Spectrum in 1877–78, with some Practical Idea of its probable Temperature of Origination," published in the Transactions of the Society.

12TH BIENNIAL PERIOD, 1880–82.—Professor JAMES GEIKIE, for his "Contributions to the Geology of the North-West of Europe," including his paper "on the Geology of the Faroes," published in the Transactions of the Society.

13TH BIENNIAL PERIOD, 1882–84.—EDWARD SANG, Esq., LL.D., for his paper "on the Need of Decimal Subdivisions in Astronomy and Navigation, and on Tables requisite therefor," and generally for his Recalculation of Logarithms both of Numbers and Trigonometrical Ratios,—the former communication being published in the Proceedings of the Society.

14TH BIENNIAL PERIOD, 1884–86.—JOHN MURRAY, Esq., LL.D., for his papers "On the Drainage Areas of Continents, and Ocean Deposits," "The Rainfall of the Globe, and Discharge of Rivers," "The Height of the Land and Depth of the Ocean," and "The Distribution of Temperature in the Scottish Lochs as affected by the Wind."

15TH BIENNIAL PERIOD, 1886–88.—ARCHIBALD GEIKIE, Esq., LL.D., for numerous communications, especially that entitled "History of Volcanic Action during the Tertiary Period in the British Isles," published in the Transactions of the Society.

16TH BIENNIAL PERIOD, 1888–90.—Dr LUDWIG BECKER, for his Paper on "The Solar Spectrum at Medium and Low Altitudes," printed in Vol. XXXVI. Part I. of the Society's Transactions.

17TH BIENNIAL PERIOD, 1890–92.—HUGH ROBERT MILL, Esq., D.Sc., for his Papers on "The Physical Conditions of the Clyde Sea Area," Part I. being already published in Vol. XXXVI. of the Society's Transactions.

18TH BIENNIAL PERIOD, 1892–94.—Professor JAMES WALKER, D.Sc., Ph.D., for his work on Physical Chemistry, part of which has been published in the Proceedings of the Society, Vol. XX., pp. 255–263. In making this award, the Council took into consideration the work done by Professor Walker along with Professor Crum Brown on the Electrolytic Synthesis of Dibasic Acids, published in the Transactions of the Society.

19TH BIENNIAL PERIOD, 1894–96.—Professor JOHN G. M'KENDRICK, for numerous Physiological papers, especially in connection with Sound; many of which have appeared in the Society's publications.

20TH BIENNIAL PERIOD, 1896–98.—Dr WILLIAM PEDDIE, for his papers on the Torsional Rigidity of Wires.

III. THE NEILL PRIZE.

1ST TRIENNIAL PERIOD, 1856–59.—Dr W. LAUDER LINDSAY, for his paper "on the Spermogones and Pycnides of Filamentous, Fruticulose, and Foliaceous Lichens," published in the Transactions of the Society.

2ND TRIENNIAL PERIOD, 1859–62.—ROBERT KAYE GREVILLE, LL.D., for his Contributions to Scottish Natural History, more especially in the department of Cryptogamic Botany, including his recent papers on Diatomaceæ.

3RD TRIENNIAL PERIOD, 1862–65.—ANDREW CROMBIE RAMSAY, F.R.S., Professor of Geology in the Government School of Mines, and Local Director of the Geological Survey of Great Britain, for his various works and Memoirs published during the last five years, in which he has applied the large experience acquired by him in the Direction of the arduous work of the Geographical Survey of Great Britain to the elucidation of important questions bearing on Geological Science.

4TH TRIENNIAL PERIOD, 1865–68.—Dr WILLIAM CARMICHAEL M'INTOSH, for his paper "on the Structure of the British Nemerteans, and on some New British Annelids," published in the Transactions of the Society.

5TH TRIENNIAL PERIOD, 1868–71.—Professor WILLIAM TURNER, for his papers "on the great Finner Whale; and on the Gravid Uterus, and the Arrangement of the Fœtal Membranes in the Cetacea," published in the Transactions of the Society.

6TH TRIENNIAL PERIOD, 1871–74.—CHARLES WILLIAM PEACH, Esq., for his Contributions to Scottish Zoology and Geology, and for his recent contributions to Fossil Botany.

7TH TRIENNIAL PERIOD, 1874–77.—Dr RAMSAY H. TRAQUAIR, for his paper "on the Structure and Affinities of *Tristichopterus alatus* (Egerton)," published in the Transactions of the Society, and also for his contributions to the Knowledge of the Structure of Recent and Fossil Fishes.

8TH TRIENNIAL PERIOD, 1877–80.—JOHN MURRAY, Esq., for his paper "on the Structure and Origin of Coral Reefs and Islands," published (in abstract) in the Proceedings of the Society.

9TH TRIENNIAL PERIOD, 1880–83.—Professor HERDMAN, for his papers "on the Tunicata," published in the Proceedings and Transactions of the Society.

10TH TRIENNIAL PERIOD, 1883–86.—B. N. PEACH, Esq., for his Contributions to the Geology and Palæontology of Scotland, published in the Transactions of the Society.

11TH TRIENNIAL PERIOD, 1886–89.—ROBERT KIDSTON, Esq., for his Researches in Fossil Botany, published in the Transactions of the Society.

12TH TRIENNIAL PERIOD, 1889–92.—JOHN HORNE, Esq., F.G.S., for his Investigations into the Geological Structure and Petrology of the North-West Highlands.

13TH TRIENNIAL PERIOD, 1892–95.—ROBERT IRVINE, Esq., for his papers on the action of Organisms in the Secretion of Carbonate of Lime and Silica, and on the solution of these substances in Organic Juices. These are printed in the Society's Transactions and Proceedings.

14TH TRIENNIAL PERIOD, 1895–98.—Professor COSSAR EWART, for his recent Investigations connected with Telegony.

IV. GUNNING VICTORIA JUBILEE PRIZE.

1ST TRIENNIAL PERIOD, 1884–87.—Sir WILLIAM THOMSON, Pres. R.S.E., F.R.S., for a remarkable series of papers "on Hydrokinetics," especially on Waves and Vortices, which have been communicated to the Society.

2ND TRIENNIAL PERIOD, 1887–90.—Professor P. G. TAIT, Sec. R.S.E., for his work in connection with the "Challenger" Expedition, and his other Researches in Physical Science.

3RD TRIENNIAL PERIOD, 1890–93.—ALEXANDER BUCHAN, Esq., LL.D., for his varied, extensive, and extremely important Contributions to Meteorology, many of which have appeared in the Society's Publications.

4TH TRIENNIAL PERIOD, 1893–96.—JOHN AITKEN, Esq., for his brilliant Investigations in Physics, especially in connection with the Formation and Condensation of Aqueous Vapour.

PROCEEDINGS

OF THE

STATUTORY GENERAL MEETINGS,

23RD NOVEMBER 1896,

22ND NOVEMBER 1897,

AND

28TH NOVEMBER 1898.

STATUTORY MEETING.

HUNDRED AND FOURTEENTH SESSION.

Monday, 23rd November 1896.

At the Annual Statutory Meeting,

Professor A. CAMPBELL FRASER, D.C.L., in the Chair,

The Minutes of last Annual Statutory Meeting of 25th November 1895 were read, approved, and signed.

Mr SKINNER and Professor FERGUSON having been named Scrutineers, the Ballot for the New Council commenced.

The TREASURER submitted his Accounts for the year. These, with the Auditors' Report, were read and approved.

Lord KELVIN (with power to name a substitute from the List of Fellows of the Society) was appointed to represent the Society at the British Association Commission on a National Physical Laboratory.

The Scrutineers reported that the following Council had been duly elected:—

The Right Hon. Lord KELVIN, G.C.V.O., LL.D., D.C.L., F.R.S., President.

Professor JAMES GEIKIE, LL.D., D.C.L., F.R.S.,
The Hon. Lord M'LAREN, LL.D., F.R.A.S.,
The Rev. Professor FLINT, D.D.,
Professor JOHN G. M'KENDRICK, M.D., LL.D., F.R.S.,
Professor GEORGE CHRYSTAL, M.A., LL.D.,
Sir ARTHUR MITCHELL, K.C.B., LL.D.,
} Vice-Presidents.

Professor P. G. TAIT, M.A., D.Sc., General Secretary.

Professor CRUM BROWN, F.R.S.,
JOHN MURRAY, LL.D.,
} Secretaries to Ordinary Meetings.

PHILIP R. D. MACLAGAN, F.F.A., Treasurer.

ALEXANDER BUCHAN, M.A., LL.D., Curator of Library and Museum.

COUNCILLORS.

Professor T. R. Fraser, LL.D., F.R.S.
Robert Munro, M.A., M.D.
D. Noël Paton, B.Sc., M.D.
Cargill G. Knott, D.Sc.
Sir William Turner, M.B., F.R.S.
Sir Stair Agnew, K.C.B.
Dr James Burgess, C.I.E., M.R.A.S.
John Sturgeon Mackay, M.A., LL.D.
Professor Ralph Copeland, Ph.D., Astronomer-Royal for Scotland.
Professor D'Arcy W. Thompson.
The Rev. Professor Duns, D.D.
Lieut.-Colonel Fred. Bailey (*late*), R.E.

On the motion of the Hon. Lord M'Laren, thanks were voted to the Treasurer.

On the motion of Dr Crum Brown, the Auditors were thanked and reappointed, and

On the motion of Dr Burgess, thanks were awarded to the Scrutineers.

John M'Laren, *C.*

STATUTORY MEETING.

HUNDRED AND FIFTEENTH SESSION.

Monday, 22nd November 1897.

At the Annual Statutory Meeting,

The Hon. Lord M'Laren, LL.D., Vice-President, in the Chair,

The Minutes of last Annual Statutory Meeting of 23rd November 1896 were read, approved, and signed.

Mr Robert Caird and Mr George H. Geddes having been named Scrutineers, the Ballot for the New Council commenced.

The Treasurer submitted his Accounts for the year. These, with the Auditors' Report, were read and approved.

The Scrutineers reported that the following New Council had been unanimously elected:—

The Right Hon. Lord Kelvin, G.C.V.O., LL.D., F.R.S., President.
The Hon. Lord M'Laren, LL.D., F.R.A.S.,
The Rev. Professor Flint, D.D.,
Professor John G. M'Kendrick, M.D., LL.D., F.R.S.,
Professor George Chrystal, M.A., LL.D.,
Sir Arthur Mitchell, K.C.B., LL.D.,
Sir William Turner, M.B., F.R.S., } Vice-Presidents.
Professor P. G. Tait, M.A., D.Sc., General Secretary.
Professor Crum Brown, F.R.S.,
John Murray, D.Sc., LL.D., } Secretaries to Ordinary Meetings.
Philip R. D. Maclagan, F.F.A., Treasurer.
Alexander Buchan, M.A., LL.D., Curator of Library and Museum.

COUNCILLORS.

Sir Stair Agnew, K.C.B., M.A.	The Rev. Professor Duns, D.D.
James Burgess, C.I.E., LL.D., M.R.A.S.	Lieut.-Colonel Fred. Bailey, R.E.
John Sturgeon Mackay, M.A., LL.D.	Professor James Geikie, LL.D., F.R.S.
Professor Ralph Copeland, Ph.D., Astronomer-Royal for Scotland.	Sir J. Batty Tuke, M.D.
	Mr A. Beatson Bell, Advocate.
Professor D'Arcy W. Thompson.	Professor John Gibson, Ph.D.

On the motion of Professor Tait, thanks were voted to the Treasurer, and

On the motion of Dr Crum Brown, the Auditors were thanked and reappointed, and

On the motion of Dr Burgess, thanks were awarded to the Scrutineers.

John M'Laren,
Chairman.

STATUTORY MEETING.

HUNDRED AND SIXTEENTH SESSION.

Monday, 28th November 1898.

At the Annual Statutory Meeting,

The Hon. Lord M'LAREN, LL.D., Vice-President, in the Chair,

The Minutes of last Annual Statutory Meeting of 22nd November 1897 were read, approved, and signed.

On the motion of Dr BUCHAN, Dr BURGESS and Mr WILLIAM BOYD were named Scrutineers, and the Ballot for the New Council commenced.

The TREASURER submitted his Accounts for the year. These, with the Auditors' Report, were read and approved.

The Scrutineers reported that the following New Council had been duly elected:—

The Right Hon. Lord KELVIN, G.C.V.O., LL.D., D.C.L., F.R.S., President.

The Rev. Professor FLINT, D.D.,
Professor JOHN G. M'KENDRICK, M.D., LL.D., F.R.S.,
Professor GEORGE CHRYSTAL, M.A., LL.D.,
Sir ARTHUR MITCHELL, K.C.B., LL.D.,
Sir WILLIAM TURNER, M.B., F.R.S.,
Professor RALPH COPELAND, Ph.D., Astronomer-Royal for Scotland,
} Vice-Presidents.

Professor P. G. TAIT, M.A., D.Sc., General Secretary.

Professor A. CRUM BROWN, F.R.S., LL.D.,
Sir JOHN MURRAY, K.C.B., LL.D., F.R.S.,
} Secretaries to Ordinary Meetings.

PHILIP R. D. MACLAGAN, F.F.A., Treasurer.

ALEXANDER BUCHAN, M.A., LL.D., F.R.S., Curator of Library and Museum.

COUNCILLORS.

Professor D'Arcy W. Thompson, C.B.	Professor Shield Nicholson, M.A., D.Sc.
The Rev. Professor Duns, D.D.	Professor John Gibson, Ph.D.
Lieut.-Colonel Fred. Bailey, R.E.	The Hon. Lord M'Laren, LL.D., F.R.A.S.
Professor James Geikie, LL.D., F.R.S.	C. G. Knott, D.Sc.
Sir J. Batty Tuke, M.D., D.Sc.	Dr Alexander Bruce, M.A., F.R.C.P.E.
Mr A. Beatson Bell, Advocate.	Mr James A. Wenley.

Sir John Murray, K.C.B., LL.D., F.R.S., Honorary Representative on George Heriot's Trust.

The following motion by Dr Crum Brown was submitted by direction of the Meeting of Council of 28th November 1898:—

"That the Ordinary Meetings of the Society be held on the first and third Mondays of November, January, February, March, May, June, and July, and on the first Monday in December, with the exception that when there are five Mondays in January the Meetings shall be held on the second and fourth Mondays of that month."

The Meeting decided that it be remitted to the Council for favourable consideration of the necessary arrangements.

On the motion of Mr Beatson Bell, the Auditors were thanked and reappointed.

On the motion of Professor Crum Brown, thanks were voted to the Treasurer.

On the motion of Dr Buchan, thanks were voted to the Scrutineers, and

On the motion of Sir John Murray, thanks were voted to the Chairman.

Arthur Mitchell,
Chairman.

The following Public Institutions and Individuals are entitled to receive Copies of the Transactions and Proceedings of the Royal Society of Edinburgh:—

London, British Museum.
... ... (Natural History Department), Cromwell Road.
... Royal Society, Burlington House.
... Anthropological Institute of Great Britain and Ireland, 3 Hanover Square.
... British Association for the Advancement of Science, Burlington House.
... Society of Antiquaries, Burlington House.
... Royal Astronomical Society, Burlington House.
... Royal Asiatic Society, 22 Albemarle Street.
... Society of Arts, John Street, Adelphi.
... Athenæum Club.
... Chemical Society, Burlington House.
... Institution of Civil Engineers, 25 Great George Street.
... Royal Geographical Society, Burlington Gardens.
... Geological Society, Burlington House.
... Royal Horticultural Society, South Kensington.
... Hydrographic Office, Admiralty.
... Imperial Institute.
... Royal Institution, Albemarle Street, W.
... Linnean Society, Burlington House.
... Royal Society of Literature, 20 Hanover Square.
... Royal Medical and Chirurgical Society, 20 Hanover Square.
... Royal Microscopical Society, 20 Hanover Square.
... Museum of Economic Geology, Jermyn Street.
... Royal Observatory, Greenwich.
... Pathological Society, 20 Hanover Sq.
... Royal Statistical Society, 9 Adelphi Terrace, Strand, London.
... Royal College of Surgeons of England, 4 Lincoln's Inn Fields.
... United Service Institution, Whitehall Yard.
London, University College, Gower Street.
... Zoological Society, 3 Hanover Square.
... The Editor of *Nature*, 29 Bedford Street, Covent Garden.
... The Editor of the *Electrician*, Salisbury Court, Fleet Street.
Cambridge Philosophical Society.
... University Library.
Leeds Philosophical and Literary Society.
Liverpool, University College Library.
Manchester Literary and Philosophical Society.
Oxford, Bodleian Library.
Plymouth, Marine Biological Laboratory, Citadel Hill.
Richmond (Surrey), Kew Observatory.
Sheffield, University College.
Yorkshire Philosophical Society.

SCOTLAND.

Edinburgh, Advocates Library.
... University Library.
... Royal College of Physicians.
... Highland and Agricultural Society, 3 George IV. Bridge.
... Royal Medical Society, 7 Melbourne Place.
... Royal Observatory.
... Royal Physical Society, India Buildings.
... Royal Scottish Society of Arts, 117 George Street.
... Royal Botanic Garden, Inverleith Row.
Aberdeen, University Library.
Dundee, University College Library.
Glasgow, University Library.
... Philosophical Society, 207 Bath Street.
St Andrews, University Library.

IRELAND.

Dublin, Royal Dublin Society.
... Royal Irish Academy.
... Library of Trinity College.
... National Library of Ireland.

COLONIES, DEPENDENCIES, ETC.

Bombay, Royal Asiatic Society.
... Elphinstone College.
Calcutta, Asiatic Society of Bengal.
... Geological Survey of India.
... Meteorological Department of the Government of India.
Madras, Literary Society.
Canada, Geological and Natural History Survey.
... Queen's University, Kingston.
... Royal Society of Canada, Ottawa.
... Quebec, Literary and Philosophical Society.
... Toronto, Literary and Historical Society.
... ... The Canadian Institute.
... ... University Library.
Cape of Good Hope, The Observatory.
Melbourne, University Library.
Sydney, University Library.
... Linnæan Society of New South Wales.
... Royal Society of New South Wales.
Wellington, New Zealand Institute.

CONTINENT OF EUROPE.

Amsterdam, Koninklijke Akademie van Wetenschappen.
... Koninklijk Zoologisch Genootschap.
Athens, University Library.
... Observatoire National.
Basle, Die Schweizerische Naturforschende Gesellschaft.
Bergen, Museum.
Berlin, Königliche Akademie der Wissenschaften.
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Plate I.	8th col., last row.	Right-hand upper lap should cross *over*.
Plate III.	4th „ 6th „	Upper lap should go under, over, *over*, *under*, over, etc.
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